Contribution of nuclear radial operators to β decay

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Abstract: β decay is one of the most common processes in atomic nuclei. In this work we obtain expressions for contribution to the β decay half life from nuclear radial operators. Once the analytical expressions have been obtained, we evaluate them in the three single-particle orbitals of the s-d shell, which is the relevant configuration space for the ^{23}Ne β decay into ^{23}Na, and estimate the impact of nuclear radial operators in this decay.

I. INTRODUCTION

The standard model of particle physics is an extremely successful theory which can describe a wide range of phenomena. Among them, β decay is a process that occurs in those nuclei that can gain nuclear binding energy decaying to other nuclei closer to stability. However, there are some events that the standard model cannot describe, such as gravity or the fact that neutrinos have mass. That is why several experiments are being held in order to test the predictions of the standard model. For instance, [1] has measured very accurately the β decay of ^{23}Ne into ^{23}Na in order to see if these very accurate results agree with the theoretical predictions. For that comparison, more accurate theoretical values are required as well. Thus, further corrections need to be taken into account. Those corrections require the evaluation of some operators between the initial and final states of the transition. The main goal of this project is to compute the analytical expression of some of these operators, which have a radial contribution. We also estimate numerically those contributions for the nuclear β decay of ^{23}Ne into ^{23}Na in order to obtain a more precise theoretical half life.

II. β DECAY

β decay consists of charge changing transitions where one nucleus decays into another. The most common nuclear β decays are the so called β^-, β^+ and electron capture (EC). The corresponding reactions are

$$\boldsymbol{\beta^-}: \quad (Z,N) \longrightarrow (Z+1,N-1) + e^- + \bar{\nu}_e, \tag{1}$$

$$\boldsymbol{\beta^+}: \quad (Z,N) \longrightarrow (Z-1,N+1) + e^+ + \nu_e, \tag{2}$$

$$\boldsymbol{EC}: \quad (Z,N) + e^- \longrightarrow (Z-1,N+1) + \nu_e. \tag{3}$$

The energy release as kinetic energy in the β^- is positive and, therefore, this process can take place in vacuum. However, in the processes β^+ and EC, the energy release in vacuum is negative, which means that a supply of extra energy is needed in order for the process to occur.

A. Leading Order

As shown in [2], β decays with no change in total angular momentum are called *Fermi transitions* and those where the angular momentum changes in one unit are called *Gamow-Teller transitions.* Both of them are allowed β decay processes which means that the final state leptons are emitted with an angular quantum number $l = 0$. The expression of the half-life of a nucleus is

$$t_{\frac{1}{2}} = \frac{\kappa}{f_0(B_F + B_{GT})}, \tag{4}$$

where $\kappa = 6147s$, f_0 is a phase integral which depends on the type of β decay process (β^-, β^+ or EC) and B_F and B_{GT} are the reduced transition probabilities of the Fermi and Gamow-Teller β decay respectively:

$$B_F = |(f||\hat{C}_0^V(q)||i)|^2, \tag{5}$$

$$B_{GT} = |(f||\hat{L}_1^A(q)||i)|^2, \tag{6}$$

where the elements $(f||\hat{C}_0^V(q)||i)$ and $(f||\hat{L}_1^A(q)||i)$ are the reduced matrix elements of the operators $\hat{C}_0^V(q)$ and $\hat{L}_1^A(q)$, respectively, between a final state $|f\rangle$ and an initial state $|i\rangle$. These operators are

$$\hat{C}_0^V(q) \simeq \frac{g_V}{2\sqrt{\pi}} \sum_{j=1}^{A} \tau_j^{\pm}, \tag{7}$$

$$\hat{L}_1^A(q) \simeq \frac{i g_A}{2\sqrt{3\pi}} \sum_{j=1}^{A} \vec{\sigma}_j \tau_j^{\pm}, \tag{8}$$

where $g_V = 1$ is the vector coupling constant, $g_A = 1.27$ is the axial vector coupling constant, $\vec{\sigma}$ is the spin operator, $\vec{\tau}$ the isospin and the subindex j means that the summation extends over all nucleons.

