communications physics

https://doi.org/10.1038/s42005-021-00676-3 OPEN

Author Correction: The origin of hysteresis and memory of two-phase flow in disordered media

Ran Holtzman, Marco Dentz, Ramon Planet & Jordi Ortín

Correction to: *Communications Physics* https://doi.org/10.1038/s42005-020-00492-1, published online 4 December 2020.

The Supplementary Information (Supp. Note 3) was amended to replace a minus sign with a plus in Eq. (21). This affects the expression of the effective width w_e in Eq. (22), also appearing as Eq. (7) in the article's Methods. The amended Supplementary Information (SI) includes the corrected equations and a short analysis demonstrating that the snap-off in drainage occurs at the critical external head H_c, in agreement with our simulations. The Methods in the main article was corrected in a similar manner; also, the linear approximation in Eq. (6) was replaced by the exact solution. Below we provide a detailed list of changes to Supplementary Information and main article Methods (by order of appearance):

1. **Supplementary Information** —text below Eq. (14): changed from "for $H_c \geq d - a_0\left[1 - \exp\left(-w/\ell_g\right)\right]/\rho g_e$".
   to "for $H_e > H_c$, where the critical height is $H_c = d - a_0\left[1 - \exp\left(-w/\ell_g\right)\right]/2\rho g_e$".
   And the following text has been added afterwards:
   "Beyond that point, i.e. once $H_e = H_c$, the interface snaps. This is because as long as the defect is fully wet, interfacial tension and gravity are in equilibrium with the capillary forces (Eq. [2]). When the external head is lowered below H_c, the defect becomes partially wet, gravity and surface tension overwhelm the weakened capillary force, and the interface snaps.
   The snap-off at H_c can also be demonstrated by the following analysis".
2. **Supplementary Information** —text below Eq. (16): changed
   from
   "Note that the defect becomes weaker as it becomes narrower. Thus we do not expect that the effective width goes to zero until the release of the interface, or in other words, the interface is going to be released at finite $w_e > 0$".
   To "The fact that the defect becomes weaker as it becomes narrower, provides the intuitive reasoning for the snap-off at $H_e = H_c$".
3. **Supplementary Information** —in Eq. (21) the minus sign
   was replaced with a plus sign to correct an error. Hence, the old equation appeared as

$$\frac{dw_e(H_e)}{dH_e}\left[1 - \frac{\partial f(H_e, w_e)}{\partial w_e}\right] = \frac{\partial f(H_e, w_e)}{\partial H_e}$$

   This has now been replaced by

$$\frac{dw_e(H_e)}{dH_e}\left[1 + \frac{\partial f(H_e, w_e)}{\partial w_e}\right] = \frac{\partial f(H_e, w_e)}{\partial H_e}$$

   Supplementary Information —as a consequence of the change of sign in Eq. (21), in Eq. (22) the numerator and denominator flipped, namely the last term was changed from

$$\frac{d - H_c}{d - H_e}$$

   to

$$\frac{d - H_e}{d - H_c}$$

4. **Supplementary Information** —the text below Eq. (22) was
   changed accordingly from "Thus, for $H_0 \leq H_e < H_c$ the interface height is given by Eq. [13] where w is replaced by Eq. [22]. The interface leaves the defect at H_0, which is obtained by setting $h(x = 0) = d$".
   to "Equation [22] indicates that w_e would increase with decreasing H_e, which is not possible because it cannot exceed the physical width w. Another explanation for the snap-off at H_c is that during drainage the interface is described by Eq. [13] with $a_e = a_0$.

This solution is valid as long as the defect is fully wet. The defect remains fully wet while $h\left(\pm w/2\right) \geq d$, namely while the equilibrium pressure head $H_e = \left(H + p_c^0/\rho g\right)/\sin\alpha$ remains above the critical position, $H_e > H_c$. Once the pressure head falls below this critical value, $H_e < H_c$, the defect would become partially wet, such that the effective width w_e would be smaller than the defect width w. The equilibrium position for this partially wet configuration would satisfy

$$H_e = d - \tfrac{a_0}{2\rho g_e}\left[1 - \exp\left(-w_e/\ell_g\right)\right]. \tag{23}$$

Since $w_e < w$, this implies that $H_e > H_c$, in contradiction with the above. Therefore, the interface will snap at H_c".

5. **Methods**—Eq. (6) which provided the linear approximation of η_c was replaced by the exact solution. Thus, Eq. (6) changed from

$$\eta_c = \frac{\ell_g^2\cos\theta}{b_0^2}2\left[1 - \exp\left(-w/2\ell_g\right)\right]\delta b,$$

to

$$\eta_c = \frac{a_0}{\rho g_e}\left[1 - \exp\left(-w/2\ell_g\right)\right].$$

Consequently, the following text which appeared after Eq. (6), became redundant and was removed: "where we used that $a_0/\rho g_e = 2\ell_q^2\cos\theta\delta b/b_0\left(b_0 - \delta b\right)$ and set $b_0 - \delta b \approx b_0$".

6. **Methods**—Eq. (7) is identical to Eq. (22) in the SI, and was thus corrected similarly, namely the last term was changed from

$$\frac{d - H_c}{d - H_e}$$

to

$$\frac{d - H_e}{d - H_c}$$

7. **Methods**—accordingly, text above Eq. (7) changed from
"$H_e \geq H_c = d - a_0\left[1 - \exp\left(-w/\ell_g\right)\right]/\rho g_e$. For $H_0 \leq H_e < H_c$,".
To "$H_e > H_c = d - a_0\left[1 - \exp\left(-w/\ell_g\right)\right]/2\rho g_e$. For $H_e < H_c$,"

8. **Methods**—accordingly, text below Eq. (7) changed from

"The effective height The effective height H_0 at which the interface leaves the defect is obtained by setting $h(x = 0) = d$,

$$H_0 + \tfrac{a}{\rho g_e}\left(1 - \exp\left[-\tfrac{w_e(H_0)}{2\ell_g}\right]\right) = d.$$

to

"Equation (7) indicates that w_e would increase with decreasing H_e, which is not possible because it cannot exceed the physical width w. Therefore, the interface will snap-off at H_c".

Note that Eq. (8) was eliminated, as it is no longer relevant.

The above corrections have been implemented in the HTML and PDF version of the article and the correct version of Supplementary Information document can be found as Supplementary Information associated with this Correction.

Published online: 29 July 2021

Additional information

Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s42005-021-00676-3.