Coleman-Weinberg Symmetry Breaking and Higgs Physics

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Abstract: The Higgs mechanism, at the origin of particle masses in the Standard Model, may be a quantum phenomenon. As we show, it can be triggered by quantum loops events though the classical potential has no mass scale (scale invariant). This is opposed to the Standard Model approach, where a negative mass is included in the potential. In this work, not only we make a qualitative study, but pursue some quantitative calculus by retrieving the value of the Higgs mass. Finally, we discuss new physics which this new formalism can attempt to address.

I. INTRODUCTION

The Higgs mechanism is used in the Standard Model (SM) to explain the origin of the mass of fermions and gauge bosons. Without this mechanism, bosons and fermions would be massless because of SM symmetry, which is in contradiction with measurements.

In the Higgs mechanism, a scalar field ϕ is introduced, interacting with bosons and fermions in a gauge-invariant way. The key ingredient of this mechanism is that ϕ acquires a non-zero vacuum expectation value (VEV), $v = \langle 0|\phi|0\rangle$. In turn, the spontaneous symmetry breaking (SSB) occurs.

At the VEV, the interactions of fermions and bosons with ϕ originates their mass. Moreover, the radial excitation of $\phi = v + h$ represents the Higgs particle.

The Higgs mechanism well describes the data but the SSB is only parameterized in the SM. To trig a non-zero VEV, the potential is ad-hoc tuned

$$V_0(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4 \,. \tag{1}$$

Here, μ is the mass of the particle, and λ is a dimensionless coupling constant. In the case of $\mu^2 > 0$, no SSB appears, since the minimum of the potential will be at $\phi = v = 0$. However, when $\mu^2 < 0$ (imaginary mass particle), then SSB occurs, and a non-zero VEV arises.

The SM approach to induce the SSB is highly unpleasant if we want a fundamental understanding of the electroweak scale.The Higgs mass by itself turns out to be as free-parameter, $m_H^2 = V''(v)/2 = v\lambda$.

In 1973, S. Coleman and E. Weinberg (CW) showed [1] that the SSB can still occur through radiative corrections for $\mu = 0$. In the CW mechanism, the model has one less parameter and is classically scale invariant. Moreover, the Higgs mass is directly predicted. In this work, we will review these earlier works.

First of all, in section II we introduce the tool of the effective potential, which is necessary to study the ground state of a QFT beyond tree level. In section III we will apply this formalism to the simple model of a real scalar field. In sections IV and IV A, we add a gauge interaction in the model and perform a more general and phenomenological study.

II. PATH INTEGRAL AND EFFECTIVE POTENTIAL

The analysis of the SSB from S. Coleman and E. Weinberg relies on the use of the effective potential, whose minima determines the vacua of the theory. For simplicity in this section only one real scalar field, ϕ, will be considered, but the extension to a more complicated case is straightforward.

First, we define [2] the generating functional $W(J)$

$$e^{iW(J)} = \int \mathcal{D}\phi \exp\left[i\int d^4x(\mathcal{L} + J(x)\phi(x))\right] \,, \tag{2}$$

where $J(x)$ is an external source. $W(J)$ is analog to the partition function in statistical mechanics. Instead of integrating over all thermal fluctuations, we integrate over all possible quantum fluctuations of the ϕ field.

The derivative of $W(J)$,

$$\frac{\partial W(J)}{\partial J(x)} = \frac{\int \mathcal{D}\phi \exp\left[i\int d^4x(\mathcal{L} + J(x)\phi(x))\right]\phi}{\int \mathcal{D}\phi \exp\left[i\int d^4x(\mathcal{L} + J(x)\phi(x))\right]} \equiv \phi_c(x) \,, \tag{3}$$

corresponds to the VEV of $\phi(x)$ in the presence of the external source $J(x)$, namely $\langle 0|\phi(x)|0\rangle_J$.

