Fair and Lorenz Domination Solution of GHGs Emissions in Multilateral Cooperation of Global Warming Control

XIUXIA YIN

Universitat de Barcelona - UB School of Economics

June 14, 2022

Supervisors: Josep Maria Izquierdo Aznar; Pedro Calleja Cortes.[1]

Abstract

Greenhouse gas(GHGs) emissions have resulted in global warming which further leads to climate change. To reduce GHGs emissions, governors make attempts to create multilateral agreements among countries. Their failure of establishing sustainable agreements drives researchers to find fair and equitable distribution rules. Based on Emission Egalitarianism for each individual, this paper creatively generates the generalized constraint equal awards rule (GEA), provides corresponding algorithm to compute emission allocations from the GEA rule and points out several characteristics of this rule. To determine a fairer rule, this paper introduces Lorenz dominance and proved in n-agent case that the allocation provided by the GEA rule is Lorenz-undominated by all other allocations from any other rules. Furthermore, in a simplified 2-agent case this paper confirmed the allocations derived from the GEA rule Lorenz dominates all other allocations from any other rules.

Keywords: GEA Rule, Proportional Rule, Multilateral Cooperation, Global Warming

JEL codes: D63, Q52, Q54

[1]Supervisors are listed in alphabetical order.

Contents

1 Introduction

Global warming brings climate change which threatens humans in extensive forms such as widespread flooding or extreme weather. Entering into the 21st century, global warming has attracted increasing attention from numerous governors and researchers. As the major source of global warming is the excessive emission of greenhouse gas(GHGs), numerous nations in the world have reached an agreement to reduce the increase of world temperature by abating the emission of GHGs.

To provide emission reference, scientists quantified future GHGs emissions by climate projection models with high techniques. For example, Nakicenovic et al.(2000) pointed out that the International Panel of Climate Change has simulated the cumulative CO_2 emissions of the world under alternative scenario with different population increase, economic growth and technology development. The results indicated that the accumulated CO_2 emissions of the world will reach from 1758 Gt to 2736 Gt by year 2050. Another representative work by Meinshausen et al.(2009) predicted that if cumulative CO_2 emission could be limited below 1000 Gt, the probability of warming exceeding 2℃ could be reduced to 25%. To avoid abrupt climate change and its potentially irreversible effects on ecosystems, most countries form an overwhelming consensus that the global warming should not exceed the limit of 2℃ by year 2050 relative to pre-industrial levels.

To promote the total emission reduction and determine reduction amount for each country, governors and scientists have come up with various multilateral agreements such as Kyoto Protocol, Paris Agreement and Effort Sharing Decisions. The Kyoto Protocol was terminated in 2005 since the Protocol only imposed binding emission targets on those signatories (developed countries) while many remaining (developing countries) signatories were free of emissions. Four years later, Paris Climate Agreement was created and it allowed countries to self determine emission target. However, US which represents around 15% of global GHGs emissions officially announced to withdraw from the Paris Climate Agreement. The reason was the agreement does not guarantee equity criteria since it leaves countries like India and China free to use fossil while US has to reduce emissions.

As the sustainability of multilateral agreement depends on equity and fairness of distribution rules, economists such as Duro et al.(2020) endeavor to find such rules by framing this temperature reducing task as traditional rationing problem. The classical rationing problem as stated by Thomson(2019) is a distribution problem where a set of agents have claims over a certain kind of resource and the total available amount of this resource is insufficient to satisfy all claims. Under the context of CO_2 reduction, the basic problem is therefore how to divide the GHGs emission permits among countries with different business-as-usual(BAU) claims, population size and historical emissions when the total available cumulative CO_2 emissions before crossing the 2℃ threshold is insufficient to meet claims of all countries. We define this problem as Past-dependent rationing problem.

A limited number of economists has devoted to defining appealing resolutions for CO_2 permission distribution. Giménez-Gómez et al.(2016) framed the distribution of total available emissions as conflicting claim problem similar to bankruptcy which allocates money in a bank to its creditors. Here, creditors are countries whose claims are GHG emissions and the money is the global carbon budget which is less than the total future emission claims of all countries. With properties like Equal Treatment of Equals, Anonymity, Order Preservation, Claim truncation invariance, José-Manuel Giménez-Gómez concluded that only Talmud rule could reach required cooperative consensus. Three years later, Duro et al.(2020) expanded the previous work in two aspects: include more principles such as composition up and super-modularity and check stability, equity and fairness of different allocations by Gini index, Atkinson index and coefficient of variation. Ju et al.(2021) put historical emission as well as population of each country into the classical model of CO_2 distribution and offered four variants of proportional rules as solutions.

This paper endeavors to search for much fairer and more egalitarian way to allocate the available GHGs emissions among countries or even among individuals. The motivation of defining such rule is to promote the sustainability of multilateral cooperation in global warming control since many agreements were terminated because of unfair distribution rules they adopted.

Specifically, this paper firstly describes four proportional rules when claimants are countries. The purpose is to identify the shortcomings and drawbacks of these rules, such as violation of Claim-boundedness and Non-negativity. This paper also verifies that when carbon emission permits are distributed among countries, it is not ensured that individuals of different countries and with same historical emissions as well as same claims could receive equal allocations.

For example, the first two proportional rules analyzed by this paper is the Equal per emission rule (EPE) and the Equal per capita rule(EPC). The EPE rule distributes total available CO_2 emissions proportionally to *claims* of countries while the EPC rule allocates total available CO_2 emissions proportionally to *populations* of countries. The justification of EPE rule is each unit of CO_2 emission claim should be treated equally no matter which country it belongs to while the argument of EPC rule is every every person on this planet should enjoy equal permission rights independently of their nationality. In addition, scholars such as Pan et al.(2014) and Posner and Sunstein(2009)have strenuously urged that emission rights should be distributed by reference to population instead of current emission claims.

This paper verified that these first two proportional rules could not realize the egalitarianism in carbon emission distribution for two reasons: first, these rules are designed based on emission right of country instead of individual; second, none of them has taken historical emissions into consideration. The ignorance of historical carbon emissions will result in injustice. A country with higher historical emissions might get more current-future emission allocation just because this country has huge claims or larger population size.

This paper then checks another two existing rules-the Historical equal per emission rule()HEPE) and the Historical equal per capita rule(HEPC). The HEPE rule first allocates the divisible CO_2 emissions proportional to claims, then the total historical credit per BAU emission of the country is corrected. In this rule, historical credit is based on ¨per BAU business emission¨. The HEPC rule first distributes carbon emissions proportional to populations, then adds the total historical credit of every citizen. These two rules have included the historical emissions of countries, however, they are under question of violating Claim-boundedness and Non-negativity. Some countries with high historical emissions might receive negative allocations while other countries with few claims are distributed with excessive emissions. In addition, the claimants are still countries, therefore, it can not be said that every individual in the world could be treated equally with these rules regardless of nationalities.

Egalitarianism is socially accepted and is supposed to be imposed on every individual rather than every country. Every citizen should be treated equally regardless of his nationality when it comes to the carbon emission rights. This paper verified that when the claimants are countries, it is impossible to realize such egalitarianism. Therefore, this paper treats individual as agents and inventively designs the Generalized Constraint Equal Awards rule (GEA). The GEA rule treats every individual in the world as claimant of the carbon distribution problem and takes historical emissions of every person into consideration. It has two stages of allocations, the first-stage allocation makes compensation to individual with less historical emissions. As long as there are enough available carbon emission permits, the first-stage allocation of this rule could disappear the historical emission difference. The second-stage allocation of this rule is allocation from classical rationing problem which guarantees equal treatment on every unit of remaining individual current-future claim.

Other outstanding innovations of this paper includes the proposed algorithm to compute GEA allocations, the formal proof that only the GEA rule could satisfy claim-boundedness, non-negativity and equal treatment of per-capita equals and some important characteristics of the GEA allocations. For instance, if an agent in an allocation receives its full emission claim and less final carbon emissions than another agent who receives only historical emissions, then the allocation must be GEA allocation. Or, if there is an allocation giving exactly same final allocations to every citizen, then it is must be GEA allocation.

The last but not least contribution of this paper is the usage of Lorenz domination in selecting and determining a much fairer way of CO_2 emission distribution. After the introduction of Lorenz dominance and the formal mathematical proof, this paper gives two theorems. One theorem is under the case of n-agent carbon distribution problem while the other theorem is under the case of 2-agent carbon distribution problem. The first theorem states that the GEA rule is Lorenz undominated by any other rules with n-agent case. The second theorem says the GEA rule Lorenz dominates any other rules under the case of 2-agent.

The paper is organized as follows. In Section 2, the Past-dependent rationing problem with country being claimants is described and four solution concepts are analysed. Then at the end of Section 2, the author extends the Past-dependent rationing problem and considers the Unit past-dependent rationing problem. In Section 3, properties such as Claim-boundedness, Non-negativity and Equal treatment of per capita equals are introduced. The former four rules are analyzed in terms of these properties. Section 4 focuses on the Unit Past-dependent rationing problem and the agents are changed from countries to individuals. The Generalized Constraint Equal Awards rule is defined as well as an algorithm is proposed to compute the GEA allocations in this section. Section 5 states some important characteristics of the GEA rule. These characteristics are the basis for the proof in next section. Section 6 gives and proves two theorems of this paper. The first theorem states that under case of n-agent where $n > 2$, the final allocation from the GEA rule is Lorenz undominated by any other final allocations from all other rules. The second theorem says under the case of 2-agent, the final allocation from the GEA rule Lorenz domnates any other final allocations from all other rules. Section 7 is the conclusion of this paper.

2 The Model

This paper takes theoretical framework from Ju et al.(2021) which extends mathematical description of standard rationing problem. The standard rationing problem is a distribution problem where a set of agents have claims over a certain kind of resource and the total available amount of this resource is insufficient to satisfy all claims. Under the context of multilateral cooperation in reducing the global temperature increase, Meinshausen et al.(2009) predicted the targeted accumulative GHGs emissions over period of 2000-50 should not exceed 1000 Gt if we intend to limit the probability of global warming exceeding 2℃ to be below 25%. However, when all countries go on with their BAU emissions, the total emissions until year 2050 will definitely beyond the measured targeted emissions.

In view of the fact that countries usually have different emissions in the past, this paper therefore defines a past-dependent rationing problem. Suppose there is a number of countries sharing the common responsibility of reducing global warming in our past-dependent rationing problem. The set of countries is denoted by N with $\mathrm{N} = \{1, 2, ..., n\}$. The targeted accumulative GHGs emissions of Meinshausen et al.(2009) is denoted by E and $E > 0$. The current-future emission claims of countries under BAU is denoted by c with $\left\{c \in R_+^n : c_i > 0, \forall i \in N\right\}$ while the past emissions of countries is denoted by h with $\left\{h \in R_+^n : h_i \geq 0, \forall i \in N\right\}$. Finally, the population size of any country $i \in N$ is denoted by v_i. For simplicity, the population size for any country i is constant in the past, now and future.

The total historical GHGs emissions of all countries is $\overline{h} = \sum_{i \in N} h_i$ while the total current-future emission claims is $\overline{c} \equiv \sum_{i \in N} c_i$. There is perfect replacement in reducing the total targeted GHGs emissions between the past, the current and the future emissions. Hence when all countries are under BAU, the total emissions of the society is $\overline{h} + \overline{c}$. When the total emissions $\overline{h} + \overline{c}$ of countries is larger than the targeted emissions E, every country should reduce their BAU emission, and the total reduced amount should be equal to $\overline{h} + \overline{c} - E$. In other words, the total allowable emissions before global warming crosses 2 ℃ relative to pre-industrial level until 2050 is up to $E - \overline{h}$ for the whole society.

We now consider to allocate total available emissions $E - \overline{h}$ among countries in our past-dependent rationing problem. We assume $c_i > 0$, $h_i \geq 0$ and $0 \leq E - \overline{h} \leq \overline{c}$. The problem is denoted by $\rho = (N, h, c, v, E)$ and the set of all such problems can be represented by $\mathscr{P}$. A solution of this problem is a rule $f = (x_i)_{i \in N}$ which could offer an allocation x_i regulating the permitted current-future emissions for all countries $i \in N$.

The solution satisfies efficiency $\sum_{i \in N} x_i = E - \overline{h}$. In other words, the sum of emission permits for all countries should be exactly equal to the available emissions. The domain of the solution is denoted by $D(\rho) = \left\{ x \in IR^n_+ : 0 \leq x_i \leq c_i, \forall i \in N \;\; and \;\; \sum\limits_{i \in N} x_i = E - \bar{h} \right\}$.

2.1 Solution Concepts

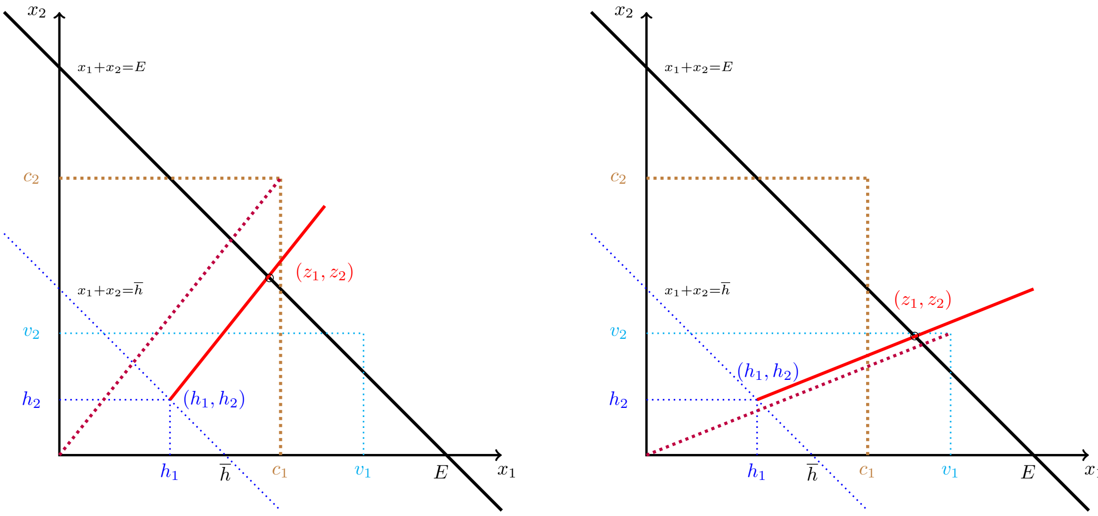
We now present in detail four solution concepts of the past-dependent rationing problem in this section. Specifically, this paper examines the following rules from Ju et al (2021): the Equal per emission rule, the Equal per capita rule, the Historical equal per emission rule, and the Historical equal per capital rule. First, we describe these rules and point out limitations as well as drawbacks of these rules, then we take two countries as example to graphically depict the allocation paths of these four rules.

