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Master thesis

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FUZZY MODEL FOR CREDIT-LINKED LIFE INSURANCE

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ABSTRACT

The aim of this project is to continue adjusting the price of life insurance linked to credits according to the profiles of the different characteristics of the insured. We will focus on the mathematical development of the Pichat Algorithm method in the field of fuzzy logic study, with the help of the Hamming distance method to find the subrelations of maximum similarity between different life insurance products. It allows us to take a new approach to measuring the similarity of various life insurance products, incorporating uncertainty management to more accurately match policies with policyholders' profiles. Therefore, we minimize the impact of the lack of information when making the decision.

KEYWORDS

- Uncertainly
- Credit-linked life insurance
- Maximum similarity subrelations
- Hamming distance
- Pichat's algorithm

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1. INTRODUCTION

1.1 Motivation

The insurance sector becomes a crucial role for the functioning of the economy: it has the function of promoting the development of the economy in society, as well as having a protective effect against any type of risk. For this reason, insurance companies have a notable and well considered tradition in the field of risk management.

However, the global economic crisis has been marked by the health crisis generated by Covid-19, which is causing a great impact worldwide. As a consequence, one of the main issues is the reduction in insurance premiums that leads to a reduction in the profits obtained in this sector, specifically in Spain.

The current situation together with the impact of this phenomenon has attracted a lot of attention and has not been ignored by the insurance industry. We not only see it reflected in customer demand, but also in the change in consumer habits and behaviours. People increasingly intend to gain more, to have maximum profitability when investing in a financial product and/or insurance.

Therefore, this leads us to investigate a topic that is fully topical in society. It is related to banks, insurance companies and individuals. To find an equilibrium point in transaction satisfaction between the three economic agents, we link the life insurance sector to loans. We consider competitive price increase factors under pressure from conditions in the general economic environment in the short and long term; the various characteristics of the policyholders; and the different profiles of insurance products.

This has the effect of guaranteeing the growth of the economy and the well-being of individuals. That is, it allows us to incorporate the importance of the concept of uncertainty to develop and to deepen those elements associated with the aforementioned research. From there, we are going to focus on the calculation of fair and equitable rates conditioned on their profiles.

1.2 Objective

The purpose of this project is to further adjust the price of life insurance according to the profiles of the different characteristics of the policyholders. We will focus on the mathematical development of the *Pichat's Algorithm* method in the Fuzzy Logic study field, with the help of the *Hamming Distance* in this case to carry out a more complete study vision.

The area of computational intelligence provides us with a wide flexibility for the treatment of the information that we use in the insurance sector. It allows us to have a new approach in measuring the similarity of various life insurance products, incorporating uncertainty management to match policies with policyholders' profiles with greater precision. Therefore, we minimize the impact of lack of information when making the decision.

This work is developed in a way that invites us to investigate a new trend that reinforces the line of traditional studies. Basically, in this work we will focus on the first part, where there is a solid theoretical approach, with the mathematical foundation of the contributions that we are going to make, together with the grouping methodology, which is one of the four bases to support the approach.

Once this work has been presented, the next step is more focused on a future path, in which we will carry out a more in-depth investigation together with the programming tool of this methodology, combining the statistical data to achieve a satisfactory result.

2. INSURANCE COMPANIES

2.1 Brief history

The word *"actuarius"* is derived from the Latin word meaning "notary or registrar", which was the person in charge of transcribing legal decisions so that they could pass into public domain. Since then, this function has been maintained until today, although the term actuary that we know has little relation to its origin. This profession is made up of people with high training and technical knowledge in mathematics and statistics, in the field of analysis, evaluation and quantification of the risks to which companies are exposed, especially in insurance companies.

The history of insurance began more than five thousand years ago. In the Ancient Age, the first impression of insurance already existed, basically when individuals faced problems when foraging. In this way, we also see it in the most advanced civilization such as the Greek, Roman or Babylonian, when there were losses of goods on their travels, which used them to cover the interests of their groups and individuals. And a little later, around 2250 BC, this practice was legalized as part of the Hammurabi Code· As an insurance, it covered any type of unforeseen risk that arose at that time.

When the Middle Ages arrived, the insurance sector took a level leap in history, it was a time of expanding commerce. From there on, life and travel insurances were born and it is considered the antecedent to current life insurance, being responsible for the rescue and safeguarded the lives of the crew and covered deaths due to illness or shipwreck. But the beginning of current insurance as we know it today, comes from the Maritime Insurance Ordinance in Barcelona (1435).

From the 18th century on, they began to consider the keys to calculating risks and probabilities and at the beginning of the 19th century, the first reinsurance company was created in Toulouse. Since then, this sector has evolved in an unstoppable way, not only showing us the adaptation capacities to cover new needs and contingencies in function of the changes in different times, but it has also taught us that insurance is already considered as a basic axis in the structure of society.

2.2 Life Insurance

2.2.1 Type of life insurance

The function of life insurance in financial activities is to guarantee the protection of the family, which objective is to protect people against the risk of death or total and permanent disability of the debtor to face unexpected situations that happen in life.

The types of life insurance have been classified in different denominations according to their different characteristics and different needs. Generally, there are the following types:

- Savings life insurance.
- Risk life insurance.
- Mixed life insurance.

Within the type of risk life insurance, there are two other modalities:

- Whole life insurance
- Temporary life insurance

Whole life insurance guarantees the payment of the capital set in the insurance contract after the death of the insured, it does not depend on when the loss occurs. On the other hand, temporary life insurance covers the risk of death during the specific period of time reflected in the policy, it is one of the modalities that is currently contracted more frequently in the market.

