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RESEARCH ARTICLE

On 'actually' and 'dthat': Truth-conditional Differences in Possible Worlds Semantics

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Abstract: Although possible worlds semantics is a powerful tool to represent the semantic properties of natural language sentences, it has been often argued that it is too coarse: with the tools that possible worlds semantics puts at our disposal, any relevant semantic difference has to be a truth conditional difference representable as a difference in intension. A case that raises questions about the ability of possible worlds semantics to make the appropriate discriminations is the distinction between rigidity and direct reference, an issue deeply connected to the representation of the behaviour of two operators: 'dthat' and 'actually'. Differences between the mode of operation of 'dthat' and 'actually' have been observed, but they have not been examined in depth. Our purpose is to explore systematically to what extent the observed differences between the two operators have truth conditional consequences that are formally representable in possible worlds semantics.

Keywords: Actuality operator; direct reference; dthat; possible worlds semantics; rigidity

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1 Possible worlds semantics

Semantics is often characterized as a theory of truth conditions. Differences in meaning between two expressions are never as clearly demonstrated as when it is possible to point out differences in the truth conditions of the sentences in which the expressions in question figure and, in spite of the emergence of other systems and approaches, possible worlds semantics continues to be the most powerful tool of formal representation in semantics, affording a precise characterization of the truth conditions of sentences as intensions, i.e., functions from possible worlds to truth values. In general, the representation of meanings in terms of functions from possible worlds to the appropriate kind of extensions delivers a compositional semantics that formally captures the semantic operation of non-syncategorematic expressions in an ordinary language.

Although possible worlds semantics is clearly a powerful tool to represent faithfully the semantic properties of natural language sentences, it has been often argued that it is too coarse. There are important properties of sentences that are not discriminated in possible worlds semantics. One interesting case that raises questions about the ability of possible worlds semantics to make the appropriate discriminations is the distinction between rigidity and direct reference, an issue deeply connected to the representation of the behaviour of two operators: 'dthat' and 'actually'. Some differences between the mode of operation of 'dthat' and 'actually' have been pointed out by several authors.[1] Our purpose here is to explore formally how different these operators really are and to what extent the observed differences do have truth conditional consequences representable in possible world semantics.

2 Rigidity and direct reference

Kripke's notion of rigidity, a notion that underpins one of the most important revolutions in semantic theory, was presented within the framework of possible worlds semantics: a rigid designator designates the same object in every possible world. Hence, 'the successor of 7' is rigid because it designates the number 8 in every possible world, whereas 'the tutor of Alexander the Great' is not

[1]Most notably by (Salmon 1991). See also (Soames 2005, 28-30) for a summary.

rigid, since someone different from Aristotle could have tutored Alexander.[2] Kripke's revolutionary claim was that names are rigid designators, whereas most definite descriptions, in particular the descriptions that were associated with names according to classical descriptivists, are not.

It has been typically assumed that a rigid designator designates nothing in a world in which the actual designatum fails to exist, following (Kripke 1971, 146) and (Kripke 1980, 49): "a designator rigidly designates a certain object if it designates that object wherever the object exists." But this has been the source of some controversy and even Kripke has suggested that some rigid designators, names in particular, designate even in worlds in which the designatum fails to exist (see (Kaplan 1989, 570, fn. 8), suggesting that names are not just rigid but *obstinately* rigid.[3] This is an important issue, and we will return to it.

A non-rigid designator can be rigidified using the intensional operator 'actually' or 'actual'. 'Actually' operates on formulas: the interpretation of *actually* ϕ at any possible world is the interpretation of ϕ at the actual world. When combined with definite descriptions, *the* x : *actually* Px (or, closer to natural language, *the actual* P) designates i in every possible world in which i exists, if *the* $x : Px$ designates i in the actual world. For instance, 'the actual tutor of Alexander the Great' designates Aristotle in every world w in which Aristotle exists, for Aristotle is the individual in w that satisfies being tutor of Alexander in the actual world.

