Dynamics of Axion Clumps in the Milky Way halo to study Dark Matter

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Abstract: In a wide variety of axion models, a relevant fraction of all Dark Matter (DM) is expected to bound in dense small-scale substructures, called axion miniclusters. Despite they enhance the local DM density, the crossing of miniclusters through Earth are rare events occurring once in 10^5 years making direct detection inefficient. Although dense, a small fraction of these structures become unbound by tidal interaction with astronomical objects, as shown mainly with stars. The perspectives of detection will depend on which fraction of axion minicluster gets disrupted and therefore, tidal interactions are reviewed in detail in this paper. As a result of the destruction, tidal axion streams are expected to form along the minicluster's previous direction in a coherent and large shape. Stream crossings are more probable events so it has been analysed if density may still be larger than the local DM density and therefore, be a real chance in direct detection systems such as haloscopes.

I. INTRODUCTION

The Standard Model (SM) is a well-justified theory that can explain how fundamental particles and electromagnetic, weak and strong nuclear forces are related. This model, however, does not contain a particle that qualifies as dark matter (DM). Dark matter is thought to be about 85% of all the matter in existence so it's pretty clear that the SM falls short of being a complete theory of fundamental interactions. Over the years, multiple candidates have been proposed as DM such as weakly interacting massive particles (WIMPs), sterile neutrinos, or little Higgs models, just to mention a few. But one of the most well-motivated candidates has shown to be the QCD axion.

Under the mathematical formulation of the QCD, charge-parity (CP) symmetry violation could be possible at the level of strong interactions. However, all experiments so far have proven that parity and CP are good symmetries at this level. This unnecessary conservation is known as strong CP problem. The most popular solution was pointed out in 1977 by Peccei and Quinn and consisted of the introduction of a CP violating term in the Lagrangian. This term appeared after a relaxation of the vacuum definition and resulted in a new parameter, θ, which defined the choice of vacuum among an infinity of other possibles values. The solution then was to alleviate the definition issue based on the obligation of the QCD vacuum energy being an absolute minimum at $\theta = 0$. As a result of this assumption, an extra global symmetry was raised in the form of a Goldstone-boson field. The particle associated with this field is the QCD axion, a low-mass 0^- bosonic particle. In addition, the QCD potential gives the axion a mass that is related to the axion decay constant f_A, an energy scale related to the scale of PQ symmetry breaking. This way, the axion mass can be expressed as:

$$m_A \approx 5,7\text{meV}\ \frac{10^9\ \text{GeV}}{f_A} \tag{1}$$

The interest in the PQ solution to the strong CP problem is considering a large decay constant (f_A). The axion, then, represents an excellent cold dark matter candidate (CDM candidate) because a large f_A ensures stability (because of the low axion mass) and the collisionless nature of dark matter (since large f_A ensures low anomalous coupling with gluons and photons). In addition, axions could be produced non-thermally in the early Universe so their velocity dispersion is small enough to be considered as a well-motivated CDM candidate.

Therefore, the axion arises as a pseudo-Goldstone boson of a new U(1) symmetry, the so-called Peccei-Quinn symmetry. This symmetry is broken when the temperature T of the universe drops below the vacuum expectation value, v_{PQ} [1]. The axion field, afterwards, takes on a random displacement between zero and 2π. This value depends on whether the symmetry is restored after inflation (post-inflationary PQ symmetry breaking scenario) or not (pre-inflationary PQ symmetry breaking scenario).

In the pre-inflation scenario, the axion field becomes homogeneous in all the observable Universe since a single casual patch is inflated away. On the contrary, if the symmetry is restored afterwards it leads to a richer context in which our Universe contains many different unconnected patches whitin the horizon coming from different values of the axion field at the time of the phase transition. Consequently, the random distribution of θ leads to different DM densities and, therefore, to overdensities $\delta \equiv \frac{\delta\rho_{DM}}{\rho_{DM}}$ in the axion field. At some point

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the self-gravity of the fluctuation comes to dominate and if $\delta \gtrsim 1$ it separates out from the cosmological expansion. This structure collapses producing gravitationally bound clumps of axions, known as axion miniclusters (AMC) [5].

