Localism in Logic: an Analysis of Chunk and Permeate Methodology

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Abstract In this paper we present a different approach in the classical debate over pluralism and monism. This approach focuses on whether the application of logic is local or global. The thesis we will defend is in favour of localism. In doing so, we will introduce the methodology of Chunk and Permeate, in order to give an account of one of the problems that localism faces.

Key words: pluralism, monism, localism, globalism, Chunk and Permeate.

1. Introduction

The aim of the present dissertation is to offer a new perspective in the classical debate, within philosophy of logic, of pluralism and monism. This perspective is introduced by means of a distinction to which, as I see it, little importance has been given, namely, localism and globalism.

Our starting hypothesis will be in favour of localism. We will defend and try to show that there are different situations that require different logics and, so, that it is much less plausible that a single logic can work in all situations.

One of the main problems of localism arises when we deal with mixed domains. Hence, we will tackle this problem by means of a method for combining logics developed by Brown and Priest (2004), the method of *Chunk and Permeate* (*C&P*).

The structure of the paper, then, is the following: in section 2 we present the most influential contemporary defences of pluralism and monism. In addition, we also introduce the concepts of localism and globalism and explain how they are related with pluralism/monism. Section 3 is devoted to the exposition and analysis of the *C&P* method, after which we will evaluate the consequences of the method for the philosophical debate in section 4. Finally, we will conclude with some remarks in section 5.

2. Is there one logic or are there many?

The purpose of this section is to set the coordinates of the analysis we will carry out hereinafter. This is, as one of the goals of the dissertation has to do with the philosophical implications of the *C&P* model –in particular, with the implications for the debate regarding pluralism/monism in logic–, we have to lay down the terms, assumptions and main theses that configure the framework of the discussion. Thus, we will have a picture with which the implications of *C&P* will be measured and compared against.

2.1. Pluralism and Monism

In order to know whether there are many logics (pluralism) or just one (monism), some clarification about what pluralism and monism mean is needed. The references we will employ for setting this debate up are Priest (2006) and Beall and Restall (2000). The justification for this choice is twofold. Firstly, they provide the most influential defences of monism and pluralism, respectively. So, they are widely taken as the best-argued accounts of each program. Secondly, despite defending opposing positions, both share a common view regarding what logic is about, namely, *logical consequence.*

Before turning to consequence relations, let us briefly make a side comment. I am not going to discuss which is the purpose or the subject matter of logic. I will simply assume Priest and B&R's position because it simplifies the debate and, also, because if there actually are various correct logics under this assumption, then pluralism is a more substantial thesis[1]. In fact, under other conceptions of what logic is about, the pluralist position seems to be even more plausible. van Benthem (2008), for instance, argues that the view of logic as being about consequence relations may had some sense when it was thought to provide the foundations of mathematics. But, since 1930s the field has changed

[1] When we use "correct logic" we mean, roughly, the logic that is most fruitful, most adequate to the data, overall simplest, etc. Thus, we do not aim to imply any metaphysical view on whether there is, or not, an objective reality that logic seeks to capture. As Priest (2006, 10. 13) points out, even though being a realist himself about logic, the criteria of correction for a logic are, in general, the same for realists and instrumentalists. For present purposes, then, we can stay neutral with respect to ontology while subscribing to the criteria of correctness.

and broadened its scope. Logic is now, van Benthem claims, about *definability*, *computation* and more (2008: 183). Indeed, van Benthem defends that the main issue of logic is "the variety of *informational tasks* performed by intelligent interacting agents, of which inference is only one among many, involving observation, memory, questions and answers, dialogue, or general communication" (2008: 182). This broadening of the subject matter of logic clearly reinforces pluralism over monism. If the domains of application of logic increase, it seems reasonable to suppose that one single logic won't do all the job. Of course, the sense of 'pluralism' might be different under this conception of logic, since it is not only that there might be different accounts of consequence that various logics systematise. It is, also, that the variety of logics might correspond to different accounts of observation, communication, and so on.

However, we are going to assume here, for the sake of simplicity of the exposition, that logic is mainly about consequence relations, that is, about what conclusions follow from what premises. The account of consequence and validity that Priest (2006) and B&R (2000) deploy is the traditional semantic one:

> (C) A conclusion, *A*, *follows from* premises, Σ, if and only if any case in which each premise in Σ is true is also a case in which *A* is true (B&R, 2000: 476).

From this definition of consequence, validity is usually defined as:

> (V) A *valid* argument is one whose conclusion is true in every case in which all its premises are true (*Ibidem*).

Now, even though Priest (2006) and B&R (2000) endorse fairly similar accounts, Priest argues for monism while B&R argue for pluralism. Let us start with the argument for pluralism.