Now, the function $\Gamma(\phi) = W(J) - \int d^4x J(x)\phi(x)$ is interesting since it satisfies:

$$\begin{aligned}\frac{\partial \Gamma(\phi)}{\partial\phi}\Big|_{\phi_c} &= \frac{\partial W(J)}{\partial\phi}\Big|_{\phi_c} - \int d^4y \frac{\partial J(y)}{\partial\phi}\Big|_{\phi_c}\phi_c(y) - J(x) \\ &= -J(x)\end{aligned}$$

where $\frac{\partial W(J)}{\partial\phi_c} = \int d^4y \frac{\partial J(y)}{\partial\phi_c}\frac{\partial W(J)}{\partial J(y)} \equiv \int d^4y \frac{\partial J(y)}{\partial\phi_c}\phi_c(y)$ from using eq. (3). In the case where $J(x)$ is zero, $\phi_c(x)$ is a minimum of $\Gamma(\phi)$ and also corresponds from eq. (3) to the true VEV of the theory. If ϕ_c is a non-zero field, then the symmetry is spontaneously broken. Now, for translational invariance, the $\Gamma(\phi_c)$ extreme, $\phi_c(x) = \phi_c$, is also a space-time constant field and $\Gamma(\phi)$ for homogeneous field is the so-called effective potential

$$V_{\text{eff}}(\phi_c) = -\frac{\Gamma(\phi_c)}{\mathcal{V}_4}, \quad \text{with } \mathcal{V}_4 = \int d^4x V_{\text{eff}}(\phi_c) \,. \tag{4}$$

The minimum of the effective potential is still ϕ_c. Actually, $\Gamma(\phi)$ is the so-called effective action, which corresponds in the classic limit to the classic action.

A. Effective potential for a real scalar field

Here we work out the general formula for $V_{\rm eff}(\phi_c)$ in the case of a real scalar field. This can be easily extended to extra scalar fields, such as complex scalar and gauge fields, which we will study in next sections. We consider the following Lagrangian

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu\phi)^2 - V_0(\phi)\,, \tag{5}$$

where $V_0(\phi)$ is the classical (initial) potential.

Let's develop the exponent on the r.h.s of eq. (2) around the stationary point ϕ_c, up to second order [4],

$$\begin{aligned} e^{iW(J)} &= \exp\left[i\int d^4x\,(\mathcal{L}[\phi_c]+J\phi_c)\right] \\ &\times\int\mathcal{D}\phi\,\exp\left[\frac{i}{2}\int d^4x\, i\Delta^{-1}(\phi_c,0)\,\phi(x)^2\right]\,, \end{aligned} \tag{6}$$

where

$$i\Delta^{-1}(\phi_c, x-y) \equiv \frac{\partial^2\mathcal{L}}{\partial\phi(x)\partial\phi(y)} \tag{7}$$

is the inverse of the propagator in configuration space. The functional integral in eq. (6) corresponds to $\mathcal{V}_4$ gaussian integrals. Then, its value is $(i\Delta^{-1}(\phi_c,0))^{-\mathcal{V}_4/2}$.

Using $\Gamma(\phi_c) = W(J) - \int d^4\, xJ\phi_c$ and eq. (4), we get

$$V_{\rm eff}(\phi_c) = V_0(\phi_c) - \frac{i}{2}\log\left(i\Delta^{-1}(\phi_c,0)\right) \tag{8}$$

and in the momentum representation

$$V_{\rm eff}(\phi_c) = V_0(\phi_c) - \frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\log\left(i\tilde{\Delta}^{-1}(\phi_c,p)\right)\,. \tag{9}$$

Note that the effective potential is different from the classic potential. Therefore, its minimum can also vary with respect to the classical case. In next sections, we study several examples for the classical potential.

III. EFFECTIVE POTENTIAL FOR Φ^4 THEORY

Here we calculate $V_{\rm eff}(\phi_c)$ in eq. (9) for the following tree-level (classical) potential in eq. (5)

$$V_0(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4\,. \tag{10}$$

where μ and λ stand for the bare couplings.

