Master in Actuarial and Financial Sciences

MASTER THESIS

Title: Controlling the life expectancy gap using joint probabilities in the design of survivor's pensions

Author: Sergio Gomez Piernas

Advisor: Mercedes Ayuso Gutiérrez

Academic year: 2020-2021

UNIVERSITAT DE BARCELONA
Facultat d'Economia i Empresa

Màster de Ciències Actuarials i Financeres

Faculty of Economics and Business

Universitat de Barcelona

Master thesis

Master in Actuarial and Financial Sciences

Controlling the life expectancy gap using joint probabilities in the design of survivor's pensions

Author: Sergio Gomez Piernas

Advisor: Mercedes Ayuso Gutiérrez

The content of this document is the sole responsibility of the author, who declares that he/she has not incurred plagiarism and that all references to other authors have been expressed in the text.

Controlling the life expectancy gap using joint probabilities in the design of survivor's pensions

Sergio Gomez Piernas

June 1st 2021

Abstract

The usage of different methodologies produces varying results in the estimation of life expectancies. This Master Thesis revolves around which are the most common approaches used for the calculation of the expected number of lived years, being the period and cohort methods, producing some important differences that are tried to be explained. The usage of period and cohort approaches incurs in different results for the expected wealth of households that are dependent on pensions received. We explain how important the concurrency of different survivorship pensions is and how it produces differences in the expected income for a couple after retirement.

Keywords: Survivorship, Concurrent pensions, Public Pension, Joint life expectancy, Household wealth

Index

1. Introduction

The aim of the survivor's pensions is to compensate for the loss of economic welfare caused by the decease of a related person. At some countries, the survivor's pension has been relegated into a secondary role, such as in Sweden, though in other countries, like Spain or Portugal, it remains as one of the most important pensions. Back in the day, the most common survivor's pension was the widow's pension, which was (and still is) mostly perceived by women. This widow's pension objective was to allow the widow to afford a living, since normally women did not work and therefore did not generate a right for a retirement pension (Ayuso and Chulia, 2018, Fuster, 2021).

Nowadays, with the participation of women in the labor market, the actual state of the widow's pension is in an increasing debate, whereas it still acts as a balance in gender equality at the later stages of life, or it should be diminishing in its importance since its gradually losing the root for what was created.

From a methodological point of view, the actuarial modeling of the survivor's pension has in its fundaments the usage of joint survivability probabilities, being the decease of one of the members of the couple what generates the right to a pension for the survivor. Different papers have revolved around the actuarial analysis of the survivor's pension, although from a different perspective than the one used in this work.

In this paper, our objective is to utilize the methodology developed at Ayuso et al (2021) and Bravo et al (2021), using individual probabilities and to extend it to joint probabilities. Therefore, differentiating from past works where the focus was on studying the pension by analyzing the individual, we will center our attention in the analysis of groups formed by two individuals, measuring life expectancies until the dissolution and the extinction of the group. In the estimation of the aforementioned joint survivability probabilities, cohort methods (BME) will be used, and we will quantify the obtained differences between these methods and those obtained from period methods, which are commonly used. This paper presents in an innovative way the gap found in the estimations for the joint probabilities obtained via the alternative proposed methodologies.

In the development of this work one of the most important aspects is to analyze the age differences between the couple members. Therefore, one of the focal points in which this study has revolved is in studying series of marriages in Spain, since 1976 to 2019, quantifying the difference in age observed on the members of the couple. This work completes the series arranged from 1976 to 2006 shown in the paper Jacinto and Hombrados-Mendieta (2011).

Additionally, we focus in the analysis on the retirement and widow's pensions in Spain, with their evolution on recent years, all from a descriptive point of view of the survivor's pensions. It can be seen that the retirement pensions for women have been increasing in recent years, with both an augment in number and value. Nonetheless, a clear asymmetry in pensions is observed, a relevant aspect that sometimes is overlooked in research and has a big relevance.

The paper is structured as follows. In the second section we briefly define the survivor's pensions and their treatment in different countries, making a detailed breakdown of the last reforms applied. A special mention is made in the case of Spain, detailing its

regulation, and the last recommendation in the context of the Pacto de Toledo Agreement (October 2020).[1]

Moving on to the third section, a descriptive analysis is presented with the complete evolution of the retirement and widow's pension in Spain, differentiating the analysis by sex. In the research we pay special attention to the perceived pension amounts (by intervals). We finally analyze the concurrence of pensions and its global effect on the full amount received after retirement by household.

Methodology used in this paper is explained and detailed in section 4. In the calculation of the joint survival probabilities, the BME methodology developed in Bravo et al. (2021) is used, which is extended now to the calculation of joint survival probabilities and joint life expectancies, considering the dissolution and the extinction of the group. In the empirical section we compare results obtained by this new methodology and those obtained via the period estimation approach, quantifying the gap between both methods. This section will be relevant when quantifying the effect of the estimation methods on the actuarial present value of the pensions that a household will receive until the couple disappears. Our objective is, therefore, to obtain a first estimation of the expected income by pension (retirement and survival pensions) in the household's economy until death of the couple. This result will be relevant if we want to understand the importance that pensions have in determining the household's wealth after retirement, a point not sufficiently analyzed in the literature (Holzmann et al. 2019, Angelini et al. 2009, Ayuso et al. 2016, Alaminos and Ayuso 2015, Alaminos and Ayuso 2016, Alaminos 2017). This paper is related only with a preliminary research, but our objective is to extend the analysis in a future research to a more detailed study by couples' profiles, trying to control by the heterogeneity of the households in terms of income and the new familiar structures that can be increasingly found in our society.

The empirical part is presented in section five where two subsections are detailed. In the first one, an analysis about differences in ages by the two members of the couple is carried out, according to data obtained from the Spanish Statistical Institute. In the second one, results for the estimation of the expected pensions income by household obtained from the BME estimation (according to different scenarios) are presented and compared with those obtained from the traditional period approach. These results allow us to quantify the gap on the estimations in terms of joint probabilities, extending the analysis carried out in Ayuso et al. (2021).

We conclude with the discussion about the main conclusions and results. Future lines of research are also presented.

2. Survivor's pension: Last reforms

In last years, we have seen an increasing debate in different countries about convenience or not of changing the survivor pension system, or about if some adjustments are needed to be done, mostly since the sustainability of the public systems is under question. We present in Table 1 a brief summary about some of the countries where survival pensions are actually operating and main points about legal requirements to be a beneficiary.

[1] Boletín Oficial de las Cortes Generales Serie D: General (congreso.es)

	Retirement Age	Survivor
Canada	65	Yes, if enough contributions have been done by the worker
Spain	66 in 2021 (67 in 2027)	Paid contributions for 500 days in the 5 years prior to the worker death or for 15 years throughout his/her entire working life
Portugal	66.5	At least 36 months of contribution by the worker
France	62	You must have been married to the deceased beneficiary (the Pacs civil partnership and cohabitation do not entitle the partner to the survivor's pension), you must be at least 55, gross annual income must not exceed 21,112 € if you live alone, or 33,779.20 € if you live with a partner
Italy	67	5 years of contribution, 3 of which in the 5 prior years to decease
Germany	67	Five years of contributions by the worker

Table 1. Different retirement ages and requirements to receive a widow pension by countries. Own preparation with Employment Social Affairs & Inclusion, European Commission data.

In a more global view of the matter, if we take a closer look to the document *Pensions at a Glance* 2019, we can observe that we are currently living in a society where the demographic trend is to increase the individual life expectancy, mainly in the OECD countries. Main reforms in pensions over the last years, from 2017, are related to limiting the increase in the retirement age or expanding early-retirement options (Italy, the Netherlands and the Slovak Republic); raising the retirement age (Estonia); enhancing work incentives (Belgium, Canada and Denmark); increasing the level or expanding the coverage of first-tier pensions, the first layer of old-age social protection (Austria, France, Mexico and Slovenia); increasing benefits while reducing contributions for low earners (Germany); suspending the adjustment of pension benefits with demographic changes (Spain); bringing public sector pension benefits more in line with private sector benefits (Norway); changing the contribution rates (Hungary, Iceland and Lithuania) or expanding contribution options (New Zealand); expanding the coverage of mandatory pensions (Chile) or developing auto-enrolment schemes (Lithuania and Poland); and, changing tax rules for pensioners (Sweden).

