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Operator mixing in large N superconformal field theories on S^4 and correlators with Wilson loops

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Abstract: We find a general formula for the operator mixing on the $\mathbb{S}^4$ of chiral primary operators for the $\mathcal{N} = 4$ theory at large N in terms of Chebyshev polynomials. As an application, we compute the correlator of a chiral primary operator and a Wilson loop, reproducing an earlier result by Giombi and Pestun obtained from a two-matrix model proposal. Finally, we discuss a simple method to obtain correlators in general $\mathcal{N} = 2$ superconformal field theories in perturbation theory in terms of correlators of the $\mathcal{N} = 4$ theory.

Keywords: 1/N Expansion, Supersymmetric gauge theory, Wilson, 't Hooft and Polyakov loops

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Contents

1 Introduction

In $\mathcal{N} = 2$ superconformal field theories (SCFT's), correlators of a special type of scalar operators known as chiral primary operators (CPO's) can be computed exactly from a deformed matrix model obtained by adding suitable source terms to the theory on $\mathbb{S}^4$ [1]. Since the construction involves a conformal map from $\mathbb{S}^4$ into $\mathbb{R}^4$, and because the regulated theory on $\mathbb{S}^4$ breaks the $\mathrm{U}(1)_R$ symmetry, mixings among operators with different R-symmetries are possible. Alternatively, since the radius of the $\mathbb{S}^4$ sets a scale which can soak up dimensions, operators of different dimensions can mix. These mixings can be viewed as a manifestation of the conformal anomaly.

In order to take care of the operator mixings, Gerchkovitz et al. [1] proposed a Gram-Schmidt procedure to disentangle the correct operators corresponding to those in $\mathbb{R}^4$. Such procedure was carried out explicitly in [2] for $\mathcal{N} = 4$ Super Yang-Mills theory (SYM) in the large N limit, where the structure of the CPO's dramatically simplifies as only single-trace operators are kept. In particular, it was shown how the expected free-field result is recovered. This is a consequence of a celebrated non-renormalization theorem [3–7].

As discussed in [1], the coefficients encoding the mixings among operators are subject to an ambiguity; as they are defined modulo the addition of holomorphic and antiholomorphic functions of the couplings. This ambiguity can nevertheless be removed by taking appropriate derivatives. Thus, the resulting differentiated mixing coefficients should be observables, very plausibly linked to new manifestations of the conformal anomaly. It is therefore natural to expect the structure of the operator mixings (modulo the holomorphic ambiguity) to be very interesting.

In this note we explore the structure of the operator mixing, focusing in the case of $\mathcal{N} = 4$ SYM in the large N limit (we provide some results for $\mathcal{N} = 2$ superconformal QCD

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as well). As we will see, the mixing coefficients nicely fit into a pattern corresponding to the Chebyshev polynomials (see eq. (2.18)). We will show that this form is exactly what is needed in order to recover the expected correlation functions from the matrix model [2]. Moreover, these operator mixings in the form of Chebyshev polynomials are also exactly what is needed to recover the expected form of the correlators between CPO's and Wilson loops [8–12], thus providing a highly non-trivial check of the relevance of the mixing structure.

It is worth emphasizing that the correlator between a CPO and a Wilson loop has been previously computed in [8–12]. Since the propagators evaluated on the loop become position independent, the computation can be done through a certain two-matrix model encoding the combinatorics of the Feynman diagrams. Interestingly, such matrix model is related to 2d Yang-Mills theory [13]. Here we shall arrive at the same result through a completely different path, thus suggesting an intriguing large N "duality" between matrix models.

This paper is organized as follows: we start section 2.1 with a brief review of the construction of [1] to compute correlators of CPO's and explain the relevance of the operator mixing. After this review section we jump into the new results found in this paper. In section 2.2 we then specialize to $\mathcal{N} = 4$ SYM and establish the generic expression (2.18) encoding such operator mixing. In section 3 we turn to the computation of the correlator of a Wilson loop and a CPO. Following the same logic, we uncover a new representation for these correlators in terms of the $\mathbb{S}^4$ matrix model with the insertion of operators in the form of the Chebyshev polynomials, as in eq. (2.18). This reproduces the same result obtained in [8] in a highly non-trivial way, and confirms the general form of the operator mixing on $\mathbb{S}^4$. In section 4 we turn to $\mathcal{N} = 2$ conformal SQCD, for which we compute the analogous correlator among a Wilson loop and a CPO. Along the way, we find that correlators of CPO's in $\mathcal{N} = 2$ superconformal QCD can be written in terms of correlators of $\mathcal{N} = 4$ CPO's and prove a conjecture [2] about the 2-point function for CPO's in $\mathcal{N} = 2$ SQCD with arbitrary SU(N) gauge group. We close in section 5 with concluding remarks.

2 Operator mixing in $\mathbb{S}^4$ from correlators of chiral primary operators

2.1 Review

Let us start with a brief review of the procedure presented in [1] to extract, from a deformed version of the $\mathbb{S}^4$ partition function, correlators for chiral primary operators; as well as its recent application to the computation of large N correlators in $\mathcal{N} = 4$ SYM and $\mathcal{N} = 2$ superconformal QCD in [2].

Superconformal primary operators, annihilated by the superconformal supersymmetry generators S^i_α and $\bar{S}^i_{\dot{\alpha}}$, play a prominent role in $\mathcal{N} = 2$ theories. Among those, chiral operators (CPO), annihilated as well by the Poincaré supercharges of one chirality $\bar{Q}^i_{\dot{\alpha}}$, are of special relevance. In particular, strong arguments suggest that these operators are always Lorentz scalars satisfying the BPS bound $\Delta = \frac{R}{2}$, being R the U(1)$_R$ charge. This implies that their OPE is non-singular with constant structure functions, endowing these operators with a ring structure. Very recently, there has been much progress in

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understanding the structure of such ring [1, 2, 14–18]. In particular, in [1] it was argued that correlators of CPO's in $\mathbb{R}^4$ can be extracted from the partition function of a deformed version of the theory on $\mathbb{S}^4$. This is possible due to a Ward identity on the $\mathbb{S}^4$, which relates the integrated correlator of particular combination including the top component of the $\mathcal{N}=2$ chiral superfield where the CPO lives, denoted by $\mathcal{C}$, with the (unintegrated) correlator of CPO's $\mathcal{O}$ at North and South poles of the sphere (see [1, 17, 18] for more details). Explicitly,

$$\left(\frac{1}{32\pi^2}\right)^2\int_{\mathbb{S}^4} d^4x\sqrt{g(x)}\int_{\mathbb{S}^4} d^4y\sqrt{g(y)}\,\langle\mathcal{C}_n(x)\overline{\mathcal{C}}_m(y)\rangle_{\mathbb{S}^4}=\langle\mathcal{O}_n(N)\overline{\mathcal{O}}_m(S)\rangle_{\mathbb{S}^4}\,. \tag{2.1}$$

