Constraints on neutron-electron coupling inside Neutron Stars.

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Abstract: Neutron star cores reach high values of density where nuclear matter can interact via weakly short range forces. We consider Neutron star in the $T \approx 0$ approximation, and matter to be in $\beta - equilibrium$ and composed by neutrons, protons and electrons. In the core region of the Neutron Star we add, within the Hartree-Fock approximation, a hypothetical coupling between neutrons and electrons mediated by a dark boson, and we constrain the magnitude of this interaction by comparing the resulting mass-radius relation with the recent data of the massive Neutron Star PSR J0740+6620, obtaining a restriction on the coupling strength when attractive interaction is considered.

I. INTRODUCTION

Physics, including particle physics, is a constantly evolving science. The introduction in the latter half of the 20th century of the Standard Model (SM), a theory describing weak, strong and electromagnetic interactions, gave us an explanation of three out of the four known elemental forces. Furthermore, a classification of the particles in quarks, leptons, antiquarks, antileptons and gauge bosons (plus the Higgs boson) has been established within the SM.

However, we still can not explain the whole phyisics within the SM. There are unexplained physics phenomena, and some predictions beyond the SM have been made. In this project we will focus on one of these predictions: Non-linearity on isotope shift in Yb measurements is not well fitted in the SM yet, and could be explained with a neutron-electron interaction beyond the SM, as discussed in Ref.[1].

Limiting the mass and the strength of the neutron-electron coupling mediated by a hypothetical dark boson is the main objective of this work.

In order to constrain this coupling, we will study the effect that, eventually, the neutron-electron interaction has on the Neutron star (NS) structure. In a very dense system, like a NS, we expect to see effects of even a weakly low-range interaction, making NS core's a good environment to treat this problem.

We will consider the dark boson only to act at high densities (above 0.5 times the nuclear saturation density, $n_0 = 0.16 fm^{-3}$) so the neutron-electron interaction will only be treated in the NS core. We will assume the core matter to be composed by neutrons, electrons and protons. For the crust, we use the equation of state HD01 from Ref.[2] (describing the matter from the surface to $0.5n_0$).

The bounds on the coupling will be made by comparing, for different values of the interaction, whether the mass-radius relation of the NS differ significantly from the recent data of mass ($2.072^{+0.067}_{-0.066} M_\odot$) and equatorial radius ($12.39^{+1.30}_{-0.98}$ Km) of the massive Neutron Star *PSR J0740+6620* (see Ref.[3]).

II. NEUTRON STAR CORE: EQUATION OF STATE

We will focus on the EOS of the NS core (approximately densities above $0.5n_0$ [2]), which we assume is in $\beta - equilibrium$. This means there is an equilibrium between the neutron decay and electron capture reactions,

$$\begin{aligned} n &\rightarrow p + e + \bar{v}_e\,, \\ p + e &\rightarrow n + v_e\,. \end{aligned} \tag{1}$$

In the zero temperature limit, we treat the NS to be neutrino transparent, as the mean free path of the neutrinos is larger than the NS size (see Ref.[4]). In this project we do not take into account other weak interactions that involve other particles, because some particles, as positrons, can be neglected or because some particles, as muons, do not have a qualitative impact on the final results and neglecting them will simplify the expressions, making easier its interpretation.

The equilibrium between weak interactions, Eq.(1), lead us to the relation between the chemical potential μ of the particles: $\mu_n = \mu_p + \mu_e$ and $\mu_e = \mu_p$

Imposing the conservation of the baryon number, n, and the charge neutrality, one finds the expressions $n = n_p + n_n$ and $n_e = n_p$ which have to be fulfilled. Furthermore one can define the particle fractions $x_p = \frac{n_p}{n}$, $x_n = \frac{n_n}{n}$ and $x_e = \frac{n_e}{n}$.

In order to find our EOS of the core we use the approximate analytic expression used for uniform matter [4],

$$E_N(u, x_p) = W(u) + S(u)(1 - 2x_p)^2\,, \tag{2}$$

where $u = \frac{n}{n_0}$ and E_N is the binding energy per nucleon. The $W(u)$ term is the energy per nucleon of symmetric nuclear matter and $S(u)$ represents the symmetry energy (difference in the binding energy between symmetric and pure neutron matter). The total energy per nucleon of the system is $E = E_0 + E_N + E_e$, where E_0 is the energy per baryon corresponding to the rest mass of neutron and proton, and E_e is the energy per baryon due to electrons (which are treated as a relativistic free Fermi gas).