Definition 1 *The Equal per emission rule (EPE), f^{EPE}, assigns for each problem $\rho \in \mathscr{P}$ and every country $i \in N$:*

$$f_i^{EPE}(\rho) = \tfrac{c_i}{c}(E - \overline{h}).$$

The EPE rule distributes total available CO_2 emissions, $E - \overline{h}$, to each country based on the ratio of its claim to all claims of participating countries. The rule is derived from the traditional proportional rule for classical rationing problem which is mentioned by Curiel et al.(1987). The distribution of EPE rule only takes current-future emission claims into account. It does not consider any other elements such as population or historical emissions. Justice and fairness are not realized in this rule. For instance, a country with higher pollution in the past still can receive more current-future emission permits from this rule as long as it has higher emission claims. Or, a country with smaller population size might declare more current-future claims to receive more permits than another country with larger population size.

Figure (a) in below illustrates the allocation path corresponding to the EPE rule for two countries. As indicated by the red bold line, the allocation path starts from the point (h_1, h_2). Hence, both countries end up with their historical emissions when there is no available emissions. When E becomes greater than $\overline{h}$, the EPE rule proportionally distributes all available emissions among two countries according to their current-future claims. So the slope of the path is ${}^{c_2}/_{c_1}$. The final allocation for both countries indicated by (z_1, z_2) is the addition of a country´s rule-determined current-future emission permits, $f^{EPE}(\rho)$, and its historical emissions. This figure clearly shows the EPE rule is in favor of countries with higher claims.



(a) The EPE Rule-Two Countries.

(b) The EPC Rule-Two Countries.

Definition 2 *The Equal per capita rule (EPC), f^{EPC}, assigns for each problem $\rho \in \mathscr{P}$ and every country $i \in N$:*

$$f_i^{EPC}(\rho) = \tfrac{v_i}{\overline{v}}(E - \overline{h}).$$

The EPC allocates GHGs emissions to a country by the ratio of its population to the whole population of all participating countries. Hence, the total available emissions, $(E - \overline{h})$, is equally distributed among all citizens regardless their nationalities. However, this EPC rule is also in violation of distribution fairness in the sense that it has ignored claims and historical emissions of a country. Therefore, a country with very few claims might receive emission permits beyond its claims due to its larger population size. On the contrary, a country who declares higher claims might be allocated with fewer emissions. In addition, the rule takes no consideration of historical emissions. For example, a country who has polluted much more in the past might get higher emission permits under this rule due to its larger population size.

Figure (b) shows the allocation path by the EPC rule with an example of two countries. Indicated by the red bold line, the path starts with point (h_1, h_2) when two countries get zero emission permits as there is no available emissions to distribute. As E grows to be greater than $\overline{h}$, the available emissions, $E - \overline{h} > 0$, are distributed proportionally to the two countries based on their population size. Therefore, the slope of the path is v_2/v_1. The final allocations for these two country are represented by point (z_1, z_2). For instance, country 1 gets emission permits from the EPC rule, $f^{EPC}(\rho)$, and its historical emissions h_1. This figure graphically shows that the EPC rule is in favor of countries with larger population size.

Until now, the EPE rule and the EPC rule have disregarded the historical emissions of each country. However, historical accountability has been supported by scholars such as Neumayer(2000) and Baer(2002) since they consistently believe that those who caused the environmental damage in the first instance should compensate it. According to the polluter-pays-principle those who had excessive historical emissions in the past should be required to make compensations to others; in addition, Baer(2002) advocated that accounting for historical emissions ensures the equality of opportunity which means every person in current or future gets to enjoy same emission rights as his counterpart in past, vise versa.

With consideration of historical emissions, the next two rules analyzed by this paper are the Historical equal per emission rule and the Historical equal per capita rule.

Definition 3 *The Historical equal per emission rule(HEPE),f^{HEPE}, assigns for each problem $\rho \in \mathscr{P}$ and every country $i \in N$:*

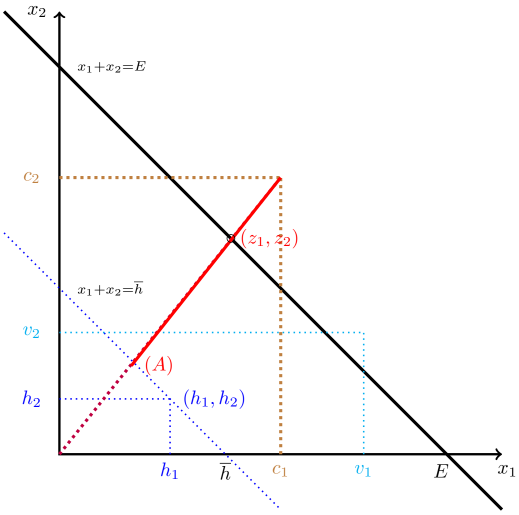
$$f_i^{HEPE}(\rho) = \frac{c_i}{\bar{c}}(E - \bar{h}) + c_i\left(\frac{\bar{h}}{\bar{c}} - \frac{h_i}{c_i}\right).$$

The HEPE rule firstly assigns baseline emission permits to each country by the EPE rule, then each country is held account for his historical credit ¨per BAU emission¨. The average global historical emissions per BAU emission is denoted by $\bar{h}/\bar{c}$. The historical emissions per BAU emission of a country i is denoted by h_i/c_i. The rule indicates if country i´s historical emission per BAU emission is higher than average, then this country should pay back since it has polluted more in the past. The total reduction from baseline allocation should be equal to $c_i \cdot \left(\frac{\bar{h}}{\bar{c}} - \frac{h_i}{c_i}\right)$.

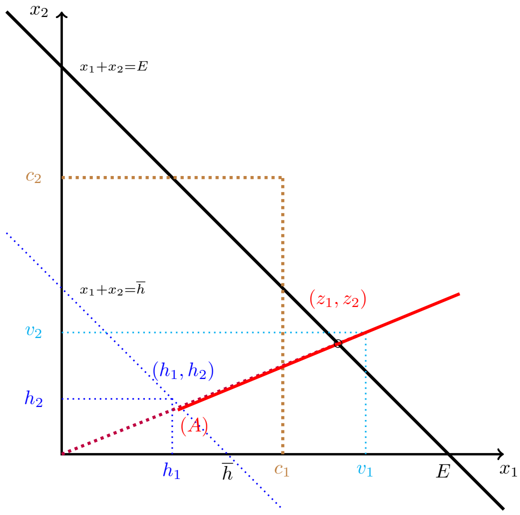
It is important to notice that $f_i^{HEPE}(h, c, v, E) = \frac{c_i}{\bar{c}}(E - \bar{h}) + c_i\left(\frac{\bar{h}}{\bar{c}} - \frac{h_i}{c_i}\right) = \frac{c_i}{\bar{c}} \cdot E - h_i$. So the HEPE rule can also be interpreted as follows: first, the targeted emissions, E, are allocated proportionally among countries according to their claims. Then the historical emissions of a country is reduced.

The outstanding drawback of this HEPE rule is countries with higher historical emissions might get negative allocations. This means these countries have to make compensations to others through channels such as buying permits or cleaning the existing CO_2 in the air. This negativity in allocation will increase the difficulty of reaching a sustainable multilateral agreement in reducing global warming.

The following figure (c) shows the allocation path of the HEPE rule by the red bold line for the same case of two countries. Notice that when there is no available emissions, $E - \bar{h} = 0$, the path no longer starts from point (h_1, h_2). It starts with point A indicating country 1 with higher pollution in the past will have to make compensations to country 2 who consumed much less emissions in the past. When the targeted emissions, E, is big enough, the final allocations of these two countries could be positive and are in proportion with countries´ claims. The path therefore takes slope of $c_2\backslash c_1$.



(c) The HEPE Rule-Two Countries.



(d) The HEPC Rule-Two Countries.

Definition 4 *The Historical equal per capita rule (HEPC), f^{HEPC}, assigns for each problem $\rho \in \mathscr{P}$ and every country $i \in N$:*

$$f_i^{HEPC}(\rho) = \tfrac{v_i}{\overline{v}}(E - \overline{h}) + v_i\Big(\tfrac{\overline{h}}{\overline{v}} - \tfrac{h_i}{v_i}\Big).$$

The HEPC rule is a modification of the former EPC rule. Country i has historical emission debts if its per capita historical emission, h_i/v_i, is above average per capita historical emission of all countries $\overline{h}/\overline{v}$. Its historical emission debts equal to amount of $v_i\Big(\frac{\overline{h}}{\overline{v}} - \frac{h_i}{v_i}\Big)$ should be reduced from its EPC shares. Hence, the HEPC rule first allocates the available emissions to countries proportionally to their population size, then each country´s historical credits are corrected.

It is worthy to notice that $f_i^{HEPC}(\rho) = \frac{v_i}{\overline{v}}(E - \overline{h}) + v_i\Big(\frac{\overline{h}}{\overline{v}} - \frac{h_i}{v_i}\Big) = \frac{v_i}{\overline{v}}E - h_i$. In words, the rule first distributes the targeted emissions E proportionally among countries with respect to their population size, then the historical emissions of a country is deducted. The rule also could give a country negative allocations when the country has high historical emissions. Countries do not want to receive negative allocations and this will drive them to withdraw from multilateral agreement which adopts this rule.

Figure (d) shows the location path corresponding to the HEPC rule for the same two countries. When there is no available emissions, $E - \bar{h} = 0$, the path indicated by the red bold line starts with point A where country 1 has to make compensation to country 2 by way of permits purchase or pollution cleaning even its emissions in the past is lower than that of country 2. This is because the population size of country 1 is much smaller than country 2 and the HEPC rule is in favor of countries with larger population size. When the available emissions,$E - \bar{h}$, is large enough, both countries get positive emission permits from the rule. Final permitted emissions indicated by point (z_1, z_2) include the EPC shares $\frac{v_i}{\bar{v}}(E - \bar{h})$ which is corrected by historical emission debts $v_i \cdot \left(\frac{\bar{h}}{\bar{v}} - \frac{h_i}{v_i}\right)$ and historical emissions h_i.

2.2 Solution Concepts from Individual Perspective

The available GHGs emissions could be treated as global public goods and there is always debate in climate ethics regarding how the emission permits should be divided. One of the most popular principle recently advocated by many scholars such as Baatz and Ott (2017), Christian Seidel (2009) and Gosseries (2005) is Egalitarianism.

From the perspective of egalitarianism, Torpman (2019) advocated the carbon emission rights belong to every individual and any distribution rule is supposed to treat individual equally regardless of nationality. As Broome (2012) said "no one in the world has a stronger claim to this resource [i.e. permits to emit greenhouse gas] than anyone else, so it should be divided equally between people ".

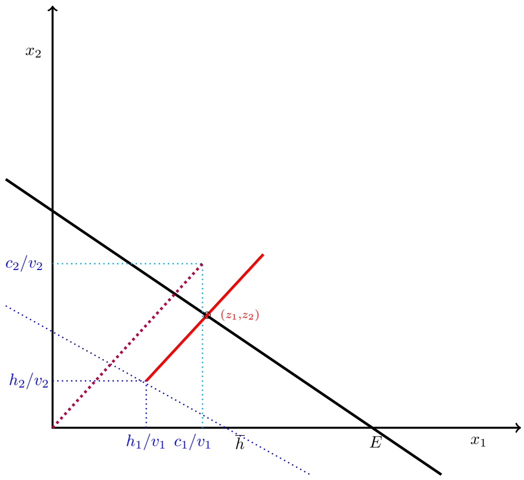
Suppose each country distributes allocations from the former four rules equally among its citizens, every citizen thus obtains per capita allocations through this way. This paper then analyzes per capita allocations under the former four rules and check whether egalitarianism is realized in perspective of per capita allocations.

For the EPE rule, the per capita reward of each individual j in country i is set as

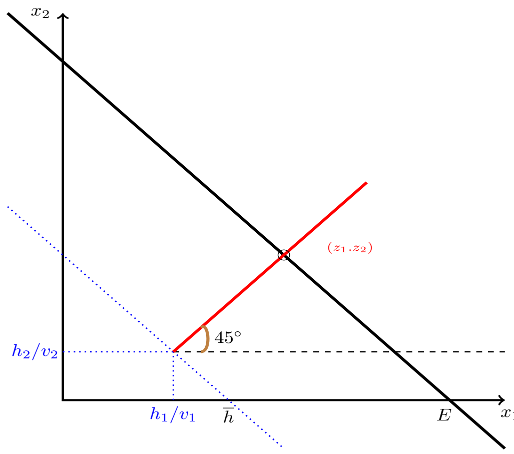
$$f_j^{EPE}(\rho) = \frac{f_i^{EPE}(\rho)}{v_i} = \frac{c_i}{v_i}\frac{(E - \bar{h})}{\bar{c}} = \frac{c_j}{\bar{c}}(E - \bar{h}), \quad where \;\; c_j = \frac{c_i}{v_i} \;\; \forall j \in N_i,$$

where N_i is the set of individuals in country i. In perspective of individuals, the EPE rule distributes available emissions among individuals from different countries proportionally to their per capita claims.

The below Figure (e) shows the allocation path for two individuals from two countries. As indicated by the red bold line, when there is no available emissions, both individuals get zero emission permits. When the available emissions, $E - \bar{h}$, is positive, the EPE rule distributes all available emissions proportionally to individuals´ per capita claims. The figure shows that the EPE rule does not lead to egalitarianism unless every individual in the world own exactly same per capita claims. In addition, the EPE rule under the perspective of individuals also disregard historical per capita emissions and the distribution is in favor of high per capita emissions.



