Master in Actuarial and Financial Sciences

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Title: Analysis of fiscal-financial profitability of annuities

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ABSTRACT

In this paper, we perform an analysis about the importance of life annuities as a complementary income to public pension and the effect of taxation on the profitability of them. We have studied life annuities and taxation, as well as the effect that the former has on this profitability and, subsequently, we have performed an empirical application through life techniques. The results show that life annuities are one of the best products to improve the quality of life at retirement age and that they have a real tax advantage, which makes them very attractive in the Spanish market.

KEY WORDS

Life annuities | Taxation | Income | Retirement age | Fiscal-financial profitability

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1. INTRODUCTION

Older people need to have a sufficient level of income to live after retirement, however long it lasts. This will depend on life expectancy, that is, the average number of years a given cohort is expected to live from a given age at any point in time (RAE, 2022). But people could live longer than expected, which leads us to the longevity risk, that is, the risk that one or all people live longer than expected and, therefore, the risk that the reserves constituted and/or resources of another type for the payment of pensions (retirement, widowhood, orphanhood and disability) are insufficient for their purpose, as they are based on mortality tables with survival hypotheses lower than the real ones. (Afi, 2017)

As life expectancy has increased in recent decades and is expected to continue growing in the coming years, it is important to cover the longevity risk, because, if there is not longevity risk coverage it is possible that the beneficiary's resources may be exhausted prior to his death, but this phenomenon can be protected by life annuities. (Wolf, 2022)

Further, the tax advantage of life annuities makes it a much more attractive product to complement the public pension, but what is the real tax advantage of this product?

Thus, the fundamental objective of this thesis is the analysis of the fiscal-financial profitability of annuities. Likewise, other secondary objectives are proposed: i) it is intended to obtain a better understanding of the life annuities and their role in the Spanish market; ii) to know the taxation that is attached to annuities in Spain; iii) learn how to introduce the taxation variable in the profitability of annuities; iv) carry out an empirical application that provides us conclusions about the effect of the fiscal-financial profitability on annuities.

The results indicate that life annuities are a good product to complement the pension received at retirement age, since, in addition to increasing the income received, it provides a tax advantage that other products on the Spanish market do not offer.

The rest of the thesis is structured as follows: in the second section, a literature review is carried out, describing the annuities and their importance in the supplementary provision of public pensions in Spain. In the third section, the Spanish taxation is explained, as well as the fiscal-financial profitability of life annuities. Subsequently, in section 4, the empirical application is carried out. Finally, the main conclusions are shown in section 5.

2. LIFE ANNUITIES

In this section of the paper, a brief review of the literature is made in relation to the topics that are addressed in this thesis. Specifically, the main works that analyze life annuities and their role in complementary pension in Spain are summarized, focusing on the following papers: i.) (Afi, 2017); ii.) (Valero, 2013); iii.) (Galdeano and Herce, 2018); iv.) (Unespa, 2018); v.) (Finkelstein and Poterba, 1999); vi) (OECD, 2016).

2.1. DEFINITION

It is easy to define a life annuity as an income for life, but, according to the Real Academia Española, "a life annuity consists of a random contract in which one party transfers to another a sum or capital with the obligation to pay a pension to the transferor or to a third person during the life of the beneficiary" (Diccionario de la lengua española, 2022). Therefore, we could say that annuities are life insurance in which a flow of income is generated for life with an agreed frequency, normally monthly (BBVA, 2022). These incomes constitute a very important instrument nowadays because they serve as a complementary pension to be able to maintain, in the retirement period, the standard of living that the saver who contracts the life annuity had in his working stage, as will be shown in the following section.

Life annuities can be classified considering the following aspects:

i. Premiums payment frequency: the premium that the insured has to pay to the insurer can be a single payment or a periodic payment (fixed or variable): i.) the prior payment of the premium made by the taker of life annuities is a single amount; ii.) the premium payment made by the life annuity policyholder is made in installments, that is, in various amounts, which may be of the same or different amount over a period of accumulation.

ii. Type of annuity disbursement: The periodic annuity disbursement made by the annuity provider may be a fixed or variable amount: i.) the insurer pays a nominal amount guaranteed fixed; ii.) the insurer pays an amount that varies depending on different possibilities, which may be linked to inflation or vary depending on the performance of the underlying assets, for example, a stock market index.

iii. Initial date for receiving the income: the disbursement period can begin immediately after contracting the insurance or at a predetermined future date, for example, when the insured turns 65, with immediate life annuities being those in which the beneficiary begins to receive income immediately after the signing of the policy and deferred life annuities those that begin to be received from a contractually stipulated future date.

iv. Number of people covered: Annuity insurance can cover a single person or it can take the form of a reversible life annuity contract in which benefits are received while one of the two or more people covered by the policy lives: i.) in the case of a single-person income, the income will be received exclusively by a single beneficiary, while he is still alive; ii.) in an annuity with reversion, in the event of the death of the main beneficiary, a second beneficiary is designated, who will continue receiving a previously established percentage of the rent.

v. Additionally, life annuities can be contracted with or without guarantee in the event of the death of the insured: i.) with guarantee, an annuity will be received under the terms established in the contract while the insured is still alive, but if he dies within the term of the contract, his beneficiaries will receive an amount as death capital; ii.) without guarantee, an income will be received, higher than in the previous case, in the terms established in the contract while the insured is still alive, but if he dies within the validity of the contract, his beneficiaries will not receive any type of benefit.

vi. Way of acquiring the insurance: the acquisition of an annuity insurance can be done directly by a person, through a collective contract, normally through the employer, or through a pension plan, an Insured Pension Plan or a collective insurance that implements pension commitments.

2.2. USEFULNESS OF LIFE ANNUITIES IN SPANISH RETIREMENT

Retirement is one of the key moments in the life cycle that affects all workers, and constitutes one of the pillars of the longest and most intense tradition of social protection in all the countries of the world, whatever their level of development. In general, the main source of income after retirement is the pension provided by a public system, although it is common that the retired person has a reduction in income, since the salary before retirement is usually higher than the amount of the public retirement pension obtained. (European Commission, 2022)

Currently, the “problem” with pensions is that due to the increasing in the life expectancy, our retirement life cycle is getting longer, and our active life cycle is getting shorter (Galdeano and Herce, 2018). Older people need to have a sufficient level of income to live the rest of their life after retirement, however long it lasts, therefore, the acquisition of a product that increases the income provided by the Spanish public pension system seems necessary.

Both Social Security and the Private Pension Systems are based on a very simple principle: the effort to make contributions during the active cycle, and their conversion into retirement income during the passive period. These annuities, by definition, have the nature of "life annuities", but must be backed by sufficient resources. (European Commission, 2021)

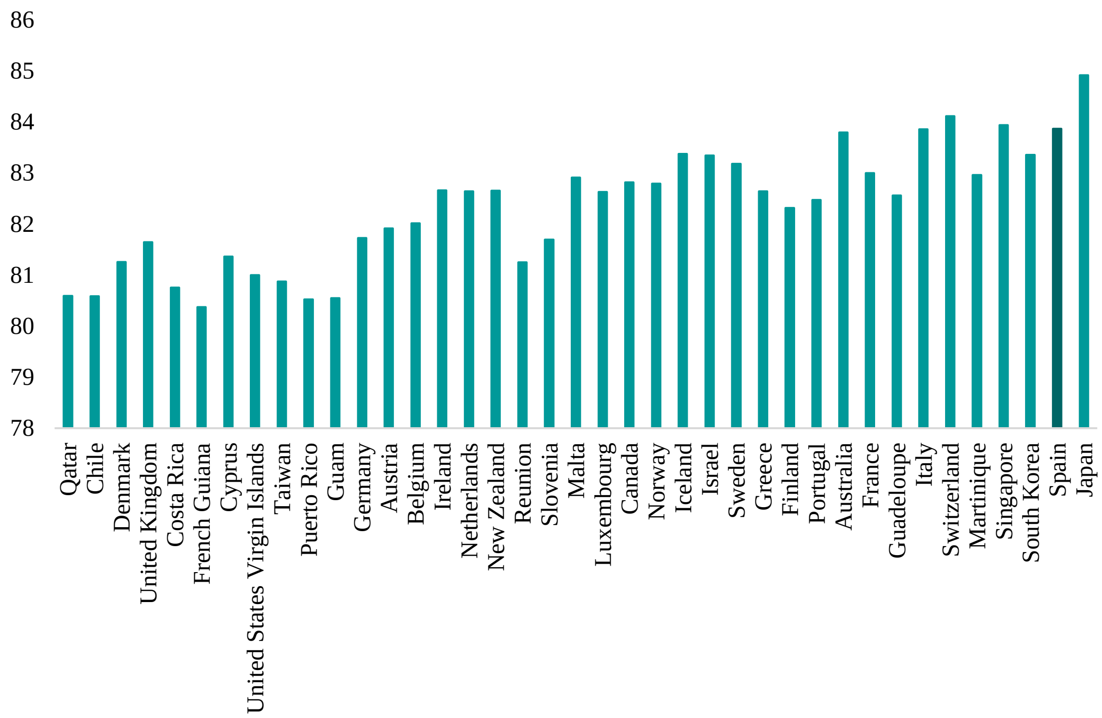
In this context, life annuities, formulated on the basis of the actuarial principle practiced by the insurance industry, are sustainable, since the resources for their payment are accumulated, preserved and made profitable from the first contribution to the last payment made. By definition, the available resources are exhausted exactly when the last payment is made at the time of the death of the annuity holder. (McKinsey, 2020)

However, the "life annuities" paid by Social Security in Spain do not adhere to this principle, because they are financed with current resources provided by current contributors, based on the principle of intergenerational solidarity, therefore, there is no accumulated capital nor, in general, strict financial-actuarial correspondence with the contributions. This means that public pensions are annuities only in the sense that each system promises to pay them until the death of the holder. (MAPFRE Economics, 2021)

Due to the historical delay in adaptation that Social Security systems accumulate, for example, in terms of retirement age, many of these systems present enormous financial imbalances and levels of implicit debt in the future that call into question the sustainability, sufficiency, or both of public pensions in the medium and long term.