It is very useful to define the energy density $\epsilon = \epsilon_0 +$

$\epsilon_N + \epsilon_e$, where ϵ_0 corresponds to the rest mass density ($n_n m_n$ and $n_p m_p$), $\epsilon_N = nE_N$ is the binding energy density and ϵ_e is the energy density of an electron relativistic free Fermi gas.
Neutron and proton chemical potentials are obtained from $\mu_n = \frac{\partial(\epsilon_N+\epsilon_0)}{\partial n_n}$ and $\mu_p = \frac{\partial(\epsilon_N+\epsilon_0)}{\partial n_p}$.

A. Approximate treating of $\beta - equilibrium$

With the aim of finding the core EOS we need to express Pressure as function of density, n, through the expression $P = n\frac{\partial \epsilon}{\partial n} - \epsilon$. As ϵ depends not only on n but also on x_p, we demand a relation of the form $x_p(n)$.
For this purpose, considering the $\beta - equilibrium$ condition $\mu_e = \mu_n - \mu_p$ and making the assumption that $m_n \approx m_p$, we find the expression

$$\mu_e = \mu_n - \mu_p = 4S(u)(1-2x_p)\,. \tag{3}$$

Treating electrons as an ultra relativistic free Fermi gas we have $\mu_e = \hbar c \sqrt[3]{3\pi n_e}$. Now we can rewrite Eq.(3) as

$$x_p = \frac{(4S(u)(1-2x_p))^3}{3\pi^2 n(\hbar c)^3}\,. \tag{4}$$

In this approximate relation one can clearly see that the particle fractions depend on the symmetry energy $S(u)$, so $\beta - equilibrium$ depends only on $S(u)$ and not on $W(u)$. For a given $S(u)$ one can get $x_p(n)$ solving Eq.(4), and therefore obtain the corresponding EOS.

B. Heiselberg and Hjorth-Jensen model

Heiselberg and Hjorth-Jensen (HH) exposed an analytical expression for $S(u)$ and $W(u)$ [5] to fit the EOS proposed by Akmal *et al.* [6]. For the symmetric matter term, they choose a functional form

$$W(u) = \epsilon_0 u \frac{u-2-\delta}{1+\delta u}\,, \tag{5}$$

whereas the symmetry energy is a power-law

$$S(u) = Ju^\gamma\,. \tag{6}$$

In this work we take the following parameters:

- $\epsilon_0 = -15.8$ MeV [5], is the binding energy per nucleon at the saturation density n_0, ignoring the Coulomb energy.
- $J = 32$ MeV [5], is the symmetry energy of the nuclear matter at saturation density n_0.
- δ is the *softness* parameter, which is related to the compressibility of the NS. At saturation point, for symmetric matter, we have $K_0 = 9n^2\frac{\partial^2 e}{\partial n^2} = \frac{18e_0}{1+\delta}$. It is taken as $\delta = 0.1$ ($K_0 = 258$ MeV) for the best fit with data [3].
- γ is related to L, which is the derivative of the symmetry energy J at the saturation point n_0. $L = 3n_0\frac{\partial S}{\partial n} = 3\gamma S_0$, so increasing γ will entail a faster increase of the symmetry energy with density. With the intention of treating different types of EOS, we will consider three possible values of $\gamma = 0.5, 0.6, 0.7$ corresponding to $L = 48$ MeV, 57.6 MeV, 67.2 MeV.

