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Renormalization of gauge-invariant operators and the axial anomaly

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The renormalization properties of gauge-invariant composite operators that vanish when the classical equations of motion are used (class II^a operators) and which lead to diagrams where the Adler-Bell-Jackiw anomaly occurs are discussed. It is shown that gauge-invariant operators of this kind do need, in general, nonvanishing gauge-invariant (class I) counterterms.

I. INTRODUCTION

Composite operators have long been used in field theories, mainly in deep-inelastic analysis, through Wilson's operator-product expansion.[1] With the appearance of the Shifman-Vainshtein-Zakharov sum rules[2] as a useful way to extend the domain of applicability of quantum chromodynamics to low-energy phenomenology, the renormalization properties of gauge-independent composite operators have been the object of renewed interest.[3,4]

Composite operators can be divided into three classes. Class I contains those operators which are invariant under classical gauge transformations and which do not vanish when one uses the covariant equations of motion (we will qualify this later). The operators that, being formally gauge invariant, vanish by virtue of the equations of motion form class II^a. Finally, class II^b groups the non-gauge-invariant operators.[5]

In general, bare operators cannot be made finite by means of multiplicative renormalization, but rather a composite operator can mix with others of the same dimension and Lorentz structure along the renormalization procedure. The problem of mixing among composite operators has been dealt with by several authors[5–7] and, in the light of their work, the following properties are known.

(i) In ordinary covariant gauges class-I operators mix not only among themselves, but also with classes II^a and II^b. On the contrary, in the background-field gauge,[8] where gauge invariance is retained in the external (classical) field, class-I operators do not mix with class II^b.[5]

(ii) The submatrix concerning the renormalization of class-I operators among themselves, $Z_{\mathrm{I\,I}}$, is gauge independent. However, even in the background-field gauge, the contribution to a renormalized class-I operator coming from class-II operators, $Z_{\mathrm{I\,II}}$, is gauge dependent.

(iii) The renormalization of class-II operators among themselves, $Z_{\mathrm{II\,II}}$, involves gauge-dependent coefficients.

Finally, it is claimed by Kluberg-Stern and Zuber[5] and Deans and Dixon[7] that for the renormalization of class-II operators one does not need any class-I counterterm (i.e., $Z_{\mathrm{II\,I}}=0$). Nevertheless, this claim, being proven for class-II^b operators, relies on a formal manipulation of functional integrals that could be incorrect for some class-II^a operators as we will see. This is indeed the case when the Adler-Bell-Jackiw anomaly[9] plays a role.

II. GENERATING FUNCTIONAL

For simplicity's sake let us consider the Yang-Mills Lagrangian with massless fermions

$$\mathscr{L}=-\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a-\frac{1}{2a}(\partial^\mu A^a_\mu)^2+\partial_\mu\bar\varphi_a D^{\mu ab}\varphi_b + i\bar\psi^\alpha \not{D}_{\alpha\beta}\psi^\beta\,. \tag{1}$$

$\mathscr{L}$ is invariant under the Becchi-Rouet-Stora[10] (BRS) transformations

$$\begin{aligned}
&\Delta A^a_\mu = D^{ab}_\mu\varphi_b\delta\lambda\,,\\
&\Delta\bar\varphi^a=\frac{1}{a}\partial^\mu A^a_\mu\delta\lambda\,,\\
&\Delta\varphi^a=\tfrac{1}{2}gf^{abc}\varphi_b\varphi_c\delta\lambda\,,\\
&\Delta\psi=-gT^a\varphi^a\psi\delta\lambda\,,\\
&\Delta\bar\psi=-ig\bar\psi T^a\varphi^a\delta\lambda\,,
\end{aligned} \tag{2}$$

$\delta\lambda$ being an anticommuting infinitesimal parameter. We now consider the generating functional of complete Green's functions. We add several operators O_i coupled to the corresponding sources,

$$W[j,J,\bar J,\phi]=\int[dA][d\varphi][d\bar\varphi][d\psi][d\bar\psi]\exp\left[i\int d^4x(\mathscr{L}+\phi_iO_i+j^a_\mu A^\mu_a+\bar J_\alpha\psi^\alpha+\bar\psi_\alpha J^\alpha)\right]. \tag{3}$$

