Group-decision making with induced ordered weighted logarithmic aggregation operators

Víctor G. Alfaro-García[a,*], José M. Merigó[b,c], Anna M. Gil-Lafuente[d], Rodrigo Gómez Monge[e]
[a] *Facultad de Contaduría y Ciencias Administrativas, Universidad Michoacana de San Nicolás de Hidalgo, Gral. Francisco J. Múgica S/N, 58030, Morelia, 58030, México.*
[b] *School of Information, Systems and Modelling, Faculty of Engineering and Information Technology, University of Technology Sydney, 81 Broadway, Ultimo, NSW 2007, Australia*
[c] *Departamento de Control de Gestión y Sistemas de Información, Universidad de Chile, Av. Diagonal Paraguay 257, 8330015, Santiago, Chile*
[d] *Facultat d'Economia i Empresa, Universitat de Barcelona Avinguda Diagonal, 690, 08034, Barcelona, España*
[e] *Facultad de Economía Vasco de Quiroga, Universidad Michoacana de San Nicolás de Hidalgo, Gral. Francisco J. Múgica S/N, 58030, Morelia, 58030, México.*

Abstract. This paper presents the induced generalized ordered weighted logarithmic aggregation (IGOWLA) operator, this operator is an extension of the generalized ordered weighted logarithmic aggregation (GOWLA) operator. It uses order-induced variables that modify the reordering process of the arguments included in the aggregation. The principal advantage of the introduced induced mechanism is the consideration of highly complex attitude from the decision makers. We study some families of the IGOWLA operator as measures for the characterization of the weighting vector. This paper presents the general formulation of the operator and some special cases, including the induced ordered weighted logarithmic geometric averaging (IOWLGA) operator and the induced ordered weighted logarithmic aggregation (IOWLA). Further generalizations using quasi-arithmetic mean are also proposed. Finally, an illustrative example of a group decision-making procedure using a multi-person analysis and the IGOWLA operator in the area of innovation management is analyzed.

Keywords: OWA operator; Logarithmic aggregation operators; Induced aggregation operators; Group decision making; Innovation management.

1. Introduction

Aggregation operators are becoming very popular in the literature, especially in the areas of economics, statistics and engineering [1]. Currently, the literature presents an extensive amount of aggregation operators [2–4]. The ordered weighted average (OWA) operator [5] stands as one of the most disseminated aggregation operator in the field. The OWA operator proposes a parameterized family including the maximum, the minimum and the average. This classic operator has been widely applied from applications in expert systems, group decision making, neural networks, data base systems, to fuzzy systems [6,7].

Yager and Filev [8] introduced an extension to the OWA operator, the induced ordered weighted average (IOWA) operator. This extension allows a broader treatment and representation of complex information. The introduced mechanism applies a reordering process to the arguments, here, a set of order-induced variables determines the order of the aggregation. This reordering mechanism of the IOWA operator has attracted much attention, motivating a broad diversity of applications [9,10] e.g. [11] develops new families of IOWA operators. In [12] dissimilarity functions are included in the analysis, [13] study the use of fuzzy numbers, [14] consider intuitionistic fuzzy information, and [15] analyze hesitant fuzzy sets and the Shapley framework. [16,17] develop induced aggregation operators with linguistic information, [18] with distance measures, [19] with heavy operators and moving averages, [20] with Bonferroni and heavy operators, [21] with prioritized operators and [22] with distances and multi-region operators.

An interesting generalization of the OWA operator results when applying quasi-arithmetic means in the aggregation process. The outcome is the quasi-arithmetic ordered weighted aggregation (Quasi-OWA) operator [23]. This operator combines a wide range of mean operators, including the generalized mean, the OWA operator, the ordered weighted geometric (OWG) operator, and the ordered weighted quadratic averaging (OWG) operator, among others. Some of the most representative extensions of the Quasi-OWA operator are, e.g., the uncertain induced quasi-arithmetic OWA (Quasi-UIOWA) operator [24], the combined continuous quasi-arithmetic generalized Choquet integral aggregation operator [25] and the quasi intuitionistic fuzzy ordered weighted averaging operator [26], among others.

Zhou and Chen [27] propose a generalization of the ordered weighted geometric averaging (OWGA) operator based on an optimal model. The introduced operator is the generalized ordered weighted logarithmic aggregation (GOWLA) operator. This contribution includes a set of parameterized families, such as the step generalized ordered weighted logarithmic averaging (Step-GOWLA) operator, the window generalized ordered weighted logarithmic averaging (Window-GOWLA) operator, and the S-GOWLA, among others. A further generalization of the GOWLA operator is that introduced by Zhou, Chen, and Liu [28] designated the generalized ordered weighted logarithmic proportional averaging (GOWLPA) operator. Some generalizations of this operator are the generalized hybrid logarithmic proportional averaging (GHLPA) operator and the quasi ordered weighted logarithmic partial averaging (Quasi-OWLPA) operator. Following the trend of developing aggregation operators based on optimal deviation models, Zhou, Chen, and Liu [28] introduce the generalized ordered weighted exponential proportional aggregation operator (GOWEPA), which is further generalized to develop the generalized hybrid exponential proportional averaging (GHEPA) operator and the generalized hybrid exponential proportional averaging-weighted average (GHEPAWA) operator. Recently, Zhou, Tao, Chen, and Liu [29] have introduced an additional generalization to the GOWLA designated the generalized ordered weighted logarithmic harmonic averaging (GOWLHA) operator, including the generalized hybrid logarithmic harmonic averaging (GHLHA) operator and the generalized hybrid logarithmic harmonic averaging weighted average (GHLHAWA) operator.