For any company, especially for insurance companies, the fundamental rule is to insure the risks that arise in the contracting period, both internal risks and external risks. They do the checks periodically to see if the best deal in the market is reached in terms of prices, restrictions and protections.

2.2.2 Credit-linked life insurance

In general, the greater access of people to credit through the offer of different financial instruments is one of the great changes that we currently find. As well as in mortgage loans for the purchase of real estate, vehicles, leasing contract to general consumer loans, credit cards, etc., all that is used in our daily lives.

In addition, as personal debts increase, the risk will increase to meet the payments of its indebtedness, especially, in the specific case of loans granted for the acquisition of the house.

To face this type of social situation, insurance companies not only play an important role in all developed economic activity, but also have a collaborative function. They need to propose performance measurement tools under conditions of uncertainty, which allow better decision-making, in order to improve the level of effectiveness in the area of adjusting pricing.

The important economic and social functions that insurance performs, help support the economic development of all levels of society, and are embodied to avoid or reduce the negative consequences that can occur when those who are obliged to repay the credit disappear. In addition, it is usually the person who generates the necessary income to pay off the debt, therefore in this case it is essential to have the support of a life insurance.

As we have already mentioned in the previous point, within the specific case of temporary risk life insurance, we focus on life insurance associated with mortgages. On many occasions, life insurance for the mortgage includes a type of temporary risk life insurance, which function is to cover the death or total disability of the insured during the time that the loan is taken out.

The owner of the mortgage is the insured and the beneficiary is the bank that receives the compensation. The objective is to comply with the payment of the debts obtained by financial institutions. In this way, the insurance will not allow the heirs or other family members to carry the weights of the loan.

In addition, the key of insurance is to offer a product that satisfies the interests of creditors, and on the other hand, to relieve the debtor's heirs from the enormous burden they bear, and to find the proper balance for financial entities and the adverse economic impact on their assets when they cannot cope with this situation.

Consequently, the debtor will have peace of mind by contracting this policy for life insurance. To guarantee the debit balance of a loan, it can be contracted individually or collectively, depending mainly on the type of debt assumed and the conditions established by the financial institution. In this work we focus on the individual form, in the next chapter we analyse the elements that make up this form.

3. METHODOLOGY

3.1 Concept of Fuzzy Logic

Fuzzy Logic (LD) ("Fuzzy Logic", in English) was developed by Lotfi A. Zadeh in 1965. It is a theory that combines with the concepts of traditional Boolean logic. It is a way of expressing subjective, imprecise, ambiguous and uncertain knowledge in better described and specified knowledge.

Therefore, it is a multivalent logic that allows intermediate values to define nuances and express results in the form of structured segments of information.

It is a necessity in our current environment, to find real solutions to problems where there is vagueness and inconcretion. The birth of fuzzy logic has taken on greater importance in its applications that are expanding in economic, social, political and industrial areas, among others.

Fuzzy logic systems are characterized by:

- supporting uncertain data
- being intuitive and easy to understand
- being flexible
- being tolerant with missing data
- it is based on the expression of human language
- takes into account the experience of experts on the problem
- it is able to model nonlinear functions of some complexity
- unifying linguistic expressions with numerical data

All of this serves to help the model to better adapt to a human-like way of thinking. It allows working with expressions of a semantic nature used on a daily basis that can later be expressed numerically, through fuzzy sets in a more natural way and closer to human thinking.

3.2 Theoretical Development

One of the characteristics of the products of insurance companies is that there are similar products under the same central line, and there is a certain degree of similarity between them. The grouping method consists of assembling the elements with a certain degree of similarity between these products to measure the discrepancies between them. In certain circumstances, this method can be very useful for companies that must make the right decisions. These tools can be used by companies when market demands arise.

In grouping theory, Kaufmann and Gil Aluja (1999) have developed a scheme for the solution of this problem based on Moore families and Galois reticules. In this section we are going to carry out a series of analysis on the different life insurance products linked to the mortgage loans of different companies, looking for the definitions of their characteristics and the similarity between them.

The steps to carry out the theoretical approach are the following:

- To define the elements to group.
- To analyze life insurance products.
- The criteria used to describe the elements.
- Finding the distance from each element to the rest.
- To order the matrices.

3.2.1 Defining the elements to group

The elements of influence at the time of contracting life insurance products can be many: The personal situation, the marital status, as well as the type of activity carried out when the risk arises… Here, we have chosen the elements together with the products in an estimated way, to make the "matching" in these components. In this way, we emphasize the concept of uncertainty in fuzzy logic theory.

The purpose of this section is to present tools that can be used to make economic decisions when the environment cannot be measured at all or when the environment is not adequately measured. Each time, we try to make the most out of the information available, either objectively or subjectively. This will lead us to use subjectivity frequently when we lack the objectivity that we so desire. We try to use the currently available knowledge to build up the more economical model or as close to reality as possible.