The key concept in the definitions of consequence and validity, the one that B&R exploit in their argumentation, is the concept of *case*. The authors maintain that for (C) to completely define logical consequence, the cases have to be specified. That is, specifying the cases provides the truth conditions for the sentences of a given language. In this sense, to specify a case is to provide an explanation of what it is for a sentence of a given language to be true in that case. The pluralist thesis, then, is the following: there are different ways in which cases can be specified and different specifications give rise to different logics that are equally correct (2000: 477-478).

So, for instance, one might construe cases as *Tarskian models* which would interpret a language of first-order logic by giving truth conditions to its sentences recursively, in the usual way. This would give rise to *classical logic*[2]. But, as we said, there are other specifications of cases which give rise to different logics. B&R (2000) provide two interesting examples of non-classical logics obtained in such a way: *relevance* and *intuitionistic* logics. Let us illustrate the point with the former.

Relevance logic is obtained by specifying cases as *situations*. To say it briefly, a situation might be understood as a restricted part of a world which might be incomplete[3]. That is, a part of a world that can make true some claims, but where it might not be the case that for every sentence φ, either φ or ¬φ are true. So, a situation might be indeterminate with respect to the truth or falsity of some sentences. Now, this has a straightforward consequence; it is easy to see that we already have a disagreement between the classical and the relevant logicians with respect to, at least, one inference. While in classical logic it is valid to infer φ∨¬φ from α (being φ∨¬φ logically valid, in classical logic, it can be inferred from no premise at all), that very same inference is not *relevantly* valid, inasmuch as we can provide a situation in which α is true without neither φ nor ¬φ being true (B&R, 2000: 483).

The way B&R explain this discrepancy nicely illustrates the sense of their pluralism:

> The virtue of a pluralist account is that we can enjoy the fruits of relevant consequence as a guide to inference without feeling guilty whenever we make an inference which is not relevantly valid. With classical consequence you know you will not make a step from truth to falsehood, assuming, with most philosophers, that *possible* worlds are complete and consistent. With relevant consequence, the strictures are tighter; you know you will not make a step from one that is true in a situation to something not true in it (but which might be true outside it). This is a tighter canon to guide reasoning (B&R, 2000: 484).

Hence, B&R consider that there is not a real rivalry between classical and relevant validity. They can perfectly coexist, since both specifications of cases illuminate different aspects of logical consequence and validity.

[2] It is also possible to specify cases as *worlds*, which is another approach within classical logic concerned with validity as *necessary truth preservation*.
[3] It can also be inconsistent, indeed, but we are not going to consider such a situation in our example.

Then, it seems as if by construing the notion of case in different ways, we could obtain different logics that systematise distinct levels or features of consequence relations in ordinary reasoning. If logic is about consequence relations, there are various aspects of consequence that require different logics in order to be captured.

As we said above, Priest (2006) agrees with B&R both in the subject matter of logic and in the conceptions of consequence and validity. Nevertheless, he argues in favour of monism. It is important to specify, however, in which sense and with respect to what Priest defends monism. So, Priest does not deny pluralism relative to pure logics (cf. Priest (2006), section 12.2, for the distinction between pure and applied logic). It is a banality that there are many pure logics. Neither does he deny theoretical pluralism: that there are different logics that compete for being the most suitable for a given domain (2006: 196). What Priest denies is that there are a variety of logics that are equally good for its *canonical application*: the analysis of reasoning. That is, Priest's monism is to the effect that there can be just one correct applied logic in the analysis of reasoning, i.e. in the analysis of what follows from what (*ibidem*)[4].

Thus, the claim that marks his opposition with B&R is that "it is only truth-preservation over *all* situations [cases] that is, strictly speaking, validity" (2006: 202). That is, according to Priest, validity is not something that can be relativized to this or that domain. Logic must work "come what may" (2006: 202). Nevertheless, Priest accepts that we might have to reason, say, classically in some domains, despite classical logic not being the correct logic. But this does not imply, according to Priest, that there are different logics for different situations, it means that we can adopt some contingent features of particular domains to recover classical validity enthymematically (2006: 198) (More on this in section 4).

Now, there is a point in need of clarification here, since B&R also want to hold that logic works come what may (B&R, 1999: 14), but the sense in which they hold it is rather different. To my mind, the best explanation they provide for supporting together the theses that logic works come what may and pluralism, is that the universal, 'any case',

[4] In what follows, then, we should have in mind that Priest's monism is restricted to the canonical application of logic. When we speak about 'different domains' or 'all domains' we are not referring to other domains of application on top of the analysis of reasoning, but to the different domains within the canonical application (mathematical reasoning, reasoning about middle sized objects, reasoning about counterfactuals, etc.).

in (C) does not quantify over a fixed domain, as Priest suggests, but that there is a variation in the domain over which the universal quantifies.