The aim of this paper is the introduction of the induced generalized ordered weighted logarithmic aggregation (IGOWLA) operator. The newly introduced operator is an extension of the optimal deviation model [27] adding the order-induced variables that change the previous ordering mechanism of the arguments. The introduction of this mechanism seeks a broader representation of the complexity in certain scenarios.

We study a series of properties and families of the operator such as the induced ordered weighted logarithmic geometric averaging (IOWLGA) operator, the induced ordered weighted logarithmic harmonic averaging (IOWLHA) operator, and the induced ordered weighted logarithmic aggregation (IOWLA) operator, among others. Furthermore, we present some extensions of the operator, first, using quasi-arithmetic means, obtaining the quasi induced generalized ordered weighted logarithmic aggregation operator (Quasi-IGOWLA) operator; second, utilizing moving averages, we develop the induced generalized ordered weighted logarithmic moving average (IGOWLMA) operator.

This paper also proposes an illustrative example to show the main characteristics of the IGOWLA operator. The example includes a multi-person decision-making analysis in the field of innovation management. The application seeks to exemplify a strategic decision-making process where a series of experts need to assess and choose new productos from a portfolio of options. The case includes a highly complex attitudinal character from management. Results show a clear difference in the aggregation when applying order-induced variables instead of using traditional operators. The operator can be useful for other decision-making applications in business, such as human resource management, strategic decision making, marketing, etc.

This paper is organized as follows. Section 2 presents basic concepts of the OWA, IOWA, Quasi-IOWA, and GOWLA operators. Section 3 presents the IGOWLA operator, the characterization of the weighting vector and families. Section 4 presents the extension of the Quasi-IGOWLA operator. In Section 5 proposes an illustrative application of a decision-making procedure utilizing the IGOWLA. Finally, Section 7 summarizes the concluding remarks of the paper.

2. Preliminaries

In the present section, we briefly review some of the principal contributions in the field of aggregation operators. Specifically, we describe the OWA operator, the induced OWA operator, the Quasi-IOWA operator and the GOWLA operator.

2.1. The OWA operator

The ordered weighted averaging operator introduced by Yager [5] proposes a family of aggregation operators that have been used in a plethora of applications (see, e.g., [7]). The OWA operator can be defined as follows:

Definition 1. An OWA operator is a mapping $OWA: R^n \to R$, which has an associated n vector $w_j = (w_n)^T$, where $w_j \in [0,1]$, and $\sum_{j=1}^{n} w_j = 1$. Accordingly:

$$OWA(a_1, ..., a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the *j*th largest of the arguments a_i.

It has been demonstrated that the OWA operator is commutative, idempotent, bounded and monotonic [5]. Furthermore, we can obtain the ascending OWA or the descending OWA by generalizing the direction of the reordering process [30].

2.2. The induced OWA operator

The induced ordered weighted averaging operator, introduced by [8] presents an extension of the OWA operator. This extension allows a reordering process that is defined by order-induced variables u_i rather than the traditional ordering constructed from the values of the a_i arguments.

Definition 2. An IOWA operator of dimension n is a mapping $IOWA: R^n \to R$, associated with a weighting vector W of dimension n such that $\sum_{j=1}^{n} w_j = 1$, $w_j \in [0,1]$, and a set of order-inducing variables u_i, following the next formula:

$$IOWA(\langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where $(b_1, \ldots, b_n)$ is $(a_1, \ldots, a_n)$ reordered in decreasing values of the u_i. Note that u_i are the order-inducing variables and the a_i are the argument variables.

2.3. The *Quasi*-IOWA operator

The quasi-arithmetic induced ordered weighted aggregation (Quasi-IOWA) operator presents an extension of the Quasi-OWA operator. The main difference is the reordering process; in this case, order-induced variables dictate the complex reordering of the arguments. The Quasi-IOWA operator can be defined as follows:

Definition 3. A Quasi-IOWA operator of dimension n is a mapping $QIOWA: R^n \times R^n \to R$ that has an associated weighting vector W of dimension n such that $w_j \in [0,1]$ for all j, and $\sum_{j=1}^{n} w_j = 1$, following the next formula:

$$QIOWA(\langle u_n, a_n \rangle) = g^{-1} \left\{ \sum_{j=1}^{n} w_j g(b_j) \right\}, \tag{3}$$

where b_j is the a_i value of the Quasi-IOWA pair $\langle u_i, a_i \rangle$ having the largest u_i, u_i is the order-inducing variable, a_i is the argument, and $g(b)$ is a strictly continuous monotonic function.

