Multifractal behaviour of the interevent times in decision-making process when facing gender-based violence situations

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Abstract: The application of physics in social spheres gives explanations to some collective phenomena as human decision-making processes. The aim of this work is to study this kind of process when facing gender-based violence situations. Personal data, decision made and time taken are obtained from a public experiment in which individuals confronted with this kind of situations have to decide if they would intervene or not. Experimental data are explained by analogy to the valley model through the study of the fractal behaviour of the interevent times. As multifractality is observed, social and personal factors seem to influence the decision-making process.

I. INTRODUCTION

Understanding the times involved in collective decision-making processes can help us to identify the personal factors involved in individuals' decision-making when confronted with certain situations and how they are influenced by others' actions [1]. If all the individuals in a decision-making process behaved in the same way and decisions were independent from one another, events would be randomly distributed in time and the process could be approximated by a Poisson process [2]. In a Poisson process the time between two consecutive decisions, known as interevent time τ, follows an exponential distribution with characteristic parameter given by the interevent time average. However, various studies show that human decision-making processes are not Poissonian [2]. Instead, the interevent times are long-tailed distributed (the density probability function is a power law). This means that there are periods where many really fast decisions are made, separated by long intervals with no decisions. Distributions of this kind appear when decisions are based on some priority [1] and show a distinctive feature of human beings. In these cases, fractality in the interevent time probability density function is observed (self-similar pattern in different scales).

A convenient way to study decision-making processes is by looking at the interevent time distribution, which is characterized by its q moments. In particular, the q moments give information about the fractal behaviour of the distribution. If the relation

$$< \tau^q >\sim L^{f(q)}$$

is satisfied for some appropriate scale L, there is fractal behaviour. When $f(q)$ is linear, the timing process is said to be monofractal [3], otherwise it is multifractal. This is due to the fact that, if $f(q)$ is linear, it is possible to determine the full distribution from just one of the q moments with a simple regression.

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In this work, we analyze human decision-making when facing gender-based violence situations in public spaces. Studying how people react as observers to these situations can give us some clues about how personal and social circumstances shape the way decisions are made and the time it takes to reach them. This is the purpose of the experiment carried out by NUS Teatre, the feminist group of Elisava School of Design and the OpenSystems research group.

The comprehensive study of the fractal behaviour of the interevent times in the aforementioned experiment is the central topic of this work. To explain the results obtained, the valley model is proposed.

II. EXPERIMENT

The experiment consisted in exposing 234 individuals, divided in groups of six, to four different gender-based violence situations in a public space [4]. Each situation was exemplified with a monologue interpreted by an actor playing the role of an observer. The situations are the following:

- Situation A: visible discomfort is observed in a man seeing two men kissing in a terrace.
- Situation R: a woman accuses a person of not using the correct toilet.
- Situation S: a person complains about a woman breastfeeding in a library.
- Situation N: a girl is hounded by a man in the street.

After the performances, the participants had to choose, via a digital device, how they would act in the actors' positions. The options were: intervene decisively (decision C), intervene in a moderate manner (decision I) or not to intervene (decision D). Decisions made by each participant were available to the whole group in real time. Their personal data (gender, age, decision made, studies...) were collected at the end of the experiment.

The experiment was part of the *"Consciències a la plaça"* project and was performed in Mercat de Sant Antoni, Plaça Sortidor, Plaça J.M. Folch i Torres and Plaça Orfila (located in Barcelona).

III. VALLEY MODEL

The valley model developed by Scher and Montroll [5] describes the relaxation time of photocurrents created in amorphous materials. We can think of the situation as having potential wells in which carriers may fall. A conditional density $\psi(\tau|\epsilon)$, which is assumed to be a simple exponential, is introduced to compute the pausing time density. It characterizes the probability that a carrier spends a time interval t within a potential well of depth ϵ given by a random variable of density $\rho(\epsilon)$ [5]. Homogeneity of the potential wells leads to monofractality in the pausing time distribution [6]. It can be shown that multifractality is obtained by taking a "stretched exponential" of the form

$$\rho(\epsilon) = \frac{1}{2\sigma\Gamma(1+1/\alpha)} \exp\left(-\left|\frac{\epsilon-\mu}{\sigma}\right|^{\alpha}\right), \qquad (1)$$

