Monetary aggregates and business cycle in the Argentine economy. A small open economy model with endogenous money multiplier and currency substitution

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Abstract

The procyclical nature of monetary aggregates has been one of the main topics of debate in monetary economics. Most of literature adopts some strategy that involves nominal rigidities or market imperfections in order to explain the causal role that monetary factors play in determining real economic activity. In this thesis I provide a novel explanation to the positive relationship between monetary and real variables in a context of full flexible prices and competitive markets. By adding currency substitution into a standard small open economy RBC model with endogenous money multiplier, the model manages to replicate the positive relationship between monetary aggregates and output in the Argentine economy under different exchange rate regimes. The model also explains the switch in the correlation between the money multiplier and output when exchange rate regime changes. Under a fixed exchange rate regime, the reverse causality mechanism works, and the positive correlation between monetary aggregates and output is driven by the money multiplier. When exchange rate is full flexible, the money multiplier reacts negatively to the productivity shock, and the positive correlation is explained by the currency substitution effect.

JEL classification: E32, E41, E51, F41.

Keywords: Monetary Aggregates, Real Business Cycle, Money Multiplier, Endogenous Money, Currency Substitution, Small Open Economy, Argentina.

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1. INTRODUCTION

The procyclical nature of monetary aggregates has been one of the main topics of debate in monetary economics. The classical study of Friedman and Schwartz (1963) represents probably the most influential empirical evidence that monetary factors play a causal role in output fluctuations. Based on almost 100 years of U.S. data, Friedman and Schwartz show that faster money growth tends to be followed by increases in output above trend, and slowdowns in money growth tend to be followed by declines in output. This evidence was updated by Belongia and Ireland (2016), who examined U.S. data from 1967 to 2013 and found large and positive correlations between monetary aggregates and GDP, with money leading output[1].

In order to explain the causal role that monetary factors play in determining real economic activity, most of the academic literature adopts some strategy that involves nominal rigidities (in prices and wages) or imperfections in some markets. This was the strategy adopted from the first models of nominal wage contracts (Fisher, 1977; Taylor, 1980)[2] to the most recent literature of New Keynesian Real Business Cycle (NK-RBC) models (Smets and Wouters, 2003; Galí and Monachelli, 2005)[3].

From an opposite point of view, Tobin (1970), King and Plosser (1984), and Freeman and Huffman (1991) argue that causality runs from real economic activity to money. Tobin (1970) suggests that the observed movements in money are likely systematic responses of the monetary authorities to changes in the real economy. King and Plosser (1984) and Freeman and Huffman (1991) make the case that movements in broad monetary aggregates, such as M2, result from the endogenous responses of the banking sector to cyclical fluctuations in the demand for deposits.

By employing a dynamic general equilibrium model with an endogenous money multiplier, Coleman (1996) quantitatively evaluates this 'reverse causality' mechanism. He compares the simulated moments generated by the model with U.S. economy data from 1959q1 to 1994q2, and he finds that this channel generates positive contemporaneous correlations between broad monetary aggregates, such as M1 or M2, and output.

A close approach to Coleman´s, but in a more parsimonious way, was done by Freeman and Kydland (2000). In their model, individuals purchase consumer goods using cash and bank deposits. There are two transaction costs that are necessary to determine the demand for money and the division of money balances. One is the cost of replenish money balances each time a transaction occurs, and the other is a fixed cost of using deposits. In this way, smallest purchases are made with cash, and the largest

[1] The authors employ the Divisia measures of money supply advocated by Barnett (1980) that differentially weighs the various components added together in the standard measure M2, where the weights depend on the user cost of each component.
[2] Fisher (1977) introduces an overlapping labor contracts mechanism, and Taylor (1980) considers staggered wages contracts.
[3] Galí and Monachelli (2005) introduce sticky nominal prices, and Smets and Wouters (2003) consider both sticky nominal prices and wages. In these models, the sticky variables adjust following a Calvo (1983) mechanism.

purchases are made with deposits. A positive technology shock increases the size of all purchases and, as deposits are preferred for larger purchases, households increase the deposits-to-cash ratio. Given that banks invest part of the deposits of individuals on physical capital, interest rate on deposits increases and then deposits receive an extra boost. At the end, the deposit-to-cash ratio, the money multiplier and the money supply increase. The increase in deposits expands the capital stock as well and, consequently, the output in the following period. Using this simple model without assuming nominal rigidities or markets imperfections, Freeman and Kydland manage to replicate the correlation between monetary variables and real variables for the U.S. economy.

Following this research agenda, Trupkin, et al. (2017) quantitatively evaluates the reverse causality mechanism for the Argentine economy during the period 1993-2014. They use the Freeman and Kydland´s model calibrated for that country, and show that this model can explain the positive correlation between monetary aggregates and output. However, it cannot replicate properly the behavior of the money multiplier observed in the stylized facts. In particular, the negative correlation with the output observed during the period of the flexible exchange rate is in the opposite direction to that predicted by the model. Table 1 shows the correlation coefficients of the cyclical components of monetary aggregates (M2 and M3), domestic currency deposits (D) and the money multiplier (mm), all of them against real GDP[4].

Table 1. Correlations with output (HP filter, lambda = 1600)

	1993:1 2001:4	2003:1 2019:4
$M2$	0.908	0.354
$M3$	0.907	0.572
D	0.913	0.486
mm	0.223	-0.182

This thesis is motivated by the missing in the literature exposed by Trupkin, et al. in relation to the behavior of the money multiplier in the Argentine economy. Given the inflationary problem that the Argentine economy has shown since the end of the currency board regime, as well as the association between the monetary aggregates and domestic prices, this study becomes relevant[5]. Between 2004 and 2019, the average inflation rate was 6.1% quarterly (25.2% annual), and the growth rate of M2/gdp was 5.8% (23.9% annual). The correlation coefficient between quarterly inflation and the growth rate of M2/gdp was 0.62.

[4] The correlation coefficients are not exactly the same as the ones estimated in Trupkin et al. (2017) since the Argentine data series were adjusted during 2016 (For more details follow the link https://www.indec.gob.ar/ftp/documentos/sintesis_gestion_indec_2015_2019.pdf).
[5] The negative effects of inflation on social welfare are well known from the literature (See Cooley and Hansen, 1989; Lucas, 1994, 2001; Dotsey and Ireland, 1996).

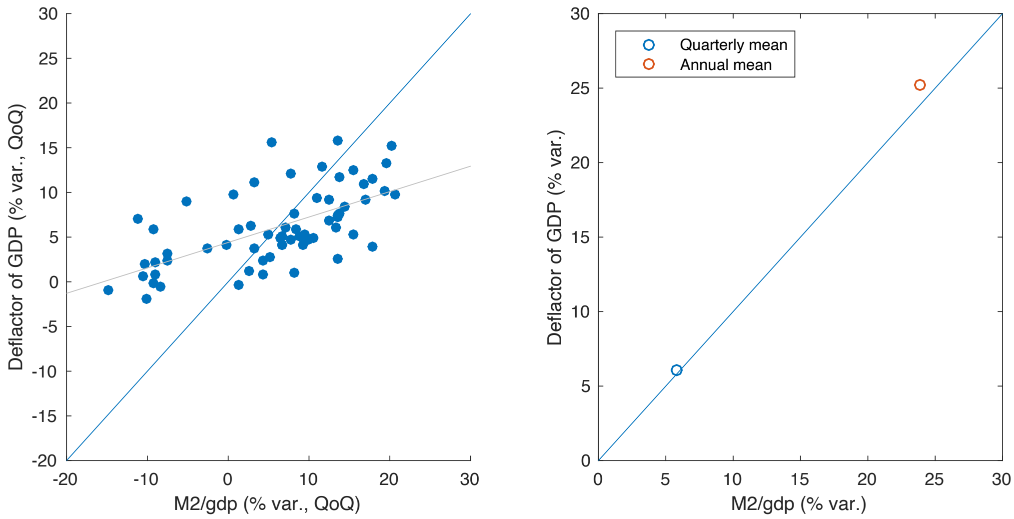

Figure 1. Inflation and money supply (short-run and long-run relationship), 2004q1-2019q4

The main objective in this thesis is to understand the relationship between monetary aggregates and the business cycle in the Argentine economy. The results obtained here may be relevant to understand better the dynamics of monetary aggregates, and consequently to design a more effective monetary policy.

The research question is if adding currency substitution into a standard small open economy RBC model with endogenous money multiplier, it may account for the relationship between the monetary aggregates and the business cycle observed in the Argentine economy. As a main hypothesis, the argument is that the currency substitution effect plays a crucial role on the real money balance decisions of the individuals, and thus it allows to explain the behavior of the money multiplier and the relationship between monetary aggregates and output in this economy.

In order to ask the research question, I extend the model of Freeman and Kydland to a small open economy version with currency substitution[6]. The incorporation of currency substitution is motivated by the not negligible participation of foreign currency deposits in the Argentina financial system, and the exchange rate regime shock as well. During the currency board period, the participation of foreign currency (U.S. dollar) deposits in the total private deposits remained relatively stable 31.3% on average. In the floating exchange regimen period, its participation grew from 2% at the beginning of 2003 to 40% at the end of 2019[7].

[6] The term 'currency substitution' means that a foreign currency gradually displaces the domestic currency as a mean of payments, and it is widespread in countries that have resorted to the inflation tax over long periods of time (see, for instance, Calvo and Vegh, 1992).

[7] A commonly used indicator proxy for currency substitution is the share of foreign-currency-denominated assets in the private sector's financial wealth (defined as M2 plus foreign-currency-denominated assets) (See Savastano, 1992). Since there is no data for foreign currency cash holdings, I calculate the ratio using only deposits.

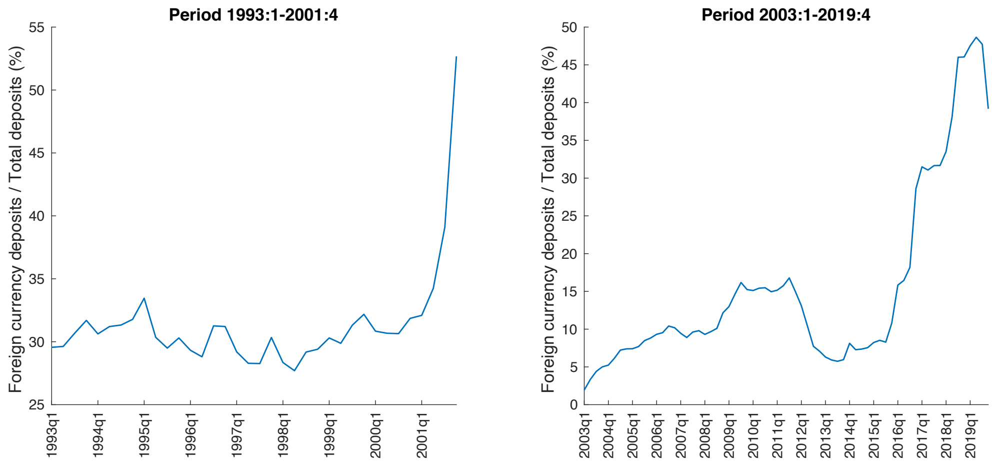

Figure 2. Foreign currency private sector deposits participation in total private sector deposits.

In the same way as the reverse causality literature, I assume totally flexible prices and quantities[8]. The monetary framework of the model is as follows. The individuals purchase consumer goods by using cash and deposits liquidity aggregators, both including domestic and foreign currency. Decisions to use each portfolio depends on the cost associated with each means of payment. The model incorporates two transaction costs. One is the cost of making money balances (necessary to determine the demand for money), since these can be replenished during each period at the expense of leisure time each time a transaction occurs. The other transaction cost is the fixed cost of using deposits (necessary to determine the division of money balances between cash and deposits). In this way, smallest purchases are made with cash, and the largest purchases are made with deposits. The individuals also choose the proportion of domestic and foreign monetary assets within each portfolio taking into account the elasticity of substitution between currencies.

I present two versions of the model (Model 1 and Model 2), in order to set the monetary policy and the exchange rate regime according to the two periods of study. Model 1 corresponds to the currency board period, where the nominal exchange rate is fixed; the monetary base growth rate is constant at steady state, but deviations from its steady state depends on the economic growth; and there are only productivity shocks. Model 2 corresponds to the floating exchange rate regime, where the nominal exchange rate is assumed full flexible (Purchasing Power Parity holds on a period-by-period basis); the monetary base growth rate is constant at steady state, but deviations from its steady state depends on past deviations and a random disturbance. Then, in Model 2 there are both productivity and monetary shocks. The rest of the model is exactly the same as Model 1. The models are calibrated according to the structural parameters of the Argentine economy for each period, and taking usual parameters values from the literature as well.

[8] Figure 1 shows the positive relationship between monetary aggregates and inflation in the Argentine economy, both in the short and long run, therefore flexible prices assumption seems to be reasonable.

The performance of the model is evaluated throughout the impulse response functions (IRF) generated by the shocks, and then comparing the statistics of stochastic simulations with the stylized facts. The correlation coefficients, standard deviations, and autocorrelation coefficients were calculated by using the cyclical components of the series, which were obtained by applying the Hodrick and Prescott (1981,1997) filter. The dataset of Argentine economy corresponds to 1993q1-2001q4 and 2003q1-2019q4, and it was obtained from the Central Bank of the Argentine Republic (BCRA), the Ministry of Economy of the Argentine Republic (MECON), the National Institute of Statistics and Census of the Argentine Republic (INDEC), the International Monetary Fund (IMF), and the World Bank (WB)[9].

The model in this thesis manages to replicate the main stylized facts of the Argentine economy during both periods characterized by different exchange rate regimes. It explains the positive relationship between monetary aggregates and output, and the different behavior of the money multiplier. In a context of fixed exchange rate regime, the reverse causality mechanism works, and the positive correlation is driven by the money multiplier. A positive shock on output increases the deposit-to-cash ratio in domestic currency, the money multiplier grows, and it drives the expansion of domestic money supply. When exchange rate is full flexible, the money multiplier reacts negatively to the productivity shock, and the positive correlation between monetary aggregates and output is driven by the currency substitution effect. This is so because individuals rebalance both portfolios towards cash and deposits in domestic currency (due to domestic prices fall and the real exchange rate remains constant), but it occurs more strongly in the case of cash liquidity aggregator than in the deposit aggregator (because elasticity of substitution is higher in the cash currency portfolio). Consequently, both the deposit-to-cash ratio in domestic currency and the money multiplier decrease. However, the money supply remains procyclical (although less correlated with output) due to the increase in the real monetary base as a consequence of the fall in domestic prices.

This thesis contributes to the existing literature in two aspects. On the theoretical side, it provides a novel explanation to the positive relationship between monetary and real variables in a context of full flexible prices and competitive markets. It is done by extending the Freeman and Kydland´s model to a small open economy version with currency substitution. On the empirical side, this thesis provides a new evidence to understand the relationship between monetary aggregates and business cycle in the Argentine economy. In particular, the model explains the switch in the sign of the correlation between the money multiplier and the output during the floating exchange rate period, something that was missing in the literature.

[9] See the Annex for a more detailed explanation of the data set, as well as the methodology used to obtain the stylized facts and the statistics from the stochastic simulations of the model.

