Dark matter streams from tidally stripped axion minihalos

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Abstract: In this report, the motion of a dark matter minihalo around the Milky Way is studied, focusing on an encounter with a star that modifies the orbits of the axions inside the minihalo. The number of axions escaping from the minihalo due to the tidal acceleration caused by the star is simulated, analyzing their distribution around the minihalo and their evolution.

I. INTRODUCTION

Dark matter (DM) is a hypothetical form of matter with gravitational effects similar to those of ordinary baryonic matter. The difference is that DM does not interact with electromagnetic radiation, so there is no absorption, reflection or emission, which makes it unobservable by means other than gravity, unless there is some other weak interaction of dark matter yet to be discovered. As it can not be seen, its effects are studied. There are three possible types of DM: hot (HDM), warm (WDM) and cold (CDM), the latter used in this report. The Cold Dark Matter candidates include Weakly Interacting Massive Particles (WIMPs), axions and primordial black holes. Here, the case of QCD axions predicted by the Peccei-Quinn model is considered to solve the strong Charge-Parity (CP) problem in particle physics [1].

Axion minihalos (MH) are gravitationally bound substructures of axions. Tidal streams form due to the disruption of these structures [2]. These minihalos start forming at a minimum mass of order 10^{-12} times the solar mass and are formed by a standard gravitational inestability, among other structures on all scales. During the Milky Way's (MW) timelife, minihalos can traverse the MW disk multiple times, opening the possibility to form tidal streams caused by interactions with stars. Minihalos can also go through without interacting, if they are not close enough to stars. Tidal streams expand over a large volume compared to the minihalo they arise from.

A dark matter minihalo orbits around the Milky Way under the influence of tidal forces inflicted by both the disk and the halo of the galaxy. When it reaches the disk and is close enough to a star to notice its gravitational field, an encounter happens. Then, the minihalo follows an hyperbolic trajectory in the star's reference system, while the orbits of the axions inside are perturbated by a difference in accelerations between the center of mass of the minihalo and the axions (tide generated by the star). The interaction with the star can make some axions escape, allowing them to gain a kinetic energy that exceeds the potential of the minihalo that was previously keeping them inside. At this point, some axions are still bound, orbiting inside the minihalo, while others escape, orbiting around the galaxy on their own. Even though they are no longer bound, they form a stream close to the minihalo orbit. Studying the energy changes before and after the interaction, the number of escaping axions and the evolution in time of their distribution is calculated.

II. PREVIOUS CONSIDERATIONS

Axions are treated as classical particles, because the de Broglie wavelength (λ_{dB}) is negligible compared to the minihalo half mass radius (a_{MH}),

$$\lambda_{dB} = \frac{h}{p} \ll a_{MH} \ , \tag{1}$$

where h is Planck's constant and p is the particle's momentum ($p = mv$). Considering an axion with $m = 1\,\mathrm{eV/c^2}$ and $v = 300\,\mathrm{km/s}$, and a minihalo with $a_{MH} = 1\,\mathrm{au} = 4.848 \cdot 10^{-9}\,\mathrm{kpc}$,

$$\lambda_{dB} \approx 4 \cdot 10^{-17}\,\mathrm{kpc} \ll \mathrm{a_{MH}} \ . \tag{2}$$

The encounter rate used in this report is one during a time range of 10^{10} years. The mass of the star is the same as the sun's, $M_s = 1M_{\odot} = 1.988 \cdot 10^{30}$ kg [3], and the minihalo's is $M_{MH} = 10^{-12}M_{\odot}$. Less bound axions are considered, increasing the number of escaping axions, namely, the minihalo disruption (more axions in the tidal stream). The minihalo mass loss due to the perturbation is so small that its potential is considered constant.

III. FORMULATION

For a system of N particles, the lagrangian formulation is used. Thus, the three-dimensional Hamiltonian in the phase space is written in terms of generalized coordinates q_j, generalized momenta p_j and time t:

$$H(q_j, p_j, t) = \sum_k \dot{q_k} p_k - L(q_j, \dot{q_j}, t) \ , \tag{3}$$

where j,k = 1, ..., 3N and $p_j = \partial L / \partial \dot{q_j}$.