B. Corrections

Recently, [3] has obtained the expressions for the correction of the half-life:

$$t_{\frac{1}{2}}^{-1} = \frac{f_0}{\kappa}\Big[B_F(1+\delta_F) + B_{GT}(1+\delta_{GT})\Big], \tag{9}$$

where δ_F and δ_{GT} are the Fermi and Gamow-Teller shape corrections, respectively. The shape corrections appearing in (9), as shown in [3], have the following expression:

$$\delta_{GT} \simeq \frac{2}{3}\frac{Q_\beta}{q}\left[\sqrt{2}\frac{(f||\hat{M}_1^V(q)||i)}{(f||\hat{L}_1^A||i)} - \frac{(f||\hat{C}_1^A(q)||i)}{(f||\hat{L}_1^A||i)}\right], \quad (10)$$

$$\delta_F \simeq -2\frac{Q_\beta}{q}\frac{(f||\hat{L}_0^V(q)||i)}{(f||\hat{C}_0^V||i)}, \quad (11)$$

where $Q_\beta = \Delta M - m_e$ with ΔM the difference between the mass of the initial nuclei and the final one, and m_e is the electron mass. The operators appearing in (10) and (11) are defined in [3], as well as the ones in (7)-(8), for any J. Taking $J = 0$ for Fermi processes, and $J = 1$ for Gamow-Teller processes we end up with

$$\hat{L}_0^V(q) \simeq -\frac{g_V}{12\sqrt{\pi}}\frac{q}{m_N}\sum_{j=1}^{A}\left[3 - 2\vec{r_j}\vec{\nabla}\right]\tau_j^{\pm}, \quad (12)$$

$$\hat{M}_1^V(q) \simeq \frac{i}{2\sqrt{6\pi}}\frac{q}{m_N}\sum_{j=1}^{A}\left[g_V\vec{L_j} + \mu\vec{\sigma_j}\right]\tau_j^{\pm}, \quad (13)$$

$$\hat{C}_1^A(q) \simeq -\frac{ig_A}{2\sqrt{3\pi}}\frac{q}{m_N}\sum_{j=1}^{A}\left[\vec{r_j}(\vec{\sigma}_j\vec{\nabla}) + \frac{1}{2}\vec{\sigma_j}\right]\tau_j^{\pm}, \quad (14)$$

where q is the transferred momentum, $m_N = 938.919$ MeV is the mass of a nucleon, which is the average between the proton mass, $m_p = 938.272$ MeV, and the neutron mass, $m_n = 939.565$ MeV and $\mu \simeq 4.7$ is the isovector magnetic moment. $\vec{r}$ is the radial operator and $\vec{\nabla}$ the nabla operator, The products $\vec{r_j}\vec{\nabla}$ and $\vec{\sigma}_j\vec{\nabla}$ in equations (12) and (14) are tensor products coupled to $J = 0$.

C. Analytical evaluation of nuclear radial operators

From [2] we know the reduced matrix elements of the operators appearing in (7),(8) and (13):

$$(n'l'\frac{1}{2}j'||\mathbb{I}||nl\frac{1}{2}j) = \hat{j}\delta_{nn'}\delta_{ll'}\delta_{jj'}, \quad (15)$$

$$(n'l'\frac{1}{2}j'||\vec{\sigma}||nl\frac{1}{2}j) = \delta_{nn'}\delta_{ll'}(-1)^{l+j'+\frac{3}{2}}\sqrt{6}\hat{j}\hat{j}'\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & 1\\ j & j' & l\end{Bmatrix}, \quad (16)$$

$$(n'l'\frac{1}{2}j'||\vec{L}||nl\frac{1}{2}j) = \delta_{nn'}\delta_{ll'}(-1)^{l'+j+\frac{3}{2}}\hat{j}\hat{j}'\times$$
$$\times\sqrt{l(l+1)(2l+1)}\begin{Bmatrix}l' & l & 1\\ j & j' & \frac{1}{2}\end{Bmatrix}, \quad (17)$$

where $\hat{j} = \sqrt{2j+1}$ and the brackets appearing in (16) and (17) are the $6j$ symbols which are related to the coupling of three angular momenta [2].

