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DECISION MAKING PROCESSES OF NON-LIFE INSURANCE PRICING USING FUZZY LOGIC AND OWA OPERATORS

Abstract. *Setting a commercial premium for an insurance policy is a complex process, even, though statistical tools provide fairly reliable information on the behavior of the frequency and cost of claims differentiated by risk profiles reflected in pure premium calculations. However lately setting the price the customer must pay has not been easy, because of the uncertainty of, having to use subjective criteria to analyze how demand may be affected by different price alternatives and economic situations. This article aims to develop this process in two stages. The first stage is carried out with the opinion of experts applied to uncertain numbers and Ordered Weighted Average (OWA) operators to assess the overall benefits of each profile to choose the best alternative. The second stage, which uses Heavy OWA (HOWA) operators, is based on the results obtained in the first stage and chooses a general price alternative for all profiles.*

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JEL Classification: D81, G22, M12, M51

1. Introduction

The process of non-life insurance pricing begins with an estimate of the pure premium using statistical methods, to which extra security is added to ensure solvency. Then, other surcharges are added to cover the costs of internal and external management. Thus far, these decisions are based on costs and may vary depending on the risk profile of the insured, taking into account previous history. However, establishing the commercial premium that customers will pay depends on the profit margin defined by the insurance company in accordance with its business strategy. This margin is usually a percentage of the premium plus the surcharges, which becomes a problem of great complexity, because the value of the commercial premium directly affects the demand and hence the overall profits. This margin is even more in those cases when the coverage is a risk and where the uncertainty is higher. Consider the following situations:

- A commercial premium is low and has a low unit profit margin, but it may be more attractive in the market and increase the demand and possibly result in greater overall profits.
- A high commercial premium has a higher profit margin, but it makes the product less competitive in the market demand and possibly decreases the overall profits.
- A decrease or increase in large portfolios insured by the two previous situations can change the risk profile of customers and lead to further adjustments to avoid possible bankruptcy.
- Economic growth may increase or decrease demand in a recession as consumer behavior is affected primarily by price and quality factors.

On the other hand, it is not easy to reach a consensus between marketing and financial departments to set pricing policy, because the former prefers lower prices to make products easier to sell, and the latter prefers higher prices to ensure better reliability. The key to deciding a pricing strategy will depend on how demand affects the total benefits and, in addition having products segmented by risk profiles will make the situation different for each of them.

All of this leads us to considerable difficulty in making forecasts on demand and profits in this problem because of the large uncertainty in different situations. In this case we will consider the following factors as the most relevant:

- The different situations of the economy and its expected behavior in the future for a given market.
- The experience of experts in the market segments determines the relevance of their opinions.

Therefore, the proposed method makes decisions about the pricing strategy for commercial premium rates in two stages, by providing better treatment of the opinions of experts using uncertain numbers and by applying information aggregation OWA operators.

2. Preliminaries

This section explains the main concepts of fuzzy logic and information aggregation OWA operators used by the proposed method.

2.1 Fuzzy Logic

Fuzzy logic (Fuzzy Logic in English) was introduced by Lotfi Zadeh (1965) as a way of dealing with ambiguous information, inaccurate or incomplete work by allowing intermediate values to be expressed in a range or membership function. Since its introduction, Fuzzy Logic has received much attention from the scientific community and there are now tens of thousands of researchers studying aspects of this method. For an overview of this theory, see Dubois and Prade (1980), Kaufmann and Gupta (1985) and Kaufmann and Gil-Aluja (1987). At present, we have designed a range of extensions and generalized this concept for a variety of applications in many scientific fields among that are decision making (Merigó and Casanovas, 2010a), engineering, statistics, finance (Merigó and Casanovas, 2011a) and strategic management (Merigó and Gil-Lafuente, 2009).

2.2 Uncertain Numbers

Uncertain numbers were introduced by Moore (1966) for, the purpose of evaluating the uncertainty through confidence intervals, in which there is a bottom end, a top end and a value or interval of maximum presumption, such that:

- Confidence interval $[a_1, a_2]$: Corresponds to a set of values greater than or equal to a_1 and less than or equal to a_2.
- Confidence Triplet$[a_1, a_2, a_3]$: Corresponding to a set of values greater than or equal to a_1 and less than or equal to a_3, but is considered most likely to be close to a_2.
- Confidence Quadruple$[a_1, a_2, a_3, a_4]$: Corresponding to a set of values greater than or equal to a_1 and less than or equal to a_4, but most likely is in the presumption maximum subinterval $[a_2, a_3]$.