Moving on to a more domestic view of the study, in the 2011 review of the Pacto de Toledo, we can find some comments on the different recommendations made to increase the sustainability of the public pensions in Spain whilst maintaining its social core and character. One of these recommendations was the "integral reformulation of the death and survivor pensions, in line with what was demanded at Pacto de Toledo 2003, which has not been totally implemented.[2]

One of the main demands is related to the improvement of the actual widow pensions, maintaining its contributive character, focusing on the individuals that their main source of income is this widow pension. To attain this, some proposals are related to the

[2] Page 89.

percentage of the regulatory base increase. It is also explained that the contribution period of the deceased should be taken into account when calculating this pension, in order to homogenize the different contributive pensions in the country.

The *OECD Pensions Outlook* (chapter 7, OECD 2018) recommends the limitation on the granting scenarios of the widow's pension, in an attempt to homogenize and get a more equal treatment of all the beneficiaries of the different pensions that are granted in the OECD countries. There is a special mention to the fact that in Spain, if one of the married couple dies, the widow has the right to be granted a widow's pension, even though they are still active and working. As stated in this report, this may generate inequity between individuals, in line with those analyzed in Alaminos and Ayuso (2019). The OECD is recommending to carry out some changes in the Spanish widow's pension system, a pension that represents the 2.3% of the Spanish GDP, and approximately the 25% of the total pension payments.

The requirements for receiving the widow pension in Spain, as is stated by the Seguridad Social system,[3] are the following ones: i) If the decease is caused by common disease, 500 days in an uninterrupted period of 5 years immediately prior to the decease date; ii) If the decease is caused by an accident, being it a work accident or not, or to professional decease, no prior period of contribution is required. If the decease happened before the wedding and caused by common disease, the survivor will need to accredit the existence of couple's children. If there are no couple's children, the wedding had to be celebrated a year prior to the decease. If the marriage had not been celebrated a year prior to the decease, a period of connivance, added to the married period, that must surpass a total of two years can be accredited in order to receive the widow pension.

In general terms, the widow pension in Spain, as stated by the Seguridad Social system, is calculated by applying a 52% percentage on the regulatory basis, but since January 2019, it is a 60% if the person is aged greater than 65 years old, he/she has no right to receive a Spanish or foreign public pension, does not receive work income for self-employment or paid employment, and does not receive investment or property income, capital gains or income from economic activities, greater than 7,569 € per year. It is even a 70% of the regulatory base if, over the pension granting period, the pensioner has family responsibilities, the widow pension constitutes the principal (>50% of the total incomes) or unique income, and the annual income for every concept does not surpass, as of January 2019, 18,539.40€ per year.

The regulatory base depends on which was the situation of the deceased. If the deceased was a pensioner, the regulatory base will be the same that determined the pension that the deceased was receiving, adding the revalorizations occurred since the pension cause data. If the deceased was an active worker, and with a common contingencies decease, the regulatory base will be the ratio between 28 and the aggregation of the contribution basis over an uninterrupted 24-month period in the last 15 years. If the worker deceased by non-work accident and did not have an uninterrupted 24-month contribution period over the last 15 previous years, the regulatory base will be the most beneficial between the resulting if he was an active worker and the ratio between 28 and the aggregation of the minimum contribution basis over the immediate 24 previous months.

[3] Seguridad Social: Prestaciones / Pensiones de Trabajadores (seg-social.es)

To finish this section, it is remarkable to denote the way of calculating and the requirements to be granted a widow pension in a neighboring country with a newly defined contribution-based pension system such us is the case of Italy (CERP Collegio Carlo Alberto, 2008). In this case, the requirements to be granted a widow's pension is that the deceased has had at least 5 years of contribution, having 3 of them in the immediate prior years to the decease. Therefore, there is no difference between being a pensioner or a worker.

The calculation is quite similar to Spain, being calculated as a percentage of the deceased pension. The percentage to be applied to the resulting pension of the deceased is a 60 %, but if this widow pension exceeds the minimum pension, the percentage will be reduced by a 25 % if it exceeds 3 times the minimum pension, 40% if it exceeds 4 times the minimum pension, and 50 % if it exceeds 5 times the minimum pension, resulting in applying a 35 %, 20 % and 10% respectively over the deceased pension in order to calculate the widow's pension.

3. Descriptive analysis

3.1 Pension entries

To properly analyze the impact that the survivor pensions have in our country, we have analyzed the most recent data for the period between 2014 and 2019, taking a proper look at the entries and mean quantities for the retirement (Table 2 and Figure 1) and widow pensions (Table 3 and Figure 2).

	2014	2015	2016	2017	2018	2019
Men	193,692	184,537	189,498	187,295	191,748	178,072
Women	104,801	105,177	118,008	122,410	136,408	125,322

Variation	2015	2016	2017	2018	2019
Men	-4.73%	2.69%	-1.16%	2.38%	-7.13%
Women	0.36%	12.20%	3.73%	11.44%	-8.13%

Table 2. New retirement pensioners by year and variation. Own preparation with Seguridad Social data

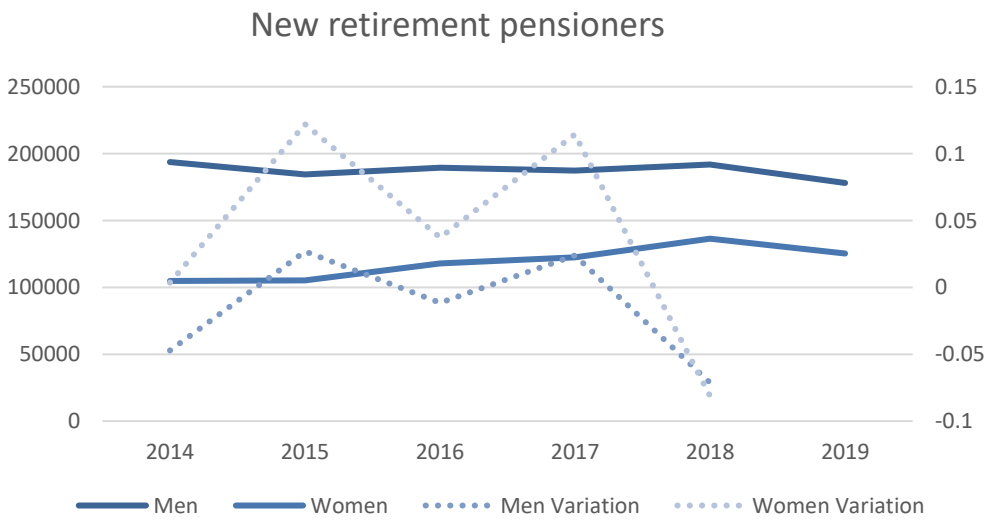

Figure 1. Retirement entries by year and variations. Own preparation with Seguridad Social data

We can observe that entries in the retirement pension from 2014 to 2019 in Spain were quite steady, but with an increase in the case of women. We can see, especially in terms of variation, that both sexes follow a similar trend along the years, both increasing or decreasing, albeit in different magnitudes.

	2014	2015	2016	2017	2018	2019
Men	15,205	15,466	15,036	15,478	16,162	16,191
Women	114,486	117,100	115,790	116,181	118,833	115,923

Variation	2015	2016	2017	2018	2019
Men	1.72%	-2.78%	2.94%	4.42%	0.18%
Women	2.28%	-1.12%	0.34%	2.28%	-2.45%

Table 3. New widow pensioners by year and variation. Own preparation with Seguridad Social data

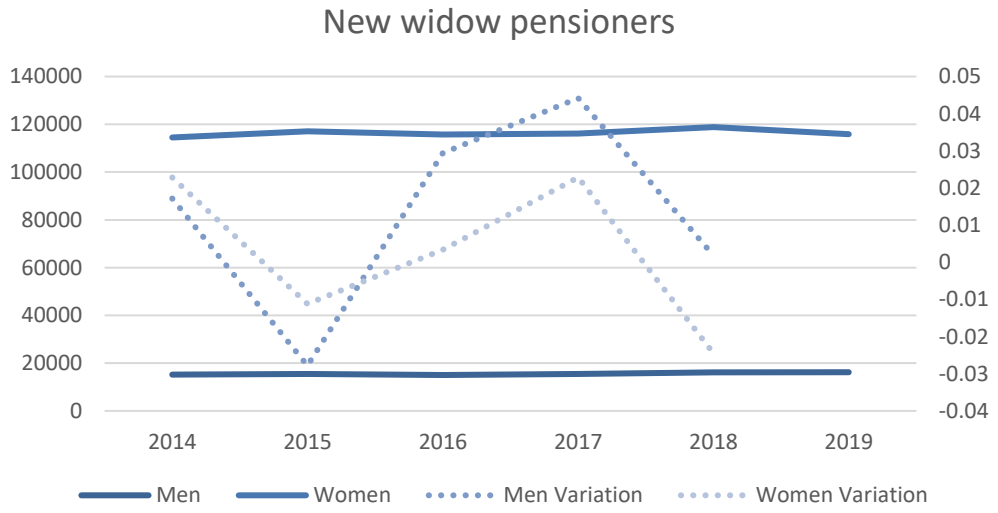

Figure 2. Widow entries by year and variations. Own preparation with Seguridad Social data

As we observe at Figure 2, variations for the new widow pensions are also following the same directions by gender, with a different magnitude. The most important aspect to mention regarding this figure is the big difference in terms of nominal quantities between the women's and men's widow entries, being the latter notably lower, with a difference for more than 100.000 cases (entries of women are seven and a half times entries of men).