As I have already mentioned, the elements and the life insurance products for grouping can be numerous. In this table, we have chosen some elements that have greater significance and serve as an example to facilitate the approach of the model:

Table 1. The main elements of life insurance

Elements	Explanation
• **Age of decease**	Age is one of the factors that directly affects the price associated with the requested life insurance products. Logically, at an older age, a greater risk of decease is assumed, therefore, the price of the product varies according to the different age groups.
• **The lending system**	The lending, in other words, can be defined as the outstanding capital of the mortgage. We have considered it as the next element, by its consequence. The function of life insurance linked to the mortgage, the presentation has a decreasing trend with age. That is, the younger the insured dies, the greater the outstanding payable capital.
• **The insurance time**	The third element that I consider very important is the insurance time. It means which is the elapsed time between the moment of contracting the insurance and when the decease of the insured person occurs.
• **Profession**	The profession of the insured is one of the essential elements for life insurance products. It has a similar effect than the age of decease, directly associated with the risk of life. Obviously, whoever has a profession of risk, will get a higher fee than anyone having a profession of no risk.
• **Health condition**	Finally, we have the health status indicator. Obviously, contracting the policy when in good health condition is when it costs the least. The health history of the insured must also be taken into account.

Source: elaboration according to Unespa report

3.2.2 Analysis of life insurance products

Table 2: Product Comparison

Classification	**$P_1^{(*)}$**	**$P_2^{(*)}$**	**$P_3^{(*)}$**	**$P_4^{(*)}$**	**$P_5^{(*)}$**
Company	*VidaCaixa*	*Mapfre*	*Zúrich*	*Generali*	*Axa*
Products	*Seviam*	*Mortgage amortization insurance*	*Klinc*	*Generali Vida Mortgages*	*Life Project*
Coverages	• *Decease* • *Absolute and permanent disability*	• *Decease* • *Absolute and permanent disability*	• *Decease* • *Absolute and permanent disability*	• *Decease* • *Absolute and permanent disability*	• *Decease from any cause..* • *Complementary:* - *Absolute permanent disability for any reason* - *Serious diseases*
Insurance payment method	• *Single premium if the insurance is temporary* • *Annual or monthly premium for annual renewable insurance*	• *Annual premium with a fractional form of payment..* • *Possibility of paying a single premium.*	• *Periodic premium based on the age of the insured and the capital contracted.*	• *Single payment method.*	• *Annual, semi-annual, quarterly or monthly premium.*
Insured capital	• *Maximum of € 30,000* • *Minimum:* - *€ 500 if the premium is unique* - *€ 3,000 if the premium is anual* - *€ 6,000 if the premium is monthly*	• *Minimum capital of € 6,000*	• *Depending on the age of the insured and the insured capital contracted.*	• *Minimum of € 18,000.* • *Maximum of € 750,000.*	• *Depending on the insured capital contracted.*
Insurance duration	• *Annual renewable* • *Temporary (single premium)*	• *Same duration as the mortgage loan to which it is linked* • *Maximum of 40 years*	• *Temporary annual renewable*	• *Temporary annual renewable*	• *Temporary annual renewable*
Additional requirement	• *Maximum hiring is 62 years.*	• *Maximum hiring is 67 years.*	• *Maximum hiring is 65 years.*	• *Maximum up to 65 years.*	• *Maximum hiring is 80 years.*

Source: own elaboration based in different products from different insurance companies[1]

[1] Extracted from Vida Caixa, Mapfre, Zurich, Generali and Axa official websites. (AXA, s.f.) (Zurich, s.f.) (Generali, s.f.)

3.2.3 Criteria used to describe the elements

In the financial market, like the insurance market we are in, when you buy a home with a mortgage guarantee through bank loans, life insurance is often interconnected.

Creating a home is a lifetime project, and mortgages are a part of it. The time horizon can extend up to several decades. Therefore, it is important to have this consolidated base to cover the expenses related to the loan.

Logically, this bet is based on the criteria of survival, since most mortgages are established at an early age. One of the basic elements in calculating the loan, is knowing if they will live long enough to pay the payments. Other conditions when calculating the loan are the benefits that serve to protect personal assets when the loss occurs.

In this section, we are going to list a series of criteria that I have considered the most important in life insurance linked to mortgages according to the Unespa report (2019): "The fatality, at the worst moment, life insurance benefits related to mortgages", together with the Unespa report (2020) entitled "Life insurance and mortgages". These reports have been prepared by 22 insurance entities, whose data have been carried by some 8,400 family members. Accordingly, in most cases, Spaniards use loans to get their houses. The main collateral for the loans was the property acquired. Therefore, building a home is usually one of the largest debt operations you can ever get.

The transformation of data into information in [0,1] has been carried out in a proportional way to the preference, they are the values corresponding to its level of preference in a directly proportional way expressed in [0,1]:

- Age of decease

Table 3: Distribution of deceases according to the age of the insured.

Age of decease	***Percentage***	***Transformation of data***
Less than 35 years	*3,19%*	*0,09*
Between 36 and 45 years	*15,66%*	*0,47*
Between 46 and 56 years	*29,86%*	*0,9*
Between 56 and 65 years	*33,17%*	*1*
Over 65 years	*18,12%*	*0,54*

Source: information from Unespa

- Lending System

Table 4: Distribution of lending according to age group

Age ranges	***Percentage***	***Transformation of data***
Less than 35 years	*3.4%*	*0,09*
Less than 35 years	*25.8%*	*0,73*
Between 46 and 55 years	*35.1%*	*1*
Between 56 and 65 years	*27.1%*	*0,77*
Over 65 years	*8.6%*	*0,24*

Source: information from Unespa

- The insurance time

Table 5: Decease covered by mortgage-related life insurance according to insurance time categories.