(e) The EPE Rule-Two Individuals.



(f) The EPC Rule-Two Individuals.

For the EPC rule, the per capita rewards of each individual j in country i is denoted by

$$f_j^{EPC}(\rho) = \frac{f_i^{EPC}(\rho)}{v_i} = \frac{E - \bar{h}}{\bar{v}}, \quad \forall j \in N_i,$$

where N_i is the set of all individuals in country i. In words, country i equally distributes all its emission permits from the EPC rule to its citizens. The above figure (f) shows the allocation path of the EPC rule for two individuals from different countries. The red bold line indicates that under the perspective of individual, the EPC rule treats individuals from different countries equally. The path thereby coincides with 45° line. Though the EPC rule realized the distribution egalitarianism, it disregards the historical per capita emissions and there might be situation in which a person could receive emission permits more than his per capita claims.

For the HEPE rule, the per capita rewards of each individual j in country i will be

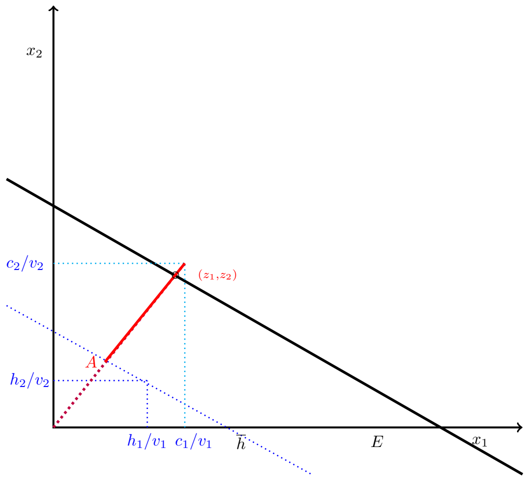
$$f_j^{HEPE}(\rho) = \frac{f_i^{HEPE}}{v_i} = \frac{c_i}{v_i}(\frac{E}{\bar{c}}) - \frac{h_i}{v_i} = \frac{c_j}{\bar{c}}E - h_j, \quad where \;\; c_j = \frac{c_i}{v_i} \;\; and \;\; h_j = \frac{h_i}{v_i}, \quad \forall j \in N_i.$$

where N_i is the set of all individuals in country i. In words, country i equally distributes its emission permits from the HEPE rule to its citizens.

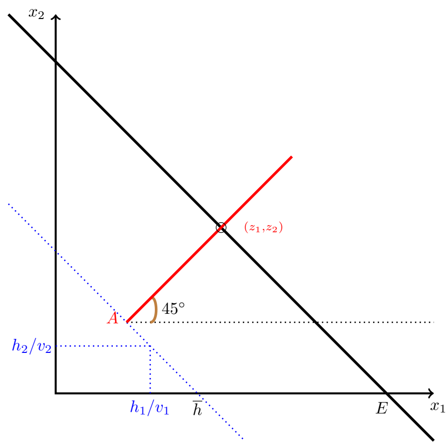
The figure (g) in below shows the allocation path for two individuals from different countries of the HEPE rule. The red bold line starts with point A when there is no available emissions.

At this point, individual 1 has much more emissions in the past than individual 2, therefore, the rule asks him to make compensations to individual 2 by channels of emission permits purchase or pollutes cleaning. When there is enough available emissions, the rule first proportionally distributes all available emissions among two individuals based on their per capita claims, then corrects individuals´ allocation by his/her historical per capita emissions.

As shown by the figure(g), the HEPE rule under the perspective of individuals could not guarantee egalitarianism and is in favor of high per capita claims. Besides, there might be situation where an individual get negative emission permits.



(g) The HEPE Rule-Two Individuals.



(h) The HEPC Rule-Two Individuals.

For the HEPC rule, the per capita rewards of each individual j in country i will be

$$f_j^{HEPC}(\rho) = \frac{f_i^{EPC}(\rho)}{v_i} = \frac{E}{\bar{v}} - \frac{h_i}{v_i} = \frac{E}{\bar{v}} - h_j, \quad where \ \ h_j = \frac{h_i}{v_i}, \quad \forall j \in i.$$

Literally, country i distribute equally what it receives from the HEPC rule to all its citizens. Figure (h) illustrates the allocation path for two individuals from different countries corresponding to the HEPC rule. The red bold line shows individual 1 has to make compensations to individual 2 for his excessive emissions in the past when there is no available emissions. When there is enough available emissions, the rule allocate equal amount to every individuals regardless of their nationalities. Therefor, the path coincides with 45° line.

The HEPC rule under perspective of individuals somehow realizes the egalitarianism. However, the figure shows that the per capita allocations one receives from this rule might exceed his/her per capita claims. Also, when the available emissions is little, the rule might give negative allocations to those individuals who have high emissions in the past.

Since this paper endeavors to create an egalitarian solution to the past-dependent rationing problem $\rho = (N, h, c, v, E)$ in perspective of individual, it is necessary to re-exam the problem from the individual point of view. Notice that when the past-dependent rationing problem ρ is defined on agent of countries, it is not possible to compare Lorenz domination and check egalitarianism in perspective of individuals.

Therefore, this paper considers a per capita extension of $\rho = (N, h, c, v, E)$ denoted by $\rho^{pc} \in \mathscr{P}$. Let $N = \{1, 2, ..., n\}$ be the set of countries and N_i be the set of inhabitants of $i \in N$. In the per-capita extension, agents are individuals, therefore, the set of agents is $\{N_1 \cup N_2 \cup ... \cup N_n\}$. Then, set $E^{pc} = E$. Set $v_j^{pc} = 1$ for all $j \in N_i$ and all $i \in N$. And set $c_j^{pc} = \frac{c_i}{v_i}$ for all $j \in N_i$ and all $i \in N$. Finally, set $h_j^{pc} = \frac{h_i}{v_i}$ for all $j \in N_i$ and all $i \in N$.

It is worthy to remark two important observations. First, for all the former EPE, EPC, HEPE and HEPC rules and all $\rho \in \mathscr{P}$, it holds that

$$\frac{f_i(\rho)}{v_i} = f_j(\rho^{pc}) \quad for \quad all \quad j \in N_i \quad and \quad all \quad i \in N.$$

Second, an allocation of ρ^{pc} is a vector that assigns emission permits to individuals rather than countries. This allows us to compare allocations from an egalitarianism perspective and by means of Lorenz domination.

Finally, observe that if a Past-dependent rationing problem is such that $v_i = 1 \ \forall i \in N$, population is vacuous. So we can write the problem as $\rho = (N, h, c, E)$. In the following we will denote a problem like this as *Unit past-dependent rationing problem.*

3 Properties of Solution Concepts

In standard rationing problem, a well-behaved solution should satisfy minimal requirements such as claim-boundedness, non-negativity and equal treatment of per-capita equals. These properties, on one hand, are generally accepted in the literature of fair allocation and no rules violating them would be socially accepted, on the other hand, provide a chance to compare different rules and identify solutions by means of the properties they satisfy.

3.1 Claim-boundedness and Non-negativity

Claim-boundedness requires that the GHGs emission permits allocated to each agent are no more than its current-future emission claims. In multilateral cooperation of reducing global warming, solutions satisfying Claim-boundedness will help to guarantee that no agent receives emission permits more than its necessities. Mathematically,

Principle 1 (Claim-boundedness). For any problem $\rho = (N, h, c, v, E)$ and every $i \in$ N, $f_i(\rho) \leq c_i$.

The next Proposition establishes the behavior of the former four rules regarding to this property.

Proposition 1 *The EPE rule satisfies Claim-boundedness while the HEPE rule, the EPC rule and the HEPC rule do not.*

Proof: For the EPE rule, let $(N, h, c, v, E) = \rho$, we have: $f_i{}^{EPE}(\rho) = \frac{c_i}{\bar{c}}(E - \bar{h})$ and $0 \leq (E - \bar{h}) \leq \bar{c}$. Therefore, $\frac{(E-\bar{h})}{\bar{c}} \leq 1$ and $f_i{}^{EPE}(\rho) = c_i \cdot \frac{(E-\bar{h})}{\bar{c}} \leq c_i$.

For the HEPE rule, define $\rho = (N, h, c, v, E)$ by: there are two countries with claim vector $c = (c_1, c_2) = (1, 4)$, historical emission vector $h = (h_1, h_2) = (0.1, 2)$ and the population size $(v_1, v_2) = (2, 1)$, the total claim is $\overline{c} = 5$ and the total historical emission is $\overline{h} = 2.1$, the targeted emissions is $E = 6$. The total available emissions is $(E - \overline{h})$=3.9.

According to the HEPE rule, $f_1{}^{HEPE}(\rho) = \frac{c_1}{\bar{c}} \cdot E - h_1 = \frac{1}{5} \cdot 6 - 0.1 = 1.1$ and $f_2{}^{HEPE}(\rho) = \frac{c_2}{\bar{c}} \cdot E - h_2 = \frac{4}{5} \cdot 6 - 2 = 2.8$ while $c_1 = 1$ so $f_1{}^{HEPE}(\rho) > c_1$.

For the EPC rule, consider the same example. $f_1{}^{EPC}(\rho) = \frac{v_i}{\bar{v}} \cdot (E - \bar{h}) = \frac{2}{3} \cdot 3.9 = 2.6$ and $f_2^{EPC}(\rho) = \frac{v_i}{\bar{v}} \cdot (E - \bar{h}) = \frac{1}{3} \cdot 3.9 = 1.3$ while $c_1 = 1$ so $f_1{}^{EPC}(\rho) > c_1$.

For the HEPC rule, take the same example. ${f_1}^{HEPC}(\rho) = \frac{v_1}{\bar{v}} \cdot E - h_1 = \frac{2}{3} \cdot 6 - 0.1 = 3.9$ and ${f_2}^{HEPC}(\rho) = \frac{v_2}{\bar{v}} \cdot E - h_2 = \frac{1}{3} \cdot 6 - 2 = 0$ While $c_1 = 1$ so ${f_1}^{HEPC}(\rho) > c_1$. □

Non-negativity sets a lower bound for the solution concepts. In the allocation of GHGs emission permits, non-negativity requires that no country receives negative amount of pollution permits. This is helpful to reach multilateral agreements since it ensures that no country needs to either buy emission permits or reduce CO_2 in the air.

Principle 2 *(Non-negativity).* For any problem $\rho = (N, h, c, v, E)$ and every $i \in N$, we have $f_i(\rho) \geq 0$.

The next proposition states the behavior of the former four rules regarding to the property of Non-negativity.

Proposition 2 *The EPE rule and the EPC rule satisfy non-negativity while the HEPE rule and the HEPC rule do not.*

Proof: it is straight forward that ${f_i}^{EPE}(\rho) = \frac{c_i}{\bar{c}}(E - \overline{h}) \geq 0$ for all $\rho \in \mathscr{P}$ and ${f_i}^{EPC}(\rho) = \frac{v_i}{\bar{v}}(E - \overline{h}) \geq 0$ for all $\rho \in \mathscr{P}$.

For the HEPE and the HEPC rules, we prove they do not satisfy non-negativity by the following example : define $\rho = (N, h, c, v, E)$ as there are only two countries with historical emissions $h = (h_1, h_2) = (0.1, 2)$, current-future claims $c = (c_1, c_2) = (1, 2)$, and population size $v = (v_1, v_2) = (1, 2)$. The targeted emissions $E = 2.7$ and the total available emissions $E - \overline{h} = 0.6$.

According to the HEPE rule, ${f_2}^{HEPE}(\rho) = \frac{c_2}{\bar{c}} \cdot E - h_2 = \frac{2}{3} \cdot 2.7 - 2 = -0.2$

and ${f_1}^{HEPE}(\rho) = \frac{c_1}{\bar{c}} \cdot E - h_1 = \frac{1}{3} \cdot 2.7 - 0.1 = 0.8$. So ${f_2}^{HEPE}(\rho) < 0$.

According to the HEPC rule, ${f_2}^{HEPC}(\rho) = \frac{v_2}{\bar{v}} \cdot E - h_2 = \frac{2}{3} \cdot 2.7 - 2 = -0.2$

and ${f_1}^{HEPC}(\rho) = \frac{v_1}{\bar{v}} \cdot E - h_1 = \frac{1}{3} \cdot 2.7 - 0.1 = 0.8$. So $f_2^{HEPC}(\rho) < 0$. □

3.2 Equal Treatment of Per-Capita Equals

The next important principle is equal treatment of equals which declares that identical agents in rationing problem should be treated equally and receive the same amount of the available resources. The principle expresses an ethical matter of impartiality and can be regarded as

¨Symmetry¨ among agents. Traditional equal treatment of equals considers only the claims of agents and requires to give the same award to agents with same claims. In this way, similar agents in term of claims are rewarded equally without any priority and excluding other non-relevant information such as names, religion, etc.

However, in our Past-dependent rationing problem of CO_2 emissions distribution, information such as population, historical emissions are equally important as claims. Besides, it is well believed that citizens with equal historical per-capita emissions and identical per capita BAU emissions should be treated with equal moral rights. Therefore, this paper follows Ju et al.(2021) and defines Equal treatment of per-capita equals as follows:

Principle 3 *(Equal treatment of per-capita equals (ETEPC)).* For any problem $\rho \in \mathscr{P}$, and each pair $i, j \in N$, $f_i(\rho)/v_i = f_j(\rho)/v_j$ whenever $c_i/v_i = c_j/v_j$ *and* $h_i/v_i = h_j/v_j$.

Though this paper takes same definition of ETEPC as Ju et al.(2021), the behavior of the former four rules regarding to this property is different from what Ju et al.(2021) concluded. The next proposition says all rules satisfy ETEPC.

Proposition 3 *All the four rules satisfy Equal treatment of per-capita equals.*

Proof: for the EPE rule, when $\frac{c_i}{v_i} = \frac{c_j}{v_j}$, $\frac{f_i^{EPE}(\rho)}{v_i} = \frac{c_i}{v_i} \cdot \frac{(E-\bar{h})}{\bar{c}} = \frac{c_j}{v_j} \cdot \frac{(E-\bar{h})}{\bar{c}} = \frac{f_j^{EPE}(\rho)}{v_j}$ for all $\rho \in \mathscr{P}$. Therefore, the EPE rule satisfies ETEPC.

