OPEN ACCESS
Fractals, Vol. 30, No. 5 (2022) 2240162 (16 pages)

DOI: 10.1142/S0218348X22401624

FORECASTING STOCK MARKET CRASHES VIA REAL-TIME RECESSION PROBABILITIES: A QUANTUM COMPUTING APPROACH

DAVID ALAMINOS,[*,†,§,**] M. BELÉN SALAS[†,‡,¶,**]
and MANUEL A. FERNÁNDEZ-GÁMEZ[†,‡,‖]

Received October 11, 2021
Accepted January 2, 2022
Published June 27, 2022

Abstract

A fast and precise prediction of stock market crashes is an important aspect of economic growth, fiscal and monetary systems because it facilitates the government in the application of suitable policies. Many works have examined the behavior of the fall of stock markets and have built models to predict them. Nevertheless, there are limitations to the available research, and the literature calls for more investigation on the topic, as currently the accuracy of the models remains low and they have only been extended for the largest economies. This study provides a comparison of quantum forecast methods and stock market declines and, therefore, a new prediction model of stock market crashes via real-time recession probabilities with the power to accurately estimate future global stock market downturn scenarios is achieved. A 104-country sample has been used, allowing the sample compositions to take into account the regional diversity of the alert warning indicators. To obtain a robust model, several alternative techniques have been employed on the sample under study, being Quantum Boltzmann Machines, which have obtained very good prediction results due to their ability to remember features and develop long-term dependencies from time series and sequential data. Our model has large policy implications for the appropriate macroeconomic policy response to downside risks, offering tools to help achieve financial stability at the international level.

Keywords: Stock Market Crashes; Forecasting; Quantum Computing; Quantum Neural Networks; Quantum Support Vector Regression; Systemic Risk.

[**]Corresponding authors.

1. INTRODUCTION

Predicting stock market crashes (a large drop in asset price) and forecasting recessions (periods of falling economic activity) is of vital importance for decision-makers and private economic operators alike.[1,2] Even though recessions are occasional events, they happen with negative potential consequences for both individuals and businesses.[3] On the one hand, they lead to high levels of unemployment and a large drop in production.[3,4] On the other hand, they reduce corporate profits and cause bankruptcy, loss of tax revenues, and budget deficits.[5] Therefore, the beginning of a downturn is the key to consider at the macroeconomic level, and yet, forecasting its occurrence and the stock market decline are still real defiance.[6]

On the other hand, recessions often precede equity market crashes. The recession prediction of many economic analysts and forecasters is found on the idea that movements in the price of stocks are predictive of the future course of later moves in the activity level of the economy, i.e. future growth rates of the economy. Thus, a large stock market decline, in the nonintervention of central banks, because there has not been a forecast of this event, will be accompanied by a deep economic slowdown.[7,8] Moreover, a world financial crisis can arise from some domestic or regional market disruptions, developing into a global economic crisis as a result of the interlinked nature of financial markets. Hence, as markets cannot adjust quickly to any shocks in their functioning, the intervention of regulators to restore financial stability becomes even more imperative.[9]

In the more current literature, several recession prediction models are prominent.[3–6,10–14] These models have shown that, although the yield curve, which is the differential for long- and short-term interest charges, is among the most widely analysed predictors of recession, macroeconomic and financial variables may be employed to forecast recessions.[10] On the other side, several prediction stock market crashes models have been developed in the latest literature.[9,15–20] The Bond-Stock Earnings Yield Differential (BSEYD) model has a noteworthy track history of forecasting significant stock market crashes worldwide over the last 50 years. However, the BSEYD model has been criticized, even though it has successfully predicted market reversals, as it has not accounted for the time-varying risk premium in the process of portfolio screening.[2] Moreover, there are still limitations in their levels of accuracy, even though these models have high explanatory power, and only concentrate on developed economies. Therefore, Yang *et al.*[5] suggested a future topic for research to explore in more depth the machine learning techniques to improve recession forecasting and select the most relevant predictors. Available models have employed different methodologies for predicting recessions, but without comparing them to establish the one that best predicts economic recessions. Therefore, the literature calls for a different recession prediction model, in particular new ones that offer a more accurate fit to global scenes, and that make comparisons between approaches to obtain better and more accurate results.[3,6,21,22]

Moreover, the results obtained by several studies validate the suitability, both in terms of explaining and of potential ranking, of overall models for estimating economic downturns and stock market crises events compared to multi-country models or single-country data. Thus, Dovern and Huber[23] demonstrated that Bayesian vector autoregression (BVAR) models provide more reliable recession likelihood predictions than individual country-specific BVARs or single-variate models. They conclude that in a globalized world, it may be worth taking into account data from other economies to better anticipate the future regime of the business cycle. For their part, Garratt *et al.*[24] compared a worldwide vector autoregression (VAR) model of expected and actual outcomes with competing models to evaluate the interdependencies between countries and the dependence of forecasts. They find that multinational data and surveys are needed to fully comprehend the impact of global expectations and interactions on predictions. Besides, Dichtl *et al.*[19] concluded that multivariate shock prediction models are of potential advantage over univariate methods, because they can incorporate multiple variables simultaneously and can identify and explain the complex relationships in the existing underlying economic conditions that precede substantial stock market declines.

In order to improve the accuracy of forecasting stock market crashes models, a comparison of methodologies has been carried out in this study to predict stock market crashes via real-time recession probabilities and, as a result, a new model that will lead to better estimates on the likelihood of a downturn and, therefore, a stock market crash, will occur in the future. This model can predict over the world, reaching accuracy levels of over 96%. The sample to build this model was 161 countries, making classification by regions: Africa and the Middle East, Latin America, South and East Asia, the United States, and Europe. In the construction of the stock market crash prediction model, several methods have been employed to achieve a robust model, which has been tested with all the techniques that have been successfully used in the literature. Especially, the methods of Quantum Support Vector Regression, Quantum Boltzmann Machines (QBMs), and Quantum Neural Networks (QNNs) have been used, and the QBMs have obtained the highest levels of accuracy.

As our model predicts stock market crashes via real-time recession probabilities, our investigation can be interesting for the researches in the field of mathematical modeling and applied sciences. To demonstrate the dynamic behaviors of many real-life phenomena, recent studies have applied the fractional calculus in different fields such as mechanics, physics, mathematics, and biology. Jajarmi *et al.*[25] developed a new and general fractional Lagrangian approach to estimate the complex behaviors of a capacitor microphone case study. They concluded that this approach supports the generalized fractional model's benefit in extracting the latent aspects of the system under study. Erturk *et al.*[26] employed the concept of the fractional Lagrangian approach to describe the motion of a beam on a nanowire. They showed that the fractional Euler–Lagrange equation allows a considerably better assessment of the hidden features of the real-world system studied. Baleanu *et al.*[27] investigated a new fractional model of COVID-19 by considering the effects of isolation and quarantine, showing that a particular case of general fractional formula provides a better fit to the real data compared to the other classical and fractional models. Baleanu *et al.*[28] studied a new chaotic system including quadratic and cubic nonlinearities as well as the Caputo fractional derivative. Therefore, our research can be of relevance for studies in the areas of mathematics, physics, health sciences, biotechnology, electricity, etc.