The rest of the thesis is structured as follows. In section two, I present the theoretical model and the functional forms. The deterministic steady state is presented in section three, and the calibration strategy in section four. In section five, I show the performance of the model and the main results. Finally, in section six, I expose the conclusions of the thesis. In the annex I include the description of the data set, the methodology for calculating the stylized facts and the statistics of stochastic simulations of the model, and additional quantitative results.

2. THE MODEL

The present model assumes totally flexible prices and quantities. Consumer goods are purchased using cash and deposits liquidity aggregators, both including domestic and foreign currency. Decisions to use this deposits and cash portfolios depend on the cost associated with using each means of payment. The model only incorporates two transaction costs. One is the cost of making money balances (time) (Baumol, 1952; Tobin, 1956), necessary to determine the demand for money and to make the speed of money endogenous. Specifically, money balances can be replenished during each period at the expense of leisure time each time a transaction occurs. The other transaction cost is the fixed cost of using deposits (i.e check-clearing), necessary to determine the division of money balances between cash and deposits (which pay an interest rate). In equilibrium, deposits offer a better rate of return than cash, but the fixed cost of using deposits creates a demand for cash despite its low rate of return. In this way, while the smallest purchases are made with cash, the largest purchases are made with deposits. In addition, the individuals choose the proportion in which domestic and foreign monetary assets within each portfolio are acquired taking into account a fixed share parameter and the elasticity of substitution between currencies. Faced with these two transaction costs and the currency substitution possibility, together with other factors that vary throughout the cycle, households make decisions that determine the money multiplier, and consequently, the money supply.

3.1. Households

The economy is populated by an infinite number of identical households with an infinite life horizon and preferences determined by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \, u \left[min \left(\frac{c_t(j)}{(1-\omega)j^{-\omega}} \right), l_t \right] \qquad (1)$$

where l_t denotes leisure time, and $c_t(j)$ consumption at period t given by the Leontief function. This utility function implies that the household distributes consumption through a continuum of goods $c_t(j)$ ordered by size and indexed by $j \in [0,1]$. The Leontief ordering implies

$$c_t(j) = (1-\omega) j^{-\omega} c_t^* \qquad (2)$$

where c_t^* is the total consumption at period t, which can be verified by integrating equation (2) from $j = 0$ to 1.

In this way, substituting (2) in (1) we obtain the standard utility function of the representative household[10]:

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U(c_t^*, l_t) \qquad (3)$$

Assets available to households are non-intermediated physical capital (a_t), bank deposits in domestic currency (D_t), bank deposits in foreign currency (D_t^*), cash in domestic currency (M_t), cash in foreign currency (M_t^*), and net foreign debt position (B_t). This last asset allows households to transfer resources over time, which gives the possibility that domestic absorption differs from production.

Non-intermediated capital and foreign bonds are illiquid. Households use a combination of bank deposits and cash to buy consumer goods $c_t(j)$. Liquid monetary assets are organized by configuring a portfolio for cash and another for deposits through constant elasticity of substitution (CES) aggregators:

$$\Omega^M(M_t, e_t M_t^*) \qquad (4)$$

$$\Omega^D(D_t, e_t D_t^*) \qquad (5)$$

where e_t is the nominal exchange rate, Ω^M is the domestic-foreign currency portfolio aggregator, and Ω^D is the domestic-foreign deposits portfolio aggregator.

I include these two liquidity aggregators in order to set a generalized currency substitution approach. Substitution between domestic and foreign currency is analogous to the one used by Vegh (1995) and Holdman and Neanidis (2006). The deposits liquidity aggregator, however, is something different to existing literature. Özbilgin (2012) models currency substitution by assuming that households arrange their deposit liquidity aggregator between domestic deposits and foreign currency. This strategy is consistent with the view that foreign currency is held in developing countries as store of value (Calvo and Vegh, 1992). However, in a context where domestic interest rate is high enough to compensate the inflation rate differential, agents would not substitute foreign currency for interest-bearing domestic assets (Tanzi and Blejer, 1982), even more if they have the opportunity to hold foreign currency interest-bearing deposits.

Similar to previous literature, I assume that it is costless for households to use domestic and foreign currency for purchase, whereas a real fixed cost is incurred when foreign and domestic deposits are

[10] The setting above when $\omega = -1$ is provided by Freeman and Kydland (2000).

used. Since there is a fixed cost if deposits are used to make purchases, the rate of return on deposits net of transaction costs converges to an infinite negative value as the size of the purchases (j) converges to zero. Thus, there is a value of j^* below which money is preferred for making purchases, and above which deposits are preferred. As a result, the demand for liquid assets of the representative household is given by the following expressions:

$$n_t \frac{\Omega_t^M(M_t, e_t M_t^*)}{P_t} = \int_0^{j^*} c_t(j)dj = \int_0^{j^*} (1-\omega) j_t^{*-\omega} c_t^* dj = j_t^{*1-\omega} c_t^* \qquad (6)$$

$$n_t \frac{\Omega_t^D(D_t, e_t D_t^*)}{P_t} = \int_{j^*}^1 c_t(j)dj = \int_{j^*}^1 (1-\omega) j_t^{*-\omega} c_t^* dj = \left(1 - j_t^{*1-\omega}\right) c_t^* \qquad (7)$$

where P_t is the domestic price index.

At the beginning of each period, households choose their monetary balances with which they will purchase goods for that period. They define the relationship between cash and deposits that they consider appropriate, and it remains constant throughout the period. By making n purchases of consumer goods during the period, the households replenish money balances n times[11]. Each time it rearranges its money balances, the households incur κ units of time, such that the total time spent on transactions in period t is equal to κn_t[12].

The fact that purchases of consumer goods must be made using money or deposits functions as a "cash-in-advance" constraint insofar as the level of consumption determines the demand for monetary balances. However, there are at least three differences to consider. First, the consumption of the period is carried out with the monetary balances of that period and not with those acquired in the preceding period. Second, the velocity of money (n) is not constant but is endogenously determined. Third, both bank deposits and cash can be used to buy consumer goods, and the ratio between these is freely chosen.

The household´s budget constraint in period t is given by:

$$c_t^* + a_{t+1} + \phi(a_{t+1} - a_t) + \frac{M_t}{P_t} + \frac{e_t {M^*}_t}{P_t} + \frac{D_t}{P_t} + \frac{e_t {D^*}_t}{P_t} + r_{t-1}^b \frac{e_{t-1} B_{t-1}}{P_{t-1}} + \tau(1 - j_t^*) =$$

$$= w_t h_t + r_t a_t + r_t^d \frac{D_{t-1}}{P_{t-1}} + \frac{M_{t-1}}{P_t} + \frac{X_t}{P_t} + r_{t-1}^* \frac{e_t D_{t-1}^*}{P_t} + \frac{e_t M_{t-1}^*}{P_t} + \frac{e_t B_t}{P_t} \qquad (8)$$

or equivalently with variables in real terms

[11] n also represents the velocity of money circulation and it is endogenously determined.

[12] This transaction technology was initially introduced by Baumol (1952) and Tobin (1956), although the valuation of the cost in temporal terms was proposed by Karni (1973).

$$c_t^* + a_{t+1} + \phi(a_{t+1} - a_t) + \frac{M_t}{P_t} + \frac{e_t M^*{}_t}{P_t} + \frac{D_t}{P_t} + \frac{e_t D^*{}_t}{P_t} + r_{t-1}^b \frac{e_{t-1} B_{t-1}}{P_{t-1}} + \tau(1 - j_t^*) =$$

$$= w_t h_t + r_t a_t + r_t^d \frac{D_{t-1}}{P_{t-1}} + \frac{1}{\pi_t} \frac{M_{t-1}}{P_{t-1}} + \frac{X_t}{P_t} + r_{t-1}^* \frac{\varepsilon_t}{\pi_t} \frac{e_{t-1} D_{t-1}^*}{P_{t-1}} + \frac{\varepsilon_t}{\pi_t} \frac{e_{t-1} M_{t-1}^*}{P_{t-1}} + \frac{e_t B_t}{P_t} \qquad (9)$$

where interest rates r_t, r_t^d, r_{t-1}^*, r_{t-1}^b stand for the gross real rate of non-intermediated physical capital, the gross real rate of domestic deposits, the gross rate foreign deposits, and the gross real cost of foreign debt, respectively, at period t[13]. $\phi(.)$ is an increasing function governing the adjustment cost of capital, $\tau(1 - j_t^*)$ is the total transaction cost, w_t is the real wage, X_t is the lump sum transfers from the government, and B_t is the net external debt position. ε_t is the nominal exchange rate depreciation of domestic currency and π_t is the domestic inflation, given by

$$\varepsilon_t = \frac{e_t}{e_{t-1}} \qquad (10)$$

and

$$\pi_t = \frac{P_t}{P_{t-1}} \qquad (11)$$

The budget constraint states that the total of wage income, capital income, transfers from the government, and the value of financial assets carried over from the previous period plus the net interest receipts on these assets must finance the sum of consumption, the new asset holdings, the capital adjustment costs, and the transaction costs incurred in the current period.

The time available per household is normalized to 1. Households use the time available for leisure (l_t), work (h_t) and to replenish monetary balances $(n_t \kappa)$:

$$1 = h_t + l_t + n_t \kappa \qquad (12)$$

Finally, households are assumed to be subject to the following sequence of borrowing constraints that prevents them from engaging in Ponzi games[14]:

$$\lim_{f \to \infty} E_t \frac{b_{t+f}}{\prod_{s=0}^{f} r_s^b} \leq 0 \qquad (13)$$

The problem facing the representative household is to maximize (3) subject to (6), (7), (9), (10), and (11) by choosing streams of a_{t+1}, M_t, D_t, $M^*{}_t$, $D^*{}_t$, B_t, c_t^*, n_t, h_t, and j_t^*, given sequences of w_t, r_t^d, r_{t-1}^b, r_{t-1}^*, r_t, e_t, P_t, and X_t.

The Lagrangian corresponding to the household´s maximization problem is

[13] Interest rates are expressed as interest factors (for instance, r=1.04 implies an interest rate of 4%).

[14] This limit condition states that the household's debt position must be expected to grow at a rate lower than the interest rate in the long run.

$$
\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & u(c_t^*, h_t + n_t\kappa) + \lambda_t^1 \left[n_t \frac{\Omega_t^M(M_t, e_t M_t^*)}{P_t} - j_t^{*\,1-\omega} c_t^* \right] \\
& + \lambda_t^2 \left[n_t \frac{\Omega_t^D(D_t, e_t D_t^*)}{P_t} - \left(1 - j_t^{*\,1-\omega}\right) c_t^* \right] \\
& + \lambda_t^3 \Bigg[w_t h_t + r_t a_t + r_t^d \frac{D_{t-1}}{P_{t-1}} + \frac{M_{t-1}}{P_t} + \frac{X_t}{P_t} + r_{t-1}^* \frac{e_t D_{t-1}^*}{P_t} + \frac{e_t M_{t-1}^*}{P_t} + \frac{e_t B_t}{P_t} - c_t^* \\
& - a_{t+1} - \phi(a_{t+1} - a_t) - \frac{M_t}{P_t} - \frac{e_t M^*{}_t}{P_t} - \frac{D_t}{P_t} - \frac{e_t D^*{}_t}{P_t} - r_{t-1}^b \frac{e_{t-1} B_{t-1}}{P_{t-1}} - \tau(1 - j_t^*) \Bigg] \Bigg\}
\end{aligned}
$$

where $\beta^t \lambda_t^i$ denotes the Lagrange multiplier associated with the i sequential constraint.

The first-order conditions are the transversality conditions on assets holding with equality (11), the budget constraint (8), the liquid assets demand equations (6) and (7), and the following conditions from the choice of $c_t^*, a_{t+1}, h_t, j_t^*, n_t, M_t, M_t^*, D_t, D_t^*, B_t$, respectively

$$U_{c,t} = \lambda_t^3 + \lambda_t^1 j_t^{*(1-\omega)} + \lambda_t^2 \left[1 - j_t^{*(1-\omega)}\right] \qquad (14)$$

$$\beta \lambda_{t+1}^3 [r_{t+1} + \phi(a_{t+2} - a_{t+1})] = \lambda_t^3 [1 + \phi(a_{t+1} - a_t)] \qquad (15)$$

$$-U_{h,t} = \lambda^3{}_t w_t \qquad (16)$$

$$\lambda^3{}_t \, \tau = c_t^* \, (1-\omega) \, j_t^{*(-\omega)} \, (\lambda^1{}_t - \lambda^2{}_t) \qquad (17)$$

$$-U_{h,t} \, \kappa = \lambda^1{}_t \, \Omega_t^M + \lambda^2{}_t \, \Omega_t^D \qquad (18)$$

$$-\beta \, \lambda^3{}_{t+1} \frac{1}{\pi_{t+1}} = \lambda^1{}_t \, n_t \, \Omega_{m,t}^M - \lambda^3{}_t \qquad (19)$$

$$-\beta \, \lambda^3{}_{t+1} \frac{\varepsilon_{t+1}}{\pi_{t+1}} = \lambda^1{}_t \, n_t \, \Omega_{m^*,t}^M - \lambda^3{}_t \qquad (20)$$

$$-\beta \, \lambda^3{}_{t+1} \, r^d{}_{t+1} = \lambda^2{}_t \, n_t \, \Omega_{d,t}^D - \lambda^3{}_t \qquad (21)$$

$$-\beta \, \lambda^3{}_{t+1} \, r^* \frac{\varepsilon_{t+1}}{\pi_{t+1}} = \lambda^2{}_t \, n_t \, \Omega_{d^*,t}^D - \lambda^3{}_t \qquad (22)$$

$$\beta \, \lambda^3{}_{t+1} \, r^b{}_t = \lambda^3{}_t \qquad (23)$$

where $U_{i,t}$ denotes the partial derivative of function U with respect to variables $i = \{c_t^*, h_t\}$, $\Omega_{i,t}^M$ denotes the partial derivative of function Ω^M with respect to variables $i = \{M_t, M_t^*\}$, and $\Omega_{i,t}^D$ denotes the partial derivative of function Ω^D with respect to variables $i = \{D_t, D_t^*\}$.

Before proceeding, note that these FOCs make sense. From equation (15) and(23), we can obtain the typical equation of interest rate equivalence in small open economy models

$$r^b{}_t = \bar{r}_{t+1} = \frac{r_{t+1} + \phi(a_{t+2} - a_{t+1})}{1 + \phi(a_{t+1} - a_t)} \qquad (24)$$

where the gross real cost of foreign debt at t+1 $\left(r^{b}{}_{t}\right)$ must be equal to the effective gross real rate of return on non-intermediated physical capital including capital adjustments costs $(\bar{r}_{t+1})$.

The intra-temporal leisure-consumption trade-off can be obtained through the following steps. On the one hand, taking $\lambda^3{}_t$ from equation (16) and substituting it into equation (14), we can obtain

$$U_{c,t} = \frac{-U_{h,t}}{w_t} + \lambda_t^1 \, j_t^{*(1-\omega)} + \lambda_t^2 \left[1 - j_t^{*(1-\omega)}\right]$$

On the other hand, substituting liquid assets equations into equation (18), we get

$$-U_{h,t}\kappa \frac{n_t}{c_t^*} = \lambda^1{}_t \, j_t^{*1-\omega} + \lambda^2{}_t\left(1 - j_t^{*1-\omega}\right)$$

Then, combining these two equations, the leisure-consumption trade-off is given by

$$\frac{-U_{h,t}}{U_{c,t}}\left(1 + \frac{w_t \, \kappa \, n_t}{c_t^*}\right) = w_t \qquad (25)$$

The right-hand side of equation (25) is the opportunity cost of leisure (the real wage), and the left-hand side is the marginal rate of substitution between leisure and labor, but multiplied by a factor that measure the cost of replenishment money balances in terms of total consumption. This additional factor comes from the fact that households not only spend time to work for consumption, but also, they need an additional leisure time to replenish liquid assets necessary to pay purchases. Note that, if time cost of replenish money balances were nil, we would obtain the typical leisure-consumption condition.

