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A new approach to the Malmquist bias

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Abstract. The Malmquist bias, found in magnitude-limited samples, is studied from a new point of view. An analytical approach is developed taking into account a more realistic space distribution than in the traditional Malmquist approximation, thus leading to accurate determination of this bias. In addition to the well known dependence on the dispersion of absolute magnitude it is found to be a function of other parameters of the sample.

Key words: methods: statistical – stars: fundamental parameters

1. Introduction

Since accurate trigonometric parallaxes are available only for nearby stars, the study of the absolute magnitude of a sample of stars usually relies on statistical methods, which are very sensitive to the selection criteria used to elaborate the sample. The limiting magnitude of the survey is one of the most severe observational constraints; however magnitude-limited samples are very common.

It is well known since the work of Malmquist (1936) that the mean absolute magnitude of such a sample is biased. This bias is, at first order, proportional to the dispersion of the intrinsic distribution of absolute magnitude. Malmquist assumed a Gaussian form for this intrinsic distribution and obtained a series development to approximate the observed distribution. The coefficients of this development depend on the spatial distribution of the stars in the sample and its determination requires a good knowledge of star counts for the type of stars involved.

The method we propose is more direct. Unlike Malmquist's method, ours needs to assume the form of the spatial distribution of the stars, but this is a minor drawback because it is usually known approximately.

2. A new approach to the Malmquist bias

We consider that we can characterize a given homogeneous sample of stars by its intrinsic distribution in absolute magnitude $\varphi_M(M)$. We will use a Gaussian distribution:

$$\varphi_M(M) \;=\; K \exp\left[-\frac{1}{2}\left(\frac{M-M_o}{\sigma_M}\right)^2\right] \tag{1}$$

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A reasonable choice for the spatial distribution taking into account the flattening of the galactic system is:

$$\varphi_x(x) \;=\; K' \exp\left(-\frac{|z|}{Z_o}\right) \tag{2}$$

where the z axis is perpendicular to the galactic disk – Z_o being the scale height – and we have assumed a homogeneous distribution in x and y. Other distributions can also be easily introduced.

If we assume that the distributions in M and x are independent, the probability of having a star with an absolute magnitude between M and M+dM in a position between x and $x+dx$ without any selection criteria is:

$$P_1(M,x) \;=\; \frac{1}{c_1}\,\varphi_M(M)\,\varphi_x(x)\,dM\,dx \tag{3}$$

where c_1 is the normalization constant.

But in the case of a sample which is limited in apparent magnitude we have a cut for stars with $m > m_{lim}$. We can take this into account by introducing a Heaviside function $\Theta(m-m_{lim})$, so we will have:

$$P_2(M,x) \;=\; \frac{1}{c_2}\,\varphi_M(M)\,\varphi_x(x)\,\Theta(m-m_{lim})\,dM\,dx \tag{4}$$

where c_2 is the new normalization constant.

The mean absolute magnitude in the non-selection case will be:

$$M_o \;=\; \int_{-\infty}^{\infty} dM \int_x dx\,\frac{M}{c_1}\,\varphi_M(M)\,\varphi_x(x) \tag{5}$$

while in the magnitude-limited case we will have:

$$\bar{M} \;=\; \int_{-\infty}^{\infty} dM \int_x dx\,\frac{M}{c_2}\,\varphi_M(M)\,\varphi_x(x)\,\Theta(m-m_{lim}) \tag{6}$$

The difference between the two mean magnitudes is the bias we are looking for:

Table 1. Values of ΔM as given by Eq. (11) for different sets of values of $(m_{lim}-M_o, \sigma_M, Z_o)$