We provide at minimum three additional advances to the available literature. First, we introduce new explicative forecasting variables of stock market crashes, verifying the relevance of these variables, not taken into account until now. These significant variables vary across regions because of differences in business cycles, economic advantages, and production patterns.[6,9,23] The contribution of new predictors has considerable policy implications for decision-makers, who will be aware of which indicators offer accurate, reliable, and potential forecasts of stock market crisis events. Our model has a major economic influence on the appropriateness of macroeconomic and political rules in the face of downside risks, delivering tools to support the achievement of global financial and monetary stability and recognizing price trends that may lead to a market downturn. Stock market prediction is a great challenge because an unforeseen recession is likely to cause major difficulties for the community, such as joblessness, falling asset prices, declining business earnings, and bankruptcies. Hence, precise forecasting of the downturn and stock market fall will allow governments, businesses, and individuals to make choices to reduce their adverse effects to a minimum. This gives solid support suggesting that our analysis provides a suitable point of departure for the development of an early alert mechanism. Moreover, given that such measures tend to take some time to have an impact on the economic environment, it is necessary that the stock market downturns can be predicted well in advance.

Second, we achieved better estimation precision than that achieved in earlier researches with pioneering techniques. Artificial intelligence methods give us a breakthrough due to improved accuracy in stock market declines forecasting. The computational techniques used in the present study have been chosen from a series of previous literature works where they show that these methodologies have a great predictive capacity and offer extensive explanatory information on economic events in economic and financial predictions.

Third, our investigation has predicted global stock market crashes, so it is not limited to developed countries, which is of interest to policymakers anywhere in the world.

Therefore, the motivation for our study is based on the fact that although there has been research on stock market crash prediction, investors or investment professionals are still searching for the reason for stock market crashes, needing new explanatory variables to predict this phenomenon. Moreover, most studies have focused on developed economies and limitations which remain in their level of accuracy. On the other hand, previous research has not compared methodologies to determine which one best predicts economic downturns.

Finally, our proposed method has great applicability in the field of mathematical modeling, applied sciences, and especially, in finance, as it offers an important opportunity contribution to dynamic behaviors of many real-life phenomena and the systemic risk. With respect to the financial market, our model provides tools to support the achievement of global financial and monetary stability and recognizes price trends that may lead to a market crash. This suggests that our analysis provides a suitable starting point for the development of an early warning mechanism. Our results have enormous implications for policymakers' upcoming decisions, enabling them to prevent relevant negative stock market scenarios and the associated potential costs. Thus, early alert measures facilitate policymakers to detect weaknesses in the economy and to adopt precautionary actions to mitigate the potential risks that could lead to a crisis.

This study is structured as described below: Sec. 2 presents the employed methods. Section 3 introduces the variables and data involved in the investigation. Lastly, Sec. 4 discusses the results generated. The paper finishes with the concluding remarks of the research and its implications.

2. METHODS

As noted above, to answer the research inquiry, we have employed a variety of techniques in the construction of the stock market crashes forecasting model. The purpose of using a variety of methodologies is to obtain a consistent model, which is proved not just by one classifier method, but by using all those shown to be accepted in the preceding literature. Precisely, Quantum Support Vector Regression, QBMs, and QNNs have been used. A summary of the process of determining the methodological details of these classification methods is presented in the following. As a complement to interpreting the results, the method of sensitivity analysis of the variables included in this work, especially, Sobol's method,[29] is required to establish and select the level of significance of the variables utilized in the forecasting of stock market crashes, satisfying the gap introduced by past literature in the performance of the variable picking exercise.[6]

2.1. Support Vector Regression Quantum Bat Algorithm

We determine the Support Vector Regression (SVR) with a nonlinear function, $\varphi(x)$, to extract the input data, $\{(xi, yi)\}_{i=1}^{N}$, using a high-dimensional space. So, we use the linear function, f, for describing the nonlinear relationships connecting input data and output data. The linear function, f, denotes the basic description of SVR, as appears in the following equation:

$$f(x) = W^T \varphi(x) + b, \tag{1}$$

where $f(x)$ is the predicted values; $\varphi(x)$ defines the feature of the map function, with a nonlinear mapping of the input space, x; the coefficients W and b are created after minimizing the risk, as appears in the following equation:

$$R_{\text{emp}}(f) = \frac{1}{N} \sum_{i=1}^{N} L_\epsilon(Y_i, W^T \varphi(X_i) + b, \tag{2}$$

where $L_\epsilon(y, f(x))$ is the ε-insensitive loss function described as follows:

$$L_\epsilon(y, f(x)) = \begin{cases} |f(x) - y| - \varepsilon & \text{if } |f(x) - y| \geq \varepsilon, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

In order to find an optimal hyperplane, we apply the function $L_\epsilon(y, f(x))$, to optimize the distance that divides the data training into two subsets. After that, the modelization of the SVR is defined to minimize the global errors rate, which is shown in the following equation:

$$\min_{\omega, b, \mathfrak{I}^*, \mathfrak{I}} R_\varepsilon(\omega, \mathfrak{I}^*, \mathfrak{I}) = \frac{1}{2} w^T w + C \sum_{i=1}^{N} (\mathfrak{I}_i^* + \mathfrak{I}_i). \quad (4)$$

This equation serves to solve the problem of penalization of large weights while keeping the nature of $f(x)$. The other condition punishes mistakes collected by the loss function using the ε-insensitive function during the training stage. The parameter C compensates $f(x)$ and y. Training errors below ε are labeled as I_i^*, while training errors above ε are designated as I_i. At last, the SVR regression is denoted as the following equation:

$$f(x) = \sum_{i=1}^{N} (\alpha_i^* - \alpha_i) K(x_i, x) + b, \quad (5)$$

where $K(x_i x_j)$ is the kernel function, α_i^*,α_i are estimated and called Lagrangian multipliers, x_i and x_j, in the feature space, $\varphi(x_i)$ and $\varphi(x_j)$, each in order, $K(x_i, x_j) = \varphi(x_i)\varphi(x_j)$ is the kernel function applied.[30]

The bat algorithm has the following steps for intelligent search.[31] It starts with the origin and speed of the algorithm and plans the way of solving the problem space; computes the value of the optimal function of the problem; adjusts the volume and speed of the bat to optimize the function and thus find the optimal solution. The bat algorithm performs a global search and a location search to generate Support Vector Regression Quantum Bat Algorithm (SVRQBA).

In the overall research, suppose the search space is with d dimensions, at the time, t, the ith bat has its position, x_i^t, and velocity, v_i^t. At the time, $t+1$, its position, x_i^{t+1}, and velocity, v_i^{t+1}, are, respectively, made more current as shown in the following equations:

$$x_i^{t+1} = x_i^t + v_i^{t+1}, \quad (6)$$

$$v_i^{t+1} = v_i^t + (x_i^t - x_*) F_i, \quad (7)$$

where x_* is the global optimal solution, F_i is the frequency, as presented in the following equation:

$$F_i = F_{\min} + (F_{\max} - F_{\min})\beta, \quad (8)$$

where $\beta \in [0, 1]$ is a random number, $F_{\max}$ and $F_{\min}$ are every the max frequency and the min frequency of the ith bat now.

Once a solution in the global optimum has been reached, in local search, every bat generates an option in the form of a random walk as appears in the following equation:

$$x_{\text{new}}(i) = x_{\text{old}} + \lambda A^t, \quad (9)$$

where x_{old} is a random solution obtained in optimal disaggregation, A^t is the average volume in the bat population, and λ is a vector D-dimensional in $[-1, 1]$.