In order to compute analytically these operators different theorems, properties and symmetries need to be used.

Firstly, for the sake of separating the coordinate space, which is formed by the radial and the angular momentum spaces, and the spin space we use the following theorem from [2]:

$$(n'l'\frac{1}{2}j'||T_L||nl\frac{1}{2}j) = \hat{j}\hat{j}'\hat{L}\begin{Bmatrix}l' & \frac{1}{2} & j'\\ l & \frac{1}{2} & j\\ L_1 & L_2 & L\end{Bmatrix}\times$$
$$\times(n'l'||T_{L_1}||nl)(\frac{1}{2}||T_{L_2}||\frac{1}{2}), \quad (18)$$

where the brackets appearing in (18) denote a $9j$ symbol which, similarly to $6j$ symbols, are related to the coupling of four angular momenta. T_{L_1} and T_{L_2} are two tensors of rank L_1 and L_2, respectively, coupled to a tensor operator of ranks L, so that $T_L = [T_{L_1}, T_{L_2}]_L$.

Besides, the radial space and the angular momentum are independent, consequently, the reduced matrix element is directly the product of the reduced matrix elements of the radial space and the angular momentum space:

$$(n'l'||T_L||nl) = \langle n'l'|T_L|nl\rangle\,(l'||T_L||l), \quad (19)$$

where the quantum number l takes part in both spaces.

The operators appearing in (15)-(17) just operate in one space (none, spin space and angular momentum space, respectively) which implies the identity $\mathbb{I}$ operating in the rest of spaces. None of them operates in the radial space.

Further, in the derivation of the nuclear matrix elements of the operators that appear in (11) and (14), we have used the relation between the scalar and cross product with the tensorial product:

$$T_L \cdot S_L = (-1)^L\hat{L}[T_LS_L]_0, \quad (20)$$

$$T \times S = -i\sqrt{2}[T_1S_1]_1, \quad (21)$$

where T_L and S_L are two tensors of rank L.

Another important theorem that has been used is

$$(n'l'\frac{1}{2}j'||T_L||nl\frac{1}{2}j) = (-1)^{j+j'+L}\hat{L}\sum_{n'',l'',j''}\begin{Bmatrix}L_1 & L_2 & L\\ j' & j & j''\end{Bmatrix}\times$$
$$\times(n'l'\frac{1}{2}j'||T_{L_1}||n''l''\frac{1}{2}j'')(n''l''\frac{1}{2}j''||T_{L_2}||nl\frac{1}{2}j), \quad (22)$$

where T_{L_1} and T_{L_2} are two tensors of rank L_1 and L_2 respectively coupled at rank L. This theorem allows us to separate two tensors of the same operator where both of them operate in some common space.

The reduced matrix elements of $\mathbb{I}$, J (general angular momentum operator), Y_L (spherical harmonic) and $\vec{\sigma}$ (spin operator), are given in [2]:

$$(j'||\mathbb{I}||j) = \hat{j}\delta_{jj'}, \tag{23}$$

$$(j'||J||j) = \hat{j}\delta_{jj'}\sqrt{j(j+1)(2j+1}, \tag{24}$$

$$(l'||Y_L||l) = (-1)^{l'}\frac{\hat{l}\hat{l'}\hat{L}}{\sqrt{4\pi}}\begin{pmatrix} l' & L & l \\ 0 & 0 & 0 \end{pmatrix} \tag{25}$$

$$(\frac{1}{2}||\vec{\sigma}||\frac{1}{2}) = \sqrt{6}, \tag{26}$$

where the element between parenthesis in (25) is a $3j$ symbol, similarly to $6j$ and $9j$, is a coefficient related to the coupling of two angular momenta. They are closely connected to the Clebsch-Gordan coefficients.