$m_{lim}-M_o$	σ_M / Z_o	0.4	0.6	0.8	1.0	1.2	1.4	$m_{lim}-M_o$	σ_M / Z_o	0.4	0.6	0.8	1.0	1.2	1.4
0	100	0.22	0.49	0.87	1.35	1.92	2.58	8	100	0.16	0.36	0.63	0.98	1.39	1.88
	200	0.22	0.49	0.87	1.36	1.95	2.64		200	0.18	0.40	0.69	1.06	1.49	1.98
	300	0.22	0.49	0.88	1.37	1.97	2.66		300	0.19	0.42	0.73	1.11	1.56	2.07
	400	0.22	0.49	0.88	1.37	1.97	2.67		400	0.20	0.43	0.76	1.16	1.62	2.13
1	100	0.22	0.48	0.86	1.33	1.89	2.53	9	100	0.15	0.34	0.61	0.95	1.36	1.84
	200	0.22	0.49	0.87	1.35	1.94	2.61		200	0.17	0.37	0.65	1.00	1.42	1.90
	300	0.22	0.49	0.87	1.36	1.95	2.64		300	0.18	0.39	0.69	1.05	1.48	1.97
	400	0.22	0.49	0.88	1.37	1.96	2.65		400	0.18	0.41	0.71	1.09	1.53	2.03
2	100	0.21	0.48	0.84	1.30	1.84	2.45	10	100	0.15	0.34	0.60	0.93	1.34	1.82
	200	0.22	0.49	0.86	1.34	1.91	2.56		200	0.16	0.35	0.62	0.96	1.37	1.85
	300	0.22	0.49	0.87	1.35	1.93	2.60		300	0.17	0.37	0.65	1.00	1.41	1.90
	400	0.22	0.49	0.87	1.36	1.95	2.63		400	0.17	0.38	0.67	1.03	1.45	1.94
3	100	0.21	0.47	0.82	1.26	1.78	2.36	11	100	0.15	0.33	0.59	0.93	1.33	1.81
	200	0.21	0.48	0.85	1.32	1.87	2.49		200	0.15	0.34	0.60	0.94	1.35	1.83
	300	0.22	0.49	0.86	1.34	1.90	2.55		300	0.16	0.35	0.62	0.96	1.37	1.85
	400	0.22	0.49	0.87	1.35	1.92	2.58		400	0.16	0.36	0.63	0.98	1.39	1.88
4	100	0.20	0.45	0.79	1.21	1.70	2.25	12	100	0.15	0.33	0.59	0.92	1.33	1.81
	200	0.21	0.47	0.83	1.28	1.81	2.41		200	0.15	0.33	0.59	0.93	1.33	1.81
	300	0.21	0.48	0.85	1.31	1.86	2.48		300	0.15	0.34	0.60	0.94	1.35	1.83
	400	0.22	0.48	0.86	1.33	1.89	2.53		400	0.15	0.34	0.61	0.95	1.36	1.84
5	100	0.20	0.43	0.76	1.15	1.62	2.13	13	100	0.15	0.33	0.59	0.92	1.33	1.81
	200	0.21	0.46	0.81	1.24	1.74	2.30		200	0.15	0.33	0.59	0.92	1.33	1.81
	300	0.21	0.47	0.83	1.28	1.81	2.40		300	0.15	0.33	0.59	0.93	1.33	1.81
	400	0.21	0.48	0.84	1.30	1.84	2.45		400	0.15	0.34	0.60	0.93	1.34	1.82
6	100	0.18	0.41	0.71	1.09	1.53	2.03	14	100	0.15	0.33	0.59	0.92	1.33	1.81
	200	0.20	0.44	0.78	1.19	1.66	2.19		200	0.15	0.33	0.59	0.92	1.33	1.81
	300	0.21	0.46	0.81	1.24	1.73	2.29		300	0.15	0.33	0.59	0.92	1.33	1.81
	400	0.21	0.47	0.82	1.26	1.78	2.36		400	0.15	0.33	0.59	0.93	1.33	1.81
7	100	0.17	0.38	0.67	1.03	1.45	1.94	15	100	0.15	0.33	0.59	0.92	1.33	1.80
	200	0.19	0.42	0.74	1.12	1.57	2.08		200	0.15	0.33	0.59	0.92	1.33	1.81
	300	0.20	0.44	0.77	1.18	1.65	2.18		300	0.15	0.33	0.59	0.92	1.33	1.81
	400	0.20	0.45	0.79	1.22	1.70	2.25		400	0.15	0.33	0.59	0.92	1.33	1.81