2.2. Quantum Boltzmann Machines

A Boltzmann Machine (BM) is a linked chart of (N) binary neurons.[32,33] These neurons can be seen (they modulate some aspect of the data distribution) or unseen (they are not linked to data and are just used to capture features of the data distribution). Each network has 2^N states possible, and the sampling likelihood of a state $s = (s_1, \ldots, s_N)$ the model is shown as given in the following equation:

$$p(s) = \frac{e^{-E(s)}}{Z}, \quad (10)$$

where Z is the partition function, and E is an energy function described as shown in the following equation:

$$E(s) = -\sum_{s_i \in S} b_i s_i - \sum_{s_i, s_j \in S} W_{ij} s_i s_j, \quad (11)$$

where parameter b denotes the linear "bias" in every neuron and parameter W defines the "weight" of the linkage among two neurons. To compute these parameters to maximize the log-likelihood (L), the gradient descent method and the learning rate η are applied to obtain the model parameter as defined in the following equations:

$$\Delta W_{ij} = \frac{1}{\eta}[(s_i s_j)D - (s_i s_j)M], \tag{12}$$

$$\Delta b_i = \frac{1}{\eta}[(s_i)D - (s_i)M]. \tag{13}$$

In Eq. (12), the values of $(s_i s_j)$ and (s_i) correspond to the values of the expectations about the data (D) and model (M) distributions. The model could be formed if the first-order moments, (s_i), and second-order moments, $(s_i s_j)$, were equal for the data and model distributions. This algorithm takes advantage of quantum effects to adopt an initial quantum system and convert it into a Hamiltonian, one where the system would have to remain in the ground state to obtain QBM.[34]

The aim is to produce a better approximation of $(s_i)M$ and $(s_i s_j)M$ than classical heuristics. This technique appears more natural in the Hamiltonian (H) form described in the following equation:

$$H(S) = -\sum_{s_i \in S} h_i S_i - \sum_{s_i, s_j \in S} J_{ij} S_i S_j, \tag{14}$$

S is the vector of spin states of the qubits, h_i are the bias terms in every qubit, and J_{ij} is the (anti)ferromagnetic attachments in the qubits.

2.3. Quantum Neural Networks

Wan *et al.*[35] illustrated the opportunities of joining the single computational capabilities of convolutional neural networks and quantum computing. This can lead to a computational approach with a strong potential for forecasting. Qubit is the smallest unit of information in quantum computation, being a probabilistic representation. A qubit can be at "1" or "0" or any superposition of the two.[36,37] The state of the qubit can be described in the following equation:

$$|\psi\rangle = \alpha|o\rangle + \beta|1\rangle, \tag{15}$$

where α and β are the numbers that trigger the amplitude of the appropriate states such that $|\alpha|^2 + |\beta|^2 = 1$. It is specified as a pair of numbers $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right]$. The angle θ is the specification characterizing geometrical aspects and is determined as: $\cos(\theta) = |\alpha|$ and $\sin(\theta) = |\beta|$. Quantum gates can be used to adjust probabilities as a result of weight enhancement.[37,38] A definition of a rotation gate is described in the following equation:

$$U(\Delta\theta) = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix}. \tag{16}$$

The state of the qubit can be improved by employing the quantum gate described above. The implementation of the spin gate in a qubit is described as below:

$$\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \tag{17}$$

QNN is suggested to predict stock market crashes. The process starts with a quantum hidden neuron from the state $|o\rangle$, which sets the superposition as shown in the following equation:

$$\sqrt{p}|O\rangle + \sqrt{1} - p|1\rangle \quad \text{with } O \leq |p| \leq 1, \tag{18}$$

where p represents the random likelihood of initiating the system in the state $|O\rangle$. The classical neurons are introduced by generating random numbers.[37,38] The output of the quantum neuron is given as indicated in the following equation:

$$v_j = f\left(\sum_{i=1}^{n} w_{ji} * x_i\right), \tag{19}$$

where f is a problem-dependent sigmoid or Gaussian function.[37] The network output is designed as shown in the following equation:

$$y_k = f\left(\sum_{j=1}^{l} w_{jk} * v_j\right). \tag{20}$$

The required output is the o_k equivalent to squared error is shown in the following equation:

$$E_k^2 = \frac{1}{2}|y_k - o_k|^2. \tag{21}$$

The training process proceeds along with the rules of the feed-forward back-propagation algorithm. The update of the output layer weight is expressed in the following equation:

$$\Delta w_{jk} = \eta e_k f' v_j. \tag{22}$$

2.4. Sensitivity Analysis

Despite the important explanatory power of data mining techniques, it is essential to measure the hit of variables, especially when there are many of them involved in the process. This is carried out through an analysis of sensitivity. This is an analysis that seeks to establish the independent variables' relative importance to the dependent variable.[39,40] It tries to narrow down the models to highly significant variables and exclude the less significant ones. This process tries to compare the aggregate of features in the model; a variable is considered more important than another variable if its variance raises. The Sobol method[29] is employed to disaggregate the total final variance $V(Y)$ supplied by the group of equations given in the following equation:

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>1} V_{ij} + \cdots + V_{1,2,\ldots,k}, \tag{23}$$

where $V_i = V(E(Y|X_i)$ and $V_{ij} = V(E(Y|X_i, X_j)) - V_i - V_j$.

The indexes of sensitivity are identified by $S_i = V_i/V$ and $S_{ij} = V_{ij}/V$, denoting S_{ij} as the impact of the interaction among two variables. The Sobol breakdown permits estimating a completed sensitivity index S_{Ti}, calculating the addition of total sensitivity effects involving the no dependent variables. The objective of this method is to figure out which part of the variation of the model results depends on every input variable used, whether it is a single variable or an integration among several variables.[41]

As a summary of the training process of this type of methodology, follow the steps described in the diagram below (Fig. 1).

3. DATASETS AND VARIABLES

The target sample consists of 104 countries in the period 1974–2019, making classification by regions (Africa and the Middle East, Latin America, South and East Asia, the United States, and Europe). We have divided the available sample set into 70% for the training datasets, 10% for the validation datasets, and 20% for the test datasets. The training data are used for model building, the validation data serve as an "early stop" to check for potential

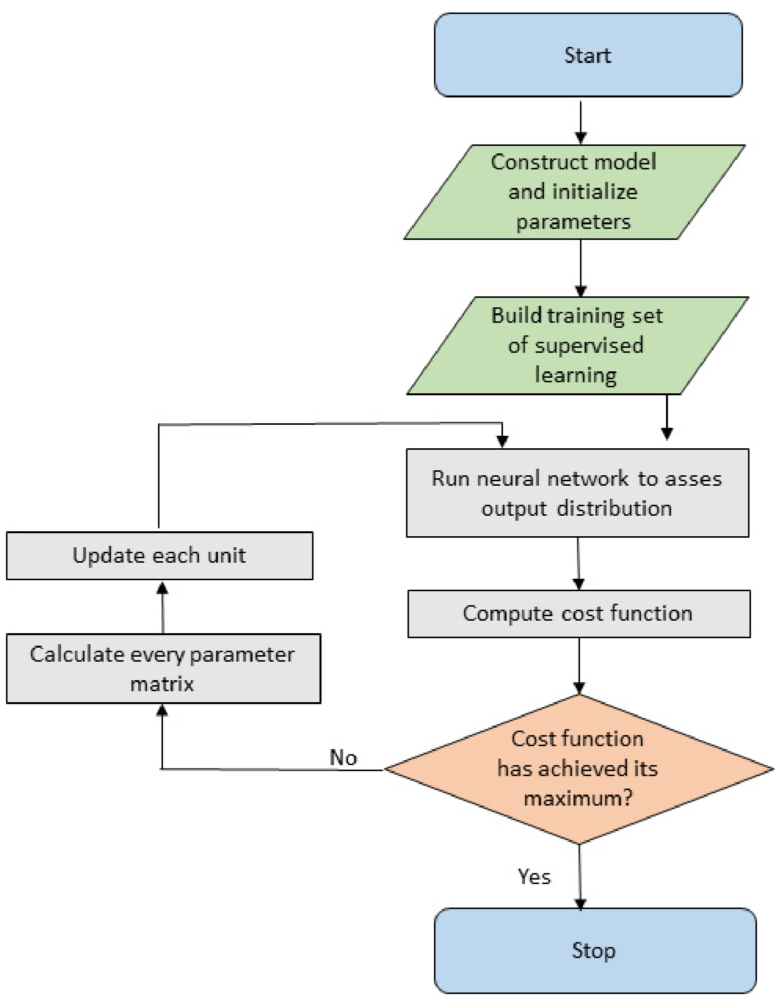

Fig. 1 Flow chart.