However, the argument for domain variation is not that obvious, so further reasons must be provided in its favour. A positive argument for variation stresses the imprecision of the notion of case. By its own, 'case' does not inform us about under which conditions a sentence is true, because cases can be specified in different types of cases. Therefore, logic –or logics– is obtained only after such specifications are provided, since (C) does not determine just one such specification. Another argument, this one negative, highlights the consequences of taking the domain of quantification as fixed: which inferences, if any, would be valid in all cases? This question is difficult to answer, in part because it is not clear how many possible classes of cases there are. The suggestion by B&R is that the only plausible candidate for being truth-preserving over all cases, in the sense of Priest, is the *identity* inference, $\alpha \vdash \alpha$ (2000: 490). Should we admit, then, that the only valid argument is identity? That there is only one logic whose subject matter is whether α follows from α?

2.2. Localism and Globalism

As we have previously stated, one of our objectives is to motivate the distinction between localism and globalism as a way of enriching the debate around pluralism and monism. The localist thesis states that there is a multiplicity of types of objects that configure various domains of discourse, in such a way that reasoning about these different kinds of objects requires adopting different logics. Globalism, on the other hand, is the position defending that the implementation of logic is global, in the sense that logical laws and valid arguments must be applicable regardless of the content (the different objects we might reason about).

As far as I know, the distinction is used by Haack (1978, Ch. 12) and Priest (2006, section 10.14) but with different nuances. As we conceive it, the distinction is orthogonal to that of pluralism/monism. So, this yields, *prima facie*, four possible theoretical options available in the debate:

Globalism-Monism: there is just one correct logic and it is neutral with respect to the domain to which it is applied.

Globalism-Pluralism: there are a variety of logics that are equally correct and their application is global, i.e. independent of the objects of reasoning.

Localism-Monism: Different domains of discourse require different logics, but there is only one correct logic for each domain.

Localism-Pluralism: Different domains of discourse require different logics and there might be various equally correct logics for a given domain.

Our starting hypothesis is in favour of localism; in favour of there being different types of objects of reasoning that require different logics. So, our theoretical options in the debate would be the last two. Let us, for the moment, leave open which kind of localism is more plausible and make a couple of comments on how Priest and B&R's position fit these options.

Firstly, Priest's position seems to correspond clearly to globalism-monism, since he defends both that there is just one correct logic and that it works come what may. That is, independently of the domain of reasoning to which it is applied. B&R's position, however, does not so obviously belong to a single option. According to Field's (2009) interpretation, for instance, B&R's pluralism, though interesting, "falls *far* short of the kind of pluralism that says that advocates of apparently competing *all-purpose* logics don't really disagree" (2009: 346). Putting it in our terms, Field claims that B&R's pluralism is not one in which every equally correct logic applies globally. I do agree with Field in this respect. To my mind, the option that better fits B&R's account is localism-pluralism, allowing that some domains to which different logics apply might overlap. Take, for example, the domain of mathematics. On B&R's view, it makes sense to use both intuitionistic and classical logics within mathematics, while relevant logic has no clear application there (2000: 485). In this sense, at least relevant logic wouldn't be global, and this is enough to take them as defending a form of localism-pluralism.

The relevant point for us is that localist theses, intuitive as they might be, have to face an important challenge; a challenge that Priest (2006) himself raises and that can be summarised as follows: one might defend that there are a variety of domains that require different logics. But there are cases in which one reasons about the interaction of different

domains, with premises about different kinds of objects coming from those domains. What kind of logic do we use, then? An underlying logic for both domains? This would give reasons for thinking that there is a logic of global application. Maybe a new logic specific for that domain of interaction? But which one? The intersection of the logics involved in each of the interacting domains might be too weak to be of any utility. Moreover, it should be taken into account that if we start trying to mix the connectives of different logics some of them may collapse. The intuitionistic conditional, for instance, collapses into classical conditional under the presence of classical negation (Priest, 2006: 199).

Ironic as it may sound, Priest, together with M. B. Brown (Brown and Priest 2004) developed a strategy for "handling the application of different logics in combination" (Priest, 2014: 333). The philosophical implications of this model for the debate that concerns us are not clear, as we will see, despite Priest (2014), a dyed-in-the-wool monist, presents the model as providing an explanation that logical pluralists could employ in meeting the challenge. Let us introduce the model.

3. The model of Chunk and Permeate

In order to provide a clear but rigorous presentation of the methodology of *C&P*, we will mostly rely on Priest (2014), where he offers the most comprehensible and systematic account of it. To begin with, we should make a minor point regarding terminology. Priest refers to *C&P* as "methodology", "structure" and "model", as far as I can tell. I guess this ambiguity is neither problematic nor confusing, so I will probably be qualifying *C&P* in those terms too.

