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Cosmic censorship in two-dimensional gravity

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A weak version of the cosmic censorship hypothesis is implemented as a set of boundary conditions on exact semiclassical solutions of two-dimensional dilaton gravity. These boundary conditions reflect low-energy matter from the strong coupling region and they also serve to stabilize the vacuum of the theory against decay into negative energy states. Information about low-energy incoming matter can be recovered in the final state but at high energy black holes are formed and inevitably lead to information loss at the semiclassical level.

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I. INTRODUCTION

Black hole physics provides a setting for the study of the interplay between general relativity and quantum mechanics. In particular, it appears difficult to reconcile the apparently thermal evaporation of a black hole formed in gravitational collapse with the Hamiltonian evolution of pure quantum-mechanical states [1,2]. In recent months considerable effort has been put into developing a semiclassical description of black hole evolution in two-dimensional dilation gravity coupled to conformal matter [3-16]. This simplified context shares important features with more realistic four-dimensional black hole physics. The original model proposed by Callan, Giddings, Harvey, and Strominger [3] (CGHS) has singular classical solutions, which describe the formation of a black hole by incoming matter, and the Hawking emission from this background geometry can be obtained from the conformal anomaly of the matter fields [17,3]. CGHS further suggested a semiclassical description of the back reaction of the Hawking radiation on the geometry by introducing anomaly-induced terms into the equations of motion of the model.

The quantum corrected CGHS equations have not been solved in closed form[1] but soon after the original work of CGHS it was shown that gravitational collapse always leads to a curvature singularity at a certain critical value of the dilaton field in their theory [4,5]. Modifications of the model have since been found, where the formation of a singularity in collapse can be avoided [9]. Furthermore, an improved treatment of the quantum theory [10-15] has led to models where the semiclassical equations can be solved exactly, as first exhibited by Bilal and Callan [11] and de Alwis [12].

In this paper we will argue that having a critical value of the dilaton, where a singularity forms in collapse, is really a blessing in disguise, and may prove essential for a consistent formulation of a quantum theory of two-dimensional black holes. The possibility that no singularity forms in gravitational collapse is appealing but unfortunately all solvable two-dimensional models suggested to date which exhibit this feature suffer from serious instabilities [11,12,14].

In a previous paper [13] we proposed a particular solvable model which has some desirable features built into it. The linear dilaton vacuum is a solution. Furthermore, all static solutions with negative Arnowitt-Deser-Misner (ADM) energy have naked singularities. We were able to follow the evolution of a black hole, formed by the gravitational collapse of a shock wave, using an exact solution of the semiclassical equations. As expected, the collapsing matter forms a spacelike singularity at a critical value of the dilaton field inside an apparent horizon. As the black hole evaporates the apparent horizon recedes and after a finite proper time it meets the singularity. At that point the singularity is no longer cloaked by the horizon and the future evolution of the geometry is not uniquely determined. In [13] we showed that there exist boundary conditions which match the final state of the black hole evolution onto the linear dilaton vacuum. With this choice of boundary conditions the geometry is nonsingular after the black hole evaporation is complete.

Here we will study this model further. A two-dimensional version of the cosmic censorship hypothesis [19] will play a central role in our considerations. At the end point of Hawking evaporation of a black hole, when the apparent horizon meets the singularity, a region of strong curvature becomes visible to outside observers. This constitutes a violation of the cosmic censorship hypothesis, but a fairly mild one, with the naked singularity being an isolated event.[2] If we choose other boundary

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[1]Numerical results on black hole evolution in the CGHS model have been obtained in [8,16,18].

[2]We thank A. Strominger for discussions on this point.

conditions, which do not match the black hole solution onto the vacuum when the evaporation is completed, the black hole singularity becomes timelike at that point and persists in the future geometry. We will, however, insist that physical configurations do not develop such extended naked singularities and use this requirement to determine physical boundary conditions in the strong coupling region. In order words, we will implement a weak form of the cosmic censorship hypothesis which states that curvature singularities in our semiclassical geometries must be hidden behind an apparent horizon, except for isolated events such as the end point of Hawking evaporation of a black hole.