problems such as overfitting. And finally, the test data test the level of success achieved by the models created. Five hundered random data sets are randomly selected and cross-validated 10 times.[42]

In the present study, the eight sixteen-core Intel Core i7-10700 processor has been used as computing resources to make estimates. The code for the estimation of our methods has been performed by Python (3.8 version), with the support of the libraries such as NumPy and QisKit to create the mathematical routines and quantum processing.[43–45]

We define the dependent variable as a binary crash indicator, CI_{t+1}, which assumes the value of one in the event of a fall in the stock market during month $t+1$, and take the value of zero otherwise.[19] We obtain the binary *crash indicator* CI_{t+1} separately for each country by comparing stock market return $\mathrm{ret}_{\mathrm{stock},t+1}$ during month $t+1$ with a historical threshold based on profitability $\mathrm{HRT}_{\mathrm{stock},t+1}$. This threshold is defined as the 5th percentile based on the historical stock market return distribution over a 10-year rolling window. The data are from the FRED database and the SNL (S&P Global Market Intelligence) website. Accordingly, a crash during month $t+1$ is indicated if the respective stock market return falls below the chosen threshold:

$$\mathrm{CI}_{t+1} = \begin{cases} 1 & \text{if } \mathrm{ret}_{\mathrm{stock},t+1} - \mathrm{HRT}_{\mathrm{stock},t+1<0}, \\ 0 & \text{otherwise.} \end{cases}$$

On the other side, we employed 32 independent variables, classified into Domestic Prices, Domestic Real Macro, Commodities and Financial, and Open Economy/Labor, as possible predictors of stock market crashes (Table 1). These variables are extensively employed in the prior literatures.[1,5,6,9,10,14,19,46,47] This macroeconomic and financial information is taken from the FRED database, World Bank Open Data, and International Monetary Fund.

Most of the variables in Domestic Prices are indexes, since they represent the current price for a specific good, service, or stock, such as Consumer Price Index, Volatility Index, S&P Index, NASDAQ 100 Index, UK 100 Index, DAX 30 Index, Real House Price Index, Current Business Climate Index, and Consumer Confidence Index. Domestic Real macro variables are macroeconomic variables, mostly measured as a percentage of GDP (Industrial Production, Exports and Imports Goods Services, Trade, Gross Fixed Capital Formation). As commodity variables, the price of oil and gold have been considered. The financial variables selected are indicators that express financial health, such as the Interest Rate Spread, Long-Term Government Bond Yields 10-year, Real Interest Rate, Effective Federal Funds Rate, and 3-Month Non-Financial Commercial Paper Rate. Finally, we have introduced as open economy variable, e.g. Real Effective Exchange Rate Index, and as labor variables the indicators that show the level of employment in an economy, such as Labor Force, Employment to Population Ratio, Unemployment, Compensation Employees, and Employees Non-Farm Payrolls.

4. RESULTS

4.1. Empirical Results

Tables 2–5 display the level of precision, root mean square error (RMSE), mean absolute percentage error (MAPE), and variables significant for each of the models by region. The precision level in the three models is always above 92.41% and RMSE and MAPE levels are suitable. The QBMs model is the one that has achieved the highest level of accuracy (96.22% on average by regions), followed by the model of SVRQBA method with 94.59% overall by regions. Overall, these results indicate a greater probability of prediction than prior studies. For instance, the work of Chatzis *et al.*[9] shows an accuracy of about 78%. In the study by Davig and Hall,[6] precision of about 74% is revealed. In the work of Aye *et al.*,[3] it approaches 80%, and in the report of Dichtl *et al.*,[19] it is nearly 80%. Table 4 displays the average of the percentage errors. Lastly, Table 5 reports the most significant variables by the method after the application of the Sobol test.

Moreover, Fig. 2 presents further details on the explicative variables. In all three models, the GDP variable has been considered explanatory in each region (Africa and the Middle East, Latin America, South and East Asia, the United States, Europe, and Global). In Africa and the Middle East samples, the variables GDP, NASI, and MCDC are repeated in the three models. In Latin American countries, the variables reiterated in all the models have been GDP, GFCF, NASI, and VIX. In the case of South and East Asia, the variables repeated in all the models are GDP, X, NASI, PER, and VIX. In the United States, the variables GDP, NASI, CCI, and SPI appear in the three models. And in Europe, the variables reiterated in the three models have been GDP and DAXI.

Table 1 Independent Variables.

Code	Description	Source
Domestic Prices		
ICP	Inflation Consumer Prices (annual %)	Yang *et al.* (2018)
CPI	Consumer Price Index (2010 = 100)	Ng (2012)
VIX	CBOE Volatility Index	Berge (2015)
SPI	S&P 500 Index	Fossati (2015)
NASI	NASDAQ 100 Index	Chatzis *et al.* (2018)
UKI	UK 100 Index	Chatzis *et al.* (2018)
DAXI	DAX 30 Index	Chatzis *et al.* (2018)
TLSP	Taxes Less Subsidies On Products (current US$)	Goodhead and Parle (2019)
GS	Gross Savings (% of GDP)	Davig and Hall (2019)
RHPI	Real House Price Index	Ng (2012)
MCDC	Market Capitalization Domestic Companies (% of GDP)	Ng (2012)
PER	The Price-to-Earning Ratio	Lleo and Ziemba (2017)
CBCI	Current Business Climate Index	Chatzis *et al.* (2018)
CCI	Consumer Confidence Index	Dichtl *et al.* (2021)
Domestic Real Macro		
GDP	GDP growth (%)	Davig and Hall (2019)
IP	Industrial production (% GDP)	Berge (2015)
X	Exports Goods Services (% of GDP)	Goodhead and Parle (2019)
M	Imports Goods Services (% of GDP)	Goodhead and Parle (2019)
T	Trade (% of GDP)	Davig and Hall (2019)
GFCF	Gross Fixed Capital Formation(% of GDP)	Goodhead and Parle (2019)
Commodities and Financial		
ISPREAD	Interest Rate Spread (lending Rate Minus Deposit Rate, %)	Berge (2015)
LTBY	Long-Term Government Bond Yields: 10-year	Ng (2012)
RIR	Real Interest Rate (%)	Ng (2012)
EFFR	Effective Federal Funds Rate	Ng (2012)
CPR	3-Month Nonfinancial Commercial Paper Rate	Fossati (2015)
M2	Broad Money (% of GDP)	Berge (2015)
M2G	Broad Money Growth (annual %)	Yang *et al.* (2018)
M1	Money Supply (2015 = 100)	Yang *et al.* (2018)
OP	Oil Price	Béllego and Ferrara (2017)
GP	Gold Price	Béllego and Ferrara (2017)
Open Economy/Labor		
LF	Labor Force	Goodhead and Parle (2019)
EPR	Employment to Population Ratio, 15+, total(%)	Goodhead and Parle (2019)
UE	Unemployment (%Total Labor Force)	Goodhead and Parle (2019)
CE	Compensation Employees (current LCU)	Goodhead and Parle (2019)
EP	Employees Nonfarm Payrolls	Fossati (2015)
RER	Real Effective Exchange Rate Index (2010 = 100)	Davig and Hall (2019)
GCE	General Government Final Consumption Expenditure (% of GDP)	Goodhead and Parle (2019)

