



# The aggregate productivity slowdown: A system approach

Rafael Serrano-Quintero

Department of Economics, University of Barcelona and Barcelona Economic Analysis Team (BEAT), Avinguda Diagonal 696, Barcelona, 08034, Spain



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## ABSTRACT

I revisit the productivity slowdown debate by estimating the capital-labor elasticity and the bias of technical change for the U.S. economy under four different models of technical change. One with constant growth rates, one with a structural break in the constant growth rates, one in which growth is linear, and one with flexible time-varying growth rates. I find evidence in support of non-constant growth rates of factor-augmenting technical change. Labor-augmenting technical change growth rates are decelerating, while capital-augmenting technical change is non negligible but vanishes quickly.

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## 1. Introduction

Motivated by the long-run constancy of the growth rate of GDP per capita, the assumption of constant growth rates of total factor productivity or factor-augmenting technical change is ubiquitous in the economics literature. However, labor productivity and total factor productivity have slowed down (Gordon, 2010; Fernald, 2014; Byrne et al., 2016). Philippon (2022) argues and provides evidence that TFP growth is, in fact, additive and not exponential. Thus, he concludes that this slowdown is more likely a result of the linear growth trend than a change in the exponential growth rate.

In this paper, I revisit the productivity slowdown debate by estimating factor-augmenting technical change processes for the aggregate U.S. economy under four scenarios. Constant growth rates, constant growth rates with a structural break, time-varying growth rates based on a Box-Cox specification (Box and Cox, 1964), and an additive growth specification as in Philippon (2022). To estimate factor-augmenting technical change, I follow (Klump et al., 2007) and use a normalized supply-side system where I estimate the elasticity of substitution between capital and labor together with the bias of technical change. I extend the results by Klump et al. (2007) considering a longer time period and estimating two additional models for technical change.

In terms of results, the model with a structural break yields the best overall fit followed by the Box-Cox model. This provides supporting evidence in favor of non-constant and decelerating growth rates of technical change. Under the Box-Cox specification, capital-augmenting technical change vanishes quickly over time, while labor-augmenting technical change shows mildly decelerating growth rates. This is consistent with capital-augmenting technical change being transitory and not the main driver of

growth (Acemoglu, 2003). All the models with non-constant growth rates suggest that both factor augmenting productivities are, in fact, decelerating. The Box-Cox model suggest that labor-augmenting technical change is decelerating at a faster pace than what the estimates of Klump et al. (2007) suggested, which is also supported by the structural break model.

I assess the contribution of the capital and labor augmenting technical change by estimating TFP using a Kmenta approximation (Kmenta, 1967) which is as a weighted average of both factor-augmenting technical change processes. I find that the dynamics of labor productivity are captured better by the structural break and the Box-Cox models, although the additive model provides a good fit. The model with constant growth rates has the poorest performance.

The rest of the paper is organized in the following sections. Section 2 introduces the estimation system; Section 3 describes the data; Section 4 shows the results; Section 5 concludes.

## 2. The supply-side system approach

To estimate the aggregate elasticity of substitution and the bias of technical change, I consider the system approach introduced by Klump et al. (2007). I follow (de La Grandville, 1989) and Klump et al. (2007) and explicitly normalize the production function for estimation. León-Ledesma et al. (2010) show that explicit normalization of the production function helps with the identification of the parameters. This normalization procedure consists on rewriting the production function in a consistent index form by choosing baseline values for labor ( $H_t$ ), capital ( $K_t$ ), output ( $Y_t$ ), and technology levels ( $\Gamma_t^i$ ). Thus, the production function explicitly normalized is given by

$$Y_t = Y_0 \left[ \pi_0 \left( \frac{\Gamma_t^K K_t}{\Gamma_0^K K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \pi_0) \left( \frac{\Gamma_t^H H_t}{\Gamma_0^H H_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (1)$$

E-mail address: [rafael.serrano@ub.edu](mailto:rafael.serrano@ub.edu).

where,

$$\pi_0 = \frac{r_0 K_0}{Y_0}$$

$\pi_0$  is the capital share evaluated at the baseline point. I follow (Herrendorf et al., 2015) by defining the normalization point as  $Y_0 = \bar{Y}$ ,  $K_0 = \bar{K}$ ,  $H_0 = \bar{H}$ ,  $\pi_0 = \bar{\pi}$ , and  $t_0 = \bar{t}$  where  $\bar{K}$ ,  $\bar{H}$ ,  $\bar{\pi}$ , and  $\bar{t}$  are the geometric averages of capital, labor, capital income share, and the time index, respectively.

