Flavor mixing, gauge invariance, and wave-function renormalization

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We clarify some aspects of the LSZ formalism and wave-function renormalization for unstable particles in the presence of electroweak interactions when mixing and CP violation are considered. We also analyze the renormalization of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix which is closely related to wave-function renormalization. We critically review earlier attempts to define a set of ''on-shell'' wave-function renormalization constants. With the aid of extensive use of the Nielsen identities complemented by explicit calculations we corroborate that the counterterm for the CKM mixing matrix must be explicitly gauge independent and demonstrate that the commonly used prescription for the wave-function renormalization constants leads to gauge parameter dependent amplitudes, even if the CKM counterterm is gauge invariant as required. We show that a proper LSZ-compliant prescription leads to gauge independent amplitudes. The resulting wave-function renormalization constants necessarily possess absorptive parts, but we verify that they comply with the expected requirements concerning CP and CPT. The results obtained using this prescription are different (even at the level of the modulus squared of the amplitude) from the ones neglecting the absorptive parts in the case of top decay. The difference is numerically relevant.

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I. INTRODUCTION

One of the pressing open problems in particle physics is to understand the origin of the CP violation phase and family mixing. In the minimal standard model (SM) the information about these quantities is encoded in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. In this work we shall denote this matrix by K_{ij}.

As is well known, some of the entries of this matrix are remarkably well measured, while others (such as the K_{tb}, K_{ts}, and K_{td} elements) are poorly known and the only real experimental constraint comes from unitarity requirements. A lot of effort in the last decade has been invested in this particular problem and this dedication will continue in the foreseeable future aiming to precision in the charged current sector comparable to that already reached in the neutral sector. As a guide, let us mention that the expected accuracy in $\sin 2\beta$ after the CERN LHCb is expected to be beyond the 1% level, and a comparable accuracy is expected by that time from the ongoing generation of experiments (BaBar, Belle) [1].

In the neutral sector it is totally mandatory to include electroweak radiative corrections to bring theory and experiment into agreement. Tree level results are incompatible with experiment by many standard deviations [2]. Obviously we are not there yet in the charged current sector, but in a few years electroweak radiative corrections will be required in the studies analyzing the ''unitarity'' of the CKM matrix.[1]

These corrections are of several types. With an on-shell scheme in mind, we need counterterms for the electric charge, Weinberg angle, and wave-function renormalization (WFR) for the W gauge boson. We shall also require WFR for the external fermions and counterterms for the entries of the CKM matrix. The latter are in fact related in a way that will be described below [3]. Finally, one needs to compute the one particle irreduceble (1PI) vertex parts of the different processes one is interested in.

In the on-shell scheme, all counterterms can be expressed as combinations of self-energies [4]. These are standard and well known at one loop in perturbation theory, and in some cases, at least for the leading pieces, up to two loops in the SM. However, a long standing controversy exists in the literature concerning the appropriate way to define both an external WFR and CKM counterterms. The issue becomes involved because we are dealing with particles which are unstable (and therefore the self-energies develop branch cuts; even gauge dependent ones in the SM) and because of mixing.

Several proposals have been put forward in the literature to define appropriate counterterms both for the external legs and for the CKM matrix elements. The original prescription for a WFR diagonalizing the on-shell propagator was introduced in [5]. In [6] the WFR ''satisfying'' the conditions of [5] was derived. However, since [6] does not take care about the branch cuts present in the self-energies those results must be considered only consistent up to absorptive terms. Later it was realized [7] that the on-shell conditions defined in [5] were inconsistent and in fact impossible to satisfy for a minimal set of renormalization constants[2] due to the imaginary branch cuts present in the self-energies. The author of [7] circumvented this problem by introducing a prescription that

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[1]The CKM matrix is certainly unitary, but the physical observables that at tree level coincide with these matrix elements certainly do not necessarily fulfil a unitarity constraint once quantum corrections are switched on.

[2]By minimal set we mean a set where the WFR of $\bar{\Psi}_0=\bar{\Psi}\bar{Z}^{1/2}$ and $\Psi_0=Z^{1/2}\Psi$ are related by $\bar{Z}^{1/2}=\gamma^0 Z^{1/2\dagger}\gamma^0$.

de facto eliminates such branch cuts, but at the price of not diagonalizing the propagators in flavor space.

Ward identities based on the $SU(2)_L$ gauge symmetry relate WFR and counterterms for the CKM matrix elements [3]. In [8] it was seen that if the prescription of [6] was used in the counterterms for the CKM matrix elements, the results were in violation of gauge invariance. As we have just mentioned, the results in [6] do not deal properly with the absorptive terms appearing in the self-energies; which in addition happen to be gauge dependent. In spite of the problems with the prescription for the WFR given in [6], the conclusions reached in [8] are correct: a necessary condition for gauge invariance of the physical amplitudes is that counterterms for the CKM matrix elements K_{ij} are by themselves gauge independent. This condition is fulfilled by the CKM counterterm proposed in [8] as it is in minimal subtraction [3,9].

Other proposals to handle CKM renormalization exist in the literature [9–11]. In all this work either the external WFR proposed originally in [6] or [7] is used, or the issue is sidestepped altogether. In either case the absorptive part of the self-energies (and even the absorptive part of the 1PI vertex part in one particular instance [10]) is not taken into account. As we shall see doing so leads to physical amplitudes—*S*-matrix elements—which are gauge dependent, and this is irrespective of the method one uses to renormalize K_{ij} provided the redefinition of K_{ij} is gauge independent and preserves unitarity.

Because of the structure of the imaginary branch cuts it turns out, however, that the gauge dependence present in the amplitude using the prescription of [7] cancels in the modulus squared of the physical *S* matrix element in the SM. This cancellation has been checked numerically by the authors in [12]. In this work we shall provide analytical results showing that this cancellation is exact. However, the gauge dependence remains at the level of the amplitude.

Is this acceptable? We do not think so. Diagrams contributing to the same physical process outside the SM electroweak sector may interfere with the SM amplitude and reveal the unwanted gauge dependence. Furthermore, gauge independent absorptive parts are also discarded by the prescription in [7]. These parts, contrary to the gauge dependent ones, do not drop in the squared amplitude as we shall show. In addition, one should not forget that the scheme in [7] does not deliver on-shell renormalized propagators that are diagonal in flavor space.

This work is dedicated to substantiating the above claims. We shall compute the gauge dependence of the absorptive parts in the self-energies and the vertex functions. We shall see how the requirements of gauge invariance and proper on-shell conditions (including exact diagonalization in flavor space) single out a unique prescription for the WFR The problem is presented in detail in the next section. The explicit expressions for the renormalization constants are given in Secs. III and IV. Implementation for *W* and top decay are shown in Sec. V. A technical discussion where extended use of the Nielsen identities has been done to extract the gauge dependence of all absorptive terms is presented in Sec. VI and it can be omitted by readers not interested in these details. In Sec. VII and VIII we return to *W* and top decay to implement the previous results, and finally we conclude in Sec. IX.

II. STATEMENT OF THE PROBLEM AND ITS SOLUTION

We want to define an on-shell renormalization scheme that guarantees the correct properties of the fermionic propagator in the $p^2 \to m_i^2$ limit and at the same time renders the observable quantities calculated in such a scheme gauge parameter independent. In the first place up- and down-type propagators have to be family diagonal on shell. The conditions necessary for that purpose were first given by Aoki *et al.* in [5]. Let us introduce some notation in order to write them down. We renormalize the bare fermion fields Ψ_0 and $\overline{\Psi}_0$ as

$$\Psi_0 = Z^{1/2}\Psi, \quad \overline{\Psi}_0 = \overline{\Psi}\,\overline{Z}^{1/2}. \tag{2.1}$$

For reasons that will become clear through the discussion, we shall allow Z and $\overline{Z}$ to be independent renormalization constants.[3] These renormalisation constants contain flavor, family and Dirac indices. We can decompose them into

$$Z^{1/2} = Z^{u\ 1/2}\tau^u + Z^{d\ 1/2}\tau^d, \quad \overline{Z}^{1/2} = \overline{Z}^{u\ 1/2}\tau^u + \overline{Z}^{d\ 1/2}\tau^d, \tag{2.2}$$

with τ^u and τ^d the up and down flavor projectors, and furthermore each piece in left and right chiral projectors L and R, respectively,

$$Z^{u\ 1/2} = Z^{uL\ 1/2}L + Z^{uR\ 1/2}R, \quad \overline{Z}^{u\ 1/2} = \overline{Z}^{uL\ 1/2}R + \overline{Z}^{uR\ 1/2}L. \tag{2.3}$$

Analogous decompositions hold for $Z^{d\ 1/2}$ and $\overline{Z}^{d\ 1/2}$. Because of radiative corrections the propagator mixes fermion of different family indices. Namely

$$iS^{-1}(p) = \overline{Z}^{1/2}[\not{p} - m - \delta m - \Sigma(p)]Z^{1/2},$$

where the bare self-energy Σ is nondiagonal and is given by $-i\Sigma = \Sigma 1\mathrm{PI}$. Within one-loop accuracy we can write $Z^{1/2} = 1 + \frac{1}{2}\delta Z$ etc. Introducing the family indices explicitly we have

$$iS_{ij}^{-1}(p) = (\not{p} - m_i)\,\delta_{ij} - \hat{\Sigma}_{ij}(p),$$

where the one-loop renormalized self-energy is given by

$$\hat{\Sigma}_{ij}(p) = \Sigma_{ij}(p) - \frac{1}{2}\delta\overline{Z}_{ij}(\not{p} - m_j) - \frac{1}{2}(\not{p} - m_i)\,\delta Z_{ij} + \delta m_i \delta_{ij}. \tag{2.4}$$

Since we can project the above definition for up- and down-type quarks, flavor indices will be dropped in the following

[3] This immediately raises some issues about Hermiticity, which we shall deal with below.

and will only be restored when necessary. Recalling the following on-shell relations for Dirac spinors ($p^2 \to m_i^2$)

$$(\not{p} - m_i) u_i^{(s)}(p) = 0,$$

$$\overline{u_i^{(s)}}(p)(\not{p} - m_i) = 0,$$

$$(\not{p} - m_i) v_i^{(s)}(-p) = 0,$$

$$\overline{v_i^{(s)}}(-p)(\not{p} - m_i) = 0, \tag{2.5}$$

the conditions [5] necessary to avoid mixing will be[4]

$$\hat{\Sigma}_{ij}(p) u_j^{(s)}(p) = 0 \quad (p^2 \to m_j^2)$$

(incoming particle), (2.6)

$$\overline{v_i^{(s)}}(-p) \hat{\Sigma}_{ij}(p) = 0 \quad (p^2 \to m_i^2)$$

(incoming antiparticle), (2.7)

$$\overline{u_i^{(s)}}(p) \hat{\Sigma}_{ij}(p) = 0 \quad (p^2 \to m_i^2)$$

(outgoing particle), (2.8)

$$\hat{\Sigma}_{ij}(p) v_j^{(s)}(-p) = 0 \quad (p^2 \to m_j^2)$$

(outgoing antiparticle) (2.9)

where no summation over repeated indices is assumed and $i \neq j$. These relations determine the nondiagonal parts of Z and $\overline{Z}$ as will be proven in the next section. Here, as a side remark, let us point out that the need of different ''incoming'' and ''outgoing'' WFR constants was already recognized in [13]. Nevertheless, that paper was unsuccessful in reconciling the on-shell prescription with the presence of absorptive terms in the self-energies. However, since its results are concerned with the leading contribution of an effective Lagrangian, no absorptive terms are present and therefore the conclusions still hold.

