Groundwater extraction for irrigation purposes: the case of asymmetric players*

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Abstract

In this article the author studies the differential game applied to the groundwater resource exploited in a common property regime. The game is solved for two agents, who differ in their water demand (demand asymmetry) as well as in time-preferences (discount rates asymmetry). The author tries to investigate if the cooperation is more beneficial than non-cooperation and if an increase in asymmetry has influence on the efficiency of the solutions. The results show that the more heterogeneous the agents are, the less inefficient is the allocation of water under non-cooperation. Moreover, the cooperation can be considered beneficial for the environment, but not for the agents: a player with lower demand suffers losses in terms of welfare, when the demand asymmetry is high.

Keywords: Groundwater resource, asymmetric players, subgame perfect equilibria, differential game

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1 Introduction

Groundwater is one of the essential life resources not only for humans, but for the whole ecosystem in general. Water running underground constitutes 98% of world's fresh water, supplying with drinking water half of the world's needs [19]. Moreover, groundwater is also used for irrigation of the crops (sumps up to 43% of the total irrigation water use [3]), for consumption in urban areas (for example, as drinking water) or serves as an important component of some industrial processes. As for the ecosystem, groundwater recharges lakes, rivers and wetlands, providing water to flora and fauna of the area.

However, the growing population and expansive industrial activities increased the pressure on water quantity and quality. As a result of excessive withdrawals of water, the problem of water scarcity arose together with the problem of degradation of water quality.

As stated in Esteban and Albiac (2011) [8], water scarcity has become widespread in most arid and semiarid regions around the world: for instance, from groundwater depletion suffer Indus–Ganges (India, Pakistan, Bangladesh, Nepal) basins and the Ogallala aquifer in the North America, as well as aquifers in Northern China plain and Europe (Spain, France and Portugal, as examples).

Access to the groundwater is usually limited to owners of the land overlying aquifers, therefore the groundwater is exploited in a common property regime, as explained in Roseta-Palma (2003) [20]. The problem with common pool resources is that it is quite difficult to establish clear property rights. Hence, access to the resource by competing users creates some externalities, as cost and strategic externalities, which lead to the inefficiency in exploitation. The cost externality arises because pumping by on user lowers the water table and, therefore, increases the cost of extraction for all other users of the aquifer. The strategic externality is a result of competition for the limited resource among farmers, since, as mentioned above, the property rights are not well defined (see Rubio and Casino [22]).

This study particularly tries to investigate if cooperation of the groundwater users can be beneficial for the environment (in terms of groundwater stock) as well as for the users themselves (in terms of welfare), when agents differ in their water demand and time preferences, and how these differences influence the stock, extractions and welfare of the agents.

One of the main studies on the groundwater management for irrigation use was published in 1980 by Gisser and Sanchez [12]. In this classic paper authors compared optimal control solution (socially optimal with one player making a decision, as for example a social planner) with the competitive solution (free market, no-control) in terms of stock and water extractions. The latter solution comes from a static optimisation problem, where agents are considered to behave myopically, that is making a decision over a short period of time without taking into account the impact of other agents' decision on the stock of the aquifer. The authors concluded that when the area of the aquifer is big enough the use of control is not justified. This effect is known as Gisser-Sanchez Effect (GSE), and it has been discussed and challenged in subsequent works (see, for example, Esteban and Albiac (2011) [8]).

Using a differential game of several farmers, further studies were comparing the open loop solution and the feedback solution with the socially optimal one, when a manager of an aquifer chooses the extraction program, which maximises the present value of profits of all farmers (for example, Negri (1989) [17] or Rubio and Casino (2011) [22]). Note that open loop solution is said to capture only the pumping (cost) externality, whereas the feedback solution supposedly captures both cost and strategic externalities. Authors showed that when agents are identical, taking into account strategic externality increases the inefficiency of the solution, *i.e.* the feedback solution is more inefficient than the open loop solution, when comparing with the socially optimal water allocation.

Introduction of asymmetries allows to obtain models that can better describe real world situations. For example, Roseta-Palma and Brasão (2004) [21] considered asymmetries in demand and costs, and Erdlenbruch *et al.* (2007) [7] focused on asymmetry in opportunity costs of resource harvesting. Only quite recently the idea of the comparison between the non-cooperative and cooperative solutions for agents with asymmetric time-preferences have been introduced in works of Marín-Solano and Shevkoplyas (2011) [16], de-Paz *et al.* (2013) [18] and Ekeland *et al.* (2013) [5], as examples. The problem is that once we depart from the assumption of unique and constant discount rates, the problem of time-inconsistency appears, as mentioned in Strotz (1955) [23]. Therefore, standard dynamic optimization techniques fail to obtain an optimal time-consistent solution. It means that the solution computed at time t is no longer optimal for $t' > t$, and modified dynamic programming equations are required. The study of de Frutos Cachorro *et al.* (2020) [11], the most similar to this work, analysed both demand and discount rate asymmetries in the context of groundwater use for different purposes, comparing cooperation versus non-cooperation equilibria. The results of this study showed that cooperation is more efficient in terms of stock than non-cooperative solutions, but that in terms of personal welfare the cooperation is not always profitable. Moreover, authors find that higher asymmetry does not necessarily mean higher inefficiency (actually, higher asymmetry of discount rates leads to lower inefficiency).

The contribution of the study can be summarised in two points. First, in this study agents are supposed to have different water demand as opposed to the classical studies of Gisser and Sanchez (1980) [12], Rubio and Casino (2001) [22], and Esteban and Albiac (2011) [8], where agents were considered to be identical in their demand for the water resource. Also unlike Roseta-Palma and Brasão [21] and de Frutos Cachorro *et al.* (2020) [11], who have already considered the asymmetry in water demand for different uses (farming and public supply), the agents in this study are using the groundwater for the same purpose - irrigation of crops - and have the same elasticity of demand (both agents are farmers and they differ in the land size - hence the difference in the amount of water in demand, but not in demand elasticity). Second, as the size of land of two farmers (a small farmer and a big farmer) is not the same there is no reason to assume that the future discount rates will be the same, as well stated in de-Paz *et al* [18]. So, here we also depart from classical studies in assuming different discount rates of agents. Moreover, in contrast to de Frutos Cachorro *et al.* (2020) [11], who considered different discount rates of agents before the occurrence of a regime shift in the aquifer (change of the natural recharge rate), in this paper different discount rates are applied to the infinite planning horizon.

To find an answer to the research question stated in the beginning of this section, firstly, the model with two asymmetries has been resolved analytically for the non-cooperative and cooperative cases. Both cases were solved using the dynamic optimization techniques, as opposed to optimal control techniques (Pontryagin Maximum Principle), used in Negri (1989) [17], Rubio and Casino (2001) [22], or Esteban and Albiac (2011) [8]. Second, the results in terms of stock, extractions and individual and group welfare were compared between two solutions to be able to make some comments on potential inefficiency of the non-cooperative one. Finally, the theoretical model was applied to real data on the aquifer situated in Spain (Western La Mancha aquifer). Using numerical simulations, the influence of two asymmetries on the stock, extractions and welfare was considered (separately as well as jointly).

The work is organised as follows: in Section 2 the model is introduced, in Section 3 the model is solved using the non-cooperation and cooperation frameworks, in Section 4 the results of the numerical application are presented, and in the Section 5 the main results and conclusions are stated.

2 Description of the model

In this work, the model of Rubio and Casino (2001) [22] is adapted to the case of asymmetric players. The assumption of farmers being identical made in this classical study is challenged in this work. Indeed, it is quite natural to assume that farmers using the water of the same aquifer for irrigation of their crops (that are not necessarily the same) may posses lands of different sizes and, therefore, have different demand for water. The model is carefully described below.

2.1 Revenues, costs and aquifer dynamics

In this model two types of users are considered: a big farmer (farmer owing a big plot of land) and a small farmer (farmer owing a small plot of land), denoted as $i \in \{b, s\}$, respectively. A big farmer may represent a big landowner as well as a big company, producing some agricultural goods. Similarly, a small farmer may refer to a small landowner or a small agricultural business. Let us assume that both farmers use the groundwater for the same purpose, particularly for irrigation. In this sense, this work follows the lead of most papers that analyze the groundwater over-exploitation problems, as Gisser and Sanchez (1980) [12] or Rubio and Casino (2001) [22].

For simplicity, let us also assume that these farmers produce the same type of crop. Moreover, farmers compete in a competitive market, so that the price of the crop (p) equals its marginal cost, which, in turn, is associated with the market price of the input. Taking into account no other expenses of the crop creation, the price of water equals its marginal product and is, therefore, identified with p.

Let the demand function for the big farmer be a negatively sloped linear function, as in previous literature: $g_b = a - bp$, where $a, b > 0$, and p is a price of water. Then, the revenue function of a big farmer obtained by integrating the inverse demand function is given by:

$$\int_{g_b} p(x)dx = \int_{g_b} \frac{a-x}{b} dx = \frac{a}{b} g_b - \frac{1}{2b} g_b^2$$

In this study, let us assume that the demand for water from the small farmer is represented by a fraction of the demand of the big farmer: $g_s = \theta(a - bp)$, with $0 < \theta \leq 1$.

The revenue function in this case will be:

$$\int_{g_s} p(x)dx = \int_{g_s} \frac{a - \frac{x}{\theta}}{b} dx = \frac{a}{b} g_s - \frac{1}{2\theta b} g_s^2$$

In the study the value of θ can vary between 0 and 1, however, no previous assumptions on the exact value of this parameter are made. There are no limits on how different the land possessions and, therefore, the water demand can be. However, when proceeding to numerical applications for a particular aquifer, some reasonable guesses can be made, according to the available data on the land ownership structure of the aquifer territory.

Following the previous literature, in this study we assume that the marginal cost of water extraction is linear function in the stock of the aquifer G (or the amount of water that can be stored). Total costs of extraction of farmer $i \in \{b, s\}$ depend on the quantity of the water extracted:

$$C_i = (z - cG)g_i, \qquad z, c > 0,$$

where z is a fixed cost and the maximum marginal cost of extraction, and c is the slope of the marginal pumping cost function (see, for example, Gisser and Sanchez (1980) [12], Negri (1989) [17], Rubio and Casino (2001) [22]). In this study it seems reasonable to suggest that the fixed costs of extraction and the marginal pumping costs are the same for both agents as a result of competition in the industry of well installation (in case they use different wells) or as a result of using the same well.

The dynamics of the aquifer (or of the stock of the aquifer) is given by a differential equation:

$$\dot{G} = r - (1 - \gamma) \sum_i g_i, \qquad i \in \{b, s\},$$

where r is a natural recharge rate and γ is the return flow coefficient, $\gamma \in [0, 1)$.

A natural recharge is basically water that moves from the land surface to the aquifer (with rain or melted snow, for example). The return flow coefficient, in turn, describes the proportion of water returned to the aquifer from the cultivated area, which depends on the quality and the type of the soil. In this work, it is assumed that both players have their fields with the soil that have the same properties.

2.2 Different discount rates

It is a common knowledge that future gains should be discounted, as humans tend to enjoy the goods better sooner than later (but suffer mischances later than sooner). Arising from the financial literature, basic models suggest that the discount rate is constant through time and that the discount factor should be of the exponential form $e^{-\delta t}$. In this work, for simplicity, the idea of constant over the whole planning horizon discount rates is maintained, even though there are studies showing that it is not necessarily the case, and economic agents might discount nearest future harder than the far-horizon future (see Loewenstein *et al.* (2002) [15]).

In this study it is considered, however, that two agents have different discount rates, as opposed to the standard assumption of the unique discount rate for both players as in Rubio and Casino (2001) [22], and Gisser and Sanchez (1980) [12]. It seems reasonable to assume that big farmers have advantages in terms of financial facilities with generally higher turn over, and that the probability of survival for this type of producer is higher. Therefore, the agricultural business for big landowners appears to be more stable, and hence they are more patient and secure about the future. These facts translate in the use of a lower discount rate of time preference by the big landowner (ρ_b) in comparison with a small farmer (ρ_s), so that $\rho_b \leq \rho_s$. Another important feature of this study is that the discount rates are set to be different during the infinite planning horizon, unlike in the work of de Frutos Cachorro *et al.* (2020) [11], where different discount rates were used until the occurrence of a shock.