Evolution of the mean retirement and widow pensions in the analyzed period is presented in table 4, where we can observe significant differences by sex. Variations along the analyzed period 2014-2019 are presented at figure 3 (mean retirement pensions) and figure 4 (mean widow pensions).

	Retirement			Widow		
	Men	**Women**	**Gap**	**Men**	**Women**	**Gap**
2014	1,165.05	712.09	452.96	473.41	635.84	-162.43
2015	1,188.35	733.48	454.87	477.81	642.82	-165.01
2016	1,211.19	756.80	454.39	482.06	650.52	-168.46
2017	1,232.83	781.10	451.73	485.71	659.66	-173.95
2018	1,261.56	811.46	450.10	492.30	678.88	-186.58
2019	1,312.42	858.21	454.21	508.56	729.11	-220.55

Table 4. Mean retirement and widow pensions, by gender. Period 2014-2019. Own preparation with Seguridad Social data

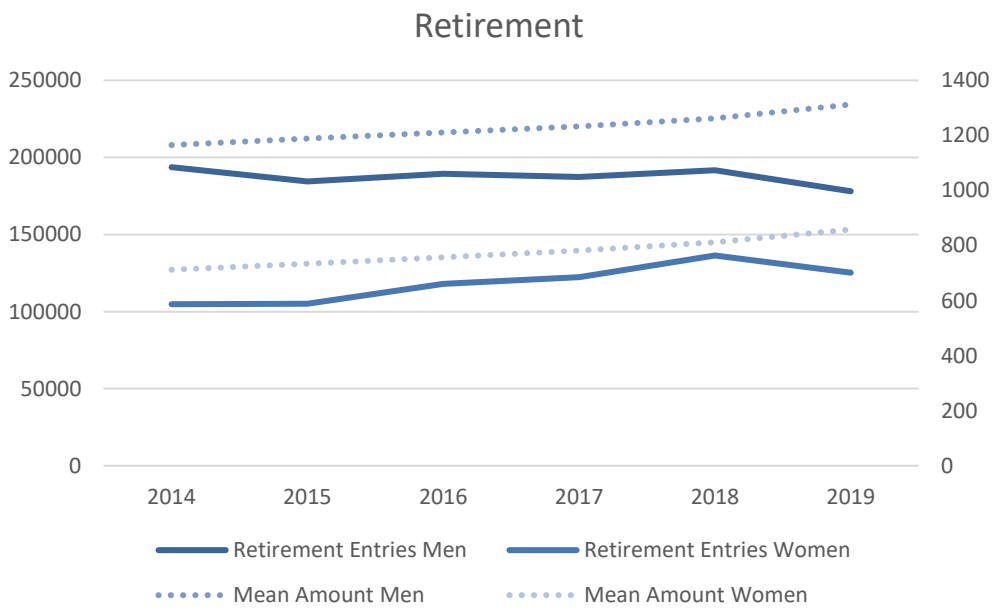

Figure 3. New retirement entries (left axis) and mean retirement pension -first pension- (right axis) by year. Own preparation with Seguridad Social data

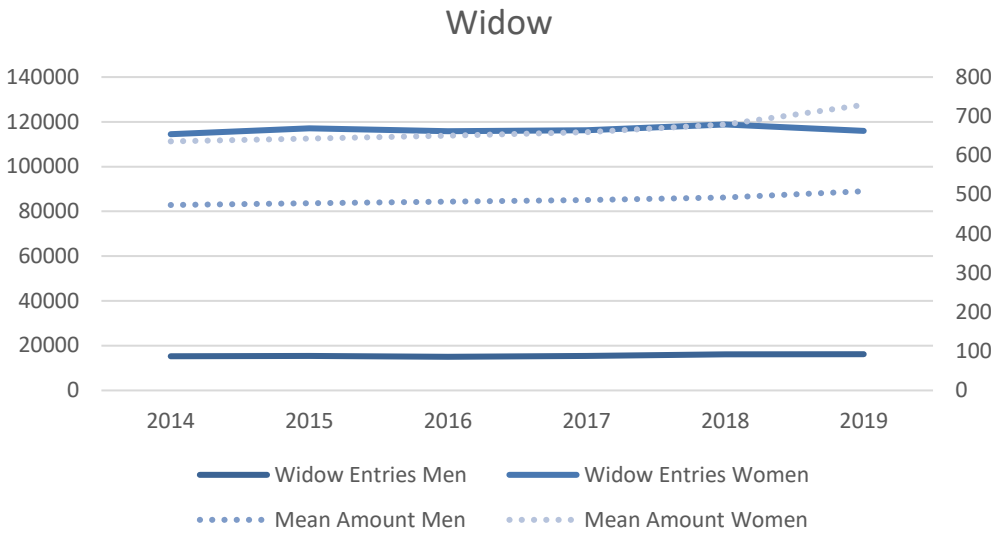

Figure 4. New widow entries (left axis) and mean widow pension -first pension- (right axis)) by year. Own preparation with Seguridad Social data

As we can observe at figure 3 the retirement pension for men is higher than for women, being the gap always around 450 euros. Even though the quantities for both sexes increase, the gap remains. In the case of the widow pensions (figure 4) the women's mean value is higher, and the gap increases over time, while the entries remain still for both sexes.

3.2 Pensions by amount intervals

We present at figures 5 and 6 distributions of the total retirement pensions by amount intervals and gender in Spain, as observed in March 2021. It is important to note that for both sexes the most common quantity found for retirement pensions is from 650.01 to 700 euros. However, we can see as distribution for men shows a right asymmetry which does not appear in case of women. In this last case a higher concentration of values is observed in the left side of the distribution. There is a total difference of 1,307,504 pensions being granted to men over women, representing the 60.65% of the two sexes sum being granted for men.

Figure 5. Distribution of the men retirement pensions by amount intervals in Spain, March 2021. Own preparation with Seguridad Social data

Figure 6. Distribution of the women retirement pensions by amount intervals in Spain, March 2021. Own preparation with Seguridad Social data

When we observe the same distributions for widow pensions, behavior is the opposite, with a right asymmetry in the case of men that disappear for women (figures 7 and 8).

Figure 7. Distribution of the men widow pensions by amount intervals in Spain, March 2021. Own preparation with Seguridad Social data

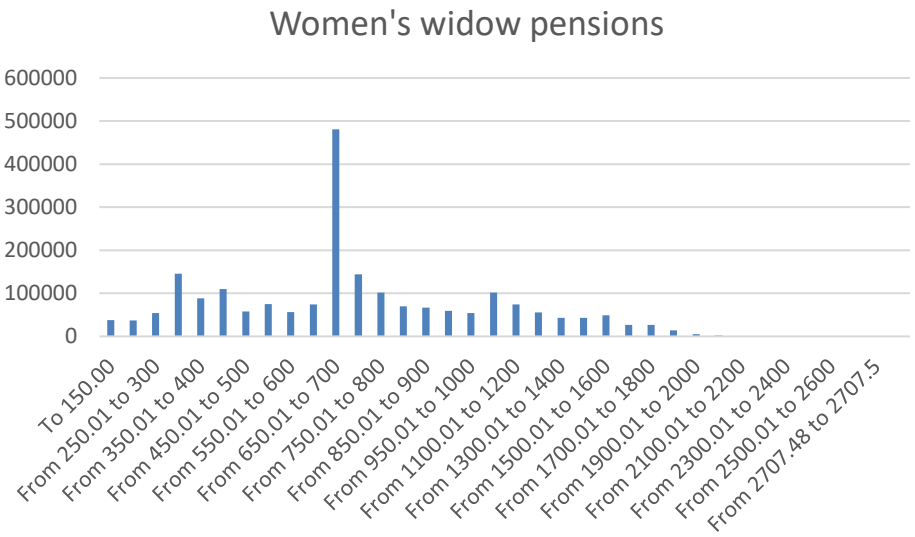

Figure 8. Distribution of the women widow pensions by amount intervals in Spain, March 2021. Own preparation with Seguridad Social data

In terms of widow pensions, we can clearly see the difference in numbers between sexes, being women granted a total of 2,158,661 pensions, which represents the 91.91% of the total.

