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Classical solutions in two-dimensional string theory and gravitational collapse

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A general solution to the $D=2$ one-loop β-function equations including tachyonic back reaction on the metric is presented. Dynamical black hole (classical) solutions representing gravitational collapse of tachyons are constructed. A discussion on the correspondence with the matrix-model approach is given.

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Recently there has been an increasing interest in understanding the quantum-mechanical paradoxes associated with Hawking radiation in the context of simple two-dimensional models containing black holes (see Ref. [1] and references therein). The knowledge of exact results coming from discrete approaches [2–7] also converts two-dimensional string theory in a simple laboratory to explore the quantum physics underlying black hole geometries (for recent discussions in this direction see Refs. [8–10]). The black hole interpretation of some classical solutions of $D=2$ string theory [11] was discovered in Ref. [12] by using an exact conformal field theory constructed from a gauged Wess-Zumino-Witten (WZW) model.

In physical situations the black hole should appear by gravitational collapse of the only propagating mode of the $D=2$ critical string theory, which at low energies is $\eta \equiv e^{-\phi}T$, where ϕ is the dilaton and T is the tachyon. In Ref. [10] it was observed that there is a natural effective geometry which emerges in scattering processes in the Das-Jevicki collective field theory [3] which for a distant observer (far away from the horizon) appears as a black hole configuration. But in view of the simplicity of the exact classical S matrix derived in the matrix-model approach one may be concerned about the actual existence of dynamically formed black holes. The fact that the static black hole survives after α' corrections, as found in Refs. [13,14], is encouraging, but unfortunately it is not enough to prove that there is gravitational collapse in the model. Note that "shock-wave" configurations constructed by a simple matching of the "exact" static metric of Ref. [13] with the linear dilaton vacuum cannot be implemented without an exact account for all higher powers of T. Here we will explicitly construct solutions representing transitions between the linear dilaton vacuum and the static black hole solutions by dynamical gravitational collapse of tachyons. The essential assumption involved in these solutions is that part of the incoming energy is not reflected. As discussed below, if a pulse is instead fully reflected there is a transitory period during which there is a black-hole-type configuration, but the final state is the linear dilaton vacuum. The resulting picture is in agreement with the matrix-model approach.

To leading α' order, the tree-level string effective action for the metric, dilaton, and tachyon is given by

$$S=\int d^2x\sqrt{-G}\left[e^{-2\phi}\left[R+4\partial_\mu\phi\partial^\mu\phi+c-\partial_\mu T\partial^\mu T+\frac{2}{\alpha'}T^2\right]+O(T^3)\right], \quad (1)$$

where $c=-8/\alpha'$.

The equations of motion, neglecting $O(T^3)$ terms in the action (1), are given by (henceforth we set $\alpha'=2$)

$$R_{\mu\nu}+2D_\mu\partial_\nu\phi=\partial_\mu T\partial_\nu T\ , \quad (2)$$

$$4(\partial\phi)^2-2\nabla^2\phi-T^2-4=0\ , \quad (3)$$

$$\nabla^2 T-2\partial_\mu\phi\partial^\mu T+T=0\ . \quad (4)$$

If $T=0$ the general solution is the Witten black hole. Around this background the field $\eta=e^{-\phi}T$ becomes massless far away from the black hole. The static solution to these equations with nonvanishing T was found in Ref. [15].

It is convenient to work in the gauge in which the dilaton is linear, $\phi=-x$, and the metric has the diagonal form

$$ds^2=A\,dt^2+B\,dx^2\ . \quad (5)$$

Then the 01, 00, 11 components of Eq. (2) and the dilaton equation (3) take the form

$$\frac{\dot B}{B}=\dot T T'\ , \quad (6)$$

$$\frac{A}{2}R-\frac{A'}{B}=\dot T^2\ , \quad (7)$$

$$\frac{B}{2}R+\frac{B'}{B}=T'T'\ , \quad (8)$$

$$4(1-B)-(\ln|B/A|)'=BT^2 \quad (9)$$

(as conventional, primes and dots denote derivatives with respect to x and t, respectively). In this gauge the curvature is

$$R=-\frac{1}{\Delta}\left[\frac{d}{dx}\left[\frac{A'}{\Delta}\right]+\frac{d}{dt}\left[\frac{\dot B}{\Delta}\right]\right],\quad \Delta=\sqrt{-AB}\ . \quad (10)$$

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From Eq. (6) we get

$$B=f(x)\exp\left[\int^{t}dt\,\dot{T}T'\right]. \tag{11}$$

Combining Eqs. (7) and (8) we find

$$(\ln|AB|)'=T'T'-\frac{B}{A}\dot{T}^2, \tag{12}$$

from where it follows

$$AB=-g(t)\exp\left[\int^{x}dx\left[T'T'-\frac{B}{A}\dot{T}^2\right]\right]. \tag{13}$$

The arbitrary function $g(t)$ can be set to 1 by a time reparametrization.