From eq. (10), $\Delta^{-1}(\phi_c, x-y)$ in eq. (7) is given by

$$i\Delta^{-1}(\phi_c, x-y) = -\left(\partial_\mu\partial^\mu + m(\phi_c)^2\right)\delta(x-y)$$

where $m(\phi_c)^2 = \mu^2 + 3\lambda\phi_c^2$ plays the role of the (Klein-Gordon) effective mass of a scalar particle. The corresponding Fourier transform, $\partial_\mu = ip_\mu$, is

$$i\tilde{\Delta}^{-1}(\phi_c,p) = p^2 - m(\phi_c)^2\,. \tag{11}$$

Now, the expression of $V_{\rm eff}$ in eq. (9) reads as

$$V_{\rm eff}(\phi_c) = V_0(\phi_c) - \frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\,\log(p^2 - m(\phi_c)^2) \tag{12}$$

However, this integral is divergent in $\mathbb{R}^4$ and has to be regularized. We use dimensional regularization to care for divergences: we analytically extend the number of dimensions to D (having in mind that, after performing the integral, we will need $D\to 4$). To ensure the correct dimensions of the integral above, the integration measure $d^4p/(2\pi)^4$ has to be replaced by $d^Dp/(2\pi)^D M^{4-D}$ and

$$\begin{aligned} V_{\rm eff} &= V_0(\phi_c) - \frac{iM^{4-D}}{2}\int\frac{d^Dk}{(2\pi)^D}\log(k^2 - m(\phi_c)^2) \\ &= V_0(\phi_c) - \frac{1}{2}\frac{M^4}{(4\pi)^{D/2}}\Gamma\left(-\frac{D}{2}\right)\left(\frac{M^2}{m(\phi_c)^2}\right)^{-D/2}\,. \end{aligned}$$

See in appendix A) for complete calculation. This expression is divergent because of the term $\Gamma(-D/2) \simeq 1/(4-D) - \gamma/2 + 3/4$. It is useful to substitute $D = 4-2\varepsilon$ and expand in terms of ε

$$V_{\rm eff}(\phi_c) = V_0(\phi_c) - \frac{1}{4}\frac{m(\phi_c)^4}{(4\pi)^2}\left(\frac{1}{\hat{\varepsilon}} - \log\frac{m(\phi_c)^2}{M^2}\right) \tag{13}$$

where $1/\hat{\varepsilon} = 1/\varepsilon - \gamma + \log 4\pi + 3/2$. Note that the divergence term is proportional to $m(\phi_c)^4 = (3\lambda\phi_c^2 + \mu^2)^2$, namely has the form of a constant, quadratic term, and quartic term of ϕ_c. Indeed, it corresponds to the renormalization of the bare parameters μ^2 and λ (and cosmological constant) of the original potential in eq. (10), i.e. $\mu^2 \to \mu^2 + \delta_{\mu^2}$ and $\lambda \to \lambda + \delta_\lambda$. Then, only we need to shift $V_0(\phi_c) \to V_0(\phi_c) + V_{c.t}(\phi_c)$ in $V_{\rm eff}(\phi_c)$, where [5]

$$V_{c.t}(\phi_c) = \frac{\delta_{\mu^2}}{2}\phi_c^2 + \frac{\delta_\lambda}{4}\phi_c^4 = \frac{1}{2}\left(\frac{\delta_{\mu^2}}{\delta_\lambda^{1/2}} + \frac{\delta_\lambda^{1/2}}{2}\phi_c^2\right)^2 = \delta_{m(\phi_c)}.$$

Here, a ϕ_c-independent term has been dropped out corresponding to an overall shift of the potential. Now, by using the $\overline{\rm MS}$ subtraction, i.e. $\delta_{m(\phi_c)} = m(\phi_c)^4/(4\pi)^2/(4\hat{\varepsilon})$, the renormalized potential reads then as

$$V_{\rm eff}(\phi_c) = \frac{\mu^2}{2}\phi_c^2 + \frac{\lambda}{4}\phi_c^4 + \frac{1}{4}\frac{m(\phi_c)^4}{(4\pi)^2}\log\frac{m(\phi_c)^2}{M^2}\,. \tag{14}$$