To obtain the diagonal parts Z_{ii}, $\overline{Z}_{ii}$, and δm_i one imposes mass pole and unit residue conditions (to be discussed below). Here it is worth making one important comment regarding the above conditions. First of all we note that in the literature the relation

$$\overline{Z}^{1/2} = \gamma^0 Z^{(1/2)\dagger} \gamma^0 \tag{2.10}$$

is taken for granted. This relation is tacitly assumed in [5] and explicitly required in [7]. Imposing Eq. (2.10) would guarantee Hermiticity of the Lagrangian written in terms of the renormalized physical fields. However, we are at this point concerned with external leg renormalization, for which it is perfectly possible to use a different set of renormalization constants [even ones that do not respect the requirement (2.10)], while keeping the Lagrangian Hermitian. In fact, using two sets of renormalization constants is a standard practice in the on-shell scheme [4], so one should not be concerned by this fact *per se*. In case one is worried about the consistency of using a set of WFR constants not satisfying Eq. (2.10) for the external legs while keeping a Hermitian Lagrangian, it should be pointed out that there is a complete equivalence between the set of renormalization constants we shall find below and a treatment of the external legs where diagrams with self-energies (including mass counterterms) are inserted instead of the WFR constants; provided, of course, that the mass counterterm satisfies the on-shell condition. Proceeding in this way gives results identical to ours and different from those obtained using the WFR proposed in [7], which do satisfy Eq. (2.10). Further consistency checks are presented in the following sections.

In any case, self-energies develop absorptive terms and this makes Eq. (2.10) incompatible with the diagonalizing conditions (2.6)–(2.9). Therefore in order to circumvent this problem one can give up diagonalization conditions (2.6)–(2.9) or alternatively the Hermiticity condition (2.10). The approach taken originally in [7] and work thereafter was the former alternative, while in this work we shall advocate the second one. The approach of [7] consists in dropping out absorptive terms from conditions (2.6)–(2.9). That is, for $i \neq j$,

$$\widetilde{\mathrm{Re}}[\hat{\Sigma}_{ij}(p)] u_j^{(s)}(p) = 0 \quad (p^2 \to m_j^2)$$

(incoming particle),

$$\overline{v_i^{(s)}}(-p) \widetilde{\mathrm{Re}}[\hat{\Sigma}_{ij}(p)] = 0 \quad (p^2 \to m_i^2)$$

(incoming antiparticle),

$$\overline{u_i^{(s)}}(p) \widetilde{\mathrm{Re}}(\hat{\Sigma}_{ij}(p)) = 0 \quad (p^2 \to m_i^2)$$

(outgoing particle),

$$\widetilde{\mathrm{Re}}[\hat{\Sigma}_{ij}(p)] v_j^{(s)}(-p) = 0 \quad (p^2 \to m_j^2)$$

(outgoing antiparticle), (2.11)

where $\widetilde{\mathrm{Re}}$ includes the real part of the logarithms arising in loop integrals appearing in the self-energies but not of the rest of coupling factors of the Feynmann diagram. This approach is compatible with the hermiticity condition (2.10) but on the other hand has several drawbacks. These drawbacks include the following:

(1) Since only the $\widetilde{\mathrm{Re}}$ part of the self-energies enters into the diagonalizing conditions the on-shell propagator remains nondiagonal.

(2) The very definition of $\widetilde{\mathrm{Re}}$ relies heavily on the one-loop perturbative calculation where it is applied. In other words $\widetilde{\mathrm{Re}}$ is not a proper function of its argument (in contrast to Re) and it is presumably cumbersome to implement in multiloop calculations.

[4]Notice that, as a matter of fact, in [5] the conditions over antifermions are not stated.

(3) As will become clear in the next sections, the on-shell scheme based in the $\widetilde{\mathrm{Re}}$ prescription leads to gauge parameter dependent physical amplitudes. The reason for this unwanted dependence is the dropping of absorptive gauge parameter dependent terms in the self-energies that are necessary to cancel absorptive terms appearing in the vertices. As mentioned in the Introduction, in the SM, the gauge dependence drops in the modulus squared of the amplitude, but not in the amplitude itself, and it could be eventually observable.

Having stated the unwanted features of the $\widetilde{\mathrm{Re}}$ approach let us briefly state the consequences of dropping condition (2.10)

(1) Conditions (2.6)–(2.9) readily determine the off-diagonal Z and $\overline{Z}$ WFRs which coincide with the ones obtained using the $\widetilde{\mathrm{Re}}$ prescription up to finite absorptive gauge parameter dependent terms.

(2) The renormalized fermion propagator becomes exactly diagonal on shell, unlike in the $\widetilde{\mathrm{Re}}$ scheme.

(3) Incoming and outgoing particles and antiparticles require different renormalization constants when computing a physical amplitude. Annihilation of particles and creation of antiparticles are accompanied by the renormalization constant Z, while creation of particles and annihilation of antiparticles are accompanied by the renormalization constant $\overline{Z}$.

(4) These constants Z and $\overline{Z}$ are in what relates to their dispersive parts identical to the ones in [7]. They differ in their absorptive parts. This might suggest to the alert reader that there could be problems with fundamental symmetries such as CP or CPT. We shall discuss this issue at the end of the paper. Our conclusion is that everything works out consistently in this respect.

For explicit expressions for Z and $\overline{Z}$ the reader should consult formulas (3.3), (3.4) and (4.10) in the next two sections. As an example of how to implement them see Sec. V. The explicit dependence on the gauge parameter (for simplicity only the W gauge parameter is considered) of the absorptive parts is given in Sec. VII.

III. OFF-DIAGONAL WAVE-FUNCTION RENORMALIZATION CONSTANTS

This section is devoted to a detailed derivation of the off-diagonal renormalization constants derived entirely from the on-shell conditions (2.6)–(2.9) and allowing for $\overline{Z}^{1/2} \neq \gamma^0 Z^{(1/2)\dagger}\gamma^0$. First of all we decompose the renormalized self-energy into all possible Dirac structures

$$\hat{\Sigma}_{ij}(p) = \not{p}[\hat{\Sigma}_{ij}^{\gamma R}(p^2)R + \hat{\Sigma}_{ij}^{\gamma L}(p^2)L] + \hat{\Sigma}_{ij}^{R}(p^2)R + \hat{\Sigma}_{ij}^{L}(p^2)L, \tag{3.1}$$

and use Eqs. (2.3), (2.4), and (3.1) to obtain

$$\begin{aligned}\hat{\Sigma}_{ij}(p) = {} & \not{p}R\left(\Sigma_{ij}^{\gamma R}(p^2) - \frac{1}{2}\delta\overline{Z}_{ij}^{R} - \frac{1}{2}\delta Z_{ij}^{R}\right) \\ & + \not{p}L\left(\Sigma_{ij}^{\gamma L}(p^2) - \frac{1}{2}\delta\overline{Z}_{ij}^{L} - \frac{1}{2}\delta Z_{ij}^{L}\right) \\ & + R\left(\Sigma_{ij}^{R}(p^2) + \frac{1}{2}(\delta\overline{Z}_{ij}^{L}m_j + m_i\delta Z_{ij}^{R}) + \delta_{ij}\delta m_i\right) \\ & + L\left(\Sigma_{ij}^{L}(p^2) + \frac{1}{2}(\delta\overline{Z}_{ij}^{R}m_j + m_i\delta Z_{ij}^{L}) + \delta_{ij}\delta m_i\right).\end{aligned} \tag{3.2}$$

Repeated indices are not summed over. Hence from Eqs. (3.2), (2.5), and (2.6) we obtain

$$\Sigma_{ij}^{\gamma R}(m_j^2)m_j - \frac{1}{2}\delta Z_{ij}^{R}m_j + \Sigma_{ij}^{L}(m_j^2) + \frac{1}{2}m_i\delta Z_{ij}^{L} = 0,$$

$$\Sigma_{ij}^{\gamma L}(m_j^2)m_j - \frac{1}{2}\delta Z_{ij}^{L}m_j + \Sigma_{ij}^{R}(m_j^2) + \frac{1}{2}m_i\delta Z_{ij}^{R} = 0.$$

Exactly the same relations are obtained from Eqs. (3.2), (2.5), and (2.9). Analogously, Eqs. (3.2), (2.5), and (2.7) [or Eq. (2.8)] lead to

$$m_i\Sigma_{ij}^{\gamma R}(m_i^2) - \frac{1}{2}m_i\delta\overline{Z}_{ij}^{R} + \Sigma_{ij}^{R}(m_i^2) + \frac{1}{2}\delta\overline{Z}_{ij}^{L}m_j = 0,$$

$$m_i\Sigma_{ij}^{\gamma L}(m_i^2) - \frac{1}{2}m_i\delta\overline{Z}_{ij}^{L} + \Sigma_{ij}^{L}(m_i^2) + \frac{1}{2}\delta\overline{Z}_{ij}^{R}m_j = 0.$$

