RAPID COMMUNICATIONS

PHYSICAL REVIEW E VOLUME 59, NUMBER 2 FEBRUARY 1999

Numerical signs for a transition in the two-dimensional random field Ising model at $T=0$

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(Received 8 January 1998; revised manuscript received 30 October 1998)

Intensive numerical studies of exact ground states of the two-dimensional ferromagnetic random field Ising model at $T=0$, with a Gaussian distribution of fields, are presented. Standard finite size scaling analysis of the data suggests the existence of a transition at $\sigma_c=0.64\pm0.08$. Results are compared with existing theories and with the study of metastable avalanches in the same model. [S1063-651X(99)50602-8]

PACS number(s): 75.10.Nr, 05.40.−a

The study of systems with quenched disorder has been a challenging problem for many years. The interplay between thermal fluctuations and disorder has a great influence on the existing phase transitions. Many systems are known to exhibit such phase diagrams highly determined by the degree of disorder (vacancies, impurities, dislocations, etc.) The most typical examples can be found in magnetism, superconductivity, structural phase transitions, etc. For such systems different models have been proposed. The Ising model with quenched disorder is one of the simplest and it has the advantage that the pure model is well known. The disorder can be of two types: (i) symmetry breaking terms, such as random-fields or random magnetic impurities, and (ii) nonsymmetry breaking, such as random-bonds, vacancies, etc. For all of the cases different probability distributions of disorder have been studied. Here we will focus on the study of the random field Ising model (RFIM) in two dimensions (2D) with a Gaussian distribution of fields. For many years there has been discussion concerning the possibility of whether a 2D model with symmetry breaking random fields exhibits order at low temperatures. The initial studies lead to a certain controversy: the Imry-Ma [1] argument suggests that the lower critical dimension, below which ferromagnetic order is destroyed, is $d_l \leqslant 2$, with $d=2$ being the limiting case. Renormalization group expansions [2] around $d=6$ lead to the ''dimensional reduction'' argument suggesting that $d_l=3$, discarding the possibility for ordering in the 2D RFIM. It has also been suggested [3] that there are different types of order for $d>1$. This controversy is probably due to the difficulty in balancing the two ingredients of such models: disorder and thermal fluctuations.

More recently a different approach to disordered systems has been proposed, namely the study of disordered systems at $T=0$, i.e., without thermal fluctuations. From a theoretical point of view this simplifies the problem without making it trivial. Moreover, several experimental systems exhibit phase transitions that can be catalogued into this ''athermal'' category: two examples are ferromagnetism at low temperatures under an external magnetic field [4], and martensitic transformations [5]. Both systems present a first-order phase transition that can be crossed by sweeping a control parameter and are greatly affected by the presence of quenched disorder. We will concentrate on the study of the 2D RFIM at $T=0$ for different values of the standard deviation σ of the Gaussian distribution of fields. Our goal is to look for signs of the existence of a phase transition at a certain σ_c from a ferromagnetic ordered state for $\sigma<\sigma_c$ to a disordered state for $\sigma>\sigma_c$. For the 3D RFIM at $T=0$, ground state studies [6,7] and renormalization group arguments [8] reveal the existence of such phase transition, but to our knowledge no results for the 2D case have been published. Figure 1 summarizes the finite size scaling study presented in this paper. Data correspond to estimations of σ_{cL}, obtained using different methods, as a function of the linear system size $L^{1/\nu}$ (where $1/\nu=0.5$ is the exponent characterizing the correlation length divergence). The standard extrapolation to $L\to\infty$, as will be discussed, renders $\sigma_c=0.64\pm0.08$ different from zero.