I estimate the model under four different assumptions for technical change. First, when technical change grows exponentially at constant rates given by Eq. (2).

$$\log\left(\frac{\Gamma_t^i}{\Gamma_0^i}\right) = \gamma_i(t - t_0) \text{ for } i \in \{K, H\} \tag{2}$$

Second, I allow for a structural break as

$$\log\left(\frac{\Gamma_t^i}{\Gamma_0^i}\right) = \gamma_i(t - t_0) + \tilde{\gamma}_i(t - \tau^i)\mathbb{I}[t \geq \tau^i] \text{ for } i \in \{K, H\} \tag{3}$$

where  $\mathbb{I}(t \geq \tau^i)$  is an indicator function taking value one if the time period is above the structural break  $\tau^i$  and zero otherwise, and  $\tilde{\gamma}_i$  captures the change in trend after period  $\tau^i$ .<sup>1</sup>

Third, I follow (Klump et al., 2007) who propose a flexible form for technical change based on a Box-Cox transformation (Box and Cox, 1964) which allows for non-constant growth rates. This particular functional form is

$$\log\left(\frac{\Gamma_t^i}{\Gamma_0^i}\right) = \frac{\gamma_i t_0}{\lambda_i} \left[ \left(\frac{t}{t_0}\right)^{\lambda_i} - 1 \right] \text{ for } i \in \{K, H\} \tag{4}$$

where  $\gamma_i$  denotes the growth rate at the baseline point, and  $\lambda_i$  controls the curvature of the function. This specification nests three particular cases depending on the value of  $\lambda_i$ . For a value of 1, it becomes (2); for a value of 0 it becomes log-linear; and for negative values it becomes hyperbolic. Third, I estimate the system under the linear functional form (5)

$$\frac{\Gamma_t^i}{\Gamma_0^i} = 1 + \gamma_i(t - t_0) \text{ for } i \in \{K, H\} \tag{5}$$

where  $\gamma_i$  gives the step increase from period  $t$  to  $t + 1$  as in Philippon (2022).

The estimating equations then consist of the production function together with the first order conditions of the firm's cost minimization problem. The markup is defined as the inverse of the real marginal cost following (Jiang and León-Ledesma, 2018). The system is given by Eqs. (6)–(8).

$$\ln\left(\frac{Y_t}{Y_0}\right) = \frac{\sigma}{\sigma - 1} \ln\left[\pi_0 \left(\frac{\Gamma_t^K K_t}{\Gamma_0^K K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1 - \pi_0) \left(\frac{\Gamma_t^H H_t}{\Gamma_0^H H_0}\right)^{\frac{\sigma-1}{\sigma}}\right] + \varepsilon_t^Y \tag{6}$$

$$\ln\left(\frac{r_t K_t}{Y_t}\right) = \ln\left(\frac{\pi_0}{1 + \mu_t}\right) + \frac{\sigma - 1}{\sigma} \ln\left(\frac{\Gamma_t^K}{\Gamma_0^K}\right) + \frac{1 - \sigma}{\sigma} \ln\left(\frac{Y_t/Y_0}{K_t/K_0}\right) + \varepsilon_t^K \tag{7}$$

$$\ln\left(\frac{w_t H_t}{Y_t}\right) = \ln\left(\frac{1 - \pi_0}{1 + \mu_t}\right) + \frac{\sigma - 1}{\sigma} \ln\left(\frac{\Gamma_t^H}{\Gamma_0^H}\right) + \frac{1 - \sigma}{\sigma} \ln\left(\frac{Y_t/Y_0}{H_t/H_0}\right) + \varepsilon_t^H \tag{8}$$

<sup>1</sup> To be consistent with the normalization,  $\tau^i \geq t_0$ .

Estimating the full system with the Box-Cox specification is challenging because it is highly non-linear. Furthermore, it has been noted that the system estimation tends to produce values of  $\gamma_K$  that are negative which are difficult to interpret (Mućk, 2017). To circumvent these issues, I estimate the system under the Box-Cox specification by doing a fine grid-search on parameter  $\lambda_K$ . In particular, I estimate the system for values of  $\lambda_K \in [-3, 3]$  and look for the minimum log-determinant of the residual covariance matrix conditional on the value of  $\lambda_K$  and on  $\gamma_K$  being greater or equal than zero.

To estimate structural break model (3), I look for the combination of  $\{\tau^H, \tau^K\}$  that yields the lowest log-determinant. I estimate the system using a two-step GMM estimator with the lags of the variables as instruments with a weighting matrix that allows for heteroskedasticity and autocorrelation of up to two lags.<sup>2</sup>

### 3. Data

I use data from the Bureau of Labor Statistics, the National Income and Product Accounts Tables, and the Fixed Assets Tables for the United States, it is yearly data and covers the period 1948 until 2015. For the construction of the series, I follow (Koh et al., 2020) closely.