Furthermore, due to supersymmetry, it holds that correlators on $\mathbb{R}^4$ satisfy that [15]

$$\langle\mathcal{O}_{n_1}(x_1)\cdots\mathcal{O}_{n_n}(x_n)\overline{O'}_m(y)\rangle_{\mathbb{R}^4}=\langle O_n(x_1)\overline{O'}_m(y)\rangle_{\mathbb{R}^4}\,,$$

for $O_n(x)\equiv\mathcal{O}_{n_1}(x)\cdots\mathcal{O}_{n_n}(x)$. Since these operators are CPO's and thus satisfy a BPS bound, R-charge conservation sets their correlator to zero unless $\Delta_O=\Delta_{O'}$. Thus, on general grounds their correlator is

$$\langle O_n(x)\overline{O'}_m(0)\rangle_{\mathbb{R}^4}=\frac{C_{nm}}{|x|^{2\Delta_O}}\delta_{\Delta_O\Delta_{O'}}\,. \tag{2.2}$$

This can now be recast as

$$|x|^{2\Delta_O}\,\langle O_n(x)\overline{O'}_m(0)\rangle_{\mathbb{R}^4}=C_{nm}\delta_{\Delta_O\Delta_{O'}}\,. \tag{2.3}$$

In order to relate the correlators on $\mathbb{R}^4$ to the correlators on $\mathbb{S}^4$, one makes use of the fact that $\mathbb{R}^4$ and $\mathbb{S}^4$ metrics are related by a conformal transformation,

$$ds^2_{\mathbb{R}^4}=\left(1+\frac{\vec{x}^2}{4}\right)^2 ds^2_{\mathbb{S}^4}\,. \tag{2.4}$$

Given that $x\to\infty$ in the $\mathbb{S}^4$ corresponds to the North pole, one has

$$\lim_{x\to\infty}|x|^{2\Delta_O}O(x)=4^{\Delta_O}\lim_{x\to\infty}\left(1+\frac{\vec{x}^2}{4}\right)^{\Delta_O}O(x)=4^{\Delta_O}O(N)\,, \tag{2.5}$$

where we used the conformal map induced by (2.4). Conversely, since $x\to 0$ corresponds to the South pole,

$$O(0)=\lim_{x\to 0}\left(1+\frac{\vec{x}^2}{4}\right)^{\Delta}O(x)=O(S)\;. \tag{2.6}$$

Thus, this allows one to relate the $\mathbb{R}^4$ correlator to the $\mathbb{S}^4$ correlator as

$$\langle\Big(\lim_{x\to\infty}|x|^{2\Delta_O}O_n(x)\Big)\,\overline{O'}_m(0)\rangle_{\mathbb{R}^4}=4^{\Delta_O}\,\langle O_n(N)\overline{O'}_m(S)\rangle_{\mathbb{S}^4}=C_{nm}\delta_{\Delta_O\Delta_{O'}}\,. \tag{2.7}$$

Substituting this formula into (2.1), one obtains

$$\left(\frac{1}{32\pi^2}\right)^2\int_{\mathbb{S}^4} d^4x\sqrt{g(x)}\int_{\mathbb{S}^4} d^4y\sqrt{g(y)}\,\langle\mathcal{C}_n(x)\overline{\mathcal{C}}_m(y)\rangle_{\mathbb{S}^4}=4^{-\Delta}\,C_{nm}\,. \tag{2.8}$$

The next problem is to determine the correlators on $\mathbb{S}^4$. A convenient way to generate these correlators is by considering a deformed version of the $\mathbb{S}^4$ partition function by adding sources τ_m for all CPO's, obtaining in this way a deformed partition function $\mathcal{Z}[\{\tau_m, \bar{\tau}_m\}]$ [1]. Here $\tau_2 = \tau_{\rm YM}$, where, as usual, $\operatorname{Im}\tau_{\rm YM} = \frac{4\pi}{g^2_{\rm YM}}$. Correlators can then be computed by

$$\frac{1}{\mathcal{Z}[\{\tau_m, \bar{\tau}_m\}]}\partial_{\tau_m}\partial_{\bar{\tau}_m}\mathcal{Z}[\{\tau_m, \bar{\tau}_m\}] = \left(\frac{1}{32\pi^2}\right)^2 \int_{\mathbb{S}^4} d^4x\sqrt{g(x)} \int_{\mathbb{S}^4} d^4y\sqrt{g(y)}\, \langle \mathcal{C}_n(x)\overline{\mathcal{C}}_m(y)\rangle_{\mathbb{S}^4}\ . \tag{2.9}$$

There is, however, one important subtlety, namely that, due to the conformal anomaly, on the $\mathbb{S}^4$ there is a highly non-trivial operator mixing. This is expected, since the $\mathbb{S}^4$ theory preserves the supergroup $osp(2|4)$, which contains the $\mathrm{SU}(2)_R$ symmetry but breaks the $\mathrm{U}(1)_R$ symmetry. Thus, mixings among different chiral primary operators are possible. Indeed, denoting by r the radius of the $\mathbb{S}^4$, a given operator O_Δ of dimension Δ on $\mathbb{R}^4$, when mapped into $\mathbb{S}^4$, generically mixes with all operators of lower dimensions in steps of 2, that is

$$O^{\mathbb{R}^4}_\Delta \to O^{\mathbb{S}^4}_\Delta + \alpha^{(2)}_\Delta \frac{1}{r^2}\, O^{\mathbb{S}^4}_{\Delta-2} + \alpha^{(4)}_\Delta \frac{1}{r^4}\, O^{\mathbb{S}^4}_{\Delta-4} + \cdots\,. \tag{2.10}$$

Due to this effect, in order to find the expected R-charge conservation on $\mathbb{R}^4$, when mapping the $\mathbb{S}^4$ computation back into the $\mathbb{R}^4$, the operator mixing must be disentangled. This can be accomplished by a Gram-Schmidt orthogonalization procedure. From now on, we set $r = 1$.