We consider the 2D RFIM on a $L\times L$ square lattice with periodic boundary conditions and with the Hamiltonian $H=-\Sigma_{i,j}^{\mathrm{nn}}S_iS_j-\Sigma_{i=1}^{N}S_ih_i$, where i and j are indices sweeping the full lattice $(i,j=1,\ldots,N=L\times L)$, the sum refers to nearest-neighbors (nn) pairs, $S_i=\pm1$ are spin variables, and h_i are independent random fields distributed according to the Gaussian probability density with $\langle h\rangle=0$ and $\langle h^2\rangle=\sigma^2$. The advantage of using a continuous distribution is that, for almost any configuration of fields $\{h_i\}$ the ground state is not

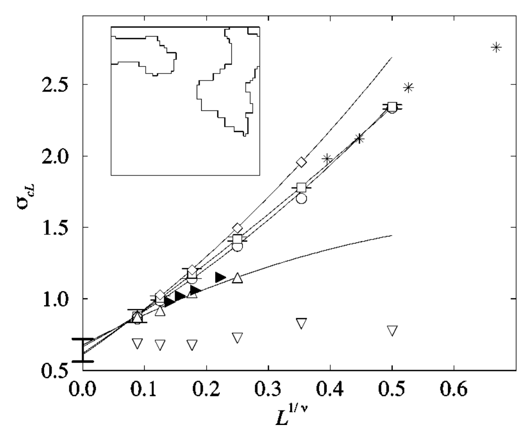

FIG. 1. σ_{cL} versus $L^{-1/\nu}$. The results have been obtained using MF at zero and higher orders, stars; exact solution of finite lattices, ○, □, ◇, △, and ▽; and studies of the metastability behavior, black triangles from Ref. [15]. Typical error bars are displayed. The inset shows an example of the ground state of a $L=64$ system, with $\sigma=1.0$

degenerated. The order parameter is the magnetization of the system defined as $m = \Sigma S_i / N$. Because the ground state is unique, thermal averages are meaningless. Since we are interested in the dependence of the system properties with the amount of disorder σ, the only possible averages with physical meaning are the ensemble averages $[\langle \cdots \rangle(\sigma)]$ performed over different realizations of the random fields with a certain fixed degree of disorder σ. Experimentally this has to be understood as averaging measurements on different samples that have been prepared with the same amount of disorder.

The zeroth-order mean field (MF) theory was proven many years ago [9]. A solution with the order parameter $\langle m \rangle(\sigma) \neq 0$ appears for $\sigma < \sigma_c = 8/\sqrt{2\pi}$, and the phase transition is continuous. Of course this MF result cannot be expected to be correct, or to reflect any dependence on dimensionality. Moreover, the MF studies can be extended to higher orders by exactly treating larger and larger clusters of spins. For thermal phase transitions this is known to extrapolate to the exact value of the critical temperature. The first-order approximation is the Bethe approximation, which consists of exactly solving a cluster of a central spin and its four nn. The method can be extended to larger clusters. We have found a continuous phase transition at $\sigma_c = 2.76$, 2.48, 2.12, and 1.98 for clusters of $N = 5$, 13, 25, and 41 spins, respectively. These results are indicated with stars in Fig. 1 (considering $L = \sqrt{N}$).

A better approach consists of looking for exact ground states by using the max-flow min-cut theorem [6]. We have designed an algorithm that solves a set of different σ values with a minimization time that grows as L^4. We have studied lattices with $L = 4$, 8, 16, 32, 64, and 128, and have taken averages over 10^5, 10^4, 10^4, 5×10^3, 10^3, and 30 realizations of random fields, respectively. The inset in Fig. 1 shows a typical example of a ground state for $\sigma = 1.0$ and $L = 64$. We have focused on the computation of different magnitudes. The order parameter has been estimated from $\langle |m| \rangle_L(\sigma)$ and $\sqrt{\langle m^2 \rangle_L(\sigma)}$. The subscript L indicates that such quantities will, in general, depend on system size. We have also measured the susceptibility as $\chi_L(\sigma) = N(\langle m^2 \rangle_L - \langle |m| \rangle_L^2)$ (which, for large σ, tends to 1 independently of L) and Binder's cumulant $g_L(\sigma) = 1 - \langle m^4 \rangle_L / (3 \langle m^2 \rangle_L^2)$. Also, the correlation length $\xi_L(\sigma)$ can be computed by fitting an exponential decay to the spin-spin correlation function.