The Euler equation can also be obtained easily. Taking $\lambda^3{}_t$ and $\lambda^3{}_{t+1}$ from equation (16), and substituting them into equation (15), we get

$$\beta \frac{-U_{h,t+1}}{w_{t+1}}\left[r_{t+1} + \phi(a_{t+2} - a_{t+1})\right] = \frac{-U_{h,t}}{w_t}\left[1 + \phi(a_{t+1} - a_t)\right]$$

Then, substituting the leisure-consumption trade-off condition, the Euler equation is

$$\beta \frac{U_{c,t+1}}{\left(1 + \frac{w_{t+1} \, \kappa \, n_{t+1}}{c_{t+1}^*}\right)}\left[r_{t+1} + \phi(a_{t+2} - a_{t+1})\right] = \frac{U_{c,t}}{\left(1 + \frac{w_t \, \kappa \, n_t}{c_t^*}\right)}\left[1 + \phi(a_{t+1} - a_t)\right] \; (26)$$

The present value of tomorrow´s consumption utility given by investing on physical capital must be equal to today´s consumption utility, taking into account the capital adjustment cost and the cost of liquid assets replenishment.

Equations (19)-(22) define the household´s portfolios decisions. They decide which mean of payment hold to purchase, by comparing the opportunity cost of making the purchase with domestic versus foreign currency/deposits.

On the one side, we can derive the optimality condition for domestic-foreign currency decision taking ${\lambda^3}_t$ from equation (15), substituting it into equation (19) and (20), and combining both equations

$$\frac{\Omega^M_{m,t}}{\Omega^M_{m^*,t}} = \frac{\bar{r}_{t+1} - \dfrac{1}{\pi_{t+1}}}{\bar{r}_{t+1} - \dfrac{\varepsilon_{t+1}}{\pi_{t+1}}} \qquad (27)$$

This condition means that relative marginal value for liquidity in the currency portfolio must be equal to the relative opportunity cost of holding each type of currency.

On the other side, we can derive the optimality condition for domestic-foreign deposits decision by substituting ${\lambda^3}_t$ from equation (15) into equation (21) and (22), and combining both equations

$$\frac{\Omega^D_{d,t}}{\Omega^D_{d^*,t}} = \frac{\bar{r}_{t+1} - {r^d}_{t+1}}{\bar{r}_{t+1} - r^* \dfrac{\varepsilon_{t+1}}{\pi_{t+1}}} \qquad (28)$$

This optimality condition implies that relative marginal value for liquidity in the domestic portfolio must be equal to the relative opportunity cost of holding each type of deposit.

Households decide which means of portfolio to purchase a given type of good j by comparing the opportunity cost of making the purchase with domestic and foreign deposits versus domestic and foreign currency,

$$\vartheta^d_t r^d_{t+1} + \left(1 - \vartheta^d_t\right) r^*_{t+1} \frac{\varepsilon_{t+1}}{\pi_{t+1}} - \Lambda^d_t - \frac{n_t \bar{r}_{t+1} \tau}{(1-\omega) j_t^{*-\omega} c^*_t} = \vartheta^m_t \frac{1}{\pi_{t+1}} + (1 - \vartheta^m_t) \frac{\varepsilon_{t+1}}{\pi_{t+1}} - \Lambda^m_t \quad (29)$$

where $\bar{r}_{t+1}$ stands for the gross return of capital net of capital adjustment costs. ϑ^d_t is the weight of domestic deposits in the domestic/foreign deposits portfolio, and ϑ^m_t is the weight of domestic currency in the domestic/foreign currency portfolio[15]. Λ^d_t and Λ^m_t are terms that correct for marginal rate of transformation between the assets including in each liquidity aggregator and are independent of the purchase size[16].

The right-hand side of equation (29) is the rate of return on the domestic and foreign currency portfolio. The left-hand side corresponds to the rate of return on the domestic and foreign deposits portfolio net of transaction costs. This equation defines the critical purchase size j^*_t so that larger purchases are made with deposits and smaller purchases are made with currency.

[15] $\vartheta^d_t = \frac{D_t}{D_t + e_t D^*_t}$; $\vartheta^m_t = \frac{M_t}{M_t + e_t M^*_t}$

[16] In equilibrium, $\Lambda^d_t = \vartheta^d_t \left(\bar{r}_{t+1} - r^*_{t+1} \frac{\varepsilon_{t+1}}{\pi_{t+1}}\right) \frac{\left(1 - \Omega^D_{d,t}\right)}{\Omega^D_{d^*,t}} + (1 - \vartheta^d_t)(\bar{r}_{t+1} - r^d_{t+1}) \frac{\left(1 - \Omega^D_{d^*,t}\right)}{\Omega^D_{d,t}}$

$$\Lambda^m_t = \vartheta^m_t \left(\bar{r}_{t+1} - \frac{\varepsilon_{t+1}}{\pi_{t+1}}\right) \frac{\left(1 - \Omega^M_{m,t}\right)}{\Omega^M_{m^*,t}} + (1 - \vartheta^m_t) \left(\bar{r}_{t+1} - \frac{1}{\pi_{t+1}}\right) \frac{\left(1 - \Omega^M_{m^*,t}\right)}{\Omega^M_{m,t}}$$

Finally, we can derive the optimality condition for times to visit the asset markets and replenish the liquid assets (n_t). Given $\lambda^1{}_t$ and $\lambda^2{}_t$ from equations (19) and (21) respectively, substituting them into equation (18), and given the optimality conditions for currency and deposits derived above, we can get

$$\beta\frac{-U_{h,t+1}}{w_{t+1}}\left\{\left(\bar{r}_{t+1}-\frac{1}{\pi_{t+1}}\right)\frac{M_t}{P_t}+\left(\bar{r}_{t+1}-\frac{\varepsilon_{t+1}}{\pi_{t+1}}\right)\frac{e_tM_t^*}{P_t}+\left(\bar{r}_{t+1}-r_{t+1}^d\right)\frac{D_t}{P_t}+\left(\bar{r}_{t+1}-r_{t+1}^*\frac{\varepsilon_{t+1}}{\pi_{t+1}}\right)\frac{e_tD_t^*}{P_t}\right\}$$
$$= -U_{h,t}\,\kappa\, n_t \qquad (30)$$

The right-hand side of equation (30) is the time cost of replenish n times the liquid assets. The left-hand side corresponds to the present value time cost of holding liquid assets during period t and t+1. When the opportunity cost of holding assets increases (decreases), it is optimal to households decrease (increase) liquid assets holdings and go to the assets markets more (less) times during the period to replenish money balances.

3.2. Firms

The firm´s problem is entirely standard. There exists a large number of firms that hire labor and rent capital to produce capital and consumption goods of every type j. They operate in perfectly competitive product and factor markets, and the production technology is given by

$$y_t = A_tF(k_t, h_t) \qquad (31)$$

Total factor productivity A_t evolves according to the following stochastic process,

$$A_{\mathrm{t}} = e^{z_{\mathrm{t}}} \qquad (32)$$

$$z_{\ t} = \rho_z z_{\ t-1} + e_{z,t} \qquad (33)$$

where $e_{z,t}$ is a white noise random variable with standard deviation σ_z and $0 < \rho_z < 1$ measures the shock persistence.

The firm's problem in each period t is to choose h_t and k_t to maximize real profits

$$\prod_t^F = y_t - r_t^k k_t - w_t h_t$$

taking the factor prices w_t and r_t^k as given. Then, first-order conditions associated with the profit maximization problem are

$$A_tF_k(k_t, h_t) = r_t^k \qquad (34)$$

$$A_tF_h(k_t, h_t) = w_t \qquad (35)$$

and the effective gross real rate of return on capital is therefore

$$r_t = r_t^k + 1 - \delta \qquad (36)$$

where $\delta \in (0,1)$ denotes the rate of depreciation of physical capital.

As usual, the capital stock k_t evolves according to

$$k_{t+1} = i_t + (1-\delta)k_t \qquad (37)$$

where i_t denotes gross investment. The physical capital available for each household to produce in period t + 1 is equal to the sum of the capital created in t and the capital stock of the previous period, net of depreciation.

3.3. Banks

The bank´s problem is easy to describe, and it is similar to Dressler (2007). There exist a large number of competitive banks which can be represented by a single representative bank, which collects deposits from households. Banks hold a fraction $\theta_d \in (0,1)$ of domestic currency deposits as legal reserves, and invest the rest in physical capital (intermediated capital). I assume for simplicity that banks invest the foreign currency deposits in the international financial market, and they obtain an interest rate lower than the real interest rate of foreign bonds, but higher than a free risk interest rate[17].

The expected proceeds of a bank in period t+1 is given by

$$\Pi^B_{t+1} = r_{t+1}(1-\theta_d)d_t + \frac{1}{\pi_{t+1}}\theta_d d_t + r^*_{t+1}\frac{\varepsilon_{t+1}}{\pi_{t+1}}d^*_t - r^d{}_{t+1}d_t - r^{d^*}_{t+1}d^*_t$$

Free entry of financial intermediaries ensures a zero profit. Then, the rate of return on domestic deposits is a linear combination of the return on physical capital and the return on domestic monetary assets:

$$r^d{}_{t+1} = (1-\theta_d)r_{t+1} + \theta_d\frac{1}{\pi_{t+1}} \qquad (38)$$

And the real interest rate of foreign currency deposits is the same as the bank obtains in international financial markets:

$$r^{d^*}_{t+1} = r^*_{t+1}\frac{\varepsilon_{t+1}}{\pi_{t+1}}$$

The clearing of the asset market for capital requires that the capital stock per household must equal the sum of capital held directly by the households and capital held by banks on behalf of each household:

[17] The interest rate obtained by the bank when investing household´s foreign currency deposits can be assimilated to an international interest rate of a similar country, but with a lower risk premium. This interest rate is set to match the steady state ratio of deposits in foreign currency over total consumption with the data.

$$k_{t+1} = a_{t+1} + (1-\theta_d)\frac{D_t}{P_t} \qquad (39)$$

Combining (39) and (37), total investment is given by

$$i_t = a_{t+1} + (1-\theta_d)\frac{D_t}{P_t} - (1-\delta)\left[a_t + (1-\theta_d)\frac{D_{t-1}}{P_{t-1}}\right]$$

3.4. Government

The government sets the nominal stock of monetary base (MB_t) in each period.

$$MB_t = \chi_t MB_{t-1} \qquad (40)$$

where χ_t is the nominal monetary base growth rate.

A process for the monetary base needs to be specified. Following Özbilgin (2012), I assume that the nominal monetary base growth rate is the sum of a constant value χ_{ss} and the deviation from its steady state $\hat{g}_t$. Then,

$$\chi_t = \chi_{ss} + \hat{g}_t \qquad (41)$$

Deviation of nominal monetary base growth rate from its steady state is defined differently in order to setting properly the monetary policy in each period of study.

Monetary base in Model 1

The first period is characterized by the currency board regime, i.e. nominal exchange rate is fixed, and the government does not control the money supply. However, according to the data, there is a strong positive relationship between the monetary base growth rate and the economic growth. In order to replicate this behavior, I assume that monetary base growth rate deviations from its steady state depends on the economic growth [18]

$$\hat{g}_t = \rho_{m1}\hat{y}_t \qquad (42.1)$$

where ρ_{m1} measures the correlation between economic growth and monetary base growth deviations from steady state value.

[18] Under this exchange rate regime, it is clear that the positive relationship between the growth rate of monetary base and economic growth comes from foreign reserves accumulation due to positive balance of payments. However, the main purpose of this study is to explain the relationship between monetary aggregates and business cycle through the money multiplier and household´s currency substitution behavior. Therefore, I adopt this simple equation for monetary base in order to avoid balance of payments dynamics and focus on the money multiplier and currency substitution effects.

Monetary base in Model 2

In the floating exchange rate period, currency board is no longer valid. Then, nominal exchange rate is flexible and the government can control the monetary base. Following Özbilgin (2012), I assume that deviations of the monetary base growth rate from its steady state depends on past deviations and a random disturbance[19]

$$\hat{g}_t = \rho_{m2}\hat{g}_{t-1} + e_{m2,t} \qquad (42.2)$$

where $e_{m2,t}$ is a white noise random variable with standard deviation σ_{m2}, and ρ_{m2} measures the persistence of the monetary shock.

Changes in the monetary base are transferred to households as lump sum of subsidies of X_t units of domestic currency to each household:

$$X_t = (\chi_t - 1)MB_{t-1} \qquad (43)$$

Money market

The clearing of the market for domestic fiat money requires that the stock of fiat money equals the combined stocks of domestic currency and reserves:

$$MB_t = M_t + \theta_d D_t \qquad (44)$$

Then, the total stock of domestic money (M2) can be expressed as the product of the monetary base and the money multiplier:

$$M2_t = MB_t\, mm_t \qquad (45)$$

where $mm_t = \left[\frac{\frac{M_t}{D_t}+1}{\frac{M_t}{D_t}+\theta_d} \right]$ is the domestic money multiplier. It is closely related to the domestic currency-to-deposit ratio, $\frac{M_t}{D_t}$, but with an adjustment for that part of the base that serves as reserves.

Finally, total domestic-foreign fiat money clearing condition is

$$\frac{M_t}{P_t} + \frac{D_t}{P_t} + \frac{e_t M^*{}_t}{P_t} + \frac{e_t D^*{}_t}{P_t} = \frac{\chi_t}{\pi_t}\frac{MB_{t-1}}{P_{t-1}} mm_t + \frac{\varepsilon_t}{\pi_t}\frac{e_{t-1}M^*_{t-1}}{P_{t-1}} + r^*_{t-1}\frac{\varepsilon_t}{\pi_t}\frac{e_{t-1}D^*_{t-1}}{P_{t-1}} \qquad (46)$$

where the right-hand side of equation (46) is the domestic and foreign currency supply in real terms, and the left-hand side corresponds to the domestic and foreign currency demand, in real terms as well. This equation is relevant to determining prices dynamics.

[19] This specification allows the monetary base growth rate to display persistence, a similar behavior that we can observe in data. The level of the monetary base growth rate, and the process that characterizes its behavior corresponds to different monetary policies.

Exchange rate policy

By definition, the real exchange rate is given by

$$rer_t = \frac{e_t P_t^*}{P_t} \qquad (47)$$

and the variation of real exchange rate is

$$vrer_t = \frac{\varepsilon_t \pi_t^*}{\pi_t} - 1 \qquad (48)$$

To setting the model according to the data, I define two different exchange rate policies. The Model 1 is set to replicate the currency board period in which the nominal exchange rate is fixed

$$e_t = 1 \qquad (49.1)$$

To the contrary, Model 2 is set to replicate the second period in which the nominal exchange rate is flexible. For simplicity, I assume that the purchasing power parity (PPP) holds on a period-by-period basis

$$e_t = \frac{P_t}{P_t^*} \qquad (49.2)$$

Then, given $\pi_t^* = 1$, variation of real exchange rate in Model 1 is

$$vrer_t = \frac{1}{\pi_t} - 1 \qquad (50)$$

and the variation of real exchange rate in Model 2 is nil since nominal exchange rate follows the domestic inflation ($\varepsilon_t = \pi_t$).

