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A MAXIMAL INEQUALITY FOR THE SKOROHOD INTEGRAL

by

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Abstract

In this paper we establish a maximal inequality for the Skorohod integral of stochastic processes belonging to the space $\mathbb{L}^F$ and satisfying an integrability condition. The space $\mathbb{L}^F$ contains both the square integrable adapted processes and the processes in the Sobolev space $\mathbb{L}^{2,2}$. Processes in $\mathbb{L}^F$ are required to be twice weakly differentiable in the sense of the stochastic calculus of variations in points (r, s) such that $r \vee s \geq t$.

1 Introduction

A stochastic integral for processes which are not necessarily adapted to the Brownian motion was introduced by Skorohod in [7]. The Skorohod integral turns out to be a generalization of the classical Itô integral, and on the other hand, it coincides with the adjoint of the derivative operator on the Wiener space. The techniques of the stochastic calculus of variations, introduced by Malliavin in [4], have allowed to develop a stochastic calculus for the Skorohod integral of processes in the Sobolev space $\mathbb{L}^{2,2}$ (see [6]).

In a recent paper ([1]) we have introduced a space of square integrable processes, denoted by $\mathbb{L}^F$, which is included in the domain of the Skorohod integral, and contains both the space of adapted processes and the Sobolev space $\mathbb{L}^{2,2}$. A process $u = \{u_t, t \in [0,T]\}$ in $\mathbb{L}^F$ is required to have square integrable derivatives $D_s u_t$ and $D^2_{r,s} u_t$ in the regions $\{s \geq t\}$ and $\{s \vee r \geq t\}$, respectively. We have proved in [1] that the Skorohod integral of processes in the space $\mathbb{L}^F$ verifies the usual properties (quadratic variation, continuity, local property) and a change-of-variable formula can also be established.

The purpose of this paper is to prove a maximal inequality for processes in the space $\mathbb{L}^F$. Section 2 is devoted to recall some preliminaries on the stochastic calculus for the Skorohod integral. In Section 3 we show the maximal inequality (Theorems 3.1 and 3.2). The main ingredients of the proof are the factorization method used to deduce maximal inequalities for stochastic convolutions (see [2])

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and the Itô formula for the Skorohod integral following the ideas introduced by Hu and Nualart in [3].

2 A class of Skorohod integrable processes

Let $(\Omega, \mathcal{F}, P)$ be the canonical probability space of the one-dimensional Brownian motion $W = \{W_t, t \in [0,T]\}$. Let H be the Hilbert space $L^2([0,T])$. For any $h \in H$ we denote by $W(h)$ the Wiener integral $W(h) = \int_0^T h(t)dW_t$. Let $\mathcal{S}$ be the set of smooth and cylindrical random variables of the form:

$$F = f(W(h_1), ..., W(h_n)), \tag{2.1}$$

where $n \geq 1$, $f \in C_b^\infty(\mathbb{R}^n)$ (f and all its derivatives are bounded), and $h_1, ..., h_n \in H$. Given a random variable F of the form (2.1), we define its derivative as the stochastic process $\{D_tF, t \in [0,T]\}$ given by

$$D_tF = \sum_{j=1}^n \frac{\partial f}{\partial x_j}(W(h_1), ..., W(h_n))h_j(t), \quad t \in [0,T].$$

In this way the derivative DF is an element of $L^2([0,T] \times \Omega) \cong L^2(\Omega; H)$. More generally, we can define the iterated derivative operator on a cylindrical random variable by setting

$$D^n_{t_1,\dots,t_n}F = D_{t_1} \cdots D_{t_n}F.$$

The iterated derivative operator D^n is a closable unbounded operator from $L^2(\Omega)$ into $L^2([0,T]^n \times \Omega)$ for each $n \geq 1$. We denote by $\mathbb{D}^{n,2}$ the closure of $\mathcal{S}$ with respect to the norm defined by

$$\| F \|^2_{n,2} = \| F \|^2_{L^2(\Omega)} + \sum_{l=1}^n \| D^l F \|^2_{L^2([0,T]^l \times \Omega)} \cdot$$

We denote by δ the adjoint of the derivative operator D that is also called the Skorohod integral with respect to the Brownian motion $\{W_t\}$. That is, the domain of δ (denoted by $\operatorname{Dom}\delta$) is the set of elements $u \in L^2([0,T] \times \Omega)$ such that there exists a constant c verifying

$$\left| E \int_0^T D_tFu_tdt \right| \leq c \| F \|_2,$$

for all $F \in \mathcal{S}$. If $u \in \operatorname{Dom}\delta$, $\delta(u)$ is the element in $L^2(\Omega)$ defined by the duality relationship

$$E(\delta(u)F) = E\int_0^T D_tFu_tdt, \quad F \in \mathcal{S}.$$

We will make use of the following notation: $\int_0^T u_t dW_t = \delta(u)$.