As a first informal approximation, we can say that the method of *C&P* offers a procedure for dealing with inconsistent information. Within classical logic, if we had an inconsistent premise set, everything would follow from it by the principle of explosion ($\alpha, \neg\alpha \vdash \beta$). *C&P*, as a paraconsistent reasoning strategy, allows us to infer non-trivial and non-arbitrary information from an inconsistent premise set. *C&P* method partitions the reasoning into discrete chunks. Each chunk may have its own logic (in principle, this is not necessary, but since we are interested in situations in which different logics have to

be combined, the *C&P* structures that interest us are those in which each chunk has a different logic) and so, we can make different derivations in each of them.

An interesting feature of *C&P* is that, on top of dividing the reasoning, it allows some flow of information between chunks. That is, there are some formulas that can be permeated from one chunk to another, what permits some kind of interaction between them.

More formally, now, *C&P* structures can be characterized as follows:

Chunks: the chunks, C_i, in which reasoning is partitioned, are theories consisting of a language, L_i, its set of closed formulas, F_i, a set of axioms, A_i, and a consequence relation, $\vdash_i$ (Priest, 2014: 333)[5].

Permeability: in order to permeate information between chunks i and j ($i \neq j$), we have a permeability filter ρ_{ij}, which is a subset of F_i. Intuitively, ρ_{ij} determines which kind of formulas we will allow to flow from C_i to C_j. Together with the permeability filter, we need a translation function, t_{ij}: $F_i \rightarrow F_j$, to transform sentences of L_i into sentences expressible in L_j.

Thus, a *C&P* structure is, $\wp = \langle \{\langle L_i, A_i, \vdash_i \rangle : i \in I\}, \{\langle \rho_{ij}, t_{ij} \rangle : i, j \in I, i \neq j\}, o \rangle$, where I is an index set, and o is the label of the output chunk, C_o, where we obtain the conclusions of the structure.

The structure operates by a series of closure and permeation steps. First, each chunk, C_i, is closed under logical consequence, $\vdash_i$. Then, the consequences of C_i that are in ρ_{ij} are translated by means of t_{ij} and added to A_j. The process of closing and permeating is repeated again ω times, and the output of the structure, $\wp$, is the output of C_o. We obtain the axioms and theorems of each chunk at the nth step recursively, as follows:

$$A_i^0 = A_i$$

$$T_i^0 = \{\alpha : A_i^0 \vdash_i \alpha\}$$

$$A_i^{n+1} = A_i^n \cup \bigcup_{i \neq j \in I} (T_j^n \cap \rho_{ji})^{t_{ji}}$$

$$T_i^{n+1} = \{\alpha : A_i^{n+1} \vdash_i \alpha\}$$

[5] Priest does not specify or restrict the kind of languages and logical consequences that chunks may have.

If the process is conducted ω times, then a formula α is a consequence of the structure, $\wp \Vdash \alpha$, iff $\alpha \in T_o^{\omega}$ (Priest, 2014: 333). It should be noted, as Brown and Priest (2015: 299) do, that the notion of consequence used here is not the usual one. The sentences in T_i^n do not determine the contents in T_o^n. In these models, the consequence relation is between *C&P* structures and some sentences. "The structure as a whole determines the content of the output chunk [C_o] at each step of the recursion" (Brown and Priest, 2015: 299).

Now, before moving on to some other issues regarding *C&P* method, let us make one observation. Right after presenting *C&P* structures (2014: 333-334), Priest points out that "the whole *C&P* mechanism can be formulated as one 'hyper-theory'" (2014: 334). If he were not a true monist, this information would probably be understood just as an innocent and interesting comment. But, at least in my view, the alleged innocent remark has the further intention of suggesting that there is a possible monist reading of this whole story. The formulation as a hyper-theory goes like this: the output chunk in the *C&P* structures has a central role in the hyper-theory. Its language is that of C_o augmented by predicates T_i, for every $i \in I$, and by names $\langle\alpha\rangle$ for every formula α of each language L_i. So, $T_i\langle\alpha\rangle$ should be read as "α is a theorem of chunk *i*". The hyper-theory, then, has the following axioms:

• for each $i \in I$, and $\alpha \in A_i$, $T_i\langle\alpha\rangle$

The logic of the theory is $\vdash_o$ plus the following rules of inference:

• for every $i \in I$, and every valid inference, $\alpha_1,\ldots, \alpha_n \vdash_i \beta$, $T_i\langle\alpha_1\rangle,\ldots, T_i\langle\alpha_n\rangle \vdash T_i\langle\beta\rangle$

• for every α in ρ_{ij}, and $i \neq j \in I$, $T_i\langle\alpha\rangle \vdash T_j\langle t_{ij}(\alpha)\rangle$

• for every α in L_o, $T_o\langle\alpha\rangle \vdash \alpha$. (Priest, 2014: 334)

We will come back to the implications of this point for the localist position later on. For the moment, there are some issues regarding *C&P* structures that have to be stressed.