Insured time	***Percentage***	***Transformation of data***
Up to 5 years	*38.92%*	*1*
Between 6 and 10 years	*17.87%*	*0,45*
Between 11 and 20 years	*30.28%*	*0,77*
Between 21 and 30 years	*6.35%*	*0,16*
Over 30 years	*6.58%*	*0,17*

Source: information from Unespa

- Profession

Table 6: Demand for life insurance according to its type and the buyer's profession

Type of profession	***Percentage of decease***	***Transformation of data***
Self-employed and merchants	*76.04%*	*0,84*
Businessmen	*76%*	*0,84*
Liberal Professionals	*77.03%*	*0,85*
Farmers and ranchers	*83.75%*	*0,92*
Managers and middle ranks	*79.07%*	*0,87*
Graduated employees	*83.45%*	*0,92*
Employees without qualifications	*74.64%*	*0,82*
Household tasks	*62.20%*	*0,68*
Retirees and pensioners	*23.13%*	*0,25*
Others	*89.47%*	*1*

Source: ICEA.

- Health Condition:. in this case, each time an insured has one of the above-mentioned health conditions, the rating factor must be reduced by 0.2. If he/she has nothing, the rating is 1, if he/she has one, the rating is 0.8, and so on.

Table 7: Life insurance according to your health risk factors

Health status	***Whether you have a record or not***	**Each risk factor decreases by 0.2**
Smoker	*Yes / no*	*0,2*
Obesity	*Yes / no*	*0,2*
Chronic disease	*Yes / no*	*0,2*
Live alone	*Yes / no*	*0,2*
Practice risk sport	*Yes / no*	*0,2*

Source: VidaCaixa Report

3.2.4 Clustering through maximum similarity subrelations

In this point we are going to find the search for the distances between the products and the elements that we have defined in the previous section. Specifically, we use the theory of fuzzy subsets such as the *Hamming Distance*. The objective is to find the dissimilarity relations and the maximum similarity subrelations between them.

3.2.4.1 Hamming distance

The *Hamming Distance* (1950) named after its inventor Richard W. Hamming, who published an article on error detection and correction. Hamming's work marked a new field of research in the area of information theory. Until today, his theory has been used for modems, memories, and even satellite communications. It has clearly shown that Hamming's code is one of the cores of coding theory and is of incredible value in lots of practical applications. In our daily life, what we can find is the code bar of a product in the supermarket, and the ISBN (International Standard Book Number) and is also very common when cataloging books.

In information theory, the effectiveness of block codes is called Hamming distance and it depends on the difference between a valid code word and another. That is, the greater the difference, the less likely that a valid code might be transformed into another valid code by a series of errors. This difference is called Hamming's distance, and it is defined as the number of bits that have to be changed to transform a valid code word into another valid code word.

Illustration:

- 100111 and 110110, the Hamming's distance is 2.
- Distancia and distinto, the Hamming's distance is 4.

Definition 3.1. The *Hamming's distance* between two vectors $x=(x_1,\cdots,x_n)$ y $y=(y_1,\cdots,y_n)$ in F_2^n, which we denote by $d(x,y)$, is defined as

$$d(x,y) = |\{i|1 \leq i \leq n, x_i \neq y_i\}| \quad (1)$$

Definition 3.2. The Hamming's weight of a vector $x=(x_1,\cdots,x_n) \in F_2^n$, which we denote by $d(x,y)$, is defined as

$$d(x,y) = |\{i|1 \leq i \leq n, x_i \neq 0\}| \quad (2)$$

From definitions 3.1 and 3.2 it follows $d(x,y) = wt(x-y)$, since if $s = d(x,y)$, then there are s coordinates in which x y y differ and $n-s$ coordinates in which x y y then coincide in the difference $x-y$ and $n-s$ zeros and s coordinates other than zero, like this $wt(x-y) = s$.

Definition 3.3. We define the intersection of binary vectors $x=(x_1,\cdots,x_n)$ and $y=(y_1,\cdots,y_n)$ as the vector $x * y=(x_1y_1,\cdots,x_ny_n)$, in which it has 1's only where x and y have them.

Lemma 3.4 $Si\ x=(x_1,\cdots,x_n),\ y=(y_1,\cdots,y_n) \in F_2^n\ and\ then,$

$$wt(x+y) = wt(x) + wt(y) - 2wt(x * y). \quad (3)$$

Properties 3.5.

$La\ función\ d: F_2 \times F_2 N \cup \{0\}\ dada\ por\ d(x,y)\ satisface\ las\ siguientes\ propiedades.$

$for\ all\ x,y,z\ en\ F_2^n.$

1. $d(x,y) \geq 0\ y\ d(x,y) = 0\ yes\ and\ only\ si\ x = y,$
2. $d(x,y) = d(y,x),$
3. $d(x,y) \leq d(x,z) + d(z,y).$

$therefor, (F_2^n, d)\ is\ a\ metric\ space.$ (López Andrade).

3.2.4.2 Fuzzy subsets theory

The fuzzy subsets theory was carried out by L.A. ZADEH (1965) of the University of California at Berkeley. This theory is based on using true data when in an environment of uncertainty. It is a fairly efficient and comfortable instrument to apply.

Given that a set A = {x} that remains in a space [a, b] that satisfies the following conditions:

When function $\mu_A\ (x)$ takes the value 1 for every element of A:

- $\mu_A\ (x) = 1\ si\ x\ \in\ [a, b]$.

When it takes the value 0 for every element of A:

- $\mu_A\ (x) = 0\ \ si\ x\ \notin\ [a, b]$.

The characteristic function of A, also called "belonging function", takes values between 0 and 1. Thus, for another set it will be blurred in any element of the space [a, b]. a level of confidence interval is given between 0 and 1. That is, from absolute non-belonging to clear belonging.