The Skorohod integral is an extension of the Itô integral in the sense that the set $L_a^2([0,T] \times \Omega)$ of square integrable and adapted processes is included into Dom δ and the operator δ restricted to $L_a^2([0,T] \times \Omega)$ coincides with the Itô stochastic integral (see [6]).

Let $\mathbb{L}^{n,2} = L^2([0,T]; \mathbb{D}^{n,2})$ equipped with the norm

$$\| v \|_{n,2}^2 = \| v \|_{L^2([0,T]\times\Omega)}^2 + \sum_{j=1}^{n} \| D^j v \|_{L^2([0,T]^{j+1}\times\Omega)}^2 .$$

We recall that $\mathbb{L}^{1,2}$ is included in the domain of δ, and for a process u in $\mathbb{L}^{1,2}$ we can compute the variance of the Skorohod integral of u as follows:

$$E(\delta(u)^2) = E\int_0^T u_t^2 dt + E\int_0^T\int_0^T D_s u_t D_t u_s ds dt. \tag{2.2}$$

Let $\mathcal{S}_T$ be the set of processes of the form $u_t = \sum_{j=1}^q F_j h_j(t)$, where $F_j \in \mathcal{S}$ and $h_j \in H$. We will denote by $\mathbb{L}^F$ the closure of $\mathcal{S}_T$ by the norm:

$$\| u \|_F^2 = E\int_0^T u_t^2 dt + E\int_{\{s\geq t\}} (D_s u_t)^2 ds dt + E\int_{\{r\vee s\geq t\}} (D_r D_s u_t)^2 dr ds dt. \tag{2.3}$$

That is, $\mathbb{L}^F$ is the class of stochastic processes $\{u_t, t \in [0,T]\}$ such that for each time t, the random variable u_t is twice weakly differentiable with respect to the Wiener process in the two-dimensional future $\{(r,s), r \vee s \geq t\}$. The space $L_a^2([0,T]\times\Omega)$ is contained in $\mathbb{L}^F$. Furthermore, for all $u \in L_a^2([0,T]\times\Omega)$ we have $D_s u_t = 0$ for almost all $s \geq t$, and, hence,

$$\| u \|_F = \| u \|_{L^2([0,T]\times\Omega)} . \tag{2.4}$$

The next proposition provides an estimate for the L^2 norm of the Skorohod integral of processes in the space $\mathbb{L}^F$.

Proposition 2.1 $\mathbb{L}^F \subset \operatorname{Dom}\delta$ *and we have that, for all* u *in* $\mathbb{L}^F$,

$$E|\delta(u)|^2 \leq 2 \| u \|_F^2 . \tag{2.5}$$

Proof:

Suppose first that u has a finite Wiener chaos expansion. In this case we can write:

$$E\left|\int_0^t u_s dW_s\right|^2 = E\int_0^t u_s^2 ds + E\int_0^t\int_0^t D_s u_\theta D_\theta u_s d\theta ds$$

$$
\begin{aligned}
&= E\int_0^t u_s^2 ds + 2E\int_0^t\int_0^\theta D_s u_\theta D_\theta u_s d\theta ds \\
&= E\int_0^t u_s^2 ds + 2E\int_0^t u_\theta \left(\int_0^\theta D_\theta u_s dW_s\right) d\theta.
\end{aligned}
$$

Using now the inequality $2\langle a,b\rangle \leq |a|^2 + |b|^2$ we obtain

$$
E|\delta(u)|^2 \leq 2E\int_0^T u_s^2 ds + E\int_0^T |\int_0^\theta D_\theta u_s dW_s|^2 d\theta. \tag{2.6}
$$

Because u has a finite chaos descomposition we have that $\{D_\theta u_s \mathbf{1}_{[0,\theta]}(s), s \in [0,T]\}$ belongs to $\mathbb{L}^{1,2} \subset \operatorname{Dom}\delta$ for each $\theta \in [0,T]$, and furthermore we have

$$
\begin{aligned}
E\int_0^T |\int_0^\theta D_\theta u_s dW_s|^2 d\theta &\leq E\int_0^T\int_0^\theta (D_\theta u_s)^2 ds d\theta \\
&+E\int_0^T\int_0^\theta\int_0^\theta (D_\sigma D_\theta u_s)^2 d\sigma ds d\theta.
\end{aligned} \tag{2.7}
$$