The Hamilton's equations used to calculate orbits are

$$\frac{\partial H}{\partial x^i} = -\frac{dp_i}{dt} \;, \quad \frac{\partial H}{\partial p_i} = \frac{dx^i}{dt} \;, \quad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t} \;, \quad (4)$$

with $x^i = (x, y, z)$ and $p_i = (p_x, p_y, p_z)$, having a total of 6N coordinates. All calculations have been done per mass unit, so $p_i = v_i$. Introducing kinetic and potential energies, the non time dependent Hamiltonian is

$$H(\vec{x}, \vec{p}) = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) + \Phi(x, y, z) \;. \quad (5)$$

IV. MODELS

Three important parts of the Milky Way are its disk (d), bulge (b) and dark halo (h). Neglecting the bulge potential for being so small compared to the others, the MW potential is $\Phi_{MW} \simeq \Phi_d + \Phi_h$. Based on established observational properties of the MW disk and halo, the models used [3] are described in the following subsections.

A. Miyamoto-Nagai model

The potential for a thin and thick disk is

$$\Phi(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2}\,)^2}} \;, \quad (6)$$

being G the gravitational constant, $M = 6 \cdot 10^{10} M_\odot$ the disk total mass, and a and b model parameters with values 5 kpc and 0.3 kpc, respectively. Cylindrical coordinates are used.

B. Navarro-Frenk-White model

Simulations of the distribution of dark matter particles suggest that the density of the dark matter halo can be approximated as a radius power law such as

$$\rho(r) = \rho_0 \left(\frac{r}{a}\right)^{-\alpha} \left(1 + \frac{r}{a}\right)^{\alpha - \beta} \;, \quad (7)$$

where spherical coordinates are used. The value of the model parameter a is 15 kpc. The NFW model for a spherical system considers $(\alpha, \beta) = (1, 3)$, leading to a potential for the dark matter halo of the form

$$\Phi(r) = -4\pi G \rho_0 a^3 \frac{ln(1 + \frac{r}{a})}{r} \;. \quad (8)$$

The speed of a star in the disk is affected by the speeds of the dark halo and the disk. Thus,

$$v_s^2 = v_d^2 + v_h^2 \;, \quad (9)$$

where speeds are calculated with the relationship between the potential derivative and the centrifugal acceleration

$$\frac{v_h^2}{r} = \frac{d\Phi_{NFW}(r)}{dr} \;, \quad \frac{v_d^2}{r} = \frac{d\Phi_{MN}(r)}{dr} \;. \quad (10)$$

Using the equations (9) and (10), and taking into consideration that the thickness of the disk is so small compared to the radius so $r \approx R$, ρ_0 is found

$$\rho_0 = \frac{v_h^2}{4\pi G a_{NFW}^3} \left[\frac{ln(1 + \frac{r}{a_{NFW}})}{r} - \frac{1}{(a_{NFW} + r)}\right]^{-1} \;. \quad (11)$$

Besides these models, there is also the one for the axions orbiting in the minihalo.

C. Plummer model

This model describes the potential of a spherical system with constant density at the center ($\propto r^2 + constant$), going to zero at greater radii ($\propto r^{-1}$). Its form is

$$\Phi = -\frac{GM}{\sqrt{r^2 + a^2}} = -\frac{GM}{a}\left(1 + \frac{r^2}{a^2}\right)^{-\frac{1}{2}} \;, \quad (12)$$

where spherical coordinates are used, being G the gravitational constant, $M = M_{MH} = 10^{-12} M_\odot$ the minihalo total mass, and $a = a_{MH} = 4.848 \cdot 10^{-9}$ kpc the Plummer scale length, in other words, it is the radius at which the mass profile projected on a two-dimensional plane is half the total mass. Using (12) and Poisson's equation, Plummer density is obtained

$$\nabla^2 \Phi = 4\pi G \rho \;, \quad (13)$$

$$\rho = \frac{3M}{4\pi a^3}\left(1 + \frac{r^2}{a^2}\right)^{-\frac{5}{2}} \;. \quad (14)$$

V. INITIAL CONDITIONS

A. Axions

The initial conditions are calculated in the minihalo reference system, using the acceptance-rejection method and a distribution function f [4], shown on the next page.

$$f = f_x f_p = \frac{\rho(x,y,z)}{M} f_p(\vec{x},\vec{p}) \ , \tag{15}$$

where $\rho(x,y,z)$ is the Plummer density.