C. Adding the dark boson: Hartree-Fock method

In order to add the effect of a coupling between neutrons and electrons we work in the Hartree-Fock approximation. The interaction will be treated with a Yukawa-like potential of the type [7],

$$V_\phi(r) = \frac{(-)^{s+1} y_e y_n}{4\pi}\frac{e^{-m_\phi r}}{r}\,, \tag{7}$$

where V_ϕ is the potential in atomic units, m_ϕ represents the mass of the dark boson and $y_e y_n$ is the neutron-electron coupling strength. The mass of the mediator m_ϕ informs us on the approximate range of the interaction, which is inversely proportional to m_ϕ [7].
In Eq.(7) the term $(-)^{s+1}$ indicates that we will consider both repulsive (+ sign) and attractive (- sign) interaction. As discussed in Ref.[7], the sign will depend on s, which is the spin of the hypothetical force carrier, and could take values $s = 0, 1, 2$. In our work we will treat this as considering both repulsive and attractive interactions.
In the Hartree-Fock approximation we do not consider the exchange term, as we are treating an interaction between two different types of particles (neutron and electron). We also assume translational invariance, which implies uniform densities.
In these approximations we obtain the contribution to the mono-particular energy due to the neutron-electron coupling, which is $U_n = \frac{(-)^{s+1}y_e y_n}{m_\phi^2}n_e$ for neutrons and $U_e = \frac{(-)^{s+1}y_e y_n}{m_\phi^2}n_n$ for electrons. Using Koopmans' theorem, the mono-particular energy can be used to express the contribution to the corresponding chemical potentials of neutrons and electrons. Adding them to Eq.(3), one finds that this contribution can be expressed as a correction to the symmetry energy in Eq.(4). In order to analyse the effect of the dark boson we define

$$S_g(u) = Ju^\gamma - \frac{un_0}{4}\frac{(-)^{s+1}y_e y_n}{m_\phi^2}\,, \tag{8}$$

which can be interpreted as a new 'effective' symmetry energy term. It is useful to define a dimensionless coupling, g_0. We propose this to be $g_0 = \frac{n_0(-)^{s+1}y_e y_n(\hbar c)^3}{4m_\phi^2 J}$ which would represent that for $g_0 = 1$ the 'effective' symmetry energy term at the saturation point, $S_g(u{=}1)$,

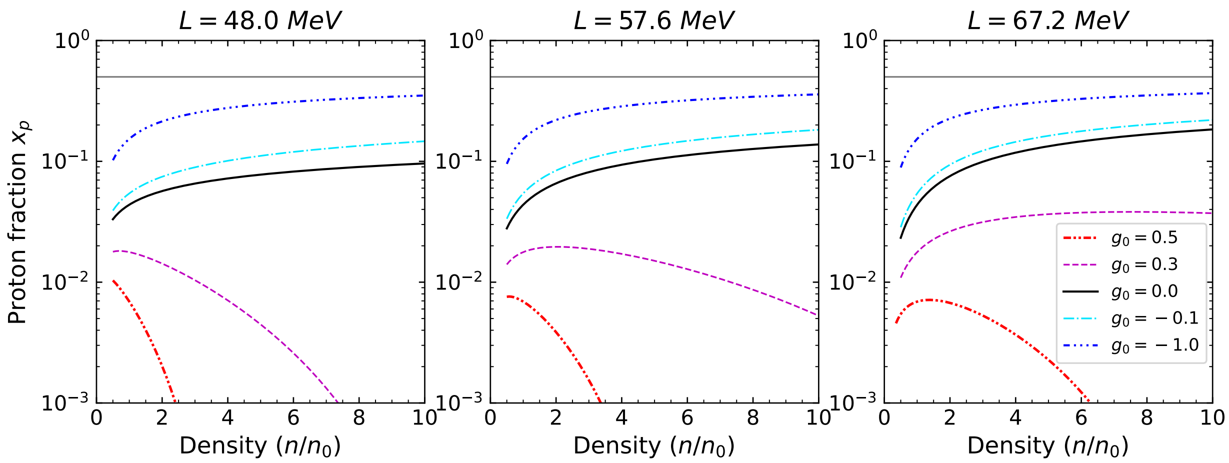

Figure 1: Dependence of proton fraction as a function of density inside the NS core, for different values of the coupling g_0 considering both attractive and repulsive interaction. It is also shown the dependence for different values of L. The gray line at the value of $x_p = 0.5$ shows the point at which the NS matter would be symmetric.

is zero, and for $g_0 = -1$ means that the 'effective' symmetry energy term at the saturation point, $S_g(u{=}1)$, is twice the value suggested by the HH model.