The best results achieved have been in the model of QBMs and show that, for the United States GDP, IP, X, NASI, PER, CCI, OP, SPI, EFFR, and RHPI are the significant variables for predicting stock market crashes. In comparison with previous studies, the Bond market yield has been one of the most widely used variables to predict the economic activity and the likelihood of a stock market decline in the United States.[9,15,47–49] For its part, Lleo and Ziemba[49] concluded that the success of the Bond market yield comes both from the information provided in government bond yields and stock prices and from the signal construction that contains a time-varying probabilistic signal threshold. In addition, the stock price index, exchange rate, oil price, gold price, and VIX variables were found to be significant in Refs. 9 and 15. According to Ref. 15, stock market swings are notoriously difficult to predict given the uncertainty related to the expectations of financial asset prices. Therefore,

Table 2 Results of Accuracy Evaluation: Classification (%).

Sample	SVRQBA			QBM			QNN		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Africa and the Middle East	95.54	94.82	93.97	97.83	97.21	96.73	93.52	93.12	92.68
Latin America	96.27	95.79	95.13	98.31	97.59	97.12	94.76	94.24	93.71
South and East Asia	96.49	95.83	96.01	98.17	96.43	95.76	93.84	93.15	92.79
The United States	95.84	95.24	94.48	97.38	97.62	97.05	93.57	93.18	92.43
Europe	95.27	94.79	94.30	96.94	96.34	95.81	94.33	94.02	93.74
Global	94.83	94.28	93.65	95.72	95.32	94.86	93.47	92.84	92.41

Note: (1) Training; (2) Validation; (3) Testing.

Table 3 Results of Accuracy Evaluation: RMSE.

Sample	SVRQBA			QBM			QNN		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Africa and the Middle East	0.16	0.28	0.32	0.07	0.10	0.18	0.18	0.20	0.22
Latin America	0.22	0.24	0.36	0.05	0.09	0.17	0.16	0.17	0.20
South and East Asia	0.22	0.27	0.30	0.10	0.13	0.21	0.22	0.25	0.27
The United States	0.18	0.21	0.25	0.08	0.12	0.18	0.18	0.21	0.24
Europe	0.19	0.22	0.26	0.07	0.13	0.16	0.19	0.21	0.24
Global	0.24	0.26	0.30	0.11	0.16	0.22	0.25	0.28	0.31

Note: (1) Training; (2) Validation; (3) Testing.

Table 4 Results of Accuracy Evaluation: MAPE.

Sample	SVRQBA			QBM			QNN		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Africa and the Middle East	0.25	0.44	0.50	0.11	0.16	0.29	0.30	0.33	0.37
Latin America	0.34	0.37	0.56	0.08	0.14	0.27	0.27	0.28	0.33
South and East Asia	0.34	0.42	0.47	0.16	0.21	0.33	0.37	0.41	0.45
The United States	0.28	0.33	0.39	0.13	0.19	0.29	0.30	0.35	0.40
Europe	0.30	0.34	0.41	0.11	0.21	0.25	0.32	0.35	0.40
Global	0.37	0.41	0.47	0.18	0.25	0.35	0.41	0.46	0.51

Note: (1) Training; (2) Validation; (3) Testing.

Table 5 Results of Accuracy Evaluation: Greater Sensitivity Variables.

Sample	SVRQBA	QBM	QNN
Africa and the Middle East	GDP, LF, NASI, DAXI, OP, ICP, MCDC	GDP, GFCF, GCE, NASI, DAXI, M2, ICP, MCDC	GDP, GFCF, GCE, NASI, PER, M2G, MCDC
Latin America	GDP, GFCF, NASI, T, VIX, RHPI	GDP, X, GFCF, NASI, PER, ICP, VIX, LTBY, RHPI	GDP, X, GFCF, ISPREAD, T, NASI, PER, VIX, LTBY
South and East Asia	GDP, IP, X, GFCF, NASI, PER, M2, VIX, MCDC	GDP, X, NASI, PER, M2G, GS, RER, VIX, MCDC	GDP, X, NASI, CCI, GS, RER, PER, VIX, RHPI
The United States	GDP, X, GCE, RER, M2G, NASI, CCI, SPI, CPR,	GDP, IP, X, NASI, PER, CCI, OP, SPI, EFFR, RHPI	GDP, IP, NASI, CCI, M2G, OP, SPI, LTBY, RHPI
Europe	GDP, IP, M, GCE, UKI, DAXI, PER, M2, T, SPI, CPR	GDP, IP, X, GCE, UKI, DAXI, M2G, CCI, RHPI	GDP, X, GFCF, DAXI, CCI, UE, M2G, LTBY, RHPI
Global	GDP, X, LF, NASI, VIX, T, CPI, M2G, PER, MCDC	GDP, IP, X, LF, NASI, PER, VIX, RER, M2G, CCI	GDP, M, NASI, PER, ISPREAD, VIX, M2G, OP, LTBY

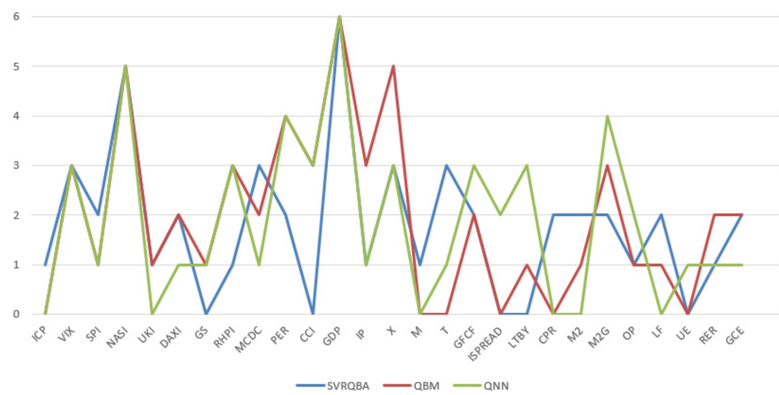

Fig. 2 Number of repetitions of the greater sensitivity variables.

he considers among the variables for predicting declines to be taken into account the VIX and S&P 500 indexes. Moreover, in addition to volatility, market participants and observers have a keen interest in the likelihood of large falls in stocks. It validates that our investigation has documented new relevant variables (GDP, PI, X, CCI, EFFR, and RHPI), confirming a new group of explicative variables not like the ones in prior works. If we highlight the domestic real macro (GDP, IP, and X), the experience of advanced economies over the past five decades suggests that, on average, these variables tend to decline around recessionary periods, making them a relevant predictor of the likelihood of a stock market decline. On the other hand, EFFR is the interest rate the banks pay for one day loans in the federal funds market. The Federal Reserve makes use of it to affect other interest rates, including those on mortgages, bank loans, and credit cards. This leads it to be the world's most important interest rate. The Federal Open Market Committee moves the reference rate to control inflation, promote employment, and keep interest rates low. These measures will keep strong economic growth, reducing the likelihood of stock market crashes.