The most common pension found in women ranges from 650.01 to 700 euros, and its men's counterpart ranges from 350.01 to 400 euros. Women have a total of 481,093 pensions in their most frequent value and men 42,397.

3.3 Concurrence of pensions by amount intervals

Moving on to the study of concurring pensions (figures 9 and 10), we find that women receive 744,442 concurrent pensions, whilst men receive 151,332 (over a total of 895,777). The most common concurrent pensions for women range from 650.01 to 700 euros, with near values in the next intervals, while for men the most found quantities range from 2707.48 to 2707.50 euros. We can see that women concentrate the higher number of both pensions. Doing the same calculations for non-concurring pensions, a total of 4,371,033 pensions are granted for men, while for women there are 3,520,104 pensions, representing a difference in favor of men of 850,929 pensions.

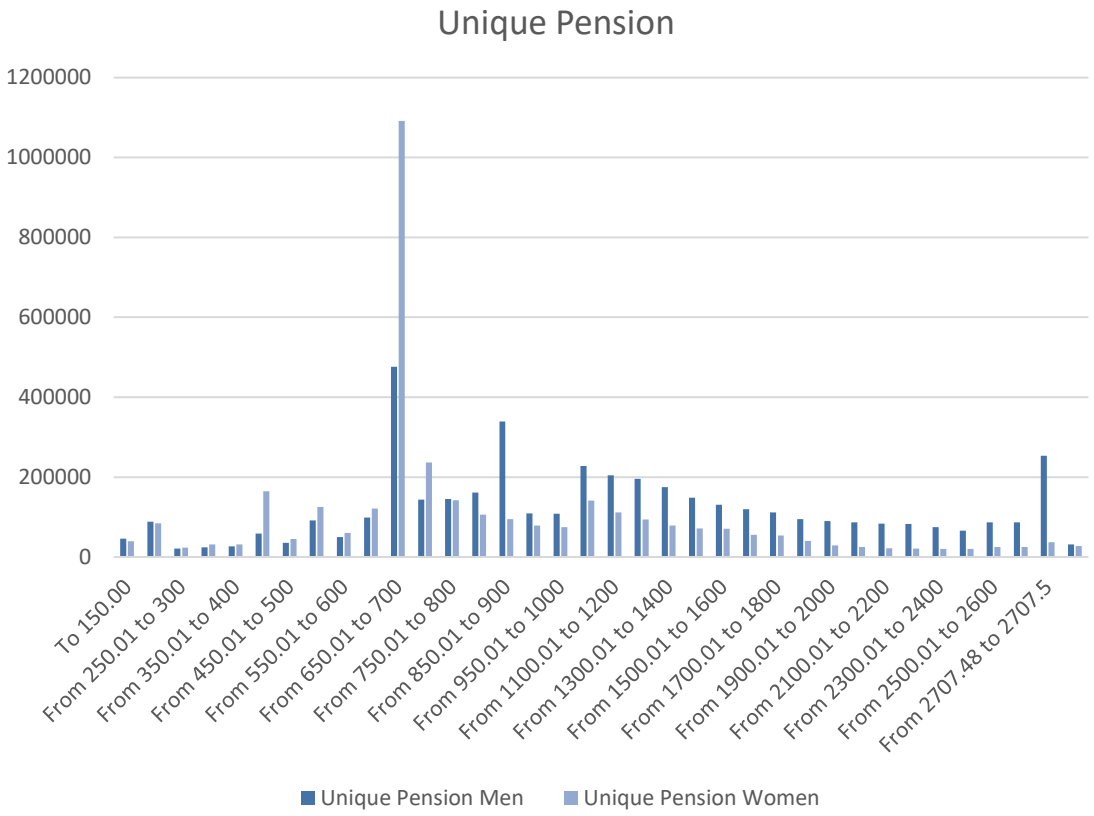

Figure 9. Distribution of pensioners with only one pension by amount intervals in Spain, March 2021. Own preparation with Seguridad Social data

Figure 10. Distribution of pensioners with more than one pension by amount intervals in Spain, March 2021. Own preparation with Seguridad Social data

4. Methodology

Our objective is to obtain joint survival probabilities until dissolution (death of the first member of the couple) and until extinction (death of the last survivor of the couple), both using a period approach and a cohort approach. We present in section 4.1 the traditional approach to obtain joint life expectancies as can be found in Ayuso et al (2007). The cohort methodological approach can be found fully detailed in Bravo et al. (2021). Our objective here is to present only a summary that helps to better understand the obtained results.

4.1 Modelling

Let $T_{\overline{xy}}$ be the random variable *Residual life of the group until the extinction*,[4] where *T(x)* and *T(y)* are the random variables residual life for the individuals aged *x* and *y*, respectively:

$$T_{\overline{xy}} = \max\big(T(x), T(y)\big).$$

Taking into account that the extinction equals the decease of both individuals, the distribution function of $T_{\overline{xy}}$, $F_{T_{\overline{xy}}}$, is:

[4] The extinction of the group happens when the last individual deceases, i.e., when the group disappears.

$$F_{T_{\overline{xy}}} = P\left(T_{\overline{xy}} \leq t\right) = {}_tq_{\overline{xy}} = {}_tq_x * {}_tq_y,$$

being the density function:

$$f_{T_{\overline{xy}}} = {}_tq_x * {}_tp_y * \mu(y+t) + {}_tq_y * {}_tp_x * \mu(x+t),$$

with ${}_tq_x$, ${}_tq_y$, the temporary probabilities of decease between *x* and *x+t*, *y* and *y+t*, respectively; being $\mu(x+t)$, $\mu(y+t)$ the force of mortality at ages *x* and *y*.

The joint life expectancy until the extinction is calculated as:

$$E\left[T_{\overline{xy}}\right] = \bar{e}_{\overline{xy}} = \int_0^\infty t * f_{T_{\overline{xy}}}(t)dt = \bar{e}_x + \bar{e}_y - \bar{e}_{xy} = \int_0^\infty \left({}_tp_x + {}_tp_y - {}_tp_{xy}\right)dt$$
$$= \int_0^\infty {}_tp_{\overline{xy}}dt.$$

The dissolution of the group happens at the first decease. If we want to center our attention exclusively in the retirement pensions (joint survivability for both individuals), we need to calculate the *Residual life of the group until the dissolution*, which in this case is the minimum between the random variable residual life for the individual aged x, and the same random variable for individual aged y,

$$T_{xy} = \min\left(T(x), T(y)\right),$$

being its distribution function,

$$F_{T_{xy}} = P\left(T_{xy} \leq t\right) = {}_tq_{xy} = {}_{1-t}p_{xy} = {}_tq_x + {}_tq_y - {}_tq_{\overline{xy}}$$

and its density function,

$$f_{T_{xy}} = {}_tp_{xy}[\mu(x+t) + \mu(y+t)]$$

being ${}_tq_{xy}$ the temporary dissolution probability for the group, ${}_tp_{xy}$ the temporary joint survivability probability, and $\mu(x+t) + \mu(y+t) = \mu(x+t, y+t)$ the joint instantaneous force of mortality.

The life expectancy until the dissolution is,

$$E\left[T_{xy}\right] = \bar{e}_{xy} = \int_0^\infty t * f_{T_{xy}}(t)dt = \int_0^\infty {}_tp_{xy}dt\text{x},$$

that we can approximate as (Ayuso et al, 2007):

$$\overset{\circ}{e}_{\overline{xy}} = \frac{1}{2} + \sum_{t=1}^{\infty} {}_tp_{\overline{xy}}$$
$$\overset{\circ}{e}_{xy} = \frac{1}{2} + \sum_{t=1}^{\infty} {}_tp_{xy}.$$

The difference between life expectancies until dissolution and life expectancies until extinction will give us the period where the widow pension will be paid to the survivor (widow pension plus retirement pension if there exists concurrence of pensions -the individual that remains alive also has the right for a retirement pension and this is not the maximum pension[5]-; or only the widow pension if the survivor does not previously work).

4.2 BME Estimation

To address the life expectancy gap between period and cohort estimates, we use the methodological approach presented in Ayuso et al. (2021) and Bravo et al. (2021) where the authors use a mixture of models in order to calculate probabilities by generations. Specifically they use an adaptive Bayesian Model Ensemble (BME), which is the assembling of different "*heterogeneous models, including six well-known parametric single population Generalized Age-Period-Cohort (GAPC) stochastic mortality models, one single-population univariate functional demographic timeseries model (the weighted Hyndman–Ullah method), one bivariate functional data model (the Regularized singular value decomposition model) and the recently proposed two-dimensional smooth constrained P-splines model, all of which can probabilistically contribute towards projecting future period and cohort life expectancy measures and the life expectancy gap*".

