On Natural Inflation with an Extra Scalar Field

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Abstract: The goal of this work is to study an extra feature of an inflationary model, the so-called Natural Inflation (NI) model, that could possibly describe the early stages of evolution of the universe. We will study the consequence of an additional coupling of the inflaton ϕ with a second scalar field χ, that respects the shift symmetry of NI for ϕ. To study the system we first present some preliminary theoretical concepts about classical field theory and about deriving the equations of motion from action principles in general relativity. Then, we will show that χ has exponentially growing solutions, such that that the inflationary period is shortened, for a given range of parameters. This could modify the observable predictions of NI.

I. INTRODUCTION

Inflationary theories were proposed as a natural and necessary solution to solve some problems that arise in the hot big bang model, the horizon problem and the flatness problem being among the most important ones [1]. Thus, an early inflationary stage in the evolution of the universe was proposed where the scale factor evolved almost exponentially and the Hubble parameter remained approximately constant. The particular inflation model studied in this work, named Natural Inflation (NI) [2], is given by a scalar field with a potential $V(\phi) = \Lambda^4(\cos\frac{\phi}{F}+1)$, allowing for a naturally flat potential, with the appropriate choice of energy scales Λ and F.

The mentioned NI model is well motivated on theoretical grounds, but does not give an optimal fit to the most recent cosmological CMB observations, from the Planck satellite [3]. Thus we investigate here if the existence of a second scalar field could modify the predictions on such observations. Such a field must be coupled respecting the symmetry of the NI model, i.e. a shift symmetry of the inflaton field ϕ. In this work we are going to consider the simplest possible coupling of this type, and we will show that this shortens the duration of inflation. We will give values for the coupling parameter and the corresponding shortening of the inflationary period.

Before diving into the examination of this particular model of inflation, the required theoretical foundations will be presented in a concrete and pedagogical manner. After giving a very brief exposition of the Euler-Lagrange equations in order to fix the notation and to make the work more self-contained, we will present the more advanced but standard topics required for the study of inflation. Those consist of the Klein-Gordon equation in a generic metric and the derivation of Einstein's field equations from an action principle. This will be done so that we can study the equations of motion and the energy-momentum tensor of the scalar fields in our inflationary setup. Throughout the work we are going to use natural Planck units, where $c = \hbar = k_b = 1$.

In section II of the work, we will briefly develop the theory needed to derive the Klein-Gordon equation in a general metric. We will also derive the Einstein field equations with an energy-momentum distribution form an action principle. In section III we will briefly review the basics of cosmology and inflation needed for the work. In section IV we will develop the modified model of NI with an extra field and give results for its parameter. Finally, in section V we will discuss some conclusions.

II. REVIEW OF THE REQUIRED THEORETICAL CONTENTS

A. Euler-Lagrange equations for scalar fields

From elementary classical mechanics, we know that the state of a system of particles with n degrees of freedom can be described by a set of n generalized coordinates taking values in the coordinate space, $(q_1, ..., q_n) \in \mathbb{R}^n$. The motion of the system will be described by a parametric curve $\gamma(t) = (q_1(t), ..., q_n(t)) \in \mathbb{R}^n$, where t represents time. In order to find this curve, also called trajectory, we can follow the approach of Lagrangian mechanics and arrive at an equation of motion involving $q_1, ..., q_n$. To do so, for every trajectory $\gamma(t) = (q_1(t), ..., q_n(t))$ we can define a Lagrangian function $\mathcal{L}(q_1, ..., q_n, \dot{q}_1, ..., \dot{q}_n, t)$ and a quantity called the action of the system, depending on the trajectory, as the following line integral.

$$\mathcal{S}(\gamma) = \int_\gamma \mathcal{L}(\gamma(t), \dot{\gamma}(t), t)dt\,. \quad (1)$$

The principle of least action postulates that actual trajectory $\gamma(t) \in \mathbb{R}^n$ taken by the system is one that gives an extreme value to the action $\mathcal{S}(\gamma)$. Using the techniques of calculus of variations one gets the Euler-Lagrange equations:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = 0,\; i = 1, ..., n. \quad (2)$$