The interesting speeds are the ones that make density constant. In addition, f does not depend on time explicitly. Thus,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f,H] = 0 \rightarrow [f,H] = 0 \ . \tag{16}$$

Considering $f = f(H)$, the Hamiltonian in spherical coordinates is

$$H = \frac{1}{2}(v_r^2 + v_\theta^2 + v_\phi^2) - |\Phi(r)| \ , \tag{17}$$

with $v_r = p_r$, $v_\theta = \frac{p_\theta}{r}$ and $v_\phi = \frac{p_\phi}{r sin\theta}$, change made in order to mantain the module of tangent vectors. Note that the potential is negative. If the particle is bound, then

$$|\Phi(r)| \geq \frac{1}{2}(v_r^2 + v_\theta^2 + v_\phi^2) \rightarrow H \in [-|\Phi|, 0] \ . \tag{18}$$

Integrating f_x in spherical coordinates, the function obtained is: $F(r,\theta,\phi) = F_1(r)F_2(\theta)F_3(\phi)$. Defining each F_i as a uniform variable u_i,

$$r = a\left(u_1^{-\frac{2}{3}} - 1\right)^{-\frac{1}{2}} \ , \quad \theta = arccos(u_2) \ , \quad \phi = 2\pi u_3 \ , \tag{19}$$

with $u_1 = u_3 = U(0,1)$ and $u_2 = U(-1,1)$. For less bound particles, $u_1 \rightarrow 1$. Knowing (r,θ,ϕ), one can calculate (x,y,z). The escaping speed of particles is

$$\frac{1}{2}mv_e^2 = |\Phi| \rightarrow v_e = \sqrt{\frac{2GM}{m}}\left(a^2 + r^2\right)^{-\frac{1}{4}} \ . \tag{20}$$

Definig new θ and ϕ values, the cartesian momenta (p_x, p_y, p_z) are obtained. The speed is smaller than v_c, close for less bound particles $\rightarrow v = uv_c$, with $u = U(0,1)$. Making some changes in (15), $f_q(q)$ is obtained

$$f_q(q) = \frac{512}{7\pi} q^2 (1-q^2)^{\frac{7}{2}} \ , \tag{21}$$

$$q = \frac{|p|}{|p_e|} = \frac{\sqrt{v_r^2 + v_\theta^2 + v_\phi^2}}{\sqrt{2|\Phi|}} \in [0,1] \ , \tag{22}$$

$$v = q\sqrt{2GM}\left(a^2 + r^2\right)^{-\frac{1}{4}} \ . \tag{23}$$

The values of q accepted by the acceptance-rejection method lead to v (23). This has been done for ten thousand axions.

B. Minihalo

The initial speed module is

$$v_{MH,0} = 240 \ \frac{\text{km}}{\text{s}} = 2.5 \cdot 10^{-7} \ \frac{\text{kpc}}{\text{yr}} \ . \tag{24}$$

The components are calculated with spherical coordinates, using random angles. The position coordinates considered are $x = y = z = 8\,\text{kpc}$.

VI. DEVELOPMENT AND RESULTS

As explained in the introduction, a dark matter minihalo orbiting around the Milky Way is studied, considering the tidal forces exerted by both the disk and the dark halo of the galaxy. The Hamilton's equations (4) are solved, using (5) with the MW's potential, Φ_{MW}. Thus,

$$\dot{x_i} = p_i \ , \quad \dot{p_i} = -\frac{\partial H}{\partial x_i} = -\left(\frac{\partial \Phi_d}{\partial x_i} + \frac{\partial \Phi_h}{\partial x_i}\right) \ . \tag{25}$$

Integrating these equations for 10^6 values of time, with initial time $-10^8\,\text{yr}$ and final time $10^8\,\text{yr}$, a phase space point is obtained for each value. All points describe the orbit of the minihalo around the galaxy in this period of time. The initial time is not essential. Substituing the data for each point in the Hamiltonian, the energy is found. These many steps of time are needed in order to calculate, through interpolation, the time when the minihalo reaches the galaxy disk ($z = 0$). If the MH is close enough to a star to notice its gravitational effects, then there is an encounter between them. Assuming it happens, the time found is $t_{int} = -4.379 \cdot 10^7\,\text{yr}$. Now, using t_{int} as final time, an integration of the equations shown before is done, where the last point obtained is the phase space point of the interaction. This point determines the position of the star. When the encounter happens, the minihalo position is the same as the star's $\rightarrow r_{int} = r_{MH} = r_s$. It also determines its polar angle and the minihalo speed, both at the galaxy reference system. The values found are

$$r_{int} = 17.687\,\text{kpc} \ , \qquad \alpha_{\text{int}} = 59.883^\circ \ , \tag{26}$$

$$v_{MH,int} = 175.316 \ \frac{\text{km}}{\text{s}} = 1.791 \cdot 10^{-7} \ \frac{\text{kpc}}{\text{yr}} \ . \tag{27}$$