2.3 Problem statement

The optimization problem of user $i \in \{b, s\}$ is to maximize his individual welfare *i.e.* the present value of his future profits in the infinite planning horizon. Therefore, if ρ_i is a discount rate of user i we must solve

$$\max_{g_i(\cdot)} \int_0^\infty F_i(G, g_i) e^{-\rho_i t} dt, \tag{1}$$

where

$$F_b(G, g_b) = \frac{a}{b} g_b - \frac{1}{2b} g_b^2 - (z - cG) g_b, \tag{2}$$

$$F_s(G, g_s) = \frac{a}{b} g_s - \frac{1}{2\theta b} g_s^2 - (z - cG) g_s, \tag{3}$$

subject to

$$\dot{G} = r - (1 - \gamma) \sum_i g_i, \tag{4}$$

with $G(0) = G_0$ given and

$$g_i \geq 0, \qquad G \geq 0, \qquad i = b, s \tag{5}$$

3 Model resolution

The objective is to solve the model under cooperation and non-cooperation by computing the corresponding subgame perfect solutions and compare the results in terms of extraction levels, stock and welfare. It is important for the further resolution of the model to assume that both agents can observe the level of the water table during the whole planning horizon.

The game in consideration has two asymmetries: first, the asymmetry in demand, expressed through $\theta \in (0, 1]$, and the second asymmetry in discounts, expressed through the relationship $\rho_b \leq \rho_s$ (for the case of complete symmetry $\rho_b = \rho_s$). Demand asymmetry has already been considered in the work of de Frutos Cachorro *et al.* (2020) [11], though in that work the aquifer exploitation for different purposes was considered, and demand functions also differed in other parameters (apart from the amount of demand *per se*), such as the demand-price elasticity. In this study we suppose that the elasticity of demand for both farmers is the same, no matter the size of the land in possession. Moreover, in the same study different discount rates were allowed for, but only within the finite time frame. Conversely, this study analyses the case of different discount rates on the infinite planning horizon.

3.1 Subgame perfect non-cooperative equilibrium

In this section we will calculate a non-cooperative solution (also known as a feedback solution), under which the agents maximise their own future profits. To be able to calculate the subgame perfect Nash equilibrium (Markov perfect equilibrium), it is important to make an assumption about the agents being able to track the level of the resource, so that the agent could make a decision on the extracted amount according to the current level of the water table. In the setting of this study it can be done by using some groundwater monitoring systems.

For calculating the subgame perfect non-cooperative Nash equilibria (SPNE), standard dynamic programming techniques are used, which are described in more detail in Dockner *et al.* (2000) [4].

The dynamic programming equation to solve by each user $i \in \{b, s\}$ is:

$$\rho_i V_i^{NC}(G) = \max_{g_i} \left\{ F_i(G, g_i) + V_i^{NC}(G)'(r - (1-\gamma)(g_i + \phi_j^{NC}(G)) \right\} \tag{6}$$

In (6) $\phi_j^{NC}(G)$ denotes the strategy of player j, for $j \neq i$. In this work the focus will be made on stationary linear (affine) strategies in this linear-quadratic differential game, so that $\phi_j^{NC}(G) = \alpha_j^{NC} G + \beta_j^{NC}$ and $V_j^{NC}(G) = A_j^{NC} G^2 + B_j^{NC} G + C_j^{NC}$.

Step 1. By plugging into (6) an expression for player's optimal strategy $\phi_i^{NC}(G)$ and his value function $V_i^{NC}(G)$ for every $i \in \{b, s\}$, and arranging terms on the right hand side separately for G^2, G and a free term, parameters of the value functions V_b^{NC} and V_s^{NC} can be expressed using parameters $\alpha_b^{NC}, \beta_b^{NC}, \alpha_s^{NC}$ and β_s^{NC}. See equations (20)-(25) in Appendix A.

Step 2. Following the procedure described in Appendix A, we conclude that parameters of the stationary extraction strategies $\alpha_b^{NC}, \alpha_s^{NC}, \beta_b^{NC}$ and β_s^{NC} should solve the following system of nonlinear equations:

$$\frac{\rho_b}{2(1-\gamma)} \left(c - \frac{\alpha_b^{NC}}{b} \right) = -\frac{(\alpha_b^{NC})^2}{2b} + c\alpha_b^{NC} - (\alpha_s^{NC} + \alpha_b^{NC}) \left(c - \frac{\alpha_b^{NC}}{b} \right) \tag{7}$$

$$\frac{\rho_s}{2(1-\gamma)} \left(c - \frac{\alpha_s^{NC}}{\theta b} \right) = -\frac{(\alpha_s^{NC})^2}{2\theta b} + c\alpha_s^{NC} - (\alpha_s^{NC} + \alpha_b^{NC}) \left(c - \frac{\alpha_s^{NC}}{\theta b} \right) \tag{8}$$

whereas given α_b^{NC} and α_s^{NC}, β_b^{NC} and β_s^{NC} should satisfy the system of linear equations:

$$\frac{1}{b} \left(\frac{\rho_b}{1-\gamma} + \alpha_b^{NC} + \alpha_s^{NC} \right) \beta_b^{NC} + \left(\frac{\alpha_b^{NC}}{b} - c \right) \beta_s^{NC} = \left(\frac{a}{b} - z \right) \left(\frac{\rho_b}{1-\gamma} + \alpha_s^{NC} \right) + \frac{r}{1-\gamma} \left(\frac{\alpha_b^{NC}}{b} - c \right) \tag{9}$$

$$\frac{1}{\theta b} \left(\frac{\rho_s}{1-\gamma} + \alpha_b^{NC} + \alpha_s^{NC} \right) \beta_s^{NC} + \left(\frac{\alpha_s^{NC}}{\theta b} - c \right) \beta_b^{NC} = \left(\frac{a}{b} - z \right) \left(\frac{\rho_s}{1-\gamma} + \alpha_b^{NC} \right) + \frac{r}{1-\gamma} \left(\frac{\alpha_s^{NC}}{\theta b} - c \right) \tag{10}$$

To describe the equilibrium we have to calculate a steady state level of the stock (G_∞^{NC}) of the aquifer, which in this case is a solution to $\dot{G} = 0$ or, taking into consideration the affine structure of the SPNE for the extraction, to $r - (1-\gamma)(\alpha_b^{NC} + \alpha_s^{NC}) G_\infty^{NC} - (1-\gamma)(\beta_b^{NC} + \beta_s^{NC}) = 0$. Therefore,

$$G_\infty^{NC} = \frac{r - (1-\gamma)(\beta_b^{NC} + \beta_s^{NC})}{(1-\gamma)(\alpha_b^{NC} + \alpha_s^{NC})} \tag{11}$$

It is more interesting, in practical terms and in terms of further policy implications, to focus on solutions that converge to a steady state, as it characterises the stable state of the system. Therefore, we must impose a condition $\alpha_b^{NC} + \alpha_s^{NC} > 0$. Additionally, looking for an interior solution and assuming that $G(t) > 0$ for all t, it is natural to impose a condition $r \geq (1-\gamma)(\beta_b^{NC} + \beta_s^{NC})$, which basically means that the water resource will not be exhausted in finite time (see de Frutos Cachorro *et al.* (2020) [11]).

The system of equations (7)-(8) can be simplified to a fourth degree equation, which can have up to four different roots. These linear quadratic differential games with two players in principle can have multiple equilibria (up to three), as Engwerda (2005) [6] showed in Theorem 8.10. However, in this model, if condition $\alpha_b^{NC} + \alpha_s^{NC} > 0$ is imposed, there exists at most one equilibrium. And if the recharge rate is too small *i.e.* $r < (1-\gamma)(\beta_b^{NC} + \beta_s^{NC})$ for the unique solution to the system (7)-(10), satisfying the condition $\alpha_b^{NC} + \alpha_s^{NC} > 0$, then no interior equilibrium exists.

Proposition 1. *In Problem (1)-(5), there exists at most one stationary linear subgame perfect non-cooperative equilibrium. If it exists, it is given by the unique solution to the system (7)-(10), satisfying the condition* $\alpha_b^{NC} + \alpha_s^{NC} > 0$.

Proof: See Appendix A.

3.2 Subgame perfect cooperative solution

If instead of maximizing their own profit, farmers decide to cooperate *i.e.* maximize their collective payoff, the result will be different. In this case a coalition of players will have to take into account their own future decisions, based on their preferences.

Moreover, as the time-preferences of two agents (big and small farmer) are set to be different, the joint preferences will be time-inconsistent. Therefore, when finding subgame perfect cooperative equilibria, we should use a non-standard technique that gives us a time-consistent solutions (Markov subgame perfect equilibria), otherwise players will have to continuously modify their calculated choices of future extractions. The equations we are using in this study were described in Marín-Solano and Shevkoplyas (2010) [16], de-Paz *et al.* (2013) [18] and Ekeland *et al.* (2013) [5].

In this case we have to solve a non-standard dynamic optimisation problem. The dynamic programming equation is:

$$\rho_b V_b^C(G) + \rho_s V_s^C(G) = \max_{g_b, g_s} \left\{ F_b(G, g_b) + F_s(G, g_s) + \left(V_b^C(G)' + V_s^C(G)' \right) (r - (1-\gamma)(g_b + g_s)) \right\}, \tag{12}$$

where as in the non-cooperative case the value function is quadratic $V_i^C(G) = A_i^C G^2 + B_i^C G + C_i^C$ and the optimal extraction strategy is an affine function $g_i^C = \phi_i^C(G) = \alpha_i^C G + \beta_i^C$ for each player $i \in \{b, s\}$.

First step in solving the problem (12) is maximising the right hand side with respect to g_b and g_s, respectively.

Therefore, the First Order Conditions are:

$$\frac{a}{b} - \frac{1}{b} g_b - (z - cG) - (2A_b^C G + B_b^C + 2A_s^C G + B_s^C)(1-\gamma) = 0$$

$$\frac{a}{b} - \frac{1}{\theta b} g_s - (z - cG) - (2A_b^C G + B_b^C + 2A_s^C G + B_s^C)(1-\gamma) = 0$$

Which leads us to the following optimal extraction strategies of the players:

$$\phi_b^C(G) = a - b(z + (1-\gamma)(B_b^C + B_s^C)) + b(c - 2(1-\gamma)(A_b^C + A_s^C))G \tag{13}$$

$$\phi_s^C(G) = \theta a - \theta b(z + (1-\gamma)(B_b^C + B_s^C)) + \theta b(c - 2(1-\gamma)(A_b^C + A_s^C))G \tag{14}$$

Knowing that the optimal extraction strategies have a linear form $\phi_i^C(G) = \alpha_i^C G + \beta_i^C$ for $i \in \{b, c\}$, we can express $\alpha_b^C, \alpha_s^C, \beta_b^C$ and β_s^C in terms of A_b^C, A_s^C, B_b^C and B_s^C.

$$\alpha_b^C = b\left(c - 2(1-\gamma)(A_b^C + A_s^C)\right), \qquad \alpha_s^C = \theta b\left(c - 2(1-\gamma)(A_b^C + A_s^C)\right) \tag{15}$$

and

$$\beta_b^C = a - zb - b(1-\gamma)(B_b^C + B_s^C), \qquad \beta_s^C = \theta\left(a - zb - b(1-\gamma)(B_b^C + B_s^C)\right) \tag{16}$$

These expressions differ from those obtained for the non-cooperative case (see Step 2 in Appendix A) because the maximisation problem is not the same. We see that parameters of the optimal extraction function in case of cooperation do not only depend on the parameters of the value function of the player himself, but also on the parameters of the value function of the other player.

Further, for calculating the coefficients A_i^C, B_i^C, C_i^C for $i \in \{b, s\}$, we have to solve the system of six equations with six unknowns, obtained by arranging the terms of G^2, G and a free term in the following equation:

$$\rho_i(A_i^C G^2 + B_i^C G + C_i^C) = F_i(G, \phi_i^C(G)) + \left[2A_i^C G + B_i^C\right]\left[r - (1-\gamma)(\phi_b^C(G) + \phi_s^C(G))\right] \tag{17}$$

More detailed equations for each player are presented in Appendix B.