It is worth noting that if the tachyon self-interactions are nonderivative, then Eqs. (11) and (13) are exact to all order in powers of T. This is because Eq. (2) is not modified by nonderivative tachyon self-interactions [but of course it is modified by two (σ model) loop corrections].

We are interested in solutions where the metric asymptotically approaches to the Minkowski metric. Thus let us assume that $A=-1+O(e^{-2x})$, $B=1+O(e^{-2x})$ and $T=e^{-x}\eta=O(e^{-x})$. Then, to this T^2 order, we can set $B/A\cong-1$ in Eq. (13) obtaining

$$AB=-\exp\left[\int^{x}dx(T'T'+\dot{T}^2)\right]. \tag{14}$$

In order to determine the arbitrary function $f(x)$ we insert Eq. (11) into the dilaton equation (9). We get $f'=2(1-f)$, i.e., $f(x)=1+Me^{-2x}$, $M=$const, and the compatibility condition

$$T^2-\dot{T}^2+2\int^{t}dt\,\dot{T}T''+4\int^{t}dt\,T'\dot{T}=0. \tag{15}$$

Since to the lowest order we have

$$T''-\ddot{T}+2T'+T=0, \tag{16}$$

the condition (15) is automatically satisfied on shell (as guaranteed by covariance). To see this explicitly multiply Eq. (16) by $\dot{T}$ and integrate over t.

Thus the general solution is given by

$$ds^2=-(1-Me^{-2x})\exp\left[\int_{\infty}^{x}dx(T'T'+\dot{T}^2)-\int_{-\infty}^{t}dt\,\dot{T}T'\right]dt^2+(1+Me^{-2x})\exp\left[\int_{-\infty}^{t}dt\,\dot{T}T'\right]dx^2,\quad \phi=-x. \tag{17}$$

If we set $T=0$ we recover the Witten black hole solution where M is the Arnowitt-Deser-Misner (ADM) mass of the black hole. Here we will be interested in black holes purely formed by gravitational collapse so we shall set M to zero.

To the present $O(T^2)$ approximation the solution (17) with $M=0$ can be written as

$$ds^2=-\left[1+\int_{\infty}^{x}dx(T'T'+\dot{T}^2)-\int_{-\infty}^{t}dt\,\dot{T}T'\right]dt^2+\left[1+\int_{-\infty}^{t}dt\,\dot{T}T'\right]dx^2. \tag{18}$$

The simplest example is that in which T is a static configuration $T=\mu e^{-x}$, $\mu=$const (or $\eta=\mu$). In this case the solution takes the form

$$ds^2=-(1-\tfrac{1}{2}\mu^2e^{-2x})dt^2+dx^2,\quad \phi=-x, \tag{19}$$

which is similar to the Witten black hole solution but not exactly the same (in the linear dilaton gauge, the Witten metric is different).

Now consider the more physical case of a dynamical gravitational collapse. Let T be an incoming localized wave packet, for example,

$$\eta(x)=\eta(x^+)=\frac{a}{\cosh(2x^+)},\quad x^+=x+t, \tag{20}$$

or $T=ae^{-x}/\cosh(2x^+)$. Then

$$\int_{-\infty}^{t}dt\,\dot{T}T'=\frac{2}{3}a^2e^{-2x}\left[1+\tanh(2x^+)-\frac{\sinh(2x^+)}{\cosh^3(2x^+)}-\frac{3}{4}\frac{1}{\cosh^2(2x^+)}\right], \tag{21}$$

$$\int_{\infty}^{x}dx(T'T'+\dot{T}^2)=-2a^2e^{2t}\left[-\frac{3}{4i}\ln\frac{1-iz}{1+iz}-\frac{7}{2}\frac{z}{z^2+1}+\frac{22}{3}\frac{z}{(z^2+1)^2}-\frac{16}{3}\frac{z}{(z^2+1)^3}\right],\quad z\equiv e^{-2x^+}. \tag{22}$$

There is an expanding horizon given by the equation $x=x_h(t)$:

$$1+\int_{\infty}^{x_h}dx(T'T'+\dot{T}^2)-\int_{-\infty}^{t}dt\,\dot{T}T'=0. \tag{23}$$

Let us now consider late times, $t\to\infty$. The leading terms in Eq. (22), $O(e^{-2x^+})$, cancel out, so

$$\lim_{t\to\infty}\int_{\infty}^{x}dx(T'T'+\dot{T}^2)=O(e^{-4x-2t})\cong 0.$$

Therefore the final metric has the form

$$ds^2=-(1-me^{-2x})dt^2+(1-me^{-2x})^{-1}dx^2,\quad m=\frac{4}{3}a^2. \tag{24}$$

This is nothing but the Witten static black hole solution. Thus the solution given by Eqs. (18), (21), and (22) describes the transition from the linear dilaton vacuum to the static black hole solution due to collapsing matter.