3.2.4.3 To find the distance in each element to the rest

At this point we are going to calculate the distance in each product from the estimated data in section 3.2.3. To make the work more simple and efficient, we suppose that:

- A 35-year-old Spanish individual, decides to take out a mortgage life insurance product to cover the risk, the said person has a 30-year mortgage with a loan of € 180,000 pending payment. His profession is entrepreneur with an intermediate level, he travels frequently and he is also a smoker. But he has a good habit of playing sports regularly, which are not considered risk sports.

The transformation of data into estimation associated with each product in a confidence interval of [0,1] is based on the information provided in sections 3.2.2 and 3.2.3, together which the opinion of the expert in this field:

Table 8: Characteristics of items associated with the products under valuation between the confidence level of [0,1].

C_i	**Characteristics**	P_1	P_2	P_3	P_4	P_5
C_1	*Age of death*	0,54	0,50	0,65	0,48	0,45
C_2	*Lending*	0,85	0,80	0,73	0,9	0,73
C_3	*Assurance time*	1	0,17	0,34	0,17	0,73
C_4	*Profession*	0,87	0,67	0,73	0,52	1
C_5	*Health condition*	0,8	0,6	0,8	0,6	0,4

Source: own elaboration

The Hamming distance between each of the products and the others will be:

$$d(P_1,P_2) = \frac{1}{4} * (|0{,}54 - 0{,}50| + |0{,}85 - 0{,}80| + |1 - 0{,}17| + |0{,}87 - 0{,}67| + |0{,}8 - 0{,}6| = 0{,}33$$

$$d(P_1,P_3) = \frac{1}{4} * (|0{,}54 - 0{,}65| + |0{,}85 - 0{,}73| + |1 - 0{,}34| + |0{,}87 - 0{,}73| + |0{,}8 - 0{,}8| = 0{,}26$$

$$d(P_1,P_4) = \frac{1}{4} * (|0{,}54 - 0{,}48| + |0{,}85 - 0{,}90| + |1 - 0{,}17| + |0{,}87 - 0{,}52| + |0{,}8 - 0{,}6| = 0{,}37$$

$$d(P_1,P_5) = \frac{1}{4} * (|0{,}54 - 0{,}45| + |0{,}85 - 0{,}73| + |1 - 0{,}73| + |0{,}87 - 1| + |0{,}8 - 0{,}4| = 0{,}25$$

$$d(P_2,P_3) = \frac{1}{4} * (|0{,}50 - 0{,}65| + |0{,}80 - 0{,}73| + |0{,}17 - 0{,}34| + |0{,}67 - 0{,}73| + |0{,}6 - 0{,}8| = 0{,}16$$

$$d(P_2,P_4) = \frac{1}{4} * (|0{,}50 - 0{,}48| + |0{,}80 - 0{,}90| + |0{,}17 - 0{,}17| + |0{,}67 - 0{,}52| + |0{,}6 - 0{,}6| = 0{,}07$$

$$d(P_2,P_5) = \frac{1}{4} * (|0{,}50 - 0{,}45| + |0{,}80 - 0{,}73| + |0{,}17 - 0{,}73| + |0{,}67 - 1| + |0{,}6 - 0{,}4| = 0{,}30$$

$$d(P_3,P_4) = \frac{1}{4} * (|0{,}65 - 0{,}48| + |0{,}73 - 0{,}9| + |0{,}34 - 0{,}17| + |0{,}73 - 0{,}52| + |0{,}8 - 0{,}6| = 0{,}23$$

$$d(P_3,P_5) = \frac{1}{4} * (|0{,}65 - 0{,}45| + |0{,}73 - 0{,}73| + |0{,}34 - 0{,}73| + |0{,}73 - 1| + |0{,}8 - 0{,}4| = 0{,}32$$

$$d(P_4,P_5) = \frac{1}{4} * (|0{,}48 - 0{,}45| + |0{,}9 - 0{,}73| + |0{,}17 - 0{,}73| + |0{,}52 - 1| + |0{,}6 - 0{,}4| = 0{,}36$$

According to the calculated results, the matrix $\underline{R}$ is obtained, considered as a Dissimilarity matrix, that is, in the main diagonal all the values are zero. And they are also symmetrical, the distance between P1 and P5 is the same distance that exists between P5 and P1.

Table 9: Dissimilarity matrix.

$\underline{R}$	P_1	P_2	P_3	P_4	P_5
P_1	0	0,33	0,26	0,37	0,25
P_2	0,33	0	0,16	0,07	0,30
P_3	0,26	0,16	0	0,23	0,32
P_4	0,26	0,07	0,23	0	0,36
P_5	0,25	0,30	0,32	0,36	0

Source: own elaboration

Table 10: Dissimilarities matrix above diagonal zero.

$\underline{R}$	P_1	P_2	P_3	P_4	P_5
P_1	0	0,33	0,26	0,37	0,25
P_2		0	0,16	0,07	0,30
P_3			0	0,23	0,32
P_4				0	0,36
P_5					0

Source: own elaboration

The similarity matrix $\hat{R}$ is obtained, it is based on the dissimilarity matrix. It is enough to obtain the complementary matrix by calculating the unit complement of each element.

$$[\hat{R}] = [1] - [\underline{R}]$$

The similarity matrix is symmetric and reflective:

Table 11: Similarity matrix

R̂	P_1	P_2	P_3	P_4	P_5
P_1	1	0,67	0,74	0,63	0,75
P_2		1	0,84	0,93	0,70
P_3			1	0,77	0,68
P_4				1	0,64
P_5					1

Source: own elaboration

R̂ =1

Table 12: Ordinaries matrix with level α=1.