For example, to calculate A_b^C and A_s^C we have to solve the following system of non-linear equations:

$$\rho_b A_b^C = 4b(1-\gamma)^2(1+\theta)A_b^C(A_b^C + A_s^C) - 2bc(1-\gamma)(A_b^C + A_s^C) - 2b(1-\gamma)^2(A_b^C + A_s^C)^2$$

$$\rho_s A_s^C = 4b(1-\gamma)^2(1+\theta)A_s^C(A_b^C + A_s^C) - 2\theta bc(1-\gamma)(A_b^C + A_s^C) - 2\theta b(1-\gamma)^2(A_b^C + A_s^C)^2$$

The full system of six equations is given in Appendix B (equations (44)-(49)). This system is quite cumbersome to solve analytically, that is why it will be solved only numerically when applying the real data on the aquifer to the theoretical model in the next section.

It remains to say a few words about the number of possible solutions and their convergence to a steady state. The equations (45) and (44) are quadratic and depend only on A_b^C and A_s^C, therefore we can be sure that there will be no more than 4 solutions for the pair (A_b^C, A_s^C). Hence, there will be maximum 4 possible solutions for B_b^C and B_s^C as the expressions (46) and (47) depend linearly on A_b^C and A_s^C. Furthermore, C_b^C and C_s^C depend linearly on B_b^C and B_s^C, so there will be maximum 4 solutions for these terms as well.

However, as previously described for the non-cooperative case, we are interested only in the solutions that converge to a steady state. This means that, according to the structure of the functions $\phi_b^C(G)$ and $\phi_s^C(G)$, $\alpha_b^C + \alpha_s^C > 0$. From (13) and (14), it is evident that the necessary inequality corresponds to:

$$b(c - 2(1-\gamma)(A_b^C + A_s^C)) + \theta b(c - 2(1-\gamma)(A_b^C + A_s^C)) > 0 \tag{18}$$

If we denote $A^C = A_b^C + A_s^C$ and rearrange the terms in (18), we can obtain the condition for convergence of the optimal solutions for the extraction rates in cooperative game:

$$A^C < \frac{c}{2(1-\gamma)} \tag{19}$$

We will check this condition, when numerically solve the model, for choosing among all possible solutions those that converge.

4 Numerical analysis: case of Western La Mancha Aquifer

In this section, the theoretical model described previously is applied to data of the Western La Mancha aquifer in the Upper Guadiana River Basin. Western La Mancha aquifer is situated in central-south Spain, it occupies around 5000 square kilometers in provinces Ciudad Real (80%), Albacete and Cuenca [1], where dry periods are frequent. Unfortunately, this aquifer has suffered from several droughts and gross mismanagement in the last decades of the 20^{th} century, which led to a decrease in the water tables impacting dramatically the wetlands in the Mancha Húmeda Biosphere Reserve [14]. Taking into account that up to 92% [2] of water extracted goes for irrigation purposes, this study is very important in terms of environmental policy implication on the possible benefits of cooperation as opposed to non-cooperation.

Parameters necessary for the simulations, previously mentioned in Esteban and Albiac (2011) [8], Esteban and Dinar (2016) [9], and de Frutos Cachorro *et al.* (2019) [10] are presented in the Table 1.

Table 1: Values of parameters for Western La Mancha aquifer

Parameter	Description	Units	Value
a	Water demand intercept	Million cubic meters / year	4403.73
b	Water demand slope	(Million cubic meters / year)^{2}Euro^{-1}	0.097
c	Pumping cost slope	Euros / Million cubic meters^2	3.162
z	Pumping cost intercept	Euros / Million cubic meters	266000
G_0	Initial stock level (in volume)	Million cubic meters	80960
r	Natural recharge rate	Million cubic meters / year	360
γ	Return flow coefficient	unitless	0.2
θ	Small farmer demand proportion	unitless	$\theta \in (0, 1]$
ρ_b	Big farmer discount rate	Year^{-1}	0.05
ρ_s	Small farmer discount rate	Year^{-1}	$\rho_s \in [0.05, 0.09]$

As mentioned before there are two asymmetries in the model: demand asymmetry expressed by θ and time-preference asymmetry expressed through different discount rates ρ_b and ρ_s.

However, a benchmark case should reflect a complete symmetry, therefore $\theta = 1$ and $\rho_b = \rho_s = 0.05$ will be used. The value of 0.05 for a discount rate is just a frequently used assumption. In the subsequent section discount rates will be manipulated to estimate the time-preference asymmetry .

Moreover, the value of θ will be also changed, firstly holding the discount rates constant for disentangling the effect of the demand asymmetry, and then varying both θ and ρ_b and ρ_s for capturing both asymmetries at once. As a reminder, θ captures the proportion of the water demand of the small farmer with respect to the big farmer's demand. It can be easily assumed, that sizes of the land may vary notably between the farmers. So, we will do the simulations for three different values of θ: $\theta = 1, \theta = \frac{1}{3}, \theta = \frac{1}{6}$.

Actually, these assumptions were built on the basis of the work of Guijarro and Sánchez (2013)

[13] on Western la Mancha aquifer. In this work authors present estimations for the incomes of regional authorities, in charge of water management, obtained through the water consumption tariffs. Breaking down to communities of water users, authors provide information on the land size of these communities that can differ in size in 3, 6 or even 10 times and can be seen as a parallel to the farmer definition used before. For example, in the Table 22, it is stated that The Union of Villarta de San Juan occupies 3000 hectares, the union San Clemente has 8780 hectares in possession and the union Socuéllamos has a 18000 hectares land plot. Therefore, the previous assumptions made on the values of θ seem quite reasonable and realistic.

4.1 Results: stock and extractions under different types of asymmetries

In this section numerical simulations are computed and the results for the evolution of stock and extraction rates are compared between the cooperative and non-cooperative case. First, the demand asymmetry effect is analysed, which is represented by variation in the parameter θ. After, the time-preference asymmetry effect is described, which is achieved by changing the parameter ρ_s, keeping the parameters ρ_b and θ constant. Finally, both effects are analysed together, and welfare comparisons are provided in order to study the efficiency of different behavioral strategies. The results are represented in Figures [1] 1, 2 and 3 and in Tables 2, 3, 4, 5, 6 and 7.

4.1.1 Demand asymmetry

In this section the effect of different levels of demand asymmetry is analysed, represented by the parameter θ *i.e.* proportion of the water demanded by the small farmer with respect to the demand of the big one. As a reminder, the lower the value of θ, the higher the asymmetry between the players.

The difference in stock and extraction rates between different game structures (cooperative and non-cooperative) during the planning horizon of 100 years can be seen in the Figure 1.

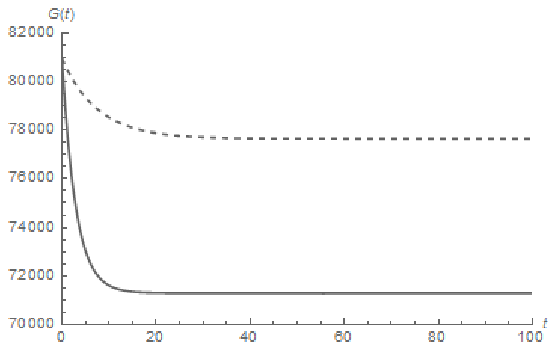
The baseline case is a complete symmetry *i.e.* $\theta = 1$ with equal discount rates ($\rho_b = \rho_s = 0.05$). Two other values of θ ($\theta = \frac{1}{3}$ and $\theta = \frac{1}{6}$) are considered in the Table 2, holding discount rates constant and equal to be able to disentangle the demand asymmetry effect.

Table 2: Stock volume (in million cubic meters) and extraction rates (in million cubic meters / year) at the steady state for different values of θ.

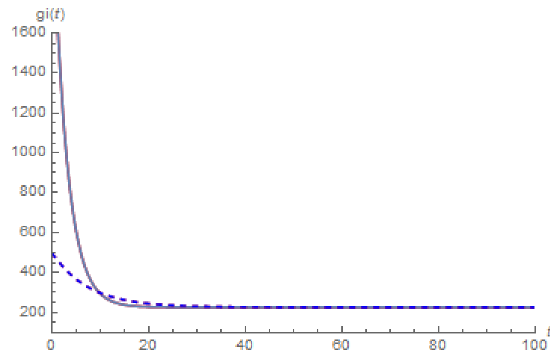
	(1) G_∞^{NC}	(2) G_∞^{C}	(3) (2)-(1)	(4) g_b^{NC}	(5) g_s^{NC}	(6) g_b^{C}	(7) g_s^{C}	(8) (4)+(5)	(9) (6)+(7)
$\theta = 1$	71278.20	77661.10	6382.90	225.00	225.00	225.00	225.00	450.00	450.00
$\theta = \frac{1}{3}$	72364.20	78029.20	5665.00	261.99	188.01	337.50	112.50	450.00	450.00
$\theta = \frac{1}{6}$	73446.40	78186.90	4740.50	296.68	153.32	385.71	64.29	450.00	450.00

[1] In the figures of this section the following colour scheme is applied. First, on the left the evolution of the stock is depicted (grey colour) and on the right side the evolution of the extraction rates both for the big (red) and small (blue) farmer is represented. Dashed lines correspond to the cooperative case, whereas solid lines refer to the non-cooperative case.

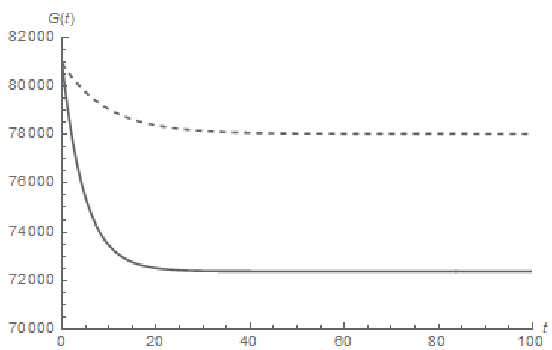
Figure 1: Simulations for the different values of θ of stock in Mm^3 (on the left) and extraction rate in $Mm^3/year$ (on the right) (see Footnote 1 for the colours code).



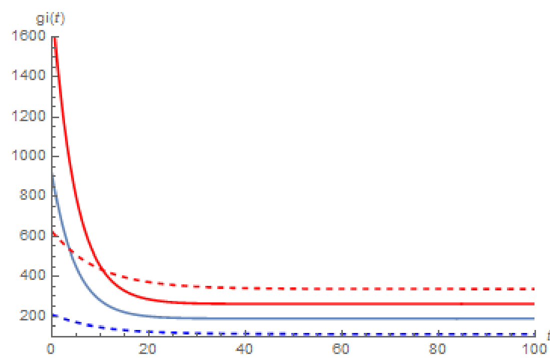
(a) $\theta = 1, \rho_b = 0.05, \rho_s = 0.05$



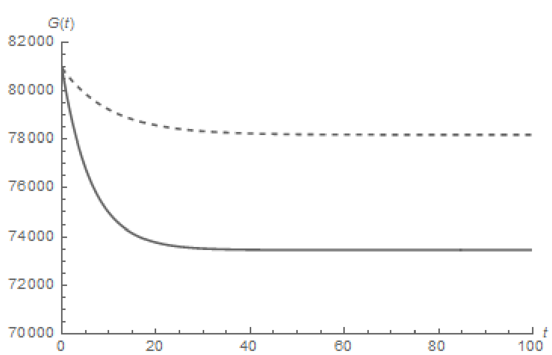
(b) $\theta = 1, \rho_b = 0.05, \rho_s = 0.05$



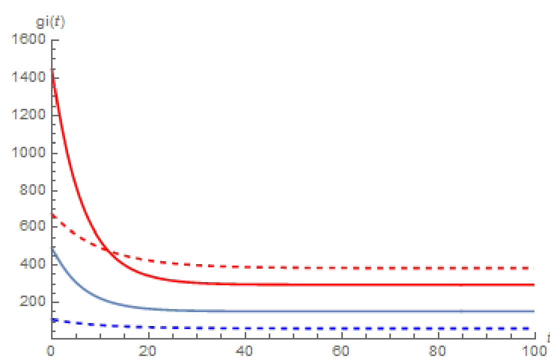
(c) $\theta = \frac{1}{3}, \rho_b = 0.05, \rho_s = 0.05$



(d) $\theta = \frac{1}{3}, \rho_b = 0.05, \rho_s = 0.05$



(e) $\theta = \frac{1}{6}, \rho_b = 0.05, \rho_s = 0.05$



(f) $\theta = \frac{1}{6}, \rho_b = 0.05, \rho_s = 0.05$

Table 3: Total extraction within the 20-year period for different values of θ and for $\rho_b = \rho_s = 0.05$.