Instead of a system of n discrete particles, we might want to consider a continuous scalar field ϕ defined over a continuous index set, $\phi(x)$. The motion of such a field will be given by a trajectory $\phi(x)(t) \in \mathbb{R}$ for each x in the index set. Usually, this index set will be $\mathbb{R}^n$ and the motion of the field will be described by the notation $\phi(x)(t) = \phi(x_0, x_1, ..., x_n) = \phi(x_i)$, where x_0 will denote time. Then we can consider a Lagrangian function $\mathcal{L}(\phi(x_i), \partial_j\phi(x_i), x_i)$ for the system. This, in fact, would be a Lagrangian density, since in order to define a Lagrangian analogous to the Lagrangian of a discrete system, we would have to take its integral over the whole index set, as $L(\phi(x_0), \partial_i\phi(x_0), x_0) = \int_\Sigma \mathcal{L}(\phi, \partial_i\phi, x_i)dx_1...dx_n$. Then, we define the action of the system for a given trajectory $\phi(x_i)$ as the integral $\int L(\phi(x_0), \partial_i\phi(x_0), x_0)dx_0$, which we can write as

$$S(\phi) = \int_{\tilde{\Sigma}} \mathcal{L}(\phi(x_i), \partial_j\phi(x_i), x_i)dx^n \,. \tag{3}$$

Here the principle of least action can be extended to continuous systems, obtaining the Euler-Lagrange equations for scalar fields:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \sum_i \frac{\partial}{\partial x_i}\frac{\partial \mathcal{L}}{\partial(\partial_i\phi)} = 0 \,. \tag{4}$$

B. Klein-Gordon equation in a general metric

In order to discuss the topics that interest us in this work, we need to formulate classical field theory in the framework of general relativity. Let then $\mathcal{M}$ be an n-dimensional Riemannian manifold with metric tensor $g(x) = g_{\mu\nu}(x)dx^\mu dx^\nu$, where the summation convention is used now and throughout the work. Let then $\phi(x)$ be a scalar field, in the sense that it is invariant under coordinate transformations. Let $\mathcal{L}(\phi, \partial_\mu\phi)$ be a Lagrangian (density) function for the field, which should again be a scalar function. We must keep in mind that now ∂_μ is in fact the covariant derivative ∇_μ. With some considerations about integration, the action ends up being

$$S(\phi) = \int_\Sigma \mathcal{L}(\phi, \nabla_\mu\phi)\sqrt{-g}dx^n \,. \tag{5}$$

Where $g = \varepsilon_{\mu_1}...\varepsilon_{\mu_n}g^{1\mu_1}...g^{n\mu_n}$ is the determinant of the metric tensor. Following the principle of stationary action we can vary now the quantity S with respect to the field ϕ. Following similar calculations but with a more careful use of Stokes' theorem in Riemannian manifolds we arrive at:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \nabla_\mu \frac{\partial \mathcal{L}}{\partial(\nabla_\mu\phi)} = 0 \,, \tag{6}$$

Which are Euler-Lagrange equations in Riemannian manifolds. Here, in order to give an example that what will be very useful later, we can consider the simplest Lagrangian function that can be constructed for a scalar field. Indeed, if we want $\mathcal{L}(\phi, \nabla_\mu\phi)$ to be a scalar depending on ϕ as well as its first covariant derivative, we can do so by considering a term of the form $(\nabla_\mu\phi)(\nabla_\nu\phi)g^{\mu\nu}$ together with a scalar function of the field which we are going to call by analogy its potential $V(\phi)$. We can see that it is indeed the simplest non-trivial form that we can give to the Lagrangian. It is customary to write it as $\mathcal{L}(\phi, \nabla\phi) = \frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi g^{\mu\nu} - V(\phi)$. Then, with this Lagrangian, the Euler-Lagrange equation takes the form:

$$\frac{dV(\phi)}{d\phi} + \Box\phi = 0 \,, \tag{7}$$

where $\Box$ is the d'Alembertian operator, defined as $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu = \nabla^\mu\nabla_\nu$. This equation is the well-known Klein-Gordon equation for a scalar field in a Riemannian manifold and corresponds to the equation of motion of the field.

