The role of heterogeneity in group cooperation

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Abstract

Maintaining cooperation in large societies has been a challenging puzzle to economists for decades. Some of the concerning problems our society is facing nowadays like global warming or air pollution stem from the existence of weak local and global cooperation. New policies to promote cooperation involve social mechanisms to reinforce the behaviours needed to extend the public goods. These mechanisms locally address a wide variety of externality problems. However, the increasingly complex interactions in society pose a new challenge. This work aims at contributing to address such a challenge by exploring the effect of peer pressure over individual decisions in social networks. Specifically, based on previous models, we analyse the role played by degree heterogeneity (i.e. diversity of individuals' number of peers) on PGG scenarios with peer-pressure. Those considerations emphasize how network position causes sensitiveness in the capacity of decision of the agents. Specifically, pressure cost unequally harms the agents in the network, negatively affecting the highly connected ones. This gap in behaviour between highly connected agents and low-connected agents do not favour the model cohesiveness. Our results suggest that heterogeneous connectivity must be considered when designing efficient and egalitarian policy tools to solve problems of externalities. These findings encourage future structural considerations in PGG with peer-pressure. (JEL: C71, H41, D62, D85)

Keywords: Public Good Games, peer-pressure, cooperation, social networks, social diversity.

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1. Introduction and background

Self-interest behaviour can lead to abusive consumption of common good resources negatively affecting society. This is well known as the tragedy of the commons. Passive smoking, air pollution or the use of common-pool resources (e.g., a lake, forest, or road) are all examples of negative externality creation produced by the aggregate actions of all agents. The issue is that society bears the cost. Consequently, since individuals share the externality triggered by their excessive consumption, incentives to cooperate are low. Those behaviours lead to the depletion of the common resources, worsening the total social welfare.

Most common policies to address this problem involve punishment and Pigouvian taxation or subsidies. Punishment adoption is an effective mechanism to ensure cooperation in public goods interactions. It reduces the defector's payoff in populations where free-riding individuals do not contribute (Hauert, C. et al., 2006). However, the cost of punishment usually outweighs the welfare obtained through cooperation (Puurtinen, M. and Mappes, T., 2009). Institutions regularly consider Pigouvian taxation or subsidies as an alternative. Taxation is introduced to discourage activities that impose adverse effects onto third parties. One example would be the carbon tax applied in many cities to make visible the hidden social cost of carbon emissions. On the other hand, Pigouvian subsidies are introduced to encourage activities involving positive externalities. For example, subsidies for recycling.

Nevertheless, both enforcement policies result imprecise in heterogeneous societies (Bicchieri and Dimant, 2019; Bramoullé et al., 2020) due to two main reasons. First, the existence of significant transaction costs and secondly the homogeneity of decisions they assume. Commonly both approaches wrongly assume that decisions do not depend on peer's actions. But not all behave the same and peers' behaviour must be considered. Social class, age, ethnicity, or other population differences causes heterogeneity in social structures. This accentuates the externality effects some individuals may experience. Some agents may try to reduce the negative externalities created and care about the action their peers undertake, while others not. Peer effects has been studied carefully to identify policies to correct those negative externalities. They allow to focus on the individual action rather than in the whole population action, creating higher efficiencies in policy making. By studying peer effects, the interaction between individual's patterns of behaviour is better understood.

In this context peer effects, as a support for Pigouvian taxation, have also gained strength (Huang, L. and Xiao, E., 2021). But in many cases Pigouvian taxation and subsidies overestimate the external cost involved reducing their effectiveness. Alternative approaches introduce social mechanisms based on peer effects to encourage pro-social behaviours. This method reduces the overestimation created under the Pigouvian mechanism and allows greater concretion. Among those mechanisms, the study of peer-pressure as a mechanism in social networks is of considerable interest.

The idea is to reduce the actions that account for the existing externality via peer-pressure. Broadly speaking, peer-pressure can be defined as the pressure to behave along certain peer-prescribed guidelines (Clasen R. and Brown B., 1985). An agent exerts some pressure over his or her peers with the aim to alter their actions, modifying the final externalities. For example, an agent may reduce its fuel consumption if relatives and friends tell him to use public transport. If everyone endeavours to raise awareness of the problem, promoting a more sustainable consumption, these problems could be solved.

On this basis, our interest resides on the impact of peer-pressure on public good consumption as an action of personal and social benefits. Peer pressure positively affects reciprocity when all agents can affect the other's payoff (Mittone, L. and Ploner, M., 2011). Therefore, peer pressure is closely related to the ties formed between individuals. Actor's pressure directly affects their closest peers. Those more distant individuals are affected through the social welfare but not directly by the actor's decision. This scenario illustrates the importance of immediate interactions. In this sense, social network structures require a deep study since connectivity plays a central role in the pressure exerted by an actor and the final externalities. Externalities involved end up being the fundamental reason to care about the social structure (Jackson, M. et al., 2016).

The study of networks using games theoretic models has stood out as the main way to understand social interactions and how the payoff an individual receives depends on the actions his or her neighbours take (Bala, V. and Goyal, S., 2000; Bloch, F. and Jackson, M., 2007; Acemoglu, D. et al., 2010). There is a growing evidence about how powerful social influences are and their use in games theory. Dawes (1980) studies norm compliance under human behaviour interaction reformulated as a prisoner's dilemma game (PDG). Other authors, compiled evidence to demonstrate "the tragedy of the commons" can be formulated as prisoner's dilemma game, since over-consumption is of the interest of every agent but also decisive in the maintenance of social welfare (Kareva, I. et al., 2013). Additionally, a considerable body of empirical evidence in evolutionary theory reveals that social behaviour can be affected by the actions of the neighbours. Chao and Elena (2017) analyse the robustness of the social mechanism against the Pigouvian one in two evolutionary games based on the PDG. Social mechanisms exhibit some advantages with respect to Pigouvian taxes. However, stability strongly depends on the relationship between peer pressure and the advantage of defectors to cooperate. He et al. (2020) identifies a convergence in actor's decisions under an increasing and unbounded peer pressure space. Sharing a common fate in the interaction seems to foster peer pressure among agents (Mittone, L. and Ploner, M., 2011). Those facts suggest that actor's decisions can be strongly conditioned by their peer's action.

This paper aims to analytically explore how heterogeneity affects social interactions on scenarios described by Public Goods Games (PGG) with peer-pressure. In particular, we are interested in the effect of degree heterogeneity (i.e. diversity on the number of peers each individual is connected to). The starting point of our research is an existing PGG model introduced by Mani et al. (2013). We selected this model because it integrates peer-pressure through social networks in a quite realistic way. The authors illustrate a new social mechanism for policy makers to face the problem created by externalities in a connected society. This model focuses on the actor's peers in the social network, giving incentives to exert pressure over them. Beyond considering a binomial scenario (as usually considered in other models), the selected model deems continuous action and pressure sets. This implies a wider definition of the pressure effects. In addition, the model provides a more realistic framework with a more interesting approach, since cooperation dynamics in networked population are localized. Every actor can observe the action their peers execute for a given pressure level. Contrary to punishment methods this model locally addresses the problem of externalities. Nevertheless, the authors only tested the model under homogeneity assumption, and opens a perfect work line for us to address our research goals.

As in the selected model, simple network structures are commonly articulated in the literature on PGG. This results in a minimisation of the influence that agent's interaction has for explaining the social cooperativeness. We posit that some network positions exhibit strategic advantages over others as Markovsky et al. (1988), showed in "Power Relations in Exchange Networks". To understand the implications agents' interactions have, we embark on an in-depth study of network structures. Our work concentrates on heterogeneous structures to disentangle the underlying consequences connectivity has. This extends previous research in PGG, providing new insights into the cooperativeness processes.

In network theory connectivity patterns are graphically represented by a set of nodes, which represent the agents in the network, and the edges, placing the potential contacts along which the pressure spreads. In the last two decades the network structure of a large range of natural and man-made system were studied. Structures as the Internet (Pastor-Satorras R. and Vespignani, A., 2004), collaboration networks (Newman, M. E., 2001; Barabási, A., et al., 2002), the World-Wide Web (Huberman, B., et al., 2000) webs of sexual contacts (Liljeros, F., et al., 2001) and others were represented by networked structures. A relevant range of literature in complex networks (Barabási A. L. and Albert R., 2002; Dorogovtsev, S., et al., 2003) has uncovered similar characteristics in many of the mentioned structures. The most remarkable common characteristic is the presence of a fat-tailed degree distribution $P(k) \sim k^{-\gamma}$. The component γ usually comprises a range interval $\gamma \in [2, 3]$ (Dorogovtsev, S and Mendes, J, 2002) imposing the existence of heterogeneity in the population structure. This peculiarity of scale free networks (SF) refers to the distribution principle of how many edges equal to the integer k are per node. The existence of edges conditions whether individuals can or cannot exert pressure among each other. Using this topology, we can deal with a heterogeneous hierarchy of vertices closest to real life, expanding both breadth and depth of our study in peer interaction.