Let us look into the effect of adding a neutron-electron coupling into the star's $\beta - equilibrium$ composition. Figure 1 shows how the x_p varies with density, obtained after computing Eq.(4) using $S_g(u)$. We show the relation $x_p(n)$ for different values of g_0 in both attractive and repulsive interaction. The left panel corresponds to $L = 48.0$ MeV, the central panel to $L = 57.6$ MeV and the right panel to $L = 67.2$ MeV. Moreover a gray horizontal line at $x_p = 0.5$ is shown in all panels, indicating the value at which we would have symmetric matter.

As seen in figure 1, if we consider attractive interaction, the neutron fraction within the core decreases and the proton fraction increases, tending to symmetrize (same number of protons and neutrons) the NS matter for large values of the coupling. For repulsive interaction the proton fraction x_p decreases as increasing g_0, and for $g_0 = 0.3$, for the case of $L = 48.0$ MeV (left panel), the NS at the center ($r = 0$) is completely composed only by neutrons. The same occurs for $g_0 = 0.5$ for $L = 67.2$ MeV (right panel). For large values above $g_0 = 0.5$ there would be only neutrons composing the core matter, which implies $x_p = 0$ (entailing that no electrons are present in this zone, and so no neutron-electron coupling is possible). This leads us to conclude that positive g_0 tends to 'neutronize' the NS matter.

III. SOLVING THE TOV EQUATIONS

Having the relation $x_p(n)$ and the core and crust EOS, we can now obtain the structure of our NS (the total mass and radius) solving, in the approximation of a relativistic non-rotation spherical NS, the Tolman–Oppenheimer–Volkoff equations (TOV)

$$\frac{dP}{dr} = -\frac{1}{r^2}\frac{(P+\epsilon)(m_< + 4\pi r^3 P)}{(1-\frac{2m_<}{r})}\,, \tag{9}$$

$$\frac{dm_<}{dr} = 4\pi r^2 \epsilon\,, \tag{10}$$

where ϵ is the energy density, $m_<$ is the enclosed mass at a given radius r and P is the pressure.

First thing to do is match the crust and core EOS. For the crust we will be using the HD01 EOS [2] (going from the surface to $0.5n_0$ density). It is important to note that the core EOS and the crust EOS have not been done considering the same type of nucleon-nucleon interaction. Moreover, in the crust EOS the dark boson effect has not been taken into consideration.

In spite of this inconsistency, the results would not change significantly when matching both into a unified EOS. As discussed in Ref.[8], this will not affect the total mass but the radius of the NS, with relative differences as large as $\approx 4\%$ when matching crust and core EOS build with different nuclear models [8]. Discussions on the thermodynamic inconsistency are also shown in Ref.[8]. The matching will be made following two methods, analysed at Ref.[8]:

1. In case the two EOS cut at a given point near the saturation density n_0, we concatenate the HD01 from the surface to the density where they cut, with the EOS of the core from this density to above.
2. In case they do not cut at any point, we use the HD01 from the surface to $0.5n_0$ (at a pressure P_0) and the core EOS from the density where the pressure is $P = P_0$ to above. The match between the two is a linear interpolation at constant pressure.