We will take the line $\phi=\phi_{\rm cr}$ to be a boundary of spacetime in the semiclassical theory.[3] In "low-energy physics" where the incoming energy flux is below a certain threshold value (which equals the rate at which a black hole evaporates[4]) no black holes are formed in scattering processes [13]. In this case the critical line remains timelike and we have to supply boundary conditions for the fields there. We will show how requiring the curvature to be finite at $\phi=\phi_{\rm cr}$, or in other words not allowing a naked singularity there, leads to the reflecting boundary conditions we suggested in our previous paper. The fact that such nonsingular boundary conditions can be found, and their physical interpretation in terms of reflecting energy, support our view that the spacetime has a boundary at $\phi=\phi_{\rm cr}$. This timelike boundary is not present in the classical theory and as a result incoming matter cannot be turned away classically from the strong coupling region regardless of how little energy it carries. The quantum corrections spontaneously generate a boundary so that low-energy matter is reflected in the semiclassical theory and a black hole is only formed if the incoming matter carries a certain minimum energy density [13]. At the same time the boundary conditions stabilize the theory against decay into negative energy configurations by implementing cosmic censorship at the critical line $\phi=\phi_{\rm cr}$.

In the following section we write down some key definitions in order to establish notation and make contact with previous work. We also point out that the functional integral over all the fields can be carried out explicitly in some models where no spacetime boundary is present. Such theories thus appear too simple to describe the physics of two-dimensional quantum gravity. However, the quantum theory becomes nontrivial with boundaries present, especially since in this case not only the two-dimensional spacetime but also the field space of the theory is bounded at the semiclassical level. In Sec. III, we consider semiclassical solutions describing general incoming matter distributions with energy flux below the threshold required to form black holes. We use cosmic censorship to derive boundary conditions at $\phi=\phi_{\rm cr}$ for this low-energy physics and then we verify that these boundary conditions are consistent with energy conservation.

[3]It is useful to keep in mind an analogy with the dimensional reduction to radial degrees of freedom of higher-dimensional gravity. In the effective two-dimensional theory spacetime has a boundary at the origin of the radial coordinate.

[4]The evaporation rate of two-dimensional black holes is independent of their mass [20,3].

II. SEMICLASSICAL MODEL

We will work with the two-dimensional dilaton gravity model we introduced in [13]. It is related to models studied by Bilal and Callan [11] and de Alwis [12] and reduces to the original CGHS theory at the classical level. The semiclassical equations are derived from the one-loop effective action

$$S=\frac{1}{\pi}\int d^2x\left[e^{-2\phi}(2\partial_+\partial_-\rho-4\partial_+\phi\partial_-\phi+\lambda^2e^{2\rho})+\frac{1}{2}\sum_{i=1}^{N}\partial_+f_i\partial_-f_i-\kappa(\partial_+\rho\partial_-\rho+\phi\partial_+\partial_-\rho)\right], \tag{2.1}$$

written here in conformal gauge, $g_{++}=g_{--}=0$, $g_{+-}=-\frac{1}{2}e^{2\rho}$. If the coefficient κ in front of the one-loop quantum correction terms has the value $\kappa=(N-24)/12$ this action defines a conformal field theory and is therefore one-loop finite [10–12]. In everything that follows we will assume that the number of matter fields is $N>24$, so that κ is positive. The first quantum correction term in (2.1) comes from the one-loop conformal anomaly and the second one is a covariant, local counterterm which we are free to add to the definition of our model. The classical theory has a symmetry generated by the current

$$j^\mu=\partial^\mu(\phi-\rho)\ . \tag{2.2}$$

The exactly soluble semiclassical models all have a corresponding symmetry but if our counterterm is added the current maintains the same simple form in terms of ρ and ϕ at the semiclassical level. By defining our model this way we simplify the analysis and interpretation of the semiclassical solutions considerably. For example, the linear dilaton solution of the classical theory is preserved in our model.

In addition to the equations derived from (2.1) we have to impose as constraints the equations of motion of the metric components which are set to zero in this gauge:

$$0=T_{\pm\pm}=\left[e^{-2\phi}+\frac{\kappa}{4}\right](4\partial_\pm\rho\partial_\pm\phi-2\partial_\pm^2\phi)+\frac{1}{2}\sum_{i=1}^{N}\partial_\pm f_i\partial_\pm f_i-\kappa(\partial_\pm\rho\partial_\pm\rho-\partial_\pm^2\rho+t_\pm)\ . \tag{2.3}$$

The nonlocal character of the conformal anomaly is expressed in the functions $t_\pm(x^\pm)$ which are to be fixed by physical boundary conditions on the matter energy-momentum tensor.