Generally, interactions are marked not only by the number of connections an agent has, the degree distribution, but also by the interrelationship with other's contacts, degree correlation. Drawing on the available literature most studies has only focused on the first, however social networks are strongly characterised by the existence of degree correlation (Krapivsky P. and Redner, S., 2001). This means that the degree of any vertex is not independent. On SF populations, most of the agents have a low number of connections, whereas a low number of highly connected agents guarantee the total connectivity of the whole population. The network displays a hub and-spoke character in those cases (Barabási, A. L., 2014), showing the disassortative structure we are interested on. An actor linked to a high-connected actor in the network may manifests remarkable differences with respect to one connected to a low-connected actor. This importantly identifies patterns presented in heterogeneous society that we consider decisive in our study of the chosen model in PGG. Those structures possess the appropriate features to uphold our insights.

The main challenge one faces in studying strategic interaction is the intrinsic complexity of heterogeneous networks. Without focusing on a concrete structure in terms of PGG, it is difficult to point out any conclusion. To capture the implications heterogeneous structures have on PGG, we disentangle the analytical method development outlined in the model by Mani et al. This thesis aims at performing an in-depth, detail examination of the analytical procedure in combination with a numerical analysis as it is usual in network sciences. We firstly study the effect of node degree on pressure profiles based on the analytical approach of the model. As a consequence, we observe a significative impact associated with degree heterogeneity. Exploring how degree disparities perform in the model, we find that not only degree heterogeneity affects the model, but also negative degree-correlation creates inequalities in the model. We recognize this as a unique opportunity to study the performance of the model under those conditions. Potential evidence of our findings is illustrated through numerical simulations in simple, yet heterogeneous, networks. We can advance that significant differences arise under degree heterogeneity, particularly in what concerns to the peer-pressure cost. As a third step, we introduce a more complex heterogeneous network to shed important light on what negative degree-correlations imply.

The paper is organized as follows. Section II introduction of the model proposed in Mani et al (2013). Section III exploration of the effect of heterogeneity and negative degree correlation. Section IV testing of observations on actual networks applying usual procedures in Network Science. Section V suggests directions for policy makers to provide more egalitarian measures. Section VI summarises the contributions made by this paper and introduces possible methodological extensions to further explore the effect of population heterogeneity on cooperation scenarios described by PGGs.

2. A PGG model with peer-pressure played in a social network

As already explained, the starting point of this work is a PGG model incorporating peer-pressure and structured interactions among actors (i.e., a social network). This section delves into the main insights of the model and the reasons behind our choice.

2.1 Model selection

Traditional punishment and taxation methods were well studied and complemented under generic population conditions in the existing literature. Recent of the mentioned research focused on the importance the population structure. Prisoner dilemma games, a useful (yet simple) model, illustrates the cooperation and defection behaviour in population. Some of the already cited evolutive analysis through the use of PDG explains the evolution of cooperation over time under diverse social norms. However, those models do not study agent's behaviour in light of social structure. We considered more suitable models to the proper study of heterogeneity effects in the population. In the first instance we studied the models proposed by Calvó-Armengol and Jackson (2010) and Marco and Goetz (2021). In the former all agents can exert peer-pressure on the rest of agents. However, they specify a binary action space. Marco and Goetz suggest under a complex pressure framework another binomial action profile, as agents can adhere to a social norm or not. The binary nature of these models limits their applicability for our research. Heterogeneity causes sensitiveness in the capacity of decision of an agent. Peer influence, and the role it plays in individual decision-making around behaviours, requires a further characterization which we do not find in those models. On the contrary, the model proposed and developed in the paper "Inducing Peer Pressure to Promote Cooperation" by Mani et al. (2013) provides a more adjusted model to our needs. The authors propose a social mechanism to address the challenge of negative externalities by inducing peer-pressure. The idea behind the model is the introduction of a new mechanism (more efficient than Pigouvian ones) for policy makers to deal with problems where local interaction is crucial. Energy efficiency illustrates a relevant policy where local interactions can boost the energy policies imposed by the institutions. In other words, their aim is to show how peer pressure can promote more efficient behaviours as a complement to public policies.

In particular, the following characteristics of the model make it especially suitable for an analysis aiming to inform public policy design:

1. Both actions taken by actors and peer-pressure intensity are not binary (i.e. yes or no), but defined in a range.
2. The model offers a framework to test peer-pressure based incentive schemes against the classical Pigouvian one.

2.2 Model description

In the selected model, the authors formulate and study how agents would exert pressure over their peers in a defined network to reduce negative externalities. They model this by using a two-stage game theoretical framework to identify the pressure and action levels each agent exerts. In the following, we present the formal model. A set of N agents is considered in a social network $S =$

(N, E), where $E \subseteq N \times N$. Let $Nbr\ (i) = \{j: (i, j) \in E\}$ be the set of peers of the actor i. The authors assume that the social network is sparse, and agents have at most K peers. An agent $i \in N$ takes an action $x_i \in \mathbb{R}_+$ and define $x \in \mathbb{R}_+^{|N|}$ be an action profile of all agents. Each agent i in the framework proposed by the authors experiences a *raw* utility from its action defined by the function $u_i : \mathbb{R}_+ \rightarrow \mathbb{R}$. This raw utility function is assumed as twice differentiable and strictly concave with a unique maximum and lower bounded first derivative that approach infinite as the action approaches zero. In the described framework, it is assumed that the utility of actor i depends both on the raw utility of its own action as well as the externalities experienced due to the actions of the rest of agents. The externality experienced by agent i due to the aggregate action of the other agents is captured by the strictly convex and increasing function $v_i : \mathbb{R}_+ \rightarrow \mathbb{R}$. In this setting the total utility an agent i has, given its own action x_i and other agents action x_{-i}, is defined as $U_i(x_i, x_{-i}) \coloneqq u_i(x_i) - v_i\left(\sum_{j \neq i} x_j\right)$. In the game social surplus is defined as the sum of utilities attained by all agents, formally presented as the linear function $S(x) \coloneqq \sum_{i \in N} U_i(x_i, x_{-i})$.

The action profile at equilibrium is given by x^* and the action that maximizes social surplus is given by the Pareto-efficient profile $\mathring{x}$. Agents have the ability to exert pressure on their peers in the population. This peer pressure is defined by the matrix $\mathbf{p} \in \mathbf{R}_+^{N \times N}$, where the element p_{ij} is the pressure the agent j receives from agent i. If no pressure is exerted, $p_{ij} = 0$. As a particularity of this model, we deal with an infinite action space, opposed to usual binary choice models. Once the pressure profiles are defined, the extended utility function is given by:

$$U_i(x_i, x_{-i}, \mathrm{p}) = u_i(x_i) - v_i\left(\sum_{j \neq i} x_j\right) - \left(\sum_{j \in Nbr(i)} p_{ji}\right)(x_i - \mathring{x}_i) - \left(\sum_{j \in Nbr(i)} p_{ij}\right) c \quad (1)$$

The final utility function describes, in addition to the raw utility and the externality, a disutility experienced by the agent i that is bilinear in the total pressure from the peers of the agent and its own chosen action. Additionally, the pressure an agent exerts screens a cost -*c*- (as the last term of the function indicates).