Despite axion detection is a challenge because the axion's coupling to SM particles is expected to be very low (fitting the definition of dark matter), the formation of miniclusters opens up new possibilities. For instance, transient radio signals could appear in encounters between AMCs and neutron stars [2]. Another example is given by Tinyakov et al. [3], when clump destruction by interaction with the MW population is taken into account. The AMC destruction, then, could leave DM streams with large enough overdensities that can be observed in ground-based detectors in a reasonable scale of time. The aim of this study, therefore, is to study the AMCs destruction in detail.

II. AXION MINICLUSTERS

Modern cosmological models conceive the formation of galaxies, clusters or all large-scale structures through the gravitational instability of small-amplitude seed density fluctuations. Similarly, large-amplitude fluctuation in the dark matter energy density would lead to the development of dark matter substructure. As previously discussed, for overdensities $\delta \gtrsim 1$ the fluctuations collapse starting in the radiation-dominated epoch and produce a dense DM object, the axion minicluster. Analytic calculations of linear perturbations have shown that the growth of small-amplitude fluctuations is limited by the cosmological expansion until after the equality epoch [4]. More precisely, matter fluctuations grow at the same time rate but before the equality epoch there was less time for collapse to proceed. Structure formation, therefore, becomes efficient at redshifts $z < z_{eq} \approx 3400$.

Similar to the standard cosmology, increasingly larger halos are built up from mergers making a hierarchical assembly of collapsed objects during $z_{eq} > z \gtrsim 20$. At $z \lesssim 20$, the structure formation proceeds as in galaxies except for a fraction of the DM is clumped in the form of many small and dense orbiting minihalos. Thus, minihalos are formed at very high redshift and later they merge into galaxies and end up in the Milky Way dark matter halo. We first focus on one individual minicluster and its destruction.

A. Tidal stripping of axion miniclusters

As developed by Kolb et al. [5, 6], assuming the simplest model for a minicluster -a spherical collapse- the final density of the virialized minicluster is:

$$\rho_{mc} \approx 140\delta^3(1+\delta)\rho_{EQ} \approx 7\cdot 10^6 \text{GeV cm}^{-3}\delta^3(1+\delta) \quad (2)$$

$$R_{mc} \approx \frac{3\times 10^7}{\delta(1+\delta)^{1/3}}\left(\frac{M}{10^{-12}M_\odot}\right)^{1/3} \text{km} \quad (3)$$

where the scale of minicluster masses is set by the total mass in axions within the Hubble radius when axion oscillations started, which is around $M_{mc} \sim 10^{-12}M_\odot$.

In the post-inflation scenario, on account of the randomness of the initial conditions over different patches, AMCs are formed with a range of overdensities and masses. About 70% of all axionic dark matter is in miniclusters with $\delta > 1$ but only about 20% is in miniclusters with $\delta \gtrsim 5$ [7]. Therefore, most miniclusters would have $\delta \sim 1$ so, given (3) and taking $M_{mc} \sim 10^{-11}M_\odot$, we can consider a 'typical' minicluster radius as $R_{mc} \sim 0.34$ UA ~ 1 UA.

Mainly, the AMC clumps can be disrupted by the gravitational field of the halo, by the collective disk field and by encounters with individual stars. However, as shown in [8], the destruction by the collective disk field is inefficient and during the typical time of passage through the disk only the interactions with individual stars are important.

The large majority of encounters occur when the separation between the AMC and the stars is many orders of magnitude larger than the typical size of the miniclusters or stars. So we should consider it is well justified to work in the 'distant-tide' approximation, later on, this assumption will be checked.

Interactions with individual stars increase the velocity dispersion of DM particles and reduce the clump's binding energy. A simple way to quantify this increase can be carried out by a simple order-of-magnitude analysis. Consequently, working in the 'distant-tide' approximation, the impulsive velocity perturbation is approximately determined by the tidal acceleration GM_*R_{mc}/b^3 that the minicluster is subjected during the characteristic time of the encounter b/v_{rel}, characterised by the relative velocity v_{rel} and the parameter of impact b:

$$\Delta v \approx \frac{GM_*R_{mc}}{b^3}\cdot\frac{b}{v_{rel}} = \frac{GM_*R_{mc}}{b^2 v_{rel}} \quad (4)$$