C. Einstein Field Equations

As another application of the ideas developed using calculus of variations, we can derive Einstein's field equations for the metric tensor g. Since $\nabla_\mu g = 0$, we have to define the Lagrangian in another way. The answer is given by the Einstein-Hilbert Lagrangian, which is defined as $\mathcal{L}(g, \nabla_\mu\nabla_\nu) = \frac{M_P^2}{2}R$, where M_P is the reduced Planck mass, with a value of $M_P \approx 2.4 \times 10^{18}$ GeV. Here, R is the Ricci scalar, a contraction of the Riemann tensor:

$$R^\beta_{\mu\nu\alpha} = \partial_\mu\Gamma^\beta_{\nu\alpha} + \Gamma^\beta_{\mu\lambda}\Gamma^\lambda_{\nu\alpha} - \partial_\nu\Gamma^\beta_{\mu\alpha} - \Gamma^\beta_{\nu\lambda}\Gamma^\lambda_{\mu\alpha} \,. \tag{8}$$

Since the Christoffel symbols are expressions on the first derivative of the metric, the Riemann tensor and its contractions are covariant expressions on the second derivatives of the metric. Studying the symmetries of the components of the Riemann tensor, it can be seen that all of its contractions are equivalent to the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu} = g^{\mu\nu}R^\beta_{\mu\alpha\nu}g_{\beta\lambda}g^{\lambda\alpha}$. Therefore, with the action defined as $S(g) = \int_\Sigma R\sqrt{-g}dx^n$, we can use the principle of stationary action again to find the equations of motion for the metric. Computing the variations for the terms R and $\sqrt{-g}$ as

$$\delta R = R_{\mu\nu}\delta g^{\mu\nu} + \nabla_\mu(g^{\mu\nu}\delta\Gamma^\beta_{\beta\nu} - g^{\beta\nu}\Gamma^\mu_{\beta\nu}) \tag{9}$$

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu} \,. \tag{10}$$

Then, using again Stokes' theorem, we finally arrive at the expression for $\delta\mathcal{S}(g) = 0$ as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0 \,. \tag{11}$$

Which are the famous Einstein field equations in the absence of energy. In order to add an energy-matter distribution, we can add an extra Lagrangian corresponding to matter, getting after normalization $\mathcal{L} = \frac{M_P^2}{2}R + \mathcal{L}_M$.

Then, the principle of least action gives us Einstein's field equation in the presence of an energy distribution:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{M_P^2}T_{\mu\nu},\ T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_M}{\delta g^{\mu\nu}}\,. \quad (12)$$

III. COSMOLOGY AND INFLATION

A. Review of cosmology

In the following sections, we are going to study inflation in the context of the hot big bang cosmological model. To do so, we are going to consider the usual cosmological solutions to the Einstein field equations (namely, the Friedmann equations), together with an extra contribution from a Lagrangian corresponding to inflation. Then, we are going to find solutions to this model and discuss its parameters.

As usual in cosmology, we are going to work with the Friedmann-Robertson-Lemaitre-Walker metric, given by

$$g = \mathrm{diag}\{-1, \frac{a^2(t)}{1-kr^2}, a^2(t)r^2, a^2(t)r^2 sin^2\theta\}\,, \quad (13)$$

where $a(t)$ is the scale factor corresponding to the expansion of the universe and k is the curvature, which by a change of coordinates can always be chosen to be $k = -1, 0, +1$. With this metric tensor, we can find solutions for the Einstein field equations in terms of the energy-momentum tensor $T_{\mu\nu}$. As usual in cosmology, we are going to suppose that the energy distribution will be that of a perfect fluid co-moving with the coordinates. That way, the energy-momentum tensor is diagonal and has the form

$$T_{00} = \rho,\ T_{ii} = pg_{ii}\,, \quad (14)$$

where ρ and p are the density and pressure of the perfect fluid and are usually related by an equation of state. A usual one is $p = w\rho$, with $w = 0$ for matter and $w = 1/3$ for radiation, two usual cases representative of two different stages of cosmological evolution. With this energy-momentum tensor and with the FRLW metric, the solution of Einstein's equations give us two independent differential equations, known as the Friedman equations, which defining the Hubble parameter as $H(t) = \frac{\dot{a}(t)}{a(t)}$ are:

$$H^2 = \frac{1}{3M_P^2}\rho - \frac{k}{a^2},\ \dot{H} = -\frac{2}{3M_P^2}(\rho + 3p) - H^2\,. \quad (15)$$