Note that the Quasi-IOWA can also be viewed as a generalized form of the IOWA operator by using quasi-arithmetic means. The Quasi-IOWA has a wide variety of particular cases [31] including, e.g., the IGOWA operator, the IOWA operator, the IOWGA operator, the IOWQA operator, and the IOWHA operator.

2.4. The GOWLA operator

The generalized ordered weighted logarithmic aggregation (GOWLA) operator [27] introduces a parameterized family of aggregation operators including the step-GOWLA operator, the window-GOWLA operator, the S-GOWLA operator and the GOWHLA operator The GOWLA operator can be formulated as follows:

$$GWLA(a_1, ..., a_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lna_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}. \quad (4)$$

If we reorder the arguments a_i, then we can define the generalized ordered weighted logarithmic averaging operator (GOWLA) as follows:

Definition 4. A GOWLA operator of dimension n is a mapping $GOWLA: \Omega^n \rightarrow \Omega$ that is demarcated by an associated weighting vector W of dimension n, satisfying $w_j \in [0,1]$ for all j, and $\sum_{j=1}^{n} w_j = 1$ and a parameter λ that moves between $(-\infty, \infty) - \{0\}$, according to the next formula:

$$GOWLA(a_1, ..., a_n) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \quad (5)$$

where b_j is the jth largest of the arguments $a_1, a_2, \ldots, a_n$. Observe that $lna_j \geq 0$. In that case, $exp\left(lna_j\right) \geq exp(0)$; therefore, $a_j \geq 1$ following the notation in Zhou and Chen [27], $\Omega = \{x | x \geq 1, x \in R\}$.

3. The induced GOWLA operator

This paper presents the induced GOWLA (IGOWLA) operator it is in fact an extension of the GOWLA operator introduced by [27]; the new formulation of the IGOWLA includes a previous reordering step, this means that the IGOWLA operator is not defined by the values and order of the arguments a_i but by order-induced variables u_i, that define the position of the arguments a_i by the values of the u_i [32]. This extension allows a generalized ordering process, where decision making can consider highly complex conditions. The IGOWLA operator can be defined as follows:

Definition 5: An IGOWLA operator of dimension n is a mapping IGOWLA: $\Omega^n \rightarrow \Omega$ defined by an associated weighting vector W such that $w_j \in [0,1]$ and $w_j \in [0,1]$ and a set of order-inducing variables u_i, according to the next formula:

$$IGOWLA(\langle u_n, a_n \rangle) = exp\left\{\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)^{\frac{1}{\lambda}}\right\}, \quad (6)$$

where λ is a parameter such that $\lambda \in (-\infty, \infty) - \{0\}$, and $(b_1, \ldots, b_n)$ is $(a_1, a_2 \ldots, a_n)$ reordered in decreasing values of the u_i. Observe that u_i are the order-inducing variables and a_i are the argument variables. Note that in this paper, we follow the original argument where $\Omega = \{x | x \geq 1, x \in R\}$.

Example 1. Assume the following collection of arguments set by their respective order-inducing variables $\langle u_i, a_i \rangle$: $\langle 3, 25 \rangle, \langle 1, 75 \rangle, \langle 6, 5 \rangle, \langle 4, 55 \rangle$. Let us assume that $W = (0.1, 0.3, 0.2, 0.4)$ and $\lambda = 2$; the aggregation will result as follows:

$$b_1 = a_3 = 5, b_2 = a_4 = 55, b_3 = a_1 = 25, b_4 = a_2 = 75,$$

$$IGOWLA(\langle 3,25 \rangle, \langle 1,75 \rangle, \langle 6,5 \rangle, \langle 4,55 \rangle) = exp\left\{\left(\begin{array}{l}(0.4 \times (ln5))^2 + (0.2 \times (ln55))^2 + \\ (0.3 \times (ln25))^2 + (0.1 \times (ln75))^2\end{array}\right)^{\frac{1}{2}}\right\} =$$

45.6804.

It is observable that the order-inducing variables u_i affect the order of the argument variables a_i in decreasing order.

It is possible to differentiate the operator between the descending induced generalized OWA (DIGOWA) operator, and the ascending induced generalized OWA (AIGOWA) operator. Regardless, the operators noted above are connected by the relationship of $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the DIGOWA operator and w^*_{n+1-j} the jth weight of the AIGOWA operator.

Proposition 1. If the weighting vector is $\sum_{j=1}^{n} w_j \neq 1$, then normalizing the weighting arguments as follows is proposed:

$$IGOWLA(\langle u_n, a_n \rangle) = exp\left\{\left(\frac{1}{W}\left(\sum_{j=1}^{n} w_j \left(lnb_j\right)^{\lambda}\right)\right)^{\frac{1}{\lambda}}\right\}. \quad (7)$$

Note that $W = \sum_{j}^{n} w_j$.