Using the above expressions we immediately obtain

$$\begin{aligned}\delta Z_{ij}^{L} = {} & \frac{2}{m_j^2 - m_i^2}[\Sigma_{ij}^{\gamma R}(m_j^2)m_i m_j + \Sigma_{ij}^{\gamma L}(m_j^2)m_j^2 \\ & + m_i\Sigma_{ij}^{L}(m_j^2) + \Sigma_{ij}^{R}(m_j^2)m_j],\end{aligned}$$

$$\begin{aligned}\delta Z_{ij}^{R} = {} & \frac{2}{m_j^2 - m_i^2}[\Sigma_{ij}^{\gamma L}(m_j^2)m_i m_j + \Sigma_{ij}^{\gamma R}(m_j^2)m_j^2 \\ & + m_i\Sigma_{ij}^{R}(m_j^2) + \Sigma_{ij}^{L}(m_j^2)m_j],\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}\delta\overline{Z}_{ij}^{L} = {} & \frac{2}{m_i^2 - m_j^2}[\Sigma_{ij}^{\gamma R}(m_i^2)m_i m_j + \Sigma_{ij}^{\gamma L}(m_i^2)m_i^2 \\ & + m_i\Sigma_{ij}^{L}(m_i^2) + \Sigma_{ij}^{R}(m_i^2)m_j],\end{aligned}$$

$$\begin{aligned}\delta\overline{Z}_{ij}^{R} = {} & \frac{2}{m_i^2 - m_j^2}[\Sigma_{ij}^{\gamma L}(m_i^2)m_i m_j + \Sigma_{ij}^{\gamma R}(m_i^2)m_i^2 \\ & + m_i\Sigma_{ij}^{R}(m_i^2) + \Sigma_{ij}^{L}(m_i^2)m_j].\end{aligned} \tag{3.4}$$

At the one-loop level in the SM we can define

$$\Sigma_{ij}^{R}(p^2) \equiv \Sigma_{ij}^{S}(p^2)m_j, \quad \Sigma_{ij}^{L}(p^2) \equiv m_i\Sigma_{ij}^{S}(p^2),$$

and therefore

$$\delta\overline{Z}^L_{ij}-\delta Z^{L\dagger}_{ij}=\frac{2}{m_i^2-m_j^2}\{[\Sigma^{\gamma R}_{ij}(m_i^2)-\Sigma^{\gamma R}_{ji}{}^*(m_i^2)]m_i m_j + [\Sigma^{\gamma L}_{ij}(m_i^2)-\Sigma^{\gamma L}_{ji}{}^*(m_i^2)]m_i^2+(m_i^2+m_j^2) \times[\Sigma^{S}_{ij}(m_i^2)-\Sigma^{S}_{ji}{}^*(m_i^2)]\}\neq 0,$$

and a similar relation holds for $\delta\overline{Z}^R_{ij}-\delta Z^{R\dagger}_{ij}$. The above nonvanishing difference is due to the presence of branch cuts in the self-energies that invalidate the pseudo-Hermiticity relation

$$\Sigma_{ij}(p)\neq\gamma^0\Sigma^\dagger_{ij}(p)\gamma^0. \tag{3.5}$$

Equation (3.5) is assumed in [5] and if we temporarily ignore those branch cut contributions our results reduce to the ones depicted in [6] or [7]. In the SM these branch cuts are generically gauge dependent as a cursory look at the appropriate integrals shows at once.

IV. DIAGONAL WAVE-FUNCTION RENORMALIZATION CONSTANTS

Once the off-diagonal WFRs are obtained we focus our attention on the diagonal sector. Near the on-shell limit we can neglect the off-diagonal parts of the inverse propagator and write

$$iS^{-1}_{ij}(p)=[\not p-m_i-\hat\Sigma_{ii}(p)]\delta_{ij} =[\not p(aL+bR)+cL+dR]\delta_{ij}, \tag{4.1}$$

and therefore after some algebra

$$-iS_{ij}(p)=\frac{\not p(aL+bR)-dL-cR}{p^2ab-cd}\,\delta_{ij};$$

in our case we have

$$\begin{aligned} a&=1-\Sigma^{\gamma L}_{ii}(p^2)+\frac{1}{2}\delta\overline{Z}^L_{ii}+\frac{1}{2}\delta Z^L_{ii},\\ b&=1-\Sigma^{\gamma R}_{ii}(p^2)+\frac{1}{2}\delta\overline{Z}^R_{ii}+\frac{1}{2}\delta Z^R_{ii},\\ c&=-\Sigma^L_{ii}(p^2)-\left(1+\frac{1}{2}\delta\overline{Z}^R_{ii}+\frac{1}{2}\delta Z^L_{ii}\right)m_i-\delta m_i,\\ d&=-\Sigma^R_{ii}(p^2)-\left(1+\frac{1}{2}\delta\overline{Z}^L_{ii}+\frac{1}{2}\delta Z^R_{ii}\right)m_i-\delta m_i. \end{aligned} \tag{4.2}$$

In the limit $p^2\to m_i^2$ the chiral structures in the numerator have to cancel ($a\to b$ and $c\to d$); this requirement leads to

$$\begin{aligned} \delta\overline{Z}^R_{ii}-\delta\overline{Z}^L_{ii}&=\Sigma^{\gamma R}_{ii}(m_i^2)-\Sigma^{\gamma L}_{ii}(m_i^2)+\frac{\Sigma^R_{ii}(m_i^2)-\Sigma^L_{ii}(m_i^2)}{m_i},\\ \delta Z^R_{ii}-\delta Z^L_{ii}&=\Sigma^{\gamma R}_{ii}(m_i^2)-\Sigma^{\gamma L}_{ii}(m_i^2)-\frac{\Sigma^R_{ii}(m_i^2)-\Sigma^L_{ii}(m_i^2)}{m_i}. \end{aligned} \tag{4.3}$$

After this, we require the inverse propagator to have a zero in its real part as $p^2\to m_i^2$

$$\lim_{p^2\to m_i^2}\mathrm{Re}(p^2b-cda^{-1})=0, \tag{4.4}$$

from which δm_i is obtained,

$$\delta m_i=-\frac{1}{2}\mathrm{Re}\{m_i\Sigma^{\gamma L}_{ii}(m_i^2)+m_i\Sigma^{\gamma R}_{ii}+\Sigma^L_{ii}(m_i^2)+\Sigma^R_{ii}(m_i^2)\}. \tag{4.5}$$

This condition defines a mass and a width that agree at the one-loop level with the ones given in [14], [15], [16], and [17]. The mass and width are defined as the real and imaginary parts of the propagator pole in the complex plane respectively. Note also that from Eqs. (4.2), (4.3), and (4.5) we have

$$\lim_{p^2\to m_i^2}(-ca^{-1})=m_i+\frac{i}{2}\mathrm{Im}(\Sigma^{\gamma R}_{ii}(m_i^2)m_i +\Sigma^{\gamma L}_{ii}(m_i^2)m_i+\Sigma^R_{ii}(m_i^2)+\Sigma^L_{ii}(m_i^2)), \tag{4.6}$$

and therefore

$$\lim_{p^2\to m_i^2}\frac{\not p(aL+bR)-dL-cR}{p^2ab-cd}=\frac{\not p+m_i-i\Gamma/2}{im_i\Gamma},$$

where the width is defined as

$$\Gamma\equiv-\mathrm{Im}(\Sigma^{\gamma R}_{ii}(m_i^2)m_i+\Sigma^{\gamma L}_{ii}(m_i^2)m_i+\Sigma^R_{ii}(m_i^2)+\Sigma^L_{ii}(m_i^2)).$$

This quantity is ultraviolet finite. In order to find the residue in the complex plane we expand the propagator around the physical mass, obtaining for $p^2\sim m_i^2$

$$S_{ij}(p)=\frac{i[\not p+m_i-i\Gamma/2+\mathcal{O}(p^2-m_i^2)]}{im_i\Gamma+(p^2-m_i^2)a^{-1}[ab+m_i^2(a'b+ab')-(c'd+cd')]}+\mathcal{O}((p^2-m_i^2)^2), \tag{4.7}$$

where $a=b$ and $c=d$ are evaluated at $p^2=m_i^2$. Hereafter primed quantities denote derivatives with respect to p^2. $\cdot\mathcal{O}[(p^2-m_i^2)^n]$ stands for nonessential corrections of order $(p^2-m_i^2)^n$. Note that the $\mathcal{O}(p^2-m_i^2)$ corrections in the numerator do not mix with the ones of the same order in the denominator since the first ones are of order Γ^{-1} and the second ones are of order Γ^{-2}. Taking into account these comments the unit residue condition amounts to requiring

$$1=\frac{a+b}{2}+m_i^2(a'+b')+(m_i-i\Gamma/2)(c'+d'), \quad (4.8)$$

from which

$$\begin{aligned}\frac{1}{2}(\delta\overline{Z}_{ii}^L+\delta\overline{Z}_{ii}^R)=&\Sigma_{ii}^{\gamma L}(m_i^2)+\Sigma_{ii}^{\gamma R}(m_i^2)-\frac{1}{2}(\delta Z_{ii}^L+\delta Z_{ii}^R)\\&+2m_i^2(\Sigma_{ii}^{\gamma L'}(m_i^2)+\Sigma_{ii}^{\gamma R'}(m_i^2))\\&+2m_i(\Sigma_{ii}^{L'}(m_i^2)+\Sigma_{ii}^{R'}(m_i^2)).\end{aligned} \quad (4.9)$$

We have already required all the necessary conditions to fix the correct properties of the on-shell propagator but still there is some freedom left in the definition of the diagonal Z's. This freedom can be expressed in terms of a set of finite coefficients α_i given by

$$\frac{1}{2}(\delta Z_{ii}^L+\delta Z_{ii}^R)=\frac{1}{2}(\delta\overline{Z}_{ii}^L+\delta\overline{Z}_{ii}^R)+\alpha_i.$$

Bearing in mind that ambiguity and using Eqs. (4.3) and (4.9) we obtain

$$\begin{aligned}\delta\overline{Z}_{ii}^L&=\Sigma_{ii}^{\gamma L}(m_i^2)-X-\frac{\alpha_i}{2}+D,\\ \delta\overline{Z}_{ii}^R&=\Sigma_{ii}^{\gamma R}(m_i^2)+X-\frac{\alpha_i}{2}+D,\\ \delta Z_{ii}^L&=\Sigma_{ii}^{\gamma L}(m_i^2)+X+\frac{\alpha_i}{2}+D,\\ \delta Z_{ii}^R&=\Sigma_{ii}^{\gamma R}(m_i^2)-X+\frac{\alpha_i}{2}+D,\end{aligned} \quad (4.10)$$

where

$$\begin{aligned}X&=\frac{1}{2}\frac{\Sigma_{ii}^R(m_i^2)-\Sigma_{ii}^L(m_i^2)}{m_i},\\ D&=m_i^2(\Sigma_{ii}^{\gamma L'}(m_i^2)+\Sigma_{ii}^{\gamma R'}(m_i^2))\\&\quad+m_i(\Sigma_{ii}^{L'}(m_i^2)+\Sigma_{ii}^{R'}(m_i^2)).\end{aligned}$$

Note that since $X=0$ at the one-loop level and choosing $\alpha_i=0$ we obtain $\delta\overline{Z}_{ii}^L=\delta Z_{ii}^L$ and $\delta\overline{Z}_{ii}^R=\delta Z_{ii}^R$. However, we have the freedom to choose $\alpha_i\neq 0$. This does not affect the mass terms or neutral current couplings, but changes the charged coupling currents by multiplying the CKM matrix K by diagonal matrices. These redefinitions do not change the physical observables provided the α_i are pure imaginary numbers. This ambiguity corresponds in perturbation theory to the well known freedom in phase redefinitions of the CKM matrix. Except for this last freedom, the on-shell conditions determine one unique solution, the one presented here, with $\overline{Z}^{1/2}\neq\gamma^0 Z^{(1/2)\dagger}\gamma^0$.