Figure 2 shows the behavior of $\sqrt{\langle m^2 \rangle_L}$ as a function of σ for different system sizes. One can estimate σ_{cL} as the inflection point of a fitted third-order polinomium. Data can be scaled using the standard finite size scaling assumption: $\sqrt{\langle m^2 \rangle_L} \sim L^{\beta} \tilde{M}[L^{1/\nu}(\sigma - \sigma_{cL})]$, where $\tilde{M}$ is the corresponding scaling function. The exponents β and ν can be estimated by fitting the power laws $\sqrt{\langle m^2 \rangle}(\sigma = \sigma_{cL}) \sim L^{\beta}$ and $d\sqrt{\langle m^2 \rangle}/d\sigma(\sigma = \sigma_{cL}) \sim L^{\beta + 1/\nu}$. One gets $\beta = -0.038 \pm 0.009$ and $1/\nu = 0.54 \pm 0.04$. The scaled data is shown in the inset in Fig. 2. A similar scheme can be applied to the study of $\langle |m| \rangle_L(\sigma)$, rendering $\beta = -0.026 \pm 0.017$ and $1/\nu = 0.54 \pm 0.05$. Susceptibility $\chi_L(\sigma)$, shown in Fig. 3, exhibits a peak at $\sigma = \sigma_{cL}$, which shifts and increases when increasing L. Data can also be scaled using $\chi_L \sim L^{\alpha} \tilde{\chi}[L^{1/\nu}(\sigma - \sigma_{cL})]$. Power law fits to the height and curvature of the peak render $\alpha = 1.89 \pm 0.03$ and $1/\nu = 0.46 \pm 0.05$. Scaled data are shown in the inset of Fig. 3. Figure 4 shows the

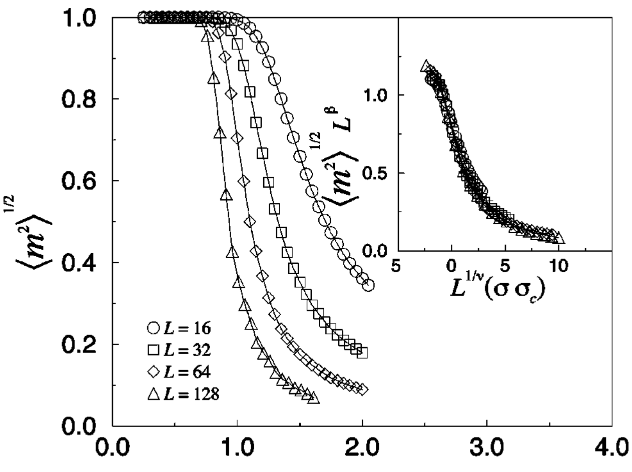

FIG. 2. Behavior of $\sqrt{\langle m^2 \rangle}(\sigma)$ for $L = 16$, 32, 34, and 128. The inset shows the same data scaled using $\beta = -0.038$ and $1/\nu = 0.50$.