Now substituting (2.7) into (2.6) we obtain

$$
\begin{aligned}
E|\delta(u)|^2 &\leq 2E\int_0^T u_s^2 ds + E\int_0^T\int_0^\theta (D_\theta u_s)^2 ds d\theta \\
&+ E\int_0^T\int_0^\theta\int_0^\theta (D_\sigma D_\theta u_s)^2 d\sigma ds d\theta \\
&\leq 2 \parallel u \parallel_F^2,
\end{aligned}
$$

which proves (2.5) in the case that u has a finite chaos descomposition. The general case follows easily from a limit argument. QED

Note that $u \in \mathbb{L}^F$ implies $u\mathbf{1}_{[r,t]} \in \mathbb{L}^F$ for any interval $[r,t] \subset [0,T]$, and, by Proposition 2.1 we have that $u\mathbf{1}_{[r,t]} \in \operatorname{Dom}\delta$.

The following results, which are proved in [1] are some basic properties for the Skorohod integral of processes u in $\mathbb{L}^F$.

(1) *(Local property for the operator δ)* Let $u \in \mathbb{L}^F$ and $A \in \mathcal{F}$ be such that $u_t(\omega) = 0$, a.e. on the product space $[0,T] \times A$. Then $\delta(u) = 0$ a.e. on A.

(2) *(Quadratic variation)* Let $u \in \mathbb{L}^F$. Then

$$
\sum_{i=1}^{n-1} (\int_{t_i}^{t_{i+1}} u_s dW_s)^2 \to \int_0^t u_s^2 ds, \tag{2.8}
$$

in $L^1(\Omega)$, as $|\pi| \to 0$, where π runs over all finite partitions $\{0 = t_0 < t_1 < \cdots < t_n = T\}$ of $[0,T]$.

The local property allows to extend the Skorohod integral to processes in the space $\mathbb{L}^F_{loc}$. That is, $u \in \mathbb{L}^F_{loc}$ if there exists a sequence $\{(\Omega_n, u^n), n \geq 1\} \subset \mathcal{F} \times \mathbb{L}^F$ such that $u = u^n$ on Ω_n for each n, and $\Omega_n \uparrow \Omega$, a.s. Then we define $\delta(u)$ by

$$\delta(u)|_{\Omega_n} = \delta(u^n)|_{\Omega_n}.$$

Suppose that u is an adapted process verifying $\int_0^T u_s^2 ds < \infty$ a.s. Then one can show that u belongs to $\mathbb{L}^F_{loc}$ and $\delta(u)$ coincides with the Itô integral of u.

Let $\mathbb{L}^F_b$ denote the space of processes $u \in \mathbb{L}^F$ such that $\| \int_0^T u_s^2 ds \|_\infty < \infty$. We have proved in [1] the following Itô's formula for the Skorohod integral:

Theorem 2.2 *Consider a process of the form* $X_t = \int_0^t u_s dW_s$, *where* $u \in (\mathbb{L}^F_b)_{loc}$. *Assume also that the indefinite Skorohod integral* $\int_0^t u_s dW_s$ *has a continuous version. Let* $F : \mathbb{R} \to \mathbb{R}$ *be a twice continuously differentiable function. Then we have*

$$\begin{aligned} F(X_t) = F(0) &+ \int_0^t F'(X_s) u_s dW_s + \frac{1}{2} \int_0^t F''(X_s) u_s^2 ds \\ &+ \int_0^t F''(X_s) \left(\int_0^s D_s u_r dW_r \right) u_s ds. \end{aligned} \tag{2.9}$$

3 Maximal inequality for the Skorohod integral process

The purpose of this section is to prove a maximal inequality for the Skorohod integral process where its integrand belongs to the space $\mathbb{L}^F$, using the ideas of [3].