Now, we focus on the case of $\mu = 0$. We want to prove that even for $\mu = 0$ SSB occurs at one-loop (namely a non-vanishing minimum) in contrast to the SM Higgs mechanism where a non-vanishing VEV arises setting $\mu^2 < 0$ in eq. (10). For $\mu = 0$, the effective potential can be concisely written into

$$V_{\rm eff}(\phi_c) = \frac{\lambda}{4}\phi_c^4\left[1 + \frac{9\lambda}{(4\pi)^2}\log\frac{3\lambda\phi_c^2}{M^2}\right] \tag{15}$$

We can easily check that the minimum of the one-loop potential is at a non-zero value, $\phi_c = v$:

$$v^2 = \frac{M^2}{3\lambda}\exp\left(-\frac{(4\pi)^2}{9\lambda} - \frac{1}{2}\right)$$

whereas the minimum of the original potential in eq. (10) for $\mu = 0$ is at $\phi_c = 0$. This is not a trivial application of the Coleman-Weinberg SSB. More specifically, the logarithm from the one-loop potential gives negative values for small values of the field corresponding to a maximum at the origin and opens up the possibility that spontaneous symmetry breaking has already occurred.

Note that the new minimum is at a value where

$$\lambda \log \frac{3\lambda v^2}{M^2} = -\frac{16}{9}\pi^2 + \mathcal{O}(\lambda)\,.$$

is large, namely the minimum is far outside the range for which the one-loop approximation is valid, since higher orders will bring higher powers of $\log \phi_c^2/M^2$ even for small couplings. The problem in this example is that we must balance terms of order λ and $\log \phi_c^2/M^2$ to find a minimum, resulting in a large logarithm. Even though we did not accomplish a spontaneous symmetry breaking for the simple interacting scalar model in eq. (10), the apparatus developed here may serve to show that the phenomena occur in slightly more advanced models.

As will be shown in the next section, if one considers a theory just slightly more complicated, with an extra coupling, it might be another story.

Before leaving this section let's discuss the delicate role of the mass scale M, appearing at one-loop level. In previous calculation, the $\overline{\text{MS}}$ scheme was used but in principle the λ subtraction could be done by different scheme. At the classical level, λ can be determined by measuring $\partial^4 V_0(\phi_c)/\partial^4\phi_c$. We can use this definition to the effective potential, namely

$$\lambda(\tilde{M}) = \frac{1}{6}\frac{\partial^4 V_{\text{eff}}(\phi_c)}{\partial^4\phi_c}\Big|_{\tilde{M}} = \lambda + \frac{9\lambda^2}{(4\pi)^2}\left(\log\frac{3\lambda\tilde{M}^2}{M^2} + \frac{25}{6}\right) \quad (16)$$

At the quantum level, we are forced to define λ at a non-vanishing scale, because the fourth derivative is ill-defined at the origin from the logarithm arising from quantum level. This is the key point of the CW mechanism. An intrinsic scale is introduced by loops even though the classical theory is scale invariant. From eq. (16), the running couplings at two different scales are related by

$$\lambda(\tilde{M}) = \lambda(M)\left[1 + \frac{9\lambda(M)}{(4\pi)^2}\log\frac{\tilde{M}^2}{M^2}\right] \quad (17)$$

In terms of $\lambda(M)$, $V_{\text{eff}}(\phi_c)$ reads as

$$V_{\text{eff}}(\phi_c) = \frac{\lambda(M)}{4}\phi_c^4\left[1 + \frac{9\lambda(M)}{(4\pi)^2}\left(\log\frac{\phi_c^2}{M^2} - \frac{25}{6}\right)\right]\,.$$

$V_{\text{eff}}(\phi_c)$ is totally invariant if we rewrite it in terms of λ at a different scale. The change of the scale M would be compensated by a change of $\lambda(\tilde{M})$ from eq. (17). The choice of the renormalization scale M is therefore totally arbitrary but non-vanishing, and will not affect the physics of the theory. Interestingly, eq. (17) can be written as

$$e^{-1/\beta\lambda(M)}M = e^{-1/\beta\lambda(\tilde{M})}\tilde{M} = \hat{M} \quad (18)$$

where $\beta = 9/(4\pi)^2$ and $\hat{M}$ is the intrinsic scale of quantum the theory, the RGE independent mass scale of the theory. Then, the minimum is simplified at $v^2 = \hat{M}^2\,\exp(11/3)$.