2-stage game

The authors study the defined model under the presence of peer-pressure as a two-stage game. In the first stage, agents choose the amount of pressure they would exert on their peers to reduce the negative externalities as a consequence of the threat of higher action in the second stage. In the second stage, agents observe the pressure their neighbours apply on themselves and then choose an action profile in response to the observed pressure. Under the model equilibrium assumptions, the optimal action response an actor takes is unique and the marginal raw utility of action for actor i at the optimal response is equal to the total pressure exerted on i, i.e., $u_i'\left(x_i^*(p_{\downarrow i})\right) = p_{\downarrow i}$. Because the equilibrium action is reached in the second stage of the game, the agents in the first stage choose the peer-pressure profile that maximizes their utility function,

$$U_i(\mathbf{p}) = u_i(x_i^*(p_{\downarrow i})) - v_i\left(\sum_{j \neq i} x_j^*(p_{\downarrow j})\right) - p_{\downarrow i} x_i^*(p_{\downarrow i}) - p_{\uparrow i} c \tag{2}$$

The model is restricted to the cases where the marginal cost of exerting pressure is neither too high nor too low. The restrictions mirror a realistic scenario. Agent may not exert any pressure under high marginal cost levels since they would find less harmful to experience the total negative externalities. On the other hand, if the cost is extremely low the agents may exert an excessive pressure on their peers. Both situations lack realism. This suggests a scenario where all agents exert pressure in a distributed way. We can frequently observe this situation in real life. A person may have serious difficulties to have the scope necessary to ban a harmful action from any neighbour. Moreover, the change in one agent's action does not imply a significative improvement in any of the other agents in the society. However, with a moderate and distributed pressure, the total externality can be reduced affecting positively the agent's total utility. Pressure on an individual to reduce his or her pollution levels is usually moderate.

Pressure profile at equilibrium

Last arguments are rescued by the authors to make two observations for the equilibrium pressure profile $\mathbf{p}^*$ after applying the Karush-Kuhn-Tucker conditions (KKT). The next propositions allow identifying the cases in which peer-pressure is exerted and over whom.

Proposition 1. For the actor $i \in N$, $j, k \in Nbr(i)$ by KKT condition to the equilibrium, an actor puts peer-pressure on a set of agents, $p_{ji}^* > 0$, if:

$$c = -v_j'\left(\sum_{k \neq j} x_k^*(\mathbf{p}^*)\right)\frac{1}{u_i''\left(x_i^*(p_{\downarrow i}^*)\right)} \tag{3}$$

In the case in which c is higher than the marginal reduction in externalities faced from exerting pressure on peer i, $p_{ji}^* = 0$. Notice that all the peers on whom the agent j exerts pressure will have the same marginal reduction in the total externality.

Proposition 2. For the actor $i \in N$, $j, l \in Nbr(i)$ by KKT condition to the equilibrium, an actor feels peer-pressure from other agents, $p_{ji}^* > 0$, if:

$$c\left|u_i''(x_i^*(p_{\downarrow i}^*))\right| = v_j'\left(\sum_{l \neq j} x_l^*(\mathbf{p}^*)\right) \tag{4}$$

In the case the equality does not fulfil, and the marginal cost multiplied by the second derivative of the raw utility of the agent receiving the pressure is higher than the marginal externality faced by the enforcer, $p_{ji}^{*} = 0$. Both propositions offer a system of equations which allow to find an equilibrium pressure profile, $\mathbf{p}^{*}$, and consequently the corresponding action profile x^{*}.

2.3. Main results obtained with this model

Peer-pressure works

In the model by Mani et al, it is highlighted that pressure exerted by individuals can help to maintain better levels of social welfare. Effectively, under the model assumptions, lower individual action profiles (i.e. lower energy consumption) are taken when peer-pressure profiles are considered. Notice that the mechanism, yet effective, in equilibrium does not bring the action to the optimal pursued level. The authors consider an illustrative example to prove their suppositions. Indeed, the action levels carried out by the agents in the social network remains lower with peer-pressure than without it. Nevertheless, the explored network structure is simple. The example considers every agent has exactly ten peers in the social network. This supposition implies that the pressure levels are the same for all agents under equilibrium.

Social mechanism vs Pigouvian one

Usually, externality effects are offset setting regulations or punishments to the defectors. On this trend, Pigouvian taxes are applied to increase social welfare, reducing negative externalities. The reward given to any actor $i \in N$ for his or her action x_i is $r_i(x_i) = u_i'(x_i^{\circ})\,(x_i^{*} - x_i)$. Therefore, under the Pigouvian mechanism an agent i receives an extra reward, which incentives him or her to choose a lower action profile.

To demonstrate the superiority of the social mechanism, the authors also consider the direct reward agents receive because of the action reduction from their peers. Concretely, $r_{ji}: \mathbb{R}_{+} \rightarrow \mathbb{R}$, describes the direct reward agent i receives as a consequence of his peer action, x_j. Thus, if agent i is connected to a set of peers, the aggregate reward he receives is given by: $\sum_{j \in Nbr(i)} r_{ji}(x_j)$ [1]. Social rewards let for a direct comparison between bot models. This mechanism compared to Pigouvian policies results in a larger reduction of negative externalities under the same subsidy budget. In particular, the authors prove that the resulting loss in social capital under the social mechanism is lower than in the equilibrium given by the Pigouvian one. Moreover, when the costs given by the peer-pressure action are low, the social surplus in the equilibrium actions increases as a consequence of an easier pressure application.

[1] Analytical method development available in Mani et al. (2013).

Summarising, the results achieved by the authors indicate the superiority of the social mechanism over the Pigouvian one.

This section shows how the authors proved analytically the strengths of the model in a network structure in which every agent has the same number of peers. As a consequence of the model conditions all actions and pressure profiles are the same. However, we believe that homogeneity is an unrealistic assumption. Here we address a more credible scenario by analysing the effect of social networks with degree heterogeneity (i.e., social structures where actors present diverse numbers of peers).

3. Exploring the effect of degree correlation

When applying game theoretical approaches on degree-heterogeneous networks, a relevant structural feature to consider is node degree correlation (Roca, CP. et al., 2010). Degree correlation indicates the relationship between node degrees in a network. If nodes with high (low) degree tend to connect to other nodes of high (low) degree, then we have a positive correlation. Otherwise (i.e. if high-degree nodes are connected to low degree ones) we talk about negative correlation. When studying peer-pressure on realistic social networks, degree correlations between nodes result crucial because it determines the pressure deviation. In this section we explore theoretically the influence of node degree and the effect of degree correlation on pressure profiles in the model under study.

3.1. Influence of node degree on pressure profile

In a first phase, the model provides an initial insight about the impact a node degree causes on its capability to exert pressure over its peers. The authors assume that the marginal cost of exerting pressure is $\frac{1}{2K}$ times lower than the reciprocal of the absolute semi-elasticity of the marginal raw utility of an actor i at the socially optimal action profile, x°. Formally,

$$c < \frac{1}{2K\left|\frac{\partial \log u_i'(x_i^{\circ})}{\partial x_i^{\circ}}\right|} = \frac{1}{2K}\left|\frac{u_i'(x_i^{\circ})}{u_i''(x_i^{\circ})}\right| \tag{5}$$

This expression significatively approach the problem high-connected individuals may face. It illustrates how the node degree importantly influences the marginal cost an agent can afford for a certain pressure level. Thus, the possible pressure an actor exerts is subordinated to the number of peers the actor has. Concretely, it is assumed that for high marginal cost levels a high-connected actor may not exert peer pressure. However, the previous assumption only states that a high-connected agent affords lower marginal cost levels of pressure but not how the agent is harmed. Thus, given a constant marginal cost level, we further explore the pressure the different connected agents exert and the existing damage.

To do that, we initially focus on the study of the effects in a neighbourhood and the spread of pressures. In the interaction between agents, peer pressures are strategic substitutes. This means that the peer pressure that an actor i exerts on any peer j is strict strategic substitute of any peer pressure not exerted on i. We are concretely interested on the strategic substitute relationship emerged within the peers connected to the same node. Formally, for any actor i and two of his peers j, k,

$$\frac{\partial^2 U_i}{\partial p_{ij} \partial p_{ik}} = -v_i''\left(\sum\nolimits_{m \neq i} x_m^* \left(p_{\downarrow m}\right)\right) \frac{\partial x_j^*\left(p_{\downarrow j}\right)}{\partial p_{ij}} \frac{\partial x_k^*\left(p_{\downarrow k}\right)}{\partial p_{ik}} < 0 \tag{6}$$

Last inequality states that the externality is concave in the action profile and the action profile is decreasing under the presence of peer pressure. When nodes are connected in same neighbourhood pressures are diverted such that the externalities are offset. Similarly, for any actor i and k with a common peer j,

$$\frac{\partial^2 U_i}{\partial p_{ij} \partial p_{kj}} = -v_i''\left(\sum\nolimits_{m \neq i} x_m^* \left(p_{\downarrow m}\right)\right) \frac{\partial x_j^*\left(p_{\downarrow j}\right)}{\partial p_{ij}} \frac{\partial x_k^*\left(p_{\downarrow k}\right)}{\partial p_{kj}} - v_i'\left(\sum\nolimits_{m \neq i} x_m^* \left(p_{\downarrow m}\right)\right) \frac{\partial^2 x_j^*\left(p_{\downarrow j}\right)}{\partial p_{ij} \partial p_{kj}} < 0$$

the peer pressures p_{ij} and p_{kj} are strict substitutes of each other. Thereby, a set of pressures of two agents linked to a same node correlate negatively. Formally,

$$\frac{\partial p_{ij}}{\partial p_{kj}} < 0 \; \forall \; k \neq i$$

We can advance that beyond the consequences the agent's own degree has, the exerted pressure strongly depends on the connections of his peers. In this context, two types of peers are defined. High node degree peers, who are well-connected agents, and low node degree peers, who are agents with few peers. Under those two dichotomies, we focus on how a well-connected agent reacts toward the interaction.