V. W^+ AND TOP DECAY

Let us now apply the above mechanism to W^+ and top decay. We write

$$W^+(q)\to f_i(p_1)\overline{f}_j(p_2), \quad (5.1)$$

$$f_i(p_1)\to W^+(q)f_j(p_2), \quad (5.2)$$

where f indicates particle and $\overline{f}$ antiparticle. The latin indices are reserved for family indices. Leptonic and quark channels can be considered with the same notation, and confusion should not arise. For the process (5.1) there are at next-to-leading order two different types of Lorentz structure:

$$\begin{aligned}M_L^{(1)}&=\overline{u}_i(p_1)\not{\varepsilon}(q)Lv_j(p_2)\quad(L\leftrightarrow R),\\ M_L^{(2)}&=\overline{u}_i(p_1)Lv_j(p_2)p_1\cdot\varepsilon(q)\quad(L\leftrightarrow R),\end{aligned} \quad (5.3)$$

where ε stands for the vector polarization of the W^+. Equivalently, for the process (5.2) we shall use

$$\begin{aligned}M_L^{(1)}&=\overline{u}_j(p_2)\not{\varepsilon}^*(q)Lu_i(p_1)\quad(L\leftrightarrow R),\\ M_L^{(2)}&=\overline{u}_j(p_2)Lu_i(p_1)p_1\cdot\varepsilon^*(q)\quad(L\leftrightarrow R).\end{aligned} \quad (5.4)$$

The transition amplitude at tree level for the processes (5.1) and (5.2) is given by

$$\mathcal{M}_0=-\frac{eK_{ij}}{2s_W}M_L^{(1)},$$

where Eq. (5.3) is used for $M_L^{(1)}$ in W^+ decay and Eq. (5.4) instead for $M_L^{(1)}$ in t decay. The one-loop corrected transition amplitude can be written as

$$\begin{aligned}\mathcal{M}_1=&-\frac{e}{2s_W}M_L^{(1)}\left[K_{ij}\left(1+\frac{\delta e}{e}-\frac{\delta s_W}{s_W}+\frac{1}{2}\delta Z_W\right)\right.\\&\left.+\delta K_{ij}+\frac{1}{2}\sum_r(\delta\overline{Z}_{ir}^{Lu}K_{rj}+K_{ir}\delta Z_{rj}^{Ld})\right]\\&-\frac{e}{2s_W}(\delta F_L^{(1)}M_L^{(1)}+M_L^{(2)}\delta F_L^{(2)}\\&+M_R^{(1)}\delta F_R^{(1)}+M_R^{(2)}\delta F_R^{(2)}).\end{aligned} \quad (5.5)$$

In this expression $\delta F_{L,R}^{(1,2)}$ are the electroweak form factors coming from one-loop vertex diagrams. The renormalization constants are given by

$$\frac{\delta e}{e} = -\frac{1}{2}[(\delta Z_2^A - \delta Z_1^A) + \delta Z_2^A]$$

$$= -\frac{s_W}{c_W M_Z^2}\Pi^{ZA}(0) + \frac{1}{2}\frac{\partial \Pi^{AA}}{\partial k^2}(0),$$

$$\frac{\delta s_W}{s_W} = -\frac{c_W^2}{2s_W^2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right)$$

$$= -\frac{c_W^2}{2s_W^2}\mathrm{Re}\left(\frac{\Pi^{WW}(M_W^2)}{M_W^2} - \frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2}\right),$$

$$\delta Z_W = -\frac{\partial \Pi^{WW}}{\partial k^2}(M_W^2),$$

and the fermionic WFR constants are depicted in Eqs. (3.3), (3.4), and (4.10) where the indices u or d must be restored in the masses. The index A refers to the photon field.

As for the δK_{ij} renormalization constants, a SU(2) Ward identity [8] fixes these counterterms to be

$$\delta K_{jk} = \frac{1}{4}[(\delta \hat{Z}^{uL} - \delta \hat{Z}^{uL\dagger})K - K(\delta \hat{Z}^{dL} - \delta \hat{Z}^{dL\dagger})]_{jk}, \quad (5.6)$$

where $\hat{Z}$ means that the WFR constants appearing in the above expression are not necessarily the same ones used to renormalize and guarantee the proper on-shell residue for the external legs, as has already been emphasised. One may, for instance, use minimal subtraction Z's for the former.

We know [18] that the combination $\delta e/e - \delta s_W/s_W$ is gauge parameter independent. All the other vertex functions and renormalization constants are gauge dependent. For the reasons stated in the Introduction we want the amplitude (5.5) to be exactly gauge independent—not just its modulus—so the gauge dependence must cancel between all the remaining terms.

In the next section we shall make use of the Nielsen identities [19–22] to determine that three of the form factors appearing in the vertex (5.5) are by themselves gauge independent, namely,

$$\partial_\xi \delta F_L^{(2)} = \partial_\xi \delta F_R^{(1)} = \partial_\xi \delta F_R^{(2)} = 0.$$

ξ is the gauge fixing parameter. We shall also see that the gauge dependence in the remaining form factor $\delta F_L^{(1)}$ cancels exactly with the one contained in δZ_W and in δZ and $\delta \overline{Z}$. Therefore to guarantee a gauge fixing parameter independent amplitude δK must be gauge independent as well.

The difficulties related to a proper definition of δK were first pointed out in [8,19], where it was realized that using the on-shell Z's of [6] in Eq. (5.6) led to a gauge dependent K and amplitude. Those authors suggested a modification of the on-shell scheme based on a subtraction at $p^2 = 0$ for all flavors that ensured gauge independence. We want to stress that the choice for δK is not unique and different choices may differ by gauge independent finite parts [12]. Note that the gauge independence of δK is in contradistinction with the conclusions of [10] and in addition these authors have a nonunitary bare CKM matrix which does not respect the Ward identity.

As we shall see, if instead of using our prescription for δZ and $\delta \overline{Z}$ one makes use of the WFR constants of [7] to renormalize the external fermion legs, it turns out that the gauge cancellation dictated by the Nielsen identities does not actually take place in the amplitude. The culprits are of course the absorptive parts. These absorptive parts of the self-energies are absent in [7] due to the use of the $\widetilde{\mathrm{Re}}$ prescription, which throws them away. Notice, though, that the vertex contribution has gauge dependent absorptive parts (calculated in the next section) and they remain in the final result.

One might think of absorbing these additional terms in the counterterm for δK. This does not work. Indeed one can see from explicit calculations that WFR constants decompose as

$$\delta Z^{Lu} = A^{uL} + iB^{uL}, \quad \delta \overline{Z}^{Lu} = A^{uL\dagger} + iB^{uL\dagger}$$
$$(L \leftrightarrow R, u \leftrightarrow d), \quad (5.7)$$

where the matrices A and B contain the dispersive and absorptive parts of the self-energies, respectively. Moreover, if one substitutes Eq. (5.7) back into Eq. (5.5) one immediately sees that a necessary requirement allowing the A^u and A^d (B^u and B^d) contributions to be absorbed into a CKM matrix counterterm of the form given in Eq. (5.6) is that A^u and A^d (B^u and B^d) were anti-Hermitian (Hermitian) matrices. By direct inspection one can conclude that all A's or B's are neither Hermitian nor anti-Hermitian matrices and therefore any such redefinitions are impossible unless one is willing to give up the unitarity of the bare K. A problem somewhat similar to that was encountered in [10] (but different in that they did not consider absorptive parts at all; the inconsistency already showed up with the dispersive parts of the on-shell scheme of [6]).

It turns out that in the SM these gauge dependent absorptive parts, leading to a gauge dependent amplitude if they are dropped, do actually cancel, at least at the one-loop level, in the modulus of the S matrix element. Thus at this level the use of $\widetilde{\mathrm{Re}}$ is irrelevant. It is also shown in Sec. VII that gauge independent absorptive parts do survive even in the modulus of the amplitude for top quark or top antoquark decay (and only in these cases). Therefore we have to conclude that the difference between using $\widetilde{\mathrm{Re}}$, as advocated in [7], or not, as we do, is not just a semantic one. As we have seen, this difference cannot be attributed to a finite renormalization of K, provided the bare K remains unitary as required by the Ward identity (5.6).