Following [1], in the recent work [2] correlators of CPO's in $\mathcal{N} = 4$ SYM have been computed solving the deformed matrix model in the large N limit. In this limit, the set of CPO's dramatically simplifies as only single-trace operators contribute. Thus, the operators in the chiral ring are of the form $O^{\mathbb{R}^4}_n = \operatorname{Tr}\phi^n$, being ϕ one of the complex scalars in the theory (that interpreted to live in the vector multiplet when the theory is viewed as an $\mathcal{N} = 2$ theory). Upon mapping into the $\mathbb{S}^4$, the operators $\operatorname{Tr}\phi^n|_{\mathbb{S}^4}$ acquire a vacuum expectation value (VEV) due to the mixing with the identity operator given by [2][1]

$$\langle \operatorname{Tr}\phi^{2r}\rangle_{\mathbb{S}^4} = N\left(\frac{\lambda}{4\pi^2}\right)^r \frac{\Gamma\left(r+\frac{1}{2}\right)}{\sqrt{\pi}\,(r+1)!} = N\left(\frac{\lambda}{4\pi^2}\right)^r \frac{(2r)!}{4^r r!(r+1)!}\,, \tag{2.11}$$

and $\langle \operatorname{Tr}\phi^{2r+1}\rangle_{\mathbb{S}^4} = 0$. It is useful to define the operator $O^{\mathbb{S}^4}_n$ as the VEV-less version of $\operatorname{Tr}\phi^n$ in $\mathbb{S}^4$, i.e.

$$O^{\mathbb{S}^4}_n = \operatorname{Tr}\phi^n|_{\mathbb{S}^4} - \langle \operatorname{Tr}\phi^n\rangle_{\mathbb{S}^4}\mathbb{1}\ . \tag{2.12}$$

Then, the large-N correlators of $O^{\mathbb{S}^4}_n$ are given by [2]

$$\langle O^{\mathbb{S}^4}_{2n}\overline{O}^{\mathbb{S}^4}_{2r}\rangle_{\mathbb{S}^4} = \left(\frac{\lambda}{4\pi^2}\right)^{n+r} \frac{\Gamma\left(n+\frac{1}{2}\right)\Gamma\left(r+\frac{1}{2}\right)}{\pi\,(n+r)\Gamma(n)\Gamma(r)}\,; \tag{2.13}$$

$$\langle O^{\mathbb{S}^4}_{2n+1}\overline{O}^{\mathbb{S}^4}_{2r+1}\rangle_{\mathbb{S}^4} = \left(\frac{\lambda}{4\pi^2}\right)^{n+r+1} \frac{\Gamma\left(n+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}{\pi\,(n+r+1)\Gamma(n+1)\Gamma(r+1)}\,. \tag{2.14}$$

[1]This VEV can also be read from the VEV of the 1/2 BPS circular Wilson loop, which at large N is given by [19, 20] $\langle W\rangle = \langle \operatorname{Tr}\exp(2\pi\phi)\rangle = \frac{2}{\sqrt{\lambda}}I_1(\sqrt{\lambda})$, by expanding the exponential (see (3.3), (3.1) below).

The map (2.10) between $O_n^{\mathbb{R}^4}$ and $O_n^{\mathbb{S}^4}$ arising from the Gram-Schimidt procedure was explicitly calculated in [2] for the first few operators. One finds

$$\begin{aligned}
O_1^{\mathbb{R}^4} &= O_1^{\mathbb{S}^4}\,; \\
O_2^{\mathbb{R}^4} &= O_2^{\mathbb{S}^4}\,; \\
O_3^{\mathbb{R}^4} &= O_3^{\mathbb{S}^4} - 3\frac{\lambda}{(4\pi)^2}O_1^{\mathbb{S}^4}\,; \\
O_4^{\mathbb{R}^4} &= O_4^{\mathbb{S}^4} - 4\frac{\lambda}{(4\pi)^2}O_2^{\mathbb{S}^4}\,; \\
O_5^{\mathbb{R}^4} &= O_5^{\mathbb{S}^4} - 5\frac{\lambda}{(4\pi)^2}O_3^{\mathbb{S}^4} + 5\frac{\lambda^2}{(4\pi)^4}O_1^{\mathbb{S}^4}\,; \\
O_6^{\mathbb{R}^4} &= O_6^{\mathbb{S}^4} - 6\frac{\lambda}{(4\pi)^2}O_4^{\mathbb{S}^4} + 9\frac{\lambda^2}{(4\pi)^4}O_2^{\mathbb{S}^4}\,; \\
&\cdots
\end{aligned} \tag{2.15}$$

With these ingredients, it was shown in [2] that the first few correlators of $O_n^{\mathbb{R}^4}$ exactly match the general formula

$$\langle O_n^{\mathbb{R}^4}(x)\overline{O}_m^{\mathbb{R}^4}(0)\rangle_{\mathbb{R}^4} = \frac{\delta_{nm}}{|x|^{2\Delta_n}}\frac{\Delta_n\,\lambda^{\Delta_n}}{(2\pi)^{2\Delta_n}}\,, \tag{2.16}$$

being λ the 't Hooft coupling. This formula coincides with the free field theory result due to a non-renormalization theorem [3–7].

2.2 A general expression for the operator mixing in large N $\mathcal{N}=4$ SYM

The mixings in (2.15) appeared in [2] for the first few operators. While the extension to arbitrary dimension is in principle straightforward, it becomes intrinsically more cumbersome to perform the Gram-Schmidt procedure. Our first task will be finding the generic form of the mixing coefficients. To that matter, upon inspection of (2.15) one can identify the general pattern underlying the mixing structure. We find that the operator mixing (2.15) is encoded in the general expression

$$O_n^{\mathbb{R}^4} \to 2\left(\frac{\lambda}{(4\pi)^2}\right)^{\frac{n}{2}}\left[T_n\left(\frac{2\pi x}{\sqrt{\lambda}}\right) - T_n(0)\right], \tag{2.17}$$

where $T_n(x)$ is the n-th Chebyshev polynomial and it is understood that the term x^k in the polynomial stands for the VEV-less operator $O_k^{\mathbb{S}^4}$, defined in (2.12). The formula can be given in a more explicit form as

$$\begin{aligned}
O_n^{\mathbb{R}^4} &= 2\left(\frac{\lambda}{(4\pi)^2}\right)^{\frac{n}{2}}\operatorname{Tr}\left[T_n\left(\frac{2\pi}{\sqrt{\lambda}}\,\phi\right)\right], \qquad n\neq 2\,, \\
O_2^{\mathbb{R}^4} &= \frac{\lambda}{(4\pi)^2}\left(2\operatorname{Tr}\left[T_2\left(\frac{2\pi}{\sqrt{\lambda}}\,\phi\right)\right]+1\right);
\end{aligned} \tag{2.18}$$

where we have used the property

$$\langle \mathrm{Tr}\left[T_n\left(\frac{2\pi}{\sqrt{\lambda}}\,\phi\right)\right]\rangle = -\frac{\delta_{2,n}}{2}\;. \tag{2.19}$$