For the EPC rule, it always holds that $\frac{f_i^{EPC}(\rho)}{v_i} = \frac{(E-\bar{h})}{\bar{v}} = \frac{f_j^{EPC}(\rho)}{v_j}$ for all $\rho \in \mathscr{P}$. Therefore, the EPC rule satisfies ETEPC.

For the HEPE rule, when $\frac{c_i}{v_i} = \frac{c_j}{v_j}$ and $\frac{h_i}{v_i} = \frac{h_j}{v_j}$, then $\frac{f_i^{HEPE}(\rho)}{v_i} = \frac{c_i}{v_i} \cdot \frac{E}{\bar{c}} - \frac{h_i}{v_i} = \frac{c_j}{v_j} \cdot \frac{E}{\bar{c}} - \frac{h_j}{v_j} = \frac{f_j^{HEPE}(\rho)}{v_j}$ for all $\rho \in \mathscr{P}$. Therefore, the HEPE rule satisfies ETEPC.

For the HEPC rule, when $\frac{h_i}{v_i} = \frac{h_j}{v_j}$, then $\frac{f_i^{HEPC}(\rho)}{v_i} = \frac{E}{\bar{v}} - \frac{h_i}{v_i} = \frac{E}{\bar{v}} - \frac{h_j}{v_j} = \frac{f_j^{HEPC}(\rho)}{v_j}$ for all $\rho \in \mathscr{P}$. Therefore, the HEPC rule satisfies ETEPC. $\square$

Table 1 of the next page summarizes the comparison among the four rules based on the properties of claim-boundedness, non-negativity and equal treatment of per capita equals. It also describes whether all relevant information is considered in these rules including claims, population size and historical emissions.

Observe that only the EPE rule satisfies Claim-boundedness, Non-negativity and Equal treatment of per-capita equals. However, the EPE rule does not take all relevant information

Table 1: Properties and Rules

Properties	**EPE**	**EPC**	**HEPE**	**HEPC**
Claim-boundedness	Yes	No	No	No
Non-negativity	Yes	Yes	No	No
Equal treatment of per-capita equals	Yes	Yes	Yes	Yes
Contain all relevant information-h,c,v	No	No	No	No

into account such as population size and historical emissions. The other three rules do not meet claim-boundedness and two of them violate Non-negativity. When a rule violates Claim-boundedness, the distribution of CO_2 emission permits might be considered unfair in a sense that there might be countries which are assigned much more emission permits than their needs while other countries with higher claims do not get enough. Non-negativity is also crucial to make sure that all countries or agents have motivation to stay in multilateral agreement. When countries are allocated with negative emission permits, they will have to buy permits from other agents or reduce the existing CO_2 in the air which will drive them to withdraw from the agreement. This is harmful for creating a sustainable multilateral agreement.

All four rules satisfy Equal treatment of per-capita equals. ETEPC is indispensable in guaranteeing the egalitarianism of a distribution rule. As long as the historical per-capita emission and per-capita BAU emission are the same for any two persons, they shold be awarded with the same CO_2 emission permits regardless of their nationalities. A rule violating of equal treatment of per-capita equals will result in discrimination among two individuals with same historical per capita emissions and per capita BAU emissions.

In addition, only part of the relevant information is considered by these four rules which might lead to distribution unfairness. For example, the equal per emission rule only considers BAU claims. This rule will allocate more CO2 emissions to countries with higher BAU emission claims ignoring the fact that they already had much more historical emissions.

A desirable solution in fair abatement of GHG emissions concerns about fairness and justice. And the prerequisite of a fair allocation rule is the inclusion of all relevant information. It also needs to satisfy Claim-boundedness, Non-negativity and Equal treatment of per-capita equals. In search of such solutions, we extended the past-dependent rationing problem by changing the agent from country to individual. Then, based on the extended rationing model proposed by Timoner and Izquierdo (2016) and treat historical emissions of agents as ex-ante conditions, this paper creatively generates a new rule which is the Generalized Constraint Equal Awards rule.

4 The Generalized Constrained Equal Awards Rule

The four rules introduced so far all treat country as agents in determining the allocation of available emissions. However, the emission rights belong to every individual and the egalitarianism in distribution should be guaranteed in terms of individual agents. In the following, this section introduces a new rule to focus on egalitarianism consideration of individuals. To this aim, the rule is defined on the set of individuals instead of on the set of countries. And this rule is firstly introduced by Timoner and Izquierdo (2016).

We define the generalized constrained equal awards rule for Unit past-dependent rationing problem denoted by $\rho = (N, h, c, E) \in \mathscr{P}$. As mentioned in the end of chapter 2, the Unit past-dependent rationing problem is just a special case of the Past-dependent rationing problem when each country only has one citizen. So it can be understood as population, v, becomes vacuous while the problem is still the Past-dependent rationing problem.

Definition 5 *The Generalized Constraint Equal Awards rule (GEA), $f^{GEA}(\rho)$, assigns for any Unit past-dependent rationing problem $\rho = (N, h, c, E)$ and every individual $i \in N$:*

$$f_i^{GEA}(N, h, c, E) = \min \left\{c_i, (\lambda - h_i)_+\right\},$$

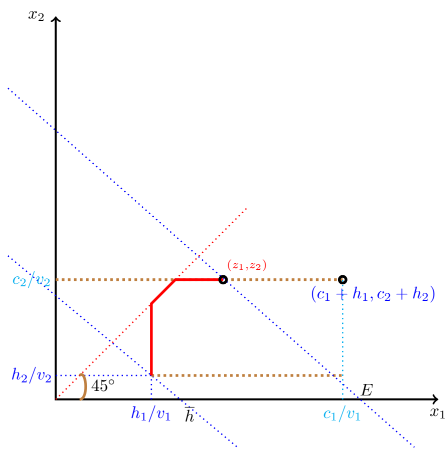
where $(\lambda - h_i)_+ = \max\{0, \lambda - h_i\}$ and λ is such that $\sum_{i\in N} f_i^{GEA}(\rho) = E - \bar{h}$.

It is worthy to point out that the above GEA rule is defined on Unit past-dependent rationing problem. For any ordinary problem $\rho = (N, h, c, v, E)$, the GEA allocation could be defined as $f_i^{GEA}(N, h, c, v, E) = v_i \cdot \min\left\{\frac{c_i}{v_i}, \left(\lambda - \frac{h_i}{v_i}\right)_+\right\} = v_i \cdot \sum\limits_{j\in N_i} f_j^{GEA}(N, h, c, E)$, for all individual $j \in N_i$, where N_i is the set of all individuals in country i.

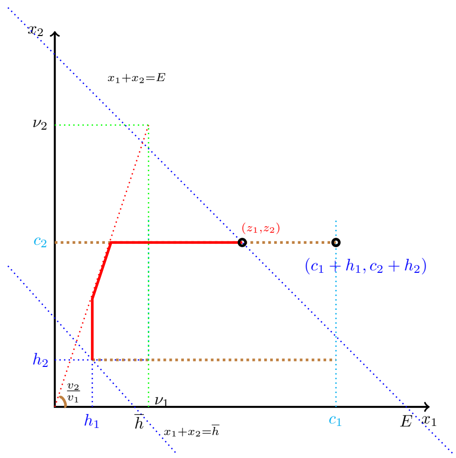
The GEA rule starts by transforming claims and historical emissions of a country to per capita claims and historical per capita emissions. Then, individuals from all countries are arranged in ascending order with respect to historical per capita emissions. The GEA firstly distributes total available carbon emissions to those individuals with less per-capita historical emissions until their per capita claims are fully satisfied or their total per capita emissions being equal to historical per capita emissions of other individuals. Then, the GEA rule distributes what remains equally among all individuals whose per-capita claims are not satisfied.

The next two figures graphically describe the allocation path corresponding to the GEA rule. In figure (i), we restrict our attention to case of only two agents while figure (j) focus on two countries with different population.

In figure(i), the claims and historical emissions of two countries are transformed to per-capita claims and historical per-capita emissions. We select one person from each country as our agent for Unit Past-Dependent Rationing problem. The historical per-capita emissions of the first individual, h_1/v_1, is much higher than that, h_2/v_2, of the second individual. The bold red line in this figure shows the allocation path of the GEA rule which firstly gives all permits to individual 2 until the rewards become equal to the historical per-capita emissions of individual 1. Then the rule allocates what remains equally among these two persons. When the claim of individual 2, c_2/v_2, is fully satisfied, all remaining available emissions go to individual 1.



(i) Allocation Path of the GEA Rule-Two Individuals.



(j) Allocation Path of the GEA Rule-Two Countries.

The figure (j) shows the allocation path of the GEA rule by countries. The country allocation is the addition of all its citizens´ allocation. As indicated by the red bold line, when there is no available emissions ($E = \bar{h}$), both countries only get their historical emissions. When there are available emissions, the GEA rule distributes emission permits between the two countries proportionally to their population size. When country 2 reaches its full claim and there are still remaining available emissions, all remaining emissions go to country 1.

4.1 Properties of the GEA Rule

In this section, we check whether the GEA rule satisfies the social accepted properties including Claim-boundedness, Non-negativity and Equal treatment of per-capita Equals.

The following proposition mathematically states the GEA rule satisfies all those three properties. In addition, this rule has taken all relevant information into consideration when decides the distribution of carbon emissions.

Proposition 4 *The GEA rule satisfies Claim-boundedness, Non-negativity and Equal treatment of per-capita equals.*

Proof: Let $\rho = (N, h, c, E)$ be a Unit past-dependent rationing problem. Then, $\forall i \in N$, we have $f_i^{GEA}(\rho) = \min\left\{c_i, (\lambda - h_i)_+\right\}$ and it is straight forward that $0 \leq f_i^{GEA}(\rho) \leq c_i$. So, the GEA rule satisfies Claim-boundedness and Non-negativity. Moreover, if $c_i = c_j$ and $h_i = h_j$, then $f_i^{GEA}(\rho) = f_j^{GEA}(\rho)$. So, the GEA rule also satisfies ETEPC. □

4.2 Proposed Algorithm for Computing the GEA allocations

The GEA rule has a nice and concise definition, however, it is not easy to understand at first sight. To make the allocation proceeding of this rule easier to catch, this paper therefore proposes the flowing corresponding algorithms to compute allocations from the GEA rule.

Let $\rho = (N, h, c, v, E)$ be an arbitrary ordinary Past-dependent rationing problem where agents are countries. We transform country historical emissions and country emission claims to per-capita historical emissions and per-capita claims, then consider the Unit past-dependent rationing problem denoted by $\rho = (N, h, c, E)$.

Let $\rho = (N, h, c, E)$ be any Unit past-dependent rationing problem. W.l.o.g, we assume $N = \{1, 2, ..., n\}$. The algorithm begins with ordering all agents with respect to their historical emissions such that $h_1 \leq h_2 \leq h_3 \leq \cdots \leq h_n$.

Step 1: set
$$\lambda = (\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_n, \lambda_{n+1})$$
$$= (h_1, h_2, h_3, \cdots, h_n, \max_{i \in N}(c_i + h_i)),$$

Step 2: obtain $t = (t_1, t_2, t_3, \cdots, t_n, t_{n+1})$,

$$\text{where } t_1 = \sum_{i \in N} \min\left\{c_i, (\lambda_1 - h_i)_+\right\},$$

$$t_2 = \sum_{i \in N} \min\left\{c_i, (\lambda_2 - h_i)_+\right\},$$

...

$$\text{Until } t_n = \sum_{i \in N} \min\left\{c_i, (\lambda_n - h_i)_+\right\},$$

$$t_{n+1} = \sum_{i \in N} \min\left\{c_i, (\lambda_{n+1} - h_i)_+\right\},$$

Step 3: Case 1: if $\exists\ j \in N$ such that $t_j = E - \bar{h}$, **algorithm stops**

$$\text{with } \lambda = \lambda_j \text{ and } f_i^{GEA}(\rho) = \min\left\{c_i, (\lambda_j - h_i)_+\right\}$$

Case 2: if not, there exists j^*, $j^* + 1 \in N$ such that $t_{j^*} < E - \bar{h} < t_{j^*+1}$, **algorithm continues** and rewards all individuals with λ_{j^*} .

$x_i = \min\left\{c_i, (\lambda_{j^*} - h_i)_+\right\} = 0\ \forall i \in [j^* + 1, n]$; set $N_0 = \{i \in N : i \geq j^* + 1\}$;

$x_i = \min\left\{c_i, (\lambda_{j^*} - h_i)_+\right\}\ \forall i \in [1, j^*]$; set $N_c = \{i \leq j^* : x_i = c_i\}$

Step 4: Define a new Unit past-dependent rationing problem with $\left(N^1, h^1, c^1, E^1\right)$, where $N^1 = N|_{N_0 \cup N_c}$, $E^1 = (E - \bar{h}) - \sum_{i \in N} x_i$, $c_i^1 = h_i + c_i - \lambda_{j*}$ and $h_i^1 = 0\ \forall i \in N^1$.

Step 5: For all $k = \left\{1, 2, ..., \left|N^1\right|\right\}$, compute $\beta^k = \frac{E^k}{\left|N^k\right|}$;

if $c_i^k \geq \beta^k$ for all $i \in N^k$, stop and set $y_i = \beta^k$ for all $i \in N^k$;

if not, set $y_i = c_i^k$ if $c_i^k < \beta^k$.

Define $N^{k+1} = N^k \backslash \left\{i \in N^k : c_i^k < \beta^k\right\}$, $E^{k+1} = E^k - \sum_{\left\{i \in N^k : c_i^k \leq \beta^k\right\}} c_i^k$, and $c_i^{k+1} = c_i^k$ for all $i \in N^{k+1}$, then go to step 5 again.

Step 6: Let k^* be such that the algorithm ends. Then

$$\lambda = \lambda_{j^*} + \beta^{k^*} \quad \text{and} \quad f_i^{GEA}(\rho) = \begin{cases} x_i & \forall i \in N_0 \cup N_c \\ x_i + y_i & \forall i \in N|_{N_0 \cup N_c} \end{cases}$$

In the appendix, we prove that when $\lambda = \lambda_{j^*} + \beta^{k^*}$, for all $i \in N_0 \cup N_c$, $f_i^{GEA}(\rho) = x_i$; and for all $i \in N|_{N_0 \cup N_c}$, $f_i^{GEA}(\rho) = x_i + y_i$.