Having the radius, the equation (11) is solved

$$\rho_0 = 2.143 \cdot 10^7 M_\odot/\text{kpc}^3 \ . \tag{28}$$

The speed of the star at r_{int} is calculated with the equations (9) and (10). The result is presented on the next page.

$$v_s = 255.618\ \frac{\text{km}}{\text{s}} = 2.612 \cdot 10^{-7}\ \frac{\text{kpc}}{\text{yr}}\ . \tag{29}$$

The gravitational field of the star introduces tidal forces, modifying the orbits of the axions inside the minihalo. These tidal forces are a perturbation that comes from a difference in accelerations between the center of mass of the minihalo (CM) and each axion. It can be seen in the following equation

$$\vec{a}_{\text{axion,CM}} = \vec{a}_{\text{axion,star}} - \vec{a}_{\text{CM,star}}\ , \tag{30}$$

where $\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$. Same expressions as (30) are obtained for $\vec{v}$ and $\vec{r}$. Having $\vec{r}$, $\vec{a} = -GM/r^3\ \vec{r}$.

The distance at which the minihalo starts noticing the tidal effects of the star is called impact parameter, and it is related with the rate of interaction between the MH and stars, $R(b)$, the density of stars per volume unit, n, and the total speed $v_T = \sqrt{v_s^2 + v_{MH}^2}$ as

$$R(b) = \frac{1}{10^{10}\ \text{yr}}\ , \qquad n = \frac{\overline{\rho}}{M_\odot} = 10^7\,\text{kpc}^{-3}\ , \tag{31}$$

$$b = \sqrt{\frac{R(b)}{n v_T \pi}} = 1.306 \cdot 10^{-5}\ \text{kpc}\ . \tag{32}$$

Aproximating the orbits of the axions inside the minihalo as circular, equating the gravitational and centrifugal forces, and using Earth data (E) [3], the rotational speed and the period are obtained

$$m\frac{v^2}{r} = \frac{GMm}{r^2} \quad \rightarrow \quad v_{ax} = v_E\sqrt{\frac{M_{MH}}{M_\odot}\frac{r_{E,\odot}}{a_{MH}}}\ , \tag{33}$$

$$v_{ax} \approx 3\ \frac{\text{cm}}{\text{s}}\ , \qquad T = 2\pi\frac{a_{MH}}{v_{ax}}\ . \tag{34}$$

The perturbation can be considered punctual because it lasts $b/v_{MH} \approx 70\,\text{yr}$, negligible compared to the axion period around the minihalo, T. Then, the axions will only suffer the tidal forces caused by the star, but not the ones caused by the galaxy. The more closer the minihalo is to the star, the more it notices the tidal forces. Polar coordinates (r, Ψ) are used. At the interaction time, $r = b$ and $\Psi = \Psi_0$ (Ψ_0 is a constant angle). The MH approaches the star in a 1D plane, with polar angle $\Psi \rightarrow 0$. When the perturbation happens, then the trajectory becomes hyperbolic (two-dimensional). The polar angle $\Psi(t)$ is obtained from the equations of hyperbolic anomalies [5]. Proceding similarly as in *Galactic Dynamics* for Kepler's potential [3], the equations of the hyperbolic orbit are the ones in (35) and (36).

$$r(\Psi) = \frac{a(1-\epsilon^2)}{1+\epsilon cos[\Psi(t)-\Psi_0]}\ , \qquad a = \frac{b^2 v_0^2}{GM_s(1-\epsilon^2)}\ , \tag{35}$$

$$tan\Psi_0 = -\frac{bv_0^2}{G(M_s+M_{MH})}\ , \qquad \epsilon = \sqrt{1+\left(\frac{bv_0^2}{GM_s}\right)^2}\ , \tag{36}$$

with (r, Ψ), the cartesian coordinates for the minihalo in the star reference system can be found. Now, integrating Hamilton's equations (4) with Plummer potential, the phase space coordinates for axions in the minihalo reference system are calculated, taking into account the tidal forces from the star (30). Thus,

$$\dot{p_i} = -\frac{GM_{MH}}{(r^2+a_{MH}^2)^{\frac{3}{2}}}x_i + (\vec{a}_{\text{axion,star}} - \vec{a}_{\text{CM,star}})\ . \tag{37}$$

The integration has been done for a total of $2 \cdot 10^5$ yr, from 10^5 yr before the closest approach to 10^5 yr after.