For its part, the results for Europe exhibit that the greater sensitivity variables to predict stock market slump are GDP, IP, X, GCE, UKI, DAXI, M2G, CCI, and RHPI. In comparison with current investigations, Bellégo and Ferrara[14] showed that financial variables have helped to give accurate and timely recession, such as term spread, exchange rates, interest rates, and commodities prices. In another paper, Dichtl *et al.*[19] demonstrated that the consumer confidence index serves as an indicator of a country's overall economic sentiment, and possesses predictive power for future stock market meltdowns. So, our study concludes that in the Euro Area new variables are validated, such as GDP, IP, X, GCE, M2G, and RHPI. Three variables of the Domestic Real Macro component stand out. On the one hand, X is, at the macroeconomic level, positive for the trade balance and the current account, favoring GDP growth. At the microlevel, exporting allows companies to diversify risks in the face of unstable domestic markets and to provide a safety cushion against possible ups and downs in the domestic, regional, or international economy. The European experience shows that companies that had developed their internationalization strategies before the downturn in domestic demand were able to survive and that many others would have been saved from closure if they had done so when the economic situation was favorable. Exports are, therefore, a relevant predictor of a possible stock market crash, adopting risk diversification policies and not concentrating only on domestic customers. Moreover, economic policymakers have to analyze the evolution of the IP, which has a very close connection with the probability of a stock market downturn.

In the case of South and East Asia, the significant variables for predicting stock market crashes are GDP, X, NASI, PER, M2G, GS, RER, VIX, and MCDC. Some works have studied the prediction of the fall of stock markets in this sample of countries. Ko *et al.*[16] suggested an alarm index to predict the Korean financial market crash and concluded that their model can be readily applied to anticipate further market declines and financial downturns for economies, where financial crises can be ranked as national or global crises. This determines that our model in the South and East Asia region provides a new set of significant variables to predict stock market crashes in this area. Of these predictors, we focus on the variables of the Domestic Prices component (NASI, PER, GS, VIX, and MCDC). GS is an economic indicator of the economy's performance, as it allows an average saving per person to be calculated and compared with the rest of the countries. It also shows the degree of a society's awareness of possible future contingencies. In turn, it indicates the number of resources that a country can dedicate to build a solid welfare state and to make productive investments both at home and abroad. GS is, therefore, a good predictor for the future probability of a stock market fall, as a country's GDP is conditioned by the behavior of consumption and savings, which typically mark the cyclical movements. VIX is an index that measures the expectation of future volatility, not the present, so in this sense, it is a very useful indicator to try to forecast what may happen in the market one month ahead. Finally, MCDC corresponds to the total value of a company's equity, based on the price at which its shares are traded. Therefore, an increase in a country's MCDC is the cause of the increase in GDP, being an important explanatory variable for detecting a possible serious stock market downturn and, therefore, a stock market decline.

On the other hand, the results for Africa and the Middle East reveal that GDP, GFCF, GCE, NASI, DAXI, M2, ICP, and MCDC are the most sensitive variables to predict stock market declines. Since no previous studies have developed predictions specifically for this sample of countries, the results of this investigation are a new idea to the literature on the forecasting of stock market downturns. Analyzing the relevant variables of the open economy/labor component, CGE is the final consumption expenditure of the general government and constitutes the main contribution of the general government to GDP, and any variation concerning the forecast of this variable will consequently affect the GDP forecast, and therefore, the probability of a stock market decline.

Our results for Latin American countries show that these countries should be attentive to the behavior of the following variables to predict the stock market fall, such as GDP, X, GFCF, NASI, PER, ICP, VIX, LTBY, and RHPI. Since there are no other prior works that elaborate forecastings specifically for this sample of countries, the results of this investigation constitute a pioneer contribution to the literature on the prediction of stock market crashes. We point out two variables of the Domestic Real Macro. First, GFCF reflects the value of fixed assets purchased or produced in a specific period by both the private and public sectors. Its evolution is important because it is considered as one of the main components of investment, and therefore, it is part of the estimation of a country's GDP. It is a useful tool when studying or analyzing the economic situation at a given time for a country, through the observation of the level of investment in new goods.

At last, it can be analyzed that the significant variables for stock market crash forecasting are GDP, IP, X, LF, NASI, PER, VIX, RER, M2G, and CCI. Previous studies on the prediction of global stock market fall that can be compared with our results. We can, however, conclude that our investigation brings new significative variables to forecast in all countries.

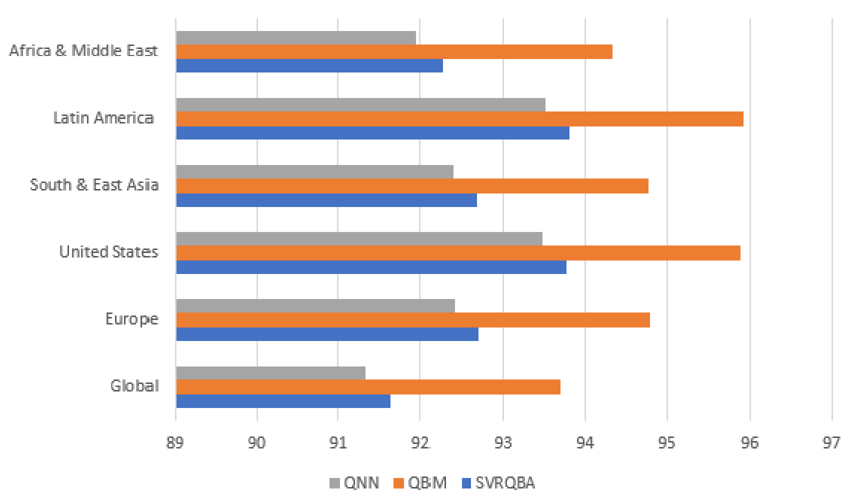

Fig. 3 Multiple-step ahead forecasts in forecast horizon $= t + 1$.

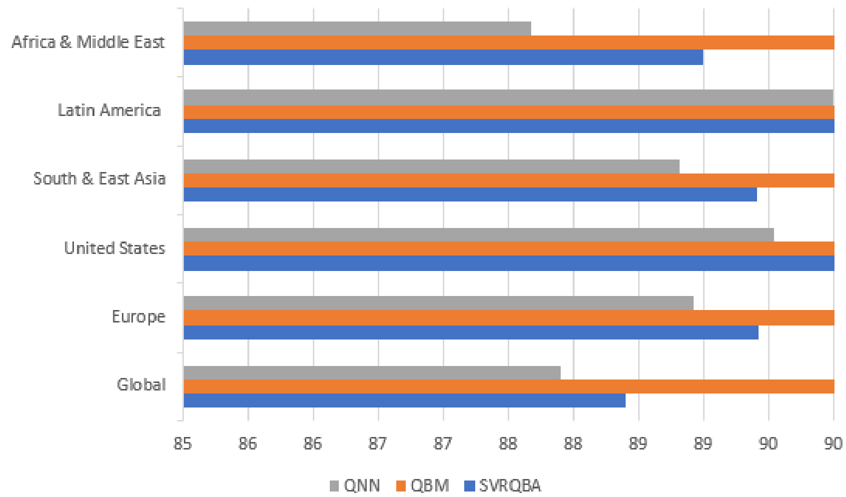

Fig. 4 Multiple-step ahead forecasts in forecast horizon = $t + 2$.