The system of equations from propositions 1 and 2 derive in an equilibrium pressure profile $\boldsymbol{p}^*$ and an action profile x^*.Uniqueness of action profile is showed in the two-stage game with peer pressure. Therefore, all agents receive the same peer pressure independently of their node degree. This suggests pressure differences. If a well-connected agent $j \in Nbr(i)$ decides to put pressure on the agent i, the pressure level must be high enough to reduce the action to the action level of the rest of neighbours that agent j is pressuring. Decomposing and analysing proposition 1 in two,

$$\underbrace{-v_j'\left(\sum_{k \neq j} x_k^*\left(p_{\downarrow k}^*\right)\right)}_{\text{A}} \underbrace{\frac{1}{u_i''\left(x_i^*\left(p_{\downarrow i}^*\right)\right)}}_{\text{B}}$$

we make two observations regarding both components. The first component (A) refers to the externality an agent j suffers. We assume all agents exert a certain pressure level to maintain the model's operation. This implies that the externality remains almost constant for every agent as $N \to \infty$ and the node degree for every agent i is at least two, $k_i \geq 2$. The second equation component (B) manifest the possible pressure enforcement. This component suggests that a low-connected nodes cause a higher pressure effort by their peers to maintain the equilibrium action. Notice, those suppositions are subject to the marginal cost of exerting peer pressure. Last arguments suggest that agents are expected to exert above average peer pressure levels in a neighbourhood with several low degree nodes. This applies to all agents, but it specially harms well-connected agents. The existence of diversity leaves survival of cooperation contingent on the feasibility of more central agents to afford excessive costs due to pressure. In this sense, strategic substitute relationship strongly applies in a heterogeneous network. Under our assumptions, a well-connected agent prioritizes their low degree connections rather than the others. Thus, under equilibrium agents have the same action sets but well-connected agents acquire high responsibilities in the exercise of pressure when they connect many low degree agents. As a consequence, their final utility will be reduced.

3.2. Effect of degree-correlation on pressure cost

In order to illustrate the pressure differences between agents and consequently the harm an agent may suffer, we work with a simplification. Assume we observe two degree-measures:

- Low degree agents, $\breve{k}$.
- High degree agents, $\hat{k}$.

This assumption, yet accepting certain homogeneity degree and simplicity, allows us to emphasize the pressure enforce needed with low degree connections. We are interested on the pressure intensity exerted on $\breve{k}$. As we pointed it out, the felt pressure by agents $\breve{k}$, is equal to the pressure exerted on the rest of agents, $p_{\downarrow\breve{k}} = p_{\downarrow\hat{k}}$. We can derive the degree correlation existing between both pressure sets such that,

$$\bar{p}_{\downarrow\breve{k}} = \frac{\breve{k}}{\breve{k}} \bar{p}_{\downarrow\hat{k}} = r\bar{p}_{\downarrow\hat{k}}$$

Further assuming all agents pressure their peers, for $r > 1$, the mean pressure an agent has to exert on a low-connected peer is in average r units higher than the one exerted on a high-connected peer. We can in advance expect that the pressure a low-connected agent receives tends to be similar from each agent exerting it. This is due to high pressure levels are required in the interactions with low degree agents, so for an agent $i \in Nbr(\breve{k})$, $\bar{p}_{\downarrow\breve{k}} \sim p_{i\breve{k}}^{max}$. Without loss of generality and for $i \in Nbr(\hat{k})$, we can expect that, $Var(p_{i\breve{k}}) \geq Var(p_{i\hat{k}})$. Thus, pressure levels exerted on high-connected agents tend to be more heterogeneous, relying on the strategic substitute's relationship and propositions 1 and 2. Therefore, the effect of p over the action set x varies according to the degree correlation.

In order to verify that we evaluate the average change on x depending on the pressure profile, $p \in [0,\ p^{max}]$. We study the action function derived from the raw utility equation, $u_i'\left(x_i^*(p_{\downarrow i})\right)$. The action resulting from the equality between the raw utility function and the total pressure exerted on agent i, is a convex and strictly decreasing function in the pressure set $p_{\downarrow i}$, as is shown in equation (6). Now we consider the first-order partial derivatives in context. To explore the required intensity of pressure on both types of agents, we focus on the average rate of change of $x_i(p_{\downarrow i})$ over the interval offered by an agent's pressure, p. For this, recall that the different quotient $\frac{x_i(p_{ji}+h)-x_i(p_{ji})}{h}$ for $x_i(p_{ji})$ of the pressure variable p_{ji} at a point tells us the rate of change per unit of pressure over the interval $[p_{ji}, p_{ji}+h]$, where h ranges in $h \in [0,\ p^{max}]$. For a low-connected agent, the pressure exerted by agent $j \in Nbr(\check{k})$, $p_{j\check{k}}$, over him to reduce his action level to x^* requires a high intensity level. This can be expressed through the average rate of change of $x_{\check{k}}(p_{\downarrow \check{k}})$ over the interval of pressure given by agent j in $p_{j\check{k}}$ (see Figure (1.a)). The given average rate of change for a low connected agent is expected to be higher than in cases of high-connected peers. For a high-connected agent, the pressure exerted by agent $j \in Nbr(\hat{k})$, $p_{j\hat{k}}$ over him to reduce his action level to x^* requires lower intensity levels than in the previous case. This is due to at least two agents will exert pressure on $\hat{k}$. Therefore, the pressure level required by the agents is lower. As before, we can characterize it through the average rate of change of $x_{\hat{k}}(p_{\downarrow \hat{k}})$ over the interval of pressure given by agent j in $p_{j\hat{k}}$. Further assuming more than two agents connected to the high-connected agent $\hat{k}$ exert pressure on him, the average rate of change becomes lower for the interval of pressure in $p_{j\hat{k}}$ (see Figure (1.b)).

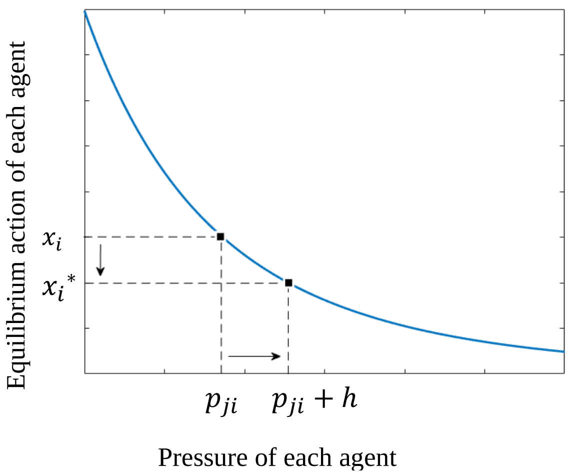

Figure 1.a | Average rate of change of the action set of a low-connected agent i under the pressure from a peer j. The effect is notorious when pressure is exerted toward a low node degree agent.
Source: Self-elaboration.