With uncertain numbers, basic operations can be performed.

If $A = [a_1, a_2, a_3]$ and $B = [b_1, b_2, b_3]$, then:

- A + B = $[a_1 + b_1, a_2 + b_2, a_3 + b_3]$
- A – B = $[a_1 - b_3, a_2 - b_2, a_3 - b_1]$
- A * k = $[k * a_1, k * a_2, k * a_3]$ where $k > 0$
- A * B = $[a_1 * b_1, a_2 * b_2, a_3 * b_3]$
- A ÷ B = $[a_1 \div b_1, a_2 \div b_2, a_3 \div b_3]$

2.3 Information Aggregation OWA Operator

Definition 1: An OWA operator is defined as a mapping of dimension *n*, $F: \Re^n \rightarrow \Re$, that has an associated weighting vector W of dimension n, $W = [w_1, w_2, \dots, w_n]^T$, that meets the following conditions:

- $w_j \epsilon [0,1]$.
- $\sum_{j=1}^{n} w_j = 1$

with,

$$f(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j \ . b_j \tag{1}$$

where b_j is the *j*th largest of the a_i.

The essence of OWA (Yager, 1988; 1992; 1994; 1998) is the rearrangement of the elements or arguments; causing aggregation in the a_j not associated with a weighting w_j but with the placement order instead.

2.4 Ascending OWA Aggregation Operator

Definition 2: An Ascending OWA (AOWA) operator is defined as a mapping of dimension *n*, $F: \Re^n \rightarrow \Re$, that has an associated weighting vector W of dimension *n*, $W = [w_1, w_2, \dots, w_n]^T$, that meets the following conditions:

- $w_j \epsilon [0,1]$.
- $\sum_{j=1}^{n} w_j = 1$

with,

$$f(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j \ . b_j \tag{2}$$

where b_j is the *j*th smallest of the a_i, such that, $b_1 \leq b_2 \leq \cdots \leq b_n$, which thus differ from the OWA where $b_1 \geq b_2 \geq \cdots \geq b_n$.

The difference between the OWA operator (Descending OWA) and the AOWA (Yager, 1993) is the way in which it manages the arguments, descending in the first and ascending in the second, respectively, and depends on the optimistic or pessimistic attitudes of the decision maker.

2.5 OWA Operators Extensions

One of the main features of these operators is the flexibility they have to adapt to different circumstances to treat a variety of problems and to combine them with other tools for decision making. So, many authors have developed multiple extensions that integrate OWA with fuzzy logic, distance measures, likelihood, and other techniques that improve the decision making process. This article only explains those extensions that are directly related to this method.

2.5.1 Uncertain OWA Operator

The Uncertain OWA Operator (UOWA) was proposed by Xu and Da (2002), for situations of uncertainty in which confidence intervals are used to present information. These intervals can be presented in different forms: confidence quadruples, when these are composed of 4-tuples (*a*, *b*, *c*, *d*), where *a* is the minimum, *d* is the maximum, and *b* and *c* are the maximum interval presumption, confidence triplets, when *b* and *c* are equal, and confidence intervals when *b* and *c* are not known.

Definition 3: The UOWA operator is defined as a mapping of dimension *n*, $F: \Omega^n \rightarrow \Omega$, with an associated vector, $w = (w_1, w_2, \dots w_n)^T$, such that:

- $w_j \epsilon [0,1]$
- $\sum_{j=1}^{n} w_j = 1$,

with,

$$UOWA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \sum_{j=1}^{n} w_j {b_j}^* \quad (3)$$

where ${b_j}^*$ is the *j*th largest $\tilde{a}_i$, $\tilde{a}_i$ $(i \epsilon N)$ are trust intervals and are defined either as simple triplets or quadruples.