The action

$$S=\int d^4x\,\mathscr{L}(x)=\hat S-\frac{1}{2a}\int d^4x[\partial^\mu A^a_\mu(x)]^2 \tag{4}$$

satisfies

$$\Delta A^a_\mu\frac{\delta\hat S}{\delta A^a_\mu}+\Delta\varphi^a\frac{\delta\hat S}{\delta\varphi^a}+\Delta\bar\varphi^a\frac{\delta\hat S}{\delta\bar\varphi^a}+\Delta\psi^\alpha\frac{\delta\hat S}{\delta\psi^\alpha}+\Delta\bar\psi^\alpha\frac{\delta\hat S}{\delta\bar\psi^\alpha}=0\,. \tag{5}$$

By using the appropriate functional derivatives and then setting all sources equal to zero, we obtain the desired Green's functions with an insertion of a composite operator.

Beyond the tree level appropriate counterterms have to be added to S in order to keep all the Green's functions finite. Let us now suppose that we are, for the time being, concerned with the renormalization of gauge-invariant operators like $F^{\mu\nu}F_{\mu\nu}$ or $\bar\psi\psi$. These operators obey an equation of the type (5),

$$\left[-D^{ab}_{\mu}\varphi_b\frac{\delta}{\delta A^a_\mu}+\tfrac{1}{2}gf^{abc}\varphi_b\varphi_c\frac{\delta}{\delta\varphi^a}+\frac{1}{a}\partial^\mu A^a_\mu\frac{\delta}{\delta\bar\varphi^a}-ig(T^a\varphi^a\psi)_\alpha\frac{\delta}{\delta\psi_\alpha}-ig(\bar\psi T^a\varphi^a)_\alpha\frac{\delta}{\delta\bar\psi_\alpha}\right]\phi_iO_i\equiv W\phi_iO_i=0\ . \tag{6}$$

We have introduced a shorthand notation W for the functional-differential operator. Clearly $W^2=0$; i.e., W is nilpotent. Equation (6) is not only reflecting the invariance of the theory under BRS transformations, and thus the fulfillment of Ward-Slavnov identities, but also is providing restrictions on the form of the permitted counterterms. When renormalizing a gauge-invariant operator, one adds gauge-invariant counterterms, but, in general, one needs (because of the breakdown of gauge invariance by the gauge-fixing term) non-gauge-invariant counterterms as well, ϕ_nN_n, provided that they conspire to build a quantity still satisfying $W\phi_nN_n=0$. The simplest possibility is obviously to take $\phi_nN_n=W\phi_nF_n$, with no restriction on F_n, provided they have suitable dimension and Lorentz and color structure. In fact, it can be shown[6] that this is the general solution.

Then the most general action satisfying Ward-Slavnov identities can be written as

$$\int d^4x\left[\mathscr{L}+\phi_iO_i+\frac{\delta\hat S}{\delta A^a_\mu}\bigg|_{\varphi=0}\phi_\kappa C^a_{\mu k}+\phi_pC^\alpha_p\frac{\delta\hat S}{\delta\psi^\alpha}+\phi_q\bar C^{\alpha}_{q}\frac{\delta\hat S}{\delta\bar\psi^\alpha}+W\phi_nF_n\right], \tag{7}$$

where the separation in classes I, II^a, and II^b is clearly exhibited. Let us recall that any gauge-invariant composite operator vanishing when the classical equations of motion are used,

$$D^{ab}_\mu F^{\mu\nu}_b+g\bar\psi T^a\gamma^\nu\psi=\frac{\delta\hat S}{\delta A^a_\nu}\bigg|_{\varphi=0}=0\ ,$$
$$i(\vec{\not D}\psi)^\alpha=\frac{\delta\hat S}{\delta\bar\psi^\alpha}=0,\quad i(\psi\overleftarrow{\not D})^\alpha=\frac{\delta\hat S}{\delta\psi^\alpha}=0\ , \tag{8}$$

is of the form

$$(D^{ab}_\mu F^{\mu\nu}_b+g\bar\psi T^a\gamma^\nu\psi)C^a_\nu(A,\bar\psi,\psi) \tag{9}$$

or

$$i\bar C^\alpha\vec{\not D}_{\alpha\beta}\psi^\beta,\quad -i\bar\psi^\alpha\overleftarrow{\not D}_{\alpha\beta}C^\beta \tag{10}$$

with C^a_ν, $\bar C^\alpha$, C^α satisfying

$$WC^a_\nu=gf^{abc}\varphi^bC^c_\nu\ ,$$
$$W\bar C^\alpha=-ig\bar C^\alpha T^a\varphi^a\ , \tag{11}$$
$$WC^\alpha=-igT^a\varphi^aC^\alpha\ .$$