R̂	P_1	P_2	P_3	P_4	P_5
P_1	1				
P_2		1			
P_3			1		
P_4				1	
P_5					1

Source: own elaboration

R̂ =0,93

.

Table 13: Ordinaries matrix with level α=0,93.

R̂	P_1	P_2	P_3	P_4	P_5
P_1	1				
P_2		1		1	
P_3			1		
P_4		1		1	
P_5					1

Source: own elaboration

$\hat{R}$ =0,84

Table 14: Ordinaries matrix with level α=0,84.

$\hat{R}$	P_1	P_2	P_3	P_4	P_5
P_1	1				
P_2		1	1	1	
P_3		1	1		
P_4		1		1	
P_5					1

Source: own elaboration

$\hat{R}$ =0,77

Table 15: Ordinaries matrix with level α=0,77.

$\hat{R}$	P_1	P_2	P_3	P_4	P_5
P_1	1				
P_2		1	1	1	
P_3		1	1	1	
P_4		1	1	1	
P_5					1

Source: own elaboration

$\hat{R}$ =0,75

Table 16: Ordinaries matrix with level α=0,75.

$\hat{R}$	P_1	P_2	P_3	P_4	P_5
P_1	1				1
P_2		1	1	1	
P_3		1	1	1	
P_4		1	1	1	
P_5	1				1

Source: own elaboration

$\hat{R}$ =0,74

Table 17: Ordinaries matrix with level α=0,74.

$\hat{R}$	P_1	P_2	P_3	P_4	P_5
P_1	1		1		1
P_2		1	1	1	
P_3	1	1	1	1	
P_4		1	1	1	
P_5	1				1

Source: own elaboration

$\hat{R}$ =0,70

Table 18: Ordinaries matrix with level α=0,70

$\hat{R}$	P_1	P_2	P_3	P_4	P_5
P_1	1		1		1
P_2		1	1	1	1
P_3	1	1	1	1	
P_4		1	1	1	
P_5	1	1			1

Source: own elaboration

$\hat{R}$ =0,68

Table 19: Ordinaries matrix with level α=0,68.

$\hat{R}$	P_1	P_2	P_3	P_4	P_5
P_1	1		1		1
P_2		1	1	1	1
P_3	1	1	1	1	1
P_4		1	1	1	
P_5	1	1	1		1

.

Source: own elaboration

$\hat{R}$ =0,66

Table 20: Ordinaries matrix with level α=0,66.

$\hat{R}$	P_1	P_2	P_3	P_4	P_5
P_1	1	1	1		1
P_2	1	1	1	1	1
P_3	1	1	1	1	1
P_4		1	1	1	
P_5	1	1	1		1

Source: own elaboration

$\hat{R}$ =0,64

Table 21: Ordinaries matrix with level α=0,64.

$\hat{R}$	P_1	P_2	P_3	P_4	P_5
P_1	1	1	1		1
P_2	1	1	1	1	1
P_3	1	1	1	1	1
P_4		1	1	1	1
P_5	1	1	1	1	1

Source: own elaboration

$\hat{R}$ =0,63

Table 22: Ordinaries matrix with level α=0,63.

$\hat{R}$	P_1	P_2	P_3	P_4	P_5
P_1	1	1	1	1	1
P_2	1	1	1	1	1
P_3	1	1	1	1	1
P_4	1	1	1	1	1
P_5	1	1	1	1	1

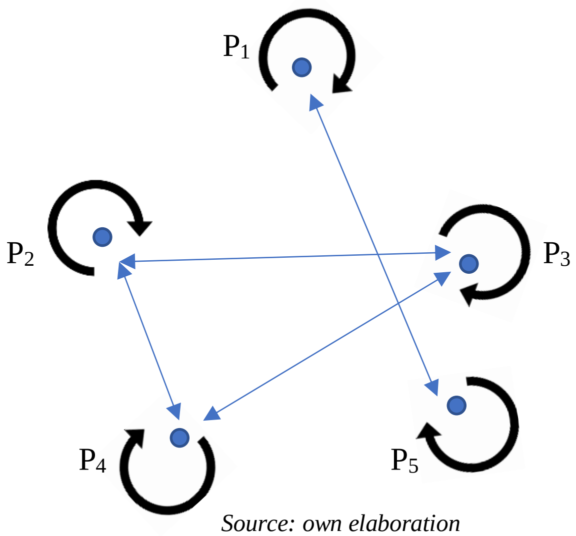
Source: own elaboration

Now, according to the estimates of the distances, we assume that the α level is equal to or less than 0.25, we choose this estimate as a reference to group the life insurance products that can form a homogeneous group according to this distance.

In this assumption of $\alpha = 0.25$ then the level of α similarities $\alpha = 0.75$, represented by the following figure:

$\alpha \geq 0{,}75$:

Figure 1



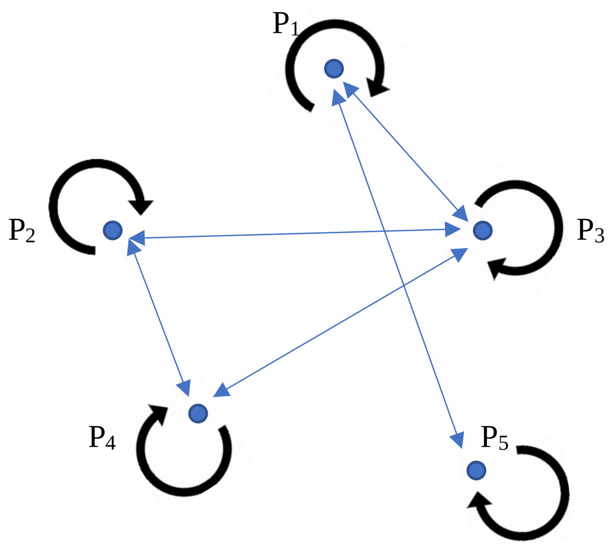
Source: own elaboration

We can clearly see in this case that the groups are formed by (P1, P5), (P2, P3, P4).