The action simplifies dramatically if we make the field redefinitions

$$\Omega = \frac{\sqrt{\kappa}}{2}\phi + \frac{e^{-2\phi}}{\sqrt{\kappa}} ,$$
$$\chi = \sqrt{\kappa}\rho - \frac{\sqrt{\kappa}}{2}\phi + \frac{e^{-2\phi}}{\sqrt{\kappa}} . \tag{2.4}$$

In terms of the new variables the action (2.1) becomes

$$S = \frac{1}{\pi}\int d^2x \left[-\partial_+\chi\partial_-\chi + \partial_+\Omega\partial_-\Omega + \lambda^2 \exp\left[\frac{2}{\sqrt{\kappa}}(\chi-\Omega)\right] + \tfrac{1}{2}\sum_{i=1}^{N}\partial_+ f_i \partial_- f_i \right] , \tag{2.5}$$

and the constraints (2.3) reduce to

$$\kappa t_\pm = -\partial_\pm\chi\partial_\pm\chi + \sqrt{\kappa}\partial_\pm^2\chi + \partial_\pm\Omega\partial_\pm\Omega + \tfrac{1}{2}\sum_{i=1}^{N}\partial_\pm f_i \partial_\pm f_i . \tag{2.6}$$

In the following section we will consider semiclassical solutions of this system.

We conclude this section with some observations about the quantum theory defined by the action (2.5). The apparent similarity to Liouville theory is somewhat deceptive because the path integral over Ω and χ can, in fact, be carried out explicitly, leading to an effective action which turns out to be identical to the classical action (2.5). To see this we first rewrite the action in terms of "light-cone" variables in field space:

$$S[\Omega_+,\Omega_-] = \frac{1}{\pi}\int d^2x \left[-\partial_+\Omega_+\partial_-\Omega_- + \lambda^2 \exp\left[\frac{2}{\sqrt{\kappa}}\Omega_-\right]\right] , \tag{2.7}$$

where $\Omega_\pm = \chi \pm \Omega$ and we have dropped the matter term from the action for the time being. Now we couple the new fields to sources and consider the generating functional

$$e^{iW[j^+,j^-]} = \mathcal{N}\int [d\Omega_+][d\Omega_-] \exp\left[iS[\Omega_+,\Omega_-] + i\int d^2x(j^+\Omega_+ + j^-\Omega_-)\right] , \tag{2.8}$$

where $\mathcal{N}$ is a normalization constant. Only Ω_- appears in the interaction term in the action (2.7) so we can carry out the functional integral over Ω_+. The result is a formal δ function involving Ω_- which means that we can also perform the second functional integral in (2.8). The connected generating functional is

$$W[j^+,j^-] = \int d^2x \left[\frac{\lambda^2}{\pi}\exp\left[-\frac{2\pi}{\sqrt{\kappa}}\int d^2x' g(x,x')j^+(x')\right] - \pi\int d^2x' j^-(x) g(x,x') j^+(x')\right] , \tag{2.9}$$

where the Green's function is a solution of

$$\frac{\partial}{\partial x^+}\frac{\partial}{\partial x^-} g(x,x') = \delta^{(2)}(x-x') .$$

The final step is to obtain the effective action via a Legendre transform

$$\Gamma[\Omega_+^{\rm cl},\Omega_-^{\rm cl}] = W[j^+,j^-] - \int d^2x(j^+\Omega_+^{\rm cl} + j^-\Omega_-^{\rm cl})$$
$$= \frac{1}{\pi}\int d^2x \left[-\partial_+\Omega_+^{\rm cl}\partial_-\Omega_-^{\rm cl} + \lambda^2\exp\left[\frac{2}{\sqrt{\kappa}}\Omega_-^{\rm cl}\right]\right] , \tag{2.10}$$

where we have used that

$$\Omega_-^{\rm cl}(x) = \frac{\delta W}{\delta j^-(x)} = -\pi\int d^2x' g(x,x') j^+(x') .$$

This formal argument is, of course, not sufficient by itself to prove that the action (2.5) receives no quantum corrections. We have to introduce regularization, choose boundary conditions on propagators, etc., to make these expressions well defined. The point, however, is simply that the kinetic terms of Ω and χ in (2.5) have opposite signs while these fields appear in a symmetric fashion in the interaction term. As a result one finds a lot of cancellation in a diagrammatic perturbation expansion. If the Ω and χ propagators are defined using identical regularization and boundary conditions they will differ only by a sign and any two diagrams which differ only by a single internal propagator will exactly cancel. This means in particular that all loop diagrams will cancel one on one and the full effective action will be generated by tree graphs as suggested by (2.10).

A key assumption in the above argument is that the Ω and χ propagators satisfy the same boundary conditions. In our semiclassical theory the critical line $\phi=\phi_{\rm cr}$ is a spacetime boundary. Quantum consistency conditions may require introducing nontrivial boundary interactions there, or even new degrees of freedom, and these will, in general, not preserve the symmetry between Ω and χ. Note also that Ω in (2.4) is bounded from below in our semiclassical theory. It takes its minimum value at $\phi=\phi_{\rm cr}$. Restricting the range of Ω in the quantum path integral leads to nontrivial physical effects. Away from the boundary quantum fluctuations can cause the dilaton field to reach its critical value in some region which is then no longer part of the spacetime. The full quantum theory will then include configurations with disconnected boundary components in the path integral. Topology change of this kind would be strongly suppressed in the weak coupling region where asymptotic observers are located but it may play an important role in the strong coupling physics near the boundary.