One of the difficulties with this method that is the arrangement of the confidence intervals, it is not always clear which one is superior to another, and thus, there is need to resort to other subjective criteria, such as the mean and, in other cases the upper confidence interval for more optimistic or bottom confidence interval for the most pessimistic, or a weighted average value of the confidence interval; for example, $(a_1 + 4a_2 + 4a_3 + a_4)/10$ in the case of a quadruple of confidence.

Definition 4: In this method, the Ascending UOWA (AUOWA) operator corresponds to an ascending sort that is defined as a mapping, $F: \Omega^n \rightarrow \Omega$, of dimension n, with an associated vector, $w = (w_1, w_2, \dots w_n)^T$, such that:

- $w_j \epsilon [0,1]$
- $\sum_{j=1}^{n} w_j = 1$,

with,

$$AUOWA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \sum_{j=1}^{n} w_j b_j^{*} \quad (4)$$

where b_j^{*} is the jth smallest $\tilde{a}_i$, $(i \epsilon N)$ are the trust intervals. The UOWA operator can be extended and generalized by various contexts (Merigó, 2011; Merigó et al. 2012; Zhou et al. 2012) including the use of general means (Merigó and Casanovas, 2011a; 2012) and quasi-arithmetic (Merigó and Casanovas, 2011b).

2.5.2 Probabilistic Uncertain OWA Operator

Definition 5: An UPOWA operator (Merigó and Wei, 2011) is defined as a mapping $F: \Omega^n \to \Omega$ of dimension n, which has an associated weight vector $w = (w_1, w_2, \dots w_n)^T$, such that, $w_j \epsilon [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, and a vector of probabilities $V = (v_1, v_2, \dots v_n)^T$, such that $v_i \epsilon [0,1]$ and $\sum_{i=1}^{n} v_i = 1$, where:

$$UPOWA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \beta \sum_{j=1}^{n} w_j b_j^{*} + (1-\beta) \sum_{i=1}^{n} v_i \tilde{a}_i \quad (5)$$

where b_j^{*} is the jth largest $\tilde{a}_i$, $\tilde{a}_i$ $(i \epsilon N)$ are confidence intervals defined either as simple triplets or quadruples.

Definition 6: In the case of the Probabilistic Uncertain Ascending OWA (AUPOWA) operator, this is defined as a mapping $F: \Omega^n \to \Omega$ of dimension n, which has an associated weight vector, $w = (w_1, w_2, \dots w_n)^T$, such that, $w_j \epsilon [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, and a vector of probabilities, $V = (v_1, v_2, \dots v_n)^T$, such that, $v_i \epsilon [0,1]$ and $\sum_{i=1}^{n} v_i = 1$, where:

$$UPOWA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \beta \sum_{j=1}^{n} w_j b_j^{*} + (1-\beta) \sum_{i=1}^{n} v_i \tilde{a}_i \quad (6)$$

where b_j^{*} is the jth smallest $\tilde{a}_i$, $\tilde{a}_i$ $(i \epsilon N)$ are confidence intervals defined either as simple triplets or quadruples. It is worth noting that the UPOWA operator can be extended by the use of distance measures (Merigó et al. 2013, Zeng et al. 2013).

2.5.3 Heavy OWA Operator

These operators were proposed by Yager (2002), and their main feature is that the sum of the weights is not equal to 1; it is between 1 and *n*. The importance of this operator is its use in situations which are mutually independent, so that the results can be produced simultaneously.

Definition 7: The HOWA operator is defined as a mapping, $F: \Re^n \to \Re$, of dimension n, with an associated vector W, such that:

- $w_j \epsilon [0,1]$

- $1 \leq \sum_{j=1}^{n} w_j \leq n$

with,

$$HOWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \tag{7}$$

where b_j, is the *j*th largest of a_i.

Definition 8: The Ascending HOWA (AHOWA) operator, which corresponds to the ascending sort of arguments, can be defined as a mapping, $F: \mathfrak{R}^n \rightarrow \mathfrak{R}$, of dimension n, with an associated vector W, such that:

- $w_j \epsilon [0,1]$
- $1 \leq \sum_{j=1}^{n} w_j \leq n$

with,

$$AHOWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \tag{8}$$

where, b_j is *j*th smallest of the a_i. Note that the HOWA operator has been extended and generalized by many studies, including the Uncertain Induced HOWA operator (Merigó and Casanovas, 2011c) and the Induced Heavy OWA distance operator (Merigó and Casanovas, 2010b).