3.1. Characterization of the weighting vector

When defining the IGOWLA operator, it is interesting to analyze the characterization of the weighting vector. Following the procedures developed by Yager [5,33] and the descriptions stated in [32] we can obtain the degree of orness or attitudinal character $\alpha(W)$, the entropy of dispersion $H(W)$, the balance $B(W)$ and the divergence $Div(W)$ for the induced logarithmic aggregation operators.

Due to the induced properties [31], the attitudinal character of the IGOWLA operator can be described from two different perspectives. If we focus on the attitudinal character, then we can use the same measure as in the OWA operator [5] because we want to measure the complex attitude, which depends solely on the weighting vector. In this case, the formulation is as follows:

$$\alpha(W) = \left(\sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1}\right)^{\lambda}\right)^{\frac{1}{\lambda}}. \quad (8)$$

Observe that $\alpha(W) \in [0,1]$. Note that the optimistic criteria are obtained when $\alpha(W) = 1$, the pessimistic criteria are given when $\alpha(W) = 0$, and the averaging criteria are obtained when $\alpha(W) = 0.5$.

Second, if we focus on the numerical values of the aggregation, then the orness measure $\alpha(W)$ should be calculated as follows:

$$a(W)^{*} = \left(\sum_{j=1}^{n} w_j e_j\right)^{\frac{1}{\lambda}}, \quad (9)$$

where e_j is the d_i value of the IGOWLA pair $\langle u_i, d_i \rangle$ having the *j*th largest u_i, u_i is the order-inducing variable, and $d_i = \left(\frac{(n-j)}{(n-1)}\right)^{\lambda}$. Note that to define the attitudinal character, we use the classical representation of the OWA operator when we do not use logarithms as inconsistencies present when $[0,1]$.

The dispersion measure $H(W)$, commonly utilized to analyze the type of information being used [5,34], can be calculated by solving the next equation:

$$H(W) = -\sum_{j=1}^{n} w_j ln(w_j). \quad (10)$$

Note that if $w_j = 1$ for any j, then $H(W) = 0$, which means that the least information is being used in the operator. Conversely, if $w_j = \left(\frac{1}{n}\right)$ for all j, then a maximum amount of information is being used.

The balance of the weighting vector can also be studied from two perspectives. If we consider the attitudinal perspective we can formulate it as follows:

$$Bal(W) = \sum_{j=1}^{n} w_j \left(\frac{n+1-2j}{n-1}\right). \quad (11)$$

However, if we consider the numerical values of the aggregation, the formulation would be as follows:

$$Bal(W)^{*} = \sum_{j=1}^{n} w_j s_j, \quad (12)$$

where s_j is the t_i value of the IGOWLA pair u_i, t_i having the *j*th largest u_i, with u_i being the order-inducing variable and $t_i = \frac{n+1-2j}{(n-1)}$. Observe that $Bal(W) \in [-1,1]$. For the minimum, $Bal(W) = -1$, and for the maximum, $Bal(W) = 1$. Note that this measure is applicable to any induced aggregation operators [8,32].

Finally, the divergence measure of the weighting vector can be obtained by:

$$Div(W) = \sum_{j=1}^{n} w_j \left(-\alpha(W) + \frac{n-j}{n-1} \right)^2 . \qquad (13)$$

Example 2. Following the arguments described in Example 1, the characterization of the weighting vector result is shown in Table 1:

Table 1. IGOWLA operator weighting vector measures

Measure	$\alpha(W)$ λ=2	$\alpha(W)^*$	$B(W)$	$B(W)^*$
Result	0.5055	0.6236	-0.2667	0.0000
Measure	$H(W)$	$Div(W)$	$Div(W)^*$	
Result	1.2799	0.1404	0.1542	

3.2. IGOWLA operator families

A group of families of the IGOWLA operator can be described when analyzing the parameter λ. Table 2 presents some of the resulting cases of special interest:

Table 2. Families of IGOWLA operators

λ	Families	Acronym
$\to 0$	Induced ordered weighted logarithmic geometric averaging operator	IOWLGA
-1	Induced ordered weighted logarithmic harmonic averaging operator	IOWLHA
1	Induced ordered weighted logarithmic aggregation operator	IOWLA
2	Induced ordered weighted logarithmic quadratic aggregation operator	IOWLQA
3	Induced ordered weighted logarithmic cubic aggregation operator	IOWLCA
$\to \infty$	Largest of the b_j , for $j = n$.	Max
$\to -\infty$	Lowest of the b_j . , for $j = n$	Min

Remark 1. Let $\lambda \to 0 \lambda \to 0$, then, the IGOWLA operator becomes the IOWLGA [27] operator:

$$IGOWLA(\langle u_n, a_n \rangle) = exp\left\{ \prod_{j=1}^{n} (lnb_j)^{w_j} \right\}, \qquad (14)$$

where $(b_1, \dots, b_n)$ is $(a_1, a_2 \dots, a_n)$ reordered in decreasing values of the u_i .