In some models with $N<24$ matter fields the singularity at $\phi=\phi_{\rm cr}$ is absent and the issue of restricting the range of Ω does not arise [9,11,12]. In this case boundary conditions are imposed on propagators in the asymptotic regions of weak and strong couplings. If they are chosen so as to preserve symmetry between Ω and χ, the quantum theory will be very simple indeed, as we argued above.

III. LOW-ENERGY PHYSICS, BOUNDARY CONDITIONS, AND COSMIC CENSORSHIP

The semiclassical equations of motion derived from the effective action (2.5) are

$$\partial_+\partial_-\chi=\partial_+\partial_-\Omega=-\frac{\lambda^2}{\sqrt{\kappa}}\exp\left[\frac{2}{\sqrt{\kappa}}(\chi-\Omega)\right]. \quad (3.1)$$

This set of equations can be explicitly integrated [11,12] and we will focus our attention on solutions which describe the response to a general distribution of incident matter. It is easily checked that the linear dilaton vacuum, which takes the form $e^{-2\phi}=e^{-2\rho}=-\lambda^2x^+x^-$ in "Kruskal" coordinates, solves the equations of motion and we will match solutions with incident matter onto this vacuum in the far past.

The classical energy flux carried by an arbitrary distribution of incoming matter is described by some $T^f_{++}(x^+)=\frac{1}{2}\partial_+f\partial_+f$ at $x^-\to-\infty$. For convenience we will assume that the matter arrives over a finite span of time so that $T^f_{++}(x^+)$ vanishes for $x^+<x_0^+$ and $x^+>x_1^+$ (see Fig. 1). The limits $x_0^+\to 0$ and $x_1^+\to\infty$ can be taken at the end of the day if desired.

At a given value of x^+ one can define the integrated incoming energy and Kruskal momentum up to that point,

$$\begin{aligned} M(x^+)&=\lambda\int_0^{x^+}dx^+x^+T^f_{++}(x^+)\,,\\ P_+(x^+)&=\int_0^{x^+}dx^+T^f_{++}(x^+)\,, \end{aligned} \quad (3.2)$$

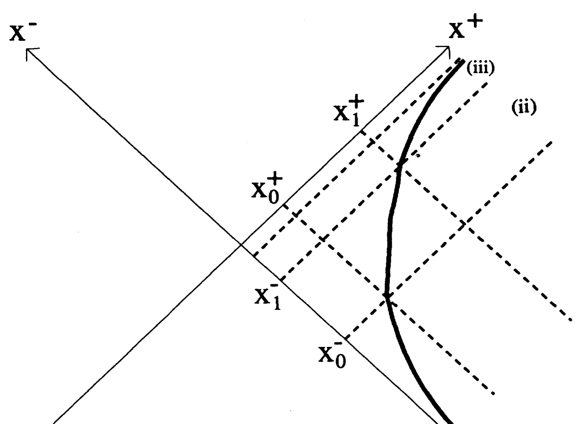

FIG. 1. Kruskal diagram of a semiclassical geometry with low-energy matter incident on the linear dilaton vacuum. The thick line is the spacetime boundary at $\phi=\phi_{\rm cr}$.

and it turns out that the incoming energy flux only enters into the semiclassical solution through these two functions. It is easy to verify that the equations of motion (3.1) and the constraints (2.6) are satisfied by

$$\Omega=\chi=-\frac{\lambda^2}{\sqrt{\kappa}}x^+\left[x^-+\frac{1}{\lambda^2}P_+(x^+)\right]+\frac{M(x^+)}{\sqrt{\kappa}\lambda}-\frac{\sqrt{\kappa}}{4}\ln(-\lambda^2x^+x^-)\,. \quad (3.3)$$

This solution is valid for all $x^-<x_0^-$ [region (i) in Fig. 1] and it reduces to the linear dilaton vacuum for $x^+<x_0^+$.