In recent decades, life expectancy, that is, the mean time left to live for an individual in a given biological population (RAE, 2022), has experienced notable advances, and we have managed to reduce the probability of dying due to factors such as: i.) medical and technological advances; ii.) the reduction in infant mortality rates; iii.) changes in nutritional habits and lifestyles; iv.) improvement in the levels of material living conditions and education; v.) the population's access to comprehensive and advanced health services. In this sense, it should be noted that Spain is the fourth country in the world, behind Japan, Switzerland and Singapore, with the highest life expectancy according to the latest world statistics published by the United Nations, as can be seen in graph 2.1., which shows the life expectancy at birth of the countries with the highest life expectancy in the year 2022.

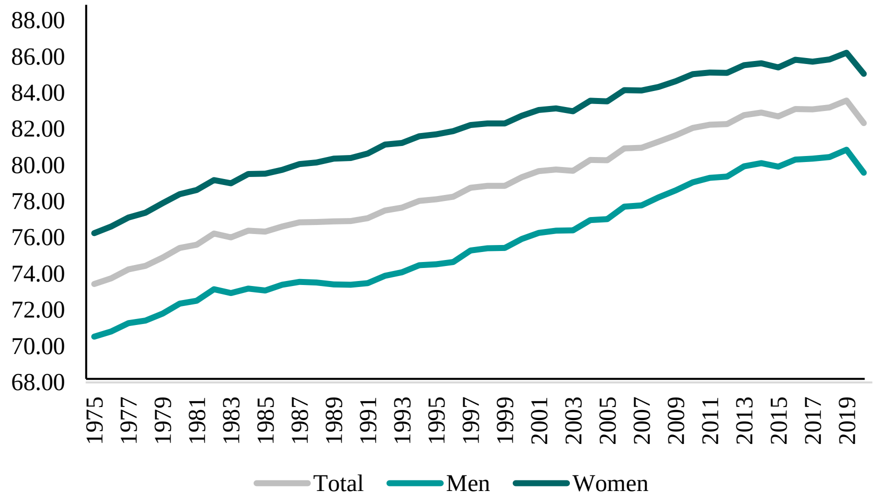
Graph 2.1. Life expectancy



Source: own elaboration with (United Nations, 2022) data.

From an individual point of view, prolonging life may be attractive, but from a socioeconomic point of view, it means compromising the retirement age, aging of the labor life and, ultimately, the economic support of retirement, since Social Security will pay retirement pensions for a longer time, regardless of the years contributed by each of the citizens. In addition, in the last ten years, in Spain, the number of pensioners has risen to reach 6,234,609 pensioners who receive a retirement benefit in March 2022 (Gobierno de España, 2022), this is due to the improvement in life expectancy in recent decades that has been remarkable and that we can see in graph 2.2.

Graph 2.2. Spanish life expectancy at birth



Source: own elaboration with (INE, 2020b) data.

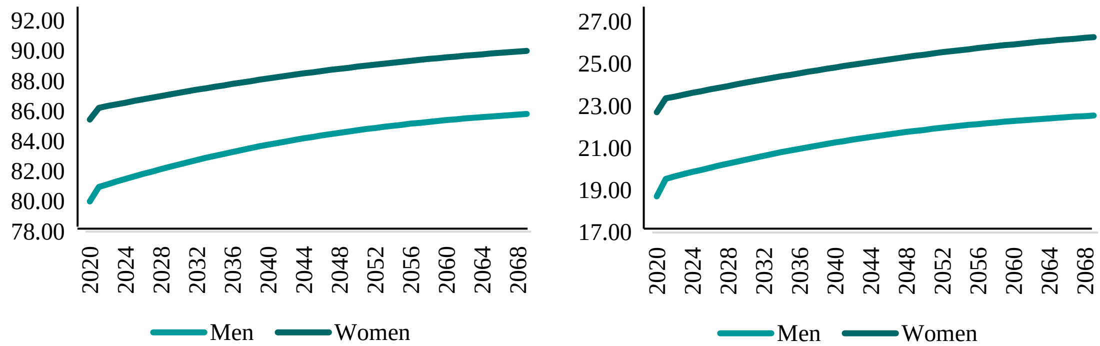
Moreover, life expectancy at 65 years has increased in more than 5 years in the past 45 years, having an approximate growth of 35%, as can be observed in table 2.1., and this trend is expected to continue, as can be seen in the projections set out in graph 2.3.

Table 2.1. Spanish life expectancy at age 65

	1975	2020	Growth
Men	13.62	18.35	34.73%
Women	16.49	22.31	35.29%
Total	15.19	20.41	34.36%

Source: own elaboration with (INE, 2020a) data.

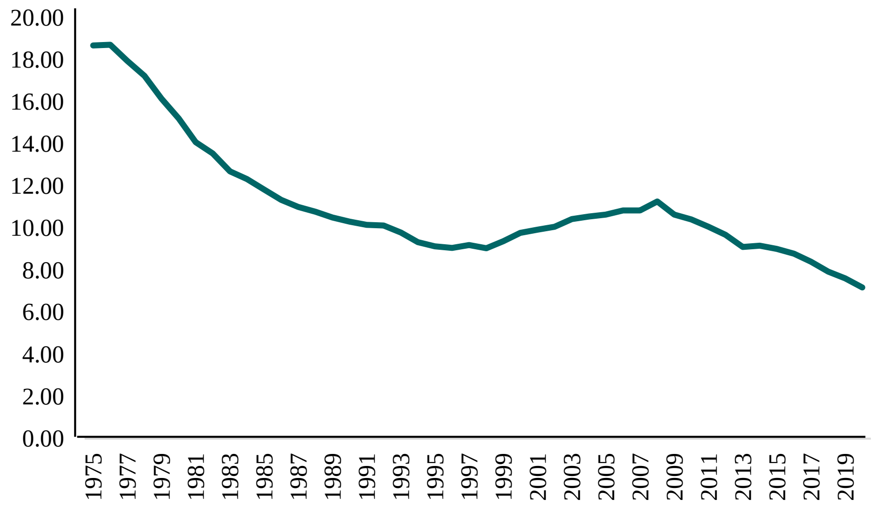
Graph 2.3. Spanish life expectancy projections at birth (left) and at 65 (right)



Source: own elaboration with (INE, 2020c) data.

This added to the modest and declining number of births, that we can observed in graph 2.4., which reduces the potential number of contributing workers, makes the sustainability of the Social Security system more difficult and, consequently, the downward pressure on the amounts of public pensions is heightened, to stabilize the system.

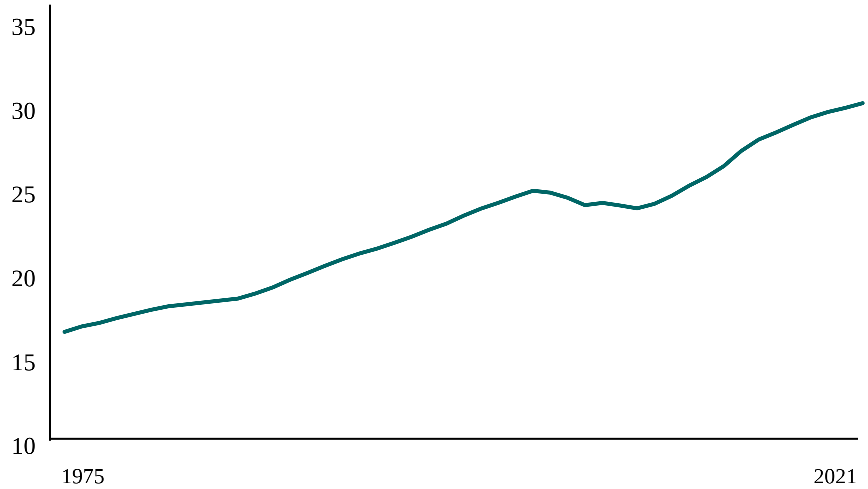
Graph 2.4. Spanish birth rate



Source: own elaboration with (INE, 2020d) data.

In this sense, according to the Instituto Nacional de Estadística, the dependency ratio, which indicates the proportion between the population over 64 years of age and the population of working age (from 16 to 64 years of age), has been increasing in recent years, as we can see in graph 2.5, which worsens the sustainability of the system.

Graph 2.5. Spanish dependency ratio (1975-2021)



Source: own elaboration with (INE, 2020e) data.

The difficulty in increasing public resources for pensions and the "problems" that have been commented make it increasingly necessary to cover the reduction in income at retirement age, so it is convenient to plan the receipt of a second retirement income in advance and consider the suitability of contracting pension savings products that, accumulating this throughout the working life, allow generating additional income during retirement. In addition, the rationalization of public pension systems around the world, to make them more sustainable, is forcing to reduce the promises of maintaining the purchasing power of pensions, which leads to the challenge of their sufficiency. It is precisely in this context that life annuities acquire an essential role compared to other products that pursue the same goal of providing retirees with supplementary income, since the life annuity guarantees that no insured outlives their savings in a very flexible, effective and efficient way (Brown et al., 2001), as will be shown in this thesis.