Using

$$\frac{\delta\hat S}{\delta A^a_\nu}=D^{ab}_\mu F^{\mu\nu}_b-gf^{abc}\partial^\nu\bar\varphi^b\varphi^c+g\bar\psi\gamma^\nu T^a\psi\ , \tag{12}$$

the generating functional (3) can be written as

$$W[j,J,\bar J\phi]=\int[dA][d\varphi][d\bar\varphi][d\psi][d\bar\psi]\exp\left[i\int d^4x\left[\mathscr{L}+\phi_iO_i+W\phi_nF_n+\phi_q\bar C^\alpha_q\frac{\delta\hat S}{\delta\bar\psi^\alpha}+\phi_pC^\alpha_p\frac{\delta\hat S}{\delta\psi_\alpha}+\frac{\delta\hat S}{\delta A^a_\mu}\phi_kC^a_{\mu k}+gf^{abc}\partial^\mu\bar\varphi^b\varphi^c\phi_kC^a_{\mu k}+j^a_\mu A^\mu_a+\bar J_\alpha\psi^\alpha+\bar\psi_\alpha J^\alpha\right]\right]\ . \tag{13}$$

Let us now perform the following change of variables:

$$A^a_\mu=A^a_\mu+\phi_kC^a_{\mu k},\quad \psi=\psi+\phi_pC_p,\quad \bar\psi=\bar\psi+\phi_q\bar C_q\ . \tag{14}$$

Equation (13) becomes (up to terms of higher order in the sources)

$$W[j,J,\bar J,\phi]=\int[dA][d\varphi][d\bar\varphi][d\psi][d\bar\psi]\mathscr{D}\exp\left[i\int d^4x\left[\mathscr{L}+\phi_iO_i+W\phi_nF_n+gf^{abc}\partial^\mu\bar\varphi^b\varphi^c\phi_kC^a_{\mu k}+\frac{1}{a}\partial^\mu A^a_\mu\cdot\partial^\lambda\phi_kC^a_{\lambda k}+j^\mu_a(A^a_\mu-\phi_kC^a_{\mu k})+\bar J_\alpha(\psi-\phi_pC_p)^\alpha+(\bar\psi-\phi_q\bar C_q)^\alpha J_\alpha\right]\right]\ . \tag{15}$$

Notice that the change (14) is easily done with field A^a_μ, since it is a local change of variables it has unit Jacobian.[11] However, this is not so simple with fermionic fields as we will see and we have introduced the Jacobian $\mathscr{D}$ of the transformation in Eq. (15).[12]

A further manipulation allows us to write Eq. (15) as

$$W[j,J,\bar{J},\phi]=\int [dA][d\varphi][d\bar{\varphi}][d\psi][d\bar{\psi}]\mathscr{D}\exp\left[i\int d^4x[\mathscr{L}+\phi_iO_i+W\phi_nF_n+W\phi_k(-\partial^\mu\bar{\varphi}^bC^b_{\mu k}) + j^\mu_a(A^a_\mu-\phi_kC^a_{\mu k})+\bar{J}^\alpha(\psi-\phi_pC_p)_\alpha+(\bar{\psi}-\phi_q\bar{C}_q)_\alpha J^\alpha]\right], \quad (16)$$

where we have used Eq. (11). Notice the two terms in Eq. (15) which have been integrated as $W\phi_k(-\partial^\mu\bar{\varphi}^bC^b_{\mu k})$.