3.5. External sector

Individuals in the economy have access to a perfectly competitive international capital market. I follow Schmitt-Grohé and Uribe (2003) and assume that the real international interest rate (r^b) at which individuals can finance themselves abroad is increasing with respect to the level of external debt[20]

$$r_t^b = r^{wr} + rp(\tilde{b}_t) \qquad (51)$$

where r^{wr} is the risk-free international interest rate, $\tilde{b}_t$ is the net real external debt per capita position, and $rp(.)$ is the country-specific interest rate premium. The function $rp\,(.)$ is assumed to be strictly

[20] This specification is called EDEIR (External Debt-Elastic Interest Rate), and it is a usual strategy used in SOE model to induce stationarity.

increasing of $\tilde{b}_t$. In equilibrium, the net real external debt per capita position is equal to the one for the representative individual:

$$\tilde{b}_t = b_t = \frac{e_t B_t}{P_t} \qquad (52)$$

By definition, the equilibrium process of the trade balance is given by

$$tb_t \equiv y_t - c_t - i_t + \frac{\phi}{2}(a_{t+1} - a_t)^2 + \tau(1 - j_t^*) \qquad (53)$$

where tb_t denotes the trade balance in period t.

Finally, the current account is given by the sum of the trade balance and net investment income

$$ca_t = tb_t - (r_{t-1}^b - 1)b_{t-1} + \left(r_{t-1}^* \frac{\varepsilon_t}{\pi_t} - 1\right)\frac{e_{t-1}D_{t-1}^*}{P_{t-1}} \qquad (54)$$

3.6. Equilibrium

A competitive equilibrium in the decentralized economy is a set of process $\{c_t^*, a_{t+1}, k_{t+1}, i_t, z_t, y_t, X_t, h_t, B_t, \hat{g}_t, MB_t, M_t, M_t^*, D_t, D_t^*, j_t^*, n_t, \lambda_t^1, \lambda_t^2, \lambda_t^3\}_{t=0}^{\infty}$, and sequences of prices $\{w_t, r_t, r_t^d, r_t^b, P_t, e_t\}_{t=0}^{\infty}$, satisfying budget constraint (9), nominal exchange rate variation (10), domestic inflation (11), no-Ponzi-game constraint holding with equality (13), first-order conditions for households (14) - (23), production function (31), productivity shock equations (32) and (33), optimality condition for firms (35) and (36), law of motion of physical capital (37), non-profit condition for banks (38), market-clearing condition of asset for capital (39), monetary policy equations (40), (41), (42.1) (or 42.2), and (43), market-clearing condition for domestic fiat money (44), market-clearing condition for total fiat money (46), exchange rate policy (49.1) (or 49.2), foreign debt interest rate equation (51), and equilibrium level of debt (52), given the world interest rate (r^{wr}), the world price level (P^*), the foreign inflation (π^*), the real interest rate of foreign currency deposits (r^*), the initial values $a_0, k_0, z_{-1}, B_{-1}, \hat{g}_{-1}, MB_{-1}, M_{-1}, M_{-1}^*, D_{-1}, D_{-1}^*, P_{-1}, e_{-1}, r_{-1}^b$, and the exogenous shocks process $\{\mathrm{e}_{\mathrm{z,t}}, \mathrm{e}_{\mathrm{m,t}}\}_{\mathrm{t=0}}^{\infty}$.

3.7. Functional forms

The household maximizes its expected utility at each moment according to the following utility function:

$$U(c_t^*, l_t) = \frac{\left(c_t^* - s\frac{\tilde{h}_t^{\zeta}}{\zeta}\right)^{1-v} - 1}{1 - v} \qquad (55)$$

where $\tilde{h}_t = 1 - l_t = h_t + n_t\kappa$ is the time devoted to work and replenishment the money balances, s is a scale parameter for time spent on market activities, ζ is the parameter that governs the intertemporal elasticity in the labor supply, and v gives the coefficient of relative risk aversion. The form of this utility function is due to Greenwood, Hercowitz, and Huffman (1988) and is typically referred to as GHH preferences[21].

Liquid monetary assets are organized by configuring a portfolio for cash and another for deposits through constant elasticity of substitution (CES) aggregators[22]:

$$\Omega^M(M_t, e_t M_t^*) = \left(\gamma_m\, {M_t}^{\xi_m} + (1-\gamma_m)\,(e_t M_t^*)^{\xi_m}\right)^{\frac{1}{\xi_m}} \qquad (56)$$

$$\Omega^D(D_t, e_t D_t^*) = \left(\gamma_d\, {D_t}^{\xi_d} + (1-\gamma_d)\,(e_t D_t^*)^{\xi_d}\right)^{\frac{1}{\xi_d}} \qquad (57)$$

The parameters γ_m and γ_d governs the share of domestic-foreign currency and domestics-foreign deposits, and ξ_m and ξ_d governs the substitutability between the two assets in each portfolio.

The capital adjustment cost function is assumed to be quadratic

$$\phi(a_{t+1} - a_t) = \frac{\phi}{2}(a_{t+1} - a_t)^2 \qquad (58)$$

where $\phi > 0$. This specification is borrowed from Mendoza (1991) and it implies that net non-intermediated capital investment, whether positive or negative, generates resource costs.

The production technology is the standard constant return to scale function of the two inputs, given by

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha} \qquad (59)$$

where $\alpha \in (0,1)$. Then, first-order conditions associated with the profit maximization problem are

$$\alpha A_t {k_t}^{(\alpha-1)} {h_t}^{(1-\alpha)} = r_t^k \qquad (60)$$

$$(1-\alpha) A_t k_t^{\alpha} h_t^{-\alpha} = w_t \qquad (61)$$

Finally, following Schmitt-Grohé and Uribe (2003), the functional form that determines the country risk premium is the following

$$rp(b_t) = \overline{rp} + \psi(e^{b_t - \bar{b}} - 1) \qquad (62)$$

where $\overline{rp} > 0$ is a permanent risk premium value, $\psi > 0$ is the additional risk premium parameter and $\bar{b}$ is the steady state level of the real value of B. At steady state ($b = \bar{b}$), and the risk premium is equal to $\overline{rp}$.

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[21] This utility function is usual in SOE models since it helps to mimic business cycles (see Uribe and Schmitt-Grohé, 2017).
[22] This function is borrowed from Selcuk (2003) and it is usual in models with currency substitution.

4. DETERMINISTIC STEADY STATE

The characterization of the deterministic steady state is interesting because it facilitates the calibration of the model, and it is a convenient starting point around which equilibrium conditions of the stochastic economy are approximated.

A deterministic economy is defined by assuming that the total factor productivity and the monetary base growth rate are constant. At steady state, nominal variables growth at the same rate as domestic prices. Then, all endogenous variables are constant in real terms.

Combining the Euler equation (26) with the interest rate equivalence (24), the international interest rate definition (51) and the risk premium equation (62), at steady state we obtain

$$1 = \beta\left[r^{wr} + \overline{rp} + \psi(e^{b_t - \bar{b}} - 1)\right]$$

I assume that

$$1 = \beta\,(r^{wr} + \overline{rp})$$

Combining these two restrictions, the steady state value of foreign debt is $b = \bar{b}$.

From equation (24), the foreign debt interest rate and the effective gross rate of return on physical capital are identical at steady state

$$r^b = r$$

The rate of return on domestic deposits is obtained from equation (38) evaluating at steady state, and it is a linear combination of return on physical capital and return on domestic monetary assets

$$r^d = (1 - \theta_d)r + \theta_d \frac{1}{\pi}$$

From equation (27), taking the derivatives using the functional forms (56), we can get the steady state ratio of domestic-foreign currency

$$\frac{M}{eM^*} = \left[\frac{\left(r - \frac{1}{\pi}\right)(1 - \gamma_m)}{\left(r - \frac{\varepsilon}{\pi}\right)\gamma_m}\right]^{\left(\frac{1}{(\xi_m - 1)}\right)}$$

In the same way, from equation (28), taking the derivatives using the functional forms (57), we can get the steady state ratio of domestic-foreign deposits

$$\frac{D}{eD^*} = \left[\frac{(r - r^d)(1 - \gamma_d)}{\left(r - r^*\frac{\varepsilon}{\pi}\right)\gamma_d}\right]^{\left(\frac{1}{(\xi_d - 1)}\right)}$$

where nominal exchange rate variation at steady state is $\varepsilon = 1$ in Model 1 and $\varepsilon = \pi$ in Model 2.

The capital-labor ratio can be obtained combining equations (26) (36) and (60) evaluating at steady state. Then,

$$\frac{k}{h} = \left[\frac{\beta^{-1} - (1-\delta)}{\alpha}\right]^{\frac{1}{\alpha-1}}$$

We can also obtain the steady state value of real wage by substituting this capital-labor ratio into equation (61)

$$w = (1-\alpha)\left[\frac{\beta^{-1} - (1-\delta)}{\alpha}\right]^{\frac{\alpha}{\alpha-1}}$$

As typical in models with one country and one sector, the steady state values of all variables cannot be pinned down given the values of the parameters. For instance, the steady state level of time devoted to work and replenishment the money balances $(\tilde{h}_t)$ can be derived from equation (25) by implementing the utility functional form (55). At steady state, $\tilde{h}$ is given by

$$\tilde{h} = \left[\frac{w}{s\left(1 + \frac{w\,\kappa\,n}{c^*}\right)}\right]^{\frac{1}{\zeta-1}}$$

where it depends not only on parameters but also on steady state values of the cost of replenishment money balances in terms of total consumption $\frac{w\,\kappa\,n}{c^*}$[23].

In order to obtain that replenishment cost, we can take equation (30) evaluated at steady state, and divide both sides by c^*

$$\beta\left[\left(r - \frac{1}{\pi}\right)\frac{M}{Pc^*} + \left(r - \frac{\varepsilon}{\pi}\right)\frac{eM^*}{Pc^*} + \left(r - r^d\right)\frac{D}{Pc^*} + \left(r - r^*\frac{\varepsilon}{\pi}\right)\frac{eD^*}{Pc^*}\right] = \frac{w\,\kappa\,n}{c^*}$$

I enforce steady state values for domestic currency-to-consumption ratio, foreign currency-to-consumption ratio, domestic deposit-to-consumption ratio, and foreign deposits-to-consumption ratio, all of them being equal to their sample average for each period.

Given the steady state ratios imposed above, we can get the values for j^* and n solving the equation system of constraints (6) and (7), and using the functional forms (56) and (57). Then,

$$j^* = \left(\frac{\frac{\Omega^D}{Pc^*}}{\frac{\Omega^M}{Pc^*}} + 1\right)^{-\frac{1}{1-\omega}}$$

[23] Note that, if time cost of replenish money balances were nil, the scale parameter for time spent on market activities would not be necessary, and we would obtain the typical steady state expression for working hours.

$$n = \frac{j^{*(1-\omega)}}{\frac{\Omega^M}{Pc^*}}$$

Given the steady state value of n and $\tilde{h}$, we can derive the time devoted to work

$$h = \tilde{h} - n\kappa$$

Then, we can obtain the steady state values for k using the capita-to-labor ratio equilibrium, y using the production function (59), and i from equation (37).

Since c^* is closely related to $\tilde{h}$, given w and n, we can derive the consumption from the steady state equation of $\tilde{h}$. Once we get c^*, all monetary variables in real terms can be derived. We can obtain the steady state value of non-intermediated physical capital a using the clearing-asset market equation for capital (39).

The steady state for domestic inflation is defined by the clearing-market condition for total domestic-foreign fiat money (46). Then,

$$\pi = \chi_{ss}\left[\frac{\frac{M}{P}+\frac{D}{P}}{\frac{M}{P}+\frac{D}{P}+\frac{eM^*}{P}+\frac{eD^*}{P}}\right] + \varepsilon\left[\frac{\frac{eM^*}{P}+r^*\frac{eD^*}{P}}{\frac{M}{P}+\frac{D}{P}+\frac{eM^*}{P}+\frac{eD^*}{P}}\right]$$

Note that it is a linear combination of monetary base growth rate and nominal exchange rate variation, weighted by the participation of domestic money and foreign money over the total fiat money available.

Finally, the steady state level of trade balance and current account can be obtained by evaluating equilibrium conditions (53) and (54) at steady state

$$tb = y - c - i + \tau(1 - j^*)$$

$$ca = tb - (r^b - 1)b + \left(r^*\frac{\varepsilon}{\pi} - 1\right)\frac{eD^*}{P}$$

5. CALIBRATION

The calibration of the model is carried out according to the structural parameters of the Argentine economy for both periods of study. As in the related business cycle literature, I adopt the calibration strategy of Mendoza (1991), similar strategy implemented by the classical literature on small open economy models such as García-Cicco, et al. (2010) and Uribe, et al. (2017). The time unit in the model is meant to be one quarter.

Table 2. Calibration of the models

Parameter	Description	Model 1	Model 2
α	Share of capital stock	0.36	0.36
δ	Rate of depreciation	0.022	0.022
β	Subjective discount factor	0.981	0.981
ζ	Utility function share scale parameter	1.6	1.6
ν	Coefficient of relative risk aversion	2	2
s	Labor supply scale parameter	4.145	4.070
r^{wr}	Gross risk-free world real interest rate	1.009	1.002
rp	Constant risk-premium spread	0.011	0.017
rp_2	Constant risk-premium spread	0.007	0.015
r^f	Gross real interest rate for foreign currency deposits	1.016	1.018
κ	Asset market trip cost	0.002	0.014
τ	Transaction cost	0.016	0.065
ϕ	Coefficient of capital adjustment cost function	0.0075	0.0068
γ_m	Share parameter in cash liquid assets aggregator	0.73	0.85
γ_d	Share parameter in deposits liquid assets aggregator	0.47	0.86
ξ_m	Substitution parameter in cash liquid assets aggregator	0.09	0.76
ξ_d	Substitution parameter in deposits liquid assets aggregator	0.01	0.01
ω	Leontief utility parameter	-1.5	-1.5
θ	Reserve requirement ratio	0.19	0.16
χ_{ss}	Gross monetary base growth rate at steady state	0.9921	1.040
π^f	Gross foreign inflation rate	1	1
ψ	Coefficient of external debt interest-rate risk premium function	0.003	0.003
$\bar{b}$	Net foreign debt position at steady state	-0.73	0.305
ρ_z	Persistence of the productivity shock	0.75	0.62
σ_z	Standard deviation of the productivity shock	0.014	0.0125
ρ_m	Coefficient of monetary base equation	0.787	0.259
σ_m	Standard deviation of the monetary shock	-	0.021

The free-risk world real interest rate r^{wr} is set according to the average of T-10y USA real interest rate for each period, i.e., 3,5% annual in Model 1, and 1% annual in Model 2.