As explained in Ayuso et al. (2021) different single models have been normally used to estimate and forecast mortality, designating which model works best according to information criteria, assuming that the model used is the most suitable for each estimation. However, this can lead to a model risk problem, causing altered inferences. To solve this, the authors propose an adaptive Bayesian Model Ensemble (BME), which includes a combination of different models as we explained before. The Bayesian model ensemble is the application of the Bayesian theory to model selection under model uncertainty. It therefore conditions the inference on the entirety of the ensemble of models.

The BME used is based on the concept of model confidence set by Hansen et al. (2011) that establish the best models according to user-specified criteria. Any potential model with relevant information could be used in the ensemble, and its introduction should be in accordance with its different pros and cons, such as having to estimate another parameter/s. A brief summary of the aforementioned used models is presented below.

Generalized Age-Period-Cohort stochastic mortality models

These models are parametric models that link a response variable with a linear predictor structure that consists of different factors that are dependent on age x; period effects, t; and year of birth effects, $c = t - x$. Included in the generalized nonlinear models group, they include a random component, a link function, a set of parameter constraints, a systematic component and time series methods.

[5] Note that the maximum for concurrence of pensions is determined by the maximum retirement pension.

Models used by Bravo et al. (2021) are the following ones:

LC: $\eta_{x,t} = \alpha_x + \beta_x * \kappa_t$

APC: $\eta_{x,t} = \alpha_x + \kappa_t + \gamma_{t-x}$

RH: $\eta_{x,t} = \alpha_x + \beta_x * \kappa_t + \beta_x * \gamma_{t-x}$

CBD: $\eta_{x,t} = \kappa_t + (x - \bar{x}) * \kappa_t$

M7: $\eta_{x,t} = \kappa_t + (x - \bar{x}) * \kappa_t + ((x - \bar{x})^2 - \sigma) * \kappa_t + \gamma_{t-x}$

Plat: $\eta_{x,t} = \alpha_x + \kappa_t + (x - \bar{x}) * \kappa_t + (\bar{x} - x)^+ + \gamma_{t-x}$.

In their work, the parameter estimates are obtained using maximum-likelihood. The ages that the authors used range from 60 to 95 years old, utilizing the country population with data from 1960 to the most recent data. Mortality rates are forecasted assuming that the age vectors α_x and β_x are constant and period indices κ_t follows a multivariate random walk with drift. General univariate ARIMA (p, d, q) models with drift were used to model the cohort indices γ_{t-x}. To estimate the better ARIMA model, the Box-Jenkins methodology was used. The ultimate age was proposed at $\omega = 125$.

Single-Population Functional time series models:

In this case the authors used the method proposed by Hyndman and Ullah (2007) which combines a nonparametric penalized regression spline with a functional principal component analysis to model and forecast log mortality rates. The authors used an extension of this model, which is the weighted Hyndman-Ullah, proposed by Shang et al. (2011) which uses geometrically decaying weights to estimate the parameters, allowing the most recent data to have more importance than the past data.

Two-dimensional smooth constrained P-splines

This method is based on the P-splines proposed by Currie et al. (2004) and Currie (2006) as a method of smoothing for generalized linear models with Poisson errors, which treats as a missing value the forecast of mortality rates. This model is known for a good accuracy in the *in-sample* forecasts but a rather low *out of sample* forecasting accuracy. The authors also considered an enhanced method by Camarda (2019) which incorporates some demographic constraints which corrects the mentioned low accuracy on forecasting future values of mortality by following a correct demographic profile.

Regularized SVD model

This method is based on Zhang et al. (2013) and Huang et al. (2009) which extended an one way functional principal component analysis into a two way functional data by regularizing right and left singular vectors in the singular value decomposition. The basis is the minimization of a regularized sum of squared reconstruction errors.

For a more detailed explanation of the used models please read the original references Ayuso et al (2021) and Bravo et al (2021).

4.3 Gap in model estimation

Following Ayuso et al (2021), but now in the context of joint probabilities, we define the *Life Expectancy (LE) gap* as:

$$LEgap=\bar{e}_{xy}^{C} - \bar{e}_{xy}^{P}$$

with $\bar{e}_{xy}^{C}$ the joint life expectancy until dissolution obtained by the BME model (cohort method) and $\bar{e}_{xy}^{P}$ the joint life expectancy until dissolution obtained by the traditional period method.

5. Results

In this section we talk about how the different methodologies used for calculating the life expectancy can affect the expected household wealth and which could be the implications for the pension system.

5.1 Age Gap

An important part of the expected household wealth is related to the dissolution of the married couple, caused by the decease of an individual, and death is correlated with age. Therefore, to properly estimate this household wealth we need to know the age of the couple. To obtain a representative value on the couple's age, we retrieved data from INE related to the Spanish marriages from 1976 to 2019 and calculate the age gap between the two members of the couple. The mean age gap is presented in figure 11, while some others statistical measures as the median and quartiles are presented in table 5 for further research.

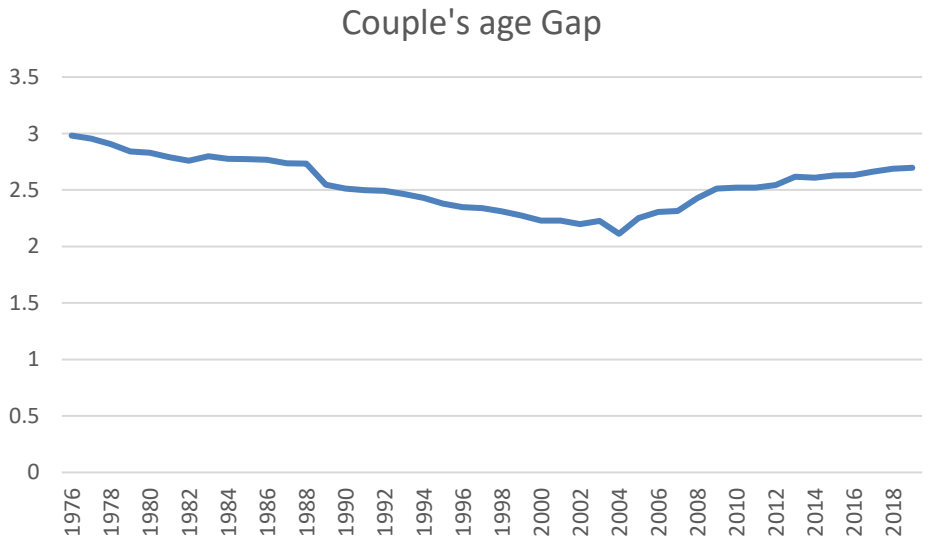

Figure 11. Age gap between men and women in marriages, Spain. From 1976 to 2019. Anonymized Marriages Series (Spanish Statistical Institute, INE).

1980		1990		2000		2010		2019	
Min	-58	**Min**	-67	**Min**	-69	**Min**	-55	**Min**	-46
1 quart	1	**1 quart**	0	**1 quart**	0	**1 quart**	0	**1 quart**	0
Median	3	**Median**	2	**Median**	2	**Median**	2	**Median**	2
Mean	2.83	**Mean**	2.51	**Mean**	2.23	**Mean**	2.52	**Mean**	2.70
3 quart	5	**3 quart**	4	**3 quart**	4	**3 quart**	5	**3 quart**	5
Max	56	**Max**	63	**Max**	55	**Max**	53	**Max**	64

Table 5. Descriptive statistics for the age gap between men and women, Spain. From 1980 to 2019. Anonymized Marriages Series (INE).

Figure 11 must be read as the difference between the men's age and the women's age in a couple that married on the year; hence, if the number is positive, it means that the male is older than the female. This figure shows us the evolution of the mean of all the couples married in every year from 1976 to 2019 in Spain. We can clearly see a descending tread in the period from 1976 to 2004, followed by an ascending tread until the last observed year. If we take a closer look at the data from 2019, we can observe that the mean of that year was 2.6981, which means that, in mean, men that married women in 2019 were a bit more than two and a half years older than the women they married. This has relevant implications in the study and estimation of the aggregated value of the pensions received, since it is commonly known that the women life expectancy is higher than the men life expectancy. This adds to the fact that women are retiring later than their couple, meaning that if men are expected to live less than women, we are adding up years of difference in the widow's pension.

Once this age gap has been established, we can proceed to calculate the expected household wealth by intertwining the age gap in a married couple, the pension's amount (retirement and survivor pensions), and the life expectancy (cohort versus period), calculated both as a couple and individually.