B. Inflation

In order to solve some issues of the hot big bang model, and early inflationary stage of the evolution of the universe was proposed, with great success [1]. This inflation is modeled with a perfect fluid with equation of state $p = -\rho$. In order to get that, we can assume the existence of a scalar field ϕ homogeneous in space (i.e., only dependent on the coordinate t). Its Lagrangian, according to section 2, will be given by $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} - V(\phi)$. Then, the Klein-Gordon equation in the FRLW metric takes the form:

$$V'(\phi) + \ddot{\phi} + 3H\dot{\phi} = 0. \quad (16)$$

Similarly, we can compute the density and pressure for ϕ from expression eq. (12) as:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \quad (17)$$

Thus, we can see that in order to have $w = -1$ we can impose the condition that $|\dot{\phi}^2| \ll |V(\phi)|$. An additional condition is required so that the inflationary stage lasts long enough for its effects to have the expected observational consequences. This condition can be stated in terms of the second derivative of the field as $|\ddot{\phi}| \ll |V'(\phi)|, |H\dot{\phi}|$. Those two conditions are usually referred to as slow-roll conditions and can be given in terms of the two slow-roll parameters $\varepsilon(\phi)$, $\eta(\phi)$:

$$\varepsilon(\phi) = \frac{1}{2}M_P^2\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \qquad \eta(\phi) = M_P^2\frac{V''(\phi)}{V(\phi)}\,. \quad (18)$$

And the slow roll conditions will be satisfied when $\varepsilon(\phi), |\eta(\phi)| \ll 1$. There is another important parameter that measures the amount of inflation of the universe during the stage were the slow roll conditions is satisfied: the number of e-folds. It is defined as $N(t) = \log\frac{a(t)}{a(t_0)}$, where t_0 is the time when inflation begins and t is a generic time. Substituting terms from the Friedmann equations and using the slow roll approximation, it can be rewritten as

$$N(\phi) = -\frac{1}{M_P^2}\int_{\phi_0}^{\phi}\frac{V(\phi)}{V'(\phi)}d\phi\,, \quad (19)$$

where $\phi_0 = \phi(t_0)$. If ϕ_1 is the value at which

$$\varepsilon(\phi_1), \eta(\phi_1) = 1\,, \quad (20)$$

most cosmological observations require that the model satisfies $N(\phi_1) > 60$. We can notice here that we can solve for ϕ_1 the equations $\varepsilon(\phi_1) = 1$ and $\eta(\phi_1) = 1$ in order to find the values at which inflation ends in this model. We must add here that due to considerations in quantum field theory, the initial condition for inflation must be taken to be

$$\phi_0 = H/2\pi\,. \quad (21)$$

IV. MODIFIED INFLATION

The inflation models that follow what was exposed in the previous section differ among themselves by means of the potential function $V(\phi)$. The potential that will be studied here is the natural potential, which has the form

$$V(\phi) = \Lambda^4 \left(\cos \frac{\phi}{F} + 1 \right) , \tag{22}$$

where Λ, F are energy scales of the model with dimensions of energy. Some motivation for this choice can be found in [2], but the argument is that it naturally gives a flat enough potential (for appropriate energy scales).

However, the observations made of the spectral index and of the tensor-to-scalar ratio from the CMB are in a small disagreement with the predictions of the model (see [4]). That's why a coupling with a second scalar field was suggested, and is the modified scalar field theory for inflation that we are going to study next.