In other words, life annuities are one of the best instruments for reducing uncertainty and maximizing wealth with respect to the income to be received at retirement age, since, in the absence of a life annuity, the retiree would have to personally manage his assets based on his life expectancy, running the risk of exhausting his assets or living below his means by miscalculating his life expectancy. Life annuities manage to mutualize deviations in life expectancy on both sides of the average and eliminate the risk of living above or below one's possibilities and of falling into poverty or inefficiency. (Akerlof, 1970)

But, technically, the most relevant aspect of life annuities is that they transfer the longevity risk from the saver to the insurance company, which is possible for the latter through the corresponding mutualization of the longevity risk in heterogeneous individuals. Faced with this determining aspect of the nature of life annuities, the financial aspects themselves are secondary, although not insignificant. (OECD, 2022)

Summarizing, the conclusions on life annuities in the field of complementary social security are the following: i.) life annuities are adapted to the reality of each insured, and there is a universe of possible annuity products among which the citizen can choose, depending on his personal, familiar, economic circumstances, etc; ii.) life annuities constitute an ideal instrument to complement the public retirement pension for maintaining the economic capacity of retirees, with the consequent positive influence on the Spanish economy and on employment; iii.) life annuities are efficient products due to the transfer of longevity risk to the insurer.

3. LIFE ANNUITIES TAXATION

In this section of the paper a brief review of the literature is made in relation to the topics that are addressed in this thesis. Specifically, the main works that analyze life annuities taxation and fiscal-financial profitability are summarized, focusing on the following papers: i.) (González-Vila et al., 2005); ii.) (González-Vila et al., 2006); iii.) (González-Vila, L., et al., 2016) ;iv.) (Edufinet, 2022a).

3.1. SPANISH TAXATION OF LIFE ANNUITIES

To know the taxation of life annuities in Spain, we have taking into account the following laws: i.) “Ley 26/2014, de 27 de noviembre, por la que se modifican la Ley 35/2006, de 28 de noviembre, del Impuesto sobre la Renta de las Personas Físicas, el texto refundido de la Ley del Impuesto sobre la Renta de no Residentes, aprobado por el Real Decreto Legislativo 5/2004, de 5 de marzo, y otras normas tributarias.”; ii.) “Ley 35/2006, de 28 de noviembre, del Impuesto sobre la Renta de las Personas Físicas y de modificación parcial de las leyes de los Impuestos sobre Sociedades, sobre la Renta de no Residentes y sobre el Patrimonio.”

The two generally applicable taxes to life annuities in Spain are Personal Income Tax (IRPF) and the Inheritance and Gift Tax (ISD). In addition, the accumulated economic rights must be taxed by the Wealth Tax (IP) when the taxpayer reaches minimum levels of assets and the Autonomous Community of residence decides not to apply discounts on this tax.

The principal of a life annuity will not be subject to tax, while the portion of income from movable capital of said benefit will be integrated into the income tax base and will be subject to withholdings on account.

It is considered, therefore, that part of the income received is return on movable capital, being its tax burden the result of applying the rate for returns on capital at a diminishing fraction of each annuity depending on the holder of the income that goes from 40% when its owner is under 40 years up to 8% when the holder is 70 or older, as can be observed in table 2.1. These percentages will be determined at the time of the constitution of the income, according to the age of the holder and will remain constant throughout its validity, so, in the case of immediate life annuities, the income for which the taxpayer is going to pay taxes will be the result of applying the following percentages to each annuity:

Table 3.1. Spanish taxation for movable capital

Beneficiary age	g_1
Less than 40 years	40%
Between 40 and 49	35%
Between 50 and 59	28%
Between 60 and 65	24%
Between 66 and 69	20%
More than 70 years	8%

Source: own elaboration with (Edufinet, 2022b) data.

Thus, it will not be taxed for the entire amount received, but only for the result of applying the corresponding percentage from the previous table to the income obtained. In this way, the result of multiplying the annuity received by the corresponding percentage becomes the return on movable capital that will be included in the tax base of savings in order to apply the tax rate according to table 3.2. (Edufinet, 2022b)

Table 3.2. Movable capital spanish retention

Tax	Retention
0 - 6.000€ -> 19% **6.000,01 - 50.000€ -> 21%** **More than 50.000,01€ -> 23%**	19%

Source: own elaboration with (Impuesto sobre la renta de las personas naturales, 2022) data.

So, given an example of a 100€ annuity, the taxes to be paid will be 7,60€ if the insured has less than 40 years old and 1,52€ if he's older than 70 years old, according to table 3.3.

Table 3.3. Taxation example

Beneficiary age	g_1	Tax (19%) per 100€
Less than 40 years	40%	7.60 €
Between 40 and 49	35%	6.65 €
Between 50 and 59	28%	5.32 €
Between 60 and 65	24%	4.56 €
Between 66 and 69	20%	3.80 €
More than 70 years	8%	1.52 €

Source: own elaboration with (Impuesto sobre la renta de las personas naturales, 2022) data.

3.2. FISCAL-FINANCIAL PROFITABILITY

The fiscal-financial profitability indicates the benefit, negative or positive, derived from a financial asset, taking into account the interest rate of the product and the tax implications, whether they are deductions, charges, etc., that the asset and its returns may have. (Fundación Mapfre, 2022)

The current economic situation in which we are immersed, in which the interest rates offered by the different financial products available in the Spanish market hardly exceed inflation, has made investment proliferate by individuals in certain products that, in addition to offer a more or less secure return, incorporate certain tax advantages in the form of bonuses, reductions or exemptions. These advantages generate a return addition that, together with the own financial profitability of the product, gives rise to what is known in the market as fiscal-financial profitability (González-Vila et al., 2006). This tax advantage increases, in fact, the profitability of the product. If we compare two insurance products with the same technical interest rate, one with tax advantages and the other without tax advantages, it seems logical to think that the one with tax advantages has a return higher than the technical interest rate due to having tax advantages.

And although the decisive factor in choosing one financial product or another is, in general, its profitability which is, for annuities, the technical interest rate, it should not be forgotten that the final remuneration will be produced, to a large extent, from the taxation of the asset. To find out how much a product earns, you have to see the fiscal-financial return, as well as the withholding applied by the Agencia Tributaria on the yields generated, thus it will allow us to know if a product has a real tax advantage. (Banc Sabadell, 2010)

According to González-Vila et al. (2006), in the Spanish financial market, they coexist two different conceptions of fiscal-financial profitability. The first understands fiscal-financial profitability of a product as the annual return that will be obtained after considering the taxation of the product and of the investor. In this case, the fiscal-financial profitability is, therefore, a net or after-tax return. On the other hand, the second conception understands fiscal-financial profitability of a product as the gross annual return that an alternative financial product should offer without any type of tax advantages so that, after taking into account the taxation of the two products and the investor, both would provide the same accumulated amount after taxes. It is, then, in this case, a gross return. In this thesis, we will calculate both profitabilities (gross and net).

4. PRACTICAL APPLICATION

In this section we will perform a practical application of the concepts that have been explained in the past sections. That is, a calculation of the value of fiscal-financial profitability of life annuities. And we will calculate that taking into account the products that two of the biggest financial companies in Spain offer: i.) (VidaCaixa, 2022); ii.) (Banc Sabadell, 2022)

In life insurance operations, the payment of the financial capital involved depends on the survival or death of the insured, so the financial capitals involved are random variables whose probabilities will have to be determined.

In order to quantify the above probabilities, there are two possibilities:

i. Work with survival models.
ii. Work with mortality tables.

We will work with the mortality tables, which contain the values of the annual probabilities of death q_x. These tables consist in discrete empirical data obtained from population censuses or from mortality observed in certain groups. In these cases, q_x is defined for integer values of x within the interval (x_0,w), where x_0 is the initial age of the group and w is the actuarial infinity or the first age in which no one from the group is alive.

But it is more convenient to work with l_x, which indicates the number of people alive at a certain age x (expressed in years) from an initial theoretical group l_{x0} of people alive at the initial age x_0, in general, equal to 1,000,000. As the life annuities that we will value are monthly, we will need tables with monthly ages that will be obtained from the annual ones assuming the hypothesis of uniform distribution of mortality within the year. In this case, the monthly life annuity is obtained by linear interpolation of the annual l:

$$l_x - l_{x+\frac{t}{12}} = \frac{t}{12} \cdot (l_x - l_{x+1}) \rightarrow l_{x+\frac{t}{12}} = \left(1 - \frac{t}{12}\right) \cdot l_x + \frac{t}{12} \cdot l_{x+1}$$

We rewrite the monthly l as $l_{x+\frac{t}{12}} = l_{12x+t}^{(12)}$ being $12x + t$ the age in months. Based on this data, we can calculate the values of the monthly survival and the death probabilities:

$$_{\frac{t}{12}}p_x = \frac{l_{x+\frac{t}{12}}}{l_x} = \frac{l_{12x+t}^{(12)}}{l_{12x}^{(12)}} \qquad _{\frac{t}{12}/\frac{1}{12}}q_x = \frac{l_{x+\frac{t}{12}} - l_{x+\frac{t+1}{12}}}{l_x} = \frac{l_{12x+t}^{(12)} - l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}}$$

Being $_{\frac{t}{12}}p_x$ the probability that the insured of age x will survive to age $x + \frac{t}{12}$ and $_{\frac{t}{12}/\frac{1}{12}}q_x$ the probability that the insured of age x will die between the ages $x + \frac{t}{12}$ and $x + \frac{t+1}{12}$.