Assumption 2: we lower the level of demand of $\alpha \geq 0.74$, we will see which the grouping's behaviour will be:

$\alpha \geq 0.74$:

Figure 2



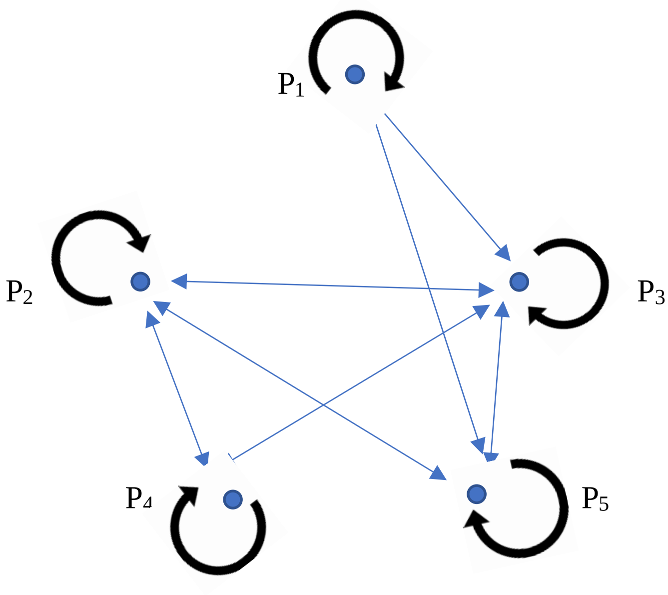
Source: own elaboration

We see that the groups are formed by (P1, P3), (P1, P5), (P2, P3, P4).

Assumption 3, we continue to lower the level of requirement of α ≥ 0.68, we will see what the grouping behaviour will be:

α ≥ 0.68:

Figure 3



Source: own elaboration

We see that the groups are formed by (P1, P3, P5), (P2, P3, P4), (P2, P3, P5).

In this intuitive way, we can obtain the maximum subrelations of similarity between different grouped products. In the next step, we will demonstrate and verify the result with the *Pichat Algorithm* theory.

3.2.4.4 Pichat's algorithm

The *Pichat's Algorithm* (1970) is a mechanism that is used in order to obtain the maximums of similarity of subrelations between the observations that we have been constituting, which are capable of transmitting through the submatrices or figures. Below we detail the steps to be carried out with this method:

a) We consider the half of the matrix corresponding to the upper part of the main diagonal.

b) We take the first row, then the second row and so on. In each of them, the zero of the index column in its corresponding empty cell is considered a Boolean variable, which is associated with the index of the row by the Boolean sum symbol.

c) Bring together the corresponding sum in each row of the Boolean product. If there is no zero in a row, it will be treated as a value of one in terms of the product.

d) Performing the Boolean product, the function obtained is represented by a minimum term, that is, that $x + x = x, x + xy = x$.

e) To obtain the result, find the complement of each element. Complementary terms will produce the largest transitive subrelations.

- The supposition that we have made previously, the ordinary matrix corresponding to the level α ≥ 0.75 is the following:

Table 23: Ordinaries matrix with level α ≥ 0,75.

R	P_1	P_2	P_3	P_4	P_5
P_1	1				1
P_2		1	1	1	
P_3			1	1	
P_4				1	
P_5					1

Source: own elaboration

- For each of the rows, it would be:

  Row P1: 1+234

  Row P2: 2 + 5

  Row P3: 3 + 5

  Row P4: 4 + 5

  Row P5 :1

- Making the corresponding product, it would be:

$$S = (1 + 234)(2 + 5)(3 + 5)(4 + 5)$$
$$= (12 + 15 + 234 + \cancel{\mathbf{2345}})(3 + 5)(4 + 5)$$
$$= (12 + 15 + 23)(3 + 5)(4 + 5)$$
$$= (\cancel{\mathbf{123}} + \cancel{\mathbf{125}} + \cancel{\mathbf{135}} + 15 + 23 + \cancel{\mathbf{235}})(4 + 5)$$
$$= (15 + 23)(4 + 5)$$
$$= (\cancel{\mathbf{145}} + 15 + 234 + 235)$$
$$= (15 + 234 + 235)$$

- The complement of each of the products:
$S'' = 234 + 15$ +14

- Finally we will have a result of maximum similarity:
(P2,P3,P4),(P1,P5),(P1,P4)

- In supposition 2, the ordinary matrix corresponding to the level α ≥ 0.74 is the following:

Table 24: Ordinaries matrix with level α ≥ 0,74.