The AMC is going to unbind if this Δv is bigger than the velocity dispersion of the gravitational tied AMC, $\sigma_{mc} = (GM_{mc}/R_{mc})^{1/2}$. Rearranging the terms, the condition $\Delta v > \sigma_{mc}$ is expressed as:

$$\Delta v > \sigma_{mc} \rightarrow \frac{GM_*R_{mc}}{b^2 v_{rel}} \gtrsim \left(\frac{GM_{mc}}{R_{mc}}\right)^{1/2} \rightarrow$$

$$b^4 \lesssim \left(\frac{GM_*R_{mc}}{v_{rel}}\right)^2\left(\frac{R_{mc}}{GM_{mc}}\right) = \frac{M_*^2}{v_{rel}^2}\frac{GR_{mc}^3}{M_{mc}} \rightarrow$$

$$b \lesssim \sqrt{\frac{M_*}{v_{rel}}}\left(\frac{GR_{mc}^3}{M_{mc}}\right)^{1/4} = \sqrt{\frac{M_*}{v_{rel}}}\left(\frac{3G}{4\pi\overline{\rho}(\delta)}\right)^{1/4} \quad (5)$$

where $\overline{\rho}(\delta) \equiv 3M_{mc}/(4\pi R_{mc}^3(\delta))$ is the mean density of the minicluster and M_* is the star mass.

However, a more detailed analysis, including how the velocity dispersion (or the binding energy) and the mean distance of the encounter depend on the internal density of the clump, leads to the addition of the α^2 and β parameters. Then, the internal energy is expressed as follows:

$$\Delta E \approx \left(\frac{2GM_*}{b^2 v_{rel}}\right)^2 \frac{M_{mc}\alpha^2 R^2}{3} \tag{6}$$

where $\alpha^2 = \langle R^2 \rangle / R_{mc}^2$ can be calculated as:

$$\alpha^2 = \frac{4\pi}{M_{mc}R_{mc}^2}\int_0^{R_{mc}} dr r^4 \rho_{int}(r) \tag{7}$$

As in the previous case, we can consider that the AMC is completely disrupted when the energy injection is greater than the binding energy, which can be parameterised as $E_{bind} = \beta G M_{mc}^2/R_{mc}$. This β parameter depends on the internal density profile such as:

$$\beta = \frac{4\pi R_{mc}}{M_{mc}^2}\int_0^{R_{mc}} dr r \rho_{int}(r) M_{enc}(r) \tag{8}$$

where $M_{enc}(r) = \int_0^r 4\pi r'^2 \rho_{int}(r')dr'$ is the mass enclosed in the radius r.

Given these definitions, we can introduce the critical impact parameter, b_{min} as in Kavanagh et al.[9]:

$$b_{min} = \sqrt{\frac{M_*}{v_{rel}}}\left(\frac{\alpha^2 G}{\beta\pi\overline{\rho}(\delta)}\right)^{1/4} \tag{9}$$

Lower b translate to larger disruptions. Thus, we can calculate the disruption cross-section of the 'strongest' encounter as $\pi b_{min}^2/M_*$ and, therefore, the break-up probability in a single passage through the Galactic disk in terms of the column mass density of stars S is:

$$p_s = \frac{\pi b_{min}^2}{M_*} S \tag{10}$$

Following [3] and [10], the column mass density of stars is taken as $S \approx 4S_\perp \approx 4\cdot(35\pm5)\ M_\odot\ pc^{-2}$. Multiplying the disruption probability in a single passage by the number of crossings n of the disk over time, we obtain the total disruption probability:

$$P(\%) \approx n\frac{4\pi b_{min}^2\ S_\perp}{M_*} \tag{11}$$

For an AMC following a potencial-law internal density distribution ($\alpha^2 = 3/11$, $\beta = 1,5$) with $\delta = 1$ that interacts with a star of $1M_\odot$ at a relative velocity of $v_{rel} = 10^{-3}c$, we obtain $b_{min} \approx 0,01$ pc and, for $n = 100$ crossings [3], we compute a total disruption probability of $P \approx 4,398\%$. Thus only a few percent of miniclusters are destroyed if we consider one single interaction. It is in this point that we justify the previous assumption of working in the 'distant-tide' approximation since b_{min} is a few orders of magnitude larger than the typical radius considered. Lower distances (lower parameters of impact) are going to lead directly to the minicluster's disruption so it will not affect our result.