IV. EFFECTIVE POTENTIAL FOR MASSIVE SCALAR QED

Now our goal is to build the effective potential for the following Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu\phi)^\dagger D^\mu\phi - \mu^2\phi\phi^\dagger - \frac{\lambda}{4}(\phi\phi^\dagger)^2, \quad (19)$$

where $D_\mu = \partial_\mu + ig\,Z_\mu$, with g the gauge self-coupling. With respect to the last section, we need to calculate $\Delta^{-1}(\phi_c, x, y)$ in eq. (9) for either the two reals components of ϕ and the physical ones of Z_μ. First, we shift the fields [6]

$$\phi = \phi_c + \frac{\sigma + i\pi}{\sqrt{2}}\,, \quad \phi^\dagger = \phi_c + \frac{\sigma - i\pi}{\sqrt{2}}\,, \quad Z_\mu = Z_\mu$$

where σ, π are two real scalar fields. Z_μ is unshifted because of Lorentz invariance, $Z_\mu^{(c)} = 0$. The $\Delta^{-1}(\phi_c, x-y)$ contributions in eq. (7) for σ and π fields is then [7].

$$\begin{aligned} i\Delta_\sigma^{-1}(\phi_c, x-y) = \frac{\partial^2\mathcal{L}}{\partial\sigma(x)\partial\sigma(y)} &= (-\partial^2 - m_\sigma^2)\delta(\vec{x} - \vec{y}) \\ i\Delta_\phi^{-1}(\phi_c, x-y) = \frac{\partial^2\mathcal{L}}{\partial\pi(x)\partial\pi(y)} &= (-\partial^2 - m_\pi^2)\delta(\vec{x} - \vec{y}) \end{aligned}$$

where $m_\sigma^2 = \mu^2 + 3\lambda\phi_c^2/2$ and $m_\pi^2 = \mu^2 + \lambda\phi_c^2/2$.

The $\Delta^{-1}(\phi_c, x, y)$ contributions to Z_μ needs some care. Because of gauge invariance, not all the Z_μ components are physical. The Z_μ quadratic part from eq. (19) reads

$$\mathcal{L}_Z = \frac{1}{2}Z^\mu\left[g_{\mu\nu}(\partial^2 + g^2\phi_c^2) - \partial_\mu\partial_\nu\right]Z^\nu \quad (20)$$

which has rank zero. For Z_μ, because of $m_Z^2 = 2g^2\phi_c^2$, only the transverse components are physical [8]. Let's define the transverse and longitudinal components of the electromagnetic field through

$$Z_T^\mu = (P_T)^\mu_\nu Z^\nu \qquad Z_L^\mu = (P_L)^\mu_\nu Z^\nu$$

where $(P_T)^\mu_\nu = g^\mu_\nu - \frac{\partial^\mu\partial_\nu}{\partial^2}$ and $(P_L)^\mu_\nu = \frac{\partial^\mu\partial_\nu}{\partial^2}$ are projection operators, i.e. $P_T^2 = P_T$, $P_L^2 = P_L$, $P_T P_L = P_L P_T = 0$. Note, P_T has rank three and P_L rank one. This is easily seen in terms of the $P_{L,T}(p)$ Fourier transform. In the rest frame, $P_T(p) = \text{Diag}(0,1,1,1)$ and $P_L(p) = \text{Diag}(1,0,0,0)$. Now, eq. (20) reads as

$$\begin{aligned} \mathcal{L}_Z &= \frac{1}{2}Z_\mu\left[(\partial^2 + m_Z^2)(P_T)^\mu_\nu + m_Z^2(P_L)^\mu_\nu\right]Z^\nu\,, \\ &= \frac{1}{2}Z_{T\mu}\left[\partial^2 + m_Z^2\right]Z_T^\mu + \frac{m_Z^2}{2}Z_{L\mu}Z_L^\mu\,. \end{aligned}$$