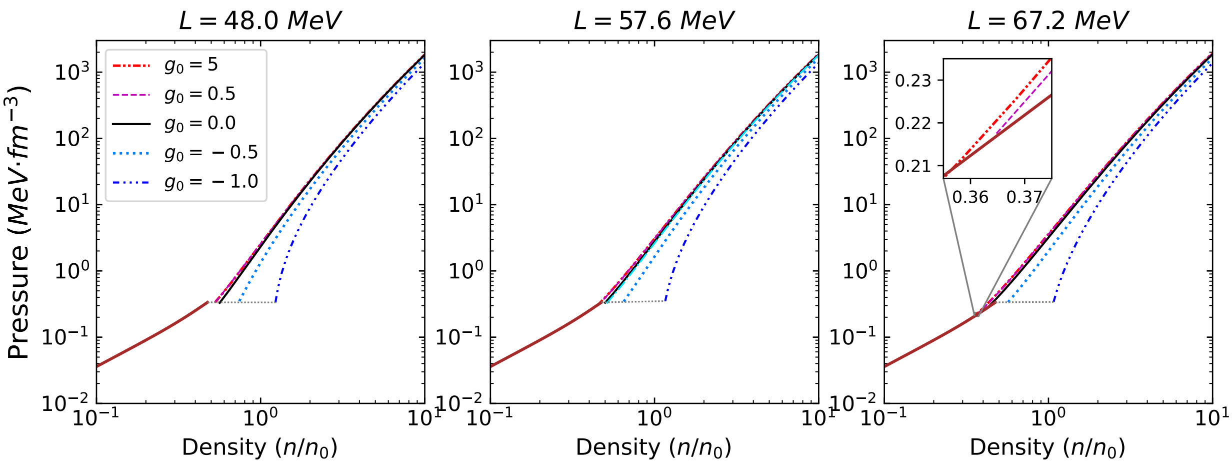

Figure 2: For $\delta = 0.1$, relation between Pressure and density for different values of L corresponding to $\gamma = 0.5$,$\gamma = 0.6$ and $\gamma = 0.7$ are shown in log-log axes. We represent the non-unified EOS for differnt values of the coupling g_0. It is also represented, with a grey densely dotted line, the lineal interpolation used in the match of the crust and core EOS.

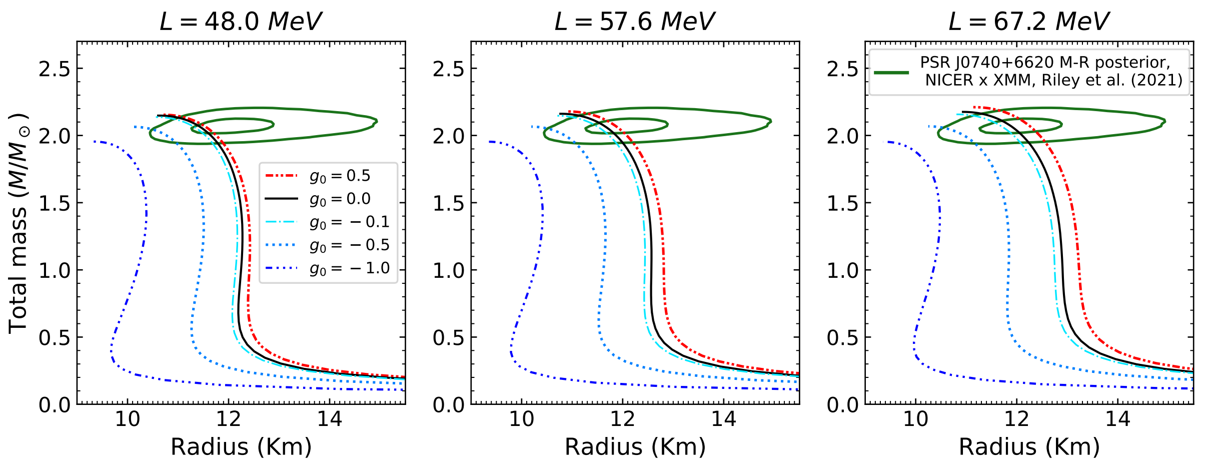

Figure 3: Mass-radius relation of the NS for different values of L and g_0, both attractive and repulsive interaction case. We show, with dark green lines, the results on mass-radius relation given by [3] from Nicer and XMM-Newton data from PSR J0740+6620. The inner dark-green ellipse represents a 68% credible region and the outer a 95% credible region.

In figure 2 we represent the relation $P(n)$ ($\equiv EOS$) used in the crust and in the core. We show this relation for different values of the coupling g_0, both attractive and repulsive interactions, and also for the three different values of L. On the right panel we show, in an inset, the intersection point between the crust and core EOS for the values of $g_0 = 0.5$ and $g_0 = 5$. These values are important: for $g_0 = 0.5$ we have that for the three values of L, the center of the NS core starts to be composed only by neutrons; for $g_0 = 5$, for all the three values of L, all the NS core is only composed by neutrons. The gray densely dotted line unifying crust and core EOS represent the lineal interpolation made at constant pressure.
Having found a unified crust-core EOS we obtain the M-R relation. The constrain is made imposing that the recent data on mass and radius from the massive Neutron star PSR J0740+6620 published at Ref.[3] have to be fulfilled by our NS mass-radius relation. These bounds are $M = 2.072^{+0.067}_{-0.066} M_\odot$ and $R = 12.39^{+1.30}_{-0.98}$ Km.
In figure 3 a representation of the mass-radius relation, for different values of the coupling g_0, is made. It is also represented for the three different values of L. On each panel the recent data of massive NS PSR J0740+6620 [3] on radius and mass is shown with dark-green lines (68% and 95% credible region respectively).
As we can see in figure 3, a conservative constrain on the coupling for the attractive interaction can be made at $g_0 = -1$ independently of the value of L, as the bounds imposed on radius and mass are clearly not being fulfilled. For positive g_0 no bounds can be established as it has a small effect on the mass-radius relation.