Example: We give the following example to illustrate the proceeding of the above algorithm. Define the Unit past-dependent problem as: there are only four countries in the abatement of greenhouse gas emissions and each country has only one citizen. Namely, $N = (1, 2, 3, 4)$. The corresponding emission claims are $c = (c_1, c_2, c_3, c_4) = (8, 1, 8, 5)$ and their historical emissions are $h = (h_1, h_2, h_3, h_4) = (2, 4, 5, 10)$. The four individuals are allowed to emit 31 Gt greenhouse gas in total, namely $E = 31$. As they already had historical emissions up to 21 Gt, the divisible GHGs emissions is only 10 Gt now, namely $E - \overline{h} = 10$.

Based on the algorithm, all four agents are arranged in ascending order with respect to historical emissions, which gives $(h_1 = 2) < (h_2 = 4) < (h_3 = 5) < (h_4 = 10)$. The maximum claims come from individual 4 which requires in total 15 Gt emissions, namely $\max(c_i + h_i) = 15$ where $i = 4$.

Step 1: set $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) = (h_1, h_2, h_3, h_4, \max_{i\in N}(c_i + h_i)) = (2, 4, 5, 10, 15)$;

Step 2: obtain $t = (t_1, t_2, t_3, t_4, t_5) = (0, 2, 4, 14, 22)$;

where $t_i = \sum_{i\in N} \min\left\{c_i, (\lambda_i - h_i)_+\right\}$, therefore, values from t_1 to t_5 are computed by

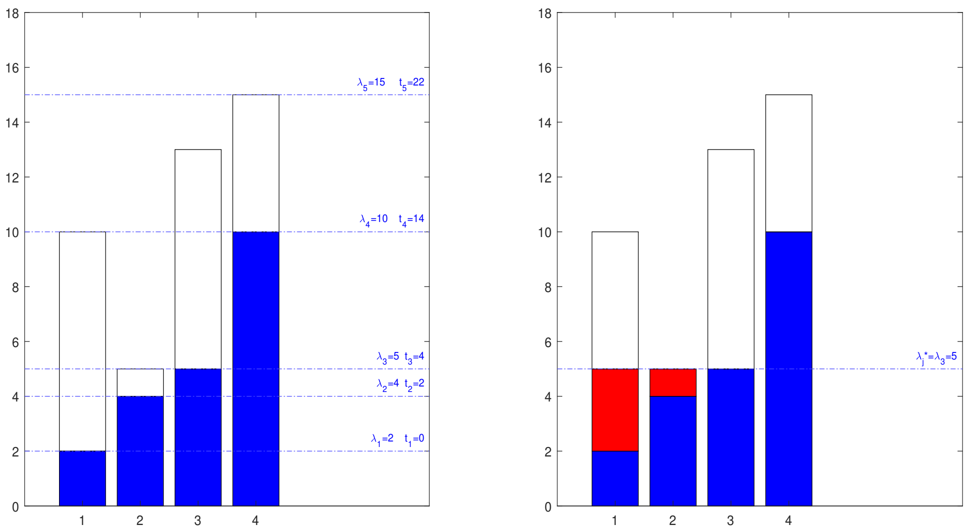
$t_1 = \min\left\{8, (2-2)_+\right\} + \min\left\{1, (2-4)_+\right\} + \min\left\{8, (2-5)_+\right\} + \min\left\{5, (2-10)_+\right\} = 0;$

$t_2 = \min\left\{8, (4-2)_+\right\} + \min\left\{1, (4-4)_+\right\} + \min\left\{8, (4-5)_+\right\} + \min\left\{5, (4-10)_+\right\} = 2;$

...

$t_5 = \min\left\{8, (15-2)_+\right\} + \min\left\{1, (15-4)_+\right\} + \min\left\{8, (15-5)_+\right\} + \min\left\{5, (15-10)_+\right\} = 22.$

Step 3: at this step, the algorithm enters into case 2 since $E - \overline{h} = 10$ is in between $t_3 = 4$ and $t_4 = 14$. our $j^* = 3$ and $j^* + 1 = 4$, all 4 agents now get their first-stage allocation $x \in IR^n_+$ with $\lambda_{j^*} = h_3 = 5$. Therefore, $x_1 = \min\left\{c_1, (\lambda_3 - h_1)_+\right\} = \min\left\{8, (5-2)_+\right\} = 3$; $x_2 = \min\left\{c_2, (\lambda_3 - h_2)_+\right\} = \min\left\{1, (5-4)_+\right\} = 1$; $x_3 = \min\left\{8, (5-5)_+\right\} = 0$; $x_4 = \min\left\{c_4, (\lambda_3 - h_4)_+\right\} = \min\left\{5, (5-10)_+\right\} = 0$. And we define $N_0 = \{4\}$ and $N_c = \{2\}$.



(k) First-stage allocation-Algorithm of the GEA rule.

As shown by the above figure, until now the algorithm gives four countries their first-stage allocation $x = (x_1, x_2, x_3, x_4) = (3, 1, 0, 0)$, denoted by the red-stacked bar in figure. And countries are divided into three sub-player sets: $N_0 = \{4\}$, $N_c = \{2\}$ and $N \backslash (N_0 \cup N_c) = \{1, 3\}$.

The total first-stage allocations are 4 units out of 10 Gt greenhouse gas. Since there remains 6 Gt greenhouse gas emissions, the algorithm then continues with step 4.

Step 4: Define a new Unit past-dependent rationing problem with $\left(N^1, h^1, c^1, E^1\right)$ where $N^1 = N|_{N_0 \cup N_c} = \{1, 3\}$, $E^1 = (E - \bar{h}) - \sum\limits_{i \in N} x_i \ = 6$, and $c^1 = [8, 8]$ while $h^1 = (0, 0)$.

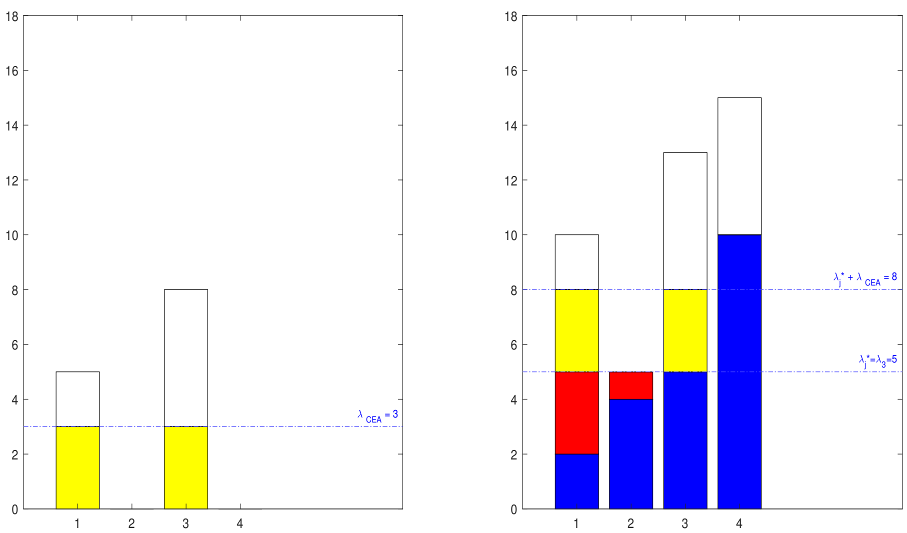
Step 5: Compute $\beta^1 = \frac{E^1}{|N^1|} = 3$;

For all $i \in N^1$, we see $c_i^1 \geq \beta^1$. So Algorithm stops and set $y_i = \beta^1 = 3$ for all $i \in N^1$;

Step 6: Algorithm ends and generates the GEA allocations of four individuals which are the addition of first-stage and second-stage distributions generated during the algorithm.

$\lambda = \lambda_{j^*} + \beta^1 = 5 + 3 = 8$ and $f^{GEA}(\rho) = \left(x_1 + y^1, x_2, x_3 + y^1, x_4\right) = (6, 1, 3, 0)$

The left part of figure(l) shows the second-stage allocation of the GEA rule. We see there are only two agents left and each of them equally receives 3 units of the remaining available GHGs emissions. The right part of figure (l) graphically shows the final total allocations of each individual. The red-stacked bar of individual 1 and 2 represents their first-stage allocations. The yellow-stacked bar of individual 1 and 3 are their second-stage allocations. The individual 4 with highest historical emissions did not receive any emissions.



(l) Second-stage Allocation-Algorithm of the GEA rule .

5 Characteristics of the Generalized Equal Awards Rule

This section presents some relevant characteristics of the GEA rule since these characteristics are essential when it comes to the formal proof of $f^{GEA}(\rho) + h$ being Lorenz undominated by any other rules in carbon emission distribution.

In order to conduct Lorenz domination comparison, we consider the allocation of individuals rather than countries. As discussed before, we restrict our attention to the Unit Past-Dependent Problem denoted by $\rho = (N, h, c, E)$ where the population is vacuous since the agent is now every individual. Let $\rho \in \mathscr{P}$, the GEA rule gives solution $f^{GEA}(\rho)$ to any problem $\rho \in \mathscr{P}$.

The first outstanding characteristic of the GEA rule states that when the final allocations of an agent is less than that of another agent, it is either because this agent has been allocated current-future emissions up to its full claim or because the other agent can not receive any positive current-future emissions. Firstly named and proved by Timoner and Izquierdo(2016), this characteristic can be mathematically stated by the following proposition.

Proposition 5-1 *Let $\rho = (N, h, c, E) \in \mathscr{P}$ and $x \in D(\rho)$. The following statements are equivalent:*

(1). $x_i = f_i^{GEA}(\rho)$.

(2). *For all $i, j \in N$ with $i \neq j$, if $x_i + h_i < x_j + h_j$, then either $x_i = c_i$ or $x_j = 0$.*

Proof: $1 \Rightarrow 2$) Suppose $\forall i \in N$, $x_i = f_i^{GEA}(\rho)$ and $0 \leq x_i \leq c_i$. Assume $\exists\, i, j \in N$ such that $x_i + h_i < x_j + h_j$, but $x_i < c_i$ and $x_j > 0$. With $x_j = \min\left\{c_j, (\lambda - h_j)_+\right\} > 0$, so $\lambda - h_j > 0$. With $x_i = \min\left\{c_i, (\lambda - h_i)_+\right\} < c_i$, so $x_i = (\lambda - h_i)_+$. Hence,
$x_i + h_i = (\lambda - h_i)_+ + h_i \geq \lambda \geq \min\{c_j + h_j, \lambda\} = \min\{c_j, (\lambda - h_j)\} + h_j = \min\left\{c_j, (\lambda - h_j)_+\right\}$ $+h_j = x_j + h_j$. Here, a contradiction has been reached since the hypothesis is $x_i + h_i < x_j + h_j$. Therefore, we conclude either $x_i = c_i$ or $x_j = 0$.

$2 \Rightarrow 1$) We begin by assuming that (2) of Proposition 4-1 holds, but $x \neq f^{GEA}(\rho)$. Then by efficiency, there exist $i, j \in N$ such that $0 \leq x_i < f_i^{GEA}(\rho) \leq c_i$ and $c_j \geq x_j > f_j^{GEA}(\rho) \geq 0$. Which means $x_i < c_i$, $\lambda - h_i \geq 0$, $x_j > 0$ and $(\lambda - h_j)_+ < c_j$. However,
$x_j + h_j > f_j^{GEA}(\rho) + h_j = (\lambda - h_j)_+ + h_j \geq \lambda \geq \min\{c_i + h_i, \lambda\}$
$= \min\{c_i, (\lambda - h_i)_+\} + h_i = f_i^{GEA}(\rho) + h_i > x_i + h_i$
So we get when $x \neq f^{GEA}(\rho)$, $x_i + h_i < x_j + h_j$ with $x_i < c_i$ and $x_j > 0$. This contradicts (2) of Proposition 5-1. Therefore, we conclude $x = f^{GEA}(\rho)$. $\square$

The second characteristic of the GEA rule states that when there exists an allocation giving all agents exactly same amount of final GHGs emissions(the rule-determined emission permits and the historical emissions), the allocation must be $f^{GEA}(\rho)+h$. Mathematically, the proposition is expressed as:

Proposition 5-2 *Let $\rho=(N,h,c,E)\in\mathscr{P}$, if there exists $x\in D(\rho)$ such that $\forall i,j\in N$ and $i\neq j$, $x_i+h_i=x_j+h_j$, then $x=f^{GEA}(\rho)$.*

Proof: Suppose $x\neq f^{GEA}(\rho)\Leftrightarrow x+h\neq f^{GEA}(\rho)+h$. Because both $f^{GEA}(\rho)$ and x satisfy efficiency, then $\exists i,j\in N$ with $i\neq j$ such that $f_i^{GEA}(\rho)+h_i<x_i+h_i=x_j+h_j<f_j^{GEA}(\rho)+h_j$. By Proposition 5.1, so either $f_i^{GEA}(\rho)=c_i$ or $f_j^{GEA}(\rho)=0$.
Case 1) if $f_i^{GEA}(\rho)=c_i$, with $x_i+h_i>f_i^{GEA}(\rho)+h_i$, then $x_i>c_i$, we reach a contradiction with $x\in D$.
Case 2) if $f_j^{GEA}(\rho)=0$, with $x_j+h_j<f_j^{GEA}+h_j$, then $x_j<0$, we reach a contradiction with $x\in D$. Therefore, we conclude $x=f^{GEA}(\rho)$. $\square$

The next feature of the GEA rule is about when any two different agents own same historical emissions. If the GEA rule allocates more emission permits to an agent than to another agent when they possessed equal amount of historical emissions, it is because the agent with less GEA allocations has reached its full emission claim. Mathematically,

Proposition 5-3 *Let $\rho=(N,h,c,E)\in\mathscr{P}$ and suppose $x=f^{GEA}(\rho)$, $\forall\ i,j\in N$ with $i\neq j$, if $h_i=h_j$ and $x_i+h_i<x_j+h_j$. Then $x_i=c_i$.*

Proof: if $x_i\neq c_i$, by Proposition 5.1, then $x_j=0$. Then $x_j+h_j=x_j+h_i=0+h_i>x_i+h_i$. Therefore, $x_i<0$ we reach a contradiction with $x\in D(\rho)$. So $x_i=c_i$. $\square$

The last observation of the GEA allocation is related to the agent with least historical emissions. As long as the available emissions, $E-\overline{h}$, is positive, the agent receives positive GEA allocations. Mathematically,

Proposition 5-4 *Let $\rho=(N,h,c,E)\in\mathscr{P}$, if $h_j=\min\limits_{i\in N}\{h_i\}$ and $E-\bar{h}>0$, then $f_j^{GEA}(\rho)>0$.*

Proof: If $f_j^{GEA}(\rho)=\min\left\{c_j,(\lambda-h_j)_+\right\}=0$, then $\lambda\leq h_j=\min\limits_{i\in N}\{h_i\}$. Then $\forall i\in N$, $f_i^{GEA}(\rho)=0$. $\sum\limits_{i\in N}f_i^{GEA}(\rho)=E-\bar{h}=0$, we reach a contradiction with $E-\bar{h}>0$. Hence, we conclude $f_j^{GEA}(\rho)>0$. $\square$

6 The Evaluation Procedure: Equity and Fairness

The prevention of global warming relies on efforts of all districts and countries. Although nearly all countries do not avoid their responsibilities and have made many attempts in reducing the total carbon emissions by creating multilateral agreements, all signed agreements unfortunately are not sustainable in promoting multilateral cooperation among countries.