Constant risk-premium for external debt rp is set in Model 2 according to the average of EMBI+Arg indicator, i.e., 7% annual. In Model 1, this parameter is set in order to get the same steady state capital-to-output ratio in both models. The risk premium for foreign currency deposits rp_2 was calibrated to match $\frac{eD^*}{Pc}$ to data. Interest rate for foreign currency deposits r^f is a constant variable given by

$$r^f = r^{wr} + rp_2$$

The discount factor β is calibrated according to steady state value of r^b. Then,

$$\beta = \frac{1}{r^{wr} + rp}$$

The inverse of the intertemporal elasticity of substitution in consumption ν and the utility function share scale parameter ζ are common values in the literature. I set them following García-Cicco, et al. (2010). The value of parameter ζ implies an elasticity of substitution equal to 1.7. The Leontief utility parameter ω is set following Özbilgin (2012). Labor supply scale parameter s is calibrated to get leisure time equal to 0.66 at steady state.

Capital-income share α is standard, and I set the same value as García-Cicco, et al. (2010). Capital depreciation rate δ is calibrated to match investment-to-output ratio according to the average value of data $\frac{i}{y} = 0.19$. Capital adjustment cost parameter ϕ is calibrated such that the model can generate observed investment volatility in the data.

The Solow´s residual is not possible to be calculated due to lack of data. As a result, the parameters that characterize the real shock process are calibrated such that the model can mimic the persistence and standard deviation of the GDP in each period. Therefore, the parameters ρ_z and σ_z are estimated using the cyclical component series of the GDP. The estimated values are close to the values observed in the business cycle literature.

The parameters of liquidity aggregators are difficult to calibrate since there is no data about the amount of foreign currency held by individuals. I follow Özbilgin (2012) to approximate the share parameter in deposits liquid assets aggregator γ_d and the share parameter in cash liquid assets aggregator γ_m with the share of domestic deposits in total deposits, and the share of checkable and savings domestic deposits in total checkable and savings deposits, respectively.

$$\gamma_d = \frac{D}{D + eD^*}$$

$$\gamma_m = \frac{M2 - M}{M2b - M}$$

where M is domestic currency in circulation, $M2$ is domestic currency in circulation plus checkable and savings domestic currency deposits, and $M2b$ is domestic currency in circulation plus total domestic-foreign checkable and savings deposits.

The substitution parameter in cash liquid assets aggregator ξ_m is calibrated to match $\frac{eM^*}{Pc}$ to data. Values in Model 1 and Model 2 are different and they imply an elasticity of substitution equal to 1 and 4, respectively. The substitution parameter in deposit liquid assets aggregator ξ_d is set following Uribe (1999). This value implies an elasticity of substitution equal to 1 and the aggregator takes the Cobb-Douglas form.

Transaction cost τ and trip to the assets market cost κ are hard to estimate. Same as I assumed in the deterministic the steady state section, I enforce steady state values for domestic currency-to-consumption ratio, foreign currency-to-consumption ratio, domestic deposit-to-consumption ratio, and foreign deposits-to-consumption ratio, all of them being equal to their sample average for each period. Then, I calibrate τ by combining equations (17), (19) and (21) evaluated at steady state[24].

$$\tau = \frac{(1-\omega) j^{*-\omega}}{n\, r}\left[\frac{\left(r - {}^{1}/_{\pi}\right)}{\frac{\Omega_m^M}{c^*}} - \frac{\left(r - r^d\right)}{\frac{\Omega_d^D}{c^*}}\right]$$

κ can be calibrated as follow. First, the ratio $\frac{\kappa}{c^*}$ is given by equation (30) evaluated at steady state and imposing the steady state ratios mentioned above

$$\frac{\kappa}{c^*} = \frac{\beta}{w\, n}\left[\left(r - \frac{1}{\pi}\right)\frac{M}{Pc^*} + \left(r - \frac{\varepsilon}{\pi}\right)\frac{eM^*}{Pc^*} + \left(r - r^d\right)\frac{D}{Pc^*} + \left(r - r^*\frac{\varepsilon}{\pi}\right)\frac{eD^*}{Pc^*}\right]$$

Second, we can obtain the steady state value for consumption-to-output ratio dividing budget constraint by output, substituting trade balance-to-output ratio and investment-to-output ratio by sample average values, and substituting $\frac{\tau}{c^*}$ from equation used above to calibrate τ[25]. Then,

$$\frac{c^*}{y} = \left(1 - \frac{i}{y} - \frac{tb}{y}\right)\left[\frac{1}{\left(1 + \frac{\tau}{c^*}(1 - j^*)\right)}\right]$$

Third, from time constraint and after some steps we can get

$$y = \frac{w\tilde{h}}{(1-\alpha) + \frac{c^*}{y}\frac{w\,\kappa\, n}{c^*}}$$

Given the steady state values for w, $\frac{c^*}{y}$, $\frac{w\,\kappa\, n}{c^*}$, and enforcing $\tilde{h} = 0.33$ as usual in literature, we obtain the steady state for output. Finally, we can obtain directly the parameter κ by combining these three steady state results[26].

Monetary base growth rate at steady state χ_{ss} is calibrated in order to match the steady state value of domestic inflation π_{ss} with the sample average of data. Foreign inflation is set nil in both periods, then $\pi^f = 1$.

The parameters that characterize the monetary policy are estimated differently in each model. In Model 1, the parameter ρ_{m1} comes from the OLS estimation of equation (42.1). In Model 2, ρ_{m2} is estimated

[24] τ is the cost of using deposits as a mean of payment. At steady state, its value should equal the benefits of using deposits, which arises from the difference in interest rates between deposits and cash. The estimated value for τ implies that the cost of using deposits for purchases amounts to 0.66% and 3% of the GDP for the Model 1 and Model 2, respectively. While the calibrated value for the currency board period is in line with previous studies (Özbilgin, 2012), the value for the second period is much higher. This difference can be explained by the lower confidence of individuals in the financial system after the economic crisis of 2001.

[25] It is possible to derive $\frac{\tau}{c^*}$ from equations (17), (19) and (21) since $\Omega_m^M(M, eM^*) \equiv \Omega_m^M\left(\frac{M}{c^*}, \frac{eM^*}{c^*}\right)$.

[26] κ value for Model 1 and Model 2 represent 6 and 17 minutes a day, respectively. Values are similar to the ones estimated in the literature (See Özbilgin, 2012).

according to OLS estimation of equation (42.2) and the standard deviation σ_m is calibrated to match the domestic inflation standard deviation to the data. The reserve requirement ratio for domestic deposits θ is calibrated according to average data in each period.

The risk-premium parameter ψ is set to a value close to cero similar to previous literature, and it ensure a stationarity solution. Net foreign debt steady state level $\bar{b}$ is calibrated to match steady state trade balance-to-output ratio with the sample average.

Finally, the nominal exchange rate in Model 1 is constant and it is set equal to 1.

6. PERFORMANCE OF THE MODEL

Once the parameters were calibrated, the dynamics responses of the models and the quantitative predictions can be explored.

The impulse-response functions allow us to appreciate the dynamics of the variables of each model in the face of a positive productivity shock of one standard deviation. Values are shown in terms of proportional deviation from their steady state value.

The responses of the main real variables of the model are identical in the model here, as they would be in an SOE-RBC model without money explicitly included. A positive productivity shock increases the level of output, and consequently, the return of physical capital and the marginal productivity of labor. The income of individuals grows, consumption increases, and the number of hours of work grows as well. Total investment initially expands due to the higher level of both, the non-intermediated physical capital and the intermediated physical capital (given by the growth of deposits in domestic currency).

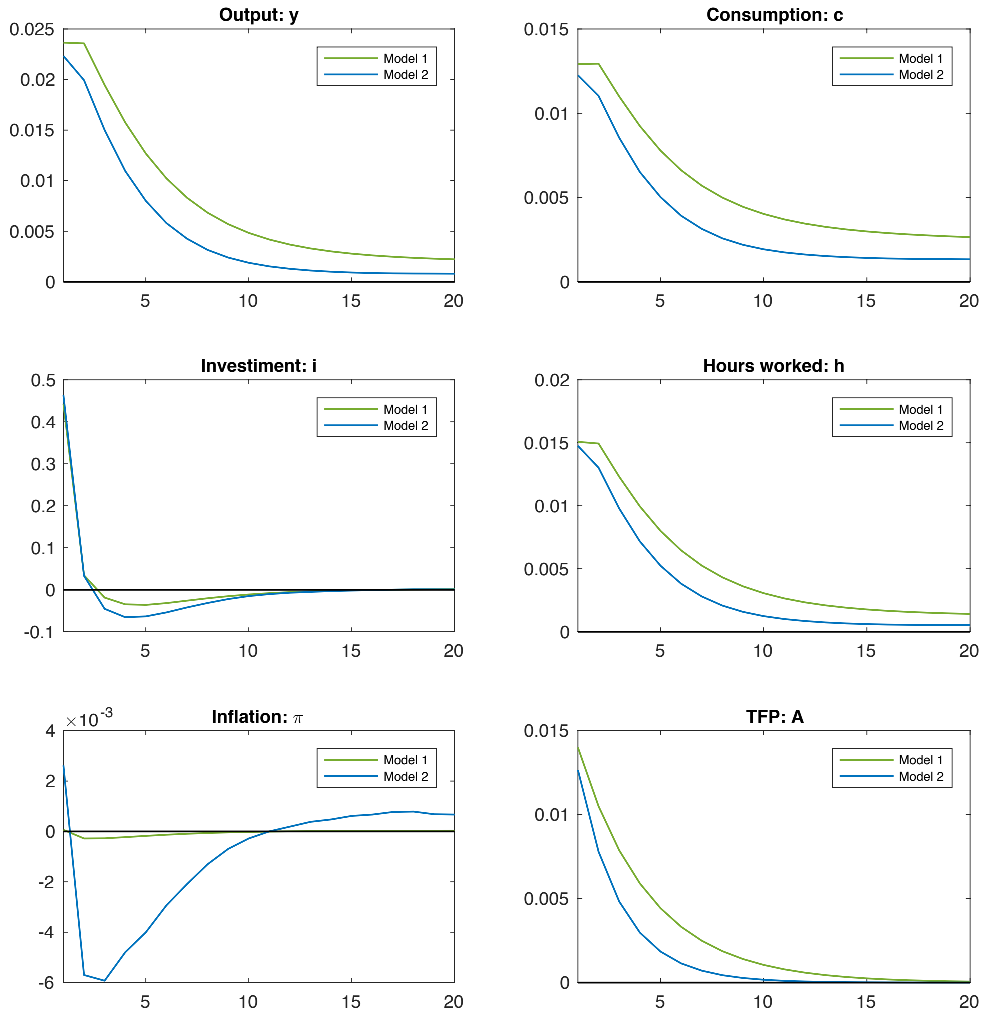

Figure 3: Impulse response functions of real variables (productivity shock).

In this model, the price level is determined as a result of two opposing effects. On the one hand, the wealth effect generated by the shock prompts households to increase their real money balances, which pushes prices down. This effect is reinforced because the higher marginal productivity of labor increases the opportunity cost of the time spent in transactions κn, further increasing the demand for real money balances in order to reduce the number of replacements n. On the other hand, the productivity shock also affects the performance of monetary assets, generating a rebalancing of portfolios. In the case of Model 1, the growth in the interest rate on deposits reduces the demand for cash, both in domestic and foreign currency. In the case of Model 2, the demand for external monetary assets is reduced. In either case, the reduction in demand for these monetary assets drives the rise in the price level. Of these two opposing effects on the price level, the first predominates (the wealth effect), which explains the countercyclical behavior of inflation in the face of a positive technological shock.

The external variables also respond as in the SOE-RBC models. The productivity shock increases the net external debt position to finance part of the increase in consumption. Trade-balance-to-output ratio deteriorates, and the deficit in current account emerges due to the higher level of external debt and the rise of international interest rate as well. In Model 1, the real exchange rate depreciation reflects the decrease in the domestic prices.

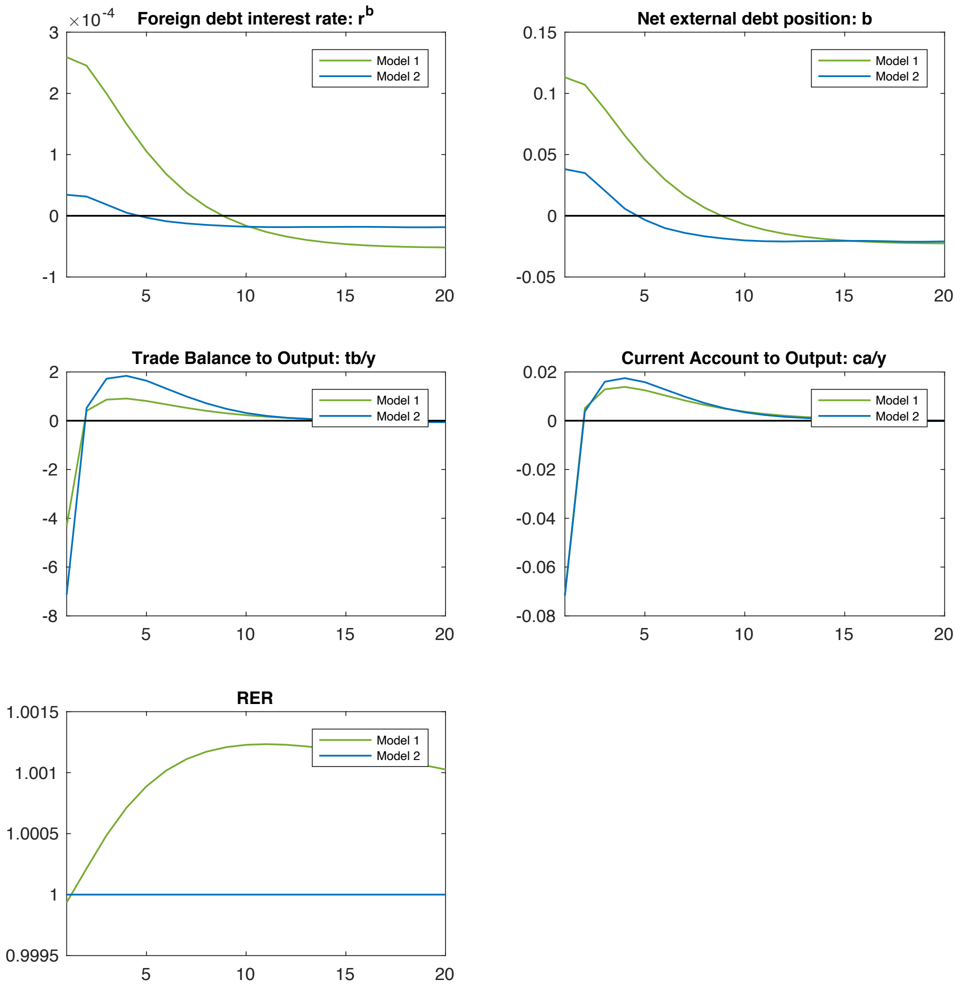

Figure 4: Impulse response functions of external sector variables (productivity shock).

The following figure shows the response of the real return on monetary assets. In Model 1, where the nominal exchange rate is fixed, the real return on domestic and foreign cash is equal to $\frac{1}{\pi}$, so the response to the shock is the same. Deposits in foreign currency also respond in the same way given that the international real interest rate they pay is constant, for which the real return is only affected by the change in domestic prices. In this way, the response of the real return on these three monetary assets is the same in Model 1. The return on domestic deposits, on the other hand, responds positively and to a greater extent, driven both by the higher return on investment and by the reduction of prices.