The measure for capital stock used in estimations is a Törnqvist aggregate of structures, equipment, and IPP capital obtained from the series for investment including private residential, private nonresidential, and government investment. Labor input is the product of nonfarm business sector average weekly hours of work and civilian employment. Output is defined as the Gross National Product. Finally, I take the estimated average markups for the U.S. from (Baqae and Farhi, 2019) which is computed as the harmonic sales-weighted average. Unfortunately, their estimated markup only goes back to 1980, so I keep the average markup constant at the 1980 value for the period 1948–1980. I use the labor share as constructed by Koh et al. (2020) and following (Jiang and León-Ledesma, 2018), I obtain the capital share as  $\frac{r_t K_t}{Y_t} = \left(1 - \frac{w_t H_t}{Y_t} \mu_t\right) \frac{1}{\mu_t}$ . Fig. 1 shows the series used in the estimation.

### 4. Results

Table 1 shows the estimated parameters for the system (6)–(8) under the four different models for technical change.

The elasticity of substitution is significantly below one in all cases and within the range of most of the estimates in the literature. The system performs well under all specifications but the best overall fit is achieved under the structural break model followed by the Box-Cox specification.

Under the constant growth rates, the structural break, and additive model specifications, capital augmenting technical change growth rates are negative. The structural break model estimates that there is a deceleration after 2001 for capital augmenting technical change and after 1974 for the labor augmenting. Under the Box-Cox specification, capital augmenting technical change is positive and statistically significant, but small. This fact together with  $\lambda_K$  being below 1 implies that capital-augmenting technical change tends to vanish quickly and is not really persistent over time. Although the structural break model slightly outperforms the Box-Cox model, the latter provides more economically sensible estimates.

<sup>2</sup> Essentially, this GMM estimator is a 3SLS estimator. It first estimates separately each equation by 2SLS and then computes a weighting matrix that allows for contemporaneous correlation of the error terms across equations. It uses this weighting matrix in the second step of GMM.

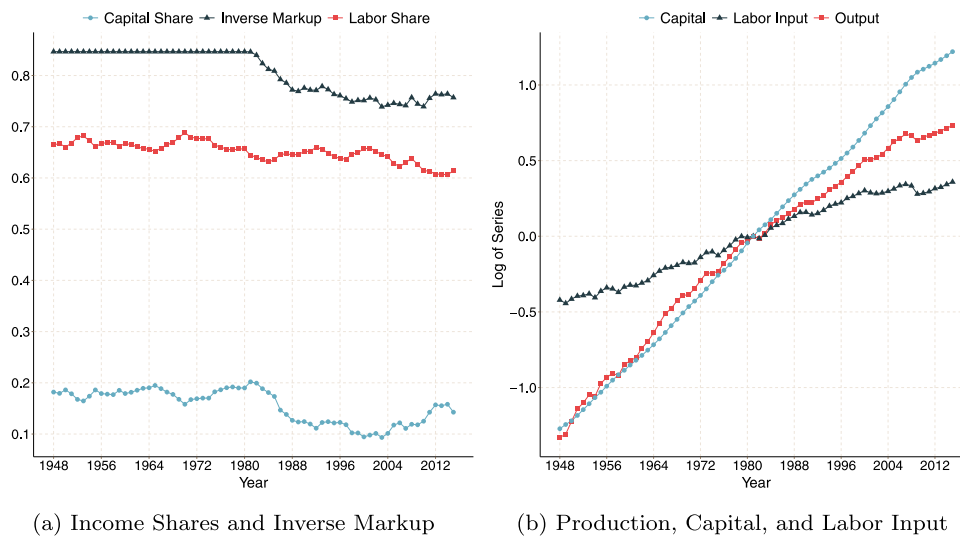


Fig. 1. Series for Estimation.

Table 1  
Estimation of the supply-side system.

	Constant T.C.	Structural Break	Box-Cox T.C.	Additive T.C.
$\sigma$	0.5912*** (0.0002)	0.2832*** (0.0006)	0.6512*** (0.0009)	0.4676*** (0.0013)
$\gamma_K$	-0.0062*** (0.0006)	-0.0017*** (0.0002)	0.0014*** (0.0004)	-0.0082*** (0.0001)
$\tilde{\gamma}_K$		-0.0357*** (0.0011)		
$\lambda_K$			0.08 [-]	
$\gamma_H$	0.0141*** (0.0002)	0.0280*** (0.0002)	0.0155*** (0.0001)	0.0147*** (0.0001)
$\tilde{\gamma}_H$		-0.0166*** (0.0002)		
$\lambda_H$			0.2155*** (0.0231)	
Structural Breaks				
$\tau^H$		1974		
$\tau^K$		2001		
Tests $p$ - values				
$\sigma = 1$	<0.001	<0.001	<0.001	<0.001
Hansen's $J$ -test	0.4441	0.4917	0.3302	0.8246
Log-Determinant	-19.3310	-23.0350	-20.5771	-19.5083