This can be proved by substituting the expressions for the VEV given in (2.11).[2]

Using (2.18) and the sphere correlators (2.13), we can now prove in full generality (2.16). To that matter, note that

$$T_n(x) = \frac{n}{2}\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}(-1)^k\,\frac{(n-k-1)!}{k!(n-2k)!}(2x)^{n-2k}\,. \tag{2.20}$$

For even n, T_n only contains even powers of x, while for odd n, T_n contains only odd powers of x. Since the correlators (2.13) vanish for $\langle O^{\mathbb{S}^4}_{2n}\overline{O}^{\mathbb{S}^4}_{2m+1}\rangle$, it follows that $\langle O^{\mathbb{R}^4}_{2n}\overline{O}^{\mathbb{R}^4}_{2m+1}\rangle = 0$.

Consider now the case of $\langle O^{\mathbb{R}^4}_{2n+1}\overline{O}^{\mathbb{R}^4}_{2m+1}\rangle_{\mathbb{R}^4}$. Substituting the Chebyshev polynomials (2.20), we find

$$\begin{aligned}\langle O^{\mathbb{R}^4}_{2n+1}\overline{O}^{\mathbb{R}^4}_{2m+1}\rangle_{\mathbb{R}^4} =\;& 4^{2n+1}(2n+1)(2m+1) \\ &\times\sum_{k=0}^{n}\sum_{q=0}^{m}\frac{(-1)^{k+q}\,g^{k+q}\,(2n-k)!\,(2m-q)!}{k!\,q!\,(2n-2k+1)!\,(2m-2q+1)!}\,\langle O^{\mathbb{S}^4}_{2(n-k)+1}\overline{O}^{\mathbb{S}^4}_{2(m-q)+1}\rangle_{\mathbb{S}^4}\,.\end{aligned} \tag{2.21}$$

where we have set, for simplicity, $\frac{\lambda}{(4\pi)^2}\equiv g$. The overall numerical factor 4^{2n+1} stands for the 4^{Δ} factor discussed above, originating from the conformal map $\mathbb{S}^4\to\mathbb{R}^4$. Substituting here the formula (2.13), we obtain

$$\begin{aligned}\langle O^{\mathbb{R}^4}_{2n+1}\overline{O}^{\mathbb{R}^4}_{2m+1}\rangle_{\mathbb{R}^4} =\;& 4^{2n+1}(2n+1)(2m+1)g^{n+m+1} \\ &\times\sum_{k=0}^{n}\sum_{q=0}^{m}\frac{(-1)^{k+q}}{k!\,q!}\,\frac{(2n-k)!\,(2m-q)!}{(n+m-k-q+1)(n-k)!^2(m-q)!^2} \end{aligned}\tag{2.22}$$

$$= (2n+1)\left(\frac{\lambda}{(2\pi)^2}\right)^{2n+1}\delta_{n,m}\,. \tag{2.23}$$

Strikingly, this exactly reproduces (2.16) for odd-dimensional operators.

Next, for even-dimensional operators the analogous computation reads

$$\begin{aligned}\langle O^{\mathbb{R}^4}_{2n}\overline{O}^{\mathbb{R}^4}_{2m}\rangle_{\mathbb{R}^4} =\;& 4^{2n}(2n)(2m)g^{n+m} \\ &\times\sum_{k=0}^{n-1}\sum_{q=0}^{m-1}\frac{(-1)^{k+q}\,(2n-k-1)!\,(2m-q-1)!}{(n+m-k-q)k!\,q!(n-k)!(n-k-1)!(m-q)!(m-q-1)!}\end{aligned} \tag{2.24}$$

$$= 2n\left(\frac{\lambda}{(2\pi)^2}\right)^{2n}\delta_{n,m}\,, \tag{2.25}$$

which exactly reproduces the general result (2.16) for even-dimensional operators.

[2]The particular combination of factors $2b^{\frac{n}{2}}T_n(\frac{x}{2b})$ goes in the literature under the name of bivariate Chebyshev polynomials.

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We have seen that the mapping between $\mathbb{R}^4$ operators and $\mathbb{S}^4$ operators in large N $\mathcal{N}=4$ SYM is given by the Chebyshev polynomials (2.18). We can now extract the explicit formulas for the operator mixing coefficients. Comparing (2.10) with (2.18), we have that

$$\alpha_n^{(2k)} = (-1)^k\, n\, \frac{(n-k-1)!}{k!(n-2k)!}\left(\frac{\lambda}{(4\pi)^2}\right)^k . \tag{2.26}$$

As discussed in [1], these coefficients are scheme-dependent and can be redefined as

$$\alpha_n^{(2k)} \to \alpha_n^{(2k)} + f_n^{(2k)}(\tau_{\rm YM}) + \bar{f}_n^{(2k)}(\bar{\tau}_{\rm YM}) .$$

Nevertheless, an unambiguous quantity, which we will denote as $A_n^{(2k)}$, can be constructed as

$$A_n^{(2k)} = \partial_{\tau_{\rm YM}}\partial_{\bar{\tau}_{\rm YM}}\alpha_n^{(2k)} . \tag{2.27}$$

In the case at hand, we find

$$A_n^{(2k)} = (-1)^k \frac{n(k+1)(n-k-1)!}{4N^2(k-1)!(n-2k)!}\frac{\lambda^{2+k}}{(4\pi)^{2+2k}} . \tag{2.28}$$

It would be extremely interesting to understand the physical significance of the $A_n^{(2k)}$, in particular, as a manifestation of the conformal anomaly.