4.2. Post-Estimations

We have implemented an iterative strategy applying multi-step forward forecasting, in which models are developed and trained for 1-step forward forecasting.[50] Once the annual data are given, at time t, forecasting is produced for time $t + 1$, and this prediction is used to estimate time $t+2$ and so successively. This implies that the data estimated for $t+1$ are treated as actual data and are joined to the end of the available data.[51] Figures 3 and 4 present the precision and error results for $t+1$ and $t+2$ predicting horizons. For $t+1$, the accuracy rank of the three models is 91.33–95.92% on average by region, achieving the highest percentage of prediction probability in the QBMs model (95.92%). For $t + 2$, this range of precision is 87.67–92.54%, being in the QBM model again where the percentage of accuracy is the highest (92.54%). These results prove that the high accuracy and robustness of the models have been obtained. So, our model can anticipate the prediction of stock market crashes because at time t a prediction is produced for time $t + 1$, and this accuracy is applied to forecast time $t + 2$, and so on.

5. CONCLUSIONS

This work has provided a comparison of methodologies to forecast stock market crashes, and a new model for predicting these events has, therefore, been generated. A sample of 161 countries has been selected for the period 1974–2019, ranking by regions: Africa and the Middle East, Latin America, South and East Asia, the United States, Europe, and a sample of countries globally. Different methods have been implemented in the building of the stock market decline prediction model to obtain a solid model. The one that has achieved the highest accuracy was the QBMs. In particular, the aim has been, on the one hand, to extend the sample to all countries in the world and, on the other hand, to improve the accuracy of prior works. The results of this study are superior to those achieved in previous literature, with an accuracy rate of 92.41–97.12%. In addition, new sensitive variables for the prediction of stock market crashes have been identified. This results in high stability in the models built at $t + 1$ and $t + 2$ prediction horizons.

In comparison to the preceding works, this research has extended the forecasting of stock market crashes further and, in addition to developed economies, all over the world. The results have presented several relevant variables for different regions as well as globally. This is an excellent achievement in the area of international finance. The findings are pertinent for policymakers, investors, central bankers, private forecasters, and economic policy practitioners in every economy. These agents want to know the indicators that deliver accurate, trustworthy, and potential forecasts of economic downturns. Our research provides significant new explanatory variables for these agents to predict the fall in economic activity. Therefore, the results obtained in our model contain essential information for policymakers, as they will take our indicators into account so that in times of economic crisis, they can avoid spill-over effects on third markets. The implications are also important for asset management practitioners, as the diversification gains may no longer exist in times of turbulence. This investigation has also brought a new stock market crash predicting model implemented employing three methods, with Boltzmann Quantum Machines being the most accurate, thereby offering to available knowledge in the area of artificial intelligence. This novel model can serve as a benchmark for setting framework macroeconomic policy and enhancing its decision-making.

In brief, our research offers an important opportunity contribution to systemic risk, as the results achieved have huge implications for the next decisions taken by the policymakers, allowing them to avoid relevant negative stock market scenarios and the potential costs involved. It helps policymakers to send early warning signals to financial markets and avoid financial crises resulting from such events. Therefore, our accurate early warning model could help activate macroprudential policy tools in time, and this, in turn, helps ensure financial stability and mitigate the impending costs of crises. Hence, early warning systems make it easier for policymakers to identify vulnerabilities in the economy and to take precautionary measures to reduce the risks that can cause a crisis. In addition, an early and precise forecast of the market gives investors the possibility of realizing profits.

Further research in this field involves the development of forecasting models that take into account political factors to assess the possible influence of economic policy management and effectiveness on the phenomenon of stock market crashes.

ACKNOWLEDGMENT

This research was funded by Universidad de Málaga.

REFERENCES

1. S. Fossati, Forecasting US recessions with macro factors, *Appl. Econ.* **47**(53) (2015) 5726–5738, doi:10.1080/00036846.2015.1058904.
2. S. Lleo and W. T. Ziemba, Does the bond-stock earnings yield differential model predict equity market corrections better than high P/E models? *Financ. Mark. Instit. Instrum.* **26** (2017) 61–123, doi:10.1111/fmii.12080.
3. G. C. Aye, C. Christou, L. A. Gil-Alana and R. Gupta, Forecasting the probability of recessions in South Africa: The role of decomposed term spread and economic policy uncertainty, *J. Int. Dev.* **31** (2019) 101–116, doi:10.1002/jid.3395.
4. S. E. Kozlowski and T. Sim, Predicting recessions using trends in the yield spread, *J. Appl. Stat.* **46**(7) (2018) 1323–1335, doi:10.1080/02664763.2018.1537364.
5. A. J. Yang, J. Xiang, H. Q. Yang and J. G. Lin, Sparse Bayesian variable selection in probit model for forecasting US recessions using a large set of predictors, *Comput. Econ.* **51**(4) (2018) 1123–1138.
6. T. Davig and A. S. Hall, Recession forecasting using Bayesian classification, *Int. J. Forecast.* **35** (2019) 848–867, doi:10.1016/j.ijforecast.2018.08.005.
7. B. Rauning and J. Scharler, Stock market volatility and the business cycle, Monetary Policy and the Economy, Oesterreichische Nationalbank (Austrian Central Bank), issue 2 (2010), pp. 54–63.
8. R. E. A. Farmer, The stock market crash really did cause the great recession, *Oxford Bull. Econ. Stat.* **77**(5) (2015) 305–9049, doi:10.1111/obes.12100.
9. S. P. Chatzis, V. Siakoulis, A. Petropoulos, E. Stavroulakis and N. Vlachogiannakis, Forecasting stock market crisis events using deep and statistical machine learning techniques, *Expert Syst. Appl.* **112** (2018) 353–371, doi:10.1016/j.eswa.2018.06.032.
10. R. Goodhead and C. Parle, Predicting recessions in the Euro area: A factor approach, Economic Letters, 2/EL/19, Central Bank of Ireland (2019), pp. 1–13.
11. U. K. Chatterjee, Bank liquidity creation and recessions, *J. Bank. Financ.* **90** (2018) 64–75, doi:10.1016/j.jbankfin.2018.03.002.
12. P. Guérin and D. Leiva-Leon, Model averaging in Markov-switching models: Predicting national recessions with regional data, *Econ. Lett.* **157** (2017) 45–49, doi:10.1016/j.econlet.2017.05.027.
13. J. Döpke, U. Fritsche and C. Pierdzioch, Predicting recessions with boosted regression trees, *Int. J. Forecast.* **33**(4) (2017) 745–759, doi:10.1016/j.ijforecast.2017.02.003.
14. C. Bellégo and L. Ferrara, Forecasting euro area recessions by combining financial information, *Int. J. Comput. Econ. Economet.* **7**(1/2) (2017) 78–94.
15. E. C. Engstrom, Forecasting stock market crashes is hard — Especially future ones: Can option prices help? FEDS Notes 2014-05-07, Board of Governors of the Federal Reserve System, U.S. (2014).
16. B. Ko, J. W. Song and W. Chang, Crash forecasting in the Korean stock market based on the log-periodic structure and pattern recognition, *Physica A* **492** (2017) 308–323, doi:10.1016/j.physa.2017.09.074.
17. H. Wang, S. Lu and J. Zhao, Aggregating multiple types of complex data in stock market prediction: A model-independent framework, *Knowledge-Based Syst.* **164** (2019) 193–204, doi:10.1016/j.knosys.2018.10.035.
18. S. Tabar and S. Sharma, A new method for predicting stock market crashes using classification and artificial neural networks, *Int. J. Bus. Data*

Analytics **1**(3) (2020) 203–217, doi:10.1504/IJBDA.2020.108697.