As seen in equation (18), the energy is negative if the potential, Plummer potential in this case, is greater than the kinetic energy, meaning that the axion speed is not enough to escape. On the contrary, if it is positive then the speed is greater or equal as the one in equation (20), enough to overcome the potential and escape from the minihalo. Studying the energy of each axion after the interaction, 10% of the total number of axions escape, while the other 90% are still bound. As approximated, the minihalo mass loss is small, so its potential is approximately constant. The escaping axions form a dark matter stream that moves around the galaxy, near the minihalo, subjected to the tidal forces from the disk and the dark halo of the MW. Thus, the motion equations they follow are those shown in (25). The equation (37) represents the axion acceleration in the minihalo system. To obtain their orbit around the galaxy, a change of reference system must be done, taking into account that some escaping axions are still slowed down by the minihalo potential. The real speed of the axions p_r in the MH reference system is

$$p_r^2 = p^2 - \frac{2GM_{MH}}{r_{axion,CM}} \quad \rightarrow \quad \vec{p_r} = \vec{p}\ \frac{p_r}{p}\ , \tag{38}$$

where p is the integrated value of equation (37). Integrating the equations in (25), using the moment of the encounter as the initial time, the time range is $[0,\ 10^8 + |t_{int}|]$ yr. The final time is the one used previously on the MH's total trajectory. The orbits of axions are obtained and plotted next to the MH's. The speed used for axions is

$$\vec{p}_{\text{axion,MW}} = \vec{p}_{\text{CM,MW}} + \vec{p_r}\ . \tag{39}$$

The following representations show all discussed above. Although they are two-dimensional figures, the final distribution and the orbits are three-dimensional.

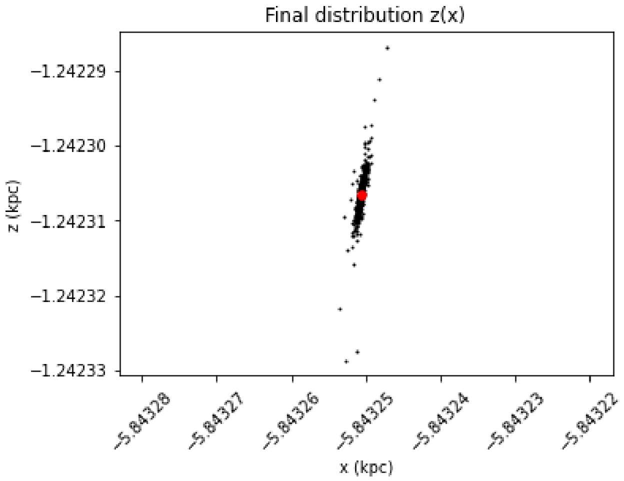

FIG. 1: Two-dimensional representation of the final distribution of escaping axions after suffering the tidal forces inflicted by the star and the MW. MH's final point is also shown in red. Projection on the plane z(x). Units are indicated on the axes.

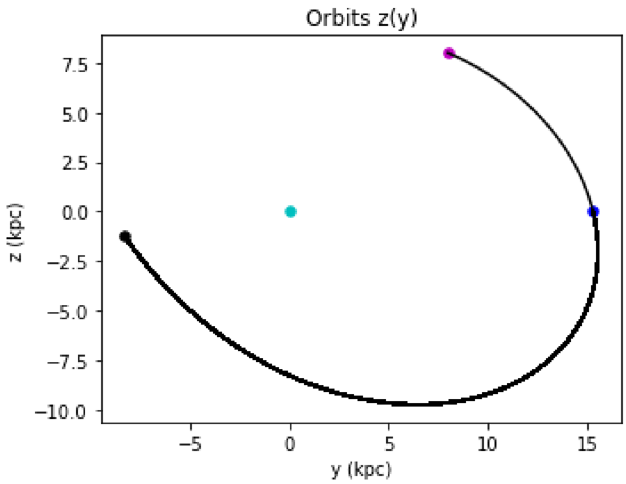

FIG. 2: Two-dimensional representation of the minihalo and stripped axion's orbits around the Milky Way. Projection on the plane z(y). Units are indicated on the figure's axes.