The regulation on which mortality tables are based in the Spanish insurance sector is the following: "Resolución de 17 de diciembre de 2020, de la Dirección General de Seguros y Fondos de Pensiones, relativa a las tablas de mortalidad y supervivencia a utilizar por las entidades aseguradoras y reaseguradoras, y por la que se aprueba la guía técnica relativa a los criterios de supervisión en relación con las tablas biométricas, y sobre determinadas recomendaciones para fomentar la elaboración de estadísticas biométricas sectoriales".

This resolution establishes two types of tables, those that exclusively include biometric risk, called second-order tables, and those that, in addition to including biometric risk, include technical surcharges that try to capture other types of risks, such as, for example, risks of model, the level of volatility, etc. These are called first-order tables.

The second-order tables are intended for the calculation of the Best Estimate Liabilities (BEL) following the Solvency II regulations and the first-order tables are used to calculate the accounting provisions or reserves and the pricing of the products. The DGSFP Resolution establishes generational tables for longevity, PER2020, and other tables for mortality, PASEM2020, differentiating these by type of product. In this thesis we will use the following tables:

i. PER2020.UX1, that are the tables used in longevity products, which are surcharged by the risks mentioned above and unisex, that is, that they take into account a sample made up of 50% men and 50% women. These tables consider the initial age to be 0 years and the actuarial infinity 120 years.

ii. PASEM2020.UX1, that are the tables used in the mortality products and which, like the previous ones, are surcharged and unisex. These tables consider the initial age to be 0 years and the actuarial infinity 109 years.

The PER2020 are generational tables and therefore the annual death probabilities for each age x depend on the year of birth of the insured. However, the PASEM2020 are non-generational tables and, therefore, the values of the annual death probabilities for each age x do not depend on the year of birth of the insured.

It should also be noted that the most common annuities in the Spanish market for complementing the public pension are usually contracted at the retirement age or even later, once the person is already retired, so these are the ones that we will value in this thesis. They have the following specific characteristics: i) Premiums payment frequency: single premium; ii) Type of annuity disbursement: fixed monthly amount; iii) Initial date for receiving the income: immediate and expired; iv) Number of people covered: single person (insured); v) Without death insurance and with death insurance equal to the single premium; vi) Way of acquiring the insurance: directly by the insured.

Therefore, we will value the following life annuities:

i. Monthly immediate and expired life annuities without death insurance (life annuities to assigned capital), in which case, the insured receives a big monthly income, but his beneficiaries do not have the right to obtain the capital sum for death and the refund of savings. (VidaCaixa, 2022)

ii. Monthly immediate and expired life annuities with death insurance that returns the single premium, this means that in case of death of the insured, his beneficiary would receive the total amount of the premium the insured paid at the time he contracted the life annuity.

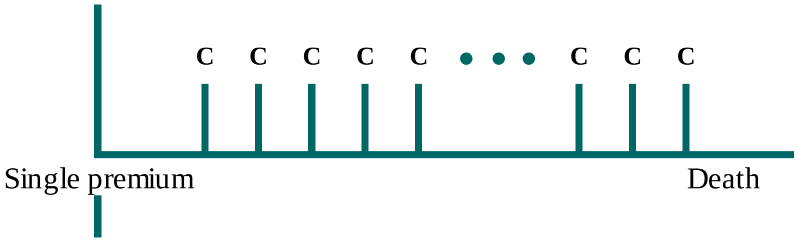
To simplify the calculations, we will assume that there are no internal management fees and no external management fees, so the single premium paid is a pure premium.

In addition, it should be mentioned that in Spain there are 287,200 people who receive a pre-retirement pension (INE, 2022), so we will consider a life annuity contracted at early retirement age or later (between 55 and 75) at single premium with a monthly fixed amount paid by the insurer. The insurer pays to the contract the technical interest rate which is denoted by I_1 and gives us the gross interest rate (before taxes) of the contract.

4.1. MONTHLY IMMEDIATE LIFE ANNUITIES WITHOUT DEATH INSURANCE

In this case, we are assuming a monthly immediate and expired annuity without death insurance, which is a product in which the insured pays a single premium at the beginning of the contract and, in exchange, he receives at the end of each month an amount of money until he dies. We can see the graphic representation in the following figure:

Figure 4.1. Monthly inmediate life annuities without death insurance graphic representation



Source: own elaboration.

In addition, we will assume that we know the import of the single premium which is equal to the savings that the insured wants to allocate to the life annuity, which is 100,000€. We will also assume that the technical interest rate is invariable, and it is equal to 1.5%, therefore, we will compute, from the initial equilibrium equation, the monthly amount C:

$$\pi = C \cdot a_x^{(12)} = \sum_{t=0}^{(120-x)12-1} C \cdot (1+I_1)^{-\frac{t+1}{12}} \cdot {}_{\frac{t+1}{12}}p_x = \sum_{t=0}^{(120-x)12-1} C \cdot (1+I_1)^{-\frac{t+1}{12}} \cdot \frac{l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}}$$

where:

- π : Single premium paid by the insured at the beginning of the contract
- C : Monthly amount that the insured will receive at the age of retirement
- x : Age of the insured at the beginning of the contract
- ${}_{\frac{t+1}{12}}p_x$: Probability that the insured of age x will reach the age $x + \frac{t+1}{12}$
- Technical bases: $I_1 = 0.015$ and mortality table PER2020.UX1

Later, in order to analyze the fiscal-financial profitability, this is, to know the influence of Spanish taxation in life annuities, we will first focus on the net interest rate which will be denoted by I_1^2 and that can be deduced from the following formula:

$$\pi = \sum_{t=0}^{(120-x)12-1} C \cdot (1 - g_1 \cdot Tax) \cdot (1 + I_1^2)^{-\frac{t+1}{12}} \cdot {}_{\frac{t+1}{12}}p_x$$

$$\pi = \sum_{t=0}^{(120-x)12-1} C \cdot (1 - g_1 \cdot Tax) \cdot (1 + I_1^2)^{-\frac{t+1}{12}} \cdot \frac{l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}}$$

Obviously I_1^2 would be lower than I_1, because this interest rate is the net interest rate that includes the effect of taxation. We will next calculate the fiscal-financial profitability, that we denote by I_1^3, which represents the gross interest rate that a product without tax advantage would have to pay for its net profitability to be the same as the one of the life annuities offered in the market. This profitability can be deduced from the following formula:

$$\sum_{t=0}^{(120-x)12-1} C\cdot(1+I_1^3)^{-\frac{t+1}{12}}\cdot {}_{\frac{t+1}{12}}p_x = \sum_{t=0}^{(120-x)12-1} C\cdot(1-Tax)\cdot(1+I_1^2)^{-\frac{t+1}{12}}\cdot {}_{\frac{t+1}{12}}p_x$$

$$\sum_{t=0}^{(120-x)12-1} C\cdot(1+I_1^3)^{-\frac{t+1}{12}}\cdot \frac{l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}} = \sum_{t=0}^{(120-x)12-1} C\cdot(1-Tax)\cdot(1+I_1^2)^{-\frac{t+1}{12}}\cdot \frac{l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}}$$

Being I_1^3 higher than I_1 and I_1 higher than I_1^2, because a product without tax advantage would have to offer a higher interest rate than a product with tax advantage to provide the same net profitability.

In table 4.1. we can see the amount of money that the insured is going to receive monthly (C), the technical interest rate used, I_1, the net interest rate, I_1^2, the fiscal-financial profitability, I_1^3, and the taxes that the saver will have to pay for each C, as well as the percentage g_1 that the insured may apply depending on his age for the calculation of these taxes.

Table 4.1. Inmediate annuities without death insurance

Age	*C*	I_1	I_1^2	I_1^3	g_1	*Taxes*
55	294.73 €	1.5%	1.1953%	2.4194%	28%	15.68 €
56	301.20 €	1.5%	1.1882%	2.4410%	28%	16.02 €
57	308.04 €	1.5%	1.1808%	2.4637%	28%	16.39 €
58	315.28 €	1.5%	1.1729%	2.4876%	28%	16.77 €
59	322.96 €	1.5%	1.1647%	2.5127%	28%	17.18 €
60	331.12 €	1.5%	1.2057%	2.5950%	24%	15.10 €
61	339.81 €	1.5%	1.1978%	2.6245%	24%	15.50 €
62	349.11 €	1.5%	1.1895%	2.6650%	24%	15.92 €
63	358.96 €	1.5%	1.1808%	2.6887%	24%	16.37 €
64	369.40 €	1.5%	1.1715%	2.7237%	24%	16.84 €
65	380.53 €	1.5%	1.1617%	2.7608%	24%	17.35 €
66	392.43 €	1.5%	1.2099%	2.8663%	20%	14.91 €
67	405.23 €	1.5%	1.2007%	2.9105%	20%	15.40 €
68	419.05 €	1.5%	1.1909%	2.9575%	20%	15.92 €
69	434.04 €	1.5%	1.1805%	3.0076%	20%	16.49 €
70	450.32 €	1.5%	1.3684%	3.2866%	8%	6.84 €
71	468.06 €	1.5%	1.3637%	3.3522%	8%	7.11 €
72	487.41 €	1.5%	1.3587%	3.4223%	8%	7.41 €
73	508.53 €	1.5%	1.3533%	3.4973%	8%	7.73 €
74	531.57 €	1.5%	1.3475%	3.5777%	8%	8.08 €
75	556.70 €	1.5%	1.3414%	3.6637%	8%	8.46 €

Source: own elaboration.

The older the insured is, the lower will be his life expectancy, so the years that the insurance company will have to pay him will be lower, and that is why the amount of money that the insured is going to receive is higher the older he is, as can be observed in table 4.1.