VI. NIELSEN IDENTITIES

In this section we derive in detail the gauge dependence of the vertex three-point function. It is therefore rather technical and it can be omitted by readers just interested in the physical conclusions. In order to have control on gauge dependence, a useful tool is provided by the so-called Nielsen

identities [20]. For such purpose in addition to the "classical" Lagrangian $\mathcal{L}_{\rm SM}$ we have to take into account the gauge fixing term $\mathcal{L}_{\rm GF}$, the Fadeev-Popov term $\mathcal{L}_{\rm FP}$, and source terms. Such source terms are the ones given by Becchi-Rouet-Stora-Tyutin (BRST) variations of matter $(\overline{\eta^u}, \eta^u, \dots)$ and gauge fields together with Goldstone and ghost fields (not including antighosts). We refer the reader to [4,19] for notation and further explanations. We also include source terms (χ) for the composite operators whose BRST variations generate $\mathcal{L}_{\rm GF}+\mathcal{L}_{\rm FP}$. Schematically,

$$\mathcal{L}=\mathcal{L}_{\rm SM}+\mathcal{L}_{\rm GF}+\mathcal{L}_{\rm FP}-\frac{1}{2\xi}\chi((\partial^\mu W^-_\mu - i\xi M_W G^-)\overline{c^+}$$
$$+(\partial^\mu W^+_\mu - i\xi M_W G^+)\overline{c^-})+\frac{ig}{\sqrt{2}}\overline{\eta^u_i}K_{ir}Ld_r$$
$$-\frac{ig}{\sqrt{2}}\overline{c^+}\,\overline{d}_r K^\dagger_{rj}R\,\eta^u_j+\overline{s^u_i}u_i+\overline{u}_j s^u_j+\overline{s^d_i}d_i+\overline{d}_j s^d_j+\cdots,$$

where the ellipsis stands for the remaining source terms. The effective action Γ is introduced in the standard manner,

$$\Gamma[\chi,\overline{\eta^u},\eta^u,\overline{u^{cl}},u^{cl},\dots]$$
$$=W[\chi,\overline{\eta^u},\eta^u,\overline{s^u},s^u,\dots]$$
$$-(\overline{s^u_i}u^{cl}_i+\overline{u^{cl}_j}s^u_j+\overline{s^d_i}d^{cl}_i+\overline{d^{cl}_j}s^d_j+\cdots), \quad (6.1)$$

with

$$e^{iW}=Z[\chi,\overline{\eta^u},\eta^u,\overline{s^u},s^u,\dots]\equiv\int D\Phi\,\exp(i\mathcal{L}). \quad (6.2)$$

From the above expressions and using (BRST) transformations we can extract the Nielsen identities for the three-point functions (see [20] for details):

$$\partial_\xi\Gamma_{W^+_\mu\overline{u}_i d_j}=-\Gamma_{\chi W^+_\mu\gamma^-_{W_\alpha}}\Gamma_{W^+_\alpha\overline{u}_i d_j}-\Gamma_{\chi\overline{u}_i\eta^u_r}\Gamma_{W^+_\mu\overline{u}_r d_j}$$
$$-\Gamma_{W^+_\mu\overline{u}_i d_r}\Gamma_{\overline{\eta^d_r}d_j\chi}-\Gamma_{\chi W^+_\mu\gamma^-_{G_\alpha}}\Gamma_{G^+_\alpha\overline{u}_i d_j}$$
$$-\Gamma_{\chi\gamma^+_{G_\alpha}\overline{u}_i d_j}\Gamma_{G^-_\alpha W^+_\mu}-\Gamma_{\chi\gamma^+_{W_\alpha}\overline{u}_i d_j}\Gamma_{W^-_\alpha W^+_\mu}$$
$$-\Gamma_{\chi W^+_\mu\overline{u}_i\eta^d_r}\Gamma_{\overline{d}_r d_j}-\Gamma_{\overline{u}_i u_r}\Gamma_{\chi W^+_\mu\overline{\eta^u_r}d_j}, \quad (6.3)$$

where we have omitted the momentum dependence and defined

$$\Gamma_{\chi\overline{u}_i\eta^u_j}\equiv\frac{\vec{\delta}}{\delta\chi}\frac{\vec{\delta}}{\delta\overline{u^{cl}_i}(p)}\frac{\delta}{\delta\eta^u_j(p)}\Gamma,$$

$$\Gamma_{\overline{\eta^u_i}u_j\chi}\equiv\frac{\delta}{\delta\overline{\eta^u_i}(p)}\frac{\vec{\delta}}{\delta u^{cl}_j(p)}\frac{\vec{\delta}}{\delta\chi}\Gamma.$$

In the rest of this section we shall evaluate the on-shell contributions to Eq. (6.3). Analogously, we can also derive Nielsen identities for two-point functions:

$$\partial_\xi\Gamma^{(1)}_{W^+_\mu W^-_\beta}=-2(\Gamma^{(1)}_{\chi W^+_\mu\gamma^-_{W_\alpha}}\Gamma_{W^+_\alpha W^-_\beta}+\Gamma^{(1)}_{\chi W^+_\mu\gamma^-_{G_\alpha}}\Gamma_{G^+_\alpha W^-_\beta}), \quad (6.4)$$

$$\partial_\xi\Gamma^{(1)}_{W^+_\mu G^-_\beta}=-2(\Gamma^{(1)}_{\chi W^+_\mu\gamma^-_{W_\alpha}}\Gamma_{W^+_\alpha G^-_\beta}+\Gamma^{(1)}_{\chi W^+_\mu\gamma^-_{G_\alpha}}\Gamma_{G^+_\alpha G^-_\beta}). \quad (6.5)$$

On shell these reduce to

$$\Gamma^{T(1)}_{\chi W^+\gamma^-_W}(M^2_W)=-\frac{1}{2}\partial_\xi\frac{\partial\Gamma^{T(1)}_{W^+W^-}}{\partial q^2}(q^2)\big|_{q^2=M^2_W}=\frac{1}{2}\partial_\xi\delta Z_W,$$
$$\Gamma^{T(1)}_{\chi W^+\gamma^-_G}(q)=0, \quad (6.6)$$

where the superscript T refers to the transverse part and the superscript (1) makes reference to the one-loop order correction.

Using these two sets of results and restricting Eq. (6.3) to the 1PI function appropriate for (on-shell) top quark decay,

$$\overline{u}_u(p_i)\epsilon^\mu(q)\partial_\xi\Gamma^{(1)}_{W^+_\mu\overline{u}_i d_j}v_d(-p_j)$$
$$=\frac{g}{\sqrt{2}}\overline{u}_u(p_i)\Big\{\Gamma_{\chi\overline{u}_i\eta^u_r}K_{rj}\not{k}L+K_{ir}\not{k}L\Gamma_{\overline{\eta^d_r}d_j\chi}$$
$$+\frac{1}{2}\partial_\xi\delta Z_W K_{ij}\not{k}L\Big\}v_d(-p_j). \quad (6.7)$$

At the one-loop level we also have the Nielsen identity

$$\partial_\xi\Sigma^u_{ij}(p)=\Gamma^{(1)}_{\chi\overline{u}_i\eta^u_j}(p)(\not{p}-m^u_j)+(\not{p}-m^u_i)\Gamma^{(1)}_{\overline{\eta^u_i}u_j\chi}(p), \quad (6.8)$$

which is the fermionic counterpart of Eqs. (6.4) and (6.5). A similar relation holds interchanging $u\leftrightarrow d$. With the use of Eq. (6.8) and an analogous decomposition to Eq. (3.1) for Γ,

$$\Gamma^{(1)}_{\chi\overline{u}_i\eta^u_j}(p)=\not{p}(\Gamma^{\gamma R(1)}_{\chi\overline{u}_i\eta^u_j}(p^2)R+\Gamma^{\gamma L(1)}_{\chi\overline{u}_i\eta^u_j}(p^2)L)$$
$$+\Gamma^{R(1)}_{\chi\overline{u}_i\eta^u_j}(p^2)R+\Gamma^{L(1)}_{\chi\overline{u}_i\eta^u_j}(p^2)L,$$
$$\Gamma^{(1)}_{\overline{\eta^u_i}u_j\chi}(p)=\not{p}(\Gamma^{\gamma R(1)}_{\overline{\eta^u_i}u_j\chi}(p^2)R+\Gamma^{\gamma L(1)}_{\overline{\eta^u_i}u_j\chi}(p^2)L)$$
$$+\Gamma^{R(1)}_{\overline{\eta^u_i}u_j\chi}(p^2)R+\Gamma^{L(1)}_{\overline{\eta^u_i}u_j\chi}(p^2)L, \quad (6.9)$$

we obtain after equating Dirac structures

$$\partial_\xi\Sigma^{u\gamma R}_{ij}(p^2)=\Gamma^{L(1)}_{\chi\overline{u}_i\eta^u_j}(p^2)-m_j\Gamma^{\gamma R(1)}_{\chi\overline{u}_i\eta^u_j}(p^2)$$
$$+\Gamma^{R(1)}_{\overline{\eta^u_i}u_j\chi}(p^2)-m_i\Gamma^{\gamma R(1)}_{\overline{\eta^u_i}u_j\chi}(p^2),$$
$$\partial_\xi\Sigma^{uR}_{ij}(p^2)=p^2\Gamma^{\gamma L(1)}_{\chi\overline{u}_i\eta^u_j}(p^2)-m_j\Gamma^{R(1)}_{\chi\overline{u}_i\eta^u_j}(p^2)$$
$$+p^2\Gamma^{\gamma R(1)}_{\overline{\eta^u_i}u_j\chi}(p^2)-m_i\Gamma^{R(1)}_{\overline{\eta^u_i}u_j\chi}(p^2), \quad (6.10)$$

and analogous expressions exchanging $L\leftrightarrow R$ and $u\leftrightarrow d$. Moreover from Eqs. (6.7) and (6.9) we obtain

$$\overline{u_u}(p_i)\epsilon^\mu(q)\partial_\xi\Gamma^{(1)}_{W^+_\mu \overline{u_i} d_j} v_d(-p_j)=\frac{g}{\sqrt{2}}\Big\{\overline{u_u}(p_i)(m_i^u\Gamma^{\gamma R(1)}_{\chi u_i \eta_r^u}(m_i^{u2})+\Gamma^{R(1)}_{\chi u_i \eta_r^u}(m_i^{u2}))K_{rj}\not{L}v_d(-p_j)+\overline{u_u}(p_i)K_{ir}\not{L}(m_j^d\Gamma^{\gamma R(1)}_{\overline{\eta_r^d} d_j \chi}(m_j^{d2})$$
$$+\Gamma^{L(1)}_{\overline{\eta_r^d} d_j \chi}(m_j^{d2}))v_d(-p_j)+\frac{1}{2}\partial_\xi\delta Z_W\overline{u_u}(p_i)K_{ij}\not{L}v_d(-p_j)\Big\}. \tag{6.11}$$

Using Eqs. (3.3), (3.4), and (6.10) one arrives at

$$m_j^u\Gamma^{\gamma R(1)}_{\eta_i^u u_j \chi}(m_j^{u2})+\Gamma^{L(1)}_{\overline{\eta_i^u} u_j \chi}(m_j^{u2})=\frac{1}{2}\partial_\xi\delta Z_{ij}^{uL}\quad (i\neq j), \tag{6.12}$$

$$m_i^u\Gamma^{\gamma R(1)}_{\chi u_i \eta_j^u}(m_i^{u2})+\Gamma^{R(1)}_{\chi u_i \overline{\eta_j^u}}(m_i^{u2})=\frac{1}{2}\partial_\xi\delta\overline{Z}_{ij}^{uL}\quad (i\neq j), \tag{6.13}$$

and once more similar relations hold exchanging $L\leftrightarrow R$ and $u\leftrightarrow d$. Notice that absorptive parts are present in the 1PI Green's functions and hence in δZ and $\delta\overline{Z}$ too. If we forgot about such absorptive parts we would have pseudo-Hermiticity; namely,

$$\Gamma^{(1)}_{\chi u_i \eta_j^u}=\gamma^0\Gamma^{(1)\dagger}_{\overline{\eta_i^u} u_j \chi}\gamma^0,$$

where $\Gamma^{\dagger}_{\overline{\eta_i^u} u_j \chi}$ means complex conjugating $\Gamma_{\overline{\eta_i^u} u_j \chi}$ and interchanging *both* Dirac and family indices. However, the imaginary branch cut terms prevent the above relation from holding and then Eq. (2.10) does not hold.