Direct reference, a different revolutionary idea introduced by Kaplan, appeared on the scene roughly around the time Kripke introduced his distinction between rigid and non-rigid designators. Unlike rigidity, direct reference relies on the Russellian picture of structured propositions. A term is directly referential just in case it contributes its referent to the proposition expressed by sentences containing it. Propositions are represented by Kaplan as n-tuples that contain the elements contributed to truth-conditional content by expressions in sentences. Hence, in (Kaplan 1978) we learn that the truth-conditional content, or proposition expressed by a sentence of the form n *is* Q, where n is directly referential, is singular, a proposition of the form $< i, Q^* >$, where i is the referent of n and Q^* is the property expressed by the predicate Q. In contrast, the propositional contribution of a non-directly referential term,

[2] This, of course, takes for granted that teaching Alexander was not an essential property of Aristotle, a plausible assumption.

[3] See (LaPorte 2018, section 1.2).

a definite description such as *the* P, is general, an attributive complex that selects at each index of evaluation w the individual relevant for the computation of truth value at w. Hence, *the* P *is* Q expresses a proposition that can be represented as $<< \text{`}\textit{the}\text{'}, P^* >, Q^* >$. The difference between singular propositions, those expressed by sentences containing directly referential terms, and general propositions can be best captured as a difference between the constituents of truth conditional content: in the case of a singular proposition, a proposition of the form $< i, Q^* >$, it is i itself that determines truth value at each index of evaluation, whereas in the case of a general proposition, a proposition of the form $<< \text{`}\textit{the}\text{'}, P^* >, Q^* >$ an attributive complex (constrained by a condition of uniqueness) selects for each index the individual that will determine truth value at that index. Even if the attributive complex $< \text{`}\textit{the}\text{'}, P^* >$ selects i at each index, the truth conditional content, i.e., that which determines truth conditions, is different in each case. Or, in other words, even if *the* P is a rigid definite description that designates i, the items determining truth conditions that correspond to *the* P and to n are quite different.

Names, indexicals and demonstratives are, according to Kaplan, paradigmatic instances of directly referential terms. But Kaplan also introduces a device that operates on definite descriptions and produces directly referential terms: the 'dthat' operator. The idea behind the mode of operation of 'dthat' can be captured if one thinks of a definite description as a demonstration, like a pointing. When a speaker ostensibly points at a person i and utters 'that P is Q', i becomes the truth conditional content of the sentence uttered. In a similar vein, if *the* P designates i, $dthat(\textit{the}\, P)$ contributes i, not the complex $< \text{`}\textit{the}\text{'}, P^* >$ to the truth conditional content. Observe, however, that the contribution of *the actual* P to content, to put it in Kaplanian terms, is not i, but rather an attributive complex corresponding to *the actual* P. Hence, the mode of operation of 'dthat' is quite different from the mode of operation of 'actual' or 'actually'. But the important question for our purposes is whether that difference generates a difference in possible-world-representable truth conditions.

3 The semantic operation of ‘actually’ and ‘dthat’

The combination of the actuality operator (A) with definite descriptions raises an interesting issue as regards the index at which the requirement of uniqueness should be satisfied. We have two prima facie plausible choices: the world of evaluation, or the actual world. Let w be a possible world, $D(w)$ the domain of individuals existing at w, @ the actual world and $Int(\phi)^w$ the interpretation of an expression ϕ at w.[4] The choices can be represented as follows:

(a) $Int(the\,x : APx)^w = i$ if $i \in D(w)$ and i is the unique individual in $D(w)$ that satisfies Px in @. $Int(the\,x : APx)^w$ is undefined otherwise.

(b) $Int(the\,x : APx)^w = i$ if $i \in D(w)$ and i is the unique individual in $D(@)$ that satisfies Px in @. $Int(the\,x : APx)^w$ is undefined otherwise.

If we put it in Russellian terms, the question is which one of these two sentences (corresponding to (a) and (b)) best captures the logical form of *the actual P is Q*, a difference that hinges on whether the uniqueness condition falls under the scope of ‘actually’:

(a’) $\exists x[APx \wedge \forall y(APy \rightarrow x = y) \wedge Qx]$

(b’) $\exists x[A(Px \wedge \forall y(Py \rightarrow x = y)) \wedge Qx]$

In (a’) we are treating *actually* P as a predicate: there is an i in w such that i is actually P and anything in w that is actually P is that very i (and i is Q).

In (b’) the condition of uniquely satisfying P is meant to be satisfied in the actual world: there is an i in w such that actually i is the only P (and i is Q).