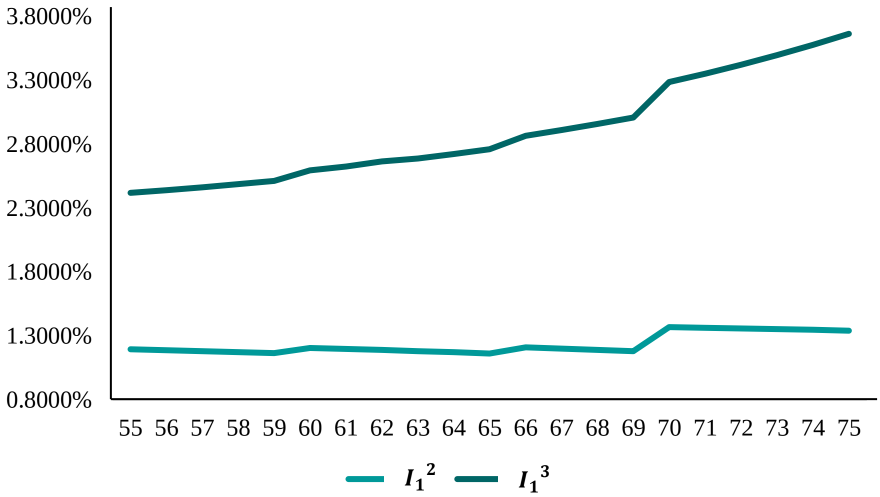
We can also see in the table that the net interest rate, I_1^2, grows in general, going from 1.1953% to 1.3414% throughout the sample, but, within each tax bracket, this interest rate decreases. This is because, the higher C is, the lower the net return will be.

As in the previous case, I_1^3, this is the fiscal-financial profitability, increases in every tax bracket and in general, going from 2.4194% to 3.6637% throughout the sample. This means that the alternative product without tax advantage would have to offer an increasing return, which, knowing that the tax advantage is increasing, makes sense, because the higher the advantage the higher will be the net profitability.

It can also be seen in this table that, in all cases, ${I_1}^3$ is greater than I_1 and I_1 is greater than ${I_1}^2$, as already mentioned in the previous section, because the net interest rate has to be lower than the gross one and the fiscal-financial profitability have to be greater than the profitability offered by the product with tax advantage.

All of this can also be seen in graph 4.1., in which the tax brackets by age can be clearly perceived, and in which we can observed the similarity of the growth of the two interest rates, ${I_1}^2$ and ${I_1}^3$.

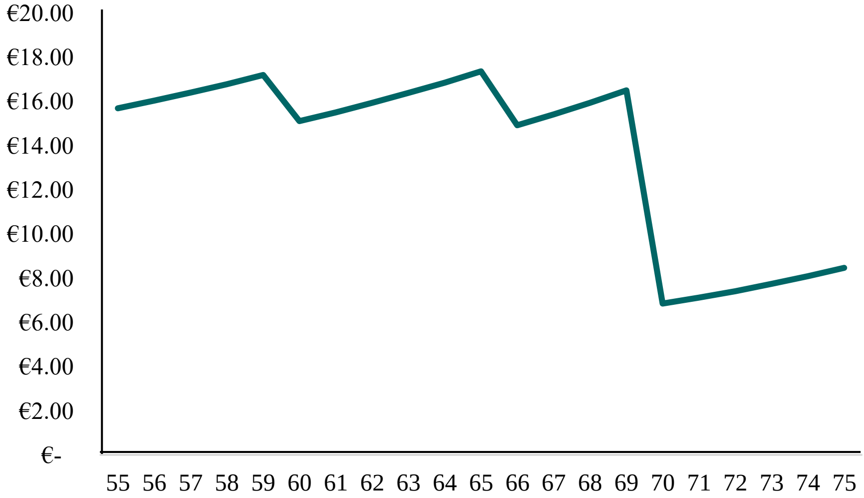
Graph 4.1. Immediate annuities without death insurance (${I_1}^2$ ***and*** ${I_1}^3$)



Source: own elaboration.

In addition, in graph 4.2., we can see that the taxes that the insured have to pay get lower the older he is, this is due to the percentage by which C is taxed, that gets smaller. Besides that, it should also be noted that these taxes get higher within the tax brackets in line with the interest rates mentioned above, despite the fact that in general it goes from 15.68€ to 8.46€. In this graph we can also clearly observe the tax brackets by age.

Graph 4.2. Immediate annuities without death insurance (Taxes)



Source: own elaboration.

4.2. MONTHLY IMMEDIATE LIFE ANNUITIES WITH DEATH INSURANCE

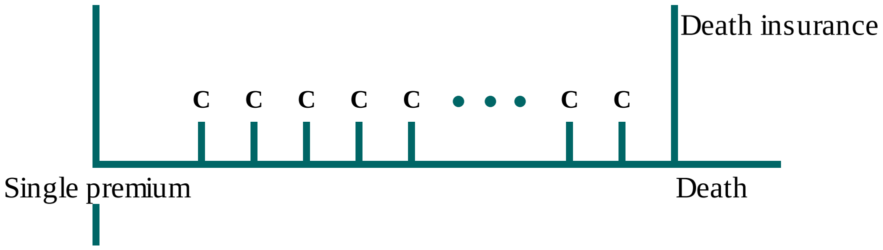
Death insurance are actuarial operations consisting of the payment of a certain amount if the contingency of death of the insured person occurs, but we can distinguish between two types of death insurance:

i. Discrete insurance, in which the benefit is paid at the end of the period of death.

ii. Continuous insurance, in which the benefit is paid at the very moment of death.

In this section we will consider a life annuity contracted at early retirement age or later at single premium with a monthly and expired fixed amount paid by the insurer. The insurer pays to the contract the technical interest rate which is denoted by I_1 and gives us the gross interest rate (before taxes) of the contract. In this case, in addition, we take into account that the insured has contracted a death insurance, so, the beneficiary of the policy would receive the total amount of money equal to the single premium that the insured paid at the beginning of the contract (π) at the end of the month when the insured dies, this is a monthly frequency discrete insurance of death. We can see the graphic representation in the following figure:

Figure 4.2. Monthly inmediate life annuities with death insurance graphic representation



Source: own elaboration.

In addition, we will assume that we know the import of the single premium which is equal to the savings that the insured wants to allocate to the life annuity, which is 100,000€. We will also assume that the technical interest rate is invariable, and it is equal to 1.5%, as in the previous case. Therefore, we will compute, from the initial equilibrium equation, the monthly amount *C*:

$$\pi = C \cdot a_x^{(12)} + \pi \cdot A_x^{(12)}$$

$$\pi = \sum_{t=0}^{(120-x)12-1} C \cdot (1+I_1)^{-\frac{t+1}{12}} \cdot {}_{\frac{t+1}{12}}p_x + \sum_{t=0}^{(109-x)12-1} \pi \cdot (1+I_1)^{-\frac{t+1}{12}} \cdot {}_{\frac{t}{12}/\frac{1}{12}}q_x$$

$$\pi = \sum_{t=0}^{(120-x)12-1} C \cdot (1+I_1)^{-\frac{t+1}{12}} \cdot \frac{l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}} + \sum_{t=0}^{(109-x)12-1} \pi \cdot (1+I_1)^{-\frac{t+1}{12}} \cdot \frac{l_{12x+t}^{(12)} - l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}}$$

where:

- π : Single premium paid by the insured at the beginning of the contract
- C : Monthly amount that the insured will receive at the age of retirement
- x : Age of the insured at the beginning of the contract
- ${}_{\frac{t+1}{12}}p_x$: Probability that the insured of age x will reach the age $x + \frac{t+1}{12}$
- ${}_{\frac{t}{12}/\frac{1}{12}}q_x$: Probability that the insured of age x dies between the ages $x + \frac{t}{12}$ and $x + \frac{t+1}{12}$.
- Technical bases: $I_1 = 0.015$ and mortality table PER2020.UX1 for chances of survival and PASEM2020.UX1 for the probability of death.

Later, in order to analyze the fiscal-financial profitability, to know the influence of Spanish taxation in life annuities, we first focus on the net interest rate which will be denoted by $I_1{}^2$ and can be deduced from the following formula:

$$\pi = \sum_{t=0}^{(120-x)12-1} C \cdot (1 - g_1 \cdot Tax) \cdot \left(1 + I_1{}^2\right)^{-\frac{t+1}{12}} \cdot {}_{\frac{t+1}{12}}p_x + \sum_{t=0}^{(109-x)12-1} \pi \cdot \left(1 + I_1{}^2\right)^{-\frac{t+1}{12}} \cdot {}_{\frac{t}{12}/\frac{1}{12}}q_x$$

$$\pi = \sum_{t=0}^{(120-x)12-1} C \cdot (1 - g_1 \cdot Tax) \cdot \left(1 + I_1{}^2\right)^{-\frac{t+1}{12}} \cdot \frac{l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}} + \sum_{t=0}^{(109-x)12-1} \pi \cdot \left(1 + I_1{}^2\right)^{-\frac{t+1}{12}} \cdot \frac{l_{12x+t}^{(12)} - l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}}$$

Obviously I_1^2 would be lower than I_1, because this interest rate includes the effect of taxation.