3 Correlators of chiral primary operators and Wilson loops in $\mathcal{N}=4$ SYM

Let us turn our attention to the correlator between a chiral primary operator and a Wilson loop [8, 9]. We consider the family of 1/8 supersymmetric Wilson loops introduced in [21–23]. These are constrained to live in an S^2 inside $\mathbb{R}^4$ at $x_4=0$ and satisfying $x_1^2+x_2^2+x_3^2=r^2$. Generically, these Wilson loops couple to three out of the six real scalars of the $\mathcal{N}=4$ SYM theory, which we may denote by X_i, as

$$W_R(C) = {\rm Tr}_R\, {\rm P}\, e^{\oint_C\left(A_j+i\epsilon_{ijk}X_i\,\frac{y^k}{r}\right)dy^j} . \tag{3.1}$$

A specially interesting class of operators are those of the form ${\rm Tr}(X_{\rm n}+iY)^n$, where $X_{\rm n}=\sum\frac{x_i}{r}X_i$ and Y stands for any of the remaining three scalars [8, 24]. These operators share 2 supersymmetries with the Wilson loop.

We may now choose the loop to lie on the maximal circle — so that it becomes 1/2 BPS — at $x^3=0$, when it couples only to X_3. Moreover, we can insert the scalar operators at $\{x_1,\,x_2,\,x_3\}=\{0,\,0,\,\pm r\}$, so that $X_{\rm n}=\pm X_3$. Then, the combination $\phi=\pm X_3+iY$ is just a CPO for which the results of [1, 2] apply. Thus, let us consider the correlator $\langle O_n^{\mathbb{R}^4}(0)W\rangle_{\mathbb{R}^4}$, with $O_n^{\mathbb{R}^4}={\rm Tr}\,\phi^n(0)$. Note that r is then the distance between the insertion point and the center of the loop. Following the same procedure as in the previous section, we can then map this into the $\mathbb{S}^4$, so that

$$\langle O_n^{\mathbb{R}^4}(0)W\rangle_{\mathbb{R}^4} = \frac{1}{r^n}\langle O_n^{\mathbb{R}^4}(S)W\rangle_{\mathbb{S}^4} . \tag{3.2}$$

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In the remainder of this section, we will compute (3.2) and then compare it against the results in [8]. To that matter, the map (2.18) will be crucial.

It is useful to first compute the vacuum expectation value of the 1/2 BPS Wilson loop, which amounts to the insertion of $\mathrm{Tr}\, e^{2\pi\phi}$. Expanding the exponential, it follows that

$$\langle W \rangle_{\mathbb{S}^4} = \sum_{k=0}^{\infty} \frac{(2\pi)^n}{n!} \langle \mathrm{Tr}\, \phi^n \rangle \ . \tag{3.3}$$

Substituting (2.11) into (3.3), we find

$$\langle W \rangle_{\mathbb{S}^4} = \frac{2}{\sqrt{\lambda}}\, I_1 \left(\sqrt{\lambda} \right) \ , \tag{3.4}$$

where I_n denotes, as usual, the modified Bessel function of the first kind. This reproduces the familiar formula for the VEV of the circular Wilson loop found in in [19, 20]. Note that we could have proceeded the other way, and read off the VEV's for the operators from the expansion of the Wilson loop in powers of λ, as the coefficient of each term would be the desired VEV. This prescription can also be used to determine the complete $1/N$ expansion of the vacuum expectation value of single trace CPO's. The exact result is the formula (2.11) multiplied by the factor $B(r,N)$ in eq. (A.9) in [20] (it can also be computed ab initio using orthogonal polynomials).

Next, consider

$$\langle O_n^{\mathbb{R}^4} W \rangle_{\mathbb{S}^4} = \sum_{r=0}^{\infty} \frac{(2\pi)^r}{r!} \langle O_n^{\mathbb{R}^4}\, \mathrm{Tr}\, \overline{\phi}^r \rangle_{\mathbb{S}^4} \ . \tag{3.5}$$

Note that in writing this we are simply using the results of localization, which permits to compute the Wilson loop in terms of local operators, (3.3). Because of localization, only the constant part of the scalar field of the vector multiplet appears in the matrix model. We also use the fact that the preserved SUSY's between the CPO and the loop are the same — and therefore we can import (3.3).

In (3.5), the operator $O_n^{\mathbb{R}^4}$ is to be interpreted through the operator mixing formula (2.18), that is, in terms of the VEV-less operators in $\mathbb{S}^4$ inside the n-th Chebyshev polynomial. Note that $\mathrm{Tr}\, \overline{\phi}^r$ does not correspond to a VEV-less operator, and hence the expressions (2.13) do not directly apply. Nevertheless, we may write $\langle O_n^{\mathbb{R}^4}\, \mathrm{Tr}\, \overline{\phi}^r \rangle_{\mathbb{S}^4} = \langle O_n^{\mathbb{R}^4} O_r^{\mathbb{S}^4} \rangle_{\mathbb{S}^4} + \langle \mathrm{Tr}\, \overline{\phi}^r \rangle_{\mathbb{S}^4} \langle O_n^{\mathbb{R}^4} \rangle_{\mathbb{S}^4}$. Since $O_n^{\mathbb{R}^4}$ is a polynomial in terms of the $O_n^{\mathbb{S}^4}$, whose VEV vanishes, the last term is zero. Therefore we can write

$$\langle O_n^{\mathbb{R}^4} W \rangle_{\mathbb{S}^4} = \sum_{r=0}^{\infty} \frac{(2\pi)^r}{r!} \langle O_n^{\mathbb{R}^4} \overline{O}_r^{\mathbb{S}^4} \rangle_{\mathbb{S}^4} \ . \tag{3.6}$$

Writing the operator $O_n^{\mathbb{R}^4}$ in terms of the Chebyshev polynomials, (2.18), using the expansion (2.20) and computing the two-point functions using (2.13), we find

$$\begin{aligned} \langle O_{2n}^{\mathbb{R}^4} W \rangle_{\mathbb{S}^4} &= 2n \sum_{s=0}^{\infty} \frac{(2\pi)^{2s}}{s!(s-1)!} \left(\frac{\lambda}{(4\pi)^2} \right)^{n+s} \sum_{k=0}^{n-1} \frac{(-1)^k (2n-k-1)!}{(n+s-k)k!(n-k-1)!(n-k)!} \\ &= 2n \sum_{s=n}^{\infty} \frac{(2\pi)^{2s}}{(s-n)!(s+n)!} \left(\frac{\lambda}{(4\pi)^2} \right)^{n+s} \end{aligned}$$

$$
\begin{aligned}
&= 2n\left(\frac{\lambda}{8\pi}\right)^{2n}\sum_{r=0}^{\infty}\frac{1}{r!(r+2n)!}\left(\frac{\lambda}{4}\right)^{r} \\
&= 2n\left(\frac{\lambda}{(4\pi)^2}\right)^{n} I_{2n}(\sqrt{\lambda})\,,
\end{aligned}
\tag{3.7}
$$