On the contrary, larger b can lead to an encounter that simply injects energy into the AMC without unbinding it. Then, multiple encounters can translate to changes in radius and mass and eventually lead to the destruction of the AMC. At this point, a deep analysis of the cumulative effect on disruption can be developed as in [9], but as mentioned in Tinyakov et al. [3] multiple non-disruptive encounters only increase the total disruption probability by a factor of about two.

We try to understand this factor regarding the main equations involved. By considering multiple encounters, we are basically making a large sum on all possible parameters of impact. Despite the number of encounters being proportional to b^2, the energy transmission is proportional to $1/b^4$ (6). Hence, the cumulative effect, also known as cumulative dynamical heating, will be dominated by small impact parameters since the dependence combining the two factors is of $1/b^2$. This basically means that the less common but higher energy injections are the relevant ones. So the multiple non-disruptive encounters enhance only the dispersion on the minicluster, adding a small fraction to the total disruption probability calculated from single encounters.

As stated in the beginning, other astronomical objects, such as gas clouds or other miniclusters, may be taken into account in the previous calculation. Regarding the total disruption probability formula (10), it is easy to see that in order to be relevant, the probability $b_{min}^2 S$ has to be maximum. Only in the case of gas clouds, this parameter is comparable to the case of stars. Nevertheless, gas clouds radii are bigger than their b_{min}, making their disruption contribution also negligible.

Finally, miniclusters structure formation involving minihalos should be considered. But, we cannot forget that we are focused on studying tidal disruption in order to quantify axion stream formation and, therefore, enhance the detection probability. Then, as pointed out by Tkachev [11] and justified in the next section, streams formed by tidal disruption of these bigger structures would lead to low amplification factors (or equivalently small contribution to the local DM density) and therefore tidal disruption of minihalos is left out of this study.

III. AXIONS STREAMS. AXION DETECTION

As seen in the previous section, generally, the gravitational interaction only leads to a few miniclusters being disrupted. As a result, tidal streams along the minicluster's trajectory are formed. Afterwards, we will see if even a small amount of streams can make a difference in direct axion detection.

The main direct axion detection experiments that would be affected by the presence of AMCs are haloscopes. These devices, firstly conceived by Sikivie in 1983, are based on the conversion of cosmic axions into detectable signals inside a resonant cavity with a strong magnetic field. The power output P_a is proportional to the local energy density of axions ρ_a times the coupling photon constant $g^2_{a\gamma\gamma}$. An AMC passing near the Earth would make a great contribution to the local DM density and, thus, enhance the power output of the cavity for a short amount of time (crossing time). But the crossing of an AMC near Earth is expected to be a rare event, as estimations show that an encounter would occur only every $10^4 - 10^6$ years. While as proposed in [3], tidal streams resulting from the miniclusters disruption could be a more promising candidate for direct detection since stream-crossing events in our neighbourhood would occur approximately every 20 years with DM densities greater than the average.

As the signal is proportional to the axion density in the stream, we can define the amplification factor as $A = \rho_{st}/\overline{\rho}$, where the mean DM density $\overline{\rho}$ can be taken as $\overline{\rho} = 0,3$ GeV cm^{-3} in the Solar neighbourhood.

As exposed by Schneider [12], the streams are coherent and long, hence the length can be estimated by taking orbits with small ellipticity, then, the stream is essentially one-dimensional with length $l(t) \sim \sigma_{mc} t$, where t is the age of the stream and σ_{mc} is the previously defined velocity dispersion of the gravitational tied AMC. The validity and consequences of this hypothesis are further discussed in the conclusions.

In this approximation, the axion density in the stream is:

$$\rho_{st} = \rho_{mc}\frac{R_{mc}}{\sigma_{mc} t} \approx 19.8 \text{ GeV cm}^{-3}\delta^{3/2}(1+\delta)^{1/2}\left(\frac{1 \text{ Gyr}}{t}\right) \quad (12)$$

All ρ_{mc}, R_{mc} and σ_{mc} can be parameterised as functions of the overdensity δ, so essentially this magnitude depend on δ and t. The numerical dependence is included following a previous reference [13].