In this work, we have used the expressions(18)-(26) to evaluate the analytical expression of the reduced matrix elements of the operators appearing in (12) and (14), in which a radial contribution appears. The results we have obtained are

$$(n'l'\frac{1}{2}j'||\vec{r}\vec{\nabla}||nl\frac{1}{2}j) = -\frac{1}{\sqrt{3}}\delta_{jj'}\delta_{ll'}\hat{j}\,\langle n'l|r\frac{\partial}{\partial r}|nl\rangle\,, \tag{27}$$

and

$$\begin{gathered}(n'l'\frac{1}{2}j'||\vec{r}(\vec{\sigma}\vec{\nabla})||nl\frac{1}{2}j)|_{j=l+\frac{1}{2}} = \sqrt{2}\hat{j}'\Bigg[\delta_{ll'}(l+1)\times \\ \begin{Bmatrix} l & j' & \frac{1}{2} \\ l+\frac{1}{2} & l+1 & 1 \end{Bmatrix}(\langle n'l|r\frac{\partial}{\partial r}|nl\rangle - l\delta_{nn'}) - \delta_{l',l+2}\times \\ \times\sqrt{(l+1)(l+2)}\begin{Bmatrix} l+2 & j' & \frac{1}{2} \\ l+\frac{1}{2} & l+1 & 1 \end{Bmatrix}\langle n'l'|r\left(\frac{\partial}{\partial r} - \frac{l}{r}\right)|nl\rangle\Bigg],\end{gathered} \tag{28}$$

$$\begin{gathered}(n'l'\frac{1}{2}j'||\vec{r}(\vec{\sigma}\vec{\nabla})||nl\frac{1}{2}j)|_{j=l-\frac{1}{2}} = \sqrt{2}\hat{j}'\Bigg[\delta_{ll'}l\times \\ \times\begin{Bmatrix} l & j' & \frac{1}{2} \\ l-\frac{1}{2} & l-1 & 1 \end{Bmatrix}(\langle n'l|r\frac{\partial}{\partial r}|nl\rangle + (l+1)\delta_{nn'}) - \delta_{l',l-2}\times \\ \times\sqrt{l(l-1)}\begin{Bmatrix} l-2 & j' & \frac{1}{2} \\ l-\frac{1}{2} & l-1 & 1 \end{Bmatrix}\langle n'l'|r\left(\frac{\partial}{\partial r} + \frac{l+1}{r}\right)|nl\rangle\Bigg].\end{gathered} \tag{29}$$

We have calculated these expressions for the nuclear radial contributions to β decay from the definition of each operator. To check our results, we have compared them to the more general expression in [4] with the appropriate values for J and taking the limit $q \to 0$.

III. EVALUATION OF THE OPERATORS IN THE S-D SHELL

In order to compare the contribution of each operator to the shape correction, in (10)-(11) we evaluate them in the s-d shell of the nuclear shell model which is the important one for ^{23}Ne and ^{23}Na. This is shown in Fig. 1, as in the case of ^{23}Na and ^{23}Ne, the first two shells, 0s and 0p, are fully occupied and the shell which

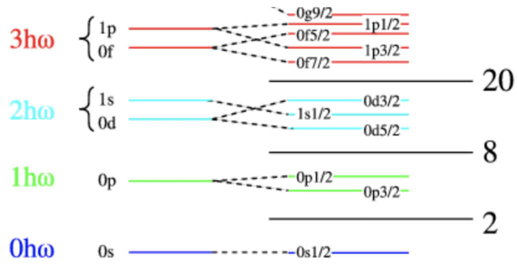

Fig 1: Harmonic oscillator single-particle levels. Initially the energy levels are degenerated in j and, depend only on n and l. After the spin-orbit coupling, those levels split. Large energy gaps divide the spectrum in shells, with the numbers of nucleons before each gap, denoted as magic numbers. From [5].

contains the levels 1s and 0d, named s-d shell, is partially occupied by nucleons. The goal is to compare between them the evaluation of the matrix elements appearing in (7) and (12) (Fermi transition operators) and the ones in (8),(13) and (14) (Gamow-Teller transition operators) in the s-d shell.