Similarly,

$$
\begin{aligned}
\langle O^{\mathbb{R}^4}_{2n+1}W\rangle_{\mathbb{S}^4} &= (2n+1)\sum_{s=0}^{\infty}\frac{(2\pi)^{2s+1}}{s!^2}\left(\frac{\lambda}{(4\pi)^2}\right)^{n+s+1}\sum_{k=0}^{n}\frac{(-1)^k(2n-k)!}{(n+s-k+1)k!(n-k)!^2} \\
&= (2n+1)\sum_{s=n}^{\infty}\frac{(2\pi)^{2s+1}}{(s-n)!(s+n+1)!}\left(\frac{\lambda}{(4\pi)^2}\right)^{n+s+1} \\
&= (2n+1)\left(\frac{\lambda}{8\pi}\right)^{2n+1}\sum_{r=0}^{\infty}\frac{1}{r!(r+2n+1)!}\left(\frac{\lambda}{4}\right)^{r} \\
&= (2n+1)\left(\frac{\lambda}{(4\pi)^2}\right)^{n+\frac{1}{2}} I_{2n+1}(\sqrt{\lambda})\,.
\end{aligned}
\tag{3.8}
$$

Thus

$$
\langle O^{\mathbb{R}^4}_{n}W\rangle_{\mathbb{S}^4} = n\left(\frac{\lambda}{(4\pi)^2}\right)^{\frac{n}{2}} I_{n}(\sqrt{\lambda})\,,
\tag{3.9}
$$

which is in perfect agreement with [8], yet through a completely different matrix model — see eq. (4.21) in [8] for $A_1 = A_2 = \frac{A}{2} = 2\pi$. The two-matrix model in [8] contains two matrices X and Y. The connection with the Gaussian matrix model used in the present paper (proposed in [20] and derived from supersymmetric localization [13]) seems to arise in the process of integrating out the Y matrix, which effectively generates the operator mixing in terms of the Chebyshev polynomial prescription. This is clear after taking the large N limit, i.e. to the level of solutions to the saddle-point equations. It would be extremely interesting to investigate the mechanism behind this duality in the most general context.

4 Correlators for chiral primary operators and Wilson loops in $\mathcal{N}=2$ superconformal SQCD

We can extend our computations of correlators between Wilson loops and CPO's to the case of other $\mathcal{N}=2$ theories. In perturbation theory, one can express correlators in any $\mathcal{N}=2$ superconformal field theory in terms of correlators of the $\mathcal{N}=4$ theory. As an illustration, we consider the example of $\mathcal{N}=2$ superconformal SCQD. For this case, the map between $\mathbb{R}^4$ and $\mathbb{S}^4$ operators was partially worked out in [2] at weak coupling. In the case of $\mathcal{N}=2$ superconformal SQCD, the map $O^{\mathbb{R}^4}\to O^{\mathbb{S}^4}$ will be given through a power-series correction to the Chebyshev polynomial prescription of $\mathcal{N}=4$ theory. This can be traced back to the fact that the partition function for $\mathcal{N}=2$ superconformal QCD is (we assume that the gauge group is U(N) for definiteness)

$$\mathcal{Z}_{\mathcal{N}=2} = \int d^N a\ \Delta(a) \frac{\prod_{i<j} H(a_i - a_j)^2}{\prod_i H(a_i)^{2N}} \left| e^{-2\pi \operatorname{Im} \tau_{\rm YM} \sum a_i^2} \right| \mathcal{Z}_{\rm inst}\,, \tag{4.1}$$

$$H(x) = \prod_{n=1}^{\infty} \left(1 + \frac{x^2}{n^2}\right)^{n^2} e^{-\frac{x^2}{n}}\,.$$

We shall consider the perturbative expansion in the zero instanton number sector, so we set $\mathcal{Z}_{\rm inst} \to 1$. A perturbative series is obtained by expanding the one-loop factor in powers of a_i. We use

$$\ln H(x) = -\sum_{n=2}^{\infty} (-1)^n \frac{\zeta(2n-1)}{n} x^{2n}\,. \tag{4.2}$$

Now we can expand $\mathcal{Z}_{\mathcal{N}=2}$ as

$$\mathcal{Z} = \mathcal{Z}_{\mathcal{N}=4} \Big\{ 1 - \zeta(3) \left(3\langle \operatorname{Tr} \phi^2 \operatorname{Tr} \overline{\phi}^2 \rangle_{S^4}^{\mathcal{N}=4} - 4\langle \operatorname{Tr} \phi^3 \operatorname{Tr} \overline{\phi} \rangle_{S^4}^{\mathcal{N}=4} \right) \tag{4.3}$$

$$-\frac{2}{3}\zeta(5) \left(10\langle \operatorname{Tr} \phi^3 \operatorname{Tr} \overline{\phi}^3 \rangle_{S^4}^{\mathcal{N}=4} - 15\langle \operatorname{Tr} \phi^4 \operatorname{Tr} \overline{\phi}^2 \rangle_{S^4}^{\mathcal{N}=4} + 6\langle \operatorname{Tr} \phi^5 \operatorname{Tr} \overline{\phi}^1 \rangle_{S^4}^{\mathcal{N}=4} \right) + \cdots \Big\}\,;$$

where $\mathcal{Z}_{\mathcal{N}=4}$ is the U(N) $\mathcal{N} = 4$ SYM partition function and $\langle \operatorname{Tr} \phi^n \operatorname{Tr} \overline{\phi}^m \rangle_{S^4}^{\mathcal{N}=4}$ refers to the 2-point function of the $\operatorname{Tr} \phi^n$, $\operatorname{Tr} \overline{\phi}^m$ operators in the $\mathcal{N} = 4$ SYM matrix model on the S^4. In this way we write the partition function for $\mathcal{N} = 2$ superconformal QCD solely in terms of quantities in $\mathcal{N} = 4$ SYM at arbitrary N. In fact, using the results in the previous sections one can see that, at large N

$$\mathcal{Z}_{\mathcal{N}=2} = \mathcal{Z}_{\mathcal{N}=4} \left\{ 1 - \frac{3N^4 \zeta(3)}{16\pi^2 \operatorname{Im} \tau_{\rm YM}^2} + \frac{30N^5 \zeta(5)}{96\pi^3 \operatorname{Im} \tau_{\rm YM}^3} + \cdots \right\}. \tag{4.4}$$