Connected Green's functions with an insertion of a composite operator will be obtained from

$$Z[j,J,\bar{J},\phi]=-i\ln W[j,J,\bar{J},\phi] \quad (17)$$

while one-particle irreducible (1PI) Green's functions will be obtained after Legendre-transforming Eq. (17). When computing 1PI Green's functions, the insertions in the external legs which are generated by the new terms in (16) coupled to the field sources which appear once the change of variables (14) is performed are exactly canceled by the (modified) propagators, which will contain the new insertions too.

Clearly, had we neglected the Jacobian we would have entirely reabsorbed class-II[a] operators in the terms $W\phi_nF_n$, so that operators which vanish by virtue of the classical equations of motion are, in spite of their gauge invariance, generated from a gauge-dependent composite operator. When this is the case both class-II[a] and class-II[b] operators can be expressed in the form $W\phi_nF_n$ and thus the results of Kluberg-Stern and Zuber indeed hold. Let us summarize, for the sake of completeness, their argument.

Consider an insertion of WF_n in a Green's function. It is given by the functional derivatives of

$$\int [dA][d\varphi][d\bar{\varphi}][d\psi][d\bar{\psi}]WF_n\exp\left[i\int d^4x(\mathscr{L}+j^\mu_aA^a_\mu+\bar{J}^\alpha\psi_\alpha+\bar{\psi}^\alpha J_\alpha+\bar{C}^a\varphi_a+\bar{\varphi}^aC_a)\right]. \quad (18)$$

Using the invariance of the action under BRS transformations, Eq. (18) can be written as

$$\int [dA][d\varphi][d\bar{\varphi}][d\psi][d\bar{\psi}]\left[j^a_\mu\frac{\delta}{\delta J^a_\mu}+\bar{C}^a\frac{\delta}{\delta K^a}+\frac{1}{a}\partial^\mu C^a\frac{\delta}{\delta j^a_\mu}-\bar{J}_\alpha\frac{\delta}{\delta\bar{K}_\alpha}+J_\alpha\frac{\delta}{\delta K_\alpha}\right]$$
$$\times F_n\exp\left[i\int d^4x[\mathscr{L}+J^\mu_aD^{ab}_\mu\varphi_b+K_a\tfrac{1}{2}gf^{abc}\varphi_b\varphi_c+\bar{K}^\alpha(igT^a\varphi^a\psi_\alpha)+(ig\bar{\psi}T^a\varphi^a)^\alpha K_\alpha+j^\mu_aA^a_\mu+\bar{J}^\alpha\psi_\alpha + \bar{\psi}^\alpha J_\alpha+\bar{C}^a\varphi_a+\bar{\varphi}^aC_a]\right]. \quad (19)$$

The operator

$$\Omega\equiv j^a_\mu\frac{\delta}{\delta J^a_\mu}+\bar{C}^a\frac{\delta}{\delta K^a}+\frac{1}{a}\partial^\mu C^a\frac{\delta}{\delta j^a_\mu}-\bar{J}_\alpha\frac{\delta}{\delta\bar{K}_\alpha}+J_\alpha\frac{\delta}{\delta K_\alpha} \quad (20)$$

being independent of the fields, can be taken out of the functional integral in (19). Notice that the auxiliary sources J^a_μ, K_a, $\bar{K}_\alpha$, and K_α, which are to be set equal to zero, are necessary to write the operator (20) in a linearized form.

After Legendre-transforming both Eqs. (18) and (19), the following Ward identity holds for 1PI Green's functions:

$$\Gamma_{WF_n}=\Omega\Gamma_{F_n}. \quad (21)$$

Recall that the right-hand side of Eq. (21) should contain the additional sources. At the one-loop level this identity has to be satisfied both for finite and divergent parts, so that the counterterms required to renormalize Γ_{WF_n} are in fact provided by Γ_{F_n}. The conclusion is obvious: Class-I counterterms are forbidden, at least at the one-loop level.