At $x^\pm=x_0^\pm$ the leading edge of the incoming matter distribution reaches the boundary at $\phi=\phi_{\rm cr}$ and the evolution of the system after that depends on the boundary conditions imposed there. There are two cases to consider. If the energy flux of the incoming matter is always smaller than the rate at which a black hole evaporates then the critical line remains timelike for $x^+>x_0^+$ [13]. This requires the inequality

$$T^f_{++}(x^+)<\frac{\kappa}{4x^{+2}} \quad (3.4)$$

to hold for all values of x^+ in Kruskal coordinates. If, on the other hand, this inequality is violated at some value of x^+ then the critical line will become spacelike there and no meaningful boundary conditions can be applied. In this case an apparent horizon will form to cloak the singularity and we have a black hole. Once the incoming flux falls below the threshold value, the apparent horizon begins to recede and, as we discussed in our previous paper [13], it will meet the singularity in a finite proper time. When the evaporation of the black hole is complete the critical line goes timelike again and we must impose boundary conditions there.

For our discussion of boundary conditions we will assume that the incoming energy flux remains below threshold at all times. This condition defines a low-energy sector of the theory without real black holes (virtual black holes will presumably appear at the quantum level) and it is of considerable interest to study the quantum theory of this sector by itself.[5] For example, it would be very interesting to determine whether quantum coherence loss occurs at a nonvanishing rate in low-energy scattering as conjectured by Hawking [2].

We will derive our boundary conditions from the cosmic censorship hypothesis as discussed in the Introduction. Imposing finite curvature at the boundary is a coordinate-invariant condition. In this section we work exclusively in Kruskal coordinates. This allows us to use special relations, such as $\Omega=\chi$, to simplify calculations but at the same time some of our formulas will appear noncovariant.

[5]A classical shock wave with $T^f_{++}(x^+)=(m/\lambda x_0^+)\delta(x^+-x_0^+)$ forms a black hole for arbitrarily small m. This does not conflict with the existence of a low-energy sector because such a concentrated flux violates the uncertainty principle and is therefore not a good description of a low-energy state.

The curvature can be expressed in terms of ϕ as

$$R=-2\nabla^2\rho = \frac{4}{1-(\kappa/4)e^{2\phi}}[\lambda^2-(\nabla\phi)^2]\ , \tag{3.5}$$

where we have used the semiclassical equations of motion. The critical line is defined as the curve of constant ϕ where

$$\Omega'(\phi)=\frac{2}{\sqrt{\kappa}}e^{-2\phi}\left[1-\frac{\kappa}{4}e^{2\phi}\right]=0$$

and the curvature will only be finite at the boundary if the dilaton satisfies

$$\nabla_n\phi=\lambda \tag{3.6}$$

there, where ∇_n denotes the invariant normal derivative at the boundary. Since $\Omega'(\phi)=0$ at the critical line it follows that requiring

$$\partial_+\Omega|_{\phi=\phi_{\rm cr}}=\partial_-\Omega|_{\phi=\phi_{\rm cr}}=0 \tag{3.7}$$

on the boundary where it is timelike is a necessary condition for finite curvature there. As we will see below, this condition uniquely determines the solution in regions (ii) and (iii) in Fig. 1. At the end of this section we will show that the resulting solution indeed has finite curvature on the boundary which means that (3.7) is also a sufficient condition when we work in Kruskal coordinates.

Let us look for a solution in region (ii) in Fig. 1 which matches continuously onto solution (3.3) which holds in region (i). The form of the new solution can only differ from (3.3) by some function of x^- alone,

$$\Omega^{(\rm ii)}(x^+,x^-)=\Omega^{(\rm i)}(x^+,x^-)+F(x^-)\ , \tag{3.8}$$

because otherwise the $++$ constraints would no longer be satisfied. The superscripts on Ω refer to the region of validity in Fig. 1. The finite curvature conditions (3.7) are sufficient to determine both $F(x^-)$ and the shape of the boundary curve $(\hat{x}^+,\hat{x}^-)$ in terms of the incoming matter distribution. For solution (3.8) these conditions imply the two relations

$$\begin{aligned}\frac{\kappa}{4}&=-\lambda^2\hat{x}^+\left[\hat{x}^-+\frac{1}{\lambda^2}P_+(\hat{x}^+)\right]\ ,\\ \sqrt{\kappa}F'(\hat{x}^-)&=\lambda^2\hat{x}^++\frac{\kappa}{4\hat{x}^-}\ .\end{aligned} \tag{3.9}$$

The first one defines the critical line and the second one can be integrated to obtain the matching function:

$$F(x^-)=\frac{\sqrt{\kappa}}{4}\ln(-\lambda^2x^-x^+)-\frac{M(\hat{x}^+)}{\sqrt{\kappa}\lambda}-\frac{\sqrt{\kappa}}{4}\ln\left[\frac{\kappa}{4}\right]\ . \tag{3.10}$$

In this way cosmic censorship determines a unique extension of the solution into region (ii). There is no discontinuity in $\partial_-\Omega$ at $x^-=x_0^-$ so the $--$ constraints in (2.6) are satisfied across the matching line and there is no shock wave carrying energy out along this null line.