Much has been discussed about the failure of a sustainable and successful international agreement on the abatement of global carbon emissions. The most believing fact is these multilateral agreements violate equity and fairness criteria. For example, Zhang et al.(2017)pointed out the withdrawal of US from Paris Agreement was due to its inequity in a sense that countries like India and China are free to use fossil while US has to reduce carbon emissions.

This paper endeavors to search for fairness and justice in distributing GHGs emissions among agents so as to promote sustainable multilateral agreements. Previous study by Otsuki(1996) proposed some criteria for distributive equity and justice such as egalitarian equivalence, freedom from envy and so on. The most popular and widely accepted criteria of justice in allocation proposal are **Lorenz domination** and **Gini index**. We therefore introduces Lorenz domination as criteria of seeking fairness among solutions. Reasons are two folds: first Lorenz domination is more widely employed than Gini index. Second, there are numerous scholars such as Bosmans and Lauwers(2011), Thomson(2012) and Giménez-Gómez and Peris (2014) studying the Lorenz dominance comparison among rules. Their conclusions could be references for our study.

In the following subsections, this paper aims to prove two theorems. First, when the number of agents is n with $1 \leq n \leq N$, the final allocation denoted by $f^{GEA}(\rho) + h$ from the GEA rule is Lorenz undominated by any other allocations from all other rules. Second, with 2-agent case, the final allocation denoted by $f^{GEA}(\rho) + h$ Lorenz dominates any other final allocations of all other rules.

To describe the Lorenz domination, we follow the description of Calleja et al.(2021). In addition, we distinguish **weakly Lorenz dominance** and **strictly Lorenz dominance**.

Definition 6: *(Weakly and Strictly Lorenz dominance)* Let $y \in R^n_+$ be the set of positive n-dimensional vectors $y = (y_1, y_2, ..., y_n)$. Denote $\hat{y} = (\hat{y}_1, \hat{y}_2, ..., \hat{y}_n)$ with coordinates from y and being arranged in a non-decreasing order, namely $\hat{y}_1 \leq \hat{y}_2 \leq ... \leq \hat{y}_n$. For any two vectors $y, z \in R^n_+$, if $\sum_{j \in N} y_j = \sum_{j \in N} z_j$, then

(a) we say y **weakly Lorenz dominates** z, $y \succeq_{\mathscr{L}} z$, iff $\sum\limits_{j=1}^{k} \hat{y}_j \geq \sum\limits_{j=1}^{k} \hat{z}_j$ for all $k = [1, 2, ...n]$;

(b) we say y **strictly Lorenz dominates** z, $y \succ_{\mathscr{L}} z$, iff when at least one of these $n-1$ inequalities will be a strict inequality.

6.1 The n-agent Case

This section introduces the first theorem of this paper with the case of n agents where $n > 2$. The theorem says that the final allocation, $f^{GEA}(\rho) + h$, from the GEA rule is an unique allocation which is Lorenz-undominated by any other final allocations from all other rules.

Since the unique Lorenz undominated rule is our GEA rule, the allocation from this rule can not be less egalitarian than allocations from other rules. Therefore, the GEA rule could be regarded as a fair and more desirable solution in carbon distribution. The theorem is mathematically stated as,

Theorem 1: Let $\rho = (N, h, c, E) \in \mathscr{P}$ and let $x \in D(\rho)$. Define a Lorenz undominated set as $UD(\rho) = \left\{x \in D(\rho) : \nexists\ y \in D(\rho) \textit{ such that } y + h \succeq_{\mathscr{L}} x + h\right\}$, then $UD(\rho) = \left\{f^{GEA}(\rho)\right\}$.

Proof: we first prove that $f^{GEA}(\rho) \in UD(\rho)$. Let us recall that for all $i \in N$

$$x_i^* = f_i^{GEA}(\rho) = \min\{c_i,\, (\lambda - h_i)_+\},$$

Therefore, we can split N into three disjoint sets:

$$\begin{aligned} N_1 &= \{k \in N : x_k^* = c_k \leq \lambda - h_k\}, \\ N_2 &= \{k \in N : 0 < x_k^* = \lambda - h_k < c_k\} \text{ and } \cdot \\ N_3 &= \{k \in N : x_k^* = 0 \geq \lambda - h_k\} \end{aligned}$$

Notice that $N_1 \cup N_2 \cup N_3 = N$. Moreover, for all $k \in N_1$, all $k' \in N_2$ and all $k'' \in N_3$,

$$c_k + h_k = x_k^* + h_k \leq \lambda = x_{k'}^* + h_{k'} \leq x_{k''}^* + h_{k''} = h_{k''}. \tag{1}$$

Now, take an arbitrary $y \in D$, $y \neq x^*$ and suppose

$$y + h \ \succeq_{\mathscr{L}}\ x^* + h$$

Since $y \neq x^*$, there exists $i \in N$ such that $y_i < x_i^*$. Notice that, since $y \in D$, either $i \in N_1$ or $i \in N_2$.

We claim $i \notin N_1$; otherwise, $i \in N_1$, but then

$$\sum_{k=1}^{|N_1|} (\widehat{y+h})_k \leq \sum_{k \in N_1} y_k + h_k < \sum_{k \in N_1} c_k + h_k = \sum_{k \in N_1} x_k^* + h_k = \sum_{k=1}^{|N_1|} (\widehat{x^* + h})_k,$$

where the equality follows by (1). But this implies that $y + h \not\succeq^{\mathscr{L}} x^* + h$ and contradicts our assumption. Hence, the claim is proved and we conclude that

$$y_k = x_k^*, \text{ for all } k \in N_1. \tag{2}$$

If $i \in N_2$, then we define $N_2^* = \{k \in N_2 : y_k < x_k^* = \lambda - h_k\}$. Notice that for all $N_1 \cup N_2^*$

$$y_k + h_k \leq x^* + h_k \leq \lambda \tag{3}$$

Moreover, for all $k \in (N_2 \setminus N_2^*) \cup N_3$ it holds $x_k \geq x_k^*$ and thus

$$y_k + h_k \geq x_k^* + h_k \geq \lambda \tag{4}$$

By (1), (3) and (4) we have

$$\begin{aligned}
\sum_{k=1}^{|N_1 \cup N_2^*|} (\widehat{y+h})_k \quad &\leq \sum_{k \in N_1 \cup N_2^*} y_k + h_k = \sum_{k \in N_1} y_k + h_k + \sum_{k \in N_2^*} y_k + h_k \\
&< \sum_{k \in N_1} c_k + h_k + \sum_{k \in N_2^*} x_k^* + h_k = \sum_{k \in N_1 \cup N_2^*} x_k^* + h_k \\
&= \sum_{k=1}^{|N_1 \cup N_2^*|} (\widehat{x^* + h})_k.
\end{aligned}$$

But this implies again that $y + h \not\succeq^{\mathscr{L}} x^* + h$ and contradicts our assumption. We conclude that $x^* + h$ cannot be Lorenz dominated.

Now it remains to prove that any other solution can be Lorenz dominated.

W.l.o.g, we assume the solution is $x + h = [x_1 + h_1, x_2 + h_2, ..., x_n + h_n]$ with coordinates being in ascending order which means $x_1 + h_1 \leq x_2 + h_2 \leq, ..., \leq x_n + h_n$. With $x \neq f^{GEA}(\rho)$, there must exist $i, j \in N$ such that $x_i + h_i < x_j + h_j$, $x_i < c_i$ and $x_j > 0$.

Suppose $j = i+1$, then $x_i + h_i < x_{i+1} + h_{i+1}$. We define $y \in R^n_+$ such that $y_k = x_k$ with $k \neq (i, i+1)$, $y_i = x_i + \varepsilon$ and $y_{i+1} = x_{i+1} - \varepsilon$. $\varepsilon \in (0, \min(c_i - x_i, x_{i+1}, x_{i+1} + h_{i+1} - (x_i + h_i))]$.

When $k = [1, 2, ..., i-1]$; since $y_k = x_k$, so we have $\sum\limits_{r=1}^{k} (x_r + h_r) = \sum\limits_{r=1}^{k} (y_r + h_r)$.

When $k = i$; $\sum\limits_{r=1}^{i} (x_r + h_r) = \sum\limits_{r=1}^{i-1} (x_r + h_r) + (x_i + h_i) < \sum\limits_{r=1}^{i-1} (y_r + h_r) + (y_i + h_i) = \sum\limits_{r=1}^{i} (y_r + h_r)$.

When $i + 1 \leq k < n$; $\sum\limits_{r=1}^{k} (x_r + h_r) = \sum\limits_{r=1}^{i-1} (x_r + h_r) + (x_i + h_i) + (x_{i+1} + h_{i+1})$

$$= \sum_{r=1}^{i-1} (y_r + h_r) + (y_i + h_i) + (y_{i+1} + h_{i+1})$$

$$= \sum_{r=1}^{k} (y_r + h_r).$$

When $k = n$; $\sum\limits_{r=1}^{k} (x^*_r + h_r) = \sum\limits_{r=1}^{k} (y_r + h_r) = E$.

Therefore, when $x \neq f^{GEA}(\rho)$, another allocation $y \in D(\rho)$ exits such that $y + h \succ_{\mathscr{L}} x + h$. $\square$

6.2 The 2-agent Case

This section introduces the second theorem of this paper with the case of 2-agent. The theorem states that the final allocation from the GEA rule Lorenz dominates any other allocations from all other rules.

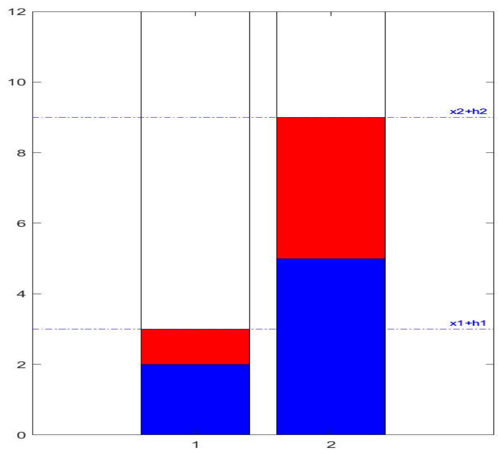
In this case, the GEA rule gives solution which Lorenze dominates any other solutions. Distribution egalitarinism is realized in the GEA rule due to the fairness and justice inhabited in allocation from this GEA rule. Mathematically,

Theorem 2: *Let $\rho = (N, h, c, E) \in \mathscr{P}$, and let $x \in D(\rho)$. With $N = 2$, the following statements are equivalent:*

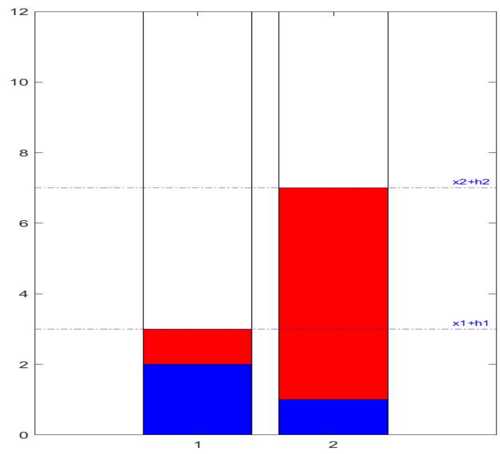
1. $x^* = f^{GEA}(\rho)$
2. $x^* + h \succ_{\mathscr{L}} x + h$ for all other $x \in D(\rho)$

Proof: W.l.o.g, we assume that $x_1 + h_1 \leq x_2 + h_2$. Depending on value of $x_1 + h_1$ and h_2, we distinguish the following two cases:

Case ① $h_2 > x_1 + h_1$



Case ② $h_2 \leq x_1 + h_1$



Based on these two cases, We define a new problem $\rho'(N', h', c', t')$ with $N' = \{1, 2\}$, the new problem has same agents with the old problem. The historical emission and the new claim vectors are $h' = (x_1 + h_1, \max\{h_2, x_1 + h_1\})$ and $c' = (c_1{}', c_2') = (c_1 - x_1, \min\{c_2, c_2 - (x_1 + h_1 - h_2)\})$ respectively. The targeted emissions is denoted by $t' = \min\{x_2, x_2 + h_2 - (x_1 + h_1)\}$.

With the new problem, apply GEA rule and obtain the allocations $(f_1^{GEA}(\rho'), f_2^{GEA}(\rho'))$. We construct a new allocation $y = (y_1, y_2) = (x_1 + f_1^{GEA}(\rho'), \max\{0, x_1 + h_1 - h_2\} + f_2^{GEA}(\rho'))$.