We will next calculate the fiscal-financial profitability, that we denote by I_1^3, which represents the gross interest rate that a product without tax advantage would have to pay for its net profitability to be the same as the one of the life annuities. This profitability can be deduced from the following formula:

$$\sum_{t=0}^{(120-x)12-1} C \cdot (1+I_1{}^3)^{-\frac{t+1}{12}} \cdot {}_{\frac{t+1}{12}}p_x + \sum_{t=0}^{(109-x)12-1} \pi \cdot (1+I_1{}^3)^{-\frac{t+1}{12}} \cdot {}_{\frac{t}{12}/\frac{1}{12}}q_x =$$

$$= \sum_{t=0}^{(120-x)12-1} C \cdot (1-\mathrm{Tax}) \cdot (1+I_1{}^2)^{-\frac{t+1}{12}} \cdot {}_{\frac{t+1}{12}}p_x + \sum_{t=0}^{(109-x)12-1} \pi \cdot \left(1+I_1{}^2\right)^{-\frac{t+1}{12}} \cdot {}_{\frac{t}{12}/\frac{1}{12}}q_x$$

$$\sum_{t=0}^{(120-x)12-1} C \cdot (1+I_1{}^3)^{-\frac{t+1}{12}} \cdot \frac{l_{12\cdot x+t+1}^{(12)}}{l_{12\cdot x}^{(12)}} + \sum_{t=0}^{(109-x)12-1} \pi \cdot (1+I_1{}^3)^{-\frac{t+1}{12}} \cdot \frac{l_{12x+t}^{(12)} - l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}} =$$

$$= \sum_{t=0}^{(120-x)12-1} C \cdot (1-\mathrm{Tax}) \times (1+I_1{}^2)^{-\frac{t+1}{12}} \cdot \frac{l_{12\cdot x+t+1}^{(12)}}{l_{12\cdot x}^{(12)}} + \sum_{t=0}^{(109-x)12-1} \pi \cdot (1+I_1{}^2)^{-\frac{t+1}{12}} \cdot$$

$$\cdot \frac{l_{12x+t}^{(12)} - l_{12x+t+1}^{(12)}}{l_{12x}^{(12)}}$$

Being $I_1^3 > I_1 > I_1^2$, because a product without tax advantage would have to offer a higher interest rate than a product with tax advantage to provide the same net profitability.

It is also important to comment that death insurance is taxed by inheritance and donation tax, but we will not consider the taxation of the death insurance because he taxes applied change depending on the lows of the different places in the country, so it will not be possible to calculate an example that would serve to every insured in Spain. Moreover, these taxes would rely on the beneficiaries, so it would not be logical to taken them into account.

In table 4.2. we can see the amount of money that the insured is going to receive monthly (C), the technical interest rate used, I_1, the net interest rate, I_1^2, the fiscal-financial profitability, I_1^2, and the taxes that the saver will have to pay for each C, as well as the percentage g_1 that the insured may apply depending on his age for the calculation of these taxes.

The older the insured is, the lower will be his life expectancy, so the years that the insurance company will have to pay him will be lower, but the higher will be the probability of death, which translates into the obligation of the insured to pay the death insurance in the short term. And that is the reason why the amount of money that the insured is going to receive is lower the older he is, as can be observed in table 4.2.

Table 4.2. Inmediate annuities with death insurance

Age	*C*	I_1	I_1^2	I_1^3	g_1	*Taxes*
55	110.14 €	1.5%	1.4219%	1.7104%	28%	5.860 €
56	110.04 €	1.5%	1.4219%	1.7103%	28%	5.854 €
57	109.95 €	1.5%	1.4218%	1.7102%	28%	5.849 €
58	109.85 €	1.5%	1.4218%	1.7101%	28%	5.844 €
59	109.77 €	1.5%	1.4218%	1.7100%	28%	5.840 €
60	109.68 €	1.5%	1.4329%	1.7215%	24%	5.002 €
61	109.61 €	1.5%	1.4329%	1.7214%	24%	4.998 €
62	109.56 €	1.5%	1.4328%	1.7213%	24%	4.996 €
63	109.49 €	1.5%	1.4328%	1.7212%	24%	4.993 €
64	109.40 €	1.5%	1.4327%	1.7212%	24%	4.989 €
65	109.31 €	1.5%	1.4327%	1.7211%	24%	4.984 €
66	109.21 €	1.5%	1.4439%	1.7326%	20%	4.150 €
67	109.12 €	1.5%	1.4438%	1.7325%	20%	4.146 €
68	109.05 €	1.5%	1.4438%	1.7324%	20%	4.144 €
69	109.00 €	1.5%	1.4438%	1.7323%	20%	4.142 €
70	108.99 €	1.5%	1.4775%	1.7670%	8%	1.657 €
71	109.01 €	1.5%	1.4775%	1.7669%	8%	1.657 €
72	109.06 €	1.5%	1.4775%	1.7668%	8%	1.658 €
73	109.15 €	1.5%	1.4774%	1.7667%	8%	1.659 €
74	109.27 €	1.5%	1.4774%	1.7666%	8%	1.661 €
75	109.40 €	1.5%	1.4774%	1.7665%	8%	1.663 €

Source: own elaboration.

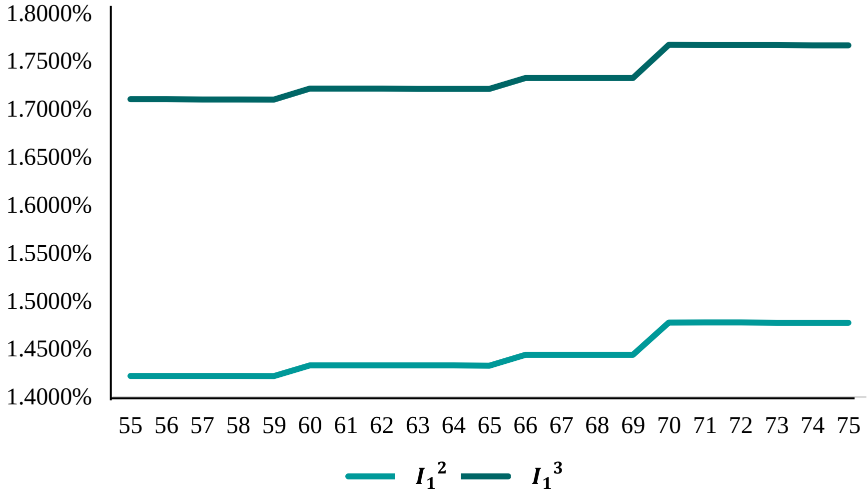
We can also see in the table that the net interest rate, I_1^2, grows in general, going from 1.4219% to 1.4774% throughout the sample, but, within each tax bracket, this interest rate decreases. This is because, the higher C is, the lower the net return will be.

As in the previous case, I_1^3, this is, the fiscal-financial profitability increases in every tax bracket and in general, going from 1.7104% to 1.7665% throughout the sample. This means that the alternative product without tax advantage would have to offer an increasing return, which, knowing that the tax advantage is increasing, makes sense, because the higher the advantage the higher will be the net profitability.

It can also be seen in this table that, in all cases, $I_1{}^3$ is greater than I_1 and I_1 is greater than $I_1{}^2$, as already mentioned in the previous section, because the net interest rate has to be lower than the gross one and the fiscal-financial profitability have to be greater than the profitability offered by the product with tax advantage.

All of this can also be seen in graph 4.3., in which the tax brackets by age can be clearly perceived, and in which we can observed the similarity of the growth of the two interest rates, $I_1{}^2$ and $I_1{}^3$.

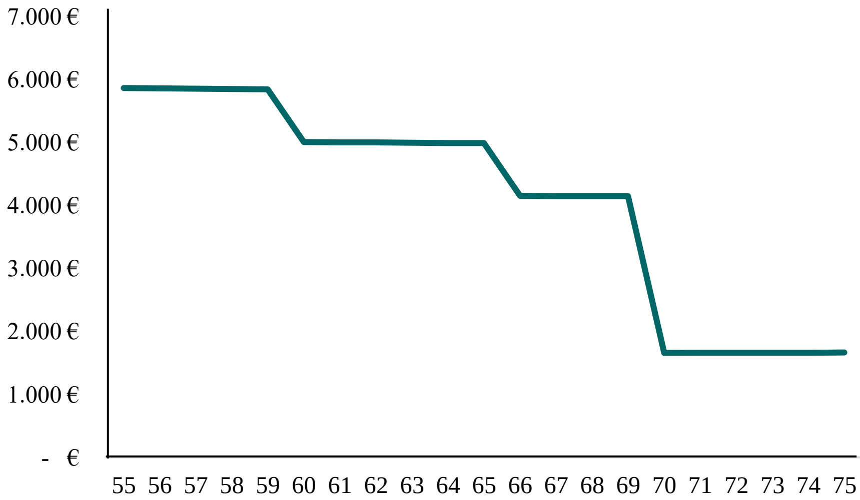
Graph 4.3. Immediate annuities with death insurance ($I_1{}^2$ *and* $I_1{}^3$)



Source: own elaboration.

In addition, in graph 4.2., we can see that the taxes that the insured have to pay get lower the older he is, this is due to the percentage by which *C* is taxed, that gets smaller, going from 5.86€ to 1.663€. In this graph we can also clearly observe the tax brackets by age.

Graph 4.4. Inmediate annuities with death insurance (Taxes)



Source: own elaboration.

5. CONCLUSIONS

In the first section of the thesis, an introductory analysis has been carried out on the variables under study. Specifically, the Spanish situation in terms of pensions has been shown. The importance of having complementary income to public pensions has been observed due to the country's situation in such terms. To be more specific, life annuities have been studied as complementary income to the public pension.

The growing financial insufficiency of public pension systems derived, to a large extent, from the progressive increase in the longevity of individuals, is forcing governments around the world, including Spain, to adopt reforms consisting of the application of generational, budgetary and actuarial factors to adjust pensions to the contributions actually paid by workers. These measures would cause a gradual downward adjustment in the rate of substitution of pensions, this is the retirement pension divided by the last salary received before retiring, that will cause growing reductions in the purchasing power of pensions that, however, must be paid for more years. And this is what leads us to the need to acquire a product that provides us with a supplementary pension, the best option for which is to hire a life annuity. We have also proven in this thesis that life annuities are adapted to the reality of each insured and have a big advantage over other pension or insurance products, which is an income for life, making it possible to maintain the economic capacity of retirees, with the consequent positive influence on the Spanish economy and on employment.