Both are possible interpretations, and many informal discussions of the role of rigidified definite descriptions fail to distinguish between the two. This is because, typically, illustrations of the behavior of ‘actual’ use definite descriptions such as ‘the actual President of the US’ a description that designates Trump in a world w in which both Trump and Clinton exist and Clinton wins the 2016 election. Since there is only one President of the US in @ and, by assumption, also in w, the question as regards the locus of satisfaction of the uniqueness requirement is not addressed.

[4]The interpretation funtion Int is defined relative to a structure, an index of evaluation and an assignment, but we are abbreviating the technical details, which are trivial.

In our view, there is a clear motivation to go for (a/a') rather than (b/b'). Consider the term 'the only actually existing person'. In a world in which Pat is the only one of the actual human beings that exists, surrounded by human possibilia, it should be true that Pat is the only actually existing human being and hence that 'the only actually existing person' designates her relative to that world.

Yet, Pat will not uniquely satisfy *Exists* x in @, and so the description in question will be denotationless in w if (b) is the interpretation of choice. Or, if (b') is the chosen representation, any sentence in which the description figures will be false:

$\exists x[A(Exists\, x \wedge \forall y(Exists\, y \rightarrow x = y)) \wedge Qx]$

for the uniqueness condition under the scope of the operator A will not be satisfied if there are in the actual world at least two people.[5]

Hence, we will take (a) and (a') to be the correct choices for the interpretation of sentences in which descriptions of the form the actual P occur.[6]

On the other hand, the interpretation of $dthat(the\, P)$ is given by the following clause:

$Int(dthat(the\, P))^w = i$ if $i \in D(@)$ and i is the unique individual in $D(@)$ that satisfies Px in @. $Int(dthat(the\, P))^w$ is undefined otherwise.

We can try to capture the logical form of sentences containing the operator 'dthat' in terms of quantifiers, taking inspiration from Russell. For a sentence such as $dthat(the\, P)$ *is* Q, the formalizations (a') or (b') will not do. When we evaluate $dthat(the\, P)$ *is* Q in any world w, we need to select an individual in the domain of the actual world, and the existential quantifier, both in (a') and (b'), search for individuals in w. Adding an actuality operator in front of the existential quantifier in (a') or (b') will not do, since then the truth of the sentence will depend on whether the individual in @ which is uniquely P in @ is Q in @, and what we really need is an individual which is uniquely P in @ and Q in w. Hence we need to be able to quantify over the domain of the actual world even when we are evaluating in a different world w.

The actuality quantifiers ($\forall^{@}, \exists^{@}$), introduced by Allen Hazen (1990), are

[5] Of course, given that we identify the domain of a world with what exists at that world, that formula is just equivalent to $\exists x[A\forall y(x = y) \wedge Qx]$.

[6] (Soames 2005, 30, fn. 22), reporting a suggestion by Ali Kazmi, proposes that what we have called (b) is a "highly intuitive reading [of sentences of the form *the actual P is Q*] in which the uniqueness condition is correctly imposed on things that have the property expressed by [P] in the world-state of the context [namely, the actual world]". Soames gives no indication why he regards this reading as highly intuitive but, as we think our example involving 'the actually existing person' shows, that reading is not adequate.

designed precisely to do that.[7] In general,

$Int(\forall^{@} x\phi(x))^w = true$ iff for all $i \in D(@)$, $Int(\phi(x)[i/x])^w = true$ (analogously for $\exists^{@}$).

With the help of those quantifiers, we can represent the sentence $dthat(the\, P)$ $is\ Q$ as:

(a*) $\exists^{@} x[APx \wedge \forall^{@} y(APy \rightarrow x = y) \wedge Qx]$

We could also use a formalization similar to (b’), namely

(b*) $\exists^{@} x[A(Px \wedge \forall^{@} y(Py \rightarrow x = y)) \wedge Qx]$,

but in this case (a*) and (b*) are equivalent. The difference between (a’) and (b’) had to do with the locus of satisfaction of the uniqueness condition. In the case of ‘dthat’ the uniqueness condition has to be satisfied in the actual world, otherwise $dthat(the\, P)$ is denotationless with respect to all indices. (b*) explicitly captures that condition by including the uniqueness condition under the scope of @. But (a*) achieves the same result, by restricting the domain of the universal quantifier to the actual world.