At this point one might be tempted to plug expressions (6.12), (6.13) into Eq. (6.11). However such relations are obtained only in the restricted case $i\neq j$. For $i=j$ Eqs. (6.10) are insufficient to determine the combinations appearing in the left-hand side (LHS) of Eqs. (6.12), (6.13) and further information is required. That is also necessary even in the actual case where the RHSs of Eqs. (6.12), (6.13) are not singular at $m_i\to m_j$ [11]. In the rest of this section we shall proceed to calculate such diagonal combinations and as a by-product we shall also cross-check the results already obtained for the off-diagonal contributions and in addition produce some new ones.

By direct computation one finds generically

$$\Gamma^{(1)}_{\chi u_i \eta_j^u}=(\not{p}m_i^u B_{ij}^u(p^2)+C_{ij}^u(p^2)+A_{ij}^u(p^2))R,$$
$$\Gamma^{(1)}_{\overline{\eta_i^u} u_j \chi}=L(\not{p}B_{ij}^u(p^2)m_j^u+C_{ij}^u(p^2)+A_{ij}^u(p^2)), \tag{6.14}$$

and analogous relations interchanging $u\leftrightarrow d$. The A function comes from the diagram containing a charged gauge boson propagator and B and C from the diagram containing a charged Goldstone boson propagator. From Eqs. (6.8) and (6.14) we obtain

$$\partial_\xi\Sigma_{ij}^{\gamma R}(p^2)=-2m_iB_{ij}(p^2)m_j,$$
$$\partial_\xi\Sigma_{ij}^{\gamma L}(p^2)=2(A_{ij}(p^2)+C_{ij}(p^2)),$$
$$\partial_\xi\Sigma_{ij}^{R}(p^2)=(p^2B_{ij}(p^2)-C_{ij}(p^2)-A_{ij}(p^2))m_j,$$
$$\partial_\xi\Sigma_{ij}^{L}(p^2)=m_i(p^2B_{ij}(p^2)-C_{ij}(p^2)-A_{ij}(p^2)). \tag{6.15}$$

The above system of equations is overdetermined and therefore some consistency identities between bare self-energies arise, namely,

$$\partial_\xi(m_i\Sigma_{ij}^R(p^2)-\Sigma_{ij}^L(p^2)m_j)=0, \tag{6.16}$$

and

$$\partial_\xi(p^2\Sigma_{ij}^{\gamma R}(p^2)+\Sigma_{ij}^{\gamma L}(p^2)m_im_j$$
$$+m_i\Sigma_{ij}^R(p^2)+\Sigma_{ij}^L(p^2)m_j)=0. \tag{6.17}$$

These constraints must hold independently of any renormalization scheme and we have checked them by direct computation. Actually the former holds trivially since, at least at the one-loop level in the SM,

$$m_i\Sigma_{ij}^R(p^2)-\Sigma_{ij}^L(p^2)m_j=0. \tag{6.18}$$

Finally, projecting Eq. (6.14) over spinors we also have

$$\overline{u_u}(p_i)\Gamma^{(1)}_{\chi u_i \eta_j^u}=\overline{u_u}(p_i)(m_i^{u2}B_{ij}^u(m_i^{u2})+C_{ij}^u(m_i^{u2})$$
$$+A_{ij}^u(m_i^{u2}))R,$$
$$\Gamma^{(1)}_{\overline{\eta_i^d} d_j \chi}v_d(-p_j)=L(B_{ij}^d(m_j^{d2})m_j^{d2}+C_{ij}^d(m_j^{d2})$$
$$+A_{ij}^d(m_j^{d2}))v_d(-p_j). \tag{6.19}$$

The RHS of the previous expressions can be evaluated in terms of the WFR via the use of Eqs. (6.15):

$$\partial_\xi(m_j^u m_i^u \Sigma_{ij}^{u\gamma R}(p^2)+p^2\Sigma_{ij}^{u\gamma L}(p^2)+m_j^u\Sigma_{ij}^{uR}(p^2)+m_i^u\Sigma_{ij}^{uL}(p^2))$$
$$=B_{ij}^u(p^2)(p^2(m_j^{u2}+m_i^{u2})-2m_j^{u2}m_i^{u2})+(2p^2-m_j^{u2}-m_i^{u2})(A_{ij}^u(p^2)+C_{ij}^u(p^2)), \quad (6.20)$$

$$\partial_\xi(\Sigma_{ij}^{d\gamma R}(p^2)m_i^d m_j^d+\Sigma_{ij}^{d\gamma L}(p^2)p^2+m_i^d\Sigma_{ij}^{dL}(p^2)+\Sigma_{ij}^{dR}(p^2)m_j^d)$$
$$=B_{ij}^d(p^2)(p^2(m_i^{d2}+m_j^{d2})-2m_i^{d2}m_j^{d2})+(2p^2-m_i^{d2}-m_j^{d2})(A_{ij}^d(p^2)+C_{ij}^d(p^2)). \quad (6.21)$$

Hence, using the off-diagonal WFR expressions (3.3), (3.4) we re-obtain

$$\overline{u}_u(p_i)\frac{1}{2}\partial_\xi\delta\overline{Z}_{ij}^{uL}R=\overline{u}(p_i)\Gamma^{(1)}_{\chi u_i\eta_j^u},$$
$$L\frac{1}{2}\partial_\xi\delta Z_{ij}^{dL}v_d(-p_j)=\Gamma^{(1)}_{\eta_i^d d_j\chi}v_d(-p_j). \quad (6.22)$$

For the diagonal WFR we use Eqs. (4.10) together with Eqs. (6.15) and (6.19), obtaining *exactly* the same result as in Eq. (6.22) with $i=j$ therein. Note, however, that since in Eq. (6.19) we have no derivatives with respect to p^2, obtaining Eq. (6.22) involves a subtle cancellation between the p^2 derivatives of the bare self-energies appearing in the definition of the diagonal WFR.

Before proceeding let us make a side remark concerning the regularity properties of the gauge derivative in Eqs. (6.20) and (6.20) in the limit $m_i\to m_j$. Note that in evaluating Eq. (6.20) at $p^2=m_i^{u2}$ and Eq. (6.21) at $p^2=m_j^{d2}$, a global factor $(m_i^{u2}-m_j^{u2})$ appears in the first equation and $(m_j^{d2}-m_i^{d2})$ in the second one. Therefore it can be immediately seen that Nielsen identities together with the information provided by Eq. (6.14) assure the regularity of the gauge derivative for the off-diagonal WFR constants when $m_i\to m_j$. Moreover, we have seen that this limit is not only regular but also equal to the expression obtained from the diagonal WFR which is not obvious *a priori* [8,11].

Replacing Eq. (6.22) in Eq. (6.7) we obtain

$$\partial_\xi(\overline{u}_u(p_i)\epsilon^\mu(q)\Gamma^{(1)}_{W_\mu^+\overline{u}_i d_j}v_d(-p_j))$$
$$=\frac{e}{2s_W}M_L^{(1)}\partial_\xi(\delta\overline{Z}_{ir}^{uL}K_{rj}+K_{ir}\delta Z_{rj}^{dL}+\delta Z_W K_{ij})$$
$$=-\frac{e}{2s_W}\partial_\xi(M_L^{(1)}\delta F_L^{(1)}+M_L^{(2)}\delta F_L^{(2)}$$
$$+M_R^{(1)}\delta F_R^{(1)}+M_R^{(2)}\delta F_R^{(2)}), \quad (6.23)$$

where Eq. (5.5) and the gauge independence of the electric charge and Weinberg angle have been used in the last equality. In the previous expression $M_{L,R}^{(i)}$ are understood with the physical momenta p_1 and p_2 of Eq. (5.3) replaced by the diagrammatic momenta p_i and $-p_j$, respectively. Note that Eq. (6.23) states that the gauge dependence of the on-shell bare one-loop vertex function cancels out the renormalization counter terms appearing in Eq. (5.5) (see Fig. 1). This is one of the crucial results and special care should be taken not to ignore any of the absorptive parts—including those in the WFR constants. As a consequence,

$$\partial_\xi\mathcal{M}_1=-\frac{e}{2s_W}M_L^{(1)}\partial_\xi\delta K_{ij},$$

and if we want a gauge independent amplitude the counterterm for K_{ij} must be separately gauge independent, as originally derived in [8].