Theorem 3.1 *Let* $2 < p < \infty$, $q > \frac{p}{2}$, $q \geq 2$ *and* $\frac{1}{q} + \frac{1}{r} = \frac{2}{p}$. *Let* $u = \{u_\theta, \theta \in [0, T]\}$ *be a stochastic process in the space* $\mathbb{L}^F$ *such that*

(i) $\int_0^T E|u_s|^{p \vee r} ds < \infty$,

(ii) $\int_{\{s \geq \theta\}} |E(D_s u_\theta)|^q ds d\theta < \infty$,

(iii) $\int_{\{r \vee s \geq \theta\}} |E(D_r D_s u_\theta)|^q dr ds d\theta < \infty$,

Then $\int_0^t u_s dW_s$ *is in* L^p *for all* $t \in [0, T]$ *and*

$$\begin{aligned} E\left(\sup_{0 \leq t \leq T} \left| \int_0^t u_s dW_s \right|^p\right) \leq K_{p,q} \Big\{ &\int_0^T E|u_s|^{p \vee r} ds \\ &+ \int_{\{s \geq \theta\}} |E(D_s u_\theta)|^q ds d\theta + \int_{\{r \vee s \geq \theta\}} E|D_r D_s u_\theta|^q dr ds d\theta \Big\}, \end{aligned} \tag{3.1}$$

where $K_{p,q}$ *is a constant depending only on* T, p *and* q.

Proof:

We will assume that $u \in \mathcal{S}_T$. The general case will follow using a density argument similar to the one in [1], pg. 8. Let $\alpha \in (\frac{1}{p}, \frac{1}{2})$. Using the fact that

$$\int_0^1 (1-u)^{\alpha-1} u^{-\alpha} du = \frac{\pi}{\sin(\alpha\pi)}$$

and applying Fubini's stochastic theorem and Hölder's inequality we obtain that

$$\begin{aligned}
& E(\sup_{0\le t\le T} | \int_0^t u_s dW_s|^p) \\
&= \frac{\sin(\alpha\pi)}{\pi} E(\sup_{0\le t\le T} | \int_0^t (\int_s^t (t-\sigma)^{\alpha-1} (\sigma - s)^{-\alpha} d\sigma) u_s dW_s|^p) \\
&= \frac{\sin(\alpha\pi)}{\pi} E(\sup_{0\le t\le T} | \int_0^t (\int_0^\sigma (\sigma - s)^{-\alpha} u_s dW_s)(t-\sigma)^{\alpha-1} d\sigma|^p) \\
&\le \frac{\sin(\alpha\pi)}{\pi} E(\sup_{0\le t\le T} \{\int_0^t | \int_0^\sigma (\sigma - s)^{-\alpha} u_s dW_s|^p d\sigma) \\
& \qquad\qquad \times | \int_0^t (t-\sigma)^{\frac{(\alpha-1)p}{(p-1)}} d\sigma|^{p-1}\}) \\
&= \frac{\sin(\alpha\pi)}{\pi} (\frac{p-1}{\alpha p - 1})^{p-1} T^{\alpha p - 1} E(\int_0^T | \int_0^\sigma (\sigma - s)^{-\alpha} u_s dW_s|^p d\sigma).
\end{aligned}$$

Let us now define for any $\sigma \in [0, T]$ the process

$$V_t^\sigma := \int_0^t (\sigma - s)^{-\alpha} u_s dW_s, \qquad t \in [0, \sigma],$$

and denote

$$C_{p,\alpha} = \frac{\sin(\alpha\pi)}{\pi} (\frac{p-1}{\alpha p - 1})^{p-1} T^{\alpha p - 1}.$$

We have proved that

$$E(\sup_{0\le t\le T} | \int_0^t u_s dW_s|^p) \le C_{p,\alpha} E(\int_0^T |V_\sigma^\sigma|^p d\sigma). \tag{3.2}$$

Now we are going to use the same ideas as in [3]. Applying Theorem 2.2 to $F(x) = |x|^p$ and taking the expectation, we obtain:

$$\begin{aligned}
E|V_t^\sigma|^p &= \frac{p(p-1)}{2} \int_0^t E[|V_s^\sigma|^{p-2} (\sigma - s)^{-2\alpha} u_s^2] ds \\
& \quad + p(p-1) \int_0^t E[|V_s^\sigma|^{p-2} (\sigma - s)^{-\alpha} u_s \int_0^s D_s u_\theta (\sigma - \theta)^{-\alpha} dW_\theta] ds \\
&=: I_1 + I_2.
\end{aligned}$$

Applying Hölder's inequality we get

$$I_1 \leq \frac{p(p-1)}{2} \int_0^t (E|V_s^\sigma|^p)^{\frac{p-2}{p}} (E|u_s|^p)^{\frac{2}{p}} (\sigma - s)^{-2\alpha} ds$$

and

$$I_2 \leq p(p-1) \int_0^t (E|V_s^\sigma|^p)^{\frac{p-2}{p}} (E|u_s \int_0^s D_s u_\theta (\sigma - \theta)^{-\alpha} dW_\theta|^{\frac{p}{2}})^{\frac{2}{p}} (\sigma - s)^{-\alpha} ds.$$