Note: Heteroskedasticity and autocorrelation robust standard errors in parentheses. \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . Parameter  $\lambda_K$  is chosen via a fine grid search. Lower panel shows  $p$ - values of the tests for  $\sigma = 1$ , Hansen's overidentification test, and the log determinant of the residual covariance.

For the labor-augmenting technical change,  $\gamma_H$  is positive and statistically significant in all specifications and always larger than  $\gamma_K$ . However, both the Box-Cox specification and the structural break models imply a deceleration in labor-augmenting technical change, since  $\lambda_H < 1$  and  $\tilde{\gamma}_H < 0$ . In fact, the estimate of  $\lambda_H$  is roughly half that of Klump et al. (2007) while  $\gamma_H$  is almost the same, which implies that labor-augmenting technical change is decelerating faster.

To assess the relative contributions of the capital and labor augmenting technologies, it is helpful to use the Kmenta approximation (Kmenta, 1967). This gives us an implied TFP that is a weighted average of both technical change processes. This approximation is given by

$$\log(TFP) = \pi_0 \log\left(\frac{\Gamma_t^K}{\Gamma_0^K}\right) + (1 - \pi_0) \log\left(\frac{\Gamma_t^H}{\Gamma_0^H}\right) - \frac{1 - \sigma}{\sigma} \frac{\pi_0(1 - \pi_0)}{2} \left(\log\left(\frac{\Gamma_t^H}{\Gamma_0^H}\right) - \log\left(\frac{\Gamma_t^K}{\Gamma_0^K}\right)\right)^2 \quad (9)$$

The panels of Fig. 2 show the relative contributions of each factor-augmenting technical change together with the Kmenta approximation. For both the additive and the structural change models, the contribution of capital augmenting technical change is negative, reducing aggregate growth. In the structural break model, there is also a reduction in the labor augmenting technical change growth rate.

To assess how well the three models track the dynamics of labor productivity, the panels of Fig. 3 show their predicted values. The structural break and the Box-Cox specification yield the lowest residual mean square error (RMSE) of 0.029 and 0.039, respectively. The additive model yields an RMSE of 0.052.

### 5. Conclusions

I revisit the productivity slowdown by estimating a supply-side system under four different models for factor-augmenting technical change. One with exponential and constant growth

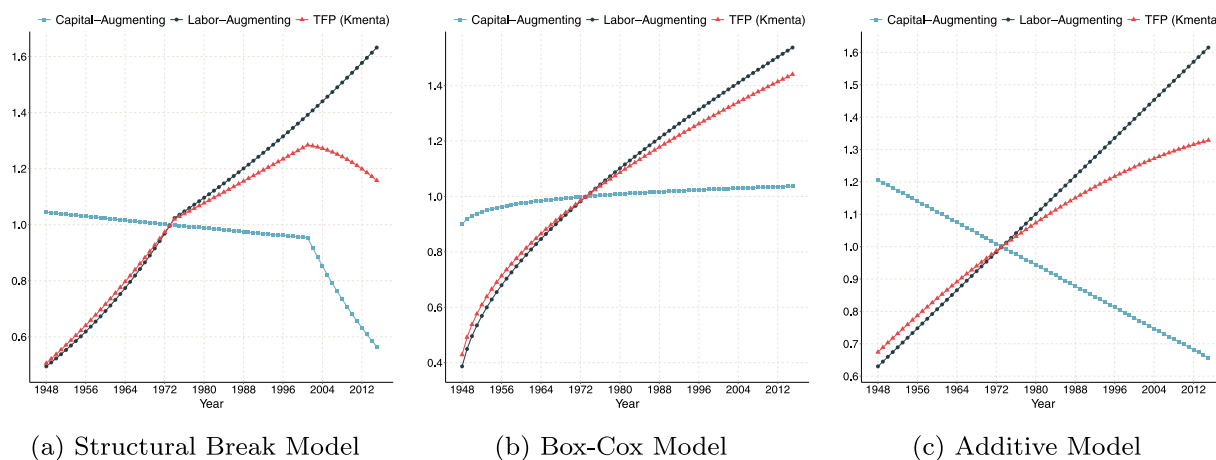


Fig. 2. Predicted Technical Change Paths.