4 A merely conceptual difference between ‘dthat’ and ‘actually’?

Clearly there is an important conceptual difference between ‘dthat’ and the ‘actually’ operator. Resorting again to the picture of structured propositions, whereas $dthat(the\, P)\ is\ Q$ expresses a singular proposition of the form $< i, Q* >$, *the actual P is Q* expresses a general proposition of the form $<< \text{‘}the\text{’}, Actual\, P* >, Q* >$ or, in other words, whereas in the case of $dthat(the\, P)\ is\ Q$, i is the object that is provided as the determiner of truth value at each index, in the case of *the actual P is Q* an attributive complex selects, among the individuals in the domain of any given world w, the unique individual that happens to be P in the actual world.

To give an example, let us suppose for the moment that Aristotle exists in all worlds: ‘the actual tutor of Alexander the Great was a philosopher’

[7] Hazen actuality quantifiers should not be confused with the ordinary quantifiers (usually misnomed ‘actualist quantifiers’, in order to distinguish them from possibilist quantifiers). As Hazen notes, ordinary quantifiers are really world-restricted, since they always range over the domain of evaluation. Hazen’s aim is different from ours: he shows that the actuality quantifiers and the operator ‘actually’ both expand the expressive power of first-order logic, and he also shows that both devices are not equivalent. Moreover, Hazen’s language does not incorporate definite descriptions.

expresses a general proposition whose truth value at an index w depends on whether the unique individual in the domain of w that happens to be actually tutor of Alexander (namely, Aristotle) is a philosopher in w. 'Dthat (the tutor of Alexander the Great) was a philosopher', on the other hand, expresses a singular proposition whose truth value at each index depends on whether Aristotle is a philosopher at that index.

This is all well, and one can see that, conceptually, there is an important difference here in how the truth conditions of the respective sentences are captured and in how truth value at an index is determined. But the difference affects the *how* not the *what*. Direct reference proponents may insist on the importance of the distinction between the two kinds of content, but the fact is that no truth conditional difference between the two sentences is manifested: the functions from possible worlds to truth values that represent their truth conditions are the same. As (Soames 2005, 28n.) puts it, "this difference between dthat-rigidified descriptions and actually-rigidified descriptions [...] all but washes away in semantic systems in which the content of an expression in a context is identified with its intension."

That there is no truth conditional difference between 'dthat (the tutor of Alexander the Great) was a philosopher' and 'the actual tutor of Alexander the Great was a philosopher', if domains are not allowed to vary, is clear. If domains do vary, interesting issues arise if we consider a world w in which Aristotle fails to exist, for decisions have to be made as regards the assignment of truth value to each of those sentences in w: false, or indeterminate. In a world in which Aristotle does not exist, he obviously is not a philosopher, so $< Aristotle, Philosopher* >$ can arguably be said to be false. On the other hand, in that very world Aristotle is not a member of the antiextension of the predicate 'philosopher' and the sentence can be considered to be indeterminate on the grounds that Aristotle is not in the range of applicability of 'philosopher'. And the same goes for $<< \text{`}\mathit{the}\text{'}, \mathit{Actual\ tutor\ of\ Alexander\ the\ Great}* >, Philosopher* >$, for there is no individual in w that can be selected by the attributive complex. No matter which way we decide to go, it is natural to think that the two sentences should suffer the same fate. True, it may be argued that the reasoning behind the assignment of falsity or indeterminacy is different for the two sentences. Sure enough, and very interesting, but let us recall that possible worlds semantics is blind to the reasoning and the narratives behind the results: differences in semantic mode of operation are intangible if they do not show up as differences in truth conditions.

However, if we think now in Russellian terms, the sentence ‘the actual tutor of Alexander the Great was a philosopher’ is false, because the claim of existence is false. But as regards ‘dthat (the tutor of Alexander the Great) was a philosopher’, there is still a choice between false and indeterminate. So if we endorse the indeterminacy of the latter, there is a truth conditional difference between ‘dthat’ and ‘actually’.