5.2 Gap in the model estimation: cohort versus period approach

Individual and joint life expectancies until dissolution and extinction have been calculated according to methodology presented in section 4. Firstly, we present results obtained applying a period approach (tables 6a and 6b). Secondly, the process is repeated using the BME approach (tables 7a and 7b). The gap between both methods is presented in table 8. By simplicity, we only present results for couples aged (69, 66), (74, 71), (79, 76) and (84, 81), being the first number that related to the man's age (the second, the woman's age).

Joint life expectancies estimates (period approach)

Age		Life expectancy		Life annuity	
Husband	**Wife**	Joint	Last	Joint	Last
69	**66**	14.35	24.71	13.85	24.21
74	**71**	10.91	20.12	10.41	19.62
79	**76**	7.87	15.74	7.37	15.24
84	**81**	5.36	11.74	4.86	11.24

Table 6a. Joint life expectancies until dissolution and extinction -period approach- Own elaboration with INE data.

Individual life expectancies estimates (period approach)

Husband		Wife	
Age	Life expectancy	**Age**	Life expectancy
83	7.28	**80**	11.05
85	6.30	**82**	9.65
87	5.41	**84**	8.35
89	4.62	**86**	7.17

Table 6b. Individual life expectancies -period approach- Own elaboration with INE data.

Multiple life survival estimates (cohort approach)

Age		Life expectancy		Life annuity	
Husband	**Wife**	Joint	Last	Joint	Last
69	**66**	15.89	27.15	15.39	26.65
74	**71**	12.40	22.51	11.90	22.01
79	**76**	9.21	18.02	8.71	17.52
84	**81**	6.49	13.82	5.99	13.32

Table 7a. *Joint* life expectancies *until dissolution and extinction* -cohort approach- Own elaboration *with INE data*.

Individual life expectancies estimates (cohort approach)

Husband		Wife	
Age	Life expectancy	**Age**	Life expectancy
85	6.49	**82**	10.12
86	6.01	**83**	9.40
88	5.12	**85**	8.07
90	4.34	**87**	6.88

Table 7b. Individual life expectancies -cohort approach- Own elaboration with INE data.

The next table shows the gap found between both approaches:

Multiple life survival estimates

Age		Life expectancy		Life annuity	
Husband	**Wife**	Joint	Last	Joint	Last
69	**66**	1.54	2.44	1.54	2.44
74	**71**	1.48	2.39	1.48	2.39
79	**76**	1.34	2.28	1.34	2.28
84	**81**	1.13	2.08	1.13	2.08

Table 8. Joint life expectancies gap until dissolution and extinction. Own elaboration with INE data.

It is important to note that we find a longer longevity of near one and a half years in the cohort approach in comparison to the period approach. This difference is reduced the greater the ages are observed. This clearly shows that the cohort approach takes into account the increasing longevity observed over the last years.

5.3 Expected household wealth based on pensions

Taking into account the observed differences in life expectancies depending on the model used for estimation (cohort versus period) we can further observe them and try to analyze as the expected household wealth will change from the retirement age of the youngest member of the couple until extinction. Some hypotheses have been established to simplify calculations:

- The annual revalorization index of pensions (both retirement and survival) is equal to the discount interest rate.
- A round approximation to the entire ages has been used in the different scenarios.

The household used for the different calculations is composed by a married couple, man and woman, being the woman 3 years younger than the man, as the round approximation to the observed mean ages gap presented in figure 11.

Our starting point is a couple aged 69 years for the man and 66 for the woman, showing the obtained results in a successive 5 years increasing aged couple. Then, according to the model used, the group is maintained until dissolution, and after that two outcomes may occur, depending on who is the first deceased individual. For Scenario A we suppose that the man is the first person who deceased, therefore the woman will be granted concurrent pensions for the remainder of her life expectancy. For Scenario B, the contrary happens, the woman is the first to be deceased whilst the man will be granted concurrent pensions.

In terms of the amount of received pensions, in subsection 5.3.1 the means for the year 2019 have been used, being the retirement pension amount 1312.42 € for men and 858.21 € for women, and the widow pensions 508.56 € for men and 729.11 € for women, respectively. In subsection 5.3.2 the maximum amounts per pensions have been used and in subsection 5.3.3 a similar procedure has been done by using minimum quantities.

5.3.1 Expected household income using mean pension amounts

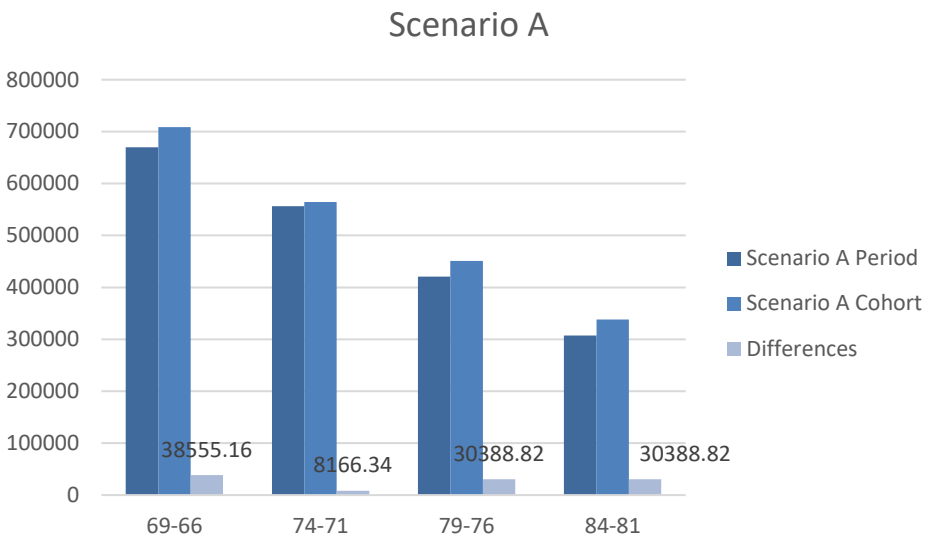

Figure 12. Expected household income for Scenario A – man dies before woman- with mean pension amounts, difference between period and cohort methods. Own preparation.

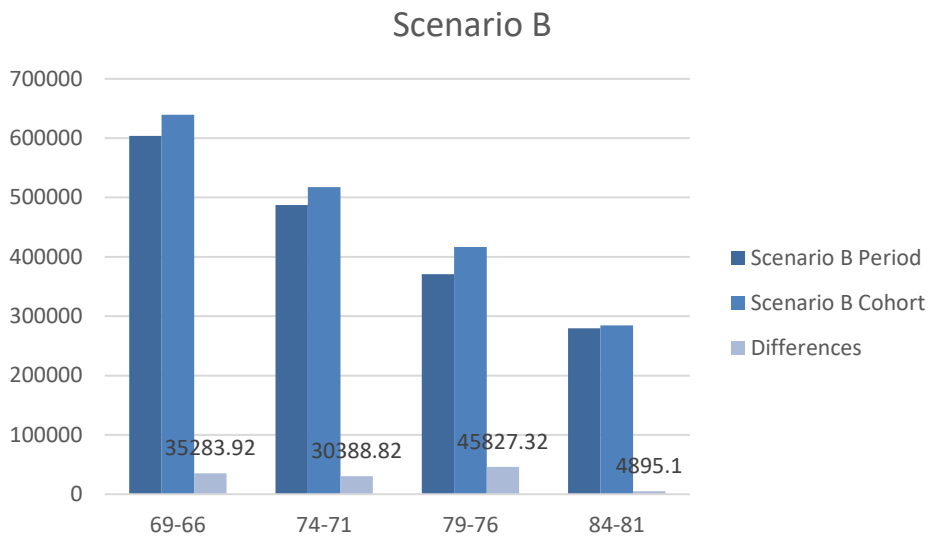

Figure 13. Expected household income for Scenario B – woman dies before man- with mean pension amounts, difference between period and cohort methods. Own preparation.

A first impression that we can take from the results is that the wealth obtained in a household is always higher in the cohort approach, and if we take further ages, the difference between estimations is reduced, according to small differences between period and cohort estimates (see subsection 5.2).[6]

[6] The scenarios include ages until 84-81 years old, in which no further estimations are done. The smaller difference between approaches in the subsequent ages made further estimations unnecessary.

For all scenarios, when the survivor is the woman, the expected household income will be higher. This phenomenon can be explained by the fact that women are expected to live longer, and their longer lives compensate differences in their retirement pension's amount that are higher when the widow's pension is aggregated.

5.3.2 Expected household income using maximum pension amounts

The same approach has been done using the maximum pension amounts, the same for both men and women. The objective is to analyze how high pension amounts affect the two proposed scenarios (figures 14 and 15).