For that purpose, we identify q with Q_β which for the β decay of ^{23}Ne is

$$\begin{aligned} q \simeq Q_\beta &= M(^{23}_{10}Ne) - M(^{23}_{11}Na) - M(e^-) \\ &= m_n - m_p + B(^{23}_{11}Na) - B(^{23}_{10}Ne) - m_e, \end{aligned} \tag{30}$$

where $B(^{23}_{11}Na)$ and $B(^{23}_{10}Ne)$ are the binding energies of the sodium and the neon respectively. From [6], we get the values $B(^{23}_{11}Na) = 186.564$ MeV and $B(^{23}_{10}Ne) = 182.979$ MeV, so that $Q_\beta = 4.369$ MeV.

Having all these values, we evaluate the matrix elements with their corresponding relative factors as shown in Table I.

The first noticeable feature is that, in the s-d shell, the operator $\vec{r}\vec{\nabla}$ is proportional to $\mathbb{I}$. This is also observable comparing (27) with (15) where the relative factor is $-\frac{1}{\sqrt{3}}\langle n'l|r\frac{\partial}{\partial r}|nl\rangle$. As in (27) there are two Kronecker's deltas in j and l, there are no levels in the s-d shell of same j and l with different n, therefore, there will be only diagonal elements. In addition, the radial integral with the same n and l in both sides gives always $\langle nl|r\frac{\partial}{\partial r}|nl\rangle = -\frac{3}{2}$. As a result we can write $\vec{r}\vec{\nabla}$ as $\frac{\sqrt{3}}{2}\mathbb{I}$. This implies that we can take $3\mathbb{I} - 2\vec{r}\vec{\nabla} = (3-\sqrt{3})\mathbb{I}$ in the s-d shell.

The zeros appearing in the 1s-0d matrix elements of the operators $\vec{\sigma}$ and $\vec{L}$ are due to the Kronecker's deltas appearing in (16)-(17). The operator $\mathbb{I}$ is diagonal as we can see in (15) and also $\vec{r}\vec{\nabla}$ due to its proportionality to the identity.

$\mathbb{I}$	$0d_{\frac{5}{2}}$	$1s_{\frac{1}{2}}$	$0d_{\frac{3}{2}}$
$0d_{\frac{5}{2}}$	2.44949	0	0
$1s_{\frac{1}{2}}$	0	1.41421	0
$0d_{\frac{3}{2}}$	0	0	2.00000

(a) Reduced matrix elements for the leading Fermi operator in the s-d shell.

$-\frac{Q_\beta}{6m_N}(3\mathbb{I} - 2\vec{r}\vec{\nabla})$	$0d_{\frac{5}{2}}$	$1s_{\frac{1}{2}}$	$0d_{\frac{3}{2}}$
$0d_{\frac{5}{2}}$	-0.00241	0	0
$1s_{\frac{1}{2}}$	0	-0.00139	0
$0d_{\frac{3}{2}}$	0	0	-0.00197

(b) Reduced matrix elements for the correction to the Fermi operator in the s-d shell.

$g_A\vec{\sigma}$	$0d_{\frac{5}{2}}$	$1s_{\frac{1}{2}}$	$0d_{\frac{3}{2}}$
$0d_{\frac{5}{2}}$	3.68082	0	-3.93495
$1s_{\frac{1}{2}}$	0	3.11085	0
$0d_{\frac{3}{2}}$	3.93495	0	-1.96747

(c) Reduced matrix elements for the leading Gamow-Teller operator in the s-d shell.

$\frac{Q_\beta}{\sqrt{2}m_N}(g_V\vec{L} + \mu\vec{\sigma})$	$0d_{\frac{5}{2}}$	$1s_{\frac{1}{2}}$	$0d_{\frac{3}{2}}$
$0d_{\frac{5}{2}}$	0.06395	0	-0.04288
$1s_{\frac{1}{2}}$	0	0.03793	0
$0d_{\frac{3}{2}}$	0.04288	0	-0.00869

(d) Reduced matrix elements for the non-radial correction to the Gamow-Teller operator in the s-d shell.