The color code for points in figure (1) is: red (MH's last point) and black (last point of all axions).

In figure (2), the color code for points is: purple (initial point, where the minihalo starts its orbit around the Milky Way), black (final point, where both the minihalo and the stripped axions's trajectories finish), dark blue (interaction point, where the minihalo reaches the disk and interacts with a star) and light blue (galactic center).

VII. CONCLUSIONS

- After the interaction between the minihalo and the star, 10% of the total number of axions that were originally inside the MH escape, causing a dark matter stream. As approximated, the minihalo mass loss is small.
- Figure (1) shows the final distribution of the stripped axions around the minihalo, for two different projections in space. The stream is longer on the orbit direction. This can be seen in the figure, where the stream is longer on the z-axis. The axions that have escaped are near the minihalo.
- In figure (2), the MH begins its trajectory and goes orbiting until it reaches the disk and an encounter happens. After, the stripped axions go round the Milky Way near the minihalo. The thick black line represents both orbits, so close to each other that at a great scale they seem to be exactly the same, but there is a distribution similar to figure (1) in each point.
- The evolution in time of the MH energy has been calculated and found to be approximately constant. In the interaction, a change in energy allows some axions to escape with $v \geq v_c$ (20).

VIII. APPENDIX

Some Python functions used for the calculations.

Acknowledgments

I would like to thank my advisor, Prof. Jordi Miralda Escudé, for introducing me to this Astrophysics line of research that I desire to follow, and for trusting and guiding me in this project. The postgraduate researcher Carles Garcia Palau explained to me some physical phenomena, and for that I am thankful. I would also like to express my gratitude to my family and friends for being supportive and always believing in me.

[1] *Cosmology Notes.* Virtual Campus UB.

[2] Peter Tinyakov, Igor Tkachev & Konstantin Zioutas. *Tidal streams from axion miniclusters and direct axion searches.* arXiv:1512.02884v2 [astro-ph.CO] 1 Feb 2016.

[3] Binney, J. Tremaine, S. (2ed.). (2008). *Galactic Dynamics. Princeton Series in Astrophysics.*

[4] Method explained by the Carles Garcia Palau.

[5] Matthew M. Peet. *Spacecraft Dynamics and Control.*

Initial conditions (Plummer) for less bound axions

In [1]:
```
# first of all the libraries needed are imported
from numpy import * # specialized in numerical calculation
from matplotlib.pyplot import * # useful for graphical representations
```

In [2]:
```
# physical variables for the minihalo
# a = 0.1 # in au -> only 3 axions scape out of 10k
a = 1.0 # in au
M = 1.0E-12 # mass in solar masses
G = 39.479 # Universal Gravitational constant in au^3/(yr^2*Msol)
```

In [3]:
```
# command random.uniform(a,b,n) -> generates an n-dim vector with random floats between a and b
# function that goes from spherical coordinates to cartesian coordinates

def spherical_to_cartesian(r,theta,phi):
    x=r*sin(theta)*cos(phi)
    y=r*sin(theta)*sin(phi)
    z=r*cos(theta)
    return x,y,z

# distribution function fq(q), where q=|p|/|pe|ϵ[0,1]
def fq(q):
    return 512.0/(7.0*pi)*q**2.0*(1.0-q**2.0)**(7.0/2.0)
```