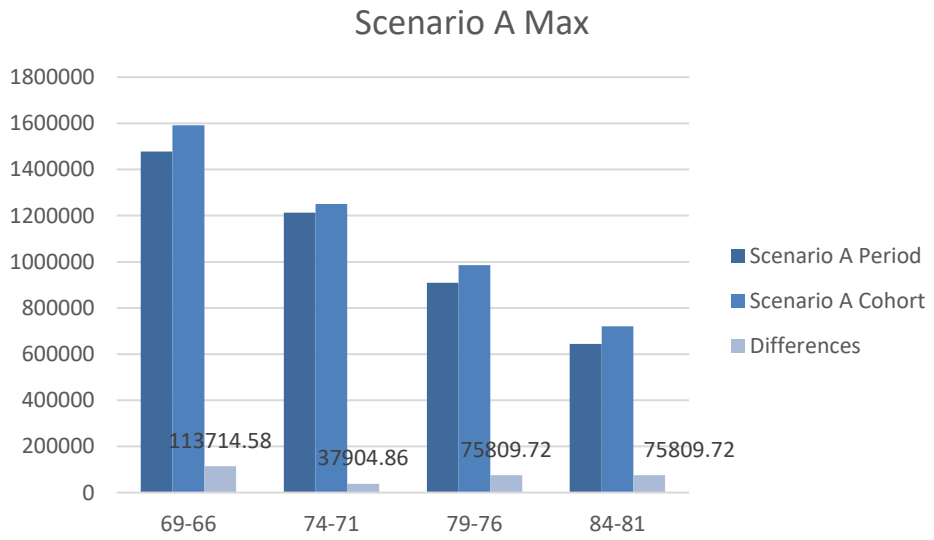

Figure 14. Expected household income for Scenario A – man dies before woman- with maximum pension amounts, difference between period and cohort methods. Own preparation.

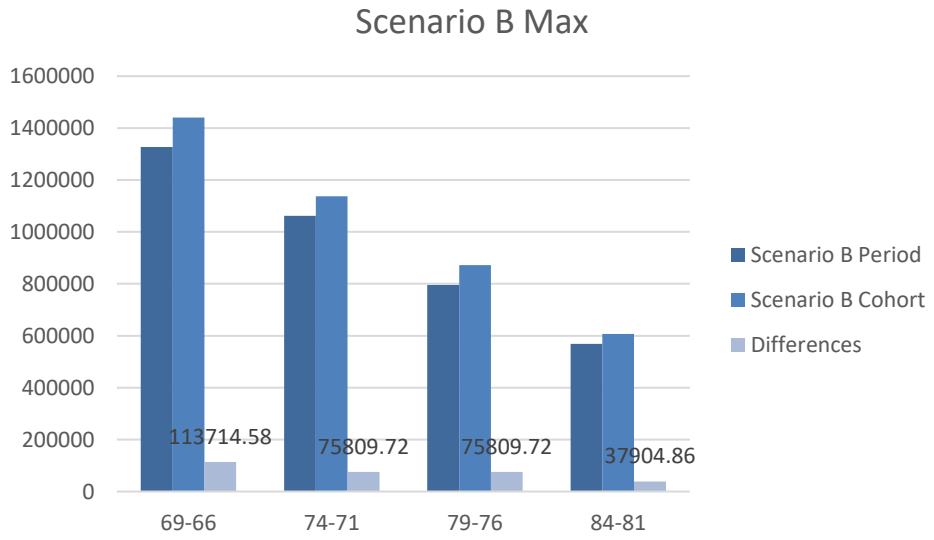

Figure 15. Expected household income for Scenario B – woman dies before man- with maximum pension amounts, difference between period and cohort methods. Own preparation.

In 2019, the maximum pension that can be granted to an individual in a single year amounts to 37,904.86 €. That means that if a pensioner is perceiving this maximum

amount for his/her retirement pension, he/she cannot obtain any payment in case of widowhood.

We can observe a similar trend in terms of the sign of the difference when compared with results obtained in section 5.3.1, showing that the wealth accumulated is higher when the cohort method is used in all the cases. It is important to note the increased difference between period and cohort methods for both scenarios, calculated with the maximum pension amount, specially in the first and second couple's intervals, for ages 69-66 and 74-71.

The wealth accumulated is higher than in the mean amount case, despite not being able to benefit from the widow's pension because they are percceiving the maximum retirement pension by law.[7]

5.3.3 Expected household income using minimum pension amounts

Finally, we take into account that both pensioners will receive only the minimum pension (figures 16 and 17), being the minimum retirement pension established at 9164.4 € per year, whilst the minimum widow pension amount is at 9655.8 € per year. This might look at a first glance as a paradox, since the widow pension is a percentage of the regulatory base but, as it is written by law, the stablished number is the minimum henceforth the minimum widow pension amount is higher than the minimum retirement pension amount.

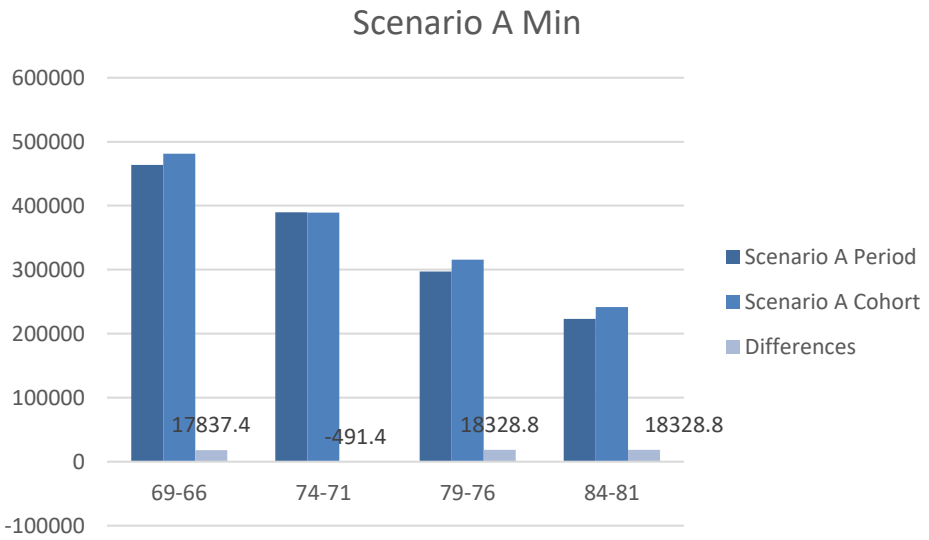

Figure 16. Expected household income for Scenario A – man dies before woman- with minimum pension amounts, difference between period and cohort methods. Own preparation.

[7] A special mention can be done for the 79-76 age interval which we observe that the expected household wealth, in spite of having different absolute values in both A and B scenarios, share the same difference between period and cohort approach, which is 75,809.72 €. If we take a closer look into that specific case, we can see that, for instance, if we suppose that the man was the first to be deceased, the couple would be together for 8 years, period case, and 9 years, cohort case. And after the dissolution of the group, in both approaches the woman lives 8 years. For the scenario B, the group dissolves at 8 years for period, and 9 for cohort, while the man lives for another 5 years in period and also 5 in cohort. Therefore, we can observe that for both cases the global difference is that a pension is being paid one more year for the cohort case.

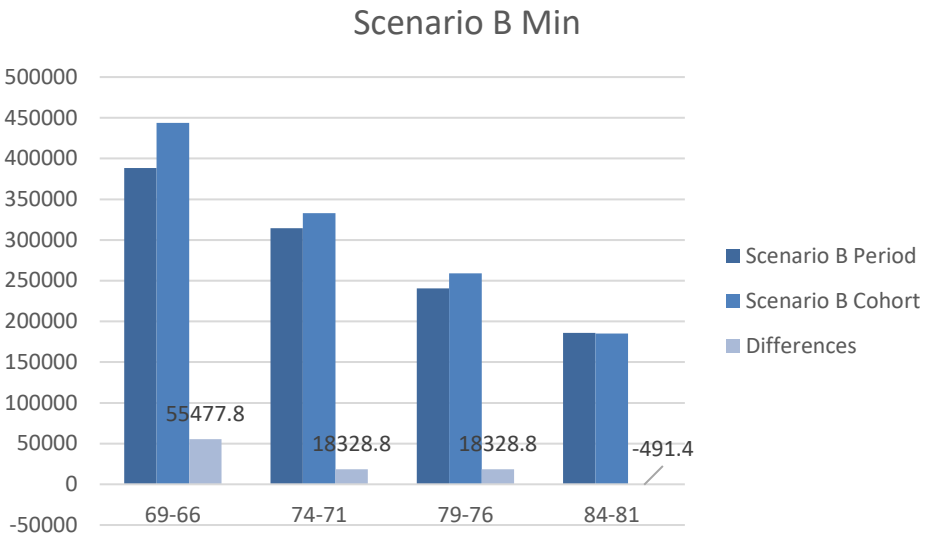

Figure 17. Expected household income for Scenario B – woman dies before man- with minimum pension amounts, difference between period and cohort methods. Own preparation.