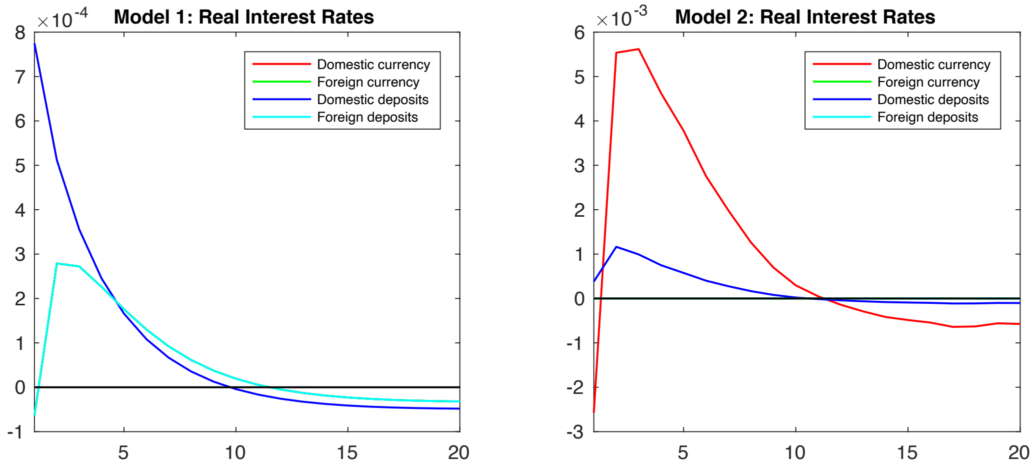

Figure 5: Impulse response functions of real interest rates (productivity shock).

In Model 2 the dynamic is different. Since the nominal exchange rate responds in the same way as domestic prices, the real return on foreign assets remains unchanged. Lower inflation strongly increases the return on domestic cash, above the return on domestic deposits.

The response of individuals to changes in the real returns is a rebalancing of the portfolios that they use to make purchases. While in Model 1 the proportion of purchases made with deposits increases (j^* decreases), in Model 2 the opposite occurs.

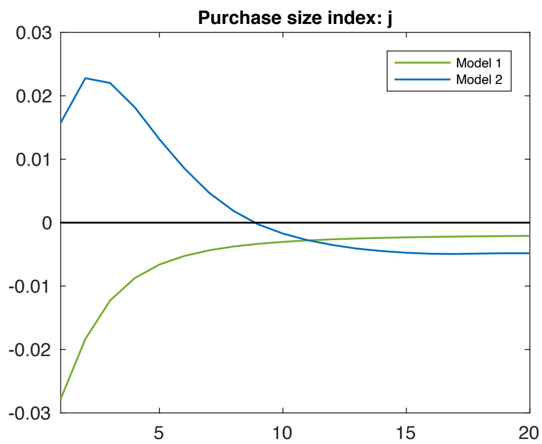

Figure 6: Impulse response function of purchase size index (productivity shock).

In Model 1, the individual reduces his holdings in cash and increases deposits, in both domestic and foreign currency. In Model 2, on the other hand, the individual rebalances his portfolios towards assets in domestic currency, both in cash and deposits.

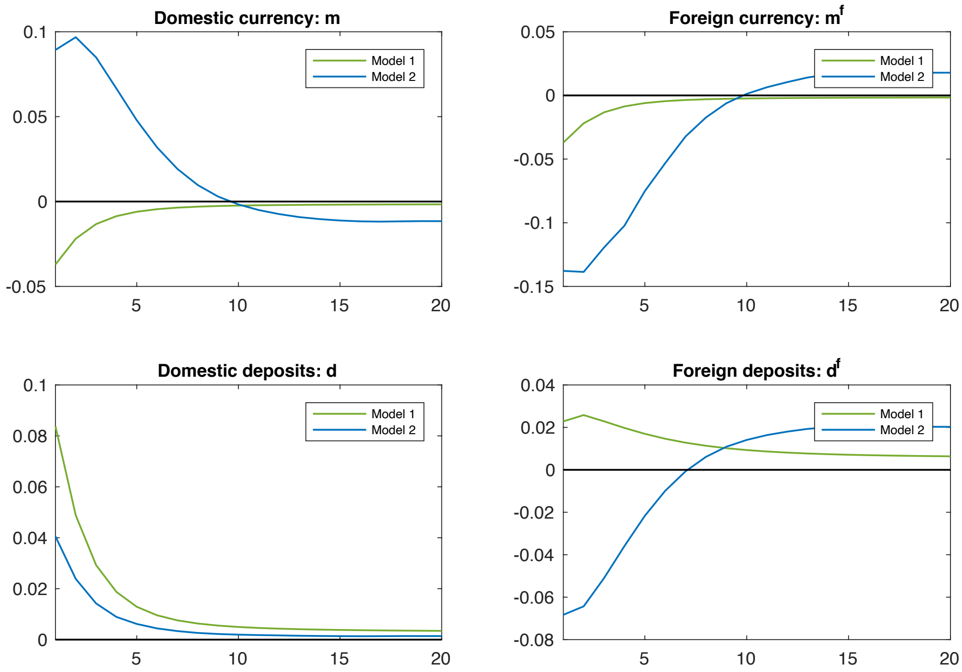

Figure 7: Impulse response functions of real monetary assets (productivity shock).

As a consequence of the rebalancing in the holding of monetary assets, the deposits-to-cash ratio in domestic assets changes, as the money multiplier and the money supply does.

In Model 1, individuals react to a positive output shock increasing the deposit-to-cash ratio in domestic currency, the money multiplier grows and it drives the expansion of domestic money supply. In addition, the monetary base is also expanded by output[27], and so the expansion of money supply is reinforced. In this model, the nominal exchange rate is fixed and consequently the currency substitution elasticity in both portfolios tends to be between 0 and 1 (there are no incentives to modify the domestic-to-foreign proportion).

In a context of full flexible exchange rate and high inflation such as in the Model 2, the currency substitution of elasticity tends to be different between the liquidity aggregators. On the one hand, individuals make larger purchases (long-term consumption, i.e. durable goods) with deposits and the elasticity of substitution in this portfolio remains similar to the case of currency board regime, which is between 0 and 1. On the other hand, the individuals make smaller purchases (short-term consumption) with cash and so the elasticity of substitution is greater than 1 in the currency portfolio (they have incentives to modify the distribution in the portfolio). The difference between the elasticities in this model changes the way individuals rearrange their different portfolios. When economy grows, liquidity needs to increase in both portfolios, and individuals rebalance both portfolios towards cash and

[27] In the Model 1, the nominal monetary base growth rate only depends positively on output. This simple specification reflects the behavior of this variable under the currency substitution scheme. When the economy grows, domestic interest rate increases and the foreign capital inflows. Then, the foreign assets held by the government increase and consequently the monetary base expands.

deposits in domestic currency, due to the falling of the domestic prices and the real exchange rate stability. However, it occurs more strongly in the case of cash liquidity aggregator than in the deposit´s aggregator, because elasticity of substitution is higher in the currency portfolio. Consequently, the deposit-to-cash ratio decreases and the money multiplier as well. Despite this counter-cyclical behavior of the money multiplier, the money supply remains procyclical (although less correlated with output) due to the increase in the real monetary base as a consequence of lower prices.

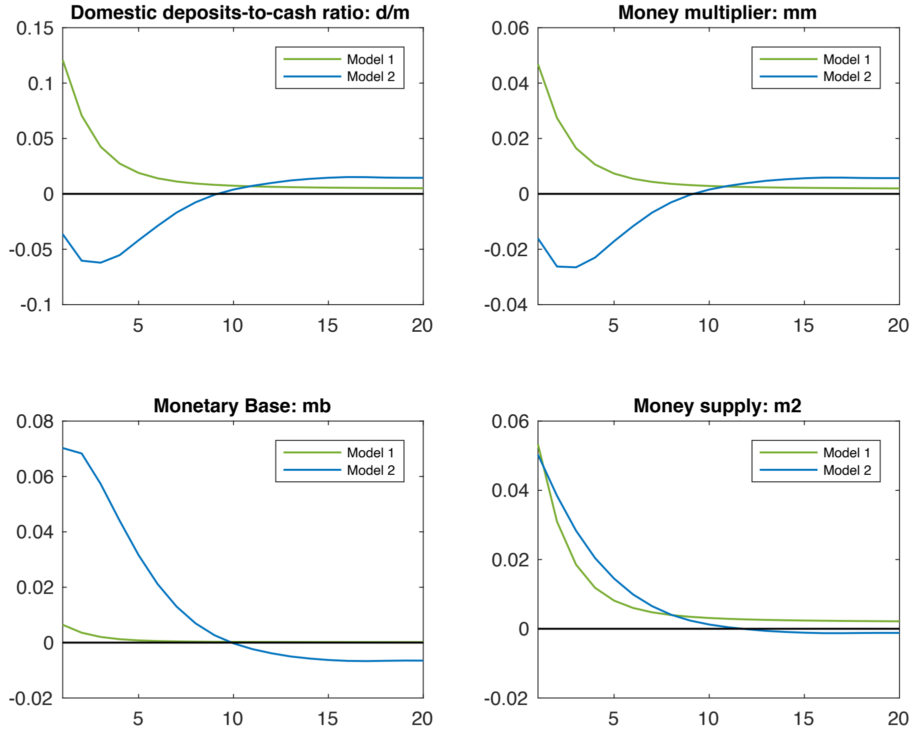

Figure 8: Impulse response functions of real monetary variables (productivity shock).

Figure 9 shows the impulse response functions of monetary nominal variables[28]. While at first the real effect prevails, then the recovery of prices does. In Model 1, the dynamics is practically the same as in the real variables given the low reaction of prices. In Model 2, monetary assets behave in a similar way to that observed in real terms, although the impact of prices partially attenuates the previous dynamics.

[28] The methodology used to calculate the IRFs and simulations of nominal variables is in the Annex.

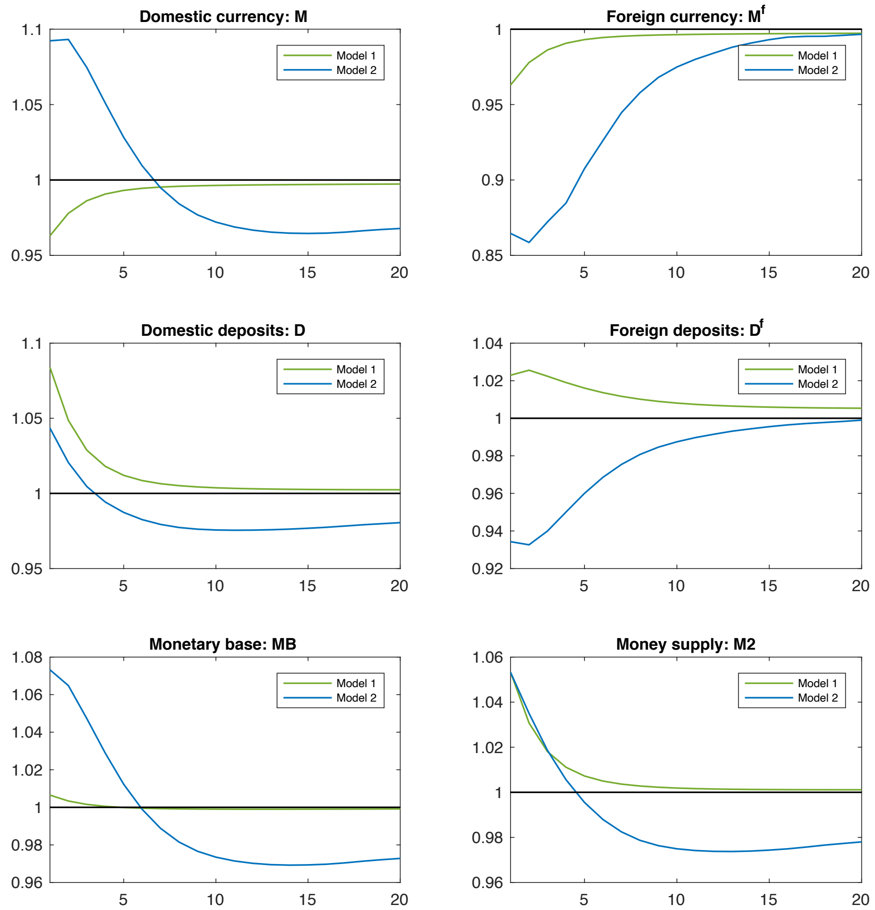

Figure 9: Impulse response functions of nominal monetary variables (productivity shock).

The main quantitative results of the model are shown in Table 3[29]. In general, the model is able to capture the correct sign and values for the correlation coefficients between the main variables and output for each period.

The positive correlation coefficient of total nominal domestic money supply (M2) with output is the main result of the model. Same as in the data, it is positive in simulations of both models, and also it is lower in Model 2 under the flexible exchange rate regime. The coefficient in the second period is close to the one observed for M3, which is a more inclusive monetary aggregator[30].

[29] Additional quantitative results such as standard deviations and correlation coefficients are in the Annex.
[30] M3 is a monetary aggregate that includes M2 and term deposits.

Table 3. Correlations with output (HP filter, lambda = 1600)

	Period 1993:1 2001:4		Period 2003:1 2019:4	
	Data	**Model 1**	**Data**	**Model 2**
y	1	1	1	1
MB	0.766	0.833	0.503	0.891
$M2$	0.908	0.884	0.354	0.785
$M3$	0.907	-	0.572	-
π	-0.022	-0.556	-0.186	-0.095
$m2$	0.907	0.887	0.425	0.992
v	-0.799	-0.585	-0.050	-0.954
mm	0.223	0.889	-0.182	-0.841
D/M	0.807	0.889	-0.230	-0.806
D	0.913	0.886	0.486	0.644
M	0.831	-0.895	0.599	0.912
c	0.968	0.998	0.891	0.998
i	0.990	0.529	0.882	0.559
a	-	0.926	-	0.942
R^d	-0.534	0.703	-0.312	-0.930
r^d_{pf}	0.372	0.943	-0.389	0.267
$r^d_{q,fm}$	-0.306	0.943	0.091	0.267
tb/y	-0.891	-0.358	-0.518	-0.397
ca/y	-0.892	-0.380	-0.296	-0.419
RER	-0.267	-0.128	-0.143	-

Domestic inflation is negatively correlated with output in both models, same as in the data. However, the coefficient is much higher in Model 1 than in Model 2. Since the opportunity cost of domestic currency in both models is the inverse of inflation, the opposite behavior of domestic currency observed in the models is explained by the different dynamics of inflation.

Velocity of money is negatively correlated with output in the models, similar to data during currency board period. It means that real money supply increases (decreases) more than output when the economy grows (falls). This effect is stronger in Model 2 since inflation is negatively correlated with output. Model 2 fails to capture the a-cyclical behavior of the velocity of circulation observed in data.

Monetary base is positively correlated with output in both models. In Model 1, this result is straightforward since the dynamic equation for its growth rate and the calibration are set to replicate this behavior. In Model 2, the nominal monetary base growth rate depends on its own growth rate in the previous period. Since the nominal variables was reconstructed using simulations of inflation and real variables, then the correlation comes from the dynamics of these baseline variables.