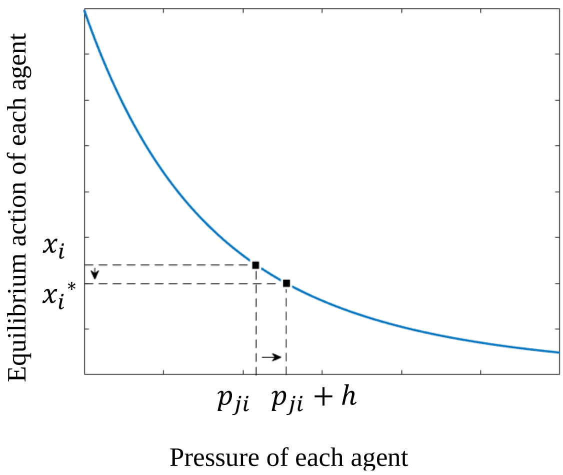

Figure 1.b | Average rate of change of the action set of a high-connected agent i under the pressure from a peer j. The effect is notably lower when pressure is exerted toward a high node degree agent.

Concluding, the pressure intensity varies depending on who is receiving it. An agent must exert higher pressure level when he connects a low degree node in order to maintain the action equilibrium level. Therefore, a high-connected agent may become harmed as the quantity of low-connected peers increases. Nevertheless, if those agents feel punished, they may not exert the required pressure. As we explained in the previous section, the model sets a reward to increase peer pressure to the social optimum level and reduce the damage suffered. Effectively, agents become rewarded by the pressure they exert on their peers to reduce their corresponding action levels. It is positively reflected in their final utility function. However, the reward function is not enough to encourage agents to exert pressure when it is costly. Thus, the pressure cost must be set in a way high-connected agent can afford the pressure exercise to reduce the harm they suffer.

The network structure exposes the grounds to be considered in the model. The maximum connection grade and the type of peers in a neighbour reveals the weaknesses and strengths of the model. With the aim to identify the possible problems that emerge with the existing node grade differences we test the degree correlation in various heterogeneous scenarios.

4. Testing the effect of degree correlation in degree-heterogeneous networks

In order to test the theoretical observations made above on the effect of node degree correlation, we need to perform an in-depth study of the effect on the model dynamics of actual degree-heterogeneous networks. The network structure of the population characterizes not only the individual's interactions but also the structure through which pressure is applied. Population structure is defined by a graph; the vertices of the graph represent the actors, and the social interactions correspond to the edges.

To reach our goal in this section, we will adopt two usual approaches applied in network theory to study the interplay between structure and dynamics. First, following works like (Santos F. et al, 2008), we consider the simplest possible configurations based on a single highly connected, central agent in a population of less connected ones. This type of network allows a simple analysis of the situation a well-connected agent may suffer. Since we do not consider further connections, this network only gives us an intuitive result which is only used as an initial approach to corroborate previous statements. Second, we consider a wider and complex network, but still only two types of agents are considered. Once this is done, we extend the analysis to a static SF network model (Goh, K.-I. et al., 2001) This network has two characteristics especially useful for our purposes: a) Its structure resembles the degree heterogeneity of real large social networks; and b) It's main structural features (including degree correlation) have been analytically estimated (Catanzaro, M. and Pastor-Satorras, R., 2005).

4.1. Study on simple networks

First, we test the effect of negative degree correlations in heterogeneous networks. For this, we begin with a simple abstraction of the Figure (2). It requires the abstraction of the connections from the central agent neighbours to the rest of nodes (b). This star-like network is characterised by the existence of only two kinds of nodes, the central agent, and the leaves.

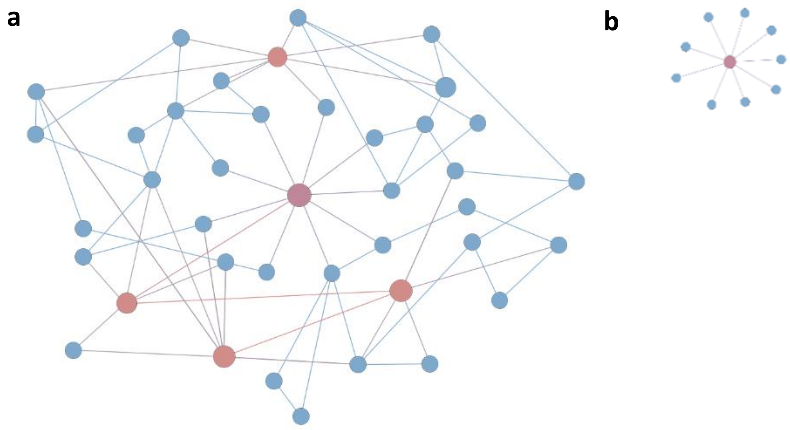

Figure 2 | Population structure and local neighbourhoods. a, Scale-free graph for a large heterogeneous population. **b**, Star-like network, representing the connections between a central agent to the rest of nodes.
Source: Self-elaboration

Consider a star of size N – a central agent or hub (H) and $N-1$ leaves (l). The set of pressure to which the agents are exposed are given by the next expressions:

$$p_{\downarrow H} \coloneqq \sum_{l} p_{lH} = (N-1)p_{lH}$$

$$p_{\downarrow l} \coloneqq \sum_{l} p_{Hl} = \frac{p_{Hl}}{N-1}$$

Since, by imposition of the model, every agent receives in equilibrium the same total pressure, the central agent, hub, becomes seriously harmed. The expected pressure each of the leaves must receive only depends on the hub, no agent plays the role for pressure to the advantage of the hub. In contrast, the leaves must exert a low pressure level in comparison with the hub. In this case, all the leaves are considered homogeneous so the pressure level they exert towards the central agent is the same. Under the consideration that all agents -hub and leaves- consume the same amount x, for any raw utility function the pressure the central agent exerts is approximately $N-1$ times the pressure each leaf applies on him,

$$p_{Hl} \cong (N-1)p_{lH}$$

For a large N, the pressure that the hub exerts is immense compared to the leaves. This situation reveals a malfunctioning of the model in a star population structure. By proposition 1, the hub will not exert any pressure unless the cost of doing so is nearly zero. This extreme situation lacks any reality and is inconsistent under the grounds of the model. However, the star-like network structure allows us to examine how damage a highly connected agent becomes when he has such a high responsibility degree in the reduction of the final externality.

The star-like network is a gross simplification of the population structure we are interested on. We consider a more useful, still simple, structure, with additional interactions. Let us start setting the dichotomy in which we consider agents with high node degree -$\hat{k}$- and with low node degree -$\check{k}$. In many real-life social networks well-connected individuals are linked to peers with a low connectivity, so it is in several SF network models, as we will see later on. As a generalization, let us then consider the agents with node degree $\hat{k}$ always connect agents with node degree $\check{k}$. In this graph characterization, we simplify the existing ties between agents, enhancing an easier illustration of the problem.

Relying in proposition 1 and 2, we consider the case where the high-connected agent (hub) is linked to all low degree node peers. Along with these propositions and by considering the relationships outside the neighbourhood, the peer pressure exerted by the hub on the low-connected peers must be enough to maintain the action level in equilibrium. By the strategic substitute relationship, the pressure the hub i exerts correlates negatively with the pressure $j \in Nbr(\check{k})$ exerts on $\check{k}$, $\frac{\partial p_{i\check{k}}}{\partial p_{j\check{k}}} < 0$. This implies that the required intensity of pressure on the connected peers becomes higher as the node degree approaches 2. With the imposed simplification we can expose the pressure levels both type of agents put toward each other. Formally we observe,

$$p_{\hat{k}\check{k}} = \frac{\hat{k}}{\check{k}} p_{\check{k}\hat{k}} = \frac{1}{r} p_{\check{k}\hat{k}}$$

Well-connected agents still suffer the degree correlation differences between nodes. Clearly, pressure ties decrease, as the low node degree agent are more connected. However, this setup requires a very low marginal cost of exerting pressure otherwise the hub will not participate.