	(2) Tg_b^C	(3) Tg_s^C	(4) Tg_b^{NC}	(5) Tg_s^{NC}	(6) (2)+(3)	(7) (4)+(5)
$\theta = 1$	6418.63	6418.63	10544.40	10544.40	12837.26	21088.80
$\theta = \frac{1}{3}$	9161.85	3053.95	12361.50	7212.36	12215.80	19573.86
$\theta = \frac{1}{6}$	10258.60	1709.77	12874.10	5124.46	11968.37	17998.56

Stock

As we can see, the steady state stock of the aquifer in cooperative case is always higher than in the non-cooperative case (values in column (3) are always greater than zero), which suggests that the cooperative behavior is beneficial for the environment in terms of stock. Moreover, the higher the asymmetry between the players is, the lower the inefficiency of the non-cooperative solution[2] is (or the lower the efficiency of the cooperative solution is) (see column (3)). The idea behind this result is that the more heterogeneous the agents are (in terms of demand), the more difficult it is to cooperate (and less sense it makes), and hence, the less effective the cooperation is, or in other words, the less ineffective the non-cooperative behavior is. Moreover, as the asymmetry increases, the total demand decreases, hence there is more water available in the aquifer. So, probably the problem of water scarcity does not appear to be as crucial as before, and the non-cooperative solution gives results that are more and more similar to the cooperative case.

It is interesting, that in the study of de Frutos Cachorro *et al.* (2020) [11] authors found the opposite result (higher the demand asymmetry, higher the inefficiency), but in their case increase in the demand asymmetry was associated to a change in the elasticity of the demand-price function for urban use with respect to agricultural use. In this work, however, the demand asymmetry is associated with the share of demand, and higher the asymmetry is, the lower the parameter θ is.

Extraction rates

It is quite logical, that the higher the demand asymmetry is, the higher the difference of extraction rates between the players is (both for the non-cooperative and cooperative cases). The difference between extraction rates of the farmers is higher in the cooperative case, which can be a consequence of the voluntary giving up the consumption of some "unnecessary water" from the part of the small farmer in favor of the big farmer owing a bigger plot of land.

Total extractions

Now, to quantify more precisely the behavior of agents within a visible time period (20 years[3]), total extractions [4] of the resource are computed. The results can be seen in the Table 3.

It is obvious that both players in the sum extract less when they cooperate rather than when they do not (compare columns (6) and (7) in Table 3), which is consistent with the previous literature (e.g. Rubio and Casino (2001) [22]). Moreover, the higher is the demand asymmetry, the lower is the inefficiency of the non-cooperative case in terms of total extractions (compare columns (6)

[2]Inefficiency of the non-cooperative behavior is defined as the difference in terms of stock between cooperation and non-cooperation at the steady state

[3]The time period of 20 years was chosen simply as an example of foreseeable future, enough to see the results of optimisation on extractions within the finite planning horizon.

[4]Total extractions are calculated as an integral of the optimal extraction function for each player between the time period $t = 0$ and $t = 20$.

and (7)). Here we understand inefficiency in terms of total extractions as an equivalent to the inefficiency in terms of stock in a short-run (as opposed to analysing the steady state (long-term) level of stock). And as we see, the influence of the asymmetry on the inefficiency in terms of stock in the short-run is akin to the long-term result.

4.1.2 Time-preference asymmetry

In this section the effect of the change in time preferences of the groundwater users is analysed. To isolate the time-preference asymmetry effect, the demand will be maintained equal for both players *i.e.* the baseline case with $\theta = 1$ will be used. We will start the analysis with the case of equal discount rates, and after will start varying values of the discount rate of the small farmer, keeping in mind that $\rho_b < \rho_s$. Therefore, we assume that the small farmer is always more impatient than the big one, because the situation of his "small business" is less stable and he is less secure about the future.

In Figure 2 three [5] simulations are represented. As soon as the demand functions are imposed to be equal ($\theta = 1$), there is a very small difference between the extraction rates of the small and big farmers (right side of the graph), and even the difference between cooperative and non-cooperative extraction is not that obvious. Nevertheless, the difference between the stock of the aquifer (left side of the graph) for cooperative and non-cooperative case is noticeable.

Stock

From the top three rows in Table 4 (correspond to $\theta = 1$), we can see that the higher the discount rate for the small farmer is (the more impatient he is), the lower the steady state level of the stock is both for cooperative and non-cooperative case (see column (2) and (3), Table 4). It can be explained by the fact that impatience of the small farmer pushes him to extract more, lowering the amount of the water available in the aquifer.

The effect of the discount asymmetry on the inefficiency of the non-cooperative solution in terms of stock is the same as for the demand asymmetry: the higher the asymmetry is, the lower the inefficiency is (see column (3)). The intuition behind is slightly different though. The idea that it is harder to cooperate when the agents are more heterogeneous still applies, only here the heterogeneity refers to the difference in time preferences. Plus, as the small farmer is more impatient, he would like to extract more than the big farmer, what he actually does in the non-cooperation case (compare columns (5) and (6)). The cooperation, however, "enforces" him to extract less than he would have preferred to satisfy his needs (compare columns (6) and (8)), hence the decreasing efficiency of the cooperative solutions in comparison with the non-cooperative one.

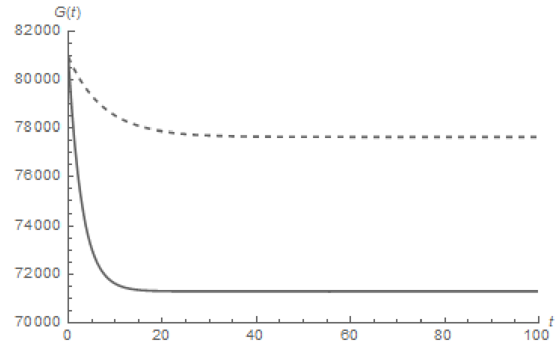
Extraction rates

For the non-cooperative case the extraction is always higher for the more impatient agent (small farmer). In other words, the player with a higher discount extracts more, and leaves less to the consumption of the other: and this effect is bigger, the higher the ρ_s is (see column (5) and (6)).

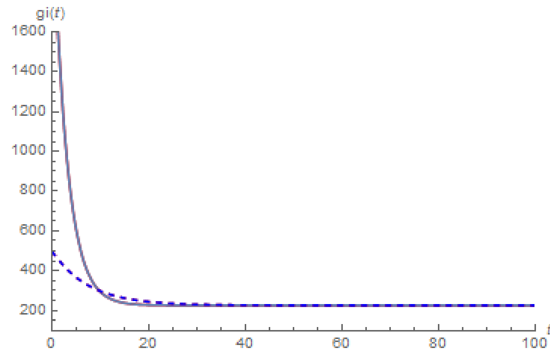
For the cooperative case the steady state extraction is the same for both players and for all values of ρ_b and ρ_s. Therefore, we can claim that there is no time-preference asymmetry effect on the steady state level of extraction for the cooperative case. One of the possible explanations might be that in cooperation what matters is the proportion of the demand going to players, and not the time preference, because initially, the purpose of cooperation is to "make up" for the impatience of one players with the patience of others.

[5] A few additional simulations for different values of ρ_s were computed and are available upon request.

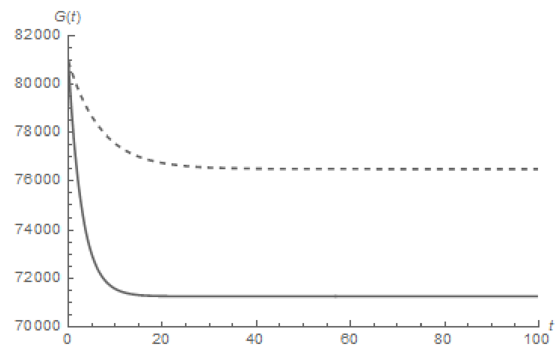
Figure 2: Simulations for the different values of ρ_s and for $\theta = 1$ of stock in Mm^3 (on the left) and extraction rate in $Mm^3/year$ (on the right).



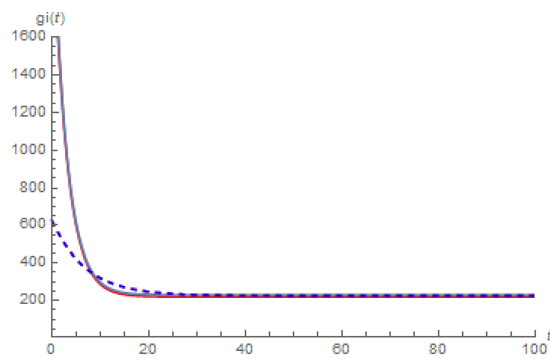
(a) $\theta = 1, \rho_b = 0.05, \rho_s = 0.05$



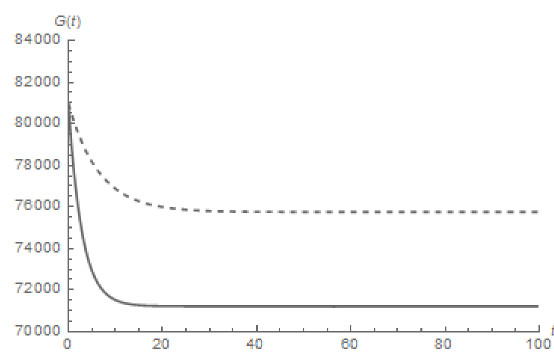
(b) $\theta = 1, \rho_b = 0.05, \rho_s = 0.05$



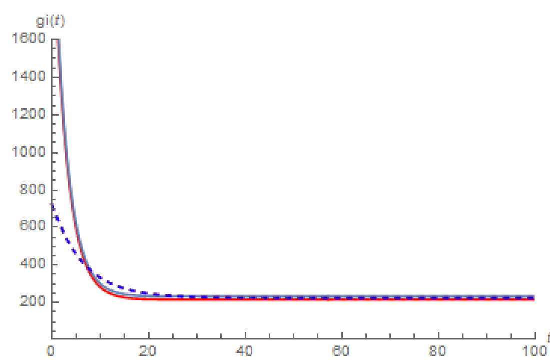
(c) $\theta = 1, \rho_b = 0.05, \rho_s = 0.07$



(d) $\theta = 1, \rho_b = 0.05, \rho_s = 0.07$



(e) $\theta = 1, \rho_b = 0.05, \rho_s = 0.09$



(f) $\theta = 1, \rho_b = 0.05, \rho_s = 0.09$

Table 4: Stock volume (in million cubic meters) and extraction rates (in million cubic meters / year) at the steady state for different values of ρ_s and for different θ: $\theta = 1$ for three upper rows and $\theta = \frac{1}{6}$ for three lower rows.