By assumption the matter stops coming in at $x^+=x_1^+$ so $M(x^+)$ and $P_+(x^+)$ receive no contribution after that. Solution (3.8) extends smoothly into region (iii) where it takes the form of a linear dilaton configuration with x^- shifted by the total incident Kruskal momentum,

$$\Omega^{(\rm iii)}(x^+,x^-)=-\frac{\lambda^2}{\sqrt{\kappa}}x^+\left[x^-+\frac{1}{\lambda^2}P_+(x_1^+)\right]-\frac{\sqrt{\kappa}}{4}\ln\left[-\lambda^2x^+\left[x^-+\frac{1}{\lambda^2}P_+(x_1^+)\right]\right]\ , \tag{3.11}$$

and there is no shock wave propagating out along $x^-=x_1^-$.

We will now give the boundary conditions implied by the cosmic censorship relations (3.7) a physical interpretation in terms of reflected energy. In Kruskal coordinates the constraints (2.6) reduce to

$$0=\sqrt{\kappa}\partial_\pm^2\chi+\tfrac{1}{2}\sum_{i=1}^{N}\partial_\pm f_i\partial_\pm f_i-\kappa t_\pm\ . \tag{3.12}$$

Let us evaluate $\partial_\pm^2\chi$ at the boundary:

$$\begin{aligned}\partial_+^2\chi(\hat{x})&=-\frac{1}{\sqrt{\kappa}}P'_+(\hat{x}^+)+\frac{\sqrt{\kappa}}{4(\hat{x}^+)^2}\ ,\\ \partial_-^2\chi(\hat{x})&=F''(\hat{x}^-)+\frac{\sqrt{\kappa}}{4(\hat{x}^-)^2}\ .\end{aligned} \tag{3.13}$$

This expression holds everywhere if we define $F(x^-)=0$ in region (i). Differentiating the first finite curvature relation in (3.7) with respect to $\hat{x}^+$ and the second one with respect to $\hat{x}^-$ leads to the relations

$$\begin{aligned}\frac{\lambda^2}{\sqrt{\kappa}}\frac{d\hat{x}^-}{d\hat{x}^+}&=-\frac{1}{\sqrt{\kappa}}P'_+(\hat{x}^+)+\frac{\sqrt{\kappa}}{4(\hat{x}^+)^2}\ ,\\ \frac{\lambda^2}{\sqrt{\kappa}}\frac{d\hat{x}^+}{d\hat{x}^-}&=F''(\hat{x}^-)+\frac{\sqrt{\kappa}}{4(\hat{x}^-)^2}\ .\end{aligned} \tag{3.14}$$

By combining this with (3.13) and using the constraints we obtain a reflection condition

$$\mathcal{T}_{--}=\left[\frac{d\hat{x}^+}{d\hat{x}^-}\right]^2\mathcal{T}_{++} \tag{3.15}$$

on the combination

$$\mathcal{T}_{\pm\pm}=\tfrac{1}{2}\sum_{i=1}^{N}\partial_\pm f_i\partial_\pm f_i-\kappa t_\pm\ . \tag{3.16}$$

Let us assume that the physical incoming energy is in the form of coherent radiation of matter fields. We then expect the outgoing radiation to consist of a coherent part and an incoherent one due to the anomaly. The reflection conditions (3.15) obtained from the cosmic censorship hypothesis do not separate the two contributions and therefore they do not supply us with unambiguous boundary conditions for the matter fields. However, it appears to be consistent with (3.15) to impose reflecting boundary conditions on the matter, for example,

$$f_i(\hat{x})=0 \ . \tag{3.17}$$

In this case the classical matter energy-momentum tensor by itself would satisfy the reflection condition and it would follow from (3.15) that

$$t_-(x^-)=\left[\frac{d\hat{x}^+}{d\hat{x}^-}\right]^2 t_+[\hat{x}^+(x^-)] \ . \tag{3.18}$$

As $x^- \to -\infty$ the semiclassical solution for Ω should approach the corresponding classical solution with the same incoming energy distribution. This determines $t_+ = 1/4x^{+2}$ and for a given timelike boundary curve (3.18) would give a unique t_- describing the anomalous component of the outgoing radiation.