Finally, since each structure $M_{L,R}^{(i)}$ must cancel separately we have that the Nielsen identities enforce

$$\partial_\xi\delta F_L^{(2)}=\partial_\xi\delta F_R^{(1)}=\partial_\xi\delta F_R^{(2)}=0.$$

VII. ABSORPTIVE PARTS

Having determined in the previous section, thanks to an extensive use of the Nielsen identities, the gauge dependence of the different quantities appearing in top quark or W decay in terms of the self-energies, we shall now proceed to list the absorptive parts of the WFR constants, with special attention to their gauge dependence. The aim of this section is to state the differences between the WFR constants given in our scheme and the ones in [7]. Recall that at one loop this difference reduces to the absorptive ($\widetilde{\mathrm{Im}}$) contribution to the δZ's. In what concerns the gauge dependent part (with $\xi\geqslant 0$) the absorptive contribution ($\widetilde{\mathrm{Im}}_\xi$) in the fermionic δZ's amounts to

$$i\widetilde{\mathrm{Im}}_\xi(\delta Z_{ij}^{uL})=\sum_h\frac{iK_{ih}K_{hj}^\dagger}{8\pi v^2 m_j^{u2}}\theta(m_j^u-m_h^d-\sqrt{\xi}M_W)$$
$$\times(m_j^{u2}-m_h^{d2}-\xi M_W^2)$$
$$\times\sqrt{((m_j^u-m_h^d)^2-\xi M_W^2)((m_j^u+m_h^d)^2-\xi M_W^2)},$$

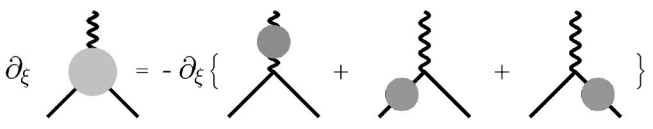

FIG. 1. Pictorial representation of the on-shell Nielsen identity given by Eq. (6.23). The blobs in the LHS. represent bare one-loop contributions to the on-shell vertex and the blobs in the RHS WFR counterterms.

$$i\widetilde{\mathrm{Im}}_\xi(\delta\overline{Z}^{uL}_{ij}) = \sum_h \frac{iK_{ih}K^\dagger_{hj}}{8\pi v^2 m_i^{u2}}\theta(m_i^u - m_h^d - \sqrt{\xi}M_W) \times (m_i^{u2} - m_h^{d2} - \xi M_W^2) \times \sqrt{((m_i^u - m_h^d)^2 - \xi M_W^2)((m_i^u + m_h^d)^2 - \xi M_W^2)},$$

$$\widetilde{\mathrm{Im}}_\xi(\delta Z^{uR}_{ij}) = \widetilde{\mathrm{Im}}_\xi(\delta\overline{Z}^{uR}_{ij}) = 0,$$

where θ is the Heaviside function and v is the Higgs vacuum expectation value. For the down δZ we have the same formulas replacing $u \leftrightarrow d$ and $K \leftrightarrow K^\dagger$. Note that using these results we can write

$$\begin{aligned} i\partial_\xi \widetilde{\mathrm{Im}}\Big[\sum_r (\delta\overline{Z}^{uL}_{ir}K_{rj} + K_{ir}\delta Z^{dL}_{rj}) + \delta Z_W K_{ij}\Big] &= K_{ij}\partial_\xi\Bigg\{\frac{i}{8\pi v^2}\Bigg[\frac{1}{m_i^{u2}}\theta(m_i^u - m_j^d - \sqrt{\xi}M_W) \times (m_i^{u2} - m_j^{d2} - \xi M_W^2) + \frac{1}{m_j^{d2}}\theta(m_j^d - m_i^u - \sqrt{\xi}M_W) \\ &\times (m_j^{d2} - m_i^{u2} - \xi M_W^2)\Bigg] \times \sqrt{((m_j^d - m_i^u)^2 - \xi M_W^2)((m_j^d + m_i^u)^2 - \xi M_W^2)} \\ &+ i\widetilde{\mathrm{Im}}_\xi(\delta Z_W)\Bigg\}. \end{aligned} \tag{7.1}$$

In the case $|m_i^u - m_j^d| \leqslant \sqrt{\xi}M_W$ the above expression reduces to

$$\partial_\xi \sum_r \widetilde{\mathrm{Im}}(\delta\overline{Z}^{uL}_{ir}K_{rj} + K_{ir}\delta Z^{dL}_{rj}) = 0, \tag{7.2}$$

while for $|m_i^u - m_j^d| \geqslant \sqrt{\xi}M_W$ we have

$$\begin{aligned} i\partial_\xi \sum_r \widetilde{\mathrm{Im}}(\delta\overline{Z}^{uL}_{ir}K_{rj} + K_{ir}\delta Z^{dL}_{rj}) &= K_{ij}\partial_\xi\Bigg\{\frac{i}{4\pi v^2}\frac{|m_i^{u2} - m_j^{d2}| - \xi M_W^2}{m_i^{u2} + m_j^{d2} + |m_i^{u2} - m_j^{d2}|} \\ &\times \sqrt{((m_j^d - m_i^u)^2 - \xi M_W^2)((m_j^d + m_i^u)^2 - \xi M_W^2)}\Bigg\}. \end{aligned} \tag{7.3}$$

Moreover the ξ dependent absorptive contribution to δZ_W ($\widetilde{\mathrm{Im}}_\xi(\delta Z_W)$) has no dependence on quark masses since the diagram with a fermion loop is gauge independent. Because of that we can conclude that the derivative in Eq. (7.1) does not vanish. Defining Δ_{ij} as the difference between the vertex observable calculated in our scheme and the same in the scheme using $\widetilde{\mathrm{Re}}$ we have

$$\Delta_{ij} \sim |K_{ij}|^2 \mathrm{Re}(i\widetilde{\mathrm{Im}}\delta Z_W) + \mathrm{Re}\Bigg\{iK^*_{ij}\sum_r [\widetilde{\mathrm{Im}}(\delta\overline{Z}^{uL}_{ir})K_{rj} + K_{ir}\widetilde{\mathrm{Im}}(\delta Z^{dL}_{rj})]\Bigg\}.$$

In the case of δZ_W one can easily check that $\widetilde{\mathrm{Im}}(\delta Z_W) = \mathrm{Im}(\delta Z_W)$, obtaining

$$\Delta_{ij} \sim \mathrm{Re}\Bigg\{iK^*_{ij}\sum_r [\widetilde{Im}(\delta\overline{Z}^{uL}_{ir})K_{rj} + K_{ir}\widetilde{\mathrm{Im}}(\delta Z^{dL}_{rj})]\Bigg\}. \tag{7.4}$$

Thus from Eq. (7.2), (7.3), and (7.4) we immediately obtain

$$\partial_\xi \Delta_{ij} \sim \mathrm{Re}\Bigg\{iK^*_{ij}\sum_r [\partial_\xi\widetilde{\mathrm{Im}}(\delta\overline{Z}^{uL}_{ir})K_{rj} + K_{ir}\partial_\xi\widetilde{\mathrm{Im}}(\delta Z^{dL}_{rj})]\Bigg\} = 0. \tag{7.5}$$

However, gauge independent absorptive parts, included if our prescription is used but not if one uses that of [7], which makes use of the $\widetilde{\mathrm{Re}}$, do contribute to Eq. (7.4). In order to see that we can take $\xi = 1$ obtaining for the physical values of the masses

$$\widetilde{\mathrm{Im}}_{\xi=1}(\delta Z^{dL}_{rj}) = 0,$$

$$\begin{aligned} \widetilde{\mathrm{Im}}_{\xi=1}(\delta\overline{Z}^{uL}_{ir}) &= \sum_h \frac{K_{ih}K^\dagger_{hr}}{8\pi v^2 m_i^{u2}}\frac{\theta(m_i^u - m_h^d - M_W)}{m_i^{u2} - m_r^{u2}} \times \sqrt{(m_i^{u2} - (M_W - m_h^d)^2)(m_i^{u2} - (M_W + m_h^d)^2)} \\ &\times \Bigg(\frac{1}{2}(m_r^{u2} + m_h^{d2} + 2M_W^2)(m_i^{u2} + m_h^{2d} - M_W^2) - (m_i^{u2} + m_r^{u2})m_h^{d2}\Bigg), \end{aligned} \tag{7.6}$$

where only the results for $i \neq j$ have been presented. Note that $\widetilde{\mathrm{Im}}_{\xi=1}(\delta\overline{Z}^{uL}_{ir}) \neq 0$ only when $i = 3$, that is, when the renormalized up particle is a top quark. In addition, since the m_r^{u2} dependence in Eq. (7.6) does not vanish, CKM phases do not disappear from Eq. (7.4), and therefore

$$\Delta_{3j} \sim \mathrm{Re}\Bigg\{iK^*_{3j}\sum_r [\widetilde{\mathrm{Im}}(\delta\overline{Z}^{uL}_{3r})K_{rj} + K_{3r}\widetilde{\mathrm{Im}}(\delta Z^{dL}_{rj})]\Bigg\} \neq 0. \tag{7.7}$$

Equations (7.5) and (7.7) show that even though the difference Δ_{3j} is gauge independent, it does not actually vanish. There are genuine gauge independent pieces that contribute not only to the amplitude, but also to the observable. As discussed, these additional pieces cannot be absorbed by a redefinition of K_{ij}. Numerically such gauge independent corrections amount roughly to $\Delta_{3j} \simeq 5 \times 10^{-3} O_{\mathrm{tree}}$ where O_{tree} is the observable quantity calculated at leading order.

VIII. *CP* VIOLATION AND *CPT* INVARIANCE

In this section we want to show that using WFR constants that do not satisfy a pseudo-Hermiticity condition does not lead to any unwanted pathologies. In particular, (a) no new sources of CP violation appear in addition to the ones already present in the SM; (b) the total widths of particles and antiparticles coincide, thus verifying the CPT theorem. Let us start with the latter, which is not completely obvious since not all external particles and antiparticles are renormalized with the same constant due to the different absorptive parts.