So the main conclusions of this section are the following: i.) life annuities are a flexible product that adapts to the insured person, having a lot of possible products in the Spanish market; ii.) life annuities constitute an ideal instrument to complement the public retirement pension, ending the problem of longevity and mitigating the risk of people running out of their savings before they die; iii.) life annuities are efficient products they manage to transfer of longevity risk to the insurance company.

In the second section of the thesis, the Spanish situation in terms of taxation of life annuities has been shown, as well as the tax advantage that this product has in Spain. To be more specific, it has been shown that life annuities are not taxed in their entirety, but by a percentage that depends on the age at the time of contracting, which, a priori, makes the product more attractive the older the insured is, because the fiscal-financial profitability, that will be calculated taking into account this percentage, will be higher.

In addition, in the third section of this thesis, we have demonstrated everything mentioned above through a practical application. We have calculated, under the same conditions, the amount of money the insured would receive and the fiscal-financial profitability the product would have depending on his age. In concrete, we have taken into account two cases, one without any type of premium refund and one with death insurance.

The results show that, in both cases, the fiscal-financial profitability is bigger than the profitability that is offered by the insurance company, as was expected, because of the tax advantage. Moreover, it has been shown that, in both cases, this advantage increases at the same time as the age of the insured does, since the percentage of taxable capital decreases.

In addition, it has been observed that the fiscal-financial profitability is higher when we are not taking death insurance into account, this is due to the influence of this death insurance in the amount of money that the insured will receive after retirement, the higher the insurance the lower the payments and, in consequence, the lower the profitability.

To summarize, in this thesis it has been observed that life annuities are a good product to complement the pension received at retirement age, since, in addition to increasing the income received, it provides a tax advantage that other products on the Spanish market do not offer, in addition to ending the risk of longevity for being a product for life.

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7. ANNEXES

##WITHOUT DEATH INSURANCE##

##55 YEARS CASE##

```
# x=55 years (UX1 1967). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:779
v<-1.015^-(t/12+1/12)
p<-lk12[661+t+1]/lk12[661]
c<-100000/sum(v*p);c      #c=294.7284

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01195316

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.0241937
```

##56 YEARS CASE##

```
# x=56 years (UX1 1966). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:767
v<-1.015^-(t/12+1/12)
p<-lk12[673+t+1]/lk12[673]
c<-100000/sum(v*p);c      #c=301.2005

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01188214

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02440964
```