$-g_A\frac{Q_\beta}{m_N}(\vec{r}(\vec{\sigma}\vec{\nabla}) + \frac{1}{2}\vec{\sigma})$	$0d_{\frac{5}{2}}$	$1s_{\frac{1}{2}}$	$0d_{\frac{3}{2}}$
$0d_{\frac{5}{2}}$	0	0	0.00373
$1s_{\frac{1}{2}}$	0	0	0
$0d_{\frac{3}{2}}$	0.00373	0	0

(e) Reduced matrix elements for the radial correction to the Gamow-Teller operator in the s-d shell.

Table I: Evaluation of the reduced matrix elements of the Fermi and Gamow-Teller operators in (7)-(8) and (12)-(14) for the three different orbitals of the s-d shell.

Another singular feature is that the diagonal matrix elements of the operator $(\vec{r}(\vec{\sigma}\vec{\nabla}) + \frac{1}{2}\vec{\sigma})$ vanish. This is due to the symmetry property [4]

$$(nl\frac{1}{2}j||T_J(q\vec{r})||n'l'\frac{1}{2}j') = (-1)^\lambda (n'l'\frac{1}{2}j'||T_J(q\vec{r})||nl\frac{1}{2}j) \tag{31}$$

where $\lambda = j' + j$ for $(\vec{r}(\vec{\sigma}\vec{\nabla}) + \frac{1}{2}\vec{\sigma})$ and $\lambda = j' - j$ for $\vec{\sigma}$. In the first case, the diagonal elements have the same j' and j, which summed gives an odd number, implying that these elements must vanish. Also because of this symmetry, in the same operator, the only two elements different from zero are symmetric. This also explains why the non-diagonal elements in $\vec{\sigma}$ different from zero are antisymmetric.

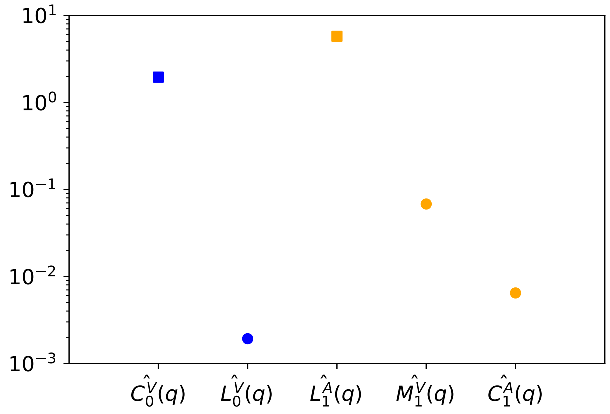

Fig 2: Absolute average value of the non-zero matrix elements of the operators appearing in (10)-(14) evaluated in the s-d shell. The first two operators (Fermi) are represented in blue and the last three (Gamow-Teller) in orange. The dominant operators are represented with squares and the rest with circles. They are all computed without the $\frac{1}{2\sqrt{\pi}}$ common factor.

For the Fermi transitions, from (11) we can estimate that the order of magnitude of the ratio $\hat{L}_0^V(q)/\hat{C}_0^V(q)$ is of the order of $Q_\beta/m_N \simeq 10^{-3}$ in agreement with the results in tables (a)-(b) and in Fig. 2.

Furthermore, for the Gamow-Teller transitions, from (10), there are two contributions: first, a non-radial operator for which we can estimate the ratio $\hat{M}_1^V(q)/\hat{L}_1^A(q)$ to be of the order of $(Q_\beta\mu)/m_N \simeq 10^{-2}$; second, a radial one with $\hat{C}_1^A(q)/\hat{L}_1^A(q)$ that we can estimate as $Q_\beta/(2m_N) \simeq 10^{-3}$. These results are in good agreement with the results in tables (c)-(e) and in figure 2. The only exception is that, $\hat{C}_1^A(q)$ is somewhat more suppressed than expected because of a cancellation between the $\vec{r}(\vec{\sigma}\vec{\nabla})$ and $\frac{1}{2}\vec{\sigma}$ terms. On grounds of that, our results suggest that it is reasonable to disregard the radial contribution as it has been done in [1].