with $\alpha > 0$ and $\sigma > 0$ (see [6]). The q moments in this case exist if $\alpha > 1$ and are given by

$$< \tau^q > \simeq e^{aq} e^{b\varphi(q)}, \qquad (2)$$

with a, b as characteristic parameters of the model and

$$\varphi(q) = \frac{1}{b_1} q[1 - \exp\left(-b_1 |q|^{1/(\alpha-1)}\right)], \qquad (3)$$

where $b_1 > 0$. Notice that, for small q's, expression (2) becomes

$$< \tau^q > \simeq e^{aq} e^{b|q|^{\alpha/(\alpha-1)}}; \qquad (4)$$

while, for large q's, we have monofractality. If a conditional density is considered, the decumulative probability function may be written as

$$\Psi(\tau) = \int_{\tau}^{+\infty} \int_{-\infty}^{+\infty} \psi(\tau'|\epsilon)\rho(\epsilon)\, d\tau' d\epsilon. \qquad (5)$$

For $\psi(\tau'|\epsilon)$ and $\rho(\epsilon)$ as above, this decumulative distribution becomes

$$\Psi(\tau) = \frac{1}{2\Gamma(1+1/\alpha)} \int_{-\infty}^{+\infty} \exp\left(-|x|^{\alpha} - \tau e^{-(\gamma x + a)}\right) dx, \qquad (6)$$

with $\gamma = \alpha \left(\frac{b}{\alpha-1}\right)^{\frac{\alpha-1}{\alpha}}$ as shown in [6].

Extrapolating to our framework, we can think of collective actions as being "interrumped" by potential wells representing the individuals, where the depth of a well could be a measure of the difficulty a person faces when making his own decision as part of a collective phenomena.

IV. FRACTAL BEHAVIOUR

In this section, we look at the fractal behaviour of the interevent times τ through the q moments. There will be monofractal behavior if

$$< \tau_i^q > = \theta_i^q e^{n_i}, \qquad (7)$$

where i indicates the group to which we refer, $\theta_i = e^{m_i}$ and m_i and n_i are the slope and the independent term, respectively, obtained from the linear fittings of data in FIG. 1 (for $q > 5$). The considered groups are the ones that arise from separating the Total group (aggregating all participants) by gender, age and decision made (I, C or D). In particular, $< \tau^q > = \theta^q e^n$ stands for the Total group.

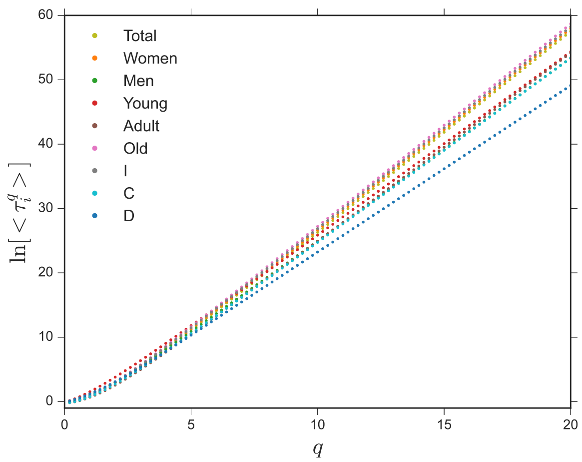

FIG. 1: Logarithm of the q moments as a function of q for all the groups.

In FIG. 1 we can observe linearity for the large q's ($q > 5$) for all the groups. To check this, we scale the q moments for each group as

$$< \hat{\tau}_i^q > \equiv < \tau_i^q > \left(\frac{\theta}{\theta_i}\right)^q \left(\frac{1}{e^{n_i}}\right). \qquad (8)$$