The essential assumption involved in the above solution is that η has only an ingoing component. More generally, if the outgoing component carries less energy than the ingoing component then the final metric will be given by Eq. (24) with m representing the energy that was not reflected. Whether all the energy is reflected or not is something that will be dictated by the higher order terms in powers of T which become important in the region $x^- = t - x \to \infty$. Let us see that if we assume that part of the energy is not reflected back then the solution (18) implies that the final state is a black hole. This is not trivial, since this solution is only valid in the region $x \gg x_h$, i.e., in the region $1 \gg m e^{-2x}$. The question is, then, whether a horizon really forms in dynamical processes or the black hole geometry is just an illusory large-distance effect. In the region $x^+ \to \pm\infty$ the tachyon goes to zero so we can exactly solve the differential equations by using the results of Refs. [13,14]. Therefore the exact final metric which at large distances matches with (24) is just the Dijkgraaf-Verlinde-Verlinde (DVV) metric (up to three-loop ambiguities pointed out in Ref. [14]) with the ADM mass equal to m:[1]

$$ds^2 = A_0(q)dt^2 - A_0^{-1}(q)dq^2, \quad A_0(q) \equiv 1 - m\frac{1}{\sinh^2(q)}, \tag{25}$$

$$\phi = -\frac{1}{2}\ln\sinh(2q) + \phi_0 = -x, \quad \phi_0 \equiv \frac{1}{4}\ln\frac{4(m+1)}{m},$$

$$R = -A_0''(q) = \frac{4m}{\sinh^4(q)}\left[\cosh^2(q) + \frac{1}{2}\right].$$

This metric has an event horizon at $x = \frac{1}{2}\ln m$ and a singularity at $x = -\infty$.

However, the scattering amplitudes for *low*-energy waves in the matrix-model approach show that everything is reflected off the wall. This seems to be the case also in the present continuum formalism, because of the following argument. By plugging the solution (18) into the tachyon equation of motion we see that it has the structure

$$T'' - \ddot{T} + 2T' + T = O_V(T^2) + O(T^3), \tag{26}$$

where $O_V(T^2) \sim e^{-2x}$ contains only a tachyon self-interaction contribution, coming from the "T^3" term in the Lagrangian. That is, the gravitational part only influences the next order $O(e^{-3x})$. But it is well known that the on-shell S matrix of low-energy tachyons due to the $O_V(T^2)$ term is unitary (see, e.g., Refs. [4,16]). Therefore, in accordance with the prediction from the matrix-model approach, an ingoing low-energy wave should undergo total reflection, since it will first feel the T^3 interaction, the gravitational forces being exponentially suppressed.

Let us consider the example

$$\eta(x) = \frac{a}{\cosh(2x^+)} + \frac{a}{\cosh(2x^-)}. \tag{27}$$

Equation (27) represents a pulse that is fully reflected. It is straightforward to compute metric (18) in this case to see that the final state is the linear dilaton vacuum. There is a transitory period during which the large-distance geometry looks like a black hole configuration. Consider now the case of an incoming "step" energy-density pulse (a constant source of ingoing energy density which turns on at some time). Even when everything is reflected, the final stationary geometry will be of the form (24) due to the constant presence of energy in the space. The picture is in fact very similar to the picture discussed in Ref. [10] in the context of Das-Jevicki collective field theory. In the view of a distant observer, there is a horizon which appears to be beyond the reflection point (this is the physical boundary of the space; the wall is not rigid, but moves in and out as the system evolves). Any nontrivial geometry produced by localized pulses disappears after the pulses have been reflected and get away from the wall, etc.[2]

To conclude, the equivalence of the continuum and discrete approaches implies that a low-energy pulse is completely reflected and hence there is not actual formation of event horizons. Equation (18) describes the evolution of the large-distance geometry in terms of the source. Transitory black-hole type configurations, with virtual horizons and singularities appearing beyond the physical boundary of the space, are present in generic physical processes. The present description breaks down for high-energy pulses which can cross the barrier.

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[1]The coordinates (t,q) are "canonical" coordinates in the Das-Jevicki field theory (in these coordinates the kinetic term of the bosonic Hamiltonian has the canonical form). In this theory an effective metric given by Eq. (24) appears in a natural way (see Ref. [10]). The coordinate q is related to the radial coordinate r of Ref. [13] by $\cosh(q) = \sqrt{1+m}\,\cosh(r)$.

[2]In the dimensional-reduction interpretation, this would correspond to a spherical shell of collapsing matter which, before reaching the Schwarzschild radius, bounces and expands due to self-interactions.

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