3.1. Analysis of the *C&P* method

The first issue I would like to go through is about a slight difference between the way *C&P* is presented in Priest (2014) and Brown and Priest (2004, 2015). In the latter works, *C&P* is presented as a method for rationally reconstructing how some people reasoned with apparently inconsistent information (infinitesimal calculus and Bohr's hydrogen atom). In these cases, they start with a set, Σ, which is inconsistent. The first step of the *C&P* method is, then, to define a *covering* on Σ, which is a set, C such that $C = \{\Sigma_i: i \in I\}$ and each Σ_i is classically consistent. In this way, we can close each Σ_i under classical logical consequence separately. This move was possible, in part, because it was assumed that the underlying logic of every chunk in the structure was classical (Brown and Priest 2004, 2015).

But, as we have seen, in Priest (2014) the purpose is different. We are not concerned here with giving a rational reconstruction of some scientific discovery who relied on inconsistent information. Our aim is, and also Priest's (2014), to provide a model that allows reasoning across domains by combining logics. So, the *C&P* structures that could serve our purpose are those in which there are at least two different logics. However, there is, in principle, no restriction as to which kind of logic the chunks might have. So, it is possible that one of the chunks' underlying logic were a paraconsistent logic. This possibility might be one of the reasons explaining the different modes of presentations in Priest (2014) and Brown and Priest (2004, 2015), since if one of the logics can be paraconsistent, then defining a covering on Σ with the criterion of classical consistency does not seem the right move[6]. This is why, I believe, Priest (2014) decides to introduce the chunks as already given; each one separated from the others with its own logic and its proper domain of application.

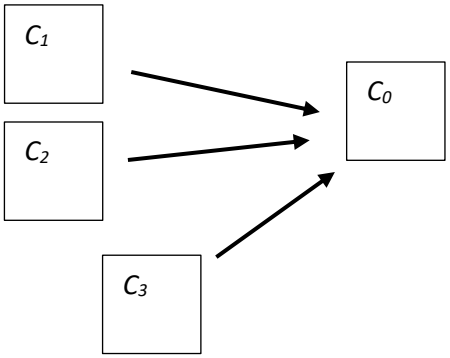
A second point worth noting has to do with the number of steps and the recursive definitions presented above. First, with respect to the number of steps that a *C&P*

[6] At least for the paraconsistent chunk, non-triviality should be the criterion.

structure needs to make in order to get the final output of C_o, I think that the following general rules or conditions can be given[7]:

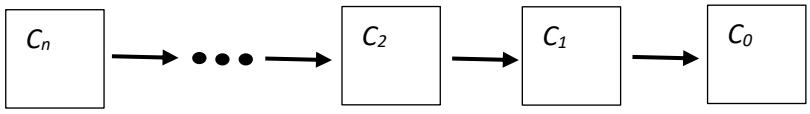
i) If $\forall i \forall j ((j \neq 0 \rightarrow \rho_{ij} = \emptyset) \wedge (i = 0 \rightarrow \rho_{ij} = \emptyset))$, $0 \leq i \leq$ n, then $T_0^1 = T_0^\omega$.

That is, if the graph of the *C&P* model is such that the only connections are between the C_is ($i \neq 0$) and C_o, then the consequences of the structure are obtained in one step. Priest (2014: 334) gives a condition stating that if the number of chunks is not infinite, the output of C_o is established after some finite n. The case we are considering would be an exception. Even if the number of chunks is infinite, the process ends in one step if the graph is of the kind considered. For instance:



ii) If $\forall i \forall j$ ($\rho_{ij} \neq \varnothing \leftrightarrow j = i$-1), $0 \leq i \leq$ n, for some finite n, then $T_0^{n-1} = T_0^\omega$.

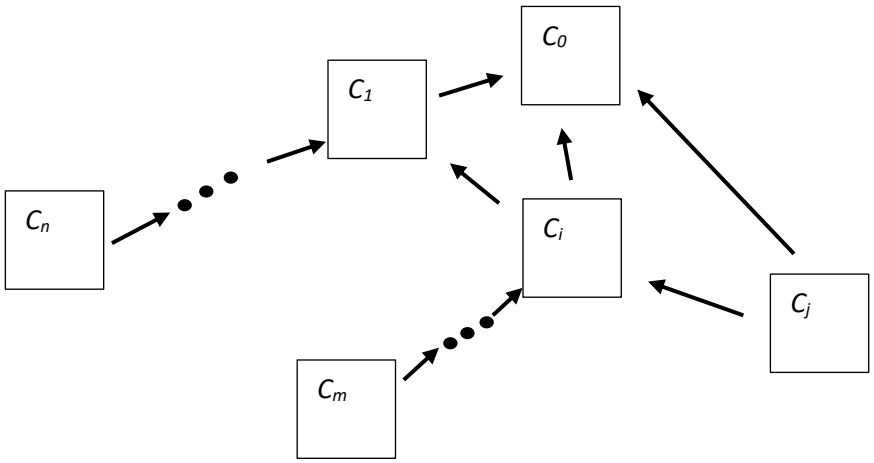
This condition states that, if there are n chunks forming a chain, such that every C_i is only connected with C_{i-1}, then the process ends and we obtain the consequences of the structure in n -1 steps.