A. The modified theory

In this modified model we postulate the existence of a second scalar field χ in the early stages of the universe which will affect the evolution of the inflaton ϕ. The idea is that this secondary scalar field could experience an exponential growth, similarly to what was proposed by [5], that included a coupling between ϕ and a gauge field. The field χ could then back-react on the first scalar field. As we will show, this will resulting in a shorter duration of inflation. The simplest interaction with a second scalar field [9] that respects the shift symmetry of ϕ is the following:

$$\mathcal{L} = \frac{1}{2} \left(\partial_\mu \phi \partial_\nu \phi + \partial_\mu \chi \partial_\nu \chi \right) g^{\mu\nu} - V(\phi) + \frac{1}{f} \chi \partial_\mu \chi \partial_\nu \phi g^{\mu\nu} , \tag{23}$$

where f is a new energy scale associated to the coupling of the field χ with ϕ. Then, for each field and assuming slow-roll conditions for ϕ, the Klein-Gordon equation takes the form:

$$\phi : \, 3H\dot{\phi} = -V'(\phi) - \frac{1}{f}(\dot{\chi}^2 + \chi\ddot{\chi} + 3H\chi\dot{\chi}) \tag{24}$$

$$\chi : \, \ddot{\chi} + 3H\dot{\chi} + \frac{1}{f} 3H\chi\dot{\phi} = 0 \, . \tag{25}$$

We can also compute the energy-momentum tensor for this Lagrangian from eq. (12), assuming slow-roll conditions for ϕ, as:

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{f}\chi\dot{\chi}\dot{\phi} + V(\phi) \tag{26}$$

$$p = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{f}\chi\dot{\chi}\dot{\phi} - V(\phi) \tag{27}$$

Our aim is, for a given choice for the parameters Λ and F (which we fix using the typical values that fit observations for NI), to find the range of values for f for which inflation is shortened and to understand quantitatively by how much it is shortened.

First, assuming that H and $\dot{\phi}$ are roughly constant one finds exponential solutions of eq. (25) of the form $\chi(t) = \chi_0 e^{\alpha t}$. Here again $\chi_0 = H/2\pi$ as in eq. (21) Substituting this into eq. (25) and solving for α while approximating $V'(\phi) \approx \bar{V}'$ as the average $\frac{V(\phi_1)-V(\phi_0)}{\phi_1-\phi_0}$ (where ϕ_1 is defined in eq. (20)) , we get the solutions:

$$\alpha_\pm = -\frac{3H}{2} \pm \frac{1}{2}\sqrt{\frac{4\bar{V}'}{f} + 9H^2} \, . \tag{28}$$

We are only interested in positive values for α, since negative exponentials yield a decaying field which does not affect the end of inflation. Clearly $\bar{V}' < 0$ from eq. (22), since the field rolls from a positive initial condition towards the minimum at $\phi = \pi F$. Therefore we can see that if $f < 0$ we are going to get $\alpha_+ > 0$ and $\chi(t)$ will grow exponentially. We will consider only $\alpha = \alpha_+$.

If we consider the terms on ϕ and the terms on χ in the right hand side of eq. (24), we can see that the term on χ acts as anti-friction, making ϕ accelerate faster. Inflation will therefore last until this contribution from χ grows to be of the same size as the contribution from the inflaton field given by $V'(\phi)$, since $\dot{\phi} \sim e^{\alpha t}$ from that point onward.

We can compute the time t_* where both contributions in the right hand side of eq. (24) are of the same size by solving for t_* in:

$$\frac{1}{f}(2\alpha^2 + 3H\alpha)\chi_0^2 e^{\alpha t_*} = -V'(\phi(t_*)) \, . \tag{29}$$

Therefore, t_* is the time when the new inflation ends. Notice that an explicit expression for $\phi(t)$ can be found by solving the differential equation eq. (16) assuming $\ddot{\phi} = 0$ (slow-roll conditions). Since once Λ and F have been fixed, the quantity t_* only depends upon the value of the coupling parameter f, we can compute the values of the end of the inflationary period for different values of f. We do so in the next section.