Denote

$$\begin{aligned} A_s \quad := \quad & \frac{p(p-1)}{2} (E|u_s|^p)^{\frac{2}{p}} (\sigma - s)^{-2\alpha} \\ & + p(p-1)(E|u_s \int_0^s D_s u_\theta (\sigma - \theta)^{-\alpha} dW_\theta|^{\frac{p}{2}})^{\frac{2}{p}} (\sigma - s)^{-\alpha} \end{aligned}$$

and $G_s = E|V_s^\sigma|^p$. Then we have that, for every $t \leq \sigma$

$$G_t \leq \int_0^t G_s^{\frac{p-2}{p}} A_s ds.$$

Using the lemma of [8], p.171 we obtain

$$G_t \leq (\frac{2}{p} \int_0^t A_s ds)^{\frac{p}{2}}.$$

Therefore

$$\begin{aligned} E|V_t^\sigma|^p \quad \leq \quad & \{(p-1) \int_0^t (E|u_s|^p)^{\frac{2}{p}} (\sigma - s)^{-2\alpha} ds \\ & + 2(p-1) \int_0^t (E|u_s \int_0^s D_s u_\theta (\sigma - \theta)^{-\alpha} dW_\theta|^{\frac{p}{2}})^{\frac{2}{p}} (\sigma - s)^{-\alpha} ds\}^{\frac{p}{2}} \\ \leq \quad & (p-1)^{\frac{p}{2}} 2^{\frac{p}{2}-1} \{\int_0^t (E|u_s|^p)^{\frac{2}{p}} (\sigma - s)^{-2\alpha} ds\}^{\frac{p}{2}} \\ & + 2^{p-1}(p-1)^{\frac{p}{2}} \{\int_0^t (E|u_s \int_0^s D_s u_\theta (\sigma - \theta)^{-\alpha} dW_\theta|^{\frac{p}{2}})^{\frac{2}{p}} (\sigma - s)^{-\alpha} ds\}^{\frac{p}{2}}. \end{aligned}$$

By Hölder's inequality we have:

$$\begin{aligned} I_3 \quad := \quad & \{\int_0^t (E|u_s \int_0^s D_s u_\theta (\sigma - \theta)^{-\alpha} dW_\theta|^{\frac{p}{2}})^{\frac{2}{p}} (\sigma - s)^{-\alpha} ds\}^{\frac{p}{2}} \\ \leq \quad & \{\int_0^t (E|u_s|^r)^{\frac{1}{r}} (E|\int_0^s D_s u_\theta (\sigma - \theta)^{-\alpha} dW_\theta|^q)^{\frac{1}{q}} (\sigma - s)^{-\alpha} ds\}^{\frac{p}{2}} \\ \leq \quad & \{\int_0^t (E|u_s|^r)^{\frac{q}{(q-1)r}} (\sigma - s)^{\frac{\alpha q}{1-q}} ds\}^{\frac{p(q-1)}{2q}} \end{aligned}$$

$$
\begin{aligned}
& \times\{\int_0^t E|\int_0^s D_s u_\theta(\sigma-\theta)^{-\alpha} dW_\theta|^q ds\}^{\frac{p}{2q}} \\
\leq \;& c_1\{\int_0^t (E|u_s|^r)(\sigma-s)^{\frac{\alpha q}{1-q}} ds\}^{\frac{2q-p}{2q}} \\
& \times\{\int_0^t E|\int_0^s D_s u_\theta(\sigma-\theta)^{-\alpha} dW_\theta|^q ds\}^{\frac{p}{2q}},
\end{aligned}
$$

for some constant c_1 depending only on p, q, α and T. Since $\frac{2q-p}{2q}+\frac{p}{2q}=1$, using the inequality $ab \leq \frac{2q-p}{2q} a^{\frac{2q}{2q-p}} + \frac{p}{2q} b^{\frac{2q}{p}}$ for $a, b \geq 0$, we have

$$
I_3 \leq c_1\{\int_0^t (E|u_s|^r)(\sigma-s)^{\frac{\alpha q}{1-q}} ds + \int_0^t E|\int_0^s D_s u_\theta(\sigma-\theta)^{-\alpha} dW_\theta|^q ds\}.
$$