Remark 2. If $\lambda = -1$, then the IGOWLA operator is reduced to the IOWLHA operator:

$$IGOWLA(\langle u_n, a_n \rangle) = \prod_{j=1}^{n} b_j^{\,w_j} . \qquad (15)$$

Remark 3. If $\lambda = 1 \lambda = 1$, then the IGOWLA operator becomes the IOWGA [27] operator:

$$IGOWLA(\langle u_n, a_n \rangle) = \prod_{j=1}^{n} b_j^{\,w_j} . \qquad (16)$$

Note that this formulation can also be presented as the IOWLA operator:

$$IOWLA(\langle u_n, a_n \rangle) = exp \sum_{j=1}^{n} w_j (lnb_j). \qquad (17)$$

Observe that similarly, if $\lambda = 1 \lambda = 1$, then we can reduce the GOWLA operator to the OWLA operator:

$$OWLA(\langle u_n, a_n \rangle) = exp \sum_{j=1}^{n} w_j (lnb_j). \qquad (18)$$

Furthermore, we can obtain the WLA operator; in this case, when $\lambda = 1 \lambda = 1$, we have:

$$WLA(\langle u_n, a_n \rangle) = exp \sum_{j=1}^{n} w_i (lna_i). \qquad (19)$$

Remark 4. If $\lambda = 2 \lambda = 2$, then the IGOWLA operator is reduced to the IOWLQA operator:

$$IOWLQA(\langle u_n, a_n \rangle) = exp\left\{ \sqrt{\left(\sum_{j=1}^{n} w_j (lnb_j)^2 \right)} \right\}. \qquad (20)$$

Remark 5. Like the IOWLQA operator, when $\lambda = 3$ $\lambda = 3$, the IGOWLA operator becomes the IOWLC operator:

$$IOWLC(\langle u_n, a_n \rangle) = exp\left\{ \left(\sum_{j=1}^{n} w_j (lnb_j)^3 \right)^{\frac{1}{3}} \right\}. \qquad (21)$$

Remark 6. If $\lambda \to \infty$, then the IGOWLA operator solution tends to the *j*th largest a_i for every pair $\langle u_i, b_i \rangle$ for all j .

$$IGOWLA(\langle u_n, a_n \rangle) = max\{(u_i, a_i)\}. \qquad (22)$$

Remark 7. If $\lambda \to -\infty$, then the IGOWLA operator solution tends to the *j*th lowest a_i for every pair $\langle u_i, b_i \rangle$ for all j .

$$IGOWLA(\langle u_n, a_n \rangle) = min\{(u_i, a_i)\}. \qquad (23)$$

Example 3. Following the arguments described in Example 1, the results for each family of the IGOWLA operator are shown in Table 3.

Table 3. Families of IGOWLA operator

λ	$\to 0$	-1	1	2
Aggregation	45.9305	42.4498	48.9898	51.6158
λ	3	∞	$-\infty$	
Aggregation	53.8483	$\to 80$	$\to 10$	

4. The Quasi-IOWLA operator

It is possible to generate an additional generalization of the general ordered weighted averaging operators by utilizing quasi-arithmetic means instead of the ordinary means (see, e.g., [32,35]). In the case of the IGOWLA operator, we suggest the use of a similar methodology to construct the Quasi-IOWLA operator.

Definition 7. A Quasi-WLA operator of dimension n is a mapping $\text{QWLA}: \Omega^n \to \Omega$ with an associated weighting vector W of dimension n such that $\sum_{n=1}^{j} w_i = 1$, $w_i \in [0,1]$, and a strictly monotonic continuous function $g(lnb)$, according to the next formula:

$$QWLA(a_1,...,a_n) = exp\ \text{g}^{-1}\left\{\left(\sum_{i=1}^{n} w_i \text{g}(lna_i)\right)\right\}, \quad (24)$$

where a_i are the set of arguments to be aggregated.

Note that if all of the weights of the QWLA are equal $\left(w_i = \frac{1}{n}\right)\forall i$, then the QWLA operator becomes the quasi-arithmetic logarithmic average (QLA).

Definition 8. A Quasi-OWLA operator of dimension n is a mapping $\text{QOWLA}: \Omega^n \to \Omega$ with an associated weighting vector W of dimension n, satisfying $\sum_{n=1}^{j} w_j = 1$, and $w_j \in [0,1]$, a set of order-inducing variables u_i, and a strictly monotonic continuous function $g(lnb)$, according to the next formula:

$$QOWLA(a_1,...,a_n) = \exp g^{-1}\left\{\sum_{j=1}^{n} w_j g(lnb_j)\right\}, \quad (25)$$

where b_j are the values a_i ordered in a decreasing way.

Definition 9. A Quasi-IOWLA operator of dimension n is a mapping $\text{QIOWLA}: \Omega^n \to \Omega$ with an associated weighting vector W of n dimension, satisfying the condition that the sum of the weights is 1, and $w_j \in [0,1]$, a set of order-inducing variables u_i, and a strictly monotonic continuous function $g(lnb)$, according to the next formula:

$$QIOWLA(\langle u_n, a_n \rangle) = \exp g^{-1}\left\{\sum_{j=1}^{n} w_j g(lnb_j)\right\}, \quad (26)$$

where b_j are the values a_i of the Quasi-IOWA pairs u_i, a_i ordered in decreasing direction of their u_i values.