Since, for instance, the correlator $\langle O_2^{\mathbb{R}^4} \overline{O}_2^{\mathbb{R}^4} \rangle_{\mathbb{R}^4} = \langle O_2^{\mathbb{S}^4} \overline{O}_2^{\mathbb{S}^4} \rangle_{\mathbb{S}^4} \sim \partial_{\tau_{\rm YM}} \partial_{\overline{\tau}_{\rm YM}} \ln \mathcal{Z}_{\mathcal{N}=2}$, we see that the correlator in $\mathcal{N} = 2$ superconformal SQCD can be written in terms of correlators in $\mathcal{N} = 4$. It is clear that, *mutatis mutandi*, this result extends to all other correlators in $\mathcal{N} = 2$ superconformal QCD.

Let us now turn to the correlator between CPO's and Wilson loops. For simplicity, we will focus on the correlator between $O_2^{\mathbb{R}^4} = \operatorname{Tr} \phi^2$ and the Wilson loop. As shown in [2], $O_2^{\mathbb{R}^4} \to O_2^{\mathbb{S}^4}$. Moreover, the correlators between $O_2^{\mathbb{S}^4}$ and any other even-dimensional operators can be read off from the formula

$$\langle O_{2n}^{\mathbb{S}^4} \overline{O}_{2m}^{\mathbb{S}^4} \rangle_{\mathbb{S}^4} = \left(\frac{\lambda}{4\pi^2}\right)^{m+n} \frac{\Gamma\left(m+\frac{1}{2}\right) \Gamma\left(n+\frac{1}{2}\right)}{\pi(m+n)\Gamma(n)\Gamma(m)} \tag{4.5}$$

$$-\frac{3}{4}\zeta(3) \left(\frac{\lambda}{4\pi^2}\right)^{m+n+2} \frac{(3+m+n+mn)\Gamma\left(m+\frac{1}{2}\right) \Gamma\left(n+\frac{1}{2}\right)}{\pi(m+1)(n+1)\Gamma(m)\Gamma(n)} + \cdots\,;$$

where the first line corresponds to the leading order term, which is identical to that of the free field theory (and consequently analogous to that of $\mathcal{N} = 4$ SYM due to the non-renormalization theorem) and the second represents the NLO correction. Correlators with odd dimensional operators vanish.

Consider the correlator

$$\langle O_2^{\mathbb{R}^4} W \rangle_{\mathbb{S}^4}^{\text{SQCD}} = \sum_{r=0}^{\infty} \frac{(2\pi)^{2r}}{(2r)!} \langle O_2^{\mathbb{S}^4} \overline{O}_{2r}^{\mathbb{S}^4} \rangle_{\mathbb{S}^4} \,. \tag{4.6}$$

Substituting the 2-point function (4.5), we find

$$\begin{aligned}
\langle O_2^{\mathbb{R}^4} W \rangle_{\mathbb{S}^4}^{\text{SQCD}} &= \frac{\lambda}{8\pi^2} \sum_{r=1}^{\infty} \frac{1}{(r-1)!(r+1)!} \frac{\lambda^r}{4^r} - \frac{3\zeta(3)\lambda^3}{512\pi^6} \sum_{r=1}^{\infty} \frac{(2+r)}{(r-1)!(r+1)!} \frac{\lambda^r}{4^r} \\
&= \frac{\lambda}{8\pi^2} I_2(\sqrt{\lambda}) \left(1 - \frac{9\zeta(3)}{64\pi^4}\lambda^2\right) - \frac{3\zeta(3)}{1024\pi^6} \lambda^{\frac{7}{2}} I_3(\sqrt{\lambda}) + \cdots \\
&= \frac{\lambda^2}{64\pi^2} + \frac{\lambda^3}{768\pi^2} + \frac{\lambda^4}{24576\pi^2} \left(1 - \frac{54\zeta(3)}{\pi^4}\right) + \cdots \,.
\end{aligned} \tag{4.7}$$

This formula generalizes the results by Giombi and Pestun to $\mathcal{N} = 2$ superconformal QCD. It can be extended to other $\mathcal{N} = 2$ superconformal field theories by following the same procedure. One may also compute the analogous correlators for CPO's of higher dimensions, though they are much more complicated because of the operator mixing. One writes $\langle O_n^{\mathbb{R}^4} W \rangle_{\mathbb{S}^4}^{\text{SQCD}} = \sum c_n \langle O_n^{\mathbb{S}^4} W \rangle_{\mathbb{S}^4}^{\text{SQCD}}$, where the first few c_n's can be read off from the explicit map given in [2].

4.1 The SU(N) case

As emphasized above, (4.3) is valid for any gauge group. Let us now particularize it and compute the partition function for $\mathcal{N} = 2$ SQCD with SU(N) gauge group at finite N. Imposing that $\operatorname{Tr}\phi = 0$, we get

$$\mathcal{Z}_{\mathcal{N}=2}^{\text{SU}(N)} = \mathcal{Z}_{\mathcal{N}=4}^{\text{SU}(N)} \left\{ 1 - 3\zeta(3) \langle \operatorname{Tr}\phi^2 \operatorname{Tr}\overline{\phi}^2 \rangle_{S^4}^{\mathcal{N}=4,\text{SU}(N)} + \cdots \right\} . \tag{4.8}$$

We stress that we will be interested on the perturbative part. This corresponds to zero instanton number (for non-zero instanton number, one needs to project out the contribution of the U(1) vector multiplet from the U(N) instanton partition function).