	Parameters	(2) G_∞^{NC}	(3) G_∞^{C}	(4) (2)-(1)	(5) g_b^{NC}	(6) g_s^{NC}	(7) g_b^{C}	(8) g_s^{C}	(9) (4)+(5)	(10) (6)+(7)
$\theta = 1$	$\rho_b = 0.05, \rho_s = 0.05$	71278.20	77661.10	6382.90	225.00	225.00	225.00	225.00	450.00	450.00
	$\rho_b = 0.05, \rho_s = 0.07$	71239.20	76510.60	5271.40	220.41	229.59	225.00	225.00	450.00	450.00
	$\rho_b = 0.05, \rho_s = 0.09$	71205.00	75739.60	4534.60	216.41	233.59	225.00	225.00	450.00	450.00
$\theta = \frac{1}{6}$	$\rho_b = 0.05, \rho_s = 0.05$	73446.40	78186.90	4740.50	296.68	153.32	385.71	64.29	450.00	450.00
	$\rho_b = 0.05, \rho_s = 0.07$	73415.80	77835.90	4420.10	294.84	155.16	385.71	64.29	450.00	450.00
	$\rho_b = 0.05, \rho_s = 0.09$	73390.30	77573.60	4183.30	293.32	156.68	385.71	64.29	450.00	450.00

Table 5: Total extraction within the 20-year period for different values of ρ_s and for different θ: $\theta = 1$ for three upper rows and $\theta = \frac{1}{6}$ for three lower rows.

	Parameters	(2) Tg_b^C	(3) Tg_s^C	(4) Tg_b^{NC}	(5) Tg_s^{NC}	(6) (2)+(3)	(7) (4)+(5)
$\theta = 1$	$\rho_b = 0.05, \rho_s = 0.05$	6418.63	6418.63	10544.40	10544.40	12837.26	21088.80
	$\rho_b = 0.05, \rho_s = 0.07$	7123.57	7123.57	10448.40	10689.80	14247.14	21138.20
	$\rho_b = 0.05, \rho_s = 0.09$	7607.74	7607.74	10362.90	10818.70	15215.48	21181.60
$\theta = \frac{1}{6}$	$\rho_b = 0.05, \rho_s = 0.05$	10258.60	1709.77	12874.10	5124.46	11968.37	17998.56
	$\rho_b = 0.05, \rho_s = 0.07$	10599.70	1766.62	12859.60	5178.71	12366.32	18038.31
	$\rho_b = 0.05, \rho_s = 0.09$	10858.90	1809.82	12847.50	5224.12	12668.72	18071.62

Total extractions

The total extractions within the period of 20 years are represented in the Table 5 (three upper rows).

For the non-cooperative case (columns (4) and (5)), when $\rho_b \neq \rho_s$ the total extractions are higher for the smaller farmer, because taking into account equal demand ($\theta = 1$), the more impatient agent extracts more.

For the cooperative case the total extractions of both players are equal, though the higher the discount asymmetry, the higher are total extractions for both of agents (columns (2) and (3)). It can be easily proved theoretically that when there is no demand asymmetry, *i.e.* $\theta = 1$, the extractions of the agents result in being the same.

Finally, the higher the discount asymmetry is, the lower the inefficiency of the non-cooperative solution is (compare columns (6) and (7)) in terms of total extractions (equivalent to stock in the short-run), which is coherent with the result for the steady state level of stock described earlier in this subsection.

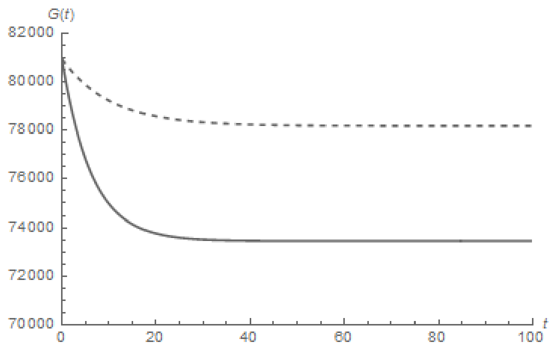
4.1.3 Considering both asymmetries at once

In this section a brief analysis of the impact of both effects, demand symmetry and time-preference asymmetry effects, on the results of the simulations is presented. That is, we will vary the discount rate of the small farmer and assume $\theta = \frac{1}{6}$ instead of $\theta = 1$. The results can be seen in Figure 3. Here we can definitely see the difference with Figure 2. Due to the demand asymmetry involved, the evolution of extractions with time is no longer the same for both players, with smaller agent extracting less both under cooperation and non-cooperation.

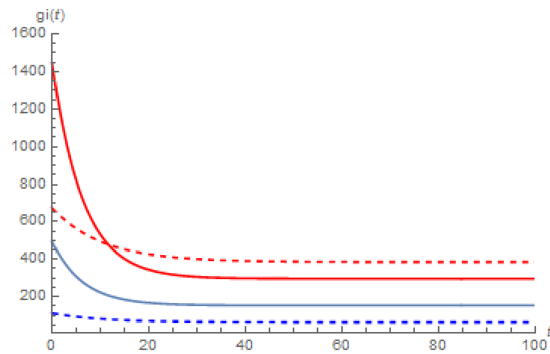
As soon as both asymmetries have the same influence on the inefficiency both in terms of stock and total extractions, it is not surprising that considering both of them at the same time leads to decrease in the inefficiency of the non-cooperative solution (see column (3) in Table 4 for the stock inefficiency, and columns (6) and (7) in Table 5, three lower rows, for the inefficiency in terms of total extractions).

Interestingly, now the extractions for the big and the small farmer differ under cooperation (columns (7) and (8) of Table 4, last three rows), even though changes in the discount rates do not make them alter, which proves again that for the cooperation it is the demand asymmetry that matters more.

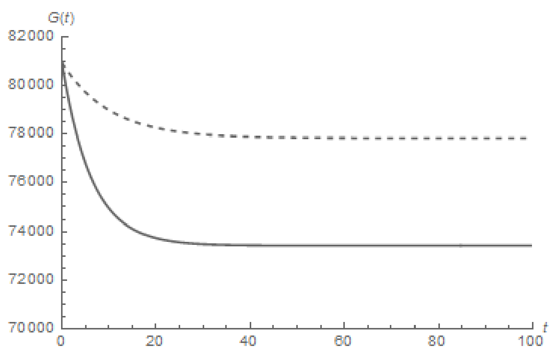
Figure 3: Simulations for the different values ρ_s and for $\theta = \frac{1}{6}$ of stock in Mm^3 (on the left) and extraction rate in $Mm^3/year$ (on the right)



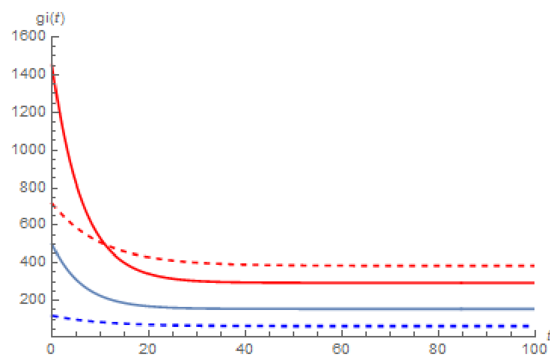
(a) $\theta = \frac{1}{6}, \rho_b = 0.05, \rho_s = 0.05$



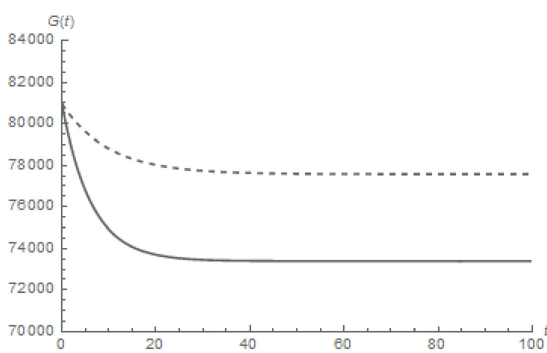
(b) $\theta = \frac{1}{6}, \rho_b = 0.05, \rho_s = 0.05$



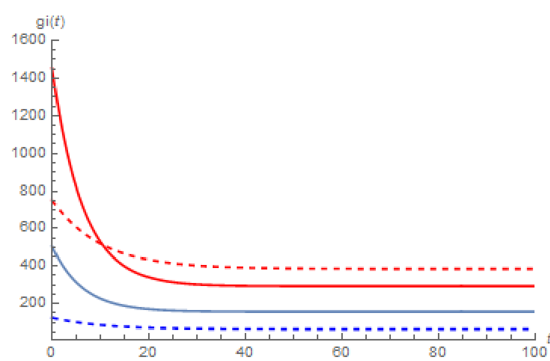
(c) $\theta = \frac{1}{6}, \rho_b = 0.05, \rho_s = 0.07$



(d) $\theta = \frac{1}{6}, \rho_b = 0.05, \rho_s = 0.07$



(e) $\theta = \frac{1}{6}, \rho_b = 0.05, \rho_s = 0.09$



(f) $\theta = \frac{1}{6}, \rho_b = 0.05, \rho_s = 0.09$

4.2 Agents' welfare: cooperation *vs.* non-cooperation

In order to study the economic efficiency of different types of behavior for every agent and solution, individual and group welfare (as a sum of agents' individual welfare) over the infinite planning horizon are computed. The welfare here is calculated as a present value of all the future profits of the agent [6]. The results are shown in Tables 6 and 7.

Demand asymmetry

Concerning the group welfare, the inefficiency of the non-cooperative solution tend to decrease with the increase in demand asymmetry (the difference between the joint cooperative and non-cooperative solutions decreases, see column (10) Table 6). From the first sight, it differs from the results obtained in de Frutos Cachorro *et al.* (2020) [11]. However, in that paper the demand asymmetry was represented by another parameter, the price elasticity, which was increasing gradually from 1 to 2, whereas in this study we decrease the parameter θ from 1 to $\frac{1}{6}$ to increase the asymmetry. So, we might expect the same result when changing the direction of asymmetry [7].

Concerning the individual welfare, we can see that when the asymmetry increases the profitability of the cooperation for the small farmer decreases, and becomes negative when θ reaches the value of $\frac{1}{6}$. On the contrary, for the bigger agent it is always profitable to cooperate as his welfare in this case is always higher in comparison with the non-cooperative case, and the highest profits are achieved when $\theta = \frac{1}{6}$. So, it might be the case that because of the big asymmetry, when cooperating, the big farmer "pulls" the extraction rates towards his own needs, and the small farmer does not gain what he could have in case of non-cooperation.

Table 6: Welfare analysis (in thousand euros) for different values of θ and with $\rho_b = \rho_s = 0.05$

Parameters	(2) V_b^C	(3) V_s^C	(4) V_b^{NC}	(5) V_s^{NC}	(6) (2)-(4)	(7) (3)-(5)	(8) (2)+(3)	(9) (4)+(5)	(10) (8)-(9)
$\theta = 1$	162448	162448	104380	104380	58068	58068	324896	208760	116136
$\theta = \frac{1}{3}$	236640	78880	159901	72275	76739	6605	315520	232176	83344
$\theta = \frac{1}{6}$	267258	44543	198145	54855	69113	-10312	311801	253000	58801

Time-preference asymmetry

For the individual welfare, the higher the discount rate ρ_s is, the lower the welfare is both for the cooperative and non-cooperative case for both agents (see columns (2)-(5) in Table 7, first three rows). It means that high impatience of just one agent is not beneficial neither for the environment (see Table 5, higher discounts lead to higher extractions) nor for the personal welfare of both agents.

Consequently, the group welfare decreases as the discount asymmetry rises - the sign of decreasing inefficiency of the non-cooperative solution, as already discussed previously.

Considering both asymmetries at once

Finally, when considering both demand asymmetry and time-preference asymmetry effects (Table 7 last three rows), cooperation becomes not profitable at all for the small farmer (see column

[6] The welfare for each player is calculated basing on the equation (1), *i.e.* using the optimal solutions for extraction and stock volume as functions of time and integrating the expression from zero to infinity.

[7] For example, instead of assuming θ as a share of the big farmer's demand that goes to the small farmer, think of it as of a a multiplicative term, that multiplies the small farmer demand to characterise the one of the big agriculturalist.

Table 7: Welfare analysis (in thousand euros) for different values of ρ_s and for different θ: $\theta = 1$ for three upper rows and $\theta = \frac{1}{6}$ for three lower rows.