Now we can estimate the Skorohod integral using Meyer's inequalities (see [5], Section 3.2) and we obtain

$$
\begin{aligned}
I_4 \;&:=\; \int_0^t E|\int_0^s D_s u_\theta(\sigma-\theta)^{-\alpha} dW_\theta|^q ds \\
&\leq\; c_2\Big\{ \int_0^t (\int_0^s (\sigma-\theta)^{-2\alpha}|E(D_s u_\theta)|^2 d\theta)^{\frac{q}{2}} ds \\
&+\; \int_0^t E(\int_0^T \int_0^s (\sigma-\theta)^{-2\alpha}|D_r D_s u_\theta|^2 d\theta dr)^{\frac{q}{2}} ds \Big\}.
\end{aligned}
$$

for some constant c_2. Hence, taking into account (3.2), we get

$$
E(\sup_{0\leq t\leq T} |\int_0^t u_s dW_s|^p) \leq c_3(I_5+I_6+I_7+I_8)),
$$

where c_3 is a constant depending on p, q, α and T, and

$$
\begin{aligned}
I_5 \;&:=\; \int_0^T (\int_0^\sigma (E|u_s|^p)^{\frac{2}{p}} (\sigma-s)^{-2\alpha} ds)^{\frac{p}{2}} d\sigma, \\
I_6 \;&:=\; \int_0^T \int_0^\sigma (\sigma-s)^{\frac{\alpha q}{1-q}} E|u_s|^r ds d\sigma, \\
I_7 \;&:=\; \int_0^T \int_0^\sigma (\int_0^s (\sigma-\theta)^{-2\alpha}|E(D_s u_\theta)|^2 d\theta)^{\frac{q}{2}} ds d\sigma, \\
I_8 \;&:=\; \int_0^T \int_0^\sigma E(\int_0^T \int_0^s (\sigma-\theta)^{-2\alpha}|D_r D_s u_\theta|^2 d\theta dr)^{\frac{q}{2}} ds d\sigma.
\end{aligned}
$$

Now using Hölder's inequality and Fubini's theorem we obtain that

$$
I_5 \;\leq\; \frac{T^{1-2\alpha}}{1-2\alpha} \int_0^T \int_0^\sigma E|u_s|^p (\sigma-s)^{-2\alpha} ds d\sigma
$$

$$
\begin{aligned}
&= \frac{T^{1-2\alpha}}{1-2\alpha}\int_0^T\left(\int_s^T(\sigma-s)^{-2\alpha}d\sigma\right)E|u_s|^p ds \\
&\leq c_4\int_0^T E|u_s|^p ds,
\end{aligned}
$$

for some constant c_4. Similarly,

$$
I_6 \leq c_5\int_0^T E|u_s|^r ds,
$$

$$
I_7 \leq c_6\int_{\{s\geq\theta\}}|E(D_s u_\theta)|^q ds d\theta,
$$

and

$$
I_8 \leq c_7\int_{\{r\vee s\geq\theta\}} E|D_r D_s u_\theta|^q dr ds d\theta,
$$

for some constants c_5, c_6 and c_7. The proof is now complete. QED

As a corollary, taking $q=2$ we have the following result:

Theorem 3.2 *Let $p\in(2,4)$, $r=\frac{2p}{4-p}$. Let $u=\{u_s, s\in[0,T]\}$ be a stochastic process in the space $\mathbb{L}^F$ such that $\int_0^T E|u_s|^r ds<\infty$. Then $\int_0^t u_s dW_s$ is in L^p for all $t\in[0,T]$ and*

$$
\begin{aligned}
E\left(\sup_{0\leq t\leq T}\left|\int_0^t u_s dW_s\right|^p\right) &\leq K_p\left\{\int_0^T E|u_s|^r ds+\int_{\{s\geq\theta\}}E|D_s u_\theta|^2 ds d\theta\right. \\
&\quad\left.+\int_{\{r\vee s\geq\theta\}}E|D_r D_s u_\theta|^2 dr ds d\theta\right\}, \qquad (3.3)
\end{aligned}
$$

where K_p is a constant depending only on p and T.

Remark: Theorem 3.2 implies the continuity of the Skorohod process $\int_0^t u_s W_s$ assuming that $u\in\mathbb{L}^F$ and $\int_0^T E|u_s|^r ds<\infty$ for some $r>2$. This result was proved in [1] using Kolmogorov continuity criterion and the technique developed in [3].

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