behavior of $\xi_L(\sigma)$. The peak gives an independent measure of σ_{cL}. The continuous line is an estimation of $\xi(\sigma)$ for $L \to \infty$ that will be discussed later. Note that the behavior of the curves is compatible with the finite size scaling hypothesis, i.e., ξ_L follows the behavior corresponding to the infinite system up to a certain $\xi_{max} = KL$. (Data are compatible with $K \simeq 0.08$). A final estimation of σ_{cL} can be obtained from $g_L(\sigma)$. For all of the studied sizes $g_L(\sigma)$ takes a low value for large σ and reaches the value 2/3 at a certain σ_{cL}. This estimation is independent of L as suggested in Ref. [7]. We want to note that $\beta \simeq 0$, which means that the order parameter increases very quickly to $m \simeq 1$ after the transition. A low-σ first-order expansion renders $1 - m \sim 10^{-11}$ for $\sigma = 0.6$. This fact may also explain why it is very difficult to measure $g_L(\sigma)$ with enough numerical accuracy to check for a crossing point, which is the standard procedure to locate the transition. Also note that $\beta \simeq 0$ and $\alpha \simeq 2$ would suggest that m exhibits a lack of self averaging [10].

The different estimations, as explained in the previous paragraph, of σ_{cL}, are plotted in Fig. 1 in front of $L^{-1/\nu}$. The

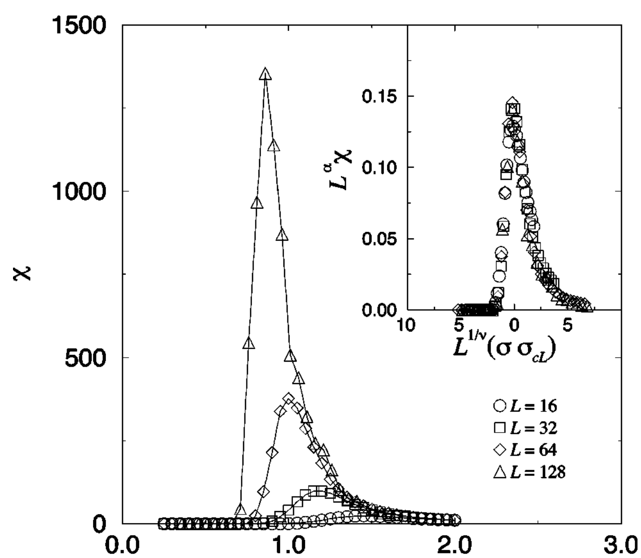

FIG. 3. Behavior of $\chi_L(\sigma)$ for $L = 16$, 32, 64, and 128. The inset shows the same data scaled using $\alpha = 1.89$ and $1/\nu = 0.46$.

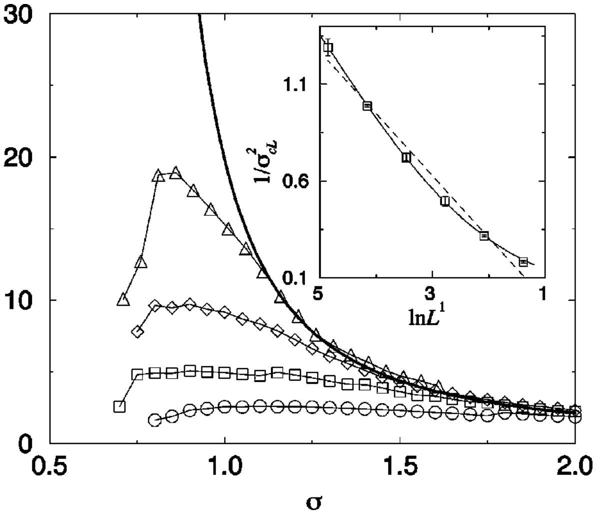

FIG. 4. Correlation length $\xi_L(\sigma)$ for systems with $L=16$, 32, 64, and 128. The continuous line shows the behavior of $\xi = A(\sigma-\sigma_c)^{-\nu}[1+C(\sigma-\sigma_c)]$ with $\sigma_c=0.64$ and $\nu=2$. The inset shows the finite size dependence of σ_{cL}^{-2} versus $\ln(L^{-1})$. Data are the same as in Fig. 1 (□). The continuous line is the standard scaling used in this work and the dashed one is the best fit of the theory by Binder *et al.* in Ref. [13].