4.1.1. Example

In order to demonstrate our results, we work with a simple modification of the illustrative example given by the authors. We consider a set of agents $N = \{1, \ldots, 111\}$ as a simple semi-heterogeneous setup depicted in Figure (3). We set the degree of the well-connected agents, $\hat{k}$, to 10. One of those as a central agent and the other ten connect to the ends of the network. On the other hand, low-connected agents possess a node degree equal to 2 when they link both types of hubs and 1 at the ends. We consider the same raw utility and externality functions considered by Mani et al. in the original model. The raw utility function is $u_i(x_i) = 12x_i^{0.8} - 4x_i$ for all $i \in N$, and the externality function of all actors is

$v_i(y) = 0.0001(y)^{1.5}$ for all $i \in N$. The marginal cost of exerting pressure must be set taking in account every agent pressure responsibility. This means that agents with more connections generally cannot afford high marginal cost of pressure. The approach given by inequality (6) sets a reasonable marginal cost in cases the social network includes heterogeneous connections. Since the optimal action profile remains the same for all agents, the marginal cost affordable by all agents is $c < 1.33$. Let us consider the marginal cost of exerting pressure $c = 1$ per unit of pressure. Following the above guidelines, the pressure that a central agent exerts toward an agent with low degree, 2 in this case, is five times higher. Conversely, the pressure that a non-central agent exerts toward the leaves at the ends will be about ten times higher. The network structure considers three types of node degree. We obtain a symmetric equilibrium under peer pressure with $x_i^*(p_{\downarrow i}^*) = 65.4$ for all $i \in N$. The peer pressure exerted on the actors is also symmetric and equal to 0.16. However, as we expected the pressure exerted by the high-connected agents is higher than the low-connected agents. Concretely, the central hub exerts a total pressure of 0.8, the non-central hubs exert a total pressure of 1.52, whereas low-connected agents exert a peer pressure of 0.032. In the increase of consumption, high-connected agents are harmed with respect to low-connected agents. In this case pressure differences do not alter significatively final utility functions since every agent is still pressuring all peers. Even so, as the action approaches the optimum social level $\mathring{x}_i$ divergences become more important as pressure differences differ even more. This example, yet simple, gives us a first numerical approach to the problem. In a more heterogeneous networked society, the differences become more severe, since not every agent may pressure all his peers. The social mechanism still demonstrates its strengths. The existing divergence in results creates inequalities which must be considered.

Both cases illustrate in a simple way the consequences emerging from a negative degree correlation. The final utility function of a well-connected agent becomes negatively affected as more low degree node agents are in his neighbourhood. This set a clear difference between high and low connected agents, but we do not know how many low degree peers an agent supports. Both social network structures were imposed, so both lacks reality. SF models gives a strong heterogeneous composition where we can test the node preferential attachment.

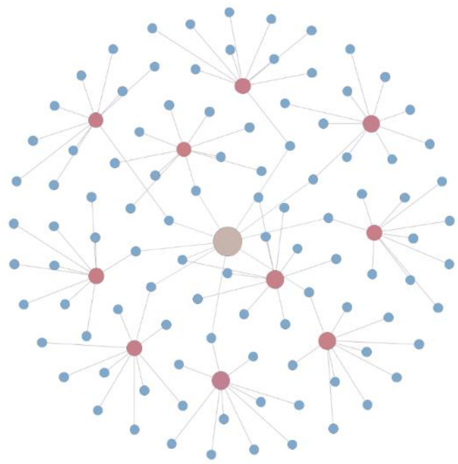

Figure 3 | Population structure and local neighbourhoods. Regular graph which mimics a population structure composed of 111 agents and three different node degrees.
Source: Self-elaboration

4.2. Analysis using a network model with heterogeneous structure

As mentioned above, SF network models reproduce patterns of degree heterogeneity of real social structures and, therefore, they are a good tool to test the agent's behaviour in population structure with degree heterogeneity. Among the SF models we focus on the static network models (SM). The election of SM is due to two main reasons. First, its simple definition and ability to create large and bounded networks. Second, its widespread use has provided further extensions and applications. In this way Goh, K. et al (2001) presented a static network model under SF parameterization, which was further generalized by Catanzaro and Pastor-Satorras (2005) based in a mapping to a hidden variables network model. Broadly speaking, hidden variables concept defines the probability unions between edges in a graph. This class of network models allows for the analytical treatment, as presented in Caldarelli, A. et al. (2002). Catanzaro and Pastor-Satorras took advantage of this to estimate analytically different structural features of a SM, including degree correlations. Consequently, we can explore the existing degree correlations with the objective of determining whether an agent is excessively harmed or not in the social mechanism.

4.2.1. Analytical estimations

As explained by Catanzaro and Pastor-Satorras, networks accounts for a fixed average degree expressed by means of $\langle k \rangle = 2m$. The parameter α is a real number in the range $\alpha \in [0, 1]$. Thus, since the parameter α ranges in that level and the distribution parameter is characterised by $P(k) \sim k^{-\gamma}$, such that

$$\gamma = 1 + \frac{1}{\alpha}$$

it is possible to generate networks with a degree exponent $\gamma \in [2, \infty]$. With the distribution already categorised, the SM generates networks with a maximum degree for the index $i = 1$, given by

$$k_{i=1} \sim 2m(1-\alpha)N^{\alpha} \tag{7}$$

Average degree. Following the steps proposed by the authors we firstly consider the behaviour of the overall average degree, analytically the average degree of the vertices with index i. Performing some approximations, the equation results in

$$\bar{k}(i) = 2m(1-\alpha)\left(\frac{i}{N}\right)^{-\alpha}(1-N^{\alpha-1}) \tag{8}$$

Degree distribution. For a SF distribution with a degree exponent given by $\gamma = 1 + \frac{1}{\alpha}$, the asymptotic behaviour of the degree distribution for k is

$$P(k) \sim k^{-1-\frac{1}{\alpha}} \tag{9}$$

Degree correlations. With the aim to evaluate the correlation linkages, the average nearest neighbour degree of the vertices with degree $k, \bar{k}_{nn}(k)$, is calculated. In the way of solving it, the average nearest neighbour degree of the vertices with index i, $\bar{k}_{nn}(i)$, is computed.

$$\bar{k}_{nn}(i) = i^{\alpha}\sum_{j=1}^{N} j^{-\alpha} \times [1 - exp\{-2m(1-\alpha)^2 N^{2\alpha-1} i^{-\alpha} j^{-\alpha}\}] \tag{10}$$

Using the expression for $\bar{k}(i)$ given in equation (8), the average degree of the nearest neighbour of the vertex with degree k, $\bar{k}_{nn}(k)$, is given in the SM by the equation

$$\bar{k}_{nn}(k) = 1 + \frac{1}{NP(k)k!}\sum_{i=1}^{N} exp\left[-\bar{k}(i)\right]\bar{k}(i)^k \bar{k}_{nn}(i). \tag{11}$$

This last expression gives us the insights for establishing the degree correlations existing in the network by numerical simulations.

4.2.2. Numerical simulations

The analytical estimations analytical estimations provided in Catanzano et al. (2005) allow us to show more clearly the occurrence of negative degree correlations in degree heterogeneous scenarios and, therefore, better assess peer-pressure imbalance in real social networks. In order to do that we generate a network of size $N = 100$. The model yields correlated networks for values $\alpha > 0.5$ in the degree distribution which approaches to the values empirically seen in real SF networks. Let us consider a network with $\alpha = 0.8$, which corresponds to a value $\gamma = 2.25$. We set a constant average degree for

$m = 2$. Results are averaged over 31 realizations under the power-law probability distribution with exponent γ. Numerical simulations enable us to depict the average degree of the nearest neighbours from expression (11). In Figure (4) we report this average for all possible node degrees existing in the network.

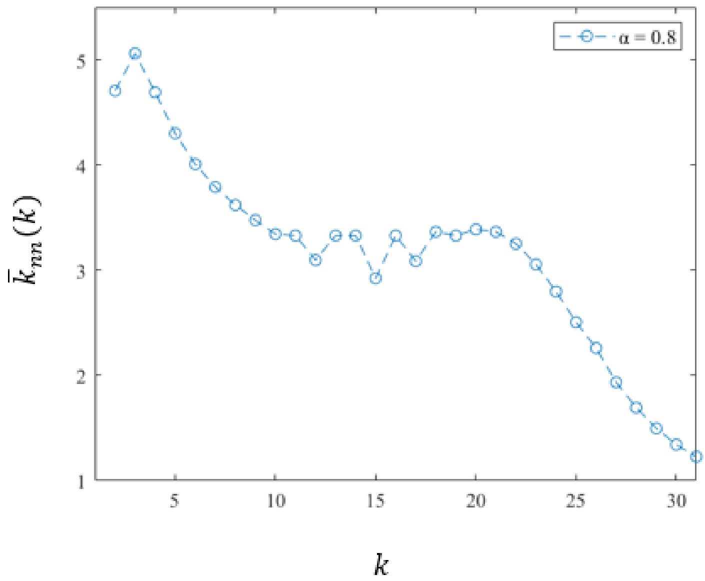

Figure 4 | Average nearest neighbour degree of the vertices with degree *k*, in the SM for a value of $\alpha = 0.8$.
Source: Self-elaboration with the established population sample.