3. Proposed Methodology

As mentioned above there are different situations that create uncertainty regarding the stated problem, which leads to resorting to expert opinions to determine the best alternative obtain the greatest total benefits. To achieve this, we propose the following process:

Stage I: Calculation of Expected Benefits of Risk Profiles and Benefit Levels.

The expected benefits are the key to making decisions about each of the pricing strategies; in this case, they are calculated by taking into account the opinion of experts for each of the segments through the following steps:

Step 1. Selection of experts: to have expert opinions that conform to reality, an expert will be selected for each segment or risk profile by assuming that each expert with prior experience will have more knowledge of the market share than the rest, so that expert 1 will correspond to the prediction of risk profile 1, expert 2 with risk profile 2, and so on.

Step 2. Selecting predictive tool: to express an opinion on future earnings given that it is a subjective value that is set in an uncertain environment, forecasts can be presented as:

- A unique value: it has been the traditional way to predict, although it is very difficult to establish and much less likely to hit it.
- Uncertain numbers: in this case, the exact prediction it unknown, but an interval, $A = [a_1, a_2]$, can be set that will contain the value that is known to be greater than a_1 and less than a_2. You can also use trusted triplets, $A = [a_1, a_2, a_3]$, where the forecasted value will be close to a_2 and is, greater than a_1 and less than a_3 or the trusted quadruples $A = [a_1, a_2, a_3, a_4]$, where the prediction is known to be greater than a_1 and less than a_4, but the chances are that it is within a_2 and a_3.

Step 3. Expert opinion: experts should give their earnings forecast for different economic scenarios and pricing strategies. These results are presented in a matrix $A\ x\ B$, where $A = (a_1, a_2, \ldots a_i)$are the different pricing strategies, and $B = (b_1, b_2, \ldots b_j)$ are economic scenarios that may occur in the future.

Step 4. Aggregation of results: the opinions of the expected benefits for each alternative were added to the different situations of the economy, taking into account criteria such as minimum, maximum, arithmetic mean, the weighted mean and different information aggregation OWA operators.

Step 5. Summary of results: the results will be summarized for each of the criteria including all segments in a matrix containing pricing alternatives and different risk profiles.

Stage II: Adding Total Results with Heavy OWA (HOWA) Operators

The results are used to make decisions on the following pricing strategy for each profile, but it is important to note that the forecasts discussed in the previous stages are independent of each other, so that, they can occur simultaneously. If a general criterion for the entire market is available, then Heavy OWA operators are most suitable. Therefore, to analyze the information and perform a total aggregation, the following steps are used:

Step 6. Selection of forecasting: the summaries of step 5 are the forecasts of the benefits using different criteria (maximum, minimum, arithmetic mean, weighted mean and OWA operators). The criteria that is chosen is the one that, according to the experts, best fits reality, and that will be the starting point in this second stage.

Step 7 Calculation of securities: in the previous stage for the forecasts, uncertain values were used through intervals, triplets or quadruples of trust, which are now defined as a representative value. For example, the interval can take the average value of ends, triplets can be set as a weighted average $(a_1 + 2a_2 + a_3)/4$ or $(2a_1 + 4a_2 + 2a_3)/8$ and quadruples can be set as $(a_1 + 2a_2 + 2a_3 + a_4)/6$ or $(a_1 + 4a_2 + 4a_3 + a_4)/10$. These definitions facilitate the aggregation process, considering that these values are already included in the uncertainty.

Step 8. Aggregation of results: the values obtained above are independent of each other and can be determined simultaneously, so there is no point in calculating values to make a decision; however, it is necessary to perform a total aggregation of results, through the Heavy OWA operators.

Step 9. Selecting the optimal alternative: results from the previous step will establish a ranking of alternatives in descending order for each of the criteria. From these data, a decision can be made to determine the most optimistic or maximum, the most pessimistic or minimum, or a conservative value that corresponds to intermediate values in ranking the criteria.