If all the groups present monofractal behaviour, when representing $\ln[< \hat{\tau}_i^q >]$ as a function of q, we should obtain the collapse of all of them to a line with slope θ. We notice from FIG. 2a that this occurs for $q \gtrsim 3$, while for $q \lesssim 3$ a non-linear behaviour is observed. To better distinguish the range of q's where there is not monofractal behaviour, we plot in FIG. 2b

$$f_F^i(q) \equiv \frac{1}{q \ln \theta_i} \ln[< \tau_i^q >] - \frac{n_i}{q \ln \theta_i} \qquad (9)$$

as a function of q. This function is defined just as a tool to further test the monofractal behaviour. Notice that, in the domain where there is monofractality, $f_F^i(q)$ must approach to 1 (due to the definition of $f_F^i(q)$ itself). We observe that this happens for $q \gtrsim 2.5$ but that for $0.4 \lesssim q \lesssim 2.5$ FIG. 2b shows deviation from this behaviour. So, for small values of q, the interevent times present multifractality for all the groups.

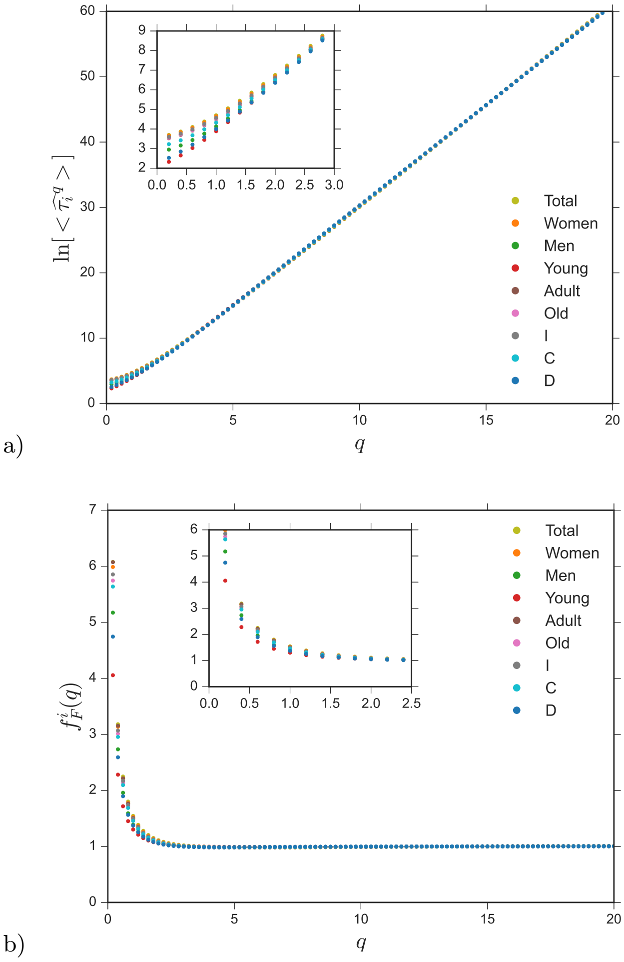

FIG. 2: a) Logarithm of the scaled q moments as a function of q for all the groups. b) Testing function $f_F^i(q)$ as a function of q for all the groups.

By analogy with the valley model, the monofractal behaviour for the large q's can be interpreted as if, within each group, there is a certain homogeneity between those individuals with large interevent times; they behave in a similar way. This behaviour is observed in all the groups considered but, from FIG. 1, not all of them have the same large interevent time average. The groups with larger slopes also have larger averages, that is, people take longer to make a decision after a previous one. If we refer to gender, this group is the one formed by women. By age, old people are the ones who spend more time in deciding after another decision has been made (this is actually the group with the largest slope; the average of the large interevent times is around $7.30s$). And by decision, people in group I have the largest interevent times.

Let's focus now on the multifractal structure for small q's. Taking the range $0.40 \lesssim q \lesssim 2.5$ in FIG. 1, we can fit, for each group, a polynomial of the form $a_i q + b_i q^{c_i}$ to $\ln[<\tau_i^q>]$ as a function of q. With such a fitting we get an expression as in (4) for the q moments in said range, with $c_i := \alpha_i/(\alpha_i - 1)$ and a_i, b_i, α_i the parameters given in TABLE I.