Taking a closer look at the scenarios, we can observe that the trend is the same as stated before. The difference between period and cohort approach is reduced at the higher ages, and also we find the same different result for Scenario A, at 74-71 ages, with even a negative difference. It is also notable the negative difference found in the 84-81 ages couples for Scenario B, in which the group dissolves at 5 years for the period approach and 6 for the cohort approach, yet the man is expected to live a year longer in the period approach, 5 against the 4 years expected by the cohort approach, resulting in a negative difference of 491.4 €.

6. Discussion

We can observe that public pensions can suppose a difficult topic in terms of their calculation, their different implications and what is their purpose.

First of all, there is not a clear methodology to follow in order to forecast the possible expenses in public pensions, since there are multiple variables that take part in this matter and they have in themselves multiple calculations that can be approached in different ways, such us in the case of the period – cohort approximation.

It is clear that utilizing one or the other can impact in the final result, which has serious implications for the forecasting of the expected expenses in public pensions, which are nowadays in the focal point of numerous debates and the daily life of politics.

When an estimation is made, there is an expectation that this calculated number can grasp, or at least approach, the real number in question. Yet problems may arise when different methods give different estimations, making the decider take action in selecting the best method available. And in these cases, where there is not a clear best method, some risks are to be taken, adopting a posture on the matter. It's for this reason that the ensemble of models is nowadays taking a lot of relevance with the aim of minimizing the risk of model and parameters.

An important matter in the public pension conversation is also how are the systems designed. Some of the public pension systems were thought a lot of years ago, in a

moment when the woman was not in the labor market. Years have passed and some of these systems have shown near to no flexibility in this matter, as can be seen in the calculations made.

The widow pension was thought in a matter of helping the woman who was not granted a retirement pension, in order to have a sufficient income to fulfill her daily necessities. And even though times are changing, the system is not adapting to this changing society we are currently living in.

Some important questions arise as a result of the research carried out. A matter to be discussed is the possible inequalities between married households and individual households, putting on the table the fact that the system cannot sustain some of these society changes whilst attaining its stablished bases, such us equality and sustainability.

As seen in the paper, a household formed by a married couple, having the man 69 years old, and the woman 66 years old, both perceiving mean retirement pensions, in the case of the dissolution of the couple caused by the decease of the man, the household is expected to have accumulated a total amount of 708446 € in a total of 26 years since the aforementioned ages until extinction; if it is the wife who firstly died, this amount would be 639183 €; a 9.8% less.

As the aforementioned example, if the maximum amount pension were to be taken, the expected total accumulated amount would be 1592004 €; if the wife were to be who firstly died, this amount would be 1440385 €, a 9.5% less.

If we did the same with the minimum pension amount, the expected total accumulated amount would be 481463 €; if the wife were to be who firstly died, this amount would be 443822 €, a 7.8% less.

Future lines of research are related to the analysis of impact of different couple compositions, mainly by quartiles of pension amount. Relationships with other variables, as education, profession or household wealth (not only by pensions) will be welcomed.

7. Bibliography

Alaminos, E. (2017). Heterogeneidad en la mortalidad y su impacto en el estado del bienestar: pensiones y dependencia. *PhD Thesis*. Universitat de Barcelona.

Alaminos, E., Ayuso, M (2015). Una estimación actuarial del coste individual de las pensiones de jubilación y viudedad: Concurrencia de pensiones del sistema de la seguridad social español. *Estudios de Economía Aplicada*. 33-3, 817-838.

Alaminos, E., Ayuso, M. (2016). Modelo actuarial multiestado para el cálculo de probabilidades de supervivencia y fallecimiento según estado civil: Una aplicación al pago de pensiones concurrentes. *Anales del Instituto de Actuarios españoles*. 22, 41-71.

Alaminos, E., Ayuso, M. (2019). Marital status, gender, mortality, and pensions: the disadvantages of being single in old age. *Revista Española de Investigaciones Sociológicas*. 165, 3-24 http://dx.doi.org/10.5477/cis/reis.165.3.

Angelini, V., Brugiavini, A., Weber, G. (2009). Ageing and unused capacity in Europe: is there an early retirement trap? *Economic Policy*. 24 (59), 463-508.

Ayuso, M., Bravo, J.M., Holzmann, R. (2016). On the heterogeneity in longevity among socioeconomic groups: scope, trends, and implications for earnings-related pension schemes. *IZA Discussion Paper* No. 10060. Institute of Labor Economics. Bonn (Germany).

Ayuso, M., Bravo, J., Holzmann, R. (2021). Getting life expectancy estimates right for pension policy: period versus cohort approach. *Journal of Pension Economics and Finance*. 20, 212-231.

Ayuso, M., Chuliá, E. (2018). Factores sociodemográficos asociados al ahorro a largo plazo: análisis mediante regresión logística. *Documentos de Trabajo Instituto BBVA de Pensiones*. 24, 1-19.

Ayuso, M., Corrales, H., Guillén, M., Pérez-Marín, A. M., Rojo, J. L. (2007). *Estadística Actuarial Vida*, Publicacions i Edicions de la Universitat de Barcelona.

BOE (2003), Informe de la Comisión no permanente para la valoración de los resultados obtenidos por la aplicación de las recomendaciones del Pacto de Toledo. Texto del Informe y votos particulares. BOE October 2 2003. Serie D 596.

BOCG (2020), Boletín Oficial de las Cortes Generales. November 10 2020 Serie D Núm 175. 59-78.

Bravo, J., Ayuso, M., Holzmann, R., Palmer, E. (2021). Addressing the life expectancy gap in pension policy. *Insurance: Mathematics and Economics.* 99, 200-221.

Camarda, C.G. (2019). Smooth constrained mortality forecasting. *Demographic Research*. 41 (38), 1091–1130.

CERP Collegio Carlo Alberto (2008), Description of the legal-institutional framework of the Italian pension system, CERP Collegio Carlo Alberto 2008, Annex 2.

Currie, I. (2006). Smoothing and forecasting mortality rates with p-splines. DP Heriot Watt University.

Currie, I., Durban, M., Eilers, P. (2004). Smoothing and forecasting mortality rates. *Stat. Model*. 4, 279–298.

European Commission, Survivor's benefits, moving & working in Europe, Employment, Social Affairs & Inclusion, European Commission. Moving & working in Europe - Employment, Social Affairs & Inclusion - European Commission (europa.eu).

Fuster, L. (2021). Las pensiones de viudedad en España. *Estudios sobre la economía española*. 2021/06. Fedea.

Hansen, P., Lunde, A., Nason, J. (2011). The model confidence set. *Econometrica*. 79, 453–497.

Holzmann, R., Ayuso, M., Alaminos, E., Bravo, J. (2019). Life cycle saving and dissaving revisited across three-tiered incomegroups: starting hypotheses, refinement through literature review, and ideas for empirical testing. *IZA Discussion Papers Series* n.12655. Institute of Labor Economics, Bonn (Germany).

Huang, J.Z., Shen, H., Buja, A. (2009). The analysis of two-way functional data using two-way regularized singular value decompositions. *Journal of the American Statistical Association.* 104 (488), 1609–1620.

Jacinto, L., Hombrados-Mendieta, I. (2011). Análisis evolutivo de la diferencia de edad en la elección de pareja de los matrimonios celebrados en España durante el período 1976-2006. *Revista de Psicología Social*. 26. 73-89.

Ministerio de Trabajo e Inmigración (2011), Informe de Evaluación y Reforma del Pacto de Toledo 2011, *Colección Seguridad Social*, Ministerio de Trabajo e Inmigración.

OECD (2018), OECD Pensions Outlook 2018, Chapter 7, OECD Publishing, Paris, https://doi.org/10.1787/pens_outlook-2018-en.

OECD (2019), Pensions at a Glance 2019: OECD and G20 Indicators, OECD Publishing, Paris, https://doi.org/10.1787/b6d3dcfc-en.

Zhang, L., Shen, H., Huang, J.Z. (2013). Robust regularized singular value decomposition with application to mortality data. *Ann. Appl. Stat.* 7 (3), 1540–1561.