R	P_1	P_2	P_3	P_4	P_5
P_1	1		1		1
P_2		1	1	1	
P_3			1	1	
P_4				1	
P_5					1

Source: own elaboration

- For each of the rows, it would be
Row P1: 1+24

Row P2: 2 + 5

Row P3: 3 + 5

Row P4: 4 + 5

Row P5 :1

- Making the corresponding product, it would be:
$S = (1 + 24)(2 + 5)(3 + 5)(4 + 5)$
$= (12 + 15 + 24 + \cancel{\mathbf{245}})(3 + 5)(4 + 5)$
$= (12 + 15 + 24)(3 + 5)(4 + 5)$
$= (123 + \cancel{\mathbf{125}} + \cancel{\mathbf{135}} + 15 + 234 + 245)(4 + 5)$
$= (15 + 123 + 234 + 245)(4 + 5)$
$= (\cancel{\mathbf{145}} + 15 + \cancel{\mathbf{1234}} + \cancel{\mathbf{1235}} + 234 + \cancel{\mathbf{2345}} + \cancel{\mathbf{245}} + 245)$
$= (15 + 234 + 245)$

- The complement of each of the products:
$S'' = 234 + 15 + 13$

- Finally we are going to obtain a result of maximum similarity:
(P2,P3,P4), (P1,P5), (P1, P3)

Finally, we analyze supposition 3, when the level of demand is reduced to $\alpha \geq 0.68$, the ordinary matrix will be:

Table 25: Ordinaries matrix with level $\alpha \geq 0{,}68$.

R	P_1	P_2	P_3	P_4	P_5
P_1	1		1		1
P_2		1	1	1	1
P_3			1	1	1
P_4				1	
P_5					1

Source: own elaboration

- For each row, it would be:
Row P1: 1+2

  Row P2: is not considered

  Row P3: is not considered

  Row P4: 4 + 5

  Row P5 :1

- Making the corresponding product, it would be:
  $S = (1+2)(4+5)$
  $= (14 + 15 + 24 + 25)$

- The complement of each of the products:
  $S'' = 235 + 234$ + 135 + 134

- Finally we will have a result of maximum similarity:
  (P2, P3, P5), (P2, P4, P5), (P1, P3, P5),(P1, P3, P4)

Observation:

Using the Pichat Algorithm, we observed that as the level of demand for α decreases, the groups of elements decrease in number but become more numerous because there are more elements that share the characteristics at the established level of demand. Comparing the three assumptions, we see that assumption 1 and 2 it is grouped into three groups, which are (P2, P3, P4), (P1, P4), (P1, P5) and (P2, P3, P4), (P1, P3), (P1, P5). While assumption 3 is grouped (P2, P3, P5), (P2, P4, P5), (P1, P3, P4), (P1, P3, P5) in four groups.

We see that in assumption 1 and 2, the products (P2, P3, P4) and (P1, P5) have been grouped together, which means that they have a relationship of maximum similarity between them. In contrast, in the product P3 and P4 they are losing their uniqueness.

In assumption 3, they are grouped in a way that is not so simple and clear, because they have 4 groups for 3 types of products, that is, the relationships become more complex between them. In addition, with the reduction of the levels of demand, there would be a greater grouping. Each time the interval of grouping the products will be larger, because it has little demand for homogeneity at this level and makes that all products can be considered without significant differences.

4. FUTURE PATHS

So far we have made the theoretical statement that we wanted. The next stage of this project at a practical level will be carried out with the help of both Python and R programming.

We use Fuzzy Logic theory combined with computational intelligence from different perspectives:

• Credibility, to see the level of payment compliance of consumers through their behaviours. From there, decisions are made by the insurance company whether to reward or penalize the client.

• Using another type of Distance, such as the *Levenshtein Distance* to compare the results obtained.

• Also from the competency point of view of the insurance companies, how can they obtain the benefits while reducing the rate prices of the policies and the way to attract clients.

• To incorporate the new tools of the computational intelligence technique to improve the programming development.

• Finally, I will carry out a study on the trends of customer profiles, to find out if it is necessary to increase the time horizon to grant a loan as longevity growth increases.

5. CONCLUSIONS

The objective of this project has been to analyze the problem of maximum magnitude to adjust the profiles of consumers among the products offered by life insurance companies. Specifically, they are life insurance products linked to the mortgage. I analyze this problem, because this phenomenon reflects great complexity in society. It is not an easy task to find the balance point, because the point of view of the consumers is different from that of the company. Obviously, their objectives in both parts are different, although they are related to each other through the same product. On the side of the consumers, they are inclined towards the comparison of the prices of the products. And on the side of the companies, it looks much more for its benefits while offering its products to consumers, in order to make decisions in an environment of uncertainty.

Using Fuzzy Logic methodology allows us to obtain the results and make a comparison between them. We see that as the levels of demand for products decrease, the maximum similarity ratio decreases, that is, homogeneity is lost between similar products. Therefore, the grouping level becomes more complex and loses the uniqueness of each product. As you lower the α levels, you will produce a larger and larger grouping. It means that having little demand for homogeneity means that they have very little significant difference between them. So it would be more complicated when the time comes to make decisions.

Although with the help of the Pichat Algorithm we can group the products with the maximum similarity between them, there is a small mishap with this method. It means that among similar products we cannot know exactly what qualified characteristics they have between them, in order to obtain this explicit result, we should return with the Galois lattice theory (18th century). Here I leave it indicated in a generalized way, because it is one of the future objectives to continue with this project.

In order to take this project further at programming level, the next step will incorporate new computational intelligence tools, so that it can better model the models in a way that is closer to reality. Finally, it is important to share new knowledge that can be included in computational intelligence techniques that involve other tools. For example, neural networks or genetic algorithms, etc., to improve the results obtained and developing computer programs that more accurately support business decisions-making.

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