The second main result of the model is the ability to explain the reversal in the correlation coefficient between money multiplier and output. In Model 1, the positive correlation observed after simulations comes from the fact that the deposit-to-cash ratio of domestic currency is positive correlated with output. In contrast, simulations of Model 2 show that the money multiplier is negatively correlated with output, and the coefficient has the same sign as we can observe in data. This switch in the sign comes from the currency substitution effect as explained above when analyzing the IRFs.

As it is expected, domestic currency deposits are positively correlated with output in both periods. The model is able to capture not only this positive correlation, but also the lower coefficient in the second period under the flexible exchange rate regime.

Real variables as consumption and investment are also high positively correlated with output in both periods. Consumption´s behavior is better captured by the model than total investment. The lower correlation coefficient of total investment in the simulated data is related to the model setting. Remember that physical capital is divided into non-intermediated and intermediated capital. The former is directly affected by productivity shocks, and so the correlation coefficient is higher than the latter. The intermediated capital is less correlated because the real interest rate paid to this capital investment is not directly related to productivity shock, but it is a linear combination of real interest rate and the inverse of inflation.

The nominal interest rate is negatively correlated with output in both periods. This negative correlation is only captured by Model 2, where prices react more than in Model 1 as it was observed in the IRFs. The positive correlation in Model 1 is a result of the much higher correlation of the real interest rate of domestic deposits to output than the same correlation of inflation.

The real interest rate of domestic deposits is positively correlated with output in both models, but these results match the data based on how the real interest rate is calculated. In Table 3, r_{pf}^{d} is the annual perfect foresight real interest rate of domestic deposits calculated using annual inflation of two periods ahead[31], and $r_{q,pf}^{d}$ is the myope real interest rate of domestic deposits calculated using inflation of the current period. The correlation in Model 1 is similar to the one observed in data using the first real interest rate. In Model 2, the positive correlation is similar to the one observed in data using the second real interest rate. It is logical to compare the correlation coefficient of Model 1 against that obtained in the data using the first estimated rate, since in this period the volatility of prices has been much lower, and consequently, individuals had a greater possibility of predicting their evolution. To the contrary, the volatility of the inflation rate in the second period has been much higher than in the first one, which makes it logical for individuals to have less ability to forecast the evolution of prices, and consequently they think of shorter-term interest rates.

Finally, the correlations of external sector´s variables are also well matched by the model. In both periods, all these variables present a negative correlation with output. Trade balance and current

[31] The results when using four periods ahead annual inflation are very similar. I decided to use two periods instead of four to consider a not so extreme case.

account correlations are better captured by the model in the second period. The real exchange rate coefficient obtained by simulations of Model 1 is also similar to the one observed in data. This coefficient is nil in Model 2 since real exchange rate is constant by construction.

7. CONCLUSIONS

The thesis shows that a small open economy model with endogenous money multiplier and currency substitution is able to explain the positive relationship between nominal money supply and the real economic variables in a full flexible price context. The model manages to replicate the main stylized facts of the Argentine economy under different exchange rate regimes.

In particular, the model explains the positive relationship between monetary aggregates and output, and the different behavior of the money multiplier in the face of the change in the exchange rate regime. When exchange rate is fixed, a positive output shock increases the deposit-to-cash ratio in domestic currency, the money multiplier grows, and it drives the expansion of domestic money supply. When exchange rate is full flexible, the deposit-to-cash ratio decreases, the money multiplier falls, and the money supply expands due to the increase in the real monetary base. The different behavior of the money multiplier is explained by the switch in the exchange rate regime, and by the way agents define the shares and the substitution elasticities in liquidity aggregators as well.

The simplicity of the model is both a strength and a weakness. In particular, the full flexible prices assumption and the PPP real exchange rate policy in Model 2 could be modified in order to study prices dynamics in a more accurate way. These modifications could help the model to explain better the lower correlation between money supply and output observed in the data during the flexible exchange rate period.

The model could also be extended in different ways. The elasticities or share parameters in the liquidity aggregators could be endogenously determined, for instance, by the level of inflation in the economy. It is an interesting extension since the model shows that the effectiveness of monetary policy to modify the money supply under a flexible exchange rate regime depends on how agents define the shares and the substitution elasticities in both, cash and deposits liquidity aggregators. The reserve requirement ratio could also be endogenous in order to match better the correlation coefficient of the money multiplier in both periods. On the production side, to divide between tradable and non-tradable goods sectors is another possible extension to study exchange rate dynamics. Adding extra shocks is also an interesting possibility. For instance, international interest rate shocks could improve the model performance on external sector variables.

The model is an interesting starting point to study the impact of the new alternative means of payments to domestic currencies. In particular, the model could be useful to evaluate the effects on the credit channel mechanism of the monetary policy in a context of currency substitution between domestic fiat currency and digital money. A possible extension could include digital currencies issued by Central Banks (CBDC). On the one hand, the incorporation of the CBDC could weaken monetary policy since it substitutes part of cash, goes directly to individuals (it does not go through the banks), and so the money multiplier does not act. On the other hand, a positive effect on monetary policy effectiveness may be the fact that reduces the costs of replenishing real balances.

Finally, on the empirical side, the parameters of the model were calibrated using the data from Argentina as well as parameters values that are common in business-cycle literature. A possible future action could be to estimate the parameters by applying Bayesian methods.

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9. ANNEX

9.1. Dataset and methodology

The data of the Argentine economy used to calculate the stylized facts in the two periods of study corresponds to 1993q1-2001q4 and 2003q1-2019q4, respectively. The series were obtained from the Central Bank of the Argentine Republic (BCRA), the Ministry of Economy (MECON), the Institute of Statistics and Census (INDEC), the International Monetary Fund (IMF), and the World Bank (WB). All series are quarterly in frequency. The monetary variables, the nominal interest rates, and the nominal exchange rate are monthly average of the data. The following table describes the variables used, with their respective units of measurement and the origin of the data in each case.

Table 4. Dataset description

Symbol	Variable	Unit	Source
Y	Gross Domestic Product	Millons of domestic currency, current values	INDEC
Pop	Population	Millons of people	INDEC (2004-2019) and WB (1993-2001)
M	Money holdings of individuals	Millons of domestic currency, current values	BCRA
MB	Monetary Base	Millons of domestic currency, current values	BCRA
M1	M + Current Account Deposits	Millons of domestic currency, current values	BCRA
M2	M1 + Saving Deposits	Millons of domestic currency, current values	BCRA
M3	M2 + Term Deposits	Millons of domestic currency, current values	BCRA
P	Price index	Deflactor of GDP	INDEC
π	Inflation	% quarterly, from price index	INDEC
m2	Real Money Supply, domestic currency	Ratio M2/P	BCRA and INDEC
v	Velocity of money, domestic currency	Ratio Y/M2	BCRA and INDEC
mm	Money Multiplier, domestic currency	Ratio M2/MB	BCRA
D/M	Deposit-to-cash ratio, domestic currency	Ratio D/M	BCRA
D	Domestic currency deposits	Millons of domestic currency, current values	BCRA
y	Gross Domestic Product	Millons of domestic currency, constant values	MECON
c	Consumption	Millons of domestic currency, constant values	INDEC
i	Investment	Millons of domestic currency, constant values	INDEC
R	Nominal Interest Rate	% annual, domestic currency term deposits 30-59 days	BCRA
tb/y	Trade Balance	% of gdp	BCRA and INDEC
ca/y	Current Account Balance	% of gdp	IFS (IMF), BCRA and INDEC
RER	Multilateral Real Exchange Rate	Index Dec2001=100	BCRA

For the period of fixed exchange rate, I decided to consider the year 1993 as the beginning due to the lack of availability of data on the real variables in previous years[32]. For the second period, I use the series from 2003 onwards, discarding the data corresponding to 2002 which is strongly influenced by economic crisis[33].

[32] The beginning of the currency board regime was established with the Convertibility Law, on March 27, 1991.
[33] 2002 data is strongly influenced by the deep economic crisis that beset the country during that year. Nominal variables present extreme values and the data availability is a problem as well.

To analyze the relationship between the different variables, the definition of the business cycle by Lucas (1977) is adopted. Given that most of the series show non-stationary behavior, they are decomposed into their stochastic trend (non-stationary) and their stationary cyclical component using the Hodrick and Prescott (1981,1997) filter[34]. Following the author´s recommendation, a value of λ=1600 is used since the series are quarterly in frequency. All variables were considered in per capita terms, with the exception of the price index, interest rates and the variables that are ratios. Before proceeding to decomposition, all series have been seasonality adjusted using the X-11 procedure[35].

The correlation coefficients, standard deviations, and autocorrelation coefficients of order 1 were obtained using the cyclical components of the series. Before using the HP-filter, all the series were expressed in logarithms with the exception of interest rates, inflation, and the ratios of external sector variables. Thus, HP-filtered cyclical components of variables in logarithms can be expressed as %, and their standard deviation can be also measured in %.

9.2. Simulation of nominal variables

For stochastic simulations of the model, I use Dynare software running on Matlab. In the Dynare code, all variables are in real terms, so I do not explicitly include the price level or the nominal variables in the equations. This is a matter of convenience, as Dynare does not solve the model with non-stationary variables, and these variables are non-stationary. Given $P_0 = 1$, I reconstruct the price level and nominal domestic currency variables for period t as follows

$$P_t = \pi_t P_{t-1}$$

$$ln\, MB_t = ln\, mb_t + ln\, P_t$$

$$ln\, M_t = ln\, m_t + ln\, P_t$$

$$ln\, D_t = ln\, d_t + ln\, P_t$$

$$ln\, M2_t = ln\, m2_t + ln\, P_t$$

where lowercase variables are variables in real terms.

The same procedure is applied to reconstruct the real exchange rate for period t, given $RER_0 = 1$.

$$RER_t = \frac{\varepsilon_t}{\pi_t} RER_{t-1}$$

[34] The Hodrick and Prescott filter is a standard procedure accepted in academia for obtaining the cyclical and trend components of economic series.
[35] I implement the package "seasonal" in R which is an interface to X-11ARIMA-SEATS (See https://cran.r-project.org/web/packages/seasonal/seasonal.pdf). For a precise description of the X-11 filter, see Ladiray and Quenneville (2001, chapter 4).

The IRF´s graphs of nominal variables are presented as proportion deviations from their steady state trend value given domestic inflation at its steady state trend as well. I reconstruct the IRF graphs as follows

$$P_{i,t} = P_{i,t-1}\,(1 + \pi_t - \pi_{ss})$$

$$MB_{i,t} = \frac{\left(mb_{i,t} + mb_{ss}\right)P_{i,t}}{MB_{ss}}$$

$$M_{i,t} = \frac{(m_{i,t} + m_{ss})\,P_{i,t}}{M_{ss}}$$

$$D_{i,t} = \frac{(d_{i,t} + d_{ss})P_{i,t}}{D_{ss}}$$

$$M2_{i,t} = \frac{(m2_{i,t} + m2_{ss})P_{i,t}}{M2_{ss}}$$

where the variable with subscript $i = \{e_z, e_m\}$ indicates the deviation of that variable from its steady state trend generated by the ***i*** shock.

According to the specifications above, nominal variables move from their steady state trend for two reasons: by prices and/or by real changes. On the one hand, if the real variable is in its steady state value, then the only way for the nominal variable to deviate from its trend is through change in prices. On the other hand, if inflation is at its steady state, $P_{i,t}$ remains stable. Then, the only way for the nominal variable to deviate from its trend is through change in the real variable.

For simplicity, I assume $P_{i,0} = 1$. It implies that nominal and real variables are identical before the shock (t=0), and the initial value for price´s deviation at period t=1 is equal to the deviation of the inflation from its steady state value.

The same procedure is applied to reconstruct the real exchange rate IRF. Given $RER_{i,0} = RER_{ss} = 1$, then the deviation of RER from its steady state trend is computed by

$$RER_{i,t} = RER_{i,t-1}\,\frac{(1 + \varepsilon_t - \varepsilon_{ss})}{(1 + \pi_t - \pi_{ss})}$$

This IRF is only possible to compute in the Model 1, where nominal exchange rate is constant. Then, the above equation reduces to the following expression

$$RER_{ez,t} = RER_{ez,t-1}\,\frac{1}{(1 + \pi_t - \pi_{ss})}$$

9.3. IRFs after a monetary shock

The impact of a monetary shock can be evaluated only in Model 2, where changes in the monetary base growth rate is a possible option for the government. In this model, the nominal monetary base grows at a constant rate and deviations from its trend is given by the monetary shock.

Money is not fully neutral in this model, albeit these real effects on output are quite small. A temporary increase in the growth rate of money (i.e., a permanent change in the trend level of the nominal money supply) lowers output, hours, investment and consumption.

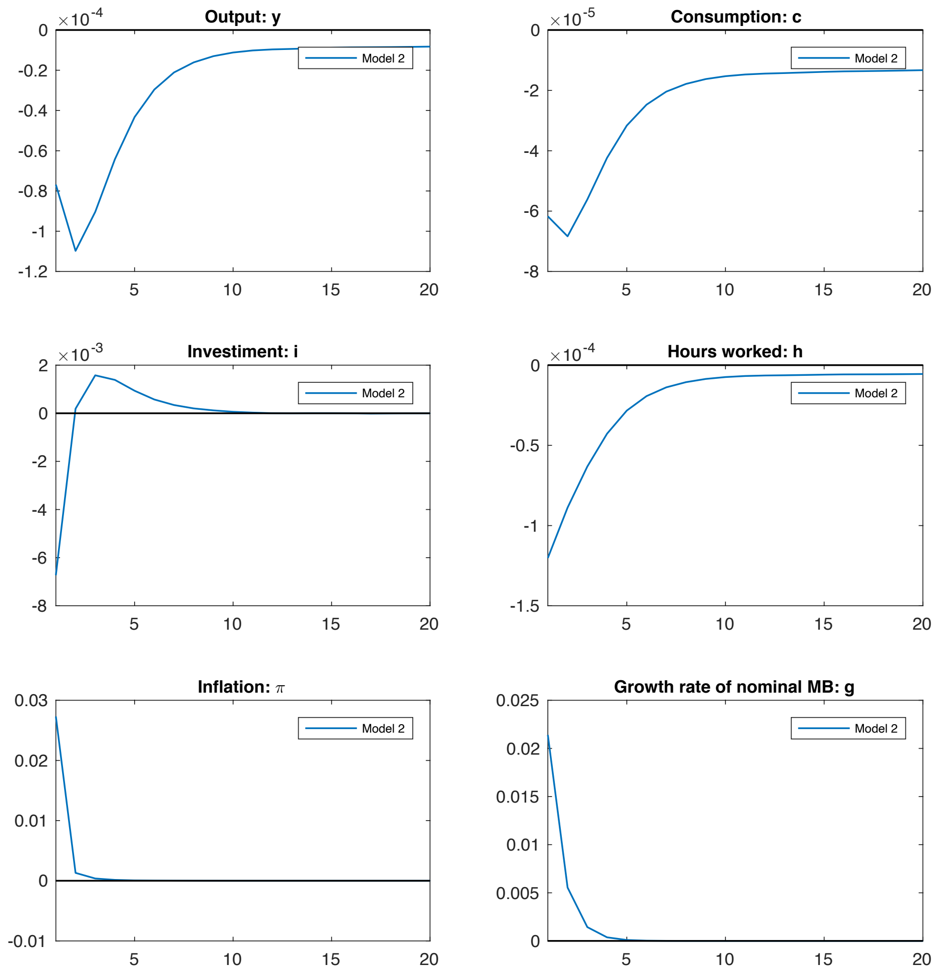

Figure 10: Impulse response functions of real variables (monetary shock).