##57 YEARS CASE##

```
# x=57 years (UX1 1965). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:755
```

```
v<-1.015^-(t/12+1/12)
p<-lk12[685+t+1]/lk12[685]
c<-100000/sum(v*p);c       #c=308.0392

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01180754

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02463666

##58 YEARS CASE##

# x=58 years (UX1 1964). Inmediate annuity with death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:743
v<-1.015^-(t/12+1/12)
p<-lk12[697+t+1]/lk12[697]
c<-100000/sum(v*p);c       #c=315.2782

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.0117291

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02487559

##59 YEARS CASE##

# x=59 years (UX1 1963). Inmediate annuity with death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:731
v<-1.015^-(t/12+1/12)
p<-lk12[709+t+1]/lk12[709]
c<-100000/sum(v*p);c       #c=322.956

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01164654

#Fiscal-financial profitability / product without tax advantage
```

```
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02512729

##60 YEARS CASE##

# x=60 years (UX1 1962). Inmediate annuity with death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:719
v<-1.015^-(t/12+1/12)
p<-lk12[721+t+1]/lk12[721]
c<-100000/sum(v*p);c       #c=331.1171

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01205671

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.0259499

##61 YEARS CASE##

# x=61 years (UX1 1961). Inmediate annuity with death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:707
v<-1.015^-(t/12+1/12)
p<-lk12[733+t+1]/lk12[733]
c<-100000/sum(v*p);c       #c=339.812

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01197822

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02624525

##62 YEARS CASE##

# x=62 years (UX1 1960). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:695
```

```
v<-1.015^-(t/12+1/12)
p<-lk12[745+t+1]/lk12[745]
c<-100000/sum(v*p);c      #c=349.1118

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01189537

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02665032

##63 YEARS CASE##

# x=63 years (UX1 1959). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:683
v<-1.015^-(t/12+1/12)
p<-lk12[757+t+1]/lk12[757]
c<-100000/sum(v*p);c      #c=358.9554

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01180783

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02688748

##64 YEARS CASE##

# x=64 years (UX1 1958). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:671
v<-1.015^-(t/12+1/12)
p<-lk12[769+t+1]/lk12[769]
c<-100000/sum(v*p);c      #c=369.4022

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.0117152

#Fiscal-financial profitability / product without tax advantage
```

```
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02723726

##65 YEARS CASE##

# x=65 years (UX1 1957). Inmediate annuity without death insurance. π= 100.000 and
i=0.015
lk12<-aprol()
t<-0:659
v<-1.015^-(t/12+1/12)
p<-lk12[781+t+1]/lk12[781]
c<-100000/sum(v*p);c       #c=380.5274

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01161706

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02760841

##66 YEARS CASE##

# x=66 years (UX1 1956). Inmediate annuity without death insurance. π= 100.000 and
i=0.015
lk12<-aprol()
t<-0:647
v<-1.015^-(t/12+1/12)
p<-lk12[793+t+1]/lk12[793]
c<-100000/sum(v*p);c       #c=392.4305

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.20*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01209938

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02866338

##67 YEARS CASE##

# x=67 years (UX1 1955). Inmediate annuity without death insurance. π= 100.000 and
i=0.015
lk12<-aprol()
t<-0:635
```

```
v<-1.015^-(t/12+1/12)
p<-lk12[805+t+1]/lk12[805]
c<-100000/sum(v*p);c      #c=405.23

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.20*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01200731

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02910459

##68 YEARS CASE##

# x=68 years (UX1 1954). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:623
v<-1.015^-(t/12+1/12)
p<-lk12[817+t+1]/lk12[817]
c<-100000/sum(v*p);c      #c=419.0543

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.20*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.0119094

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.02957461

##69 YEARS CASE##

# x=69 years (UX1 1953). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:611
v<-1.015^-(t/12+1/12)
p<-lk12[829+t+1]/lk12[829]
c<-100000/sum(v*p);c      #c=434.038

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.20*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01180517

#Fiscal-financial profitability / product without tax advantage
```

```
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.03007586

##70 YEARS CASE##

# x=70 years (UX1 1952). Inmediate annuity without death insurance. π= 100.000 and
i=0.015
lk12<-aprol()
t<-0:599
v<-1.015^-(t/12+1/12)
p<-lk12[841+t+1]/lk12[841]
c<-100000/sum(v*p);c      #c=450.3243

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01368435

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.03286616

##71 YEARS CASE##

# x=71 years (UX1 1951). Inmediate annuity without death insurance. π= 100.000 and
i=0.015
lk12<-aprol()
t<-0:587
v<-1.015^-(t/12+1/12)
p<-lk12[853+t+1]/lk12[853]
c<-100000/sum(v*p);c      #c=468.0633

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01363712

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.03352197

##72 YEARS CASE##

# x=72 years (UX1 1950). Inmediate annuity without death insurance. π= 100.000 and
i=0.015
lk12<-aprol()
t<-0:575
```

```
v<-1.015^-(t/12+1/12)
p<-lk12[865+t+1]/lk12[865]
c<-100000/sum(v*p);c      #c=487.4106

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01358672

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.03422321

##73 YEARS CASE##

# x=73 years (UX1 1949). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:563
v<-1.015^-(t/12+1/12)
p<-lk12[877+t+1]/lk12[877]
c<-100000/sum(v*p);c      #c=508.5286

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01353291

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.03497348

##74 YEARS CASE##

# x=74 years (UX1 1948). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:551
v<-1.015^-(t/12+1/12)
p<-lk12[889+t+1]/lk12[889]
c<-100000/sum(v*p);c      #c=531.5659

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.01347543

#Fiscal-financial profitability / product without tax advantage
```

```
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.03577666

##75 YEARS CASE##

# x=75 years (UX1 1947). Inmediate annuity without death insurance. π= 100.000 and i=0.015
lk12<-aprol()
t<-0:539
v<-1.015^-(t/12+1/12)
p<-lk12[901+t+1]/lk12[901]
c<-100000/sum(v*p);c       #c=556.6965

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t/12+1/12)*p)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i<-0.013414

#Fiscal-financial profitability / product without tax advantage
i3<-function(i3){sum(c*(1+i3)^-(t/12+1/12)*p)-sum(c*(1-0.19)*(1+i)^-(t/12+1/12)*p)}
uniroot(i3,c(0,1),tol=0.000000000001)
#iff2=0.03663691

##WITH DEATH INSURANCE##

##55 YEARS CASE##

# x=55 years (UX1 1967). Inmediate annuity with death insurance. π= 100.000 and i=0.015

lk12<-aprol()
t1<-0:779
v<-1.015^-(t1/12+1/12)
p<-lk12[661+t1+1]/lk12[661]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:647
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[661+t2]-lk12m[661+t2+1])/lk12m[661]
seg<-sum(u*v*q);seg          #seg=62628.72
c<-(100000-seg)/r;c          #c=110.1438

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
```

```
i2<-0.01421938

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01710389

##56 YEARS CASE##

# x=56 years (UX1 1966). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:767
v<-1.015^-(t1/12+1/12)
p<-lk12[673+t1+1]/lk12[673]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:635
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[673+t2]-lk12m[673+t2+1])/lk12m[673]
seg<-sum(u*v*q);seg  #seg=63464.92
c<-(100000-seg)/r;c          #c=110.0438

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01421892

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01710287

##57 YEARS CASE##

# x=57 years (UX1 1965). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:755
v<-1.015^-(t1/12+1/12)
```

```
p<-lk12[685+t1+1]/lk12[685]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:623
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[685+t2]-lk12m[685+t2+1])/lk12m[685]
seg<-sum(u*v*q);seg  #seg=64307.6
c<-(100000-seg)/r;c          #c=109.9466

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01421846

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01710188

##58 YEARS CASE##

# x=58 years (UX1 1964). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:743
v<-1.015^-(t1/12+1/12)
p<-lk12[697+t1+1]/lk12[697]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:611
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[697+t2]-lk12m[697+t2+1])/lk12m[697]
seg<-sum(u*v*q);seg  #seg=65156.77
c<-(100000-seg)/r;c          #c=109.8531

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01421798
```

```
#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.0171009

##59 YEARS CASE##

# x=59 years (UX1 1963). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:731
v<-1.015^-(t1/12+1/12)
p<-lk12[709+t1+1]/lk12[709]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:599
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[709+t2]-lk12m[709+t2+1])/lk12m[709]
seg<-sum(u*v*q);seg  #seg=66012.38
c<-(100000-seg)/r;c          #c=109.7651

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.28*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.0142175

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01709993

##60 YEARS CASE##

# x=60 years (UX1 1962). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:719
v<-1.015^-(t1/12+1/12)
p<-lk12[721+t1+1]/lk12[721]
```

```
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:587
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[721+t2]-lk12m[721+t2+1])/lk12m[721]
seg<-sum(u*v*q);seg  #seg=66874.36
c<-(100000-seg)/r;c           #c=109.6847

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01432896

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01721538

##61 YEARS CASE##

# x=61 years (UX1 1961). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:707
v<-1.015^-(t1/12+1/12)
p<-lk12[733+t1+1]/lk12[733]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:575
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[733+t2]-lk12m[733+t2+1])/lk12m[733]
seg<-sum(u*v*q);seg  #seg=67742.59
c<-(100000-seg)/r;c           #c=109.6145

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01432853
```

```
#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01721439

##62 YEARS CASE##

# x=62 years (UX1 1960). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:695
v<-1.015^-(t1/12+1/12)
p<-lk12[745+t1+1]/lk12[745]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:563
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[745+t2]-lk12m[745+t2+1])/lk12m[745]
seg<-sum(u*v*q);seg  #seg=68616.95
c<-(100000-seg)/r;c          #c=109.5619

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01432811

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01721344

##63 YEARS CASE##

# x=63 years (UX1 1959). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:683
v<-1.015^-(t1/12+1/12)
p<-lk12[757+t1+1]/lk12[757]
r<-sum(v*p)
```

```
#Insurance
lk12m<-aprol()
t2<-0:551
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[757+t2]-lk12m[757+t2+1])/lk12m[757]
seg<-sum(u*v*q);seg  #seg=69497.26
c<-(100000-seg)/r;c          #c=109.4912

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01432768

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01721249

##64 YEARS CASE##

# x=64 years (UX1 1958). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:671
v<-1.015^-(t1/12+1/12)
p<-lk12[769+t1+1]/lk12[769]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:539
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[769+t2]-lk12m[769+t2+1])/lk12m[769]
seg<-sum(u*v*q);seg  #seg=70383.31
c<-(100000-seg)/r;c          #c=109.4047

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01432724

#Fiscal-financial profitability
```

```
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01721155

##65 YEARS CASE##

# x=65 years (UX1 1957). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:659
v<-1.015^-(t1/12+1/12)
p<-lk12[781+t1+1]/lk12[781]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:527
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[781+t2]-lk12m[781+t2+1])/lk12m[781]
seg<-sum(u*v*q);seg  #seg=71274.81
c<-(100000-seg)/r;c          #c=109.3072

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.24*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.0143268

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01721064

##66 YEARS CASE##

# x=66 years (UX1 1956). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:647
v<-1.015^-(t1/12+1/12)
p<-lk12[793+t1+1]/lk12[793]
r<-sum(v*p)
```

```
#Insurance
lk12m<-aprol()
t2<-0:515
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[793+t2]-lk12m[793+t2+1])/lk12m[793]
seg<-sum(u*v*q);seg  #seg=72171.41
c<-(100000-seg)/r;c          #c=109.2079

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.2*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01443869

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01732585

##67 YEARS CASE##

# x=67 years (UX1 1955). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:635
v<-1.015^-(t1/12+1/12)
p<-lk12[805+t1+1]/lk12[805]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:503
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[805+t2]-lk12m[805+t2+1])/lk12m[805]
seg<-sum(u*v*q);seg  #seg=73072.67
c<-(100000-seg)/r;c          #c=109.1176

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.2*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01443832

#Fiscal-financial profitability
```

```
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01732494

##68 YEARS CASE##

# x=68 years (UX1 1954). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:623
v<-1.015^-(t1/12+1/12)
p<-lk12[817+t1+1]/lk12[817]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:491
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[817+t2]-lk12m[817+t2+1])/lk12m[817]
seg<-sum(u*v*q);seg  #seg=73978.04
c<-(100000-seg)/r;c          #c=109.0461

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.2*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01443794

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01732405

##69 YEARS CASE##

# x=69 years (UX1 1953). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:611
v<-1.015^-(t1/12+1/12)
p<-lk12[829+t1+1]/lk12[829]
r<-sum(v*p)
```

```
#Insurance
lk12m<-aprol()
t2<-0:479
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[829+t2]-lk12m[829+t2+1])/lk12m[829]
seg<-sum(u*v*q);seg  #seg=74886.84
c<-(100000-seg)/r;c          #c=109.0007

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.2*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01443756

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01732318

##70 YEARS CASE##

# x=70 years (UX1 1952). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:599
v<-1.015^-(t1/12+1/12)
p<-lk12[841+t1+1]/lk12[841]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:467
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[841+t2]-lk12m[841+t2+1])/lk12m[841]
seg<-sum(u*v*q);seg  #seg=75798.18
c<-(100000-seg)/r;c          #c=108.9867

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01477493

#Fiscal-financial profitability
```

```
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01767009

##71 YEARS CASE##

# x=71 years (UX1 1951). Inmediate annuity with death insurance. π= 100.000 and i=0.015

lk12<-aprol()
t1<-0:587
v<-1.015^-(t1/12+1/12)
p<-lk12[853+t1+1]/lk12[853]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:455
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[853+t2]-lk12m[853+t2+1])/lk12m[853]
seg<-sum(u*v*q);seg  #seg=76711
c<-(100000-seg)/r;c          #c=109.0072

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01477478

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01766914

##72 YEARS CASE##

# x=72 years (UX1 1950). Inmediate annuity with death insurance. π= 100.000 and i=0.015

lk12<-aprol()
t1<-0:575
v<-1.015^-(t1/12+1/12)
p<-lk12[865+t1+1]/lk12[865]
r<-sum(v*p)
```

```
#Insurance
lk12m<-aprol()
t2<-0:443
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[865+t2]-lk12m[865+t2+1])/lk12m[865]
seg<-sum(u*v*q);seg  #seg=77623.99
c<-(100000-seg)/r;c          #c=109.063

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01477462

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01766817

##73 YEARS CASE##

# x=73 years (UX1 1949). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:563
v<-1.015^-(t1/12+1/12)
p<-lk12[877+t1+1]/lk12[877]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:431
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[877+t2]-lk12m[877+t2+1])/lk12m[877]
seg<-sum(u*v*q);seg  #seg=78535.63
c<-(100000-seg)/r;c          #c=109.1524

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01477447

#Fiscal-financial profitability
```

```
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.0176672

##74 YEARS CASE##

# x=74 years (UX1 1948). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:551
v<-1.015^-(t1/12+1/12)
p<-lk12[889+t1+1]/lk12[889]
r<-sum(v*p)

#Insurance
lk12m<-aprol()
t2<-0:419
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[889+t2]-lk12m[889+t2+1])/lk12m[889]
seg<-sum(u*v*q);seg  #seg=79444.27
c<-(100000-seg)/r;c          #c=109.2672

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01477433

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01766621

##75 YEARS CASE##

# x=75 years (UX1 1947). Inmediate annuity with death insurance. π= 100.000 and
i=0.015

lk12<-aprol()
t1<-0:539
v<-1.015^-(t1/12+1/12)
p<-lk12[901+t1+1]/lk12[901]
r<-sum(v*p)
```

```
#Insurance
lk12m<-aprol()
t2<-0:407
u<-100000
v<-1.015^-(t2/12+1/12)
q<-(lk12m[901+t2]-lk12m[901+t2+1])/lk12m[901]
seg<-sum(u*v*q);seg  #seg=80348.25
c<-(100000-seg)/r;c          #c=109.4006

#Interest rate after taxes
i2<-function(i2){sum(c*(1-0.08*0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q)-100000}
uniroot(i2,c(0,1),tol=0.000000000001)
i2<-0.01477419

#Fiscal-financial profitability
i3<-function(i3){(sum(c*(1+i3)^-(t1/12+1/12)*p)+sum(100000*(1+i3)^-
(t2/12+1/12)*q))-(sum(c*(1-0.19)*(1+i2)^-(t1/12+1/12)*p)+sum(100000*(1+i2)^-
(t2/12+1/12)*q))}
uniroot(i3,c(0,1),tol=0.000000000001)
#i3=0.01766519
```