Please also note that the Q-IOWLA is a particular case of the IGOWLA when the strictly monotonic function is set on $g(lnb_j) = (lnb_j)^{\lambda}$. This approach is equivalent for the Quasi-WLA and the Quasi-OWLA. Therefore, all these operators share the properties studied for the IGOWLA operator; specifically, it is bounded, idempotent and commutative. However, as shown in section 3, in some cases, it is not monotonic.

Observe that we can also distinguish between the descending Quasi-DIOWLA and the ascending Quasi-AIOWLA. The relationship found between the descending and the ascending operators is $w_j = w^*_{n+1-j}$, where w_j is the j th weight of the Quasi-DIOWLA operator and w^*_{n+1-j} the jth weight of the Quasi-AIOWLA operator.

The wide range of operators that quasi-arithmetic means provide have proven to be effective when treating problems covering a wide range of complexities [36], including geometric aggregations, quadratic aggregations, and harmonic aggregations.

Proposition 2. In case of any ties, replacing the tied arguments with the quasi-arithmetic logarithmic average operator is proposed [32].

5. Group Decision Making with the IGOWLA operator

5.1. Decision-making process

The IGOWLA operator is suitable for a wide range of applications in decision making processes (see, e.g., [37–40]). Here, a decision-making application in innovation management is proposed to show the variations and benefits of the newly introduced order-induced mechanisms of the IGOWLA operator.

The main reason for selecting this topic is the presentation of information, which, in the case of innovation, has been stated to be imprecise and uncertain [41,42]. Therefore, there is the motivation to use the opinion of different decision makers or experts to find a suitable solution.

Strategic decision making in innovation management addresses diverse aspects that include not only imprecise information [43] but also a certain level of attitudinal character on the part of the decision makers, e.g., the possibility of different strategic outcomes [44], complexity and unfamiliar interactions [45], the lack of information [46], time, flexibility and control. In this sense, the use of inducing variables should aid in the complex decision-making procedure.

Innovation management considers a wide range of problems to be assessed, one of them is the correct selection new products to be developed, this from a portfolio of possible prototypes. The effectiveness with which an organization manages its new products portfolio is often a key determinant of competitive advantage [47]. Here, portfolio management deals with the allocation of the scarce resources of the business, namely: money, time, people, machinery, etc. to potential developments under uncertain conditions. The key concepts to analyze are quantity, quality, and organizational capability for new product development. The selected new products to be developed must correctly align with business objectives and balance several elements such as timespan and risk.

The process to follow in the selection of strategies in innovation management with the IGOWLA operator and the application when introducing a multi-person analysis can be summarized as follows:

Step 1. Assuming that $A=\{A_1,A_2,\ldots,A_m\}$ is a set of options, including $S=\{S_1,S_2,\ldots,S_m\}$ as a set of characteristics to be evaluated, both elements constitute the payoff matrix, $(a_{hi})_{m\times n}$. Introducing the set $E=\{e_1,e_2,\ldots,e_q\}$ as a finite group of decision makers. In this case each decision-maker has a different level of relevance such that $X=(x_1,x_2,\ldots,x_p)$ represents the weighting vector such that $\sum_{k=1}^{p}x_k=1$ and $x_k\in[0,1]$. Each decision-maker is asked to provide a personal pay-off matrix $(a_{hi})^k{}_{m\times n}$ based on its preferences.

Step 2. Based on the highly complex attitudinal character of the case, introduce a set of order-inducing variables $(u_{hi})_{m\times n}$ corresponding to each alternative h and characteristic i . Include a $W=(w_1,w_2,\ldots,w_n)$ weighting vector, make sure that this verctor satisfies the IGOWLA operator formulation, next, define a λ value to be applied in the aggregation operation.

Step 3. In this case we propose the weighted average to aggregate the information provided by the decision-makers E and the vector X . The aggregated information results in the collective payoff matrix $(a_{hi})^k{}_{m\times n}$; therefore, $a_{hi}=\sum_{k=1}^{p}x(a_{hi})^k$.

Step 4. Solve for the IGOWLA operator as described in Eq. 6. Please note that λ value is typically set as 1; however, any of the families described in section 3.2 can be used, depending on the problem analyzed.

Step 5. After solving for the IGOWLA operator, set a ranking of the alternatives; compare the results of the specific problem and propose a decision-making approach.

5.2. Illustrative Example

This paper proposes an illustrative example of the IGOWLA operator in a strategic decision-making process of portfolio management with multi-person inputs. Other business decision-making applications in the field of innovation management can be assessed e.g. knowledge management, project management, organization and structure, among others, please see [48].