Assume $x \in D(\rho)$ is an arbitrary allocation different from $f^{GEA}(\rho)$. We first prove:

$$y + h \succ_{\mathscr{L}} x + h.$$

Since $f^{GEA}(\rho) + h \neq x + h$, then $t = E - \bar{h} > 0$. $h_2' = \max(h_2, x_1 + h_1) \geq x_1 + h_1 = h_1'$, and $t' > 0$, by Proposition 5-4, $f_1^{GEA}(\rho') > 0$. Then, $y_1 + h_1 = x_1 + f_1^{GEA}(\rho') > x_1 + h_1$. By efficiency, $y_1 + h_1 + y_2 + h_2 = x_1 + h_1 + x_2 + h_2$. Therefore,

$$y + h \succ_{\mathscr{L}} x + h.$$

Since we have proved $y + h \succ_{\mathscr{L}} x + h$, it only remains to prove

$$y = f^{GEA}(\rho).$$

Case 1): if $y_1 + h_1 = y_2 + h_2$, then by Proposition 4-2, any allocation with all coordinates being exactly equal is allocation of GEA, we have $y = f^{GEA}(\rho)$;

Case 2): if $y_1 + h_1 < y_2 + h_2$, we also distinguish two subcases $h_2 > x_1 + h_1$ and $h_2 \leq x_1 + h_1$.

Sub-case 1) if $h_2 > x_1 + h_1$, then $y_2 = \max\{0, (x_1 + h_1 - h_2)\} + f_2^{GEA}(\rho') = 0 + f_2^{GEA}(\rho')$. And $h_2' = \max\{h_2, x_1 + h_1\} = h_2$, then $y_1 + h_1 < y_2 + h_2$ equals to

$$\boldsymbol{h_1' + f_1^{GEA}(\rho')} = x_1 + f_1^{GEA}(\rho') + h_1 = y_1 + h_1 < y_2 + h_2 = f_2^{GEA}(\rho') + h_2'.$$

According to proposition 5-1, either $f_1^{GEA}(\rho') = c_1'$ or $f_2^{GEA}(\rho') = 0$. If $f_1^{GEA}(\rho') = c_1'$, then $y_1 = x_1 + c_1' = c_1$. Then, by Proposition 5-1, $y = f^{GEA}(\rho)$; If $f_1^{GEA}(\rho') < c'_1$, by Proposition 5-1, $f_2^{GEA}(\rho') = 0$ and $y_2 = 0 + f_2^{GEA}(\rho') = 0$. With our assumption $y_1 + h_1 < y_2 + h_2$ and by Proposition 5-1, we have $y = f^{GEA}(\rho)$.

Sub-case 2) if $h_2 \leq x_1 + h_1$, then $h'_2 = \max\{h_2, x_1 + h_1\} = h'_1$;
$y_2 = \max\{0, (x_1 + h_1 - h_2)\} + f_2^{GEA}(\rho') = x_1 + h_1 - h_2 + f_2^{GEA}(\rho')$. So $y_1 + h_1 < y_2 + h_2$ equals to $x_1 + h_1 + f_2^{GEA}(\rho') - h_2 + h_2 = \boldsymbol{h_2' + f_2^{GEA}(\rho') > h_1' + f_1^{GEA}(\rho')} = x_1 + h_1 + f_1^{GEA}(\rho')$. Apply Proposition 5-3, then $f_1^{GEA}(\rho') = c_1'$. So $y_1 = x_1 + f_1^{GEA}(\rho') = x_1 + c_1' = x_1 + c_1 - x_1 = c_1$. With $y_1 + h_1 < y_2 + h_2$ and $y_1 = c_1$, by proposition 5-1, we have $y = f^{GEA}(\rho)$.

Case 3): $y_1 + h_1 > y_2 + h_2$, we prove this case is impossible.
Sub-case 1) if $h_2 > x_1 + h_1$, then $y_2 = \max\{0, x_1 + h_1 - h_2\} + f_2^{GEA}(\rho') = f_2^{GEA}(\rho')$ and $h_2' = \max(h_2, x_1 + h_1) = h_2 > h_1' = x_1 + h_1$. Then, $y_1 + h_1 > y_2 + h_2$ equals to $x_1 + h_1 + f_1^{GEA}(\rho') > h_2 + f_2^{GEA}(\rho') \Leftrightarrow \boldsymbol{h_1' + f_1^{GEA}(\rho') > h_2' + f_2^{GEA}(\rho')}$. By Proposition 5-1 and $f_1^{GEA}(\rho') > 0$, we have $f_2^{GEA}(\rho') = c_2' = \min\{c_2, c_2 - (x_1 + h_1 - h_2)\} = c_2$.
Because $t' = \min\{x_2, x_2 + h_2 - (x_1 + h_1)\} = x_2 \leq c_2$, so $f_2^{GEA}(\rho') = c_2 \geq t'$, which implies $f_2^{GEA}(\rho') = t'$. By efficiency, $f_1^{GEA}(\rho') = 0$. Therefore, we reach a contradiction with $f_1^{GEA}(\rho') > 0$.

Sub-case 2) if $h_2 \leq x_1 + h_1$, then,
$y_2 = \max\{0, x_1 + h_1 - h_2\} + f_2^{GEA}(\rho') = x_1 + h_1 - h_2 + f_2^{GEA}(\rho')$.
$y_1 + h_1 > y_2 + h_2 \Leftrightarrow x_1 + h_1 + f_1^{GEA}(\rho') > x_1 + h_1 - h_2 + f_2^{GEA}(\rho')$. With $h_1' = x_1 + h_1$ and $h_2' = x_1 + h_1 - h_2$. So we have $\boldsymbol{h_1' + f_1^{GEA}(\rho') > h_2' + f_2^{GEA}(\rho')}$. With $f_1^{GEA} > 0$ and by proposition 5-1, $f_2^{GEA}(\rho') = c_2 \geq t'$, which implies $f_2^{GEA}(\rho') = t'$. By efficiency, $f_1^{GEA}(\rho') = 0$. Therefore, we reach a contradiction with $f_1^{GEA}(\rho') > 0$. □

It is confirmed that the final allocation from the GEA rule Lorenz dominates any other allocations in 2-agent problem. We have to admitted that it is meaningful and feasible to generalize the above proof to problem of n agents. However, due to limited time, this paper will have to leave the proof as future work.

7 Conclusion

To control the trend of global warming, governors and scholars reach a censuses in reducing carbon emissions which is regarded as the major source of world temperature increase. The abatement of GHGs emissions relies on multilateral cooperation of all countries and districts. However, many multilateral agreements failed due to unfair carbon distribution rules.

This paper follows previous studies which framed the distribution of available carbon emissions as rationing problem and make extensions as well as improvements in defining appealing solutions for CO_2 emission distribution. Specifically, this paper firstly pointed out drawbacks of the four existing rules created by Ju et al.(2021). For example, the Equal per emission rule and the Equal per capita rule does not consider the historical emissions of countries. Moreover, the Equal per capita rule is in violation of Claim-boundedness which might give a country more emission permits than its needs. Though the Historical equal per emission rule and the Historical equal per capita rule take historical emissions into consideration, these two rules are in violation of Non-negativity which means countries might get negative emission permits. Negative emission permits is detrimental to sustainability of multilateral agreement in a way that it drives countries to drop out multilateral agreement. In addition, these rules are based on country agents. According to egalitarianism, individuals instead of countries should be treated equally. When countries are treated as agents, it is impossible to make Lorenz domination comparison so as to find a fair and egalitarian rule in perspective of individuals.

In view of drawbacks of existing rules, this paper inventively designs the Generalized Constraint Equal Awards rule where individuals are treated as agents. The GEA rule not only takes historical emissions of every individual into consideration but also satisfies Claim-boundedness and Non-negativity. Then, this paper proposes a corresponding algorithm to compute allocations from the GEA rule. To distinguish different rules in terms of distribution fairness and justice, this paper also introduces Lorenz dominance and proved under the case of n-agent problem with $n > 2$ that allocations from the GEA rule is Lorenz undominated by any other allocations from all other rules. With the case of 2-agent, this paper concludes that allocations from the GEA rule Lorenz dominates any other allocations from all other rules.

It is meaningful and practical to prove that allocations from the GEA rule Lorenz dominates any other allocations from all other rules even under the case of n-agent problem where n is greater than 2. However, due to limited time, this paper will need to leave this as future research work.

8 Appendices

8.1 Appendix A: Proof from Algorithm of the GEA rule

We prove when $\lambda = \lambda_{j^*} + \beta^{k^*}$ then $f_i^{GEA}(\rho) = \begin{cases} x_i & \forall i \in N_0 \cup N_c \\ x_i + y_i & \forall i \in N|_{N_0 \cup N_c} \end{cases}$.

Proof: for all $i \in N_0$, with $\lambda = h_{j^*} + \beta^{k^*} < h_{j^*+1}$, we have $h_i \geq \lambda$, therefore $f_i^{GEA}(\rho) = \min\left\{c_i, (\lambda - h_i)_+\right\} = 0 = x_i$; for all $1 \leq i \leq j^*$, we distinguish two cases:

Case 1) if $h_{j^*} > c_i + h_i$, then $h_{j^*} + \beta^{k^*} > c_i + h_i$, $\Rightarrow h_{j^*} + \beta^{k^*} - h_i = \lambda - h_i > c_i$, so $f_i^{GEA}(\rho) = \min\left\{c_i, (\lambda - h_i)_+\right\} = \min\left\{c_i, (h_{j^*} + \beta^{k^*} - h_i)_+\right\} = \min\left\{c_i, (h_{j^*} - h_i)_+\right\} = c_i = x_i$.

Case 2) if $h_{j^*} \leq c_i + h_i$, with $h_{j^*} \geq h_i$.

$f_i^{GEA}(\rho) = \min\left\{c_i, (\lambda - h_i)_+\right\} = \min\left\{c_i, (h_{j^*} + \beta^{k^*} - h_i)_+\right\} + \underbrace{\min\left\{c_i + h_i - h_{j^*}, 0\right\}}_{0}$

$= \min\left\{c_i, (h_{j^*} + \beta^{k^*} - h_i)\right\} + \min\left\{c_i, (h_{j^*} - h_i)\right\} - \left\{h_{j^*} - h_i\right\}$

$= \min\left\{c_i + h_i - h_{j^*}, \beta^{k^*})\right\} + \min\left\{c_i, (h_{j^*} - h_i)_+\right\}$, with $c' = c_i + h_i - h_{j^*} \forall 1 \leq i \leq j^*$,

$= \min\left\{c', \beta^{k^*}\right\} + \min\left\{c_i, (\lambda_{j^*} - h_i)_+\right\} = x_i + y_i$. $\square$

8.2 Appendix B: Matlab Code of the Algorithm for the GEA rule

We provide the following matlab code to compute allocations from the GEA rules. With the code and after inputting the number of agents n, the final allocation of the GEA rule is simulated.

The initial parameters

```
clear;
n = input('n');
h = zeros(1,n);
c = zeros(1,n);
c_total = 0;
h_total = 0;
for i=0:(n-1)
    m_2=randi([1,10],1,1);
    h(i+1)=[m_2];
    h_total=h_total+h(i+1);
    i=i+1;
end
h_order=sort(h, "ascend");

for i=0:(n-1)
    m_1=randi([1,10],1,1);
    c(i+1)=[m_1];
    c_total=c_total+c(i+1);
    i=i+1;
end

c_h_order=c+h_order;
E=randi([h_total,h_total+c_total-1],1,1);
E_h=E-h_total
```

Get all lambdas

```
lamda=zeros(1,n+1);
for i=1:n
    lamda(i)=h_order(i);
end
lamda(1,n+1)=max(c_h_order);      % obtain all lamdas of our algorithm
```

Obtain all t(i)

```
t=zeros(1,n+1);
z=0;
for i=1:n+1
    for j=1:n
        z=z+find_min(c(j),lamda(i),h_order(j));
        j=j+1;
    end
    t(i)=z;
    z=0;
    i=i+1;
end
```

Get lamda_j_star where t_j_star < E-h < t_j_star_plus_1

```
for i=1:n
    if t(i)<=(E_h) && t(i+1)>(E_h)
        j_star=i;
        lamda_j_star=h_order(i);
    else
        i=i+1;
    end
end
```

Obtain first-stage allocation

```
for i=1:n
x(i)=find_min(c(i),lamda_j_star,h_order(i));
end
```

Define the NEW Rationing problem

```
E_1=E_h-sum(x);
for i=1:n
   c_1(i)=min(c_h_order(i)-lamda_j_star,c(i));
end
for i=j_star+1:n
   c_1(i)=0;
   i=i+1;
end
for i=1:n
   while c_1(i)<0
      c_1(i)=0;
   end
   i=i+1;
end
c_2=c_1; c_1(find(c_1==0))=[];
n_c_1=numel(c_1 );
```

Obtain CEA allocation at second stage

```
CEA=zeros(1,n);
z=0;
lamda_CEA(1)=0;
max_lamda_CEA=max(c_2);
while sum(CEA)<=E_1
    z=z+1;
         for i=1:n
            CEA(i)=min(c_2(i),lamda_CEA(z));
            i=i+1;
         end
num_f=sum(CEA(:)==c_2(:));
lamda_CEA(z+1)=lamda_CEA(z)+(E_1-sum(CEA))/(n-num_f);
        if lamda_CEA(z+1)==lamda_CEA(z)
        break
        end
end
```

Output first-stage allocation, sec-stage allocation and final GEA allocation

```
clearvars -except h_order c E_h lamda t lamda_j_star x
CEA
Final_Allo=[h_order;x;CEA;c-CEA-x];
Final_Allo=Final_Allo';
Final_bar=bar(Final_Allo,'stacked');
set(Final_bar(1),'facecolor', 'blue');
set(Final_bar(2),'facecolor', 'red');
set(Final_bar(3),'facecolor', 'yellow');
set(Final_bar(4),'facecolor', 'none');
```

8.3 Appendix C: Matlab Code of the Example in Chapter 4

At chapter 4 subsection 4.3, there is an example which this paper used to illustrate the proceeding of the algorithm corresponding to the GEA rule. The following code shows how this paper gets final GEA allocations from the GEA rule with help of Matlab.