Then the generating functional with the suitable counterterms, which at the one-loop level takes the form

$$\int [dA][d\varphi][d\bar{\varphi}][d\psi][d\bar{\psi}]\exp\left[i\int d^4x(\mathscr{L}+\Delta\mathscr{L}+\phi_iO_i+\Delta\phi_iO_i+W\phi_nF_n+\Delta W\phi_nF_n+\text{field sources})\right], \quad (22)$$

can be written, introducing the bare fields and parameters, in the form

$$\int [dA][d\varphi][d\bar{\varphi}][d\psi][d\bar{\psi}]\exp\left[i\int d^4x(\mathscr{L}^0+\phi^0_iO^0_i+W\phi^0_nF^0_n+\text{field sources})\right], \quad (23)$$

where ϕ^0_i and ϕ^0_n stand for

$$\phi^0_i=\phi_i+\sum_i\Delta_{ij}\phi_j, \qquad \phi^0_n=\phi_n+\sum_j\Delta_{nj}\phi_j+\sum_m\Delta_{nm}\phi_m \quad (24)$$

and O^0_i and F^0_n denote the operators written in terms of bare fields. (Indices i,j run over the gauge-independent operators and m,n over the gauge-dependent operators.)

Notice that Eq. (24) is indeed providing the sort of mixing which is expected from the results of Kluberg-Stern and Zuber. What is relevant to us is that Eq. (23) takes exactly the same form as Eq. (16), so that one can extend the previous argument to any order.[5]

What we want to do now is to study whether things are altered when the change of Eq. (14) does not have a trivial Jacobian and thus cannot be dropped out of the generating functional (16).

III. EVALUATION THE JABCOBIAN

By performing the suitable Wick rotation in Eq. (16), we will calculate in Euclidean space-time. The Euclidean spinors ψ and $\bar\psi$ can be expanded according to[13]

$$\psi(x)=\sum a_n\varphi_n(x)\ ,\qquad \bar\psi(x)=\sum \bar b_n\varphi_n^\dagger(x)\ . \tag{25}$$

a_n and $\bar b_n$ are elements of the Grassmann algebra. φ_n are chosen to be the eigenfunctions of the equation

$$i\not{D}\varphi_n=\lambda_n\varphi_n\ . \tag{26}$$

$i\not{D}$, being now a Hermitian operator, has real eigenvalues. The solutions to Eq. (26) are taken to satisfy

$$\int d^4x\ \varphi_n^\dagger(x)\varphi_m(x)=\delta_{nm}\ . \tag{27}$$

The path-integral fermionic measure, which is properly defined by

$$\prod_n da_n d\bar b_n\ , \tag{28}$$

under the change

$$da_n'=C_{nm}da_m\ ,\qquad d\bar b_n'=d\bar b_n\bar C'_{nm}\ , \tag{29}$$

transforms as

$$\prod_n da_n d\bar b_n\to\prod_n da_n' d\bar b_n'\det[C\bar C']^{-1}\ . \tag{30}$$

To be definite consider, for instance, that we are interested in the pseudoscalar *operators* of *dimension* 4. Class II[a] contains the operators $i\bar\psi\gamma^5\vec{\not{D}}\psi$ and $-i\bar\psi\overleftarrow{\not{D}}\gamma^5\psi$, so that $\bar C_q=\bar\psi\gamma^5$ and $C_p=\gamma^5\psi$. (Our γ^5 is defined as $\gamma^5=\gamma^0\gamma^1\gamma^2\gamma^3$. In Euclidean space-time $\gamma^5=\gamma^4\gamma^1\gamma^2\gamma^3$ with $\gamma^4=i\gamma^0$.) Then

$$C_{nm}=\delta_{nm}+\int d^4x\ \varphi_n^\dagger(x)\phi_p\gamma^5\varphi_m(x)\ ,\qquad \bar C'_{nm}=\delta_{nm}+\int d^4x\ \varphi_n^\dagger(x)\phi_q\gamma^5\varphi_m(x)\ . \tag{31}$$

Since we are ultimately going to take a derivative and set $\phi=0$, we will retain terms at most linear in ϕ. So, using

$$\det[1+L]=\exp[\mathrm{Tr}\ln(1+L)]\simeq\exp(\mathrm{Tr}L)\ , \tag{32}$$

we find

$$\mathscr{D}=\det[C\bar C']^{-1}=\exp\left[-\sum_n\int d^4x\ \varphi_n^\dagger(x)(\phi_p+\phi_q)\gamma^5\varphi_n(x)\right]\ . \tag{33}$$