4. Illustrative Application

To set the value of the commercial premium, insurance policies are divided into 5 risk profiles. A group of experts, each one known for a market segment, is asked to issue a forecast for the profile that corresponds to the expected benefits of different pricing values considering different surcharge alternatives and economic scenarios, such as:
Alternative surcharges to the price:
Alternative 1: provide a profit margin (profit) of 10%
Alternative 2: provide a profit margin (profit) of 15%
Alternative 3: provide a profit margin (profit) of 20%
Alternative 4: provide a profit margin (profit) of 25%
Alternative 5: provide a profit margin (profit) of 30%

Economic scenarios:
Situation 1: Strong economic growth.
Scenario 2: Moderate economic growth.
Situation 3: Economy stable.
Situation 4: Slight economic recession.
Situation 5: Strong economic recession.

Risk profiles correspond to the characteristics that are identifiable in each segment and that influence the frequency and/or the cost of insurance claims, such as:

Table 1. Example of Risk Profiles

Segments	Gender	Age	Vehicle Range
Profile 1	Male	18 - 30	Medium
Profile 2	Male	31 - 50	Medium
Profile 3	Male	18 - 30	High
Profile 4	Male	31 - 50	High
Profile 5	Female	18 - 30	Medium
Profile 6	Female	31 - 50	Medium
Profile 7	Female	18 - 30	High
Profile 8	Female	31 - 50	High

Table 2. Risk profiles for the Application

Segments	Gender	Age	Vehicle Range
Profile 1	Male	18 - 30	Medium
Profile 2	Male	31 - 50	Medium
Profile 3	Female	31 - 50	High
Profile 4	Male	18 - 30	High
Profile 5	Female	18 - 30	High

The results are presented below:

Table 3. Expert forecast 1 for Profile 1

Alternatives	Situation 1		Situation 2		Situation 3		Situation 4		Situation 5	
Alternative 1	680	720	600	650	500	540	450	500	380	420
Alternative 2	700	750	650	680	530	570	470	510	400	450
Alternative 3	760	800	660	700	550	600	420	470	310	350
Alternative 4	780	820	620	660	500	550	400	450	250	300
Alternative 5	800	850	610	660	470	530	350	400	200	250

Table 4. Expert forecast 2 for Profile 2

Alternatives	Situation 1		Situation 2		Situation 3		Situation 4		Situation 5	
Alternative 1	800	850	760	800	700	750	700	750	650	700
Alternative 2	840	880	790	830	750	800	660	710	630	670
Alternative 3	910	960	870	910	790	840	650	700	610	650
Alternative 4	930	980	870	920	760	820	620	680	550	610
Alternative 5	920	970	860	900	740	800	600	650	520	570

Table 5. Expert forecast 3 for Profile 3

Alternatives	Situation 1		Situation 2		Situation 3		Situation 4		Situation 5	
Alternative 1	580	630	550	600	450	500	420	480	410	450
Alternative 2	600	650	570	630	470	510	440	490	390	440
Alternative 3	620	660	600	650	480	530	400	450	300	350
Alternative 4	610	640	580	620	460	500	350	400	260	310
Alternative 5	600	640	560	600	400	440	300	360	200	240

Table 6. Expert forecast 4 for Profile 4

Alternatives	Situation 1		Situation 2		Situation 3		Situation 4		Situation 5	
Alternative 1	980	1050	950	1000	920	960	870	900	800	850
Alternative 2	1090	1140	990	1050	890	940	810	850	650	750
Alternative 3	1160	1200	960	1010	880	930	800	850	580	630
Alternative 4	1170	1220	940	1000	800	850	700	740	410	470
Alternative 5	1190	1250	900	950	650	700	600	650	300	340

Table 7. Expert forecast 5 for Profile 5

Alternatives	Situation 1		Situation 2		Situation 3		Situation 4		Situation 5	
Alternative 1	300	350	280	330	250	300	240	280	230	270
Alternative 2	400	450	300	350	280	320	250	300	200	250
Alternative 3	480	520	410	450	320	360	230	270	180	230
Alternative 4	500	540	430	480	300	350	210	250	170	210
Alternative 5	510	550	420	460	270	310	190	220	110	150

Experts have also defined the weight vectors, P = (0.1, 0.2, 0.3, 0.3, 0.1) and W = (0.1, 0.15, 0.25, 0.2, 0.3), which correspond to the subjective probability of each of the situations and to an optimistic or pessimistic attitude respectively, as the cases to add information to, and a parameter, β = 0.4, which represents the relative importance of subjective information in front of the decision maker's attitude reflected in the vectors P and W. These expressions are used to calculate the following criteria:

- UMax: the confidence interval corresponds to higher value or more optimistic.
- Umin: the confidence interval corresponds to the least significant or the most pessimistic.