$i\Delta^{-1}_{Z_{T,L}}(\phi_c, x-y)$ for Z_T and Z_L are respectively

$$\frac{\partial^2 \mathcal{L}_Z}{\partial Z_T^2} = (\partial^2 + m_Z^2)\delta(\vec{x}-\vec{y}) \,, \quad \frac{\partial^2 \mathcal{L}}{\partial Z_L^2} = m_Z^2 \delta(\vec{x}-\vec{y})$$

where $T = 1, 2, 3$. After the Fourier transform, the $V_{\text{eff}}(\phi_c)$ in eq. (9) reads

$$V_{\text{eff}}(\phi_c) = V_0(\phi_c) - \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\left(\log(p^2-m_\sigma^2) + \log(p^2-m_\pi^2)\right) - \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\left(\sum_{T=1}^{3}\log(-p^2+m_Z^2) + \log m_Z^2\right)$$

The integrals are similar to the ones of the last section (the one $\propto \log m_Z^2$ shifts $V_{\text{eff}}(\phi_c)$ by a constant). Thus,

$$V_{\text{eff}}(\phi_c) = \mu^2\phi_c^2 + \frac{\lambda}{4}\phi_c^4 + \frac{1}{4}\frac{m_\sigma^4}{(4\pi)^2}\log\frac{m_\sigma^2}{M^2} + + \frac{1}{4}\frac{m_\pi^4}{(4\pi)^2}\log\frac{m_\pi^2}{M^2} + \frac{3}{4}\frac{m_Z^4}{(4\pi)^2}\log\frac{m_Z^2}{M^2}$$

Any real scalar field gives the same contribution as in the previous section, whereas Z_T counts for three.

Now, we focus on the case of $\mu = 0$

$$V_{\text{eff}}(\phi_c) = \frac{1}{4}\phi_c^4\left[\lambda \; + \frac{3g^4}{4\pi^2}\log\frac{2g^2\phi_c^2}{M^2} + \frac{\lambda^2}{64\pi^2}\left(9\log\frac{3\lambda\phi_c^2}{2M^2} + \log\frac{\lambda\phi_c^2}{2M^2}\right)\right] \tag{21}$$

As we show later, to find the minimum of the potential it is enough to work in the approximation $\lambda \lesssim g^4$. Then, we can neglect in eq. (21) the λ^2 terms with respect to the λ and g^4 ones.

$$V_{\text{eff}}(\phi_c) = \frac{1}{4}\phi_c^4\left[\lambda + \frac{3g^4}{4\pi^2}\log\frac{2g^2\phi_c^2}{M^2}\right] \tag{22}$$

By studying the zeros of the derivative

$$\frac{\partial V_{\text{eff}}}{\partial \phi_c} = \phi_c^3\left[\lambda + \frac{3g^4}{8\pi^2}\left(1 + 2\log\frac{2g^2\phi_c^2}{M^2}\right)\right] = 0\,. \tag{23}$$

we find that the minimum is at $\phi_c = v$:

$$v^2 = \frac{M^2}{2g^2}\exp\left\{-\frac{4\pi^2\lambda}{3g^4} - \frac{1}{2}\right\}.$$

This is yet another example of dimensional transmutation. The classical theory with $\mu^2 = 0$ is scale invariant, because there is no mass parameter in the theory. The scale M was introduced as the renormalization scale, while λ and g^2 are physical measured at scale M. The theory, however, turns out to develop a mass scale $\phi_c = v$ at the minimum of the potential exponentially suppressed relative to the renormalization scale. Loop corrections trig a non-zero minimum. The minimum condition can be also written as

$$\frac{\partial V_{\text{eff}}}{\partial \phi_c} = 0 \to \lambda(M) = -\frac{3g^4}{8\pi^2}\left(1 + 2\log\frac{2g^2v^2}{M^2}\right) \tag{24}$$