What is left is to evaluate

$$D(x)=\sum_n\varphi_n^\dagger(x)\gamma^5\varphi_n(x)\ . \tag{34}$$

This is easily done choosing a plane-wave basis for the $\varphi_n(x)$ (see Ref. 13). The result is

$$D(x)=\frac{g^2}{32\pi^2}N_f\tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}(x)F^a_{\alpha\beta}(x) \tag{35}$$

so that

$$\mathscr{D}=\exp\left[-\int d^4x(\phi_p+\phi_q)\frac{g^2}{32\pi^2}N_f\tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F^a_{\alpha\beta}\right] \tag{36}$$

or, in Minkowski space,

$$\mathscr{D}=\exp\left[i\int d^4x(\phi_p+\phi_q)\frac{g^2}{32\pi^2}N_fF\tilde F\right]\ . \tag{37}$$

Equation (16) now reads for pseudoscalar operators of dimension 4

$$W[j,J,\bar J,\phi]=\int[dA][d\varphi][d\bar\varphi][d\psi][d\bar\psi]\exp\left[i\int d^4x\left[\mathscr{L}+\phi_iO_i+(\phi_p+\phi_q)\frac{g^2}{32\pi^2}N_fF\tilde F+W\phi_nF_n+\text{field sources}\right]\right]\ , \tag{38}$$

where index i runs over gauge-invariant operators and index n labels gauge-dependent operators. O_i and F_n are understood to have suitable quantum numbers. Of course, $F\tilde F$ was already included in the set of class-I operators O_i, but this does not affect our argument. The proof of Kluberg-Stern and Zuber we have sketched in the preceding section still implies the lack of mixing of WF_n operators to class-I operators but now, and this is the crucial point, class-II[a] operators cannot be expressed as WF_n. On the contrary, they get a class-I contribution from the very beginning.

The renormalization argument previously used works in the same way at any order in perturbation theory, i.e., class-I operators mix among themselves and with WF_n-type operators along the renormalization procedure, whereas operators of the form WF_n mix only among themselves. Much as before, one can write the equivalent to Eq. (23) as

$$\int[dA][d\varphi][d\bar\varphi][d\psi][d\bar\psi]\exp\left\{i\int d^4x\left[\mathscr{L}^0+\phi_i^0O_i^0+(\phi_p^0+\phi_q^0)\left[\frac{g^2}{32\pi^2}N_fF\tilde F\right]^0+W\phi_n^0F_n^0+\text{field sources}\right]\right\} \tag{39}$$

with

$$\phi_i^0=\phi_i+\sum_j \Delta_{ij}\phi_j+\Delta_{ip}\phi_p+\Delta_{iq}\phi_q \;,$$
$$\phi_p^0=\phi_p+\sum_j \Delta_{pj}\phi_j, \;\; \phi_q^0=\phi_q+\sum_j \Delta_{qj}\phi_j \;, \tag{40}$$
$$\phi_n^0=\phi_n+\sum_j \Delta_{nj}\phi_j+\Delta_{np}\phi_p+\Delta_{nq}\phi_q+\sum_m \Delta_{nm}\phi_m \;.$$

As usual, indices i,j label class-I operators, indices m,n class-II operators, and p and q stand for $i\bar{\psi}\overleftarrow{\not{D}}\gamma^5\psi$ and $i\bar{\psi}\gamma^5\overrightarrow{\not{D}}\psi$, respectively.

We have put $\Delta_{pp}=\Delta_{pq}=\Delta_{qp}=\Delta_{qp}=0$ as a consequence of the nonrenormalization of the axial anomaly, but nothing prevents $F\tilde{F}$ from coupling to other gauge-invariant operators.[4]

Equations (39) and (40) are our final result. They indeed show the coupling of some class-II[a] operators to class-I operators.

IV. CONCLUSION

$\bar{\psi}\gamma^5\overrightarrow{\not{D}}\psi$ and $\bar{\psi}\overleftarrow{\not{D}}\gamma^5\psi$ do have a class-I contribution, as we have learned. In general, this can also happen with more sophisticated composite operators containing both γ^5 and $\not{D}$. In other words, and remarkably enough, the claim about the independence of class-II[a] operators on class-I operators seems to fail exactly by the same reasons that the axial anomaly occurs in field theory.

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