- UAM: the arithmetic mean of the confidence intervals for each of the alternatives.
- UPA: the weighted arithmetic mean of the confidence intervals for each of the alternatives with the weight vector *P*.
- UOWA: the aggregation of the descending sort of the confidence intervals of the alternatives using the weighting vector *W*.
- AUOWA: the aggregation of the ascending sort of the confidence intervals of the alternatives using the weighting vector *W*.
- UPOWA: the aggregation using the β factor for UOWA and (1 - β) for the UPA.

The results are shown below:

Table 8. Aggregate Results of Profile 1

Alternatives	U-Max		U-Min		UAM		UWA		UOWA		AUOWA		UPOWA	
Alternative 1	680	720	380	420	522	566	511	556	487	531	555	598	501	546
Alternative 2	700	750	400	450	550	592	540	580	514	557	583	625	530	571
Alternative 3	760	800	310	350	540	584	530	576	490	534	592	636	514	559
Alternative 4	780	820	250	300	510	556	497	544	451	499	568	613	479	526
Alternative 5	800	850	200	250	486	538	468	521	419	472	552	605	448	501

Table 9. Aggregate Results of Profile 2

Alternatives	U-Max		U-Min		UAM		UWA		UOWA		AUOWA		UPOWA	
Alternative 1	800	850	650	700	722	770	717	765	704	753	737	785	712	760
Alternative 2	840	880	630	670	734	778	728	774	711	756	760	804	721	767
Alternative 3	910	960	610	650	766	812	758	805	732	778	803	850	748	794
Alternative 4	930	980	550	610	746	802	736	793	703	760	791	846	723	780
Alternative 5	920	970	520	570	728	778	718	769	682	733	775	826	704	755

Table 10. Aggregate Results of Profile 3

Alternatives	U-Max		U-Min		UAM		UWA		UOWA		AUOWA		UPOWA	
Alternative 1	580	630	410	450	482	532	470	522	460	509	501	551	466	517
Alternative 2	600	650	390	440	494	544	486	535	468	517	517	566	479	528
Alternative 3	620	660	300	350	480	528	476	525	442	491	516	563	462	511
Alternative 4	610	640	260	310	452	494	446	489	411	455	493	532	432	475
Alternative 5	600	640	200	240	412	456	402	448	364	408	457	500	387	432

Table 11. Aggregate Results of Profile 4

Alternatives	U-Max		U-Min		UAM		UWA		UOWA		AUOWA		UPOWA	
Alternative 1	980	1050	800	850	904	952	905	948	885	930	925	975	897	941
Alternative 2	1090	1140	650	750	886	946	882	936	837	902	934	990	864	922
Alternative 3	1160	1200	580	630	876	924	870	919	814	863	938	985	848	897
Alternative 4	1170	1220	410	470	804	856	796	846	721	774	885	937	766	817
Alternative 5	1190	1250	300	340	728	778	704	754	627	675	820	872	673	722

Table 12. Aggregate Results of Profile 5

Alternatives	U-Max		U-Min		UAM		UWA		UOWA		AUOWA		UPOWA	
Alternative 1	300	350	230	270	260	306	256	302	252	297	268	315	254	300
Alternative 2	400	450	200	250	286	334	279	326	265	313	308	355	273	321
Alternative 3	480	520	180	230	324	366	313	354	290	333	359	400	304	345
Alternative 4	500	540	170	210	322	366	306	351	283	327	360	404	297	341
Alternative 5	510	550	110	150	300	338	284	321	253	291	344	383	271	309

All of these criteria are valid for making a decision regarding the strategy to pursue, making it possible to choose an alternative for each profile. For example, choosing the most optimistic value with the least optimistic value, as detailed in the following summaries:

Table 13. Maximum Results Summary

Alternatives	Profile 1		Profile 2		Profile 3		Profile 4		Profile 5	
Alternative 1	680	720	800	850	580	630	980	1050	300	350
Alternative 2	700	750	840	880	600	650	1090	1140	400	450
Alternative 3	760	800	910	960	620	660	1160	1200	480	520
Alternative 4	780	820	930	980	610	640	1170	1220	500	540
Alternative 5	800	850	920	970	600	640	1190	1250	510	550

Table 14. Minimum Result Summary

Alternatives	Profile 1		Profile 2		Profile 3		Profile 4		Profile 5	
Alternative 1	380	420	650	700	410	450	800	850	230	270
Alternative 2	400	450	630	670	390	440	650	750	200	250
Alternative 3	310	350	610	650	300	350	580	630	180	230
Alternative 4	250	300	550	610	260	310	410	470	170	210
Alternative 5	200	250	520	570	200	240	300	340	110	150

Table 15. Summary of the Results with the Arithmetic Mean

Alternatives	Profile 1		Profile 2		Profile 3		Profile 4		Profile 5	
Alternative 1	522	566	722	770	482	532	904	952	260	306
Alternative 2	550	592	734	778	494	544	886	946	286	334
Alternative 3	540	584	766	812	480	528	876	924	324	366
Alternative 4	510	556	746	802	452	494	804	856	322	366
Alternative 5	486	538	728	778	412	456	728	778	300	338

Table 16. Summary of the Results with the Weighted Average

Alternatives	Profile 1		Profile 2		Profile 3		Profile 4		Profile 5	
Alternative 1	511	556	717	765	470	522	905	948	256	302
Alternative 2	540	580	728	774	486	535	882	936	279	326
Alternative 3	530	576	758	805	476	525	870	919	313	354
Alternative 4	497	544	736	793	446	489	796	846	306	351
Alternative 5	468	521	718	769	402	448	704	754	284	321

Table 17. Summary of the Results with the UOWA Operators

Alternatives	Profile 1		Profile 2		Profile 3		Profile 4		Profile 5	
Alternative 1	487	531	704	753	460	509	885	930	252	297
Alternative 2	514	557	711	756	468	517	837	902	265	313
Alternative 3	490	534	732	778	442	491	814	863	290	333
Alternative 4	451	499	703	760	411	455	721	774	283	327
Alternative 5	419	472	682	733	364	408	627	675	253	291

Table 18. Summary of the Results with the AUOWA Operators

Alternatives	Profile 1		Profile 2		Profile 3		Profile 4		Profile 5	
Alternative 1	555	598	737	785	501	551	925	975	268	315
Alternative 2	583	625	760	804	517	566	934	990	308	355
Alternative 3	592	636	803	850	516	563	938	985	359	400
Alternative 4	568	613	791	846	493	532	885	937	360	404
Alternative 5	552	605	775	826	457	500	820	872	344	383

Table 19. Summary of the Results with the UPOWA Operators

Alternatives	Profile 1		Profile 2		Profile 3		Profile 4		Profile 5	
Alternative 1	501	546	712	760	466	517	897	941	254	300
Alternative 2	530	571	721	767	479	528	864	922	273	321

Alternative 3	514	559	748	794	462	511	848	897	304	345
Alternative 4	479	526	723	780	432	475	766	817	297	341
Alternative 5	448	501	704	755	387	432	673	722	271	309

It is important to note, that if you want to have the same pricing policy in relation to profit margins, the above criteria involve selecting alternatives for each profile. In this case, it is necessary to compute the total aggregate results using Heavy OWA operators.