In [4]:
```
# function that calculates the initial conditions following Plummer model
# F=integral(fx)=integral(p(r)/M) -> spherical integration -> r^2 sinϑ drdϑdΦ
# dividing the main integral in three parts dependening on r,ϑ i Φ, respectively, the variable as a function of a
# random float between 0 i 1 ( u ϵ U(0,1) ) is obtained
# initial conditions of some axions bound but with big orbits -> grater orbital period -> less energy
# -> less bounded, so a greater u0 is needed (almost 1.0)

def initial_conditions(seed,n):
    random.seed(seed) # the seed is for random numbers

    # Initial conditions of position
    u0=random.uniform(0.95,1.0,n) # n-dim vector with random floats between 0.999 i 1. As u0 is a vector, r is too
    u1=random.uniform(-1.0,1.0,n) # n-dim vector with random floats between -1 i 1. As u1 is a vector, theta is too
    r=a*(u0**(-2.0/3.0)-1.0)**(-1.0/2.0)
    theta=arccos(u1)
    phi=random.uniform(0.0,2.0*pi,n)

    x,y,z=spherical_to_cartesian(r,theta,phi)

    # method of rejection for the initial conditions of velocity
    fqmax=50176.0*sqrt(7.0)/(19683.0*pi) # maximum of fq(q)
    v=zeros(n) # n-dim vector with zeros -> inital condition of velocity

    j=0 # inicialization of the the counter for v

    for i in range(3*n):
        q=random.uniform(0.0,1.0)
        fm=fqmax*random.uniform(0.0,1.0) # fraction of fqmax

        if fm <= fq(q):
            v[j] = q*sqrt(2.0*M*G)*(a**2.0 + r[j]**2.0)**(-1.0/4.0)   # q multiplied to the escaping speed
            j=j+1

        if j==n: # when v[j] is full, the loop stops
            break

    u0=random.uniform(0.0,1.0,n)
    u1=random.uniform(-1.0,1.0,n)
    theta=arccos(u1) # different angles from the ones used for position
    phi=random.uniform(0.0,2.0*pi,n)
    px,py,pz=spherical_to_cartesian(v,theta,phi) # n-dim vectors because v,theta and phi are n-dim vectors

    return x,y,z,px,py,pz
```

In [5]:
```
seed=20158401
n=10000
x0,y0,z0,px0,py0,pz0=initial_conditions(seed,n)
CI=array([x0,y0,z0,px0,py0,pz0]) # initial conditions (x,y,z in au and px,py,pz in au/Myr)
save("initial_conditions_less_bound_axions.npy", CI)
```

Hamilton's equations - interaction (Plummer + tidal forces)

In [1]:
```
def Hamilton_equations_interaction(f, t):
    x, y, z, px, py, pz = f # variables needed in order to calculate dpx,... (coordinates of the axion in the minihalo RS)

    dx = px # dx means dx/dt, and px means px/m=vx (speed)
    dy = py
    dz = pz

    # coordinates of the hyperbolic orbit of the minihalo around the star
    # anomalies
    psi_m = t*(v_0**3.0)/(G*M_star)
    psi_h = arcsinh(psi_m/exc)
    psi_v = 2.0*arctan(sqrt((exc+1.0)/(exc-1.0))*tanh(psi_h/2.0))
    psi = psi_v + psi_0
    #position
    r_cm_s = (a*(1.0-exc**2.0))/(1.0+exc*cos(psi_v))
    r_cm_s_3 = r_cm_s*r_cm_s*r_cm_s
    x_cm_s = r_cm_s*cos(psi)
    y_cm_s = r_cm_s*sin(psi)

    # acceleration of the star with respect to the minihalo CM (vector) (there's a change of perspective -> sign change)
    a_cm_s_x = -G*M_star*x_cm_s/r_cm_s_3
    a_cm_s_y = -G*M_star*y_cm_s/r_cm_s_3
    a_cm_s_z = 0.0 # z_cm_s = 0.0

    x_ax_s = x + x_cm_s
    y_ax_s = y + y_cm_s
    z_ax_s = z
    r_ax_s_2 = x_ax_s*x_ax_s + y_ax_s*y_ax_s + z_ax_s*z_ax_s
    r_ax_s_3 = sqrt(r_ax_s_2)*r_ax_s_2

    a_ax_s_x = -G*M_star*x_ax_s/r_ax_s_3
    a_ax_s_y = -G*M_star*y_ax_s/r_ax_s_3
    a_ax_s_z = -G*M_star*z_ax_s/r_ax_s_3

    r_2 = x*x + y*y + z*z

    # In addition to Plummer acceleration of the axion with respect the minihalo CM, the tidal forces are added,
    # so there is the total acceleration experienced by the axion
    dpx = -G*M_dm*x/a_dm_3/(r_2/a_dm_2+1.0)**(3.0/2.0) + (a_ax_s_x - a_cm_s_x)
    dpy = -G*M_dm*y/a_dm_3/(r_2/a_dm_2+1.0)**(3.0/2.0) + (a_ax_s_y - a_cm_s_y)
    dpz = -G*M_dm*z/a_dm_3/(r_2/a_dm_2+1.0)**(3.0/2.0) + (a_ax_s_z - a_cm_s_z)

    return dx, dy, dz, dpx, dpy, dpz # it is, in fact, dx, dy, dz, ddx, ddy, ddz (magnitudes of the axion)

    # with this function the phase coordinates of the axion in the minihalo system is obtained
```