The intuition of this results is as follows. Inflation is essentially a tax on the holders of domestic currency. The more inflation there is, the more individuals who hold money are penalized. Because money is necessary to consume (i.e., the liquidity in advance constraint), in equilibrium individuals cannot hold less money than the government prints, therefore they end up substituting away from things which require money (consumption) and into things that don´t (leisure). This ends up reducing consumption, and employment. Investment also decreases at the beginning, but then it recovers since the reduction in consumption dominates the reduction in hours worked.

Trade balance and current account respond in the opposite way to what was observed with the productivity shock as a consequence of a greater drop in consumption in relation to the drop in output. The external debt, on the other hand, grows driven by the holdings of external monetary assets by individuals.

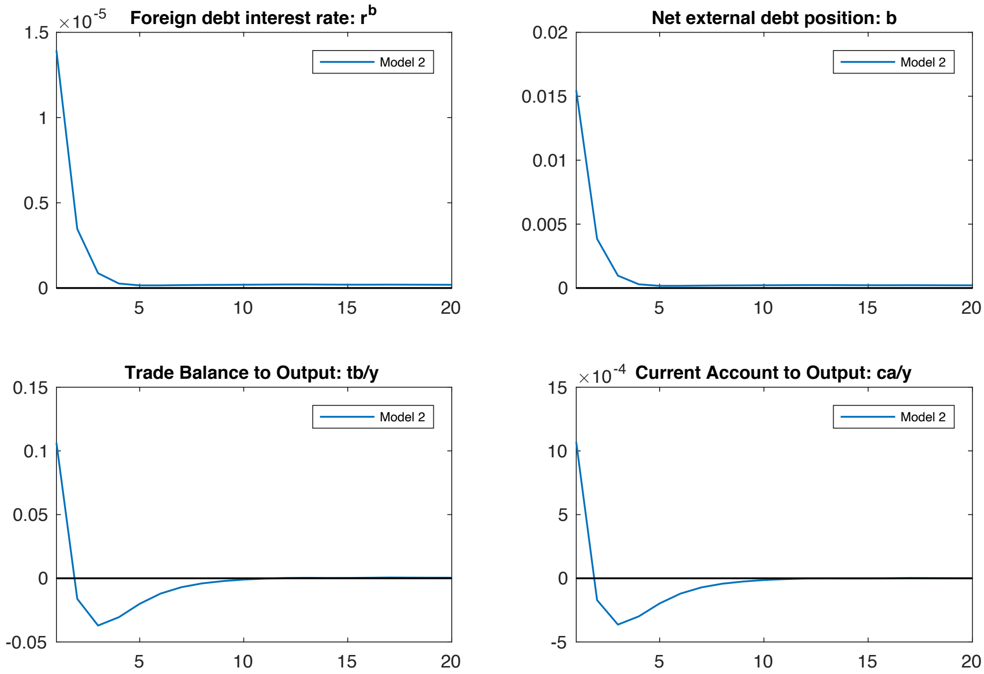

Figure 11: Impulse response functions of external sector variables (monetary shock).

Higher inflation generates a fall in the returns on domestic currency assets, to a greater extent on cash. This negative impact on cash changes the share of portfolios in favor of deposits (j^* decreases).

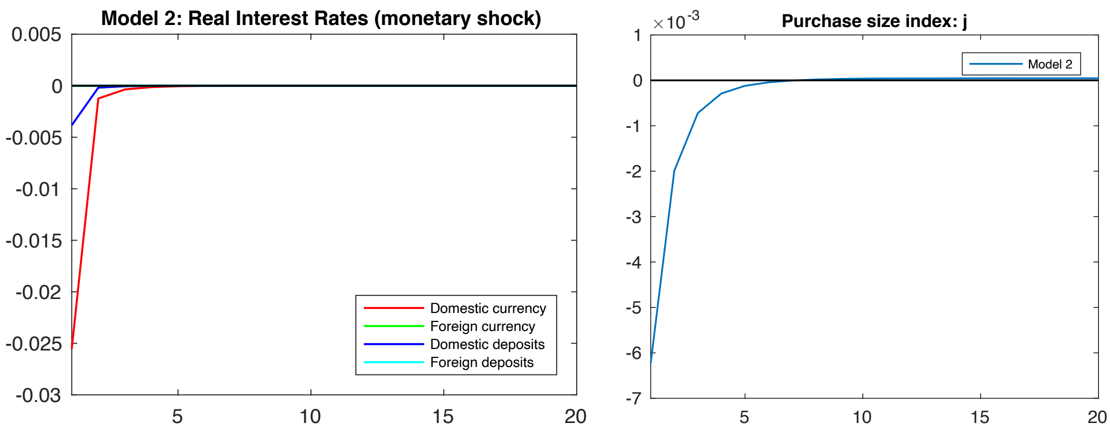

Figure 12: Impulse response functions of real interest rates and purchase size index (monetary shock).

Within each portfolio, the individual rebalances his holdings in favor of foreign currency assets, whose performance was not affected by the monetary shock given the immediate adjustment of the nominal exchange rate. Deposits in domestic currency, however, managed to grow due to the strong rebalancing towards the portfolio of deposits.

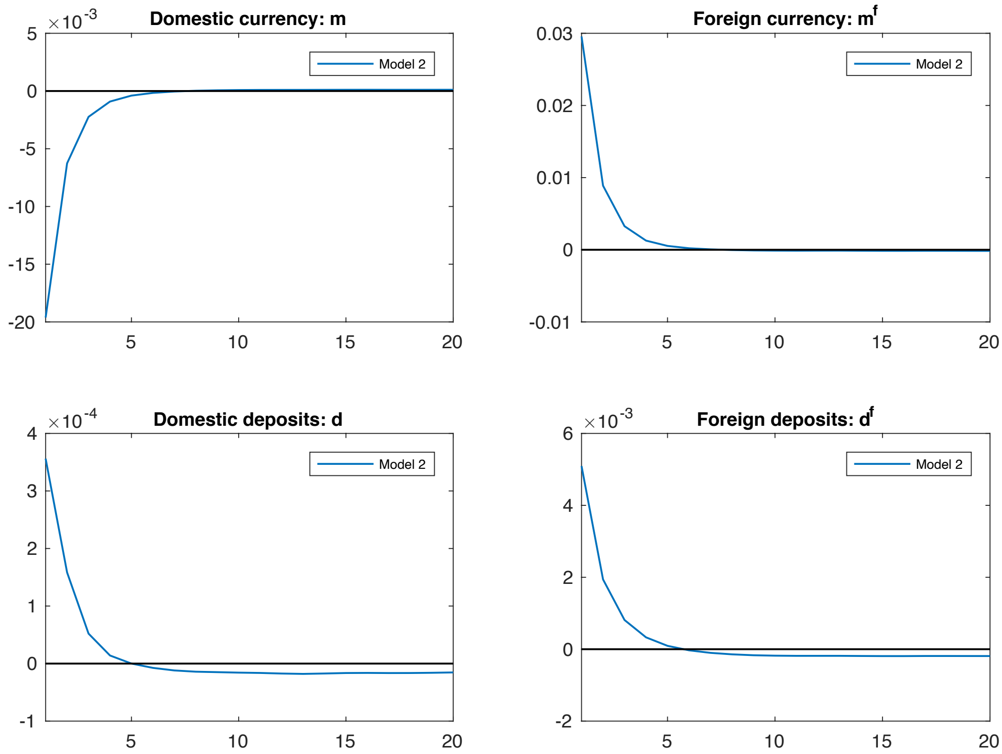

Figure 13: Impulse response functions of real monetary assets (monetary shock).

The fall in the demand for domestic currency and the slight increase in the demand for domestic deposits explain the positive response of the money multiplier. However, the positive effect of the multiplier is offset by the increase in prices. The monetary base contracts in real terms as does the domestic money supply.

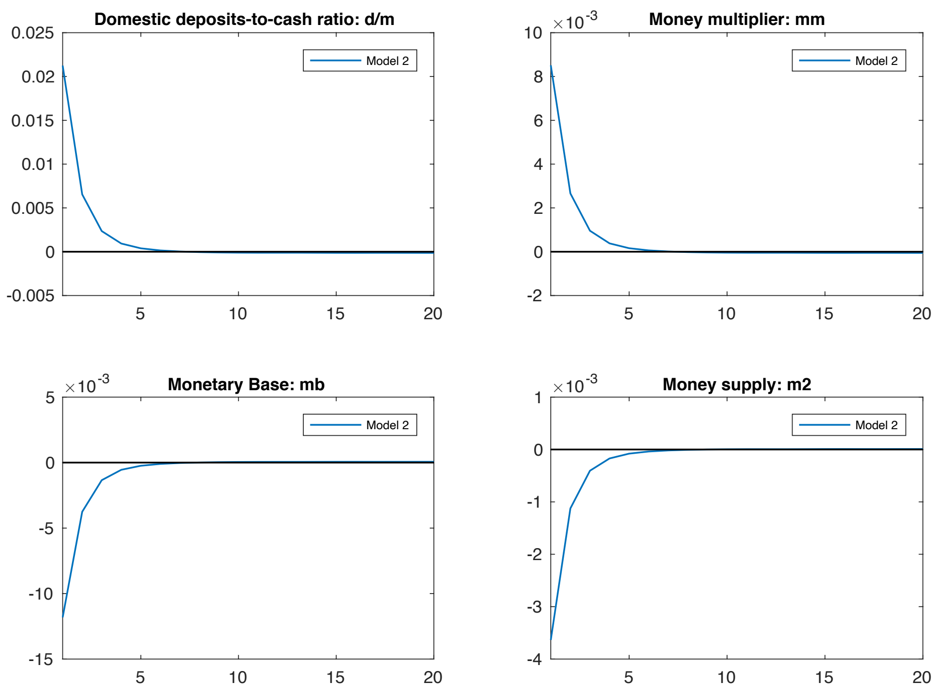

Figure 14: Impulse response functions of real monetary variables (monetary shock).

Finally, as a result of the positive monetary shock, all the nominal variables respond positively and they stabilize above its steady state trend.

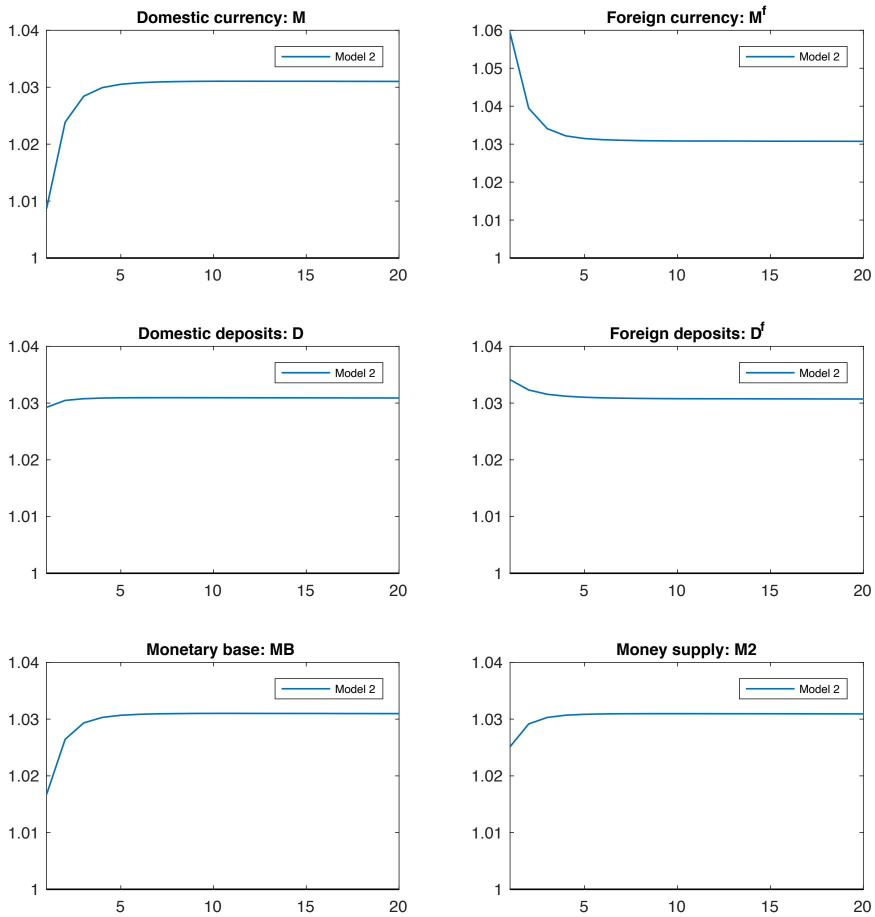

Figure 15: Impulse response functions of nominal monetary variables (monetary shock).

9.4. Additional quantitative results

Table 5: Standard deviations

	Period 1993:1 2001:4		Period 2003:1 2019:4	
	Data	**Model 1**	**Data**	**Model 2**
y	0.0329	0.0328	0.0291	0.0293
MB	0.0941	0.0070	0.0560	0.1039
$M2$	0.0919	0.0560	0.0550	0.0748
$M3$	0.1016	-	0.0434	-
π	0.0078	0.0004	0.0278	0.0279
$m2$	0.0968	0.0351	0.0773	0.0545
v	0.0674	0.0521	0.0687	0.0295
mm	0.0546	0.1207	0.0499	0.1153
D/M	0.0770	0.3637	0.0492	0.3560
D	0.1226	0.0898	0.0458	0.0646
M	0.0602	0.0369	0.0559	0.1357
c	0.0321	0.0145	0.0333	0.0125
i	0.0917	0.0917	0.0909	0.0912
a	-	0.0953	-	0.1067
R^d	0.0226	0.0002	0.0423	0.0055
r^d_{pf}	0.0495	0.0009	0.0461	0.0039
$r^d_{q,fm}$	0.0092	0.0009	0.0243	0.0039
tb/y	0.0110	0.0760	0.0202	0.0773
ca/y	0.0104	0.0757	0.0146	0.0767
RER	0.0308	0.0011	0.0658	-

Table 6: Autocorrelation of order 1

	Period 1993:1 2001:4		Period 2003:1 2019:4	
	Data	**Model 1**	**Data**	**Model 2**
y	0.712	0.648	0.649	0.665
MB	0.669	0.399	0.700	0.741
$M2$	0.707	0.373	0.768	0.687
$M3$	0.707	-	0.861	-
π	-0.201	0.556	-0.093	0.096
$m2$	0.730	0.372	0.723	0.626
v	0.739	0.214	0.764	0.584
mm	0.567	0.374	0.712	0.804
D/M	0.862	0.374	0.776	0.794
D	0.755	0.370	0.839	0.650
M	0.583	0.374	0.837	0.765
c	0.708	0.643	0.615	0.662
i	0.724	0.019	0.768	0.193
a	-	0.769	-	0.784
R^d	0.644	0.272	0.770	0.735
r^d_{pf}	0.320	0.467	0.599	0.137
$r^d_{q,fm}$	0.110	0.467	-0.087	0.137
tb/y	0.682	-0.001	0.706	0.187
ca/y	0.7045	-0.002	0.379	0.187
RER	0.727	0.906	0.678	-