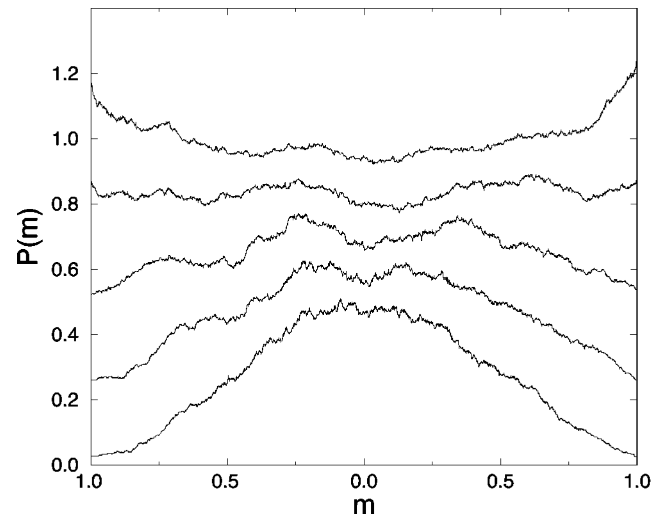

FIG. 5. Histograms of the magnetization distribution for a $L=64$ system and for $\sigma=1.20$, 1.15, 1.10, 1.05, and 1.00 (from bottom to top). The distribution evolves from a single peak at $\sigma>\sigma_c$ to a two-peak distribution for $\sigma<\sigma_c$. The curves are shifted 0.2 units each to clarify the plot.

open symbols correspond to the estimation from $\langle|m|\rangle_L(\sigma)$, (○); $\sqrt{\langle m^2\rangle_L}$, (□); $\chi_L(\sigma)$, (◇); $\xi_L(\sigma)$, (△); and $g_L(\sigma)$, (▽). In order to extrapolate the data to $L\rightarrow\infty$, we have used the standard expansion for the divergence of ξ, up to second order: $\xi\sim(\sigma-\sigma_c)^{-\nu}[1+C(\sigma-\sigma_c)]$. Now, supposing that σ_{cL} is determined by the condition $\xi=KL$, one gets $\sigma_{cL}=\sigma_c+C_1L^{-1/\nu}+C_2L^{-2/\nu}$. Such parabolic fits are also shown in Fig. 1 with continuous lines. The extrapolated σ_c all lay within $\sigma_c=0.64\pm0.08$. To get an idea of the error margins, we have also fitted the first-order expansion ($C_2=0$) leaving ν free, rendering $\sigma_c=0.65\pm0.1$ and $\nu=1.8\pm0.2$, or fixing $\nu=2$, which renders $\sigma_c=0.56\pm0.06$.

The existence of this phase transition is in apparent contradiction with previous results. It has been proved [11] that the RFIM has a unique Gibbs state in the thermodynamic limit, i.e., for a given configuration of the random fields the ground state is unique. This can be misunderstood [12] as proof that the ordered phase cannot exist. When considering the ensemble of all possible realizations of the random fields, corresponding to a certain value of σ, it may well be that the distribution of magnetizations changes from a single peak one (for large σ) to a bimodal one for small values of σ. Thus, the phase transition we are proposing should be understood as existing in this ensemble rather than for a single system for which the ground state is unique. It is true that there is an open question here concerning the size of this ensemble: in the thermodynamic limit, is there more than one realization of disorder compatible with a certain σ? We understand that given the discreteness of the Ising lattice and the continuity of the random fields one can still consider the existence of such an ensemble.