For this example, we use the data from the results obtained by the UPOWA operator (Table 19), but the computation may require additional data depending on the judgment of the decision maker. In this case an expert works with a representative value corresponding to the average value of the ends of each of the confidence intervals as shown below:

Table 20. Securities of the Operator UPOWA

Alternatives	Profile 1	Profile 2	Profile 3	Profile 4	Profile 5
Alternative 1	524	736	491	919	277
Alternative 2	550	744	503	893	297
Alternative 3	537	771	487	872	325
Alternative 4	502	751	454	792	319
Alternative 5	475	729	409	698	290

The predictions of the benefits of each alternative charge for the different profiles are independent, occurring simultaneously. Consequently, treatment can be performed with the Heavy OWA, in which the main difference compared with the other types of OWA operators is that the sum of the weights is greater than 1 ($1 \leq \sum_{j=1}^{n} w_j \leq n$). In this case we use the following vectors:

- $W = (1,1,1,1,1)$, where $\sum_{j=1}^{n} w_j = 5$, which generates a total aggregation in which forecasts are considered to be met in full.
- $W = (1,0.9,0.9,0.8,0.7)$, where $\sum_{j=1}^{n} w_j = 4.4$, which is to be used for the Heavy OWA (HOWA), with an optimistic attitude, where greater weight is given to higher values, and the Ascending Heavy OWA (AHOWA) operator with a pessimistic attitude, where greater weighting is given to lower values.
- $P = (0.9,0.7,0.8,0.9,0.8)$, which corresponds to the subjective probability forecasts of compliance and will be used in the calculation of the weighted average (HPA).

- β = 0.5, which generates the Heavy Probabilistic OWA operator (HPOWA) that, combines the subjective probability with the optimistic or pessimistic attitude; thus, $HPOWA = \beta(HOWA) + (1-\beta)HPA$.

The results obtained are shown in Table 21:

Table 21. Results Aggregate with Heavy OWA

Alternatives	HOWA-Total	HPA	HOWA	A-HOWA	HPOWA
Alternative 1	2947	2428	2639	2422	2534
Alternative 2	2987	2460	2668	2465	2564
Alternative 3	2991	2456	2665	2473	2561
Alternative 4	2818	2308	2506	2334	2407
Alternative 5	2601	2125	2312	2157	2218

If we order the alternatives in descending order for each criterion we obtain the following ranking, which will make a decision that depends of the attitude assumed by the company or those who decide the pricing strategy.

Table 22. Ranking Alternatives

Criteria	Order
HOWA - Total	A3>A2>A1>A4>A5
HPA	A2>A3>A1>A4>A5
HOWA	A2>A3>A1>A4>A5
A-HOWA	A3>A2>A1>A4>A5
HPOWA	A2>A3>A1>A4>A5

In this case, to set the value of the commercial premium, an optimistic approach would take the alternative of higher value; that it to say, an optimistic approach would choose Alternative 3 (profit margin of 20%) with HOWA-Total and A-HOWA, and Alternative 2 (profit margin of 15%) with the HPA, HOWA and HPOWA. A pessimistic view would choose the smallest value that corresponds to Alternative 5 (profit margin of 30%) with any of the criteria. However, with a more conservative attitude, one could choose an intermediate value (2nd, 3rd or 4th place) in the ranking.

Importantly, the small difference in the results between Alternatives 2 and 3 is also reflected in the rankings of the criteria and hinders the choice between these options. It is most advisable in this case to use as a criterion of the HPOWA (β = 0.5), which combines HOWA and HPA in a single operator, which is equally weighted, as a way to decide ties between alternatives.

5. Conclusions

We have developed a new methodology to determine insurance rates from a model that is not based solely on a profit margin increase from the cost structure, but also considers a strategic vision represented by the opinions of experts. Thus, this methodology establishes a degree of pricing strategy that takes into account the demand from different economic scenarios and provides a better perspective on the overall benefits, not just the benefits of each policy unit.

Using uncertain numbers through confidence intervals facilitates the work of the experts, when evaluating each of the alternatives, because it is not easy to define a precise value in situations of great uncertainty.

This two-stage methodological process permits the establishment of a separate pricing strategy for each profile through any of the criteria used in the first stage, or through a second phase that determines an overall strategy for all segments by applying the Heavy OWA operators.

The aggregation of information to obtain a global value Heavy OWA operator, allows each of the experts to make a judgment regarding the profile or segment that they know best, or have more experience working with, and thus avoids the distortions that occur when there is not enough information or experience present.

Finally, we note that this proposed methodology provides a formal procedure for making charging decisions helps to eliminate political decisions and places the emphasis on purely economic issues, either endogenous or exogenous to the firm. Thus, this methodology tends to homogenize the decision criteria for all profiles, to optimize the overall profit.

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