$$\Delta M = M_o - \bar{M} \tag{7}$$

Using spherical galactic coordinates and introducing Eqs. (1) and (2) in Eq. (6) we obtain, after integration in (M,l,b):

$$\bar{M} = \frac{2\sqrt{2}Z_o\sigma_M}{c_2}\int_0^\infty r\left(1-\exp\left(-\frac{r}{Z_o}\right)\right)\left[-\frac{\sigma_M}{\sqrt{2}}\exp\left(-\alpha_{lim}^2\right)+M_o\frac{\sqrt{\pi}}{2}\,\mathrm{erfc}(-\alpha_{lim})\right]dr \tag{8}$$

with

$$\alpha_{lim} = \frac{m_{lim}-5\log_{10}(r)+5-M_o}{\sqrt{2}\,\sigma_M} \tag{9}$$

and

$$c_2 = \sqrt{2\pi}Z_o\sigma_M\int_0^\infty r\left(1-\exp\left(-\frac{r}{Z_o}\right)\right)\mathrm{erfc}(-\alpha_{lim})dr \tag{10}$$

From Eqs. (8) and (10) the Malmquist bias is found to be:

$$\Delta M = \sigma_M\sqrt{\frac{2}{\pi}\,\frac{\int_0^\infty r\left(1-\exp\left(-\frac{r}{Z_o}\right)\right)\exp\left(-\alpha_{lim}^2\right)dr}{\int_0^\infty r\left(1-\exp\left(-\frac{r}{Z_o}\right)\right)\mathrm{erfc}(-\alpha_{lim})dr}} \tag{11}$$

Numerical integration of this expression provides the value of the bias for a given set of parameters (M_o, σ_M, Z_o, m_{lim}). As can be seen, however, it is function of only three quantities ($m_{lim}-M_o$, σ_M, Z_o). In Table (1) the bias is given for different values of these quantities.

3. Results and discussion

In Figs.(1-2) we show the dependence of the bias on σ_M and the other parameters. As stated above, it is approximately proportional to σ_M^2 but now we can see how it depends on the other parameters.

The traditional correction applied to eliminate the Malmquist bias uses the approximate formula

$$\Delta M = 1.38\sigma_M^2 \tag{12}$$

which assumes a constant space distribution function. As Malmquist (1936) shows, this hypothesis holds only for samples of dwarf stars or giant stars, the latter when selected with

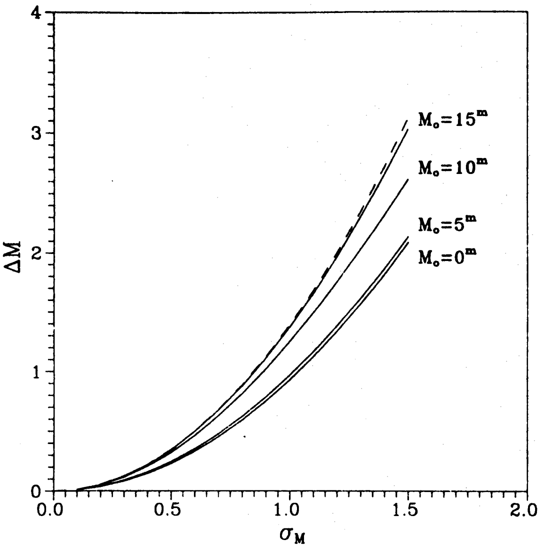

Fig. 1. The bias as a function of σ_M for different absolute magnitudes and $m_{lim} = 15^m, Z_o = 200pc$. *Dashed line*: traditional correction

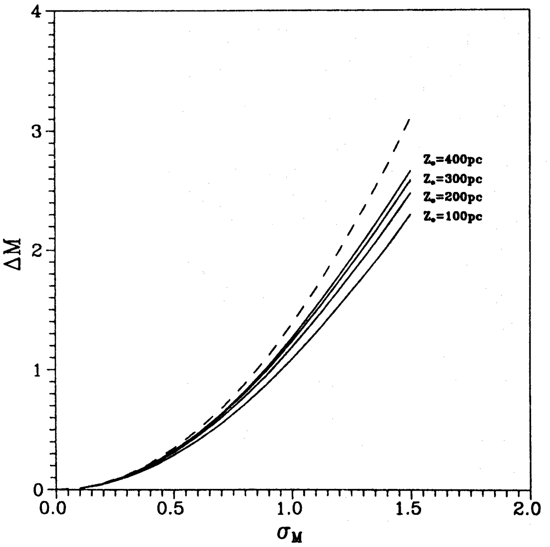

Fig. 2. The bias as a function of σ_M for different values of scale height $M_o = 0^m, mlim = 15^m$. *Dashed line*: traditional correction