The initial parameters

```
c=[8,1,8,5];
h_order=[2,4,5,10];
c_h_order=c+h_order;
E_h=10;
```

The lambda vector

```
lamda=zeros(1,5);
for i=1:4
    lamda(i)=h_order(i);
end
lamda(1,5)=max(c_h_order);          % obtain all lamdas of our algorithm
```

Obtain all t(i)

```
t=zeros(1,5);
z=0;
for i=1:5
    for j=1:4
        z=z+find_min(c(j),lamda(i),h_order(j));
        j=j+1;
    end
     t(i)=z;
      z=0;
     i=i+1;
end
```

Get lamda_j_star where t_j_star < E-h < t_j_star_plus_1

```
for i=1:4
    if t(i)<=(E_h) && t(i+1)>(E_h)
        j_star=i;
        lamda_j_star=h_order(i);
    else
        i=i+1;
    end
end
```

Obtain first-stage allocation

```
for i=1:4
x(i)=find_min(c(i),lamda_j_star,h_order(i));
end
```

Define the NEW rationing problem

```
E_1=E_h-sum(x);
for i=1:3              % j_star==3
    c_1(i)=min(c_h_order(i)-lamda_j_star,c(i));
```

```
end
  % delete players No and players Nc
c_1(find(c_1==0))=[];
n_c_1=numel(c_1);
```

Obtain allocation at second stage

```
CEA=zeros(1,n_c_1);
z=0;
lamda_CEA(1)=0;
max_lamda_CEA=max(c_1);
while sum(CEA)<=E_1
    z=z+1;
        for i=1:n_c_1
            CEA(i)=min(c_1(i),lamda_CEA(z));
            i=i+1;
        end
num_f=sum(CEA(:)==c_1(:));
lamda_CEA(z+1)=lamda_CEA(z)+(E_1-
sum(CEA))/(4-num_f);
       if lamda_CEA(z+1)==lamda_CEA(z)
       break
       end
end
lamda_GEA=lamda_j_star+lamda_CEA(z);
```

Plot first figure

```
subplot(2,2,1)
A=reshape([h_order;c], 1, []);
B=reshape(A,[2,4]);
B_1=B';
N=(1:4);
%
bar_1=bar(B_1,'stacked');
set(bar_1(1),'facecolor','blue')
set(bar_1(2),'facecolor','none')
set(gca,'XLim',[0 7]);
ylim([0,18]);
set(gca,'YTick',[0:2:18]);
label = {'\lambda_1=2    t_1=0'
   ...
   '\lambda_2=4  t_2=2'
   ...
   '\lambda_3=5  t_3=4'
   ...
   '\lambda_4=10    t_4=14'
   '\lambda_5=15     t_5=22'};
yline(lamda,'-.b',label,'FontSize',8)
hold on
```

Second figure

```
subplot(2,2,2)
c_2(1)=c_1(1);
c_2(2)=0;
c_2(3)=c_1(2);
c_2(4)=c(4);
B_1=[h_order;x;c_2]';
bar_1=bar(B_1,'stacked');
set(bar_1(1),'facecolor','blue')
```

```
set(bar_1(2),'facecolor','r')
set(bar_1(3),'facecolor','none')
set(gca,'XLim',[0 7]);
ylim([0,18]);
```

```
set(gca,'YTick',[0:2:18]);
yline(lamda_j_star,'-
.b','\lambda_j*=\lambda_3=5','FontSize',8 )
hold on
```

The third figure

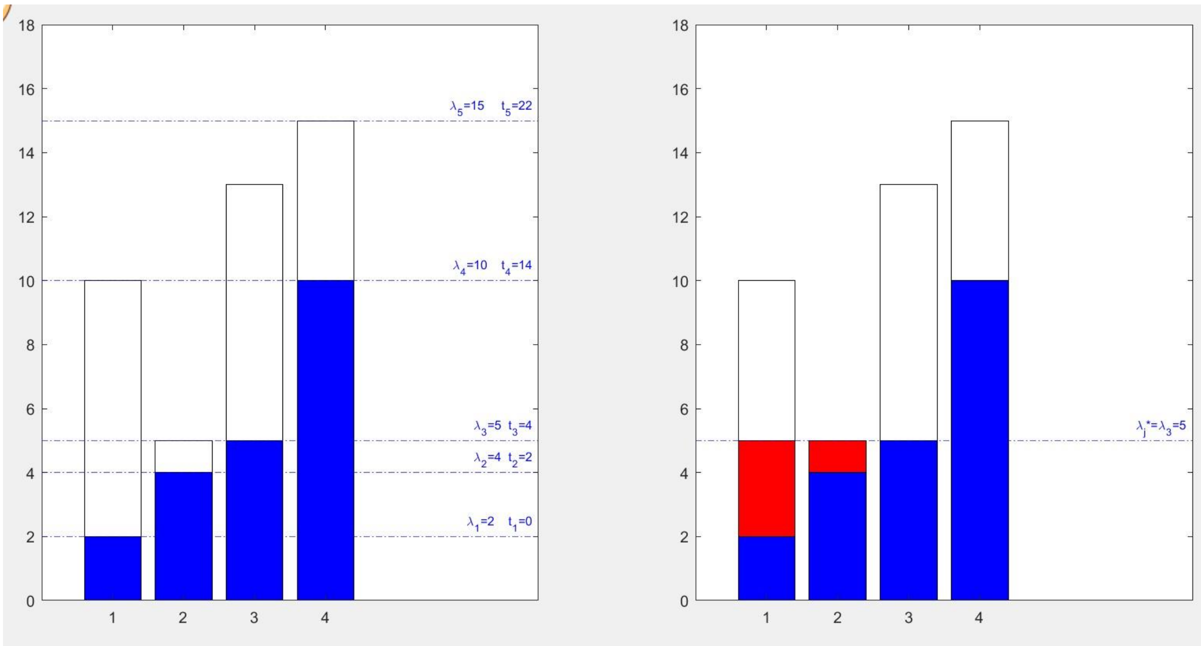
```
subplot(2,2,3)
N_CEA=(1:1:3);
CEA_section=zeros(1,2);
for i=1:2
   CEA_section(i)=lamda_CEA(z);
   i=i+1;
end
cea_matrix_3=c_1-CEA_section;
cea_matrix_0=[CEA_section;cea_matrix_3];
cea_matrix_1=cea_matrix_0';
aux=[0,0];
```

```
cea_matrix_2=[cea_matrix_1(1,:); aux;
cea_matrix_1(2,:); aux];
cea_bar=bar(cea_matrix_2,'stacked');
set(cea_bar(1),'facecolor', 'yellow');
set(cea_bar(2),'facecolor', 'none');
yline(lamda_CEA(z),'-.b','{\lambda _{CEA}}
= 3','FontSize',8)
set(gca,'XLim',[0 7]);
ylim([0,18]);
set(gca,'YTick',[0:2:18]);
hold on
```

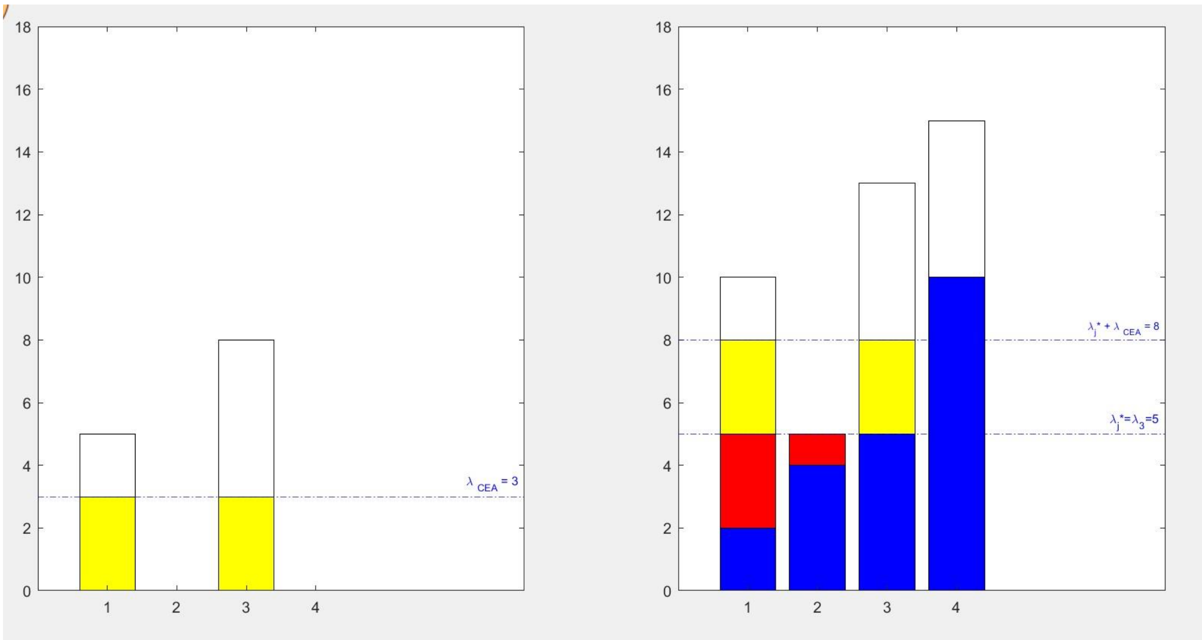
The final figure of first and second stage allocation of GEA rule

```
subplot(2,2,4)
CEA(1)=3;
CEA(2)=0;
CEA(3)=3;
CEA(4)=0;
cea_matrix_3(4)=c(4);
cea_matrix_3(2)=0;
cea_matrix_3(3)=c(3)-CEA(3);
final_matrix_0=[h_order;x;CEA;cea_matrix_3
];
final_matrix_1=final_matrix_0';
final_bar=bar(final_matrix_1,'stacked');
```

```
set(final_bar(1), 'facecolor', 'blue');
set(final_bar(2), 'facecolor', 'r');
set(final_bar(3), 'facecolor', 'y');
set(final_bar(4), 'facecolor', 'none');
yline(lamda_j_star,'-
.b','\lambda_j*=\lambda_3=5','FontSize',8)
yline(lamda_GEA,'-.b','{\lambda_j*} +
{\lambda _{CEA}} = 8','FontSize',7)
set(gca,'XLim',[0 7]);
ylim([0,18]);
set(gca,'YTick',[0:2:18]);
hold off
```



λ_5=15 t_5=22
λ_4=10 t_4=14
λ_3=5 t_3=4
λ_2=4 t_2=2
λ_1=2 t_1=0
λ_j*=λ_3=5



λ_{CEA} = 3
λ_j* + λ_{CEA} = 8
λ_j*=λ_3=5

References

Baatz, C., & Ott, K. (2017). In defense of emissions egalitarianism. *Climate justice and historical emissions*, 1–28.

Baer, P. (2002). Equity, greenhouse gas emissions, and global common resources. *Climate change policy: A survey*, 393–408.

Bosmans, K., & Lauwers, L. (2011). Lorenz comparisons of nine rules for the adjudication of conflicting claims. *International Journal of Game Theory*, *40*(4), 791–807.

Broome, J. (2012). *Climate matters: Ethics in a warming world (norton global ethics series)*. WW Norton & Company.

Calleja, P., Llerena, F., & Sudhölter, P. (2021). Constrained welfare egalitarianism in surplus-sharing problems. *Mathematical Social Sciences*, *109*, 45–51.

Curiel, I. J., Maschler, M., & Tijs, S. H. (1987). Bankruptcy games. *Zeitschrift für operations research*, *31*(5), A143–A159.

Duro, J. A., Giménez-Gómez, J.-M., & Vilella, C. (2020). The allocation of co2 emissions as a claims problem. *Energy Economics*, *86*, 104652.

Giménez-Gómez, J.-M., & Peris, J. E. (2014). A proportional approach to claims problems with a guaranteed minimum. *European Journal of Operational Research*, *232*(1), 109–116.

Giménez-Gómez, J.-M., Teixidó-Figueras, J., & Vilella, C. (2016). The global carbon budget: A conflicting claims problem. *Climatic change*, *136*(3), 693–703.

Gosseries, A. (2005). Cosmopolitan luck egalitarianism and the greenhouse effect. *Canadian Journal of Philosophy*, *35*(sup1), 279–309.

Ju, B.-G., Kim, M., Kim, S., & Moreno-Ternero, J. D. (2021). Fair international protocols for the abatement of ghg emissions. *Energy Economics*, *94*, 105091.

Meinshausen, M., Meinshausen, N., Hare, W., Raper, S. C., Frieler, K., Knutti, R., Frame, D. J., & Allen, M. R. (2009). Greenhouse-gas emission targets for limiting global warming to 2 c. *Nature*, *458*(7242), 1158–1162.

Nakicenovic, N., Alcamo, J., Davis, G., Vries, B. d., Fenhann, J., Gaffin, S., Gregory, K., Grubler, A., Jung, T. Y., Kram, T., et al. (2000). Special report on emissions scenarios.

Neumayer, E. (2000). In defence of historical accountability for greenhouse gas emissions. *Ecological economics*, *33*(2), 185–192.

Otsuki, M. (1996). Criteria for distributive equity: Characterization in terms of share value. *journal of economic theory*, *71*(2), 532–557.

Pan, X., Teng, F., & Wang, G. (2014). Sharing emission space at an equitable basis: Allocation scheme based on the equal cumulative emission per capita principle. *Applied Energy*, *113*, 1810–1818.

Posner, E. A., & Sunstein, C. R. (2009). Should greenhouse gas permits be allocated on a per capita basis. *Calif. L. Rev.*, *97*, 51.

Thomson, W. (2012). Lorenz rankings of rules for the adjudication of conflicting claims. *Economic Theory*, *50*(3), 547–569.

Thomson, W. (2019). *How to divide when there isn't enough*. Cambridge University Press.

Timoner, P., & Izquierdo, J. M. (2016). Rationing problems with ex-ante conditions. *Mathematical Social Sciences*, *79*, 46–52.

Torpman, O. (2019). The case for emissions egalitarianism. *Ethical Theory and Moral Practice*, *22*(3), 749–762.

Zhang, Y.-X., Chao, Q.-C., Zheng, Q.-H., & Huang, L. (2017). The withdrawal of the us from the paris agreement and its impact on global climate change governance. *Advances in Climate Change Research*, *8*(4), 213–219.