It is useful now to re-write $\langle \operatorname{Tr}\phi^2 \operatorname{Tr}\overline{\phi}^2 \rangle_{S^4}^{\mathcal{N}=4,\text{SU}(N)}$ in terms of VEV-less operators O_2 in the SU(N) $\mathcal{N} = 4$ theory as

$$\langle \operatorname{Tr}\phi^2 \operatorname{Tr}\overline{\phi}^2 \rangle_{S^4}^{\mathcal{N}=4,\text{SU}(N)} = \langle O_2 \overline{O}_2 \rangle_{S^4}^{\mathcal{N}=4,\text{SU}(N)} + V_2^2 \,, \tag{4.9}$$

being V_2 the VEV of $\operatorname{Tr}\phi^2$ in the SU(N) theory. Note that both V_2 and $\langle \operatorname{Tr}\phi^2 \operatorname{Tr}\overline{\phi}^2 \rangle_{S^4}^{\mathcal{N}=4,\text{SU}(N)}$ follow from the partition function $\mathcal{Z}_{\mathcal{N}=4}^{\text{SU}(N)}$ through derivatives. Recall that (see e.g. [20])

$$\mathcal{Z}_{\mathcal{N}=4}^{\text{SU}(N)} = \sqrt{2\,N \operatorname{Im}\tau_{\text{YM}}}\, \mathcal{Z}_{\mathcal{N}=4}^{\text{U}(N)} \,; \qquad \mathcal{Z}_{\mathcal{N}=4}^{\text{U}(N)} = (4\pi \operatorname{Im}\tau_{\text{YM}})^{-\frac{N^2}{2}} \, (2\pi)^{\frac{N}{2}} \, G(N+2) \,. \tag{4.10}$$

Therefore

$$\langle O_2 \overline{O}_2 \rangle_{S^4}^{\mathcal{N}=4,\text{SU}(N)} = \pi^{-2} \partial_{\tau_{\text{YM}}} \partial_{\overline{\tau}_{\text{YM}}} \ln \mathcal{Z}_{\mathcal{N}=4}^{\text{SU}(N)} = \frac{N^2-1}{8\pi^2 \operatorname{Im}\tau_{\text{YM}}^2} \,, \tag{4.11}$$

and

$$V_2 = i\pi^{-1} \partial_{\tau_{\text{YM}}} \ln \mathcal{Z}_{S^4}^{\mathcal{N}=4,\text{SU}(N)} = \frac{N^2-1}{4\pi \operatorname{Im}\tau_{\text{YM}}} \,. \tag{4.12}$$

With this at hand

$$\mathcal{Z}^{\mathrm{SU}(N)}_{\mathcal{N}=2} = \mathcal{Z}^{\mathrm{SU}(N)}_{\mathcal{N}=4} \left\{ 1 - \frac{3\zeta(3)\,(N^4-1)}{16\pi^2\,\mathrm{Im}\,\tau^2_{\mathrm{YM}}} + \cdots \right\} . \tag{4.13}$$

Let us now consider correlators for CPO's. The special case of correlators for $\mathrm{Tr}\,\phi^2$ is simple, as these insertions arise from derivatives of the YM coupling τ_{YM}. The case of the SU(N) theory is particularly interesting, since a conjecture for $\langle \mathrm{Tr}\,\phi^2\,\mathrm{Tr}\,\overline{\phi}^2\rangle_{\mathbb{R}^4}$ was put forward in [2]. Using (4.13) one finds

$$\langle \mathrm{Tr}\,\phi^2\,\mathrm{Tr}\,\overline{\phi}^2\rangle^{\mathcal{N}=2,\mathrm{SU}(N)}_{\mathbb{R}^4} = \frac{2(N^2-1)}{\pi^2\,\mathrm{Im}\,\tau^2_{\mathrm{YM}}} - \frac{9\zeta(3)\,(N^2-1)(N^2+1)}{2\pi^4\,\mathrm{Im}\,\tau^4_{\mathrm{YM}}} + \cdots , \tag{4.14}$$

which demonstrates the conjecture of [2]. In the particular case of SU(2) gauge group, this formula reproduces an earlier result given in [15] (see also [1]) and, for SU(3), SU(4), the formula (4.14) also reproduces the expressions given in [1, 16].

5 Conclusions

In this paper we have analyzed the structure of the operator mixing arising when mapping the $\mathbb{R}^4$ theory into the $\mathbb{S}^4$. Concentrating on the $\mathcal{N}=4$ SYM case, and at large N, such operator mixings are elegantly encoded into Chebyshev polynomials. This provides a general expression for the mixing coefficients, whose $\partial_{\tau_{\mathrm{YM}}}\partial_{\overline{\tau}_{\mathrm{YM}}}$ derivative is an unambiguous quantity $A_n^{(2k)}$. It would be be extremely interesting to understand if the gauge-invariant mixing coefficients $A_n^{(2k)}$ can be computed in the holographic dual setup for supergravity in $AdS_5 \times S^5$ space.

We have provided very non-trivial evidence of the relevant role of the Chebyshev polynomial mixing by computing the $\langle W O_n^{\mathbb{R}^4}\rangle_{\mathbb{R}^4}$ correlator. In [8], such correlators were computed by means of a completely different matrix model — namely the two-matrix model. Here we have reproduced the result [8] for the correlation function $\langle W O_n^{\mathbb{R}^4}\rangle_{\mathbb{R}^4}$ from first-principles, using the localized partition function and disentangling the operator mixing induced by mapping into $\mathbb{S}^4$. Thus, our result proves the conjectural relation proposed in [8] with the two-matrix model. In addition, it suggests a novel duality between two completely different matrix models at large N. It would be very interesting to explore such "duality" and understand its origins, in particular under the light of the connection of the two-matrix model to 2d Yang-Mills, and the extent to which this duality holds. In particular, it would be very interesting to understand the generic correlators $\langle W \prod_i O_{n_i}^{\mathbb{R}^4}\rangle_{\mathbb{R}^4}$. Note that the Chebyshev polynomial prescription does not take into account mixing with multitrace operators in the $\mathbb{S}^4$, which, on the other hand, might be relevant whenever there is more than one CPO in the correlator with the Wilson loop. Understanding this point would be very interesting.

We have also computed the $\langle W O_n^{\mathbb{R}^4}\rangle_{\mathbb{R}^4}$ correlator in $\mathcal{N}=2$ superconformal SQCD up to (and including) $O(\lambda^4)$ in perturbation theory. Along the way, we have seen that correlators of CPO's in $\mathcal{N}=2$ superconformal QCD can be written in terms of correlators of CPO's in $\mathcal{N}=4$ SYM. In particular, this allowed us to provide a simple proof of the formula for the finite N two-point function of the $O_2^{\mathbb{R}^4}$ operator conjectured in [2]. Further studying this connection among CPO correlators of $\mathcal{N} = 2$ SQCD and $\mathcal{N} = 4$ SYM would be very interesting.

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