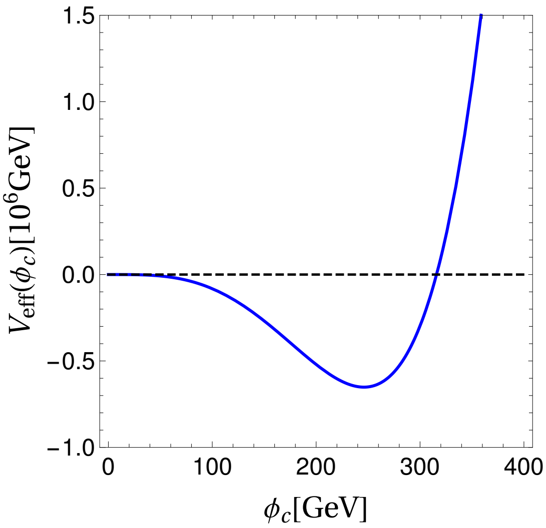

FIG. 1: The CW potential in eq.(24) setting $v = 246$ GeV. SSB occurs with a non-zero mass term.

To reduce large log corrections and make the one-loop calculation reliable, we set the renormalization scale at $M = v$. By using the experiment inputs $v = 246$ GeV and $m_Z = 91$ GeV and setting $g^2 = m_Z^2/v^2 = 0.14$, we get, at the scale $M = v$, $\lambda(v) = 1.1 \times 10^{-3}$, which is perturbative and well in the range of our approximation, i.e. $\lambda \lesssim g^4 = 2 \times 10^{-2}$.

Rewriting $V_{\text{eff}}(\phi_c)$ in eq. (22) in terms of λ from eq. (24), we obtain an expression of $V_{\text{eff}}(\phi_c)$ manifestly independent of the renormalization scale

$$V_{\text{eff}}(\phi_c) = -\frac{3g^4\phi_c^4}{32\pi^2}\left(1 - 2\log\frac{\phi_c^2}{v^2}\right).$$

Which has a maximum at $\phi_c = 0$, whereas a minimum at $\phi_c = v$, as we can see in Fig. 1.

Now we can predict the Higgs boson mass by CW potential. The Higgs mass is the value of the curvature of the effective potential around the minimum,

$$m_H^2 = \frac{1}{2}\frac{\partial^2 V_{\text{eff}}}{\partial \phi_c^2}\bigg|_{\phi_c = v} = \frac{3g^4\, v^2}{4\pi^2} \equiv \frac{3}{16\pi^2}\,\frac{m_Z^4}{v^2} \tag{25}$$

As we have earlier mentioned, this value does not depend on the value we picked for M, since its dependency vanishes compensated by the $\lambda(M)$ factor at (24). Numerically, we obtain

$$m_H = 9.5 \text{ GeV}$$

We have achieved a minimum in the perturbative regions thanks to the introduction of the g coupling. However, this m_H value does not agree with the experimental one. More complicated models are needed, by extending the Higgs sector (with extra λ self-couplings) or the gauge sectors (with much heavier bosons (see eq. (25))).

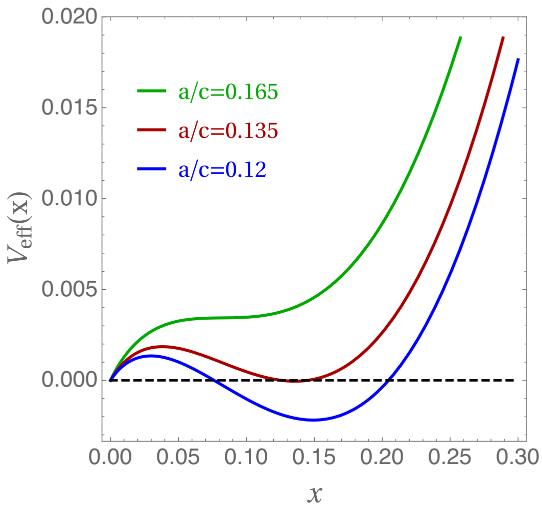

FIG. 2: Different shapes of the potential in eq.(26) depending on the value of a.