	a_i	b_i	α_i
Total	-3.78 ± 0.04	4.69 ± 0.04	7.04 ± 0.05
Women	-3.06 ± 0.03	3.98 ± 0.03	6.16 ± 0.04
Men	-4.38 ± 0.07	5.30 ± 0.07	7.81 ± 0.09
Young	-2.57 ± 0.06	4.10 ± 0.06	8.983 ± 0.115
Adult	-2.373 ± 0.015	3.053 ± 0.015	4.887 ± 0.017
Old	-2.82 ± 0.03	3.78 ± 0.03	5.88 ± 0.03
I	-2.40 ± 0.03	3.33 ± 0.03	5.33 ± 0.03
C	-6.39 ± 0.08	7.17 ± 0.08	9.92 ± 0.09
D	0.041 ± 0.004	1.153 ± 0.004	3.669 ± 0.008

TABLE I: Parameters obtained using a polynomial $a_i q + b_i q^{c_i}$ to fit the curves in FIG.1 for $0.4 \lesssim q \lesssim 2.5$.

To corroborate that the q moments of our data do follow expression (4) for $0.4 \lesssim q \lesssim 2.5$, we plot

$$\frac{1}{q} \ln[<\tau_i^q>] - a_i \qquad (10)$$

as a function of $b_i q^{1/(\alpha_i - 1)}$. FIG. 3 shows the collapse of all the groups to a line of slope 1 and independent term equal to 0 for the small values of q. Therefore, the q moments verify (4), which supports the multifractality observed in FIG. 1. Furthermore, the observed tendency of $\frac{1}{q}\ln[<\tau_i^q>] - a_i$ to a constant for larger q's bears out again the monofractal behaviour. We can also check that the multifractal formula (4) fits our data well by looking at FIG. 4.

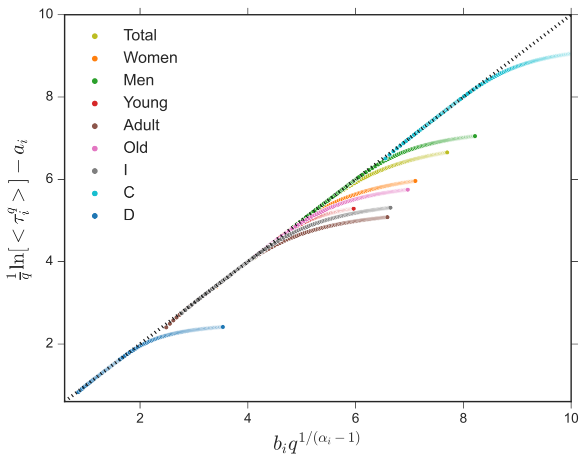

FIG. 3: Expression introduced in (10) as a function of the modified order $b_i q^{1/(\alpha_i - 1)}$ for all the groups. Black dotted line corresponds to $y = x$.

As in the valley model multifractality appears because of the heterogeneity of the potential wells, we could think of the multifractal behaviour for the small interevent times as a consequence of the heterogeneity of the individuals involved in the experiment. It seems that this heterogeneity plays an important role when people take small times in deciding after another participant.

Notice that multifractality of the data is derived from the exponent of q in (4). The parameters α_i are then related to the fractal dimension of the process, being a good indicator of the multifractal behavior. The groups with the smallest values of α_i deviate the most from monofractal behaviour. As shown in TABLE I, those groups are D and Adult; the minimum α_i is for group D, showing that within this group the heterogeneity is more important (there are more differences between the individuals). Conversely, the group with largest α_i is C, people who decide to intervene decidedly behave in a more similar way.

At this point we would like to obtain a general expression to describe the q moments for all $0.4 \lesssim q \lesssim 20$. Obviously, formula (4) for $0.4 \lesssim q \lesssim 2.5$ and linearity of $\ln[<\tau_i^q>]$ for $q \gtrsim 2.5$, should be re-obtained from such an expression in the appropriate ranges of q. By analogy with the valley model, we have fitted to the experimental curves in FIG. 1 an expression of the form $\ln[<\tau_i^q>] = a_i q + B_i q[1 - \exp(-b_{1i} q^{c_i})]$ in the range $0.4 \lesssim q \lesssim 20$ for all the groups. The parameters extracted can be found in TABLE II; we have taken $B_i := b_i/b_{1i}$ and $c_i := 1/(\alpha_i - 1)$. Considering this fitting, the q moments go as in (2).