	Parameters	(2) V_b^C	(3) V_s^C	(4) V_b^{NC}	(5) V_s^{NC}	(6) (2)-(4)	(7) (3)-(5)	(8) (2)+(3)	(9) (4)+(5)	(10) (8)-(9)
$\theta = 1$	$\rho_b = 0.05, \rho_s = 0.05$	162448	162448	104380	104380	58068	58068	324896	208760	116136
	$\rho_b = 0.05, \rho_s = 0.07$	161339	129227	102938	96262	58401	32965	290566	199200	91366
	$\rho_b = 0.05, \rho_s = 0.09$	159232	111746	101663	90650	57569	21096	270978	192313	78665
$\theta = \frac{1}{6}$	$\rho_b = 0.05, \rho_s = 0.05$	267258	44543	198145	54855	69113	-10312	311801	253000	58801
	$\rho_b = 0.05, \rho_s = 0.07$	267102	34239	197238	46106	69864	-11867	301341	243344	57997
	$\rho_b = 0.05, \rho_s = 0.09$	266774	28405	196477	40526	70297	-12121	295179	237003	58176

(7) last three rows). It means that when the demand is highly asymmetric, it is the small farmer who bares the losses under the cooperation, and the higher is the discount asymmetry the higher are the losses. It might be the case that when the agent is very impatient, the cooperation imposes restrictions on the extraction, making it lower in comparison with the amount the agent would have extracted were he allowed, and thus lowering his welfare.

As for the group welfare, the effect of both asymmetries taken into account simultaneously is not clear, even though separately both effects show negative influence on the inefficiency (inefficiency decreases with an increase in asymmetry).

4.3 Additional simulations

In this section a robustness analysis was performed in order to verify if the results stay the same, when the baseline case is changed. Here we consider a case with $\theta = 1$ and $\rho_b = \rho_s = 0.03$ as a baseline case. Hence, we assume that the big farmer is more patient during the whole planning horizon.

It is actually the case, that the main results concerning the inefficiency stay the same. An increase in the demand asymmetry, as well as in discount asymmetry leads to a decrease in the inefficiency of the non-cooperative solution in terms of stock and total extractions, which validates the robustness of the theoretical model.

It is interesting, however, that as the big farmer is more patient now, group total extractions are lower and, hence, the steady state level of the stock is higher in comparison with the case, when ρ_b was set equal 0.05. In fact the steady state level of the stock is higher than the initial level G_0, when considering both the demand asymmetry and time-preference asymmetry effects (see Figure 4 in Appendix C).

We can observe the reverse pattern [8] of the stock as well as extractions of both players for the cooperation case.

A possible explanation is that when the players are patient enough and confident about their future, *i.e.* they have quite low values of the future discount rates, the resource is "saved" for the future, and the initial extractions are actually smaller than the future ones in the case of cooperation (and the natural recharge is higher than extraction rates). And because initially the users do not overexploit the resource, the stock of the aquifer increases with respect to its initial level. It makes the cooperation in this case beneficial from the environmental point of view.

In terms of welfare, it is still not worthwhile for the small player to cooperate when the demand and discount asymmetries are combined together (see column (7) last four rows in Table 8 in Appendix C). However, due to the higher patience of the big farmer, the cooperation brings more welfare to the community in general, *i.e.* together to the big and small farmers (compare column (8) in Table 8 and Table 7). Even non-cooperative behavior is more profitable when at least one of the agents is considered to be more patient. It proves once again that the rates with which the players discount the future are very important for an effective water management, beneficial for the environment in terms of stock (though benefits for the agents are not that straightforward because even if collective welfare increases with higher patience of agents, there are still some players who are not willing to cooperate).

[8] Revers in the sense that $G^C_\infty > G_0$, whereas before $G^C_\infty < G_0$ was true.

5 Conclusions

This study shows how different types of asymmetries between groundwater users affect the exploitation of the natural resource in cooperative (subgame perfect cooperative solution) and non-cooperative (subgame perfect non-cooperative solution) cases. Particularly, two types of asymmetries are considered: an asymmetry of demand and an asymmetry of time-preferences.

The model was solved using the dynamic programming techniques. For the non-cooperative case, both the steady state level of water stock and extractions for both players were computed, whereas for the cooperative case due to cumbersome equations, these expressions were only computed numerically. The model then was applied to the real data on the Western La Mancha aquifer, which has suffered from the mismanagement and several droughts during the last decades.

First, the effects of two asymmetries were analysed separately, and then together. The main results show that the higher the asymmetry (demand or time-preferences), the smaller the inefficiency of the non-cooperative solution in terms of stock and total extractions. It differs from the results of the study of de Frutos Cachorro *et al* (2020) [11], because of some distinctions in asymmetries considered there.

Another important conclusion is that under the cooperation the time-preference asymmetry has no influence on the steady state extraction rates: they are equivalent for both players in the long-run. It means that in cooperation it does not matter how impatient the agents are, what matters is the proportion in which the water should be allocated to different farmers according to their demand requirements (defined by the size of the land).

We also have seen that even though for the environment the cooperation is always beneficial (the stock of the aquifer is always higher under cooperation), it is not so in terms of welfare, because higher the asymmetry, the less profitable it is for the smaller farmer to cooperate.

A result, which mostly contributes to the field literature, is the observed "pattern inversion" of the volume of the stock and extractions for the cooperative case, when the discount rate of the big farmer is set to be relatively low ($\rho_b = 0.03$). It is interesting how lowering the impatience of the bigger agent (at the same time keeping the natural recharge rate constant) can radically influence the outcomes. It actually results in stock of the aquifer being higher than the initial level, thus helping the natural resource to "recover". It is hard to overestimate the importance of this results for the environment.

The users' discount rates do matter when talking about water management, they reflect the situation of stability and certainty about the future, which depends on the general state of the economy. It means that the authorities should assure a stable general economic situation for farmers to be more confident and secure about the future, and hence to be more patient with water usage. Therefore, the government should think carefully not only about the agricultural policies *per se* in order to maintain proper functioning of the aquifer, but foresee the general consequences of implementation of new financial instruments and other economic policies, and their influence on producers of agricultural goods.

Another important policy implication following from this study is that, apart from introducing quotas on consumption to reduce the level of extraction (the morality of which can be argued as water is one of the life essential resources), the government should also promote the cooperation of the groundwater users, especially when the asymmetry between users is small. As we have seen, in this case the cooperation is beneficial not only for the environment in terms of stock, but for the agents' welfare as well.

One possible extension to this work includes analysing an open-loop equilibrium, which is another type of non-cooperative solution to compare the results with a subgame perfect Nash equilibrium (feedback equilibrium). The difference is that in this case agents predetermine future extractions from the beginning, and do not change their behavior according to the current state of the system. Moreover, as stated in Negri (1989) [17] and Rubio an Casino (2001) [22], the open loop solution takes into account only a cost externality, whereas the feedback solution takes into account both cost and strategic externalities. So, comparing these two solutions will help to evaluate the impact of strategic externality on the stock and extraction rates, when having asymmetric players. Another useful thing would be to include environmental externality in the cost function as in Esteban and Albiac (2011) [8], to account for the possible ecosystem damage caused by excessive extractions. Last but not least, it would be very useful to implement the theoretical results on the aquifer data from some developing country, as for example Venezuela, where the problem of general water management (not only of the optimal extraction) is quite critical [24].

Appendix

A Subgame Perfect Non-cooperative Equilibrium

Step 1.The value functions of the agents are $V_b^{NC}(G) = A_b^{NC}G^2 + B_b^{NC}G + C_b^{NC}$ and $V_s^{NC}(G) = A_s^{NC}G^2 + B_s^{NC}G + C_s^{NC}$, where the coefficients $A_b^{NC}, B_b^{NC}, C_b^{NC}$ and $A_s^{NC}, B_s^{NC}, C_s^{NC}$ are given by:

$$A_b^{NC} = \frac{-\frac{(\alpha_b^{NC})^2}{2b} + c\alpha_b^{NC}}{\rho_b + 2(1-\gamma)(\alpha_b^{NC} + \alpha_s^{NC})} \tag{20}$$

$$B_b^{NC} = \frac{\left(\frac{a}{b} - z\right)\alpha_b^{NC} - \frac{\alpha_b^{NC}\beta_b^{NC}}{b} + c\beta_b^{NC} + 2A_b^{NC}\left(r - (1-\gamma)(\beta_b^{NC} + \beta_s^{NC})\right)}{\rho_b + (1-\gamma)(\alpha_b^{NC} + \alpha_s^{NC})} \tag{21}$$

$$C_b^{NC} = \frac{-\frac{1}{2b}(\beta_b^{NC})^2 + \left(\frac{a}{b} - z\right)\beta_b^{NC} + B_b^{NC}\left(r - (1-\gamma)(\beta_b^{NC} + \beta_s^{NC})\right)}{\rho_b} \tag{22}$$

$$A_s^{NC} = \frac{-\frac{(\alpha_s^{NC})^2}{2\theta b} + c\alpha_s^{NC}}{\rho_s + 2(1-\gamma)(\alpha_b^{NC} + \alpha_s^{NC})} \tag{23}$$

$$B_s^{NC} = \frac{\left(\frac{a}{b} - z\right)\alpha_s^{NC} - \frac{\alpha_s^{NC}\beta_s^{NC}}{b} + c\beta_s^{NC} + 2A_s^{NC}\left(r - (1-\gamma)(\beta_b^{NC} + \beta_s^{NC})\right)}{\rho_s + (1-\gamma)(\alpha_b^{NC} + \alpha_s^{NC})} \tag{24}$$

$$C_s^{NC} = \frac{-\frac{1}{2\theta b}(\beta_s^{NC})^2 + \left(\frac{a}{b} - z\right)\beta_s^{NC} + B_s^{NC}\left(r - (1-\gamma)(\beta_b^{NC} + \beta_s^{NC})\right)}{\rho_s} \tag{25}$$

Step 2. From the First Order Conditions of a maximum in the right hand side of (6) we obtain respectively:

$$\frac{1}{b}\phi_b^{NC} = \left(c - 2A_b^{NC}(1-\gamma)\right)G + \frac{a}{b} - z - B_b^{NC}(1-\gamma)$$

and

$$\frac{1}{\theta b}\phi_s^{NC} = \left(c - 2A_s^{NC}(1-\gamma)\right)G + \frac{a}{b} - z - B_s^{NC}(1-\gamma)$$

Knowing the structure of the functions $\phi_j^{NC}(G)$ we get:

$$\alpha_b^{NC} = b\left(c - 2(1-\gamma)A_b^{NC}\right), \qquad \alpha_s^{NC} = \theta b\left(c - 2(1-\gamma)A_s^{NC}\right) \tag{26}$$

and

$$\beta_b^{NC} = a - zb - b(1-\gamma)B_b^{NC}, \qquad \beta_s^{NC} = \theta\left(a - zb - b(1-\gamma)B_s^{NC}\right) \tag{27}$$

From (26) and (27) we obtain:

$$A_b^{NC} = \frac{1}{2(1-\gamma)}\left(c - \frac{\alpha_b^{NC}}{b}\right), \qquad A_s^{NC} = \frac{1}{2(1-\gamma)}\left(c - \frac{\alpha_s^{NC}}{\theta b}\right)$$

$$B_b^{NC} = \frac{1}{b(1-\gamma)}\left(a - bz - \beta_b^{NC}\right), \qquad B_s^{NC} = \frac{1}{b\theta(1-\gamma)}\left(a\theta - b\theta z - \beta_s^{NC}\right)$$

The results for α_b^{NC} and α_s^{NC} are obtained by substituting the above expressions for A_b^{NC} and A_s^{NC} in (20) and (23), respectively. The results for β_b^{NC} and β_s^{NC} are obtained by substituting the above expressions for B_b^{NC} and B_s^{NC} in (21) and (24) and arranging terms.