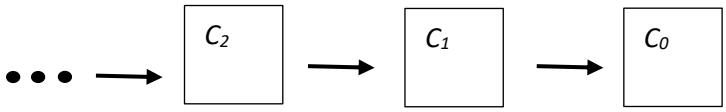
iii) It is possible, in principle, that there is more than one finite chain forming a treelike structure. In this case, the longest branch will set the number of steps

[7] We will assume that each C_i has a different logic and that every time some new information flows into one chunk, that chunk produces new consequences. Thus, our rules determine the maximum of steps needed for the system to get at the final output.

needed for reaching the end of the process. If the longest branch consists of n chunks, then we will get the final output in n-1 steps.



iv) If $\forall i \forall j$ ($\rho_{ij} \neq \varnothing \leftrightarrow j = i\text{-}1$), $0 \leq i \leq$ n, for an infinite n, then the process ends in ω steps.



There probably is some other way of stating under which conditions a graph containing cycles ends the process in ω steps, but I cannot come up with any such condition. Of course, it could be objected that there are no cases of logics with interesting applications that are combined in an infinite chain. Fair enough, but that was not the purpose of stating these conditions and, I would add, it's hard to conceive a case with applications in which the process takes ω steps to finish.

Now, moving to the point about the recursive definitions, there is an issue that could affect the general validity of the conditions we have just stated. Priest (2014) does not make any explicit restriction on the kind of logics that can underlie the different chunks. So, *prima facie*, it should be possible to have a chunk whose underlying logic is a contraction-free substructural logic (Structural rule of contraction: if $\Gamma, \alpha, \alpha \vdash \beta$ then $\Gamma, \alpha \vdash \beta$). If this is so, then the above conditions would only be valid as long as the logics involved are contractive. For notice that if they are not, we could draw new conclusions by permeating

the very same information as in the previous step and so, the process would not necessarily end when no more new information can be permeated.

As Priest presents it, it seems that he is not considering the possibility of having a contraction-free substructural logic in a chunk, inasmuch as he offers the recursive definitions in terms of sets, which do not allow for multiple instances of the same elements. Nevertheless, I guess we could generalize the recursive definitions from sets to *multisets*, in order to allow for contraction-free logics, by adding multiplicity functions, m: $\Gamma \rightarrow \mathbb{N}_{\geq 1}$, to each A_i^n and T_i^n. Thus, we obtain multisets, which are 2-tuples (A_i^n, m_j) and (T_i^n, m_k). In this way, the multiplicity function determines the number of occurrences of an element in a set.

There is a third and last point I would like to briefly talk about, namely, preservation of consistency after information flow. In Brown and Priest (2004) the authors ask whether there are any "constraints on information flow between the chunks which are sufficient (or even necessary and sufficient) for the maintenance of consistency" (386-387). Well, in my modest opinion, there are not. At least not interesting ones.[8]

The main difficulty in stating those constrains is that they will crucially depend on the concrete features of each case. Notice that it is not only constrains on the permeability filter, ρ_{ij}, that we have to consider. The translation function, t_{ij}, the configuration of the graph, the kinds of logics that might underlie each chunk, etc. play a decisive role for the question of consistency. Even the domains involved in the interaction may affect our considerations about consistency, since it is not irrelevant the fact that the logics we combine have overlapping domains or not. Intuitively, if we have, say, an intuitionistic logic for mathematical objects and a temporal logic dealing with time intervals, it seems easier to avoid contradictions when information flows, inasmuch as the logics won´t be stating φ and $\neg\varphi$ with respect to the same object.

[8] I guess a basic restriction could be this: don´t let every sentence flow to the output chunk, at least if they are not translated in such a way as to avoid inconsistencies. If the translations relativize the quantifiers to their original domains (e.g. $\forall x\,(K_1x \rightarrow \alpha)$, K_1x meaning that x is an object of kind 1; of domain 1) and the theories had a model, then the output chunk will have a model if its domain is the union of the other domains.

On top of this, the possibility of having paraconsistent logics in our chunks makes the issue of preserving consistency even more convoluted. In such a situation, it is not inconsistency which needs to be avoided, but non-triviality, and the criteria for this will be clearly different. For these reasons, I believe it is both difficult and unimportant to provide necessary and sufficient conditions for the preservation of consistency. If there are any, the big amount of variables that play a role on the issue will probably make constrains irrelevant for any concrete application. Therefore, we might say that the preservation of consistency is a problem that has to be balanced and resolved in the particular application, taking into consideration the great variability in features that *C&P* structures may exhibit from one application to another.