B. Results

Here, concrete results are presented for the model and expressions described in the previous section. Solving eq. (16) we obtain $\phi(t)$ and solving eq. (29) we get $t_*(f)$, where f is the coupling energy scale. We can therefore compute $\phi(t_*(f))$ for a range of values for $f < 0$ in order to compare them with the end of inflation ϕ_1 defined in eq. (20). We do so in fig. 1. In this way, we can find the range of f for which $\phi_*(f) < \phi_1$, resulting in a shorter inflation.

In order to compare the amount of inflation given by this modified model, we can compare $N(\phi_1)$ and $N(\phi_*(f))$. In this way, we can obtain a lower bound for which $N(\phi_*(f)) \approx 60$. We do so in fig. 2.

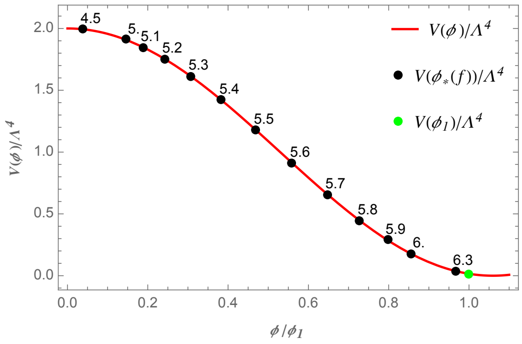

FIG. 1: Different values for $V(\phi_*(f))$ plotted over the NI potential of eq. (22). The label k for each point corresponds to $f = -kF \times 10^{-1}$. To get those results, the values $\Lambda = 1.8 \times 10^{16}$ GeV and $F = 8M_P$ [4] were used. The green point corresponds to the end of the original single-field inflation. A value of $\phi_*(f) \approx \phi_1$ is reached for $f \approx -0.65F$.

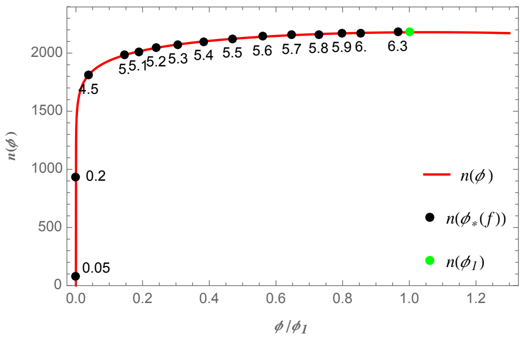

FIG. 2: Different values for $n(\phi_*(f))$ plotted over the function $N(\phi)$ calculated by solving eq. (19). The label for each point corresponds again to $f = -kF \times 10^{-1}$. To get those results, the values $\Lambda = 1.8 \times 10^{16}$ GeV and $F = 8M_P$ [4] were again used. The green point corresponds to the end of the original single-field inflation. A value of $N(\phi_*(f)) \approx 60$ is reached for $f \approx -0.005F$.

V. CONCLUSIONS

- In the first sections, we obtained the equations of motion for a scalar field both in euclidean and in Riemannian geometries. We saw how the Euler-Lagrange equations for a scalar field became the Klein-Gordon equation for a scalar field in a generic metric. We also derived from a variational principle the Einstein field equations.

- We reviewed the basic contents of cosmology and of inflationary theories, allowing us to introduce expressions and concepts used in the final section. We introduced the notions of a slow-rolling scalar field and of the number of e-folds of the inflationary expansion. The way to obtain explicitly the field and time values at the end of inflation was also indicated.

- Finally, we studied the model where a second scalar field is coupled with the main inflation field in the simplest possible way. Here we used specific values for the parameters Λ and F in order to give a range of values for the coupling parameter f for which the inflation in shortened but without going below the bound of $N \gtrsim 60$. This results were presented in fig. 1 and fig. 2. Using the values $\Lambda \approx 1.8 \times 10^{16}$GeV and $F \approx 8\bar{M}_P$ we obtained an interval for f of $-0.65F \gtrsim f \gtrsim -0.005F$.

- It must be noted that we have not obtained any result comparable with the observational data to prove whether this model makes better predictions than the previous one. This was out of the scope of the work.

Acknowledgments

I would like to thank all the people who have been beside me all these years. Specially, I would like to express my gratitude towards my parents and sister for their support and to my advisor, Alessio Notari, without whom this work would not have been possible.

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