Proof Proposition 1. Consider a simplification of (7) and (8) with $\gamma = 0$ and $\theta = 1$. Then (7) and (8) can be rewritten as:

$$\rho_b(bc - \alpha_b^{NC}) = -(\alpha_b^{NC})^2 + 2bc\alpha_b^{NC} - 2(\alpha_b^{NC} + \alpha_s^{NC})(bc - \alpha_b^{NC})$$

$$\rho_s(bc - \alpha_s^{NC}) = -(\alpha_s^{NC})^2 + 2bc\alpha_s^{NC} - 2(\alpha_b^{NC} + \alpha_s^{NC})(bc - \alpha_s^{NC})$$

Now, denote $x = \alpha_b^{NC} - bc$ and $y = \alpha_s^{NC} - bc$, then the previous equations can be rewritten as:

$$-\rho_b x = -(x^2 + 2bcx + b^2c^2) + 2bc(x + bc) + 2(x + y + 2bc)x$$
$$-\rho_s y = -(y^2 + 2bcy + b^2c^2) + 2bc(y + bc) + 2(x + y + 2bc)y$$

And rearranging the terms:

$$x^2 + (4bc + \rho_b)x + 2xy + b^2c^2 = 0 \tag{28}$$

$$y^2 + (4bc + \rho_s)y + 2xy + b^2c^2 = 0 \tag{29}$$

and following the definition of x and y, the condition for convergence will be $x + y + 2bc > 0$.

Denote $z_b = x + y + 2bc + \frac{\rho_b}{2}$ and $z_s = x + y + 2bc + \frac{\rho_s}{2}$. Then y and x can be expressed from the definitions of z_b and z_s, respectively:

$$y = z_b - x - 2bc - \frac{\rho_b}{2} \tag{30}$$

$$x = z_s - y - 2bc - \frac{\rho_s}{2} \tag{31}$$

By plugging (30) and (**??**) into (28) and (29), respectively:

$$x^2 - 2z_b x - b^2c^2 = 0 \tag{32}$$

$$y^2 - 2z_s y - b^2c^2 = 0 \tag{33}$$

So, the solution to this system of equations will be:

$$x = z_b \pm \sqrt{z_b^2 + b^2c^2}, \qquad y = z_s \pm \sqrt{z_s^2 + b^2c^2} \tag{34}$$

So, there are 4 potential solutions that it is necessary to check for the convergence *i.e.* check their stationarity.

Case 1. $x = z_b + \sqrt{z_b^2 + b^2c^2}$ and $y = z_s + \sqrt{z_s^2 + b^2c^2}$.
Then, taking into account that $z_b = x + y + 2bc + \frac{\rho_b}{2}$ and $z_s = x + y + 2bc + \frac{\rho_s}{2}$, and arranging the terms, the expression for $x + y + 2bc$ can be obtained:

$$x + y + 2bc = -2bc - \frac{\rho_b}{2} - \frac{\rho_s}{2} - \sqrt{z_b^2 + b^2c^2} - \sqrt{z_s^2 + b^2c^2}$$

This expression cannot possibly be positive (as the condition for convergence requires), because all the terms on the right hand side are negative, taking into account that $b, c > 0$ as well as $\rho_b, \rho_s > 0$. Hence, this solution will not converge.

Case 2. $x = z_b - \sqrt{z_b^2 + b^2c^2}$ and $y = z_s + \sqrt{z_s^2 + b^2c^2}$.
Following the same procedure as above, it is easily obtained:

$$x + y + 2bc = -2bc - \frac{\rho_b}{2} - \frac{\rho_s}{2} + \sqrt{z_b^2 + b^2c^2} - \sqrt{z_s^2 + b^2c^2}$$

However, $\sqrt{z_b^2 + b^2c^2} - \sqrt{z_s^2 + b^2c^2} < 0$, because $z_s > z_b$ as soon as $\rho_s > \rho_b$ by assumption. Therefore, as before, it is not possible for the expression $x + y + 2bc$ to be greater than 0, so this solution will not converge.

Case 3. $x = z_b + \sqrt{z_b^2 + b^2c^2}$ and $y = z_s - \sqrt{z_s^2 + b^2c^2}$.
As in two cases above, following the similar steps, it is easily obtained:

$$x + y + 2bc = -2bc - \frac{\rho_b}{2} - \frac{\rho_s}{2} - \sqrt{z_b^2 + b^2c^2} + \sqrt{z_s^2 + b^2c^2}$$

In this case it is not obvious what sign the right hand side will have. So, let us check the sign of the part of the right hand side expression, precisely $-2bc - \frac{\rho_s}{2} + \sqrt{z_s^2 + b^2c^2}$.
Considering the definition of z_s it is possible to write that

$$z_s^2 + b^2c^2 = \left(x + y + 2bc + \frac{\rho_s}{2}\right)^2 + b^2c^2$$

For convergence it is required that $x + y + 2bc > 0$, and as $\rho_s > 0$ by definition, it is quite easy to see that $x + y + 2bc + \frac{\rho_s}{2} > 0$. Then, we can use a property that sum of squares is smaller than the square of the sum. More precisely:

$$\left(x + y + 2bc + \frac{\rho_s}{2}\right)^2 + (bc)^2 < \left(x + y + 2bc + \frac{\rho_s}{2} + bc\right)^2$$

Therefore, $z_s^2+b^2c^2 < (x+y+2bc+\frac{\rho_s}{2}+bc)^2$ and, consequently, $\sqrt{z_s^2+b^2c^2} < x+y+2bc+\frac{\rho_s}{2}+bc$ It means that:

$$x+y+2bc < -2bc - \frac{\rho_b}{2} - \frac{\rho_s}{2} - \sqrt{z_b^2+b^2c^2} + x + y + 2bc + \frac{\rho_s}{2} + bc$$

Which leads to

$$bc < -\frac{\rho_b}{2} - \sqrt{z_b^2+b^2c^2},$$

which is not possible because both parameters b and c are positive. Hence, this solution neither will converge.

There is one case left for consideration $x = z_b - \sqrt{z_b^2+b^2c^2}$ and $y = z_s - \sqrt{z_s^2+b^2c^2}$, which means that if the condition $x+y+2bc > 0$ is fulfilled there will be at most one solution converging to the steady state, otherwise there is no stationary linear subgame perfect non-cooperative equilibrium.

Now consider a more general case, where without loss of generality $\gamma = 0$ is assumed. Then (7) and (8) can be rewritten as:

$$\rho_b(bc - \alpha_b^{NC}) = -(\alpha_b^{NC})^2 + 2bc\alpha_b^{NC} - 2(\alpha_b^{NC} + \alpha_s^{NC})(bc - \alpha_b^{NC})$$

$$\rho_s(bc - \alpha_s^{NC}) = -(\alpha_s^{NC})^2 + 2\theta bc\alpha_s^{NC} - 2(\alpha_b^{NC} + \alpha_s^{NC})(\theta bc - \alpha_s^{NC})$$

Now, denote $x = \alpha_b^{NC} - bc$ and $y = \alpha_s^{NC} - \theta bc$, then the previous equations can be rewritten as:

$$-\rho_b x = -(x^2 + 2bcx + b^2c^2) + 2bc(x+bc) + 2\left(x+y+2bc(1+\theta)\right)x$$
$$-\rho_s y = -(y^2 + 2\theta bcy + \theta^2b^2c^2) + 2\theta bc(y+\theta bc) + 2\left(x+y+2bc(1+\theta)\right)y$$

And rearranging the terms:

$$x^2 + \left(2bc(1+\theta)+\rho_b\right)x + 2xy + b^2c^2 = 0 \tag{35}$$

$$y^2 + \left(2bc(1+\theta)+\rho_s\right)y + 2xy + \theta^2b^2c^2 = 0 \tag{36}$$

and following the definition of x and y, the condition for convergence will be $x+y+bc(1+\theta) > 0$.

Now, denote $z_b = x+y+(1+\theta)bc+\frac{\rho_b}{2}$ and $z_s = x+y+(1+\theta)bc+\frac{\rho_s}{2}$. Then y and x can be expressed from the definitions of z_b and z_s, respectively:

$$y = z_b - x - (1+\theta)bc - \frac{\rho_b}{2} \tag{37}$$

$$x = z_s - y - (1+\theta)bc - \frac{\rho_s}{2} \tag{38}$$

By plugging (37) and (38) into (35) and (36), respectively:

$$x^2 - 2z_bx - b^2c^2 = 0 \tag{39}$$

$$y^2 - 2z_sy - \theta^2b^2c^2 = 0 \tag{40}$$

So, the solution to this system of equations will be:

$$x = z_b \pm \sqrt{z_b^2 + b^2c^2}, \qquad y = z_s \pm \sqrt{z_s^2 + \theta^2 b^2 c^2} \tag{41}$$

So, there are 4 potential solutions that it is necessary to check for the convergence *i.e.* check their stationarity.

Case 1. $x = z_b + \sqrt{z_b^2 + b^2c^2}$ and $y = z_s + \sqrt{z_s^2 + \theta^2 b^2 c^2}$.

Then, taking into account that $z_b = x + y + (1+\theta)bc + \frac{\rho_b}{2}$ and $z_s = x + y + (1+\theta)bc + \frac{\rho_s}{2}$, and arranging the terms, the expression for $x + y + (1+\theta)bc$ can be obtained:

$$x + y + (1+\theta)bc = -(1+\theta)bc - \frac{\rho_b}{2} - \frac{\rho_s}{2} - \sqrt{z_b^2 + b^2c^2} - \sqrt{z_s^2 + \theta^2 b^2 c^2}$$

This expression cannot possibly be positive (as the condition for convergence requires), because all the terms on the right hand side are negative, taking into account that $b, c > 0$ as well as $\rho_b, \rho_s > 0$. Hence, this solution will not converge.

Case 2. $x = z_b + \sqrt{z_b^2 + b^2c^2}$ and $y = z_s - \sqrt{z_s^2 + \theta^2 b^2 c^2}$.

Following the same procedure as above, it is easily obtained:

$$x + y + (1+\theta)bc = -(1+\theta)bc - \frac{\rho_b}{2} - \frac{\rho_s}{2} - \sqrt{z_b^2 + b^2c^2} + \sqrt{z_s^2 + \theta^2 b^2 c^2}$$

In this case it is not obvious what sign the right hand side will have because $z_s > z_b$, but at the same time $\theta \leq 1$.

Considering the definition of z_s it is possible to write that

$$z_s^2 + \theta^2 b^2 c^2 = \left(x + y + (1+\theta)bc + \frac{\rho_s}{2}\right)^2 + \theta^2 b^2 c^2$$

For convergence it is required that $x + y + (1+\theta)bc > 0$, and as $\rho_s > 0$ by definition, it is quite easy to see that $x + y + (1+\theta)bc + \frac{\rho_s}{2} > 0$. Then, as in the simplified case, we can use a property that sum of squares is smaller than the square of the sum. More precisely:

$$\left(x + y + (1+\theta)bc + \frac{\rho_s}{2}\right)^2 + (\theta bc)^2 < \left(x + y + (1+\theta)bc + \frac{\rho_s}{2} + \theta bc\right)^2$$

Therefore, $z_s^2 + \theta^2 b^2 c^2 < \left(x + y + (1+\theta)bc + \frac{\rho_s}{2} + \theta bc\right)^2$ and, consequently, $\sqrt{z_s^2 + \theta^2 b^2 c^2} < x + y + (1+\theta)bc + \frac{\rho_s}{2} + \theta bc$

It means that:

$$x + y + (1+\theta)bc < -(1+\theta)bc - \frac{\rho_b}{2} - \frac{\rho_s}{2} - \sqrt{z_b^2 + b^2c^2} + x + y + (1+\theta)bc + \frac{\rho_s}{2} + \theta bc$$

Which leads to

$$bc < -\frac{\rho_b}{2} - \sqrt{z_b^2 + b^2c^2},$$

which is not possible because both parameters b and c are positive. Hence, this solution neither will converge.

Case 3. $x = z_b - \sqrt{z_b^2 + b^2c^2}$ and $y = z_s + \sqrt{z_s^2 + b^2c^2}$.
As in two cases above, following the similar steps, it is easily obtained:

$$x + y + (1+\theta)bc = -(1+\theta)bc - \frac{\rho_b}{2} - \frac{\rho_s}{2} + \sqrt{z_b^2 + b^2c^2} - \sqrt{z_s^2 + \theta^2 b^2 c^2}$$

If $\theta = 1$, then for sure $\sqrt{z_s^2 + \theta^2 b^2 c^2} > \sqrt{z_b^2 + b^2c^2}$, and so the right hand side of the expression above is negative, and it is impossible for the solution to converge to the steady state.