4. The *C&P* model in the debate over pluralism, monism, localism and globalism

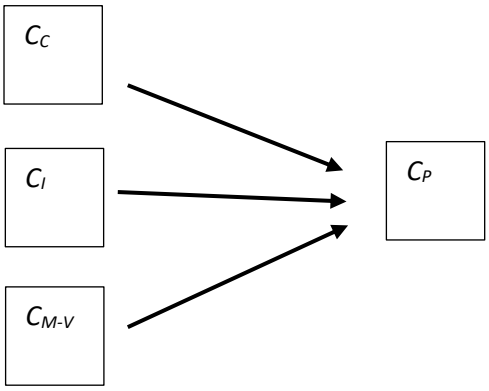
As we said at the beginning, one of our main goals is to evaluate the consequences of *C&P* –as a model for combining logics– for the debate over pluralism and monism (enriched with the distinction localism/globalism). As I see it, there is a first straightforward observation that can be made: Priest (2014) presents the method of *C&P* as a way of explaining, from a kind of pluralist standpoint, how do we reason in situations of interacting domains and which logical apparatus do we use. The kind of pluralism in which Priest is thinking there, is a pluralism of a particular sort, namely, one maintaining that "there are different kinds of situations, and different logics (or consequence relations) may be appropriate for reasoning about them" (Priest, 2014: 331). A little thought, however, suffices to see that this kind of pluralism corresponds to what we called "localism".

Now, if *C&P* methodology provides reasons in favour of the pluralist conception to which Priest (2014) refers, we would have arguments for localism, which would not settle the issue further, i.e. they would not decide over localism-pluralism or localism-monism. Our two types of localism could have been encompassed by some traditional pluralist conceptions, but, at the end of the day, what counts are the theses each option

defends and the arguments they deploy in defending them. It does not matter whether pluralism is vindicated, or whether someone wants to call "pluralism" to what we refer as "localism". That is just a matter of labels. To my mind, our distinction does not introduce further theoretical options arbitrarily. On the contrary, it identifies significant divergences that can offer a better understanding of the essential elements of the debate.

Then, does the *C&P* method offer reasons for believing in the validity of localism? Well, I would say that, despite not giving conclusive reasons, of course, it can be viewed as another element in favour of it. We must admit, though, the difficulty of quantifying the scope of the *C&P* method for the debate. A source of uncertainty in this respect has to do with the applicability of the method itself. We already pointed out that the method was conceived as a way of rationally reconstructing the reasoning processes in situations involving inconsistent information. This, however, only provides us with an answer to the challenge raised against localism: how to explain the reasoning when domains, with locally correct logics, interact. So, the *C&P* method helps in addressing the problem of mixed domains. Albeit this might be enough for preferring localism over globalism if, overall, there are better reasons for localism, we can still try to theorize about a broader hypothetical situation. A situation in which we can provide a *C&P* model encompassing every sub-domain of the canonical application of logic.

So, let us suppose, simplifying and idealizing, that intuitionistic logic, classical logic and a given many-valued logic, for instance, are the logics of the sub-domains exhausting the domain of reasoning. Let's suppose, furthermore, that the *C&P* model that captures this situation has a paraconsistent logic underlying the output chunk, C_o (as we will present the model later as a hyper-theory, let's concede this to Priest). The model, then, could be like this:



In principle, there could be more flow of information between chunks, but, to simplify, let us keep it like that. Moreover, we will suppose that the permeability filters, ρ_{ij}, and the translation functions, t_{ij}, are such that non-triviality is conserved (non-triviality because the logic of C_o is paraconsistent). So, the question is, within this hypothetical situation: do we have reasons for thinking that localism is the best account of canonically applied logics?

The immediate answer seems to be 'yes', after all, we have given a localist model that would explain the whole picture. Nevertheless, I guess that a globalist like Priest, would still have at least two possible moves available. We will consider them in turn:

1) A first possible response can be that, despite we have given a *C&P* model, we can interpret it as a single hyper-theory. Under this interpretation, the logic that matters in the end, the real logic, is the logic of the output chunk (a paraconsistent logic in our example, as Priest, in fact, believes). The problem of this reading, I would say, is that if you look at the axioms of the hyper-theory, you realize that the need of the other logics still persists. Indeed, the only way of adding the valid inferences $\alpha_1,\ldots, \alpha_n \vdash_i \beta$ as $T_i\langle\alpha_1\rangle,\ldots, T_i\langle\alpha_n\rangle \vdash T_i\langle\beta\rangle$, is by having previously worked out those valid inferences in each sub-domain with its own logic. Even more, consider that for some sentences α, β, and γ the following inference is intuitionistically valid, $\alpha, \beta \vdash_I \gamma$. So, we add to our hyper-theory this inference: $T_I\langle\alpha\rangle, T_I\langle\beta\rangle \vdash T_I\langle\gamma\rangle$. But the problem is, now, that our hyper-theory does not really capture anything about mathematical objects or about our reasoning on mathematical objects, in this case. Rather, it captures features about intuitionistic logic, what makes the hyper-theory, in part, a theory of intuitionistic logic. This, however, does not seem a consequence which globalists want to go along with.
2) The second possible reply has already been mentioned, since it is one of the arguments Priest himself endorses. The basic idea around which the argument is construed, is that there is a core of valid inferences that every canonically applied logic shares. This is the only logic, since the others would be obtained enthymematically by appealing to the different contingent properties of each sub-domain. So, for instance, if there are situations in which objects are not self-identical, then self-identity is not a logical law. But, in the appropriate domain, the domain of macro-objects, one could use self-identity by appealing to the