Given this strong form of the reflecting conditions a distant observer would be able to completely reconstruct the initial state from the outgoing radiation and no information would be lost in low-energy physics at the semiclassical level. Quantum fluctuations could still cause the boundary curve to go spacelike and lead to information loss. It should be stressed that while these strong reflection conditions, imposed directly on the matter fields, are consistent with the cosmic censorship hypothesis, they do not follow from it and it is possible that they are not the appropriate boundary conditions at $\phi=\phi_{\rm cr}$.

Returning to the weaker form of the reflecting conditions (3.15) we can check their consistency by computing the total energy radiated out to $x^+ \to \infty$. First of all, there should be no outgoing radiation in region (iii) in Fig. 1 where the solution is vacuumlike. Evaluating $\mathcal{T}_{--}$ there gives

$$\mathcal{T}^{\rm (iii)}_{--} = -\frac{\kappa}{4}\frac{1}{[x^- + (1/\lambda^2)P_+(x_1^+)]^2} \ . \tag{3.19}$$

This vacuum contribution must be subtracted from $\mathcal{T}_{--}$ to obtain the outgoing energy in a given region and the total radiated energy is

$$E_{\rm out} = -\lambda\int_{-\infty}^{x_1^-} dx^- \left[x^- + \frac{1}{\lambda^2}P_+(x_1^+)\right] \times\left[\mathcal{T}_{--} + \frac{\kappa}{4}\frac{1}{[x^- + (1/\lambda^2)P_+(x_1^+)]^2}\right] . \tag{3.20}$$

The weight factor of $-[x^- + (1/\lambda^2)P_+(x_1^+)]$ appears because we are not using asymptotically Minkowskian coordinates. A straightforward calculation shows that $E_{\rm out}$ precisely equals the total incoming energy $M(x_1^+)$ so our boundary conditions appear to conserve energy.

Energy conservation can also be checked using a definition of total energy in terms of the asymptotic curvature, analogous to the one introduced in [8]:

$$m(x^-) = \lim_{x^+ \to \infty} \frac{1}{4\lambda}\left[1 - \frac{\kappa}{4}e^{2\phi}\right] e^{-2\phi}R \ . \tag{3.21}$$

This definition gives the correct mass for classical black hole solutions and tends to the total incoming energy as $x^- \to -\infty$. The formulas below are streamlined by including the factor of $[1-(\kappa/4)e^{2\phi}]$ in the definition of the energy. It can be added at no cost since it goes to 1 in the limit $x^+ \to \infty$. Using the semiclassical equations of motion for ϕ and ρ the following expression is obtained for the rate of change of the energy:

$$\begin{aligned}\frac{dm}{dx^-} &= \lim_{x^+ \to \infty} \frac{2}{\lambda}e^{-2\rho}\partial_+\phi\left[e^{-2\phi} - \frac{\kappa}{4}\right](2\partial_-^2\phi - 4\partial_-\rho\partial_-\phi)\\ &= \lim_{x^+ \to \infty} \frac{2}{\lambda}e^{-2\rho}\partial_+\phi[\mathcal{T}_{--} + \kappa(\partial_-\rho\partial_-\rho - \partial_-^2\rho)] \ ,\end{aligned} \tag{3.22}$$

where we used the constraints (2.3) to obtain the second equality. The term accompanying $\mathcal{T}_{--}$ inside the square brackets subtracts off the same vacuum contribution as in (3.20). By inserting the explicit semiclassical solution and integrating over x^- one can easily check that $m(x^-)$ goes to zero as we enter region (iii).

Since conditions (3.7), which we used to determine the solution in region (ii), appear to be weaker than (3.6), which is expressed in terms of the original dilaton field, one might worry that we have not guaranteed finite curvature at the boundary. However, the semiclassical geometry, which results from imposing (3.7), is, in fact, nonsingular as the critical line is approached. To see this, we consider a point near the boundary, $(x^+,x^-)=(\hat{x}^+ + \delta x^+, \hat{x}^- + \delta x^-)$, and evaluate the curvature in the limit of vanishing $\delta x^\pm$. We need to show that the expression in the square brackets in (3.5) is of order $\delta x^\pm$. In order to obtain $\partial_\pm\phi(\hat{x})$ we expand Ω around $\Omega(\phi_{\rm cr})$ in two different ways:

$$\begin{aligned}\Omega(\phi) &= \Omega(\phi_{\rm cr}) + \tfrac{1}{2}\Omega''(\phi_{\rm cr})(\phi-\phi_{\rm cr})^2 + \cdots\\ &= \frac{\sqrt{\kappa}}{4}\left[1 - \ln\frac{\kappa}{4}\right] + \frac{\sqrt{\kappa}}{2}(\phi-\phi_{\rm cr})^2 + \cdots \ ,\end{aligned} \tag{3.23}$$

and

$$\begin{aligned}\Omega(x^+,x^-) &= \Omega(\hat{x}) + \tfrac{1}{2}\partial_+^2\Omega(\hat{x})\delta x^{+2} + \partial_+\partial_-\Omega(\hat{x})\delta x^+\delta x^-\\ &\quad + \tfrac{1}{2}\partial_-^2\Omega(\hat{x})\delta x^{-2} + \cdots \ .\end{aligned} \tag{3.24}$$