If $0 < \theta < 1$, then the situation is not that obvious. However, we ca n consider $z_b^2 + b^2c^2 = \left(x + y + (1+\theta)bc + \frac{\rho_b}{2}\right)^2 + b^2c^2$. Then, because $x + y + (1+\theta)bc + \frac{\rho_b}{2} > 0$, we can write:

$$\left(x + y + (1+\theta)bc + \frac{\rho_b}{2}\right)^2 + (bc)^2 < \left(x + y + (1+\theta)bc + \frac{\rho_b}{2} + bc\right)^2$$

Therefore, $z_b^2 + b^2c^2 < \left(x + y + (1+\theta)bc + \frac{\rho_b}{2} + bc\right)^2$ and, consequently, $\sqrt{z_b^2 + b^2c^2} < x + y + (1+\theta)bc + \frac{\rho_b}{2} + bc$

It means that:

$$x + y + (1+\theta)bc < -(1+\theta)bc - \frac{\rho_b}{2} - \frac{\rho_s}{2} - \sqrt{z_s^2 + \theta^2 b^2 c^2} + x + y + (1+\theta)bc + \frac{\rho_b}{2} + bc$$

Which leads to

$$\theta bc < -\frac{\rho_s}{2} - \sqrt{z_s^2 + \theta^2 b^2 c^2},$$

which is not possible because all parameters θ, b and c are positive. Hence, this solution neither will converge.

There is one case left for consideration $x = z_b - \sqrt{z_b^2 + b^2c^2}$ and $y = z_s - \sqrt{z_s^2 + \theta^2 b^2 c^2}$, which means that if the condition $x + y + (1+\theta)bc > 0$ is fulfilled there will be at most one solution converging to the steady state, otherwise there is no stationary linear subgame perfect non-cooperative equilibrium.

B Subgame Perfect Cooperative Equilibrium

Plugging the expressions for optimal extractions (13) and (14) into (12) and writing down the equation separately for each player, we obtain:

$$\begin{aligned}
\rho_b(A_b^C G^2 + B_b^C G + C_b^C) =& [2A_b^C G + B_b^C][r - (1-\gamma)(1+\theta)\{a + b(z + (1-\gamma)(B_b^C + B_s^C)) + \\
&+ b(c - 2(1-\gamma)(A_b^C + A_s^C))G\}] + \\
&+ [a - b(z + (1-\gamma)(B_b^C + B_s^C)) + b(c - 2(1-\gamma)(A_b^C + A_s^C))G] \\
&[\frac{a}{b} - (z - cG) - \frac{1}{2b}\{a - b(z + (1-\gamma)(B_b^C + B_s^C)) + \\
&+ b(c - 2(1-\gamma)(A_b^C + A_s^C))G\}]
\end{aligned} \tag{42}$$

$$\rho_s(A_s^C G^2 + B_s^C G + C_s^C) = [2A_s^C G + B_s^C][r - (1-\gamma)(1+\theta)\{a + b(z + (1-\gamma)(B_b^C + B_s^C)) + b(c - 2(1-\gamma)(A_b^C + A_s^C))G\}] + [\theta a - \theta b(z + (1-\gamma)(B_b^C + B_s^C)) + \theta b(c - 2(1-\gamma)(A_b^C + A_s^C))G] [\frac{a}{b} - (z - cG) - \frac{1}{2\theta b}\{\theta a - \theta b(z + (1-\gamma)(B_b^C + B_s^C)) + \theta b(c - 2(1-\gamma)(A_b^C + A_s^C))G\}] \quad (43)$$

From these cumbersome expressions we can derive a system of 6 equations that we will have to solve in order to define $A_b^C, A_s^C, B_b^C, B_s^C, C_b^C$ and C_s^C.

$$\rho_b A_b^C = 4b(1-\gamma)^2(1+\theta)A_b^C(A_b^C + A_s^C) - 2bc(1-\gamma)(A_b^C + A_s^C) - 2b(1-\gamma)^2(A_b^C + A_s^C)^2 \quad (44)$$

$$\rho_s A_s^C = 4b(1-\gamma)^2(1+\theta)A_s^C(A_b^C + A_s^C) - 2\theta bc(1-\gamma)(A_b^C + A_s^C) - 2\theta b(1-\gamma)^2(A_b^C + A_s^C)^2 \quad (45)$$

$$\rho_b B_b^C = 2A_b^C[r - (1-\gamma)(1+\theta)\{a + b(z + (1-\gamma)(B_b^C + B_s^C)) + bc\}] + 2B_b^C b(1-\gamma)^2(1+\theta)(A_b^C + A_s^C) + [a - b(z + (1-\gamma)(B_b^C + B_s^C)) + bc](c + (1-\gamma)(A_b^C + A_s^C)) - 2b(1-\gamma)(A_b^C + A_s^C)[\frac{a}{2b} - z + \frac{1}{2}(z + (1-\gamma)(B_b^C + B_s^C)) - \frac{1}{2}c] \quad (46)$$

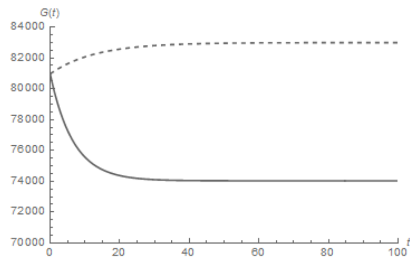
$$\rho_s B_s^C = 2A_s^C[r - (1-\gamma)(1+\theta)\{a + b(z + (1-\gamma)(B_b^C + B_s^C)) + bc\}] + 2B_s^C b(1-\gamma)^2(1+\theta)(A_b^C + A_s^C) + \theta[a - b(z + (1-\gamma)(B_b^C + B_s^C)) + bc](c + (1-\gamma)(A_b^C + A_s^C)) - 2\theta b(1-\gamma)(A_b^C + A_s^C)[\frac{a}{2b} - z + \frac{1}{2}(z + (1-\gamma)(B_b^C + B_s^C)) - \frac{1}{2}c] \quad (47)$$

$$\rho_b C_b^C = B_b^C[r - (1-\gamma)(1+\theta)\{a + b(z + (1-\gamma)(B_b^C + B_s^C)) + bc\}] + [a - b(z + (1-\gamma)(B_b^C + B_s^C)) + bc] [\frac{a}{2b} - z + \frac{1}{2}(z + (1-\gamma)(B_b^C + B_s^C)) - \frac{1}{2}c] \quad (48)$$

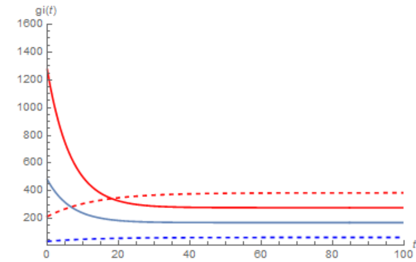
$$\rho_s C_s^C = B_s^C[r - (1-\gamma)(1+\theta)\{a + b(z + (1-\gamma)(B_b^C + B_s^C)) + bc\}] + \theta[a - b(z + (1-\gamma)(B_b^C + B_s^C)) + bc] [\frac{a}{2b} - z + \frac{1}{2}(z + (1-\gamma)(B_b^C + B_s^C)) - \frac{1}{2}c] \quad (49)$$

C Additional simulations

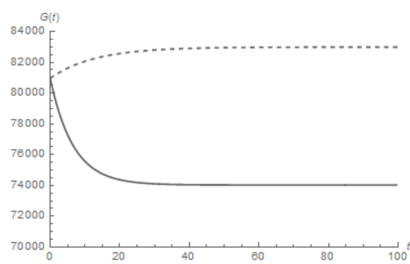
Figure 4: Simulations for the different values of ρ_b and ρ_s, and for $\theta = \frac{1}{6}$ of stock in Mm^3 (on the left) and extraction rate in $Mm^3/year$ (on the right) (with the alternative baseline case)



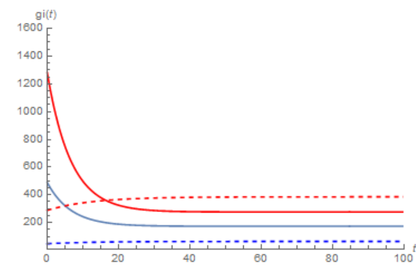
(a) $\theta = \frac{1}{6}, \rho_b = 0.03, \rho_s = 0.03$



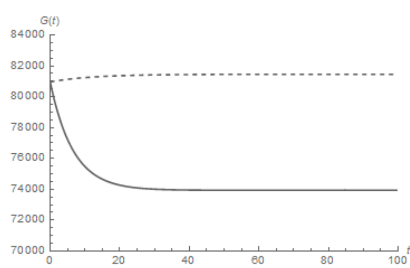
(b) $\theta = \frac{1}{6}, \rho_b = 0.03, \rho_s = 0.03$



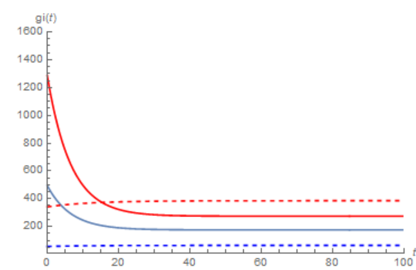
(c) $\theta = \frac{1}{6}, \rho_b = 0.03, \rho_s = 0.05$



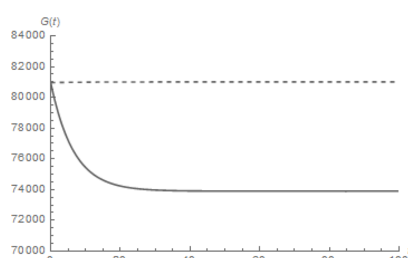
(d) $\theta = \frac{1}{6}, \rho_b = 0.03, \rho_s = 0.05$



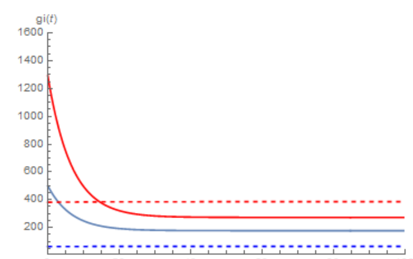
(e) $\theta = \frac{1}{6}, \rho_b = 0.03, \rho_s = 0.07$



(f) $\theta = \frac{1}{6}, \rho_b = 0.03, \rho_s = 0.07$



(g) $\theta = \frac{1}{6}, \rho_b = 0.03, \rho_s = 0.09$



(h) $\theta = \frac{1}{6}, \rho_b = 0.03, \rho_s = 0.09$

Table 8: Welfare analysis (in thousand euros) for different values of ρ_s and for different θ: $\theta = 1$ for four upper rows and $\theta = \frac{1}{6}$ for four lower rows (for the alternative baseline case).

	Parameters	(2) V_b^C	(3) V_s^C	(4) V_b^{NC}	(5) V_s^{NC}	(6) (2)-(4)	(7) (3)-(5)	(8) (2)+(3)	(9) (4)+(5)	(10) (8)-(9)
$\theta = 1$	$\rho_b = 0.03, \rho_s = 0.03$	259451	259451	122005	122005	137446	137446	518902	244010	274892
	$\rho_b = 0.03, \rho_s = 0.05$	252991	159632	119558	106030	133433	53602	412623	225588	187035
	$\rho_b = 0.03, \rho_s = 0.07$	243119	125550	117445	97542	125674	28008	368669	214987	153682
	$\rho_b = 0.03, \rho_s = 0.09$	233925	108280	115598	91708	118327	16572	342205	207306	134899
$\theta = \frac{1}{6}$	$\rho_b = 0.03, \rho_s = 0.03$	436142	72690	256372	84720	179770	-12030	508832	341092	167740
	$\rho_b = 0.03, \rho_s = 0.05$	435138	41827	254061	62194	181077	-20367	476965	316255	160710
	$\rho_b = 0.03, \rho_s = 0.07$	433294	30024	252198	51302	181096	-21278	463318	303500	159818
	$\rho_b = 0.03, \rho_s = 0.09$	431330	23876	250662	44507	180668	-20631	455206	295169	160037

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