enthymeme ‘all macro-objects are self-identical’ (Priest, 2006: 200). Yet, the correct logic would still be the weakest one not containing self-identity.

There are some things one can object, though. Firstly, Priest is arguing for a really strong thesis, namely, that the only correct logic is that composed by the valid inferences that are in the intersection of every canonically applied logic. The answer he gives to those objecting that such a logic would be vacuous is that no one has shown that to be the case. Furthermore, he believes that “not all principles of inference fail in some situation” (2006: 203). According to him, the principle of conjunction elimination (from ‘p and q’ we may infer ‘p’) holds in every situation. Hence the intersection of the valid principles in every canonically applied logic would be neither empty nor trivial.

It is easy to notice that the arguments of both sides are quite tentative and this latter argument by Priest is not an exception. Surprisingly, the example he gives, after saying that conjunction elimination holds in every situation, is rather ordinary, namely, “the sky is blue and the sun is shining” (2006: 203). But situations are diverse and it is not obvious that conjunction elimination will hold in all of them. For instance, under some interpretations of quantum mechanics, the correct description of “Schrödinger’s cat” thought experiment, before opening the chamber, is that the cat is alive and dead. However, from it being alive and dead we cannot infer that it is dead neither that it is alive. It would not be correct to affirm one of the conjuncts alone.

On top of particular examples in which conjunction elimination might not hold, we actually have some connexive logics that reject this principle as a valid inference. One of the reasons for rejecting it is that conjunction elimination is incompatible with Aristotle’s Thesis ($\neg(\neg p \rightarrow p)$), a characteristic theorem of connexive logics (Thompson, 1991: 249-250)[9]. Of course, Priest could try to respond to particular examples and even argue that connexive logics are not well motivated. But this is something he should convincingly do. As I see it, the burden of the proof is more on his side. He is the one claiming for such a strong thesis,

[9] In particular, Thompson (1991) argues that connexivism rejects conjunction elimination because of the formula ‘$(p \wedge \neg(p \rightarrow p)) \rightarrow p$’, “for this formula asserts that ‘p’ follows from ‘p’, even under the condition that it does not!” (Thompson, 1991: 253).

while localists have in their favour the empirical evidence of practice in logic, or philosophical logic. New logical systems with canonical applications arise at a stretch. For avoiding or solving a paradox, for formalising reasoning about future contingents, about vague terms, and so on. Against this evidence, what Priest assumes seems to be too much. The thesis implicitly assumes that the domain of reasoning cannot increase, since the more logics we have for new sub-domains the more difficult will be to have a non-vacuous core of valid inferences. It also assumes that we know which the locally applied logics that we will consider for determining the valid inferences in all situations are.

With such assumptions and against the tendency that the evidence reveals, Priest's thesis appears as an, in principle, possibility. But this is too weak against the plausibility of localism. In fact, it is equally possible, in principle, that every valid inference of each situation corresponds to a contingent property of that situation. That is, that there is no essential property[10] at all and, therefore, no inference which is valid in all situations.

Everything considered, I am inclined to say that the *prima facie* intuitive claim in favour of localism is reinforced by further reflection. Even if the *C&P* method's scope is not sufficient for capturing the whole picture of canonically applied logics, it is at least a tool for explaining the cases of reasoning about interacting domains. This, together with the other considerations, makes localism a much more plausible theoretical position.

5. Conclusion

A first remark that should be made is that we have not attempted to provide a full response to the debate and, therefore, our arguments do not suffice for articulating a complete theoretical position. More concretely, they do not decide over localism-pluralism and localism-monism. Obviously, this is not something positive but if at least we have given

[10] In the sense of being a property of every situation.

good reasons in favour of localism and against globalism, then we have made some progress in a more precise formulation of the debate.

Finally, I would like to point out that the methodology of *C&P* is not the only available option for localism as a tool for solving the problem of mixed domains. So even if one wanted to reject the *C&P* method, localism would have enough options (juxtaposition, algebraic fibring, etc.) for remaining, everything considered, as a more plausible and reasonable position than globalism.

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