A. Studies of the CW potential

Now we are interested in studying some features of the CW potential for $\mu^2 \geq 0$, but still disregarding the terms proportional to λ^2. To this purpose, it is useful to regard the potential as function of $x = 2g^2\phi_c^2/M^2$

$$\hat{V}(x) = \frac{V(x)}{M^4} = ax + bx^2 + cx^2 \log x \tag{26}$$

where $a = \mu^2/2g^2M^2$, $b = \lambda/2(2g^2)^2$ and $c = 3/64\pi^2$. In our analysis, we apply $b \simeq c$ and vary a, namely μ. We can check in Fig. 2 that two different regimes appear. For $a/c \gtrsim 0.135$, the potential has a minimum at $x = 0$ (no dynamical generation of mass happens). In this case, the μ^2 term is much larger than λ, so that the quantum corrections can't balance the μ^2. However, for $a/c \lesssim 0.135$, a new minimum appears at a non-zero value. Being an absolute minimum, it would be the ground state of theory and would explain the dynamical generation of mass. In this case, the μ^2 term is comparable to λ and, thus, quantum corrections can account for the SSB.

V. CONCLUSIONS AND PERSPECTIVES

In this paper we studied several examples of dimensional transmutation, and the following dynamical generation of the Higgs mass. In order to do so, we used the basics of QFT and renormalization group, and mastered the computation of effective potentials for different lagrangians. We have been able to explain how SSB can appear due only to radiative corrections (with the absence of a negative mass term which is a typical strategy in the Higgs SM). Finally, we also have obtained a quantitative value for the Higgs mass which is far away from the measured value, due to the simplicity of the model. More complicated models are needed: either extending the Higgs sector (with extra λ self-couplings) or the gauge sectors (with much heavier bosons -see eq. (25)-). This is far away from the present work. However, it has proven that this methodology can be used in research to try and explain the measured mass of the Higgs.

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[2] M. Peskin and D. Schroeder, "An Introduction to QFT," 1995. Addison-Wesley, Reading, USA, 1995.
[3] R.Jackiw, Phys. Rev. D9, 1686 (1974).
[4] Only the loops up to first order are taken into account
[5] Since we are interested in the renormalization of potential, we only need two counter-terms. We don't need the counter-terms of kinetic terms or, equivalently, of the field renormalization constant.
[6] Thanks to the gauge invariance, we can take ϕ_c to be real without a loss of generality.
[7] We use the Landau gauge, $\partial_\mu Z^\mu = 0$ and the $A^\mu \partial_\mu \pi$ term vanishes upon integration by parts.
[8] In the Landau gauge, $Z_{L\mu} = P_{L\mu\nu} Z^\nu = 0$ plays no role.

Appendix A: Effective potential integral

Here, we explicitly calculate the following integral

$$\int \frac{d^Dk}{(2\pi)^D} \log(k^2 - m) = \frac{d}{d\alpha} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2 - m^2)^\alpha}\bigg|_{\alpha=0}$$

This integral has two reals branching points, $k = \pm m$. We integrate over the Euclidean space (Wick rotation), i.e. $k_E = (ik^0, \vec{k})$ to move the poles to the imaginary axis

$$\frac{d}{d\alpha} \int \frac{i\, d^Dk_E}{(2\pi)^D} \frac{(-1)^\alpha}{(k_E^2 + m^2)^\alpha}\bigg|_{\alpha=0} =$$
$$= \int d\Omega_{D-1} \frac{d}{d\alpha} \int \frac{i\, dk_E}{(2\pi)^D} \frac{(-1)^\alpha k_E^{D-1}}{(k_E^2 + m^2)^\alpha}\bigg|_{\alpha=0} =$$
$$= -\frac{i}{2} \frac{1}{(4\pi)^{D/2}} \Gamma\left(-\frac{D}{2}\right) \left(m^2\right)^{D/2}$$

where $d^Dk_E = d\Omega_{D-1} k_E^{D-1} dk_E$ are spherical coordinates in the D.