5 Attributions of existence

Arguably, a truth conditional difference between sentences containing a standard rigid designator and sentences containing an obstinate rigid designator shows up in attributions of existence. Recall that an obstinately rigid designator refers even in worlds in which its designatum fails to exist, whereas a standard rigid designator (or a persistent designator, in Salmon’s terminology) does not designate anything in worlds in which its designatum fails to exist.[8] ‘Dthat’, arguably, operates as an obstinate rigidifier, since the designatum of $dthat(the\, P)$ is the object relevant for the evaluation of sentences in which $dthat(the\, P)$ figures, independently of whether that designatum exists or does not exist at a given index. *The actual* P, on the other hand, is a standard rigid designator. As Scott Soames has noted:

> A dthat-rigidified description, $dthat[the\, x : Fx]$, which designates an object o in the world-state of the context [the actual world], designates o in all world-states, even those in which o does not exist. By contrast, [...] $the\, x : actually\, Fx$ will fail to designate anything at a world-state in which o does not exist. (Soames 2005, 28-29)

Focusing now on attributions of existence, if r is a standard rigid designator, there is a choice as to how sentences of the form r *exists* are evaluated in worlds in which r does not denote. The sentences may be considered false (which appears to be the more natural choice) or they can be interpreted as indeterminate if we abide by the general principle that sentences with denotationless terms are indeterminate. But if r is obstinately rigid, there is no

[8] See (Salmon 1991, 34) for the definition of obstinacy and persistence.

choice; r designates even in worlds in which its designatum fails to exist, and there is no question that in such worlds r *exists* is false.[9]

The difference observed between obstinate rigid designators and standard rigid designators as regards attributions of existence should be expected to occur in the presence of 'dthat' and of 'actually: 'dthat (the tutor of Alexander the Great)' and 'the actual tutor of Alexander the Great' both designate Aristotle in the actual world. But in a world w in which Aristotle does not exist, 'dthat (the tutor of Alexander the Great) exists' would be assigned the truth value False, the same truth value that corresponds to 'Aristotle exists', for those sentences express a complete and evaluable singular proposition $< Aristotle, Exists* >$. But in such a world, the actualized description fails to denote, and the sentence 'the actual tutor of Alexander the Great exists' can be evaluated as indeterminate or as false. So, a truth conditional difference between sentences of the form *the actual P is Q* and sentences of the form $dthat(the\, P)$ *is* Q may manifest itself.

Observe, however, that the case for a truth-conditional difference depends on accepting a specific policy on the treatment of sentences with denotationless terms. And it depends also on accepting 'exists' as a predicate in the language, a controversial choice that some may find objectionable. If we reject the predicative status of 'exists' and paraphrase its ocurrences in favor of quantification, the sentence 'the actual tutor of Alexander the Great exists' will be as false in w as 'dthat (the tutor of Alexander the Great) exists', since any representation of the logical form of 'the actual tutor of Alexander the Great exists' will start with an existential quantifier ranging over the domain of w, and no individual in w is actually tutor of Alexander. So, even in the case of attributions of existence, it is doubtful that the two operators generate sentences that differ in truth value across possible worlds. In fact, this is not a phenomenon that arises because of the use of 'dthat' and 'actually'. Compare 'Aristotle exists' with 'the offspring of gametes X and Y exists.' Let us say that the description 'the offspring of gametes X and Y' refers rigidly to Aristotle. Whereas the name is obstinately rigid, the description is only standardly rigid, however, the formulas $\exists x x = Aristotle$ and $\exists x[Ox \wedge \forall y(Oy \rightarrow x = y)]$ are both false in a world in which Aristotle does not exist. So the phenomenon just observed generalizes to all obstinate and all standard rigid designators.

Attributions of existence do not conclusively provide a way of discriminat-

[9]See also (Gómez-Torrente 2006, 250-251), and (Besson 2009, sect. 2.4), for related discussions on attributions of existence.

ing between obstinately and standardly rigid designators and, in particular, between ‘dthat’ and ‘actually’. There is, however, a truth conditional difference generated by the two operators, one that has not been so widely discussed, and that it is easy to capture without relying on dubious commitments.

6 A truth-conditional difference

Let us consider the following case: in the actual world @ there are many people that smoke. A description such as ‘the smoker’ is improper and fails to denote. Suppose now that w is a world in which only one of the actual smokers, let us call him a, exists. a may be or may not be a smoker in w. But in w ‘the actual smoker’ designates a, for a is the unique individual in the domain of w that satisfies the open formula $actual\ smoker(x)$, in virtue of satisfying $smoker(x)$ in @. Hence ‘the actual smoker’ fails to denote in @, but it does acquire a denotation in w, for in w there is after all a unique individual in the domain that is selected by the attributive complex ‘actual smoker’: in w, a is an actual smoker.[10] Resorting to the picture of propositions used by Kaplan, we can say that the proposition expressed by ‘the actual smoker is a man’, the general proposition $<<\text{‘}the\text{’}, Actual\,Smoker* >\ \ , Man* >$, will be false or indeterminate in @, but will be true in w.