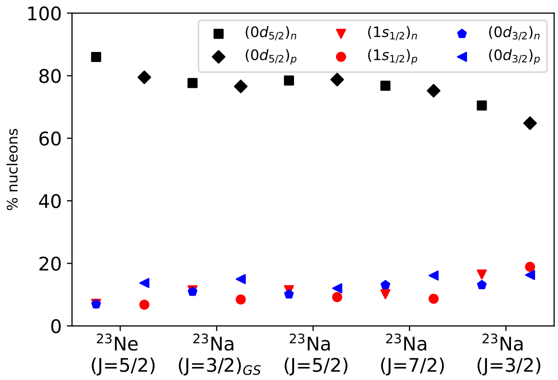

Fig 3: Percentage of the total nucleons in the s-d shell that occupy the different orbitals for the ground state of the ^{23}Ne and the four lowest energy states states of ^{23}Na. All states have positive parity.

IV. NUCLEAR SHELL MODEL CALCULATION FOR ^{23}Ne AND ^{23}Na

In this section we compute the occupation of the different s-d shell orbitals for ^{23}Ne and ^{23}Na. In order to do so, we use the code *Antoine* explained in [7]. The code solves the Schrödingher's equation for the nuclear shell model

$$H_{eff}\,|\psi\rangle = E\,|\psi\rangle\,, \tag{32}$$

where $|\psi\rangle$ is a nuclear state, H_{eff} is the effective Hamiltonian in the valence space and E is the energy of the nuclear state.

In the nuclear shell model the valence space consists of all single-particle orbitals actively involved in the generation of configurations of the many-nucleon system considered. In our case, this space is the s-d shell orbitals. All the orbitals below are considered to be inert and receive the name of *core*. The reason for defining a core is that the computational effort increases very rapidly with an increasing number of single-particle orbitals included in the valence space.

Four different states of the ^{23}Na can be the final states of the beta decay of ^{23}Ne: the ground state ($J=\frac{3}{2}$) and three excited states ($J=\frac{5}{2},\frac{7}{2},\frac{3}{2}$) as shown in Fig. 3. We calculate the initial and the four possible final states using (32) and in all the nuclear states we find that the most occupied orbital, by difference, is the $0d_{\frac{5}{2}}$, the least energetic orbital according to Fig. 1, regardless if they are protons are neutrons.

Therefore, between all the reduced matrix elements in Table I, the most important one is $(0d_{\frac{5}{2}}||\hat{O}||0d_{\frac{5}{2}})$, where $\hat{O}$ is any of the five different operators. As a consequence, in Fig. 2, where the average of the different matrix elements of the different operators is represented, the element $0d_{\frac{5}{2}}-0d_{\frac{5}{2}}$ should have substantially more weight than others. In particular, the radial Gamow-Teller correction is additionally suppressed because its only non-vanishing matrix element is the $0d_{\frac{5}{2}}-0d_{\frac{3}{2}}$ one involving the $0d_{\frac{3}{2}}$ orbital, which is relatively empty in ^{23}Ne and ^{23}Na. This also supports the approximation in [1] of neglecting this operator.

V. CONCLUSIONS

In this work we give analytical expressions for the reduced matrix elements of operators which contribute to β decay, with particular emphasis on those with radial dependence. The radial matrix elements reveal non-trivial properties in the s-d shell such as a proportionality to the reduced matrix element of the identity in the case of the Fermi operator, and a symmetry property in the Gamow-Teller operator. The non-radial contribution to the Gamow-Teller shape correction is expected to be one order of magnitude bigger than the radial contribution because the first one is a correction of 2-3% while the second one is a 0.1% correction. Finally, the occupation of the different orbitals of neon and sodium suggests an additional suppression of the radial correction to Gamow-Teller transitions. This is because only the $0d_{\frac{5}{2}}-0d_{\frac{3}{2}}$ matrix element is not vanishing for this operator, and the orbital $0d_{\frac{3}{2}}$ is mostly empty in ^{23}Ne and ^{23}Na. Therefore, this work indicates that the ^{23}Ne decay receives contributions from nuclear radial operators only below the 0.1% level.

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