The period of high signal in the detector will correspond to the time interval while the detector crosses a stream. This amount again is only depending on the overdensity and the relative velocity of the original minicluster, such as:

$$\tau = \frac{2R_{mc}}{v_{rel}} \approx \frac{62 \text{ h}}{\delta(1+\delta)^{1/3}}\left(\frac{M}{10^{-12}M_\odot}\right)^{1/3}$$

where we have assumed that $v_{rel} = 10^{-3}c$.

A simple order of magnitude calculation of the passage probability can be carried out by making some simplifying assumptions [11]. We first consider the calculation for one concrete overdensity and time. Taking the probability for a randomly chosen object to be found inside a stream as $P_{in} = \overline{\rho}/\rho_{st} = A^{-1}$, the time interval between two consecutive stream crossings is $T = \tau/P_{in}F = A\tau F$, where $F \equiv P(\delta)^{-1}$ quantifies the disruption probability of the original AMC.

Considering an axion stream coming from a tidal disruption of an AMC with a typical overdensity $\delta \sim 1$ and mass $M_{mc} = 10^{-11}M_\odot$. As developed in the previous section, tidal disruption by stars is the mean contribution to the disruption probability. So as star formation peaks at a redshift between $z \sim 1.2$ and $z \sim 2$, taking a flat Universe with $\omega_m = 0.286$ and Hubble constant $H_0 = 69.6 km/s\ Mpc$, we should consider times $t \lesssim 8.5$ Gyr (taking the lower redshift value). Taking $t = 5\ Gyr$, we get $A \sim 20$, $\tau \sim 4$ days and $T \sim 2.5$ years, where we have used the approximate calculation of the total disruption probability($\sim 2 \cdot 4.398\%$) calculated in the previous section. This is an overestimated calculus, however, it states the importance of axion streams on direct detection.

In a more rigorous way, when computing the crossing rate of steams we should consider not only disruption of miniclusters distributed in different δ at different times but also different total disruption probability over different times. So the calculation becomes an integration over times and densities or, given the dependence on t of A, an integration over amplification factors and densities instead.

This calculation is carried out by Tinyakov et al. [3]. The mean number of steam encounters then is found to be $N(A) = \nu(A)\Delta t$. A simplified estimation developed by the above-mentioned authors showed that the tidal streams encounter rate can be 1 in ~ 20 years with still a relevant amplification $A \sim 10$. So this calculation gives a reasonable scale of time of detection in haloscope.

In addition, as pointed out by Zioutas et al. [14] or Gardikiotis et al. [15], the directionality of the stream can enhance stream detection. As example, streams which get occasionally align with the Sun-Earth direction and, therefore these streams will get gravitational focused at the Earth position by the Sun.

IV. CONCLUSIONS

In this paper, we have studied the tidal disruption of dark matter dense small-scale substructures, the axion miniclusters. We have argued that the main contribution comes from single and cumulative encounters with individual stars. An estimation of AMC destruction has been developed following previous papers and found to be a small fraction.

Despite only a few percent of miniclusters getting disrupted by the present time, this small fraction forms tidal streams that could enhance the local DM density and, therefore, the direct detection of the axion. As a result, the time interval crossing the stream is found to be a period of high signal in the detector. We quantify this amplification and develop an order of magnitude estimation of the time interval between successive stream crossings. This result gives a reasonable scale of time involving a relevant amplification factor, giving hopeful stream crossing rates for direct detection techniques such as haloscopes.

But we have considered the one-dimensional evolution of the stream, which we now review. In principle, the axions inside the AMC are expected to orbit the center of mass of the minicluster. Taking this effect into account, we shall consider expansion over the other two dimensions, at least during one complete orbit of the axion around the minicluster. With an approximate dispersion $\sigma_a \sim 10^{-4}$ km/s, this leads to an orbit period around $T_a \sim 10^5$ years. So we have considered that for much bigger times such as $t = 5$ Gyr the streams are approximately one-dimensional, which is partially true. More rigorously, we should consider 3D expansion until $t \sim T_a$ and after the one-dimensional evolution of this density. Our estimations suggest, then, that the stream should be still long and one-dimensional but with a density $\sim 10^{-5}$ smaller than the previously calculated. In this limit then the amplification will be lost and we would see axion streams as a more diffusive contribution filling the space with multiple streams and, therefore, much more difficult to detect with direct detection methods. This seems to be a more accurate but complex approach that could be studied in following investigations.

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