The correlation function displays a decreasing slope for the considered degree. We note a strong disassortative mixing (i.e. negative degree correlation) for extreme node degree values. Those agents with low degree nodes tend to connect in average higher connected peers than those agents with high degree nodes. These correlations are characterised by the absence of multiple connections. Only a few highly connected individuals ensure the overall connectivity of the entire population (see Figure (5)). Based on the analytical exploration from section 3, the pressure that individuals with a degree $k > 20$ exerts will be higher than those bellow the barrier. Those agents are in general connected to peers with a range connectivity $k \in [1, 2]$. The existence of isolated leaves in their neighbour increases the damage range, in that all pressure responsibility relies on the highly connected agent. On the other hand, the connectivity interval also comprises peers with two connections. As we already discuss, the equilibrium action of those peers depends even more on their connections pressure. The agent of our interest is forced to exert an amount of pressure $p_{ij} \geq \frac{p_{\downarrow j}}{2}$ for all $j \in N$ with $k = 2$, $i \in Nbr(j)$. However, those peers possess another connection which degree is contingent upon the structure of the network. This complicates the interactions study when not even the agents must pressure all their peers. Therefore, the exact calculation of the pressure impact is outside the scope of this paper, as it would require massive numerical exercises based, for instance, on computer-based experimentation (see subsection 6.2). Nevertheless, the implications of this network structure shed light on the matter and help to explain the consequences of degree differences in heterogeneous populations.

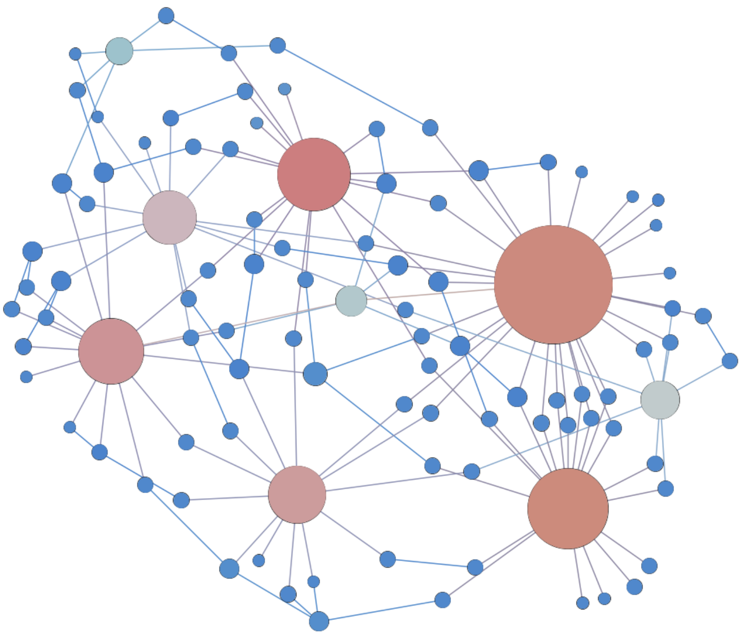

Figure 5 | Population structure and local neighbourhoods. Scale-free graph for a population of 100 individuals. *Source: Self-elaboration*

5. Policy implications

5.1. Community level policies

Research frequently reveals that diverse communities exhibit different intra-community behaviours (Laurence, J., et al., 2019). Those behaviours are strongly characterised by individuals' interactions. Social linkages can define people's conduct and vice versa. This situation has always implied that some individuals are more potentially connected than others. Those agents are usually characterised by a higher social status and consequently their decisions generate greater impacts on other behaviours. Furthermore, social connectivity has increased dramatically last decade due to the rise of digital platforms. A public speech, an official event or even a tweet from a celebrity or political personality, has an inordinate impact. For this reason, we believe policy makers must consider local interactions to delimit the negative externalities on public goods. To help addressing this challenge, we provide an exploratory analysis about the implications heterogeneous connections have on cooperation when individuals can exert pressure on each other.

The mechanism accounts for the association of community-level characteristics. Individual's interactions are local, which means that some of the decisions that affects the entire community are subordinated to their closest relationships. The intuition behind the mechanism is intuitive and realistic. Individuals exerts peer-pressure to their peers to promote decisions aligned with social wellness. Those considerations about population structure reinforce the mechanism compared to most common centralized policies. In this respect, our analysis significantly strengthens the evidence for the importance of the number of connections implies. Importantly, even over small communities, individuals' attitudes in diverse populations can change due to their connectivity patterns (i.e. the structure of their social networks). Our perception is that connectivity differences must be addresses to find the most efficient and egalitarian policy tools to solve problems of externalities.

These findings also help to understand why, on average, diversity is characterized by social gaps. Our exploratory analysis has uncovered how individuals with higher connection responsibilities are negatively impacted. As a result, one of the implications of the model is to adequately reward those individuals to compensate for pressure cost inequality. Based on our results, the intuition behind the social reward mechanism proposed in Mani et al. (2013) creates inefficiencies. Individuals are rewarded as a cooperation incentive. However, the reward system favours free-ride situations from the social budget. This attempts against the main motive of the model creating efficiency costs. With the aim to solve this issue we encouraged to make explicit the peer-pressure of each individual. In the next subsection we suggest a potentially more appropriate rewarding mechanism.

5.2. Reward function on degree heterogeneous social networks

Social mechanisms promote cooperation by rewarding individuals for the pressure exerted. In the model, a reward is given to agent i as a result of his peer, agent j's action x_j. Despite rewarding the agents and assuming the costs, the mechanism results inefficient.

Under the assumption of a heterogeneous network, peer-pressure costs seriously harm highly connected agents interacting with low node degree peers. Consequently, agents may attempt to free-ride on rewarding. In many cases agents do not pressure all their peers since the model prioritizes some peers over others. However, agents receive a reward by all their peers' low action. An imposed mechanism to avoid the "Tragedy of the commons" may results in losses in the reward budget.

To address this issue, the mechanism must consider who is pressuring on whom to correctly offsets the costs. In this line, we propose a modification in the original social reward function.

More specifically, the reward is given to agent i as a result of the rate of change of a neighbour agent j, given $p_{ij} > 0$. Formally,

$$r_{ij}(x_j) = (\alpha_j + \beta_i)(x_j'(p_{ij}) - x_j) \text{ where } \alpha_j = cu_j''(x_j^\circ) \text{ and } \beta_i = v_i'\left(\sum_{k \neq i} x_k^\circ\right)$$

where $x_j'(p_{ij})$ states for the new action level once agent i pressures his peer j. Since in equilibrium pressure decisions are known, each agent becomes rewarded by their own actions. Comparing with the initial reward, the modification component included in the function, increases the mechanism efficiency. Nevertheless, we must bear in mind that this approach may require a costly information acquisition process to set all trade-offs. Consequently, its translation into specific policy practices (and, in particular, the incorporation of such information costs) requires further study.

6. Conclusions

6.1 Summary of results

The key theme underlying social interaction is that our activities are shaped to a great extent by society-driven factors. There exists evidence that cooperation develops relatively easily between individuals when challenged with a collective threat such as an environmental crisis (Hoffman et al.; Schlager; Van Vugt). In behavioural economics and evolutionary game theory, social interaction is recognised as an important factor in social dilemmas.

The main goal of this work is to explore the effect of degree heterogeneity (a usual feature of real social networks) on scenarios described by Public Goods Games (PGG) with peer-pressure. In particular, we aim at disentangling the influence of such a structural feature on peer-pressure, a social cooperation-promoting mechanism in PGG. To do so, we selected an already proposed model (Mani et al, 2013) and analysed its dynamics under newly defined heterogeneous scenarios. The chosen model is ideal since pressure decisions are left to the agents in the network. Thus, enforcement totally relies on the local level. In this sense, individual behaviour requires more attention. Our analysis consisted on two steps. First, we theoretically explored the effect of node degree correlation (an indicator measuring the relationship between node degrees of connected individuals in a network). Secondly, we tested such observations on networks presenting desired structural characteristics. Following usual practise in Network Science, we carefully studied actors' behaviour in very simple, small networks and then we extended to larger SF networks (artificially created graphs reproducing the degree heterogeneity of real social networks).