‘Dthat (the smoker)’, on the other hand, does not designate anybody in @ either, for the description it employs fails, like an ambiguous pointing, to demonstrate a unique individual. But unlike ‘the actual smoker’, ‘dthat (the smoker)’ will not acquire a denotation in w. The role of the operator ‘dthat’ applied to a definite description is to provide the individual designated by the definite description as the element that figures in the computation of truth value at all indices of evaluation. In the case of ‘dthat (the smoker)’ we are missing that element, so the computation of truth value at w does not even get off the ground.

[10](Soames 2005, 29), crediting Ali Kazmi, mentions the peculiar behaviour of expressions of the form *the actual P* when they are improper in the actual world, since they may acquire different denotations in different worlds. Soames’ concerns have to do with how this peculiar behaviour affects the status of ‘actually’ as a rigidifier. Here we are focusing rather on how the truth-conditional behaviour of ‘actually’ can be shown to be different from the truth-conditional behaviour of ‘dthat’. It is worth noticing that it is in order to avoid the effects of this peculiar behavior that Soames proposes (Soames 2005, fn. 22) an interpretation of *the actual P is Q* that imposes the satisfaction of the condition of uniqueness in the actual world. It is attractive to do so, since in the conditions envisaged ‘the actual smoker’ would not designate in w because it would not designate in @. But, as we have argued (see section 3, and in particular fn. 6), we think that the interpretation Soames suggests is not correct.

Resorting again to the apparatus of structured propositions we can say that since 'dthat (the smoker)' fails to designate in the actual world, there is no propositional constituent provided. The proposition expressed by 'dthat (the smoker) is a man' should be a proposition of the form $< i, Man* >$, a singular proposition with an object in the subject position, an object whose role is to intervene in the computation of truth value in @ and in other indices. But there is no such object. We might say that 'dthat (the smoker) is a man' does not express a proposition. Or, following a lead by Kaplan,[11] we may say that it expresses a gappy proposition, one that can be evaluated as false or as indeterminate, in @ and also in w. But surely, the proposition in question will not be true in w.

Hence, this case suggests that the two operators can have different truth conditional impact. A more precise representation of the semantics of the two sentences discussed will further clarify the details. Following the strategy of section 3, we will first focus on a language that contains a descriptor and the operators 'dthat' and 'actually'. We will then focus on a language in which the descriptor and the operator 'dthat' are eliminated in favor of ordinary and Hazen's actuality quantifiers.

Let us consider a standard first-order language with identity, a descriptor operator 'ι', the modal operator of actuality 'A' and the operator 'dthat'. Given a variable x, a term t and a formula ϕ, $\iota x\phi$ is a term, $A\phi$ is a formula and $dthat(t)$ is a term. As usual, we will consider the language interpreted on a structure consisting of a set of possible worlds W containing the actual world @, a domain of individuals $D(w)$ for each world $w \in W$ and interpretations for constants and predicates.[12] Clauses for interpretation are the usual ones, with the new operators interpreted as:

$Int(\iota x\phi)^w = i$ iff i is the unique individual in $D(w)$ such that i satisfies ϕ in w; otherwise, $Int(\iota x\phi)^w$ is undefined.

$Int(A\phi)^w = Int(\phi)^@$

$Int(dthat(t))^w = Int(t)^@$

In this language, 'the actual smoker is a man' will be formalized as $M(\iota xASx)$ and 'dthat (the smoker) is a man' as $M(dthat(\iota xSx))$. To see the difference in truth conditions, let us consider the following structure $< W, D, Int >$:

$W = \{@, w\}$

[11]See (Kaplan 1989, 496, fn. 23).

[12]Nothing depends on the specific modal logic, so we will not represent the relation of accessibility between worlds. As above, we will also disregard the assignment function.

$D(@) = \{0, 1\}$
$D(w) = \{0, 2\}$
$Int(S)^{@} = \{0, 1\}$ $Int(S)^{w} = \{2\}$
$Int(M)^{@} = \{1\}$ $Int(M)^{w} = \{0\}$

$\iota xASx$ does not denote in @, because in $D(@)$ there are two smokers, and $\iota xASx$ denotes 0 in w, because 0 is the unique actual smoker in $D(w)$. Therefore, the sentence $M(\iota xASx)$ is either false or undetermined (we leave that unspecified) in @, but it is true in w.