Step 1. Let's assume that company Y is involved in the design of fast-moving consumer goods in the alimentary sector. The company must decide from its portfolio of new products and select one of five potential enhanced beverage concepts. Thus, we have:

- A_1 Super Sport: vitamin C with electrolytes
- A_2 High Energy: vitamin C with caffeine
- $A3$ Fast Recover: vitamins B5, B6 and B12
- A_4 0 Sugar Sport: vitamin C with electrolytes and no sugar
- A_5 AntiOx: manganese plus vitamin B3

This problem requires the inputs of several experts of the company to assure the relevance, appropriateness and a strategic alignment to the requirements of the business. The company sets 6 key factors to be analyzed in the selection process:

- S_1 Expected benefits
- S_2 Alignment to business
- S_3 Development costs
- S_4 Technical viability
- S_5 Risk
- S_6 Time to market

The experts are divided in groups of 3 (Tables 4 – 9). The first group (Table 4 and 5) has two engineering experts, the second includes two experts from marketing and sales (Table 6 and 7), and in the third group two financial experts (Table 8 and 9). The experts are asked to provide their opinion in a scale of 1 to 100, their opinions are bounded to the expected performance of each product based on the key factors selected by management. This case requires, firstly to generate a multi-aggregation process so the opinions of the groups can be aggregated. Secondly, we need to aggregate all the information into a sole collective payoff matrix. Once we obtain the matrix, we use the IGOWLA operator to generate the final results and aid the board of directors in the selection of the most suitable alternative for the elements that constitute the problem.

Step 2. Due to the complex attitudinal character of the administration, the next set of order-inducing variables are included in the problem: $U=(7,5,4,2,10,9)$. Also, the experts have considered a weighting vector $W=(0.1,0.1,0.1,0.2,0.1,0.4)$.

Step 3. For this case, we will consider the weighting vectors X, representing the different importance of each expert in the analysis. For the first group of experts we have $X_1=(0.4,0.6)$, the second group of experts $X_2=(0.7,0.3)$, and the third group of experts $X_3=(0.5,0.5)$. Please note that the collective payoff matrix has $X_4=(0.3,0.4,0.3)$. All the elements have been correctly defined, therefore we can obtain results by first aggregating the opinions of the three groups of experts using the weighted average; the results are shown in Tables 10, 11 and 12. Using this information, we now aggregate the three subgroups into a collective payoff matrix. The results are shown in Table 13.

Step 4. Solving for the IGOWLA operator families, we aggregate the collective information and obtain results. Table 14 show the final aggregations.

Step 5. The problem requires a visualization of the diverse decisions that can be generated. Therefore, we establish a ranking of the performance of each product. The preferred ordering of the alternatives is presented in Table 15. The $\succ$ symbol represents preferred to.

Table 4. Payoff Matrix – Expert 1.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	81	99	98	100	86	89
A_2	27	42	29	48	34	37
A_3	65	82	87	88	98	98
A_4	97	100	88	82	88	100
A_5	50	49	53	48	46	50

Table 5. Payoff Matrix – Expert 2.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	92	94	98	88	86	100
A_2	56	26	36	27	28	32
A_3	42	48	43	100	86	79
A_4	81	80	94	95	92	81
A_5	59	60	43	55	46	44

Table 6. Payoff Matrix – Expert 3.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	94	82	88	94	85	100
A_2	57	53	58	36	20	25
A_3	93	50	100	48	100	77
A_4	93	91	89	90	93	98
A_5	42	57	46	44	44	51

Table 7. Payoff Matrix – Expert 4.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	97	87	85	89	82	97
A_2	25	48	38	52	47	27
A_3	93	79	61	57	98	73
A_4	92	83	92	84	94	100
A_5	51	51	40	57	44	40

Table 8. Payoff Matrix – Expert 5.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	96	100	100	99	80	85
A_2	52	47	45	60	38	34
A_3	51	50	66	40	50	47
A_4	100	86	97	99	95	86
A_5	48	54	57	44	43	50

Table 9. Payoff Matrix – Expert 6.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	93	86	89	98	81	94
A_2	59	23	57	57	20	57
A_3	90	64	59	98	99	52
A_4	88	84	96	98	94	81
A_5	50	54	54	53	46	52

Table 10. Payoff Matrix – Group 1 (Experts 1 and 2).

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	87.60	96.00	98.00	92.80	86.00	95.60
A_2	44.40	32.40	33.20	35.40	30.40	34.00
A_3	51.20	61.60	60.60	95.20	90.80	86.60
A_4	87.40	88.00	91.60	89.80	90.40	88.60
A_5	55.40	55.60	47.00	52.20	46.00	46.40

Table 11. Payoff Matrix – Group 2 (Experts 3 and 4).

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	94.90	83.50	87.10	92.50	84.10	99.10
A_2	47.40	51.50	52.00	40.80	28.10	25.60
A_3	93.00	58.70	88.30	50.70	99.40	75.80
A_4	92.70	88.60	89.90	88.20	93.30	98.60
A_5	44.70	55.20	44.20	47.90	44.00	47.70

Table 12. Payoff Matrix – Group 3 (Experts 5 and 6).

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	94.50	93.00	94.50	98.50	80.50	89.50
A_2	55.50	35.00	51.00	58.50	29.00	45.50
A_3	70.50	57.00	62.50	69.00	74.50	49.50
A_4	94.00	85.00	96.50	98.50	94.50	83.50
A_5	49.00	54.00	55.50	48.50	44.50	51.00

Table 13. Collective payoff matrix.