$m_{lim} < 6^m$. The method we propose includes the dependence on (M_o,Z_o,m_{lim}) so it can provide a fairly precise estimation of the bias for a general case – the precision depending on how realistic are the distributions taken for the sample – in a very simple way.

We can compare the two corrections for F dwarfs. From Mihalas (1981) we have $M_o = 6^m$, $\sigma_M = 1^m$ and $Z_o = 200pc$ for these stars. In a sample limited by $m_{lim} = 15^m$ we obtain $\Delta M = 1.0^m$ with our expression, while Eq. (12) gives $\Delta M = 1.38^m$. We can see that the difference is not negligible.

As expected, our method reproduces the traditional approximation (12) for samples having a small distance modulus – see Fig. (1) – which are near the sun, well inside the disc, and so present an approximately homogeneous distribution.

An important advantage of our method is its flexibility: different forms of spatial distribution and/or absolute magnitude distribution – see Jaschek & Gómez (1985) – can be easily introduced. Furthermore, apparent magnitude selections other than a simple cut can also be implemented, even when dependent on l,b, etc. – e.g. in the case of HIPPARCOS data, Gómez et al. (1989) –.

4. Application to the determination of absolute magnitudes

This kind of probabilistic treatment of the Malmquist bias is being implemented in a maximum likelihood method to obtain mean parameters of samples of stars – see Luri et al. (1992) –. In Fig. (3) we can see that the mean absolute magnitudes of a set of simulated samples – limited in apparent magnitude – follow the prediction given by Eq. (8) – and are thus affected by the Malmquist bias – while the values obtained by our maximum likelihood method agree with the real mean value M_o.

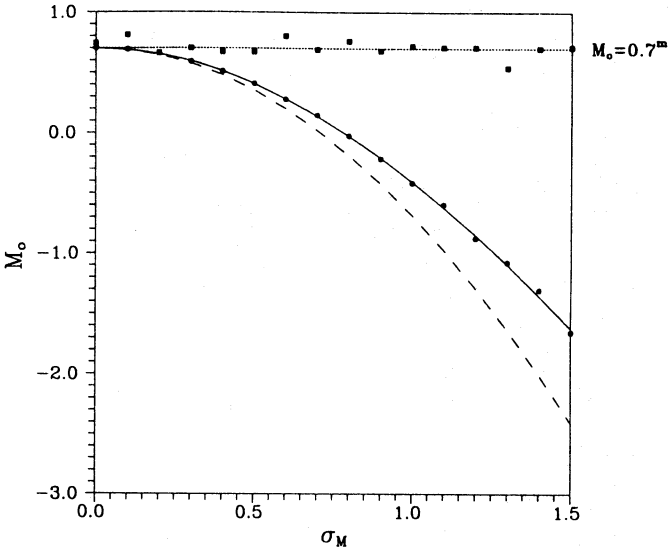

Fig. 3. Mean absolute magnitudes of simulated samples, containing 2000 stars, as a function of the σ_M with $m_{lim} = 6.5^m$, $M_o = 0.7^m$ and $Z_o = 100pc$. *Circles*: mean magnitude affected by the Malmquist bias, *squares*: values obtained by our maximum likelihood method. *Solid line*: prediction of mean magnitude according to Eq. (8) *dashed line*: prediction of mean magnitude using the traditional Malmquist correction – Eq. (12) –

In this example we have only included the effect of a cut in apparent magnitude, which causes the Malmquist bias, but in a more general case we will have different selection criteria leading to other biases. With the approach described above, the effect of selection criteria is automatically taken into account and so the method provides an estimation of the mean parameters which is free of selection biases.

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