By the semantic clause for 'dthat':

$Int(dthat(\iota xSx))^{w} = Int(\iota xSx)^{@} = i$ if i is the unique individual in $D(@)$ that satisfies Sx in @, and it is undefined otherwise.

It is crucial that in this semantic analysis the relativity to the domain of the world of evaluation disappears, so $dthat(\iota xSx)$ denotes neither in @ nor in w, and the sentence $M(dthat(\iota xSx))$ is either false or undetermined in both worlds. We see then that there is a difference in truth value status in the world w.

Let us now consider a language in which 'dthat' and the descriptor have been eliminated. In this case the sentences 'the actual smoker is a man' and 'dthat (the smoker) is a man' will be translated using our previous formalizations (a') and (a*):

$\exists x[ASx \wedge \forall y(ASy \rightarrow x = y) \wedge Mx]$
$\exists^{@} x[ASx \wedge \forall^{@} y(ASy \rightarrow x = y) \wedge Mx]$

And following the argument of section 3 it is obvious that the first sentence is true and the second is false at the index of evaluation w.

In order to highlight the special status of this case, let us now consider a different structure with the same constituents as before, with the exception of the interpretation of the predicate S, which is replaced by the following one:

$Int(S)^{@} = \{1\}$ $\qquad$ $Int(S)^{w} = \{2\}$

Now $dthat(\iota xSx)$ denotes 1 in @ and the sentence $M(dthat(\iota xSx))$ is true in @. In w, $dthat(\iota xSx)$ still denotes 1, but since 1 is not a member of the domain $D(w)$, the truth value of $M(dthat(\iota xSx))$ depends on decisions about the truth value of sentences in which a term denotes an object that fails to exist in the world in question. As it was the case in the previous structure, the description $\iota xASx$ designates 1 in @, but now it fails to designate in w, making the sentence $M(\iota xASx)$ true in @ and either false or undetermined in w, depending on general policies about the truth-value status of sentences with denotationless terms.[13]

If our policy on how to treat sentences with denotationless terms differs from our policy as regards how to treat sentences in which the denotation of a term fails to exist, then in w we would assign false to one of the sentences and undetermined to the other. A truth conditional difference is revealed in this case, but that difference only shows up because of decisions about semantic phenomena unrelated to the semantic behavior of the operators in question. By contrast, in the first structure, the truth conditional difference is independent of those policies. That, we submit, unmistakably establishes that possible worlds semantics can distinguish between the two operators.[14]

References

Besson, Corine. 2009. "Externalism, Internalism and Logical Truth." *The Review of Symbolic Logic* 2(1): 1-29. https://doi.org/10.1017/S1755020309090091

Gómez-Torrente, Mario. 2006. "Rigidity and Essentiality." *Mind* 115(458): 227-259. https://doi.org/10.1093/mind/fzl227

Hazen, Allen. 1990. "Actuality and Quantification." *Notre Dame Journal of Formal Logic* 31(4): 498-508. https://doi:10.1305/ndjfl/1093635586

Kaplan, David. 1978. "Dthat." In *Syntax and Semantics. Volume 9: Pragmatics*, edited by P. Cole, 221-243. New York: Academic Press.

Kaplan, David. 1989. "Afterthoughts." In *Themes From Kaplan*, edited by J. Almog, H. Wettstein, and J. Perry, 565-614. Oxford: Oxford University Press.

Kripke, Saul. 1971. "Identity and Necessity." In *Identity and Individuation*, edited by M.K. Munitz, 135-164. New York: New York University Press.

Kripke, Saul. 1980. *Naming and Necessity*. Cambridge, (Mass.): Harvard University Press.

LaPorte, Joseph. 2018. "Rigid Designators." *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta. https://plato.stanford.edu/archives/spr2018/entries/rigid-designators/.

Salmon, Nathan. 1991. *Reference and Essence*. Princeton: Princeton University Press.

Soames, Scott. 2005. *Reference and Description. The Case against Two-dimensionalism.* Princeton: Princeton University Press.

[13]Note that this structure is a simplified model of sentences such as 'dthat (the tutor of Alexander the Great) was a philosopher' and 'the actual tutor of Alexander the Great was a philosopher' that were discussed in section 4, with 1 playing the role of Aristotle.

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