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	92.59	90.10	92.59	94.39	83.59	95.17
A_2	48.93	40.82	46.06	44.49	29.06	34.09
A_3	73.71	59.06	72.25	69.54	89.35	71.15
A_4	91.50	87.34	92.39	91.77	92.79	91.07
A_5	49.20	54.96	48.43	49.37	44.75	48.30

Table 14. Aggregated Results

	MIN	MAX	IGOWLA λ = -1	IGOWLA λ = 1	IGOWLA λ = 2	IGOWLA λ = 3
A_1	83.59	95.17	92.08	92.11	92.12	92.14
A_2	29.06	48.93	41.08	41.34	41.46	41.58
A_3	59.06	89.35	69.64	69.84	69.94	70.04
A_4	87.34	92.79	90.92	90.93	90.94	90.95
A_5	44.75	54.96	49.68	49.73	49.75	49.77
	OWA	IOWA	GOWLA λ = -1	GOWLA λ = 1	GOWLA λ = 2	GOWLA λ = 3
A_1	89.18	92.17	89	89.05	89.08	89.11
A_2	37.15	41.77	35.96	36.38	36.59	36.8
A_3	68.34	70.27	67.49	67.75	67.89	68.02
A_4	90.04	90.95	90	90.01	90.02	90.02
A_5	47.77	49.81	47.63	47.67	47.7	47.72

Table 15. Ranking of the options

Ranking		Ranking	
MIN	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$	OWA	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$
MAX	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	IOWA	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$
IGOWLA $(\lambda=-1)$	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	GOWLA $(\lambda=-1)$	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$
IGOWLA $(\lambda=1)$	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	GOWLA $(\lambda=1)$	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$
IGOWLA $(\lambda=2)$	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	GOWLA $(\lambda=2)$	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$
IGOWLA $(\lambda=3)$	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$	GOWLA $(\lambda=3)$	$A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$

Results show that the elements have been ordered in different ways, depending directly on the operator utilized in the aggregation of the arguments. In this hypothetical case, which includes the diverse expert opinion of six persons and the highly complex attitudinal characteristics of the direction board, the exercise concludes that the concepts that should be firstly developed are products: A_1 (Super Sport) and A_4 (No Sugar Sport). Please note that the induced operators show a different ranking from the traditional ones, this indicates a clear difference when introducing order-induced mechanism to the reordering process. The aggregated results show no specific ties; therefore, the use of the proposed quasi-arithmetic means is not required in this case. Please also note that the multi-person process can be aggregated and presented in many other approaches; in this example it is assumed that the management board needed the information presented as represented in the example.

6. Conclusions

This paper presents the IGOWLA operator, it is a generalization of the GOWLA operator, therefore the introduced operator shares its main characteristics. The order-induced variables included in the formulation of the IGOWLA operators, allows an even wider representation of the possible highly complex attitude of decision makers in certain problems.

Diverse measures for characterizing the weighting vector have been analyzed; specifically, we have studied the degree of orness measure, the dispersion measure, the balance measure and the divergence measure. Note that some of these measures can be calculated from two different perspectives, depending on the attitudinal character or the numerical value of the weighting vector. Furthermore, we describe several families of the IGOWLA operator based on the λ parameter, including the IOWLGA operator, the IOWLHA operator, the IOWGA operator, the IOWLA operator, the IOWLQA operator, the IOWLC operator, and the maximum and minimum IGOWLA operators.

We introduce diverse generalizations of the IGOWLA operator. First, using the notion of quasi-arithmetic means, we introduce the QWLA operator, the QOWLA operator, and the QIOWLA operator, therefore adding the option of considering geometric aggregations, quadratic aggregations and harmonic aggregations into the process.

The IGOWLA operator has been designed to aid group decision making, and could be used in several areas, such as economics, statistics and engineering problems. This paper proposes an illustrative example of a possible utilization of the IGOWLA operator. Here, a multi-expert for strategic decision-making process in the area of innovation management is exemplified. The case deals with the assessment of a decision in portfolio management of a company. The objective is the selection of new products to be developed based on diverse characteristics of the products and the alignment to the objectives and preferences of the studied case. This example seeks to show the components of the IGOWLA operator, i.e. the order-induced variables, the construction of scenarios including the generalized lambda vector, and the option of dealing with diverse expert opinions and the highly complex attitudinal characteristics of the aggregation elements.

Further developments and research need to be assessed. Firstly, deepen the mathematical characteristics of the logarithmic properties that build the IGOWLA operator. Secondly, new extensions should be developed e.g. to assess uncertain information, i.e., fuzzy numbers, linguistic variables and interval numbers, the inclusion of distance measures and the possibility of working with heavy aggregations, the new extensions allow the construction of complex formulations that could aid decision making problems in wider scenarios. Finally, new decision-making problems in diverse fields of knowledge should be considered for the application of the newly introduced tools.

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