Another interesting point is the comparison of our results with other studies, suggesting an exponential divergence of the correlation length $\xi\sim\exp\{-B/(\sigma-\sigma_c)^{\tau}\}$. On the basis of the study of the interfaces separating regions with $m>0$ and $m<0$, Binder [13] derived a theory with $\tau=2$ and $\sigma_c=0$. We have tested the validity of this theory by studying the corresponding finite size scaling hypothesis [13,14]: $1/\sigma_{cL}^2\propto\ln(L^{-1})$. The inset in Fig. 4 shows the comparison of this behavior with the standard one we propose in this work, on top of the data corresponding to the estimations of σ_{cL} from the inflection point in $\sqrt{\langle m^2\rangle_L}$ (both fits have two free parameters). The standard theory works better. We have also tested that, assuming the standard hypothesis ($\xi_L\sim(|(\sigma-\sigma_{cL})/\sigma_{cL}|L^{1/\nu}+A)^{-\nu}$), the finite size scaling of ξ_L is better than using Binder's hypothesis. Moreover, Binder's theory proposes that, for large enough systems, the configurations with total $m\simeq0$ will be more and more frequent. We have not observed the existence of many ''slab'' configurations but have found ground states with closed domains, such as those in the inset of Fig. 1. Figure 5 shows the probability $P(m)$ obtained from the computations of a very large number of exact ground states for a system with $L=64$. Clearly, for $\sigma<\sigma_c$, the configurations with $m\simeq0$ have much less probability than the configurations with $m\simeq1$. The reason for the failure of Binder's theory could be that, in order to perform the thermodynamic limit, he uses a very anisotropic system with open boundary conditions.

Our data can be compared with the studies of the evolution of the RFIM at $T=0$, obeying a local relaxation dynamics. It has been found that, when sweeping the external field, the system evolves by avalanches between metastable states. At a certain degree of disorder σ_c the distribution of avalanches becomes critical. In Fig. 1 we show the values of the σ_{cL}, corresponding to the 2D case from Ref. [15]. The behavior is very similar to the equilibrium data. Different extrapolations to $L\rightarrow\infty$ have been reported ($\sigma_c=0.75\pm0.03$ [15], $\sigma_c=0.54\pm0.04$ [16]) but all are close to the equilibrium extrapolation. Concerning the exponents for the metastable studies, the exponent β has also been found to be very small, while previous reported values for ν are 1.6 ± 0.1 [15] and 5.3 ± 1.4 [16]. Therefore, we suggest that the metastability phenomena, found in the out-of-equilibrium studies, might be associated with a real underlying equilibrium phase transition at $\sigma<\sigma_c$ for zero external field. It should be mentioned that in the context of these out of equilibrium phase transitions, Sethna and collaborators have [16] proposed a theory with exponential divergence of ξ with $\tau=1$ and $\sigma_c=0.42\pm0.04$. Our data are not consistent with such theory. If we perform a fit in the evolution of $\sigma_{cL}(L)$, leaving σ_c and τ free, we get $\tau=0.6$ and $\sigma_c=0.25$. We can still obtain a good fit (and good scalings) by taking $\tau=1$ and $\sigma_c=0$, although we cannot provide any physical explanation for such behavior. We finally want to point out that the phase transition we have found at $T=0$ may also be related to the change in the type of growth found at $\sigma=0.33$ in the studies of the depinning transition in the same model [17].

In conclusion, we have presented a finite size scaling analysis of numerical data for systems up to $L=128$, which suggests that the RFIM with a Gaussian distribution of fields at $T=0$ exhibits a phase transition at $\sigma_c=0.64\pm0.08$. The ensemble average of the magnetization changes from $\langle m\rangle=0$ for $\sigma>\sigma_c$ to a state with $\langle m\rangle\neq0$ for $\sigma<\sigma_c$. The transition is characterized by the exponents $1/\nu=0.5\pm0.05$, $\beta=-0.03\pm0.02$, and $\alpha=1.89\pm0.03$. The possibility of exponential divergence $\xi\sim\exp(B/\sigma)$ cannot be excluded.

We acknowledge financial support from CICyT under Project Nos. mat95-0504 and mat98-0315, CRAY, and FCR. C.F. also acknowledges the Comissionat per a Universitats i Recerca (Generalitat de Catalunya) for financial support.

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