Our exploratory analysis showed that agents exhibit different responsibility stages depending on their degree and that of their peers. This is a relevant contribution to the literature on how cooperation in PGG is shaped by social linkages since previous researchers did not give so much importance to behaviour divergences. Specifically, we noted that highly connected individuals are vital to the well-functioning of the mechanism since they ensure the overall connectivity of the entire population. However, the gap in behaviour between high-connected agents (hubs) and low-connected agents (leaves) do not favour the model cohesiveness. Given a heterogenous networked population, highly connected agents' interaction results inefficient from a cost perspective. The author's analytic solution dismisses this possibility. In this respect, degree correlation enabled us to identify structural situations that could harm the social mechanism. One can imagine scenarios in which individuals with a powerful social status can influence more on others, but higher costs are assumed in exchange. Therefore, we importantly emphasize how node degree correlation causes sensitiveness in the capacity of decision of the agents. Our findings encourage future structural considerations in PGG. Not everyone responds similarly to their peers' actions.

Those considerations reinforce and reveal inefficiencies of the social mechanisms. We realise diversity strongly alters pressure deviations and we emphasize the importance of its consideration. Notwithstanding the contributions of this work, our findings must be considered with some caveats. Diversity implies differences, not only because connection status, but cultural, ethnic, or socio-economic differences can be hardly monitored. Our analysis relies on a generic structural consideration, not attempting to provide specific insights. Secondly, as in the studied model, the success of the game relies on the effective monitoring of the peer-pressure and the reward.

6.2. Extension possibilities through computational experiments

The exploratory analysis presented in this thesis (both theoretical and applied to public policy) are relevant, but still limited. Degree heterogeneity in real social networks may influence peer-pressure and cooperation in PGG scenarios in different ways. Moreover, from the perspective of public policy design, different incentive strategies should be tested.

Unfortunately, though, all these extensions are beyond the scope of this thesis, both in terms of time availability and methodological constraints. Focusing on the second aspect, the approach adopted here (i.e. mainly, an analytical exploration of a rather complex model) is a perfect starting point but should be complemented with a program of computational experiments. Consequently, roughly described, the extension of the work presented here would follow these steps:

a) Design of an agent-based model based on the one proposed in Mani et al (2013): Agent-based models (Hamill & Gilbert, 2015) is a computational modelling approach usually applied in evolutionary game theory because it allows to easily integrate individual decision-making and interactions over social networks.
b) Comparative analysis of the effect of degree heterogeneity: By running the computational model designed at step a) over networks presenting different controlled levels of degree heterogeneity, it would be possible to further understand the effect of this structural feature. A typical approach in Network Science would be comparing model executions on Erdös-Renyi (completely homogeneous) and Scale-Free (heterogeneous) networks.
c) Public policy design 'sandbox': The, computer-based, experimental environment described in the previous points could be used to test different socially-defined incentive mechanisms and compare them with classical Pigouvian ones. This would be done by running the model in a) with such different incentive mechanisms over the same network and comparing the results in terms of actors' behaviour, peer-pressure profiles, and implementation costs.

7. Referencies

Acemoglu, D., Ozdaglar, A., & ParandehGheibi, A. (2010). *Spread of (mis) information in social networks.* Games and Economic Behavior.

Adamic, L. A., Huberman, B. A., Barabási, A. L., Albert, R., Jeong, H., & Bianconi, G. (2000). *Power-law distribution of the world wide web*. Science.

Albert, R., & Barabási, A. L. (2002). *Statistical mechanics of complex networks*. Reviews of modern physics.

Bala, V., & Goyal, S. (2000). *A noncooperative model of network formation*. Econometrica.

Barabási, A. L. (2014). *Network Science Book*. Boston, MA: Center for Complex Network, Northeastern University.

Barabâsi, A. L., Jeong, H., Néda, Z., Ravasz, E., Schubert, A., & Vicsek, T. (2002). *Evolution of the social network of scientific collaborations.* Physica A: Statistical mechanics and its applications.

Barrat, A., Barthelemy, M., Pastor-Satorras, R., & Vespignani, A. (2004). *The architecture of complex weighted networks*. Proceedings of the national academy of sciences.

Bicchieri, C., & Dimant, E. (2019*). Nudging with care: The risks and benefits of social information*. Public choice.

Bloch, F., & Jackson, M. O. (2007). *The formation of networks with transfers among players*. Journal of Economic Theory.

Bramoullé, Y., Djebbari, H., & Fortin, B. (2020). *Peer effects in networks: A survey*. Annual Review of Economics.

Caldarelli, G., Capocci, A., De Los Rios, P., & Munoz, M. A. (2002). *Scale-free networks from varying vertex intrinsic fitness.* Physical review letters.

Calvó-Armengol, A., & Jackson, M. O. (2010). *Peer pressure*. Journal of the European Economic Association.

Catanzaro, M., & Pastor-Satorras, R. (2005). *Analytic solution of a static scale-free network model*. The European Physical Journal B-Condensed Matter and Complex Systems.

Chao, L., & Elena, S. F. (2017). *Nonlinear trade-offs allow the cooperation game to evolve from Prisoner's Dilemma to Snowdrift*. Proceedings of the Royal Society B: Biological Sciences.

Clasen, D. R., & Brown, B. B. (1985). *The multidimensionality of peer pressure in adolescence*. Journal of youth and adolescence.

Dawes, R. M. (1980). *Social dilemmas*. Annual review of psychology.

Dorogovtsev, S. N., & Mendes, J. F. (2013). *Evolution of networks: From biological nets to the Internet and WWW*. OUP Oxford.

Dorogovtsev, S. N., Mendes, J. F. F., & Samukhin, A. N. (2003). *Metric structure of random networks*. Nuclear Physics B.

Erdös, P., & Rényi, A. (1959). *Publications Mathematicae 6, 290*. Publ. Math. Inst. Hung. Acad. Sci.

Goh, K. I., Kahng, B., & Kim, D. (2001). *Universal behavior of load distribution in scale-free networks.* Physical review letters.

Goh, K. I., Kahng, B., & Kim, D. (2001). *Universal behavior of load distribution in scale-free networks.* Physical review letters.

Hamill, L., & Gilbert, N. (2015). *Agent-based modelling in economics*. John Wiley & Sons.

Hauert, C., Holmes, M., & Doebeli, M. (2006). *Evolutionary games and population dynamics: maintenance of cooperation in public goods games.* Proceedings of the Royal Society B: Biological Sciences.

He, G., Zhang, W., Liu, J., & Ruan, H. (2020). *Opinion dynamics with the increasing peer pressure and prejudice on the signed graph*. Nonlinear Dynamics.

Huang, L., & Xiao, E. (2021). *Peer effects in public support for Pigouvian taxation.* Journal of Economic Behavior & Organization.

Jackson, M. O., Rogers, B., & Zenou, Y. (2016). *Networks: An economic perspective.*

Kareva, I., Morin, B., & Karev, G. (2013). *Preventing the tragedy of the commons through punishment of over-consumers and encouragement of under-consumers*. Bulletin of mathematical biology.

Krapivsky, P. L., & Redner, S. (2001). *Organization of growing random networks*. Physical Review E.

Laurence, J., Schmid, K., & Hewstone, M. (2019). *Ethnic diversity, ethnic threat, and social cohesion:(re)-evaluating the role of perceived out-group threat and prejudice in the relationship between community ethnic diversity and intra-community cohesion.* Journal of Ethnic and Migration Studies.

Liljeros, F., Edling, C. R., Amaral, L. A. N., Stanley, H. E., & Åberg, Y. (2001). *The web of human sexual contacts*. Nature.

Mani, A., Rahwan, I., & Pentland, A. (2013). *Inducing peer pressure to promote cooperation*. Scientific reports.

Marco, J., & Goetz, R. (2021). *Moral ties, peer pressure and social norms of cooperation*. Available at SSRN 3814048.

Markovsky, B., Willer, D., & Patton, T. (1988). *Power relations in exchange networks*. American sociological review.

Mittone, L., & Ploner, M. (2011). *Peer pressure, social spillovers, and reciprocity: an experimental analysis*. Experimental Economics.

Newman, M. E. (2001). *Clustering and preferential attachment in growing networks.* Physical review E.

Puurtinen, M., & Mappes, T. (2009). *Between-group competition and human cooperation*. Proceedings of the Royal Society B: Biological Sciences.

Roca, C. P., Lozano, S., Arenas, A., & Sánchez, A. (2010). *Topological traps control flow on real networks: The case of coordination failures*. PLoS One.