Comparing (3.13) and (3.14) leads us to write

$$\begin{aligned}\partial_+^2\Omega(\hat{x}) &= \frac{\lambda^2}{\sqrt{\kappa}}\frac{d\hat{x}^-}{d\hat{x}^+} \ ,\\ \partial_-^2\Omega(\hat{x}) &= \frac{\lambda^2}{\sqrt{\kappa}}\frac{d\hat{x}^+}{d\hat{x}^-} \ ,\end{aligned} \tag{3.25}$$

where we have used that $\Omega=\chi$ in Kruskal coordinates. Inserting these relations, along with the equation of motion, $\partial_+\partial_-\Omega = -\lambda^2/\sqrt{\kappa}$, into (3.24) and comparing with (3.23) gives the following expression for ϕ in terms of $\delta x^\pm$:

$$\begin{aligned}\phi - \phi_{\rm cr} &= \frac{\lambda}{\sqrt{\kappa}}[\sqrt{d\hat{x}^+/d\hat{x}^-}dx^-\\ &\quad - \sqrt{d\hat{x}^-/d\hat{x}^+}\delta x^+] + \cdots \ .\end{aligned} \tag{3.26}$$

From this we can read off the values of $\partial_+\phi(\hat{x})$ and $\partial_-\phi(\hat{x})$ and insert them into (3.5) to see that our solution has finite curvature at $\phi=\phi_{\rm cr}$.

Finally it is interesting to note that the solution for Ω can be expressed in terms of the boundary curve in a surprisingly simple manner:

$$\begin{aligned}\Omega(x^+,x^-)-\Omega_{\rm cr}&=\frac{\lambda^2}{\sqrt{\kappa}}\left[-x^+x^-+\hat{x}^+(x^-)x^-\right.\\&\qquad\left.+\int_{\hat{x}^+(x^-)}^{x^+}du^+\hat{x}^-(u^+)\right]\\&=\frac{\lambda^2}{\sqrt{\kappa}}\left[-x^+x^-+x^+\hat{x}^-(x^+)\right.\\&\qquad\left.+\int_{\hat{x}^-(x^+)}^{x^-}du^-\hat{x}^+(u^-)\right].\end{aligned} \tag{3.27}$$

These relations can be obtained by integrating the expressions for $\partial_+^2\Omega(x)$ and $\partial_-^2\Omega(x)$ given above.

IV. DISCUSSION

We have shown how the cosmic censorship hypothesis leads to reflecting boundary conditions for matter energy. The matter carrying this reflected energy consists of both coherent and incoherent f fields and cosmic censorship alone does not allow us to distinguish between the two components. It is nevertheless strong enough to uniquely determine the semiclassical evolution of the geometry and dilaton field for a given distribution of low-energy incoming matter and we have checked that energy is conserved in that evolution. Other boundary conditions which do not rule out extended naked singularities presumably lead to instability. They could allow a black hole to evolve into an object carrying an arbitrary amount of negative energy and it could then continue to radiate forever happily. If we implement boundary conditions consistent with cosmic censorship an evaporating black hole returns to the vacuum configuration after a finite proper time and the Hawking emission stops [13].

The expression for the curvature (3.5) takes a simple form on the apparent horizon of a black hole:

$$R\,|_{\partial_+\phi=0}=\frac{4\lambda^2}{1-(\kappa/4)e^{2\phi}}\ . \tag{4.1}$$

From this it is clear that the curvature on the apparent horizon diverges as it approaches the singularity curve and therefore cosmic censorship is violated at the end point of the evaporation process. If the boundary conditions we have advocated here are adopted, the violation of cosmic censorship is minimal in that the naked singularity is an isolated event.

At the semiclassical level it appears that information loss can be avoided in the low-energy sector of the theory by imposing reflecting boundary conditions directly on the matter fields. This is a desirable feature of any model of real low-energy physics. If, on the other hand, the incoming energy flux is above the threshold for forming a black hole we see no way to recover any information about the initial state, except its total energy, in this semiclassical theory. It remains an interesting open question whether quantum fluctuations cause the boundary to go spacelike even in the low-energy sector and whether this leads to disastrous loss of quantum coherence [21].

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