19. H. Dichtl, D. Wolfgang and O. Tizian, Forecasting stock market crashes via machine learning (2021), pp. 1–61, https://ssrn.com/abstract=3843319 or doi:10.2139/ssrn.3843319.
20. A. Thakkar and K. Chaudhari, Fusion in stock market prediction: A decade survey on the necessity, recent developments, and potential future directions, *Inf. Fusion* **65** (2021) 95–107, doi:10.1016/j.inffus.2020.08.019.
21. R. Tian and G. Shen, Predictive power of Markovian models: Evidence from US recession forecasting, *J. Forecast.* **38**(6) (2019) 525–551, doi:10.1002/for.2579.
22. C. R. Proaño and T. Heobald, Predicting recessions with a composite real-time dynamic probit model, *Int. J. Forecast.* **30** (2014) 4898–4917, doi:10.1016/j.ijforecast.2014.02.007.
23. J. Dovern and F. Huber, Global prediction of recessions, *Econ. Lett.* **133** (2015) 81–84, doi:10.1016/j.econlet.2015.05.022.
24. A. Garratt, K. Lee and K. Shields, Forecasting global recessions in a GVAR model of actual and expected output, *Int. J. Forecast.* **32** (2016) 374–390, doi:10.1016/j.ijforecast.2015.08.004.
25. A. Jajarmi, D. Baleanu, K. Zarghami Vahid, H. Mohammadi Pirouz and J. H. Asad, A new and general fractional Lagrangian approach: A capacitor microphone case study, *Results Phys.* **31** (2021) 104950, doi:10.1016/j.rinp.2021.104950.
26. V. S. Erturk, E. Godwe, D. Baleanu, P. Kumar, J. Asad and A. Jajarmi, Novel fractional-order lagrangian to describe motion of beam on nanowire, *Acta Phys. Pol. A* **140**(3) (2021) 265, doi:10.12693/APhysPolA.140.265.
27. D. Baleanu, M. Hassan Abadi, A. Jajarmi, K. Zarghami Vahid and J. J. Nieto, A new comparative study on the general fractional model of COVID-19 with isolation and quarantine effects, *Alexandria Eng. J.* **61**(6) (2022) 4779–4791, doi:10.1016/j.aej.2021.10.030.
28. D. Baleanu, S. Zibaei, M. Namjoo and A. Jajami, A nonstandard finite difference scheme for the modeling and nonidentical synchronization of a novel fractional chaotic system, *Adv. Differ. Equ.* **2021** (2021) 308, doi:10.1186/s13662-021-03454-1.
29. A. Saltelli, Making best use of model evaluations to compute sensitivity indices, *Comput. Phys. Commun.* **145** (2002) 280–297, doi:10.1016/S0010-4655(02)00280-1.
30. C. F. Moss and S. R. Sinha, Neurobiology of echolocation in bats, *Curr. Opin. Neurobiol.* **13** (2003) 751–758, doi:10.1016/j.conb.2003.12.001.
31. M. W. Li, J. Geng, S. Wang and W. C. Hong, Hybrid chaotic quantum bat algorithm with SVR in electric load forecasting, *Energies* **10** (2017) 2180.
32. M. Benedetti, J. Realpe-Gómez, R. Biswas and A. Perdomo-Ortiz, Quantum-assisted learning of hardware-embedded probabilistic graphical models, *Phys. Rev. X* **7** (2017) 041052, doi:10.1103/PhysRevX.7.041052.
33. S. H. Adachi and M. P. Henderson, Application of quantum annealing to training of deep neural networks, preprint (2015), arXiv:1510.06356.
34. E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren and D. Preda, A quantum adiabatic evolution algorithm applied to random instances of an NP-complete problem, *Science* **292**(5516) (2001) 472–475, doi:10.1126/science.1057726.
35. K. H. Wan, O. Dahlsten, H. Kristjánsson, R. Gardner and M. S. Kim, Quantum generalisation of feedforward neural networks, *NPJ Quantum Inf.* **3** (2017) 36, doi:10.1038/s41534-017-0032-4.
36. C. P. S. Gonçalves, Quantum neural machine learning: Theory and experiments, Chapter 5, *Artificial Intelligence — Applications in Medicine and Biology*, ed. M. A. Aceves-Fernandez (IntechOpen, London, 2019), pp. 68–84, doi:10.5772/intechopen.84149.
37. R. P. Mahajan, A quantum neural network approach for portfolio selection, *Int. J. Comput. Appl.* **29**(4) (2011) 47–54.
38. M. Zidan, A. H. Abdel-Aty, M. El-shafei, M. Feraig, Y. Al-Sbou, H. Eleuch and M. Abdel-Aty, Quantum classification algorithm based on competitive learning neural network and entanglement measure, *Appl Sci.* **9** (2019) 1277, doi:10.3390/app9071277.
39. D. Delen, C. Kuzey and A. Uyar, Measuring firm performance using financial ratios: A decision tree approach, *Expert Syst. Appl.* **40** (2013) 3970–3983, doi:10.1016/j.eswa.2013.01.012.
40. D. Efimov and H. Sulieman, Sobol sensitivity: A strategy for feature selection. Mathematics across contemporary sciences, in *AUS-ICMS 2015, Springer Proceedings in Mathematics & Statistics*, Vol. 190 (Springer, Cham, 2017), pp. 57–76, doi:10.1007/978-3-319-46310-0_4.
41. X. Y. Zhang, M. N. Trame, L. J. Lesko and S. Schmidt, Sobol sensitivity analysis: A tool to guide the development and evaluation of systems pharmacology models, *CPT Pharmacometrics Syst. Pharmacol.* **4**(2) (2015) 69–79.
42. D. Alaminos, S. M. Fernández, F. García and M. A. Fernández, Data mining for municipal financial distress prediction, in *Advances in Data Mining, Applications and Theoretical Aspects*, ed. P. Perner, Lecture Notes in Computer Science (2018), pp. 296–308, doi:10.1007/978-3-319-95786-9_23.

43. C. R. Harris, K. J. Millman, S. J. Van der Walt, R. Gommers, P. Virtanen, D. Cournapeau and T. E. Oliphant, Array programming with NumPy, *Nature* **585** (2020) 357–362, doi:10.1038/s41586-020-2649-2.
44. IBM Quantum (2021), https://quantum-computing.ibm.com.
45. Qiskit, Qiskit 0.33.0 documentation (2021), https://qiskit.org/documentation.
46. E. C. Y. Ng, Forecasting US recessions with various risk factors and dynamic probit models, *J. Macroecon.* **34**(1) (2012) 112–125, doi:10.1016/j.jmacro.2011.11.001.
47. T. J. Berge, Predicting recessions with leading indicators: Model averaging and selection over the business cycle, *J. Forecast.* **34** (2015) 455–471, doi:10.1002/for.2345.
48. D. Alaminos, M. B. Salas and M. A. Fernández-Gámez, Quantum computing and deep learning methods for GDP growth forecasting, *Comput. Econ.* **59**(21) (2022) 1–27, doi:10.1007/s10614-021-10110-z.
49. S. Lleo and W. T. Ziemba, The bond-stock earnings yield differential model: Additional applications and other models for stock market crash prediction, *Quant. Finance Lett.* **4**(1) (2016) 26–34, doi:10.1080/21649502.2015.1165905.
50. I. Koprinska, M. Rana and A. Rahman, Dynamic ensemble using previous and predicted future performance for multi-step-ahead solar power forecasting, in *ICANN 2019*: *Artificial Neural Networks and Machine Learning* (2019), pp. 436–449, doi:10.1007/978-3-030-30490-4_35.
51. S. Makridakis, E. Spiliotis and V. Assimakopoulos, Statistical and machine learning forecasting methods: Concerns and ways forward, *PLoS One* **13**(3) (2018) e0194889, doi:10.1371/journal.pone.0194889.