	a_i	b_i	b_{1i}	α_i
Total	-1.39 ± 0.04	3.26 ± 0.05	0.740 ± 0.009	2.95 ± 0.02
Women	-1.12 ± 0.04	2.79 ± 0.04	0.678 ± 0.008	2.78 ± 0.02
Men	-1.31 ± 0.04,	3.24 ± 0.05	0.803 ± 0.009	2.820 ± 0.019
Young	0.010 ± 0.026	2.21 ± 0.03	0.799 ± 0.009	2.91 ± 0.02
Adult	-1.36 ± 0.04	2.78 ± 0.05	0.671 ± 0.009	2.68 ± 0.02
Old	-0.83 ± 0.03	2.41 ± 0.04	0.627 ± 0.008	2.635 ± 0.017
I	-0.96 ± 0.03	2.57 ± 0.04	0.649 ± 0.007	2.692 ± 0.017
C	-1.65 ± 0.05	3.50 ± 0.07	0.7958 ± 0.0113	2.88 ± 0.03
D	0.406 ± 0.014	0.984 ± 0.014	0.475 ± 0.006	2.360 ± 0.013

TABLE II: Parameters obtained by fitting, for each group, a function $a_i q + B_i q[1 - \exp(-b_{1i} q^{c_i})]$ to the curves in FIG.1 for $0.4 \lesssim q \lesssim 20$. The values are different from TABLE I because now we are fitting all the range of values of q.

We introduce now a function $f^i_{HMF}(q)$ that serves as a tool to prove that the mentioned generalized formula describes our data. This function is given by

$$f^i_{HMF}(q) \equiv \frac{b_{1i}}{b_i} \left[\frac{1}{q} \ln[<\tau_i^q>] - a_i \right], \qquad (11)$$

where now a_i, b_i and b_{1i} are the ones in the table above. If the q moments really follow expression (2), when plotting $f^i_{HMF}(q)$ in front of $b_{1i} q^{1/(\alpha_i - 1)}$, we will observe a curve of the form $1 - exp(-x)$. As we can see in FIG. 5, this

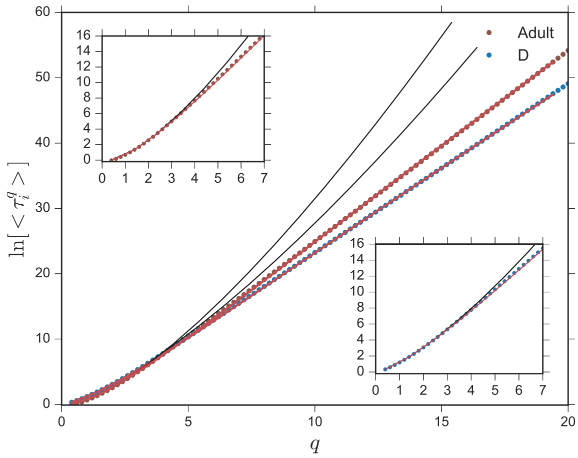

FIG. 4: Logarithm of the q moments as a function of q for groups Adult and D. Red and black solid lines correspond to regressions assuming (2) and (4), respectively.

happens for all the groups. We can conclude that indeed formula (2) describes our data for $0.4 \lesssim q \lesssim 20$. In FIG. 4 we can observe the goodness of the fitting for the groups Adult and D.

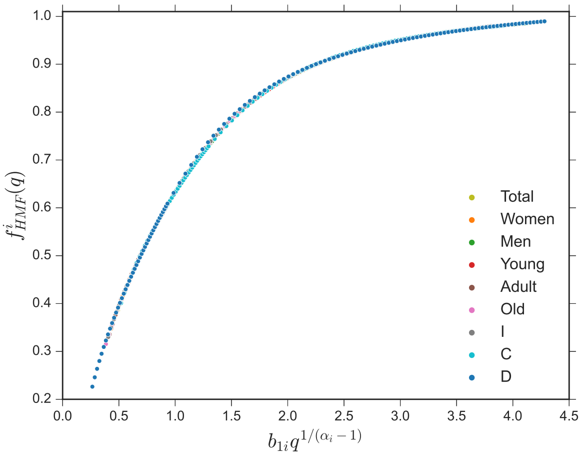

FIG. 5: Plot of f^i_{HMF} as a function of $b_{1i} q^{1/(\alpha_i - 1)}$ for all the groups.