IV. CONCLUSIONS

The addition of neutron-electron interaction inside NS core has been made within the HH model. In order to see clearly the effect of the coupling on the particle fractions we have made two approximations: $m_n \approx m_p$ and an ultra relativistic Fermi gas treatment of the electrons.
Using these procedures and approximations, we have seen the qualitative effect that a dark boson coupling would have in NS. For the NS core, an attractive neutron-electron coupling tend to symmetrize the NS matter. The opposite occurs for repulsive interaction, where for large values of the coupling we have that $x_n \rightarrow 1$ and $x_p \rightarrow 0$, $x_e \rightarrow 0$, and the dark boson 'neutronizes' the NS matter.
Analysing the final results on the mass-radius relation for different values of g_0 we can put a constraint for the neutron-electron coupling when attractive interaction is considered, taking a conservative value of $g_0 = -1$. In figure 4 this constrain is shown in terms of the relation between the coupling strength, $y_e y_n$, and the mass of the dark boson m_ϕ (the area over the red line represents the forbidden values imposed by our work). The Compton wavelength λ_C (shown on the upper axis in figure 4) gives us an idea about the range of the interaction mediated by the force carrier with mass m_ϕ. We can also see in figure 4 the constrain given by this work in comparison with constraints from other measurements [9–11]. It is interesting to see that our restriction could be useful to give an idea of a possible bound for a short-range interaction, where the constraints shown do not reach.
Furthermore, it is important to note that our restriction has been established treating both coupling strengths (y_e,y_n) together on the same problem, in comparison with the other relations shown which have been obtained multiplying bounds for the coupling y_e with bounds for the y_n coupling.
A comment has to be made when considering repulsive interaction. No constrain have been done in this case due to the fact that the dark boson tends to neutronize the NS core matter. In this scenario the NS fulfills the bounds imposed on the mass-radius relation when the NS core gets fully composed only by neutrons. Above this point, no neutron-electron interaction can be considered and so an increase on the coupling has no effect. In order to constrain the coupling for repulsive interaction, a future study on possible instabilities when the NS core 'neutronizes' (and so we stop having $\beta - equilibrium$) would have to be made.

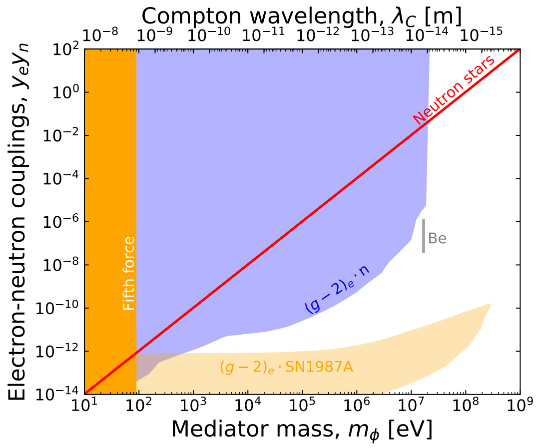

Figure 4: Neutron-electron couplings $y_e y_n$ as function of the mass mediator m_ϕ. On the upper axis is shown the corresponding Compton wavelength. Red line shows the bound imposed by our work at $g_0 = -1$. The gray bar represents the values of $y_e y_n$ at $m_\phi = 17$ MeV accomodating the Be anomaly [9]. Shaded areas correspond to the fifth force prediction (dark orange) [10], and the constrains from $(g-2)_e$ measurement combined with neutron scattering measurements (blue) and SN 1987A measurements (light orange) [11].

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