The optical theorem asserts that

$$\Gamma_t \sim \sum_f \int d\Pi_f |M(t^{(\hat{n})}(p) \to f)|^2 = 2\,\mathrm{Im}[M(t^{(\hat{n})}(p) \to t^{(\hat{n})}(p))], \tag{8.1}$$

$$\Gamma_{\bar{t}} \sim \sum_f \int d\Pi_f |M(\bar{t}^{(\hat{n})}(p) \to f)|^2 = 2\,\mathrm{Im}[M(\bar{t}^{(\hat{n})}(p) \to \bar{t}^{(\hat{n})}(p))], \tag{8.2}$$

where we have considered, just as an example, top quark $[t^{(\hat{n})}(p)]$ and anti-top quark $(\bar{t}^{(\hat{n})}(p))$ decay, with p and $\hat{n}$ being their momentum and polarization. Recalling that the incoming fermion and outgoing antifermion spinors are renormalized with a common constant [see Eq. (2.1)] as are the outgoing fermion and incoming antifermion ones, it is immediately clear that

$$M(t^{(\hat{n})}(p) \to t^{(\hat{n})}(p)) = \bar{u}^{(\hat{n})}(p) A_{33}(p) u^{(\hat{n})}(p),$$

$$M(\bar{t}^{(\hat{n})}(p) \to \bar{t}^{(\hat{n})}(p)) = -\bar{v}^{(\hat{n})}(p) A_{33}(-p) v^{(\hat{n})}(p),$$

where the minus sign comes from an interchange of two fermion operators and where the subscripts in A indicate family indices. Using the fact that

$$u^{(\hat{n})}(p) \otimes \bar{u}^{(\hat{n})}(p) = \frac{\not p + m}{2m} \frac{1+\gamma^5 \not n}{2},$$

$$-v^{(\hat{n})}(p) \otimes \bar{v}^{(\hat{n})}(p) = \frac{-\not p + m}{2m} \frac{1+\gamma^5 \not n}{2},$$

with $n = [1/\sqrt{(p^0)^2 - (\vec{p}\cdot\hat{n})^2}](\vec{p}\cdot\hat{n}, p^0\hat{n})$ being the polarization four-vector and performing some elementary manipulations we obtain

$$\begin{aligned}
\bar{u}^{(\hat{n})}(p) A_{33}(p) u^{(\hat{n})}(p) &= \mathrm{Tr}\left[\left(\frac{\not p + m}{2m}\frac{1+\gamma^5 \not n}{2}\right)[a(p^2)\not p L + b(p^2)\not p R + c(p^2) L + d(p^2) R)\right] \\
&= \frac{1}{4}\mathrm{Tr}\left\{\frac{\not p + m}{2m}\{[a(p^2)+b(p^2)]\not p + c(p^2) + d(p^2)\}\right\} \\
&= \frac{1}{4}\mathrm{Tr}\left\{\frac{-\not p + m}{2m}\{-[a(p^2)+b(p^2)]\not p + c(p^2) + d(p^2)\}\right\} \\
&= \mathrm{Tr}\left[\frac{-\not p + m}{2m}\frac{1+\gamma^5 \not n}{2}[-a(p^2)\not p L - b(p^2)\not p R + c(p^2) L + d(p^2) R]\right] \\
&= -\bar{v}^{(\hat{n})}(p) A_{33}(-p) v^{(\hat{n})}(p),
\end{aligned}$$

where we have decomposed $A_{33}(p)$ into its most general Dirac structure. We thus conclude the equality between Eqs. (8.1) and (8.2) verifying that the lifetimes of top quark and top antiquark are identical. The detailed form of the WFR constants, or whether they have absorptive parts or not, does not play any role.

Even though total decay widths for top quark and top antiquark are identical, the partial ones need not be if CP violation is present, and some compensation between different processes must take place. This issue is discussed in detail in [23]. Here we shall show that when $K = K^*$ the CP invariance of the Lagrangian manifests itself in a zero asymmetry between the partial differential decay rate of the top quark and its CP conjugate process. The fact that the external renormalization constants have dispersive parts does not alter this conclusion. This is of course expected on rather general grounds, so the following discussion has to be taken really as a verification that no unexpected difficulties arise.

To illustrate this point let us consider the top quark decay channel $t(p_1) \to W^+(p_1 - p_2) + b(p_2)$ and its CP conjugate process $\bar{t}(\tilde{p}_1) \to W^-(\tilde{p}_1 - \tilde{p}_2) + b(\tilde{p}_2)$. Let us denote the respective amplitudes by $\mathcal{A}$ and $\mathcal{B}$, which are given as

$$\mathcal{A} = \varepsilon^\mu \bar{u}^{(s_2)}(p_2) A_\mu u^{(s_1)}(p_1),$$

$$\mathcal{B} = -\tilde{\varepsilon}^\mu \bar{v}^{(s_1)}(\tilde{p}_1) B_\mu v^{(s_2)}(\tilde{p}_2),$$

where $\tilde{a}^\mu = a_\mu = (a^0, -a^i)$ for any four-vector. Considering contributions up to and including next-to-leading corrections we have

$$A_\mu = -i\frac{e}{\sqrt{2}s_W}[(\overline{Z}^{bL\ 1/2}K^\dagger Z^{tL\ 1/2} + K^\dagger\delta_V + \delta K^\dagger)\gamma_\mu L + \delta F_\mu],$$

$$B_\mu = -i\frac{e}{\sqrt{2}s_W}[(\overline{Z}^{tL\ 1/2}KZ^{bL\ 1/2} + K\delta_V + \delta K)\gamma_\mu L + \delta G_\mu],$$

with $\delta_V = \delta e/e - \delta s_W/s_W + \frac{1}{2}\delta Z_W$ and δF_μ and δG_μ are given by the one-loop diagrams. From a direct computation it can be seen that if $K = K^*$ this implies

$$\overline{Z}^{L\ 1/2} = (Z^{L\ 1/2})^T, \quad \overline{Z}^{R\ 1/2} = (Z^{R\ 1/2})^T,$$

$$\tilde{\varepsilon}^\mu \delta G_\mu = \varepsilon^\mu \gamma^2 \delta F_\mu^T \gamma^2, \tag{8.3}$$

where the superscript T means transposition with respect to all indices (family indices in the case of $Z^{L\ 1/2}$ and $Z^{R\ 1/2}$ and Dirac indices in the case of δF_μ). Using

$$i\gamma^2\overline{u^{(s)T}}(p) = sv^{(s)}(\tilde{p}),$$

$$u^{(s)T}(p)i\gamma^2 = -s\overline{v^{(s)}}(\tilde{p}),$$

where $s = \pm 1$, depending on the spin direction on the $\hat{z}$ axis, we obtain

$$\begin{aligned}
\mathcal{A} &= \frac{-ie}{\sqrt{2}s_W}\varepsilon^\mu \overline{u^{(s_2)}}(p_2)[(\overline{Z}^{bL\ 1/2}K^\dagger Z^{tL\ 1/2} + K^\dagger\delta_V + \delta K^\dagger)\gamma_\mu L \\
&\quad + \delta F_\mu]u^{(s_1)}(p_1) \\
&= \frac{-ie}{\sqrt{2}s_W}\varepsilon^\mu u^{(s_1)T}(p_1)[L((Z^{tL\ 1/2})^T K^*(\overline{Z}^{bL\ 1/2})^T + K^*\delta_V \\
&\quad + \delta K^*)\gamma_\mu^T + \delta F_\mu^T]\overline{u^{(s_2)T}}(p_2) \\
&= \frac{-s_1 s_2 ie}{\sqrt{2}s_W}\varepsilon^\mu \overline{v^{(s_1)}}(\tilde{p}_1)\gamma^2[L((Z^{tL\ 1/2})^T K^*(\overline{Z}^{bL\ 1/2})^T \\
&\quad + K^*\delta_V + \delta K^*)\gamma_\mu^T + \delta F_\mu^T]\gamma^2 v^{(s_2)}(\tilde{p}_2) \\
&= \frac{-s_1 s_2 ie}{\sqrt{2}s_W}\varepsilon^\mu \overline{v^{(s_1)}}(\tilde{p}_1)[((Z^{tL\ 1/2})^T K^*(\overline{Z}^{bL\ 1/2})^T + K^*\delta_V \\
&\quad + \delta K^*)\gamma_\mu^\dagger L + \gamma^2\delta F_\mu^T\gamma^2]v^{(s_2)}(\tilde{p}_2),
\end{aligned}$$

Now, using Eq. (8.3) we see that if no CP violating phases are present in the CKM matrix K [and therefore neither in δK, Eq. (5.6)] we obtain that $\mathcal{A} = -s_1 s_2 \mathcal{B}$ and thus

$$|\mathcal{A}|^2 = |\mathcal{B}|^2.$$

Note again that when CP violating phases are present we can expect in general nonvanishing phase-space dependent asymmetries for the different channels. Once we sum over all channels and integrate over the final state phase space a compensation must take place, as we have seen, guaranteed by unitarity and CPT invariance. Using a set of WFR constants with absorptive parts as advocated here (and required by gauge invariance) leads to different results from using the prescription originally advocated in [7]; in particular, using Eq. (7.7) for $K \neq K^*$ we expect $\Delta_{3j}^{(t\ decay)} - \Delta_{3j}^{(\overline{t}\ decay)} \neq 0$.

IX. CONCLUSIONS

Let us recapitulate our main results. We hope, first of all, to have convinced the reader that *there is* a problem with what appears to be the commonly accepted prescription for dealing with wave-function renormalization when mixing is present. The situation is even further complicated by the appearance of CP violating phases. The problem has a twofold aspect. On the one hand the prescription of [7] does not diagonalize the propagator matrix in flavor space in what pertains to the absorptive parts. On the other hand it yields gauge *dependent* amplitudes, albeit gauge *independent* moduli squared of the amplitudes. This is not satisfactory: interference with, e.g., strong phases may reveal an unacceptable gauge dependence.

The only solution is to accept WFR constants that do not satisfy a pseudo-Hermiticity condition due to the presence of the absorptive parts, which are neglected in [7]. This immediately brings about some gauge *independent* absorptive parts which appear even in the modulus squared of the amplitude and which are neglected in the treatment of [7]. Furthermore, these parts (and the gauge dependent ones) cannot be absorbed in unitary redefinitions of the CKM matrix which are the only ones allowed by Ward identities. We have checked that—although unconventional—the presence of the absorptive parts in the WFR constants is perfectly compatible with basic tenets of field theory and the standard model. Numerically we have found the differences to be important, at the order of 0.5%: small, but relevant in the future. This information will be relevant to extract the experimental values of the CKM mixing matrix.

Traditionally, wave-function renormalization seems to have been the ''poor relative'' in the standard model renormalization program. We have seen here that it is important on two counts. First, because it is related to the counterterms for the CKM mixing matrix, although the on-shell values for wave-function constants cannot be directly used there. Second, because it is crucial to obtain gauge independent S matrix elements and observables. While using our WFR constants (but not the ones in [7]) for the external legs is strictly equivalent to considering reducible diagrams (with on-shell mass counterterms), the former procedure is considerably more practical.

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