FIG. 5 shows again that it makes sense to look for an explanation to our data in the valley model; up to now, all the results have been coherent with it. To finish with, the valley model provides equation (6) as the decumulative interevent time distribution. When substituting in the latter the parameters in TABLE II for the Total group, the empirical decumulative distribution of the interevent times should resemble the theoretical one. We notice in FIG. 6 that this occurs for large q's and when q tends to zero. Even though we have not found the stretched exponential $\rho(\epsilon)$ that exactly describes all our interevent time distribution, the decumulative distribution given by the valley model explains well the asymptotic cases that actually characterize this kind of processes; better than the exponential function derived from a Poisson process (see FIG. 6).

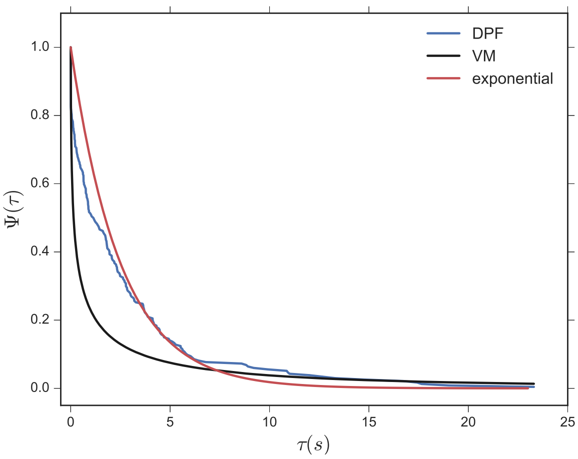

FIG. 6: Decumulative probability function of the interevent times as a function of the interevent times. DPF stands for the empirical decumulative function, VM for the theoretical one given by the valley model and the exponential shows the distribution for a Poisson process.

V. CONCLUSIONS

We have seen throughout this work that all the different groups considered behave in a self-similar way; in general, we have similar multifractality for all the groups. This result is of great importance as it implies that the decision-making process in front of a situation of gender-based violence is not Poissonian, times in which decisions are made appear not to be random.

To better understand the mechanism behind the decision-making process we have carried out a detailed study of the interevent times' fractality. Valley model has been shown to be a good candidate to explain the fractal behaviour observed. The main result obtained is that, for all the groups, there is monofractal behaviour for large values of q ($q \gtrsim 2.5$) and multifractal behaviour for the small ones ($0.4 \lesssim q \lesssim 2.5$). The valley model provides an interesting explanation for these results: when individuals take long time to decide after a previous decision, there is homogeneity; differences between the individuals due to personal factors do not play an important role in the decision process. Nevertheless, we see variations in the expected large interevent time between groups. The fact that groups Old and I are the ones with the largest interevent times seems quite coherent. Firstly, old people are, probably, the population sector least familiar with these topics, as their introduction in education is relatively new. Secondly, group I is the one formed by the people who decided to intervene in a moderate manner. This decision is the most neutral of the three possible options so it could be the one chosen by those people who didn't know how to react, which would explain why the expected time is large. Contrarily, when the interevent times are small, heterogeneity within a group has to be considered, personal factors have influenced the decision times. We already commented that, according to the parameter α_i, some groups present more heterogeneity than others. The reason why C is the most homogeneous group could be that, people who would intervene decisively if facing these situations, are mainly characterized by this intended decision; they are clear about what to do before seeing the others actions.

Taking all the above into account, we conclude that the way in which individuals decide when facing a gender-based violence situation, if they are exposed to the way the others behave, is based on some priority and it seems that there are some social or personal conditionings influencing the times involved. Although we have seen some patterns of behavior, with the study carried out here it is still not possible to assure which are these influencing factors. Nevertheless, to be able to claim that people are conditioned by some aspects is an important step on the way to understand social sensitivity towards gender-based violence. This can be really useful to provide better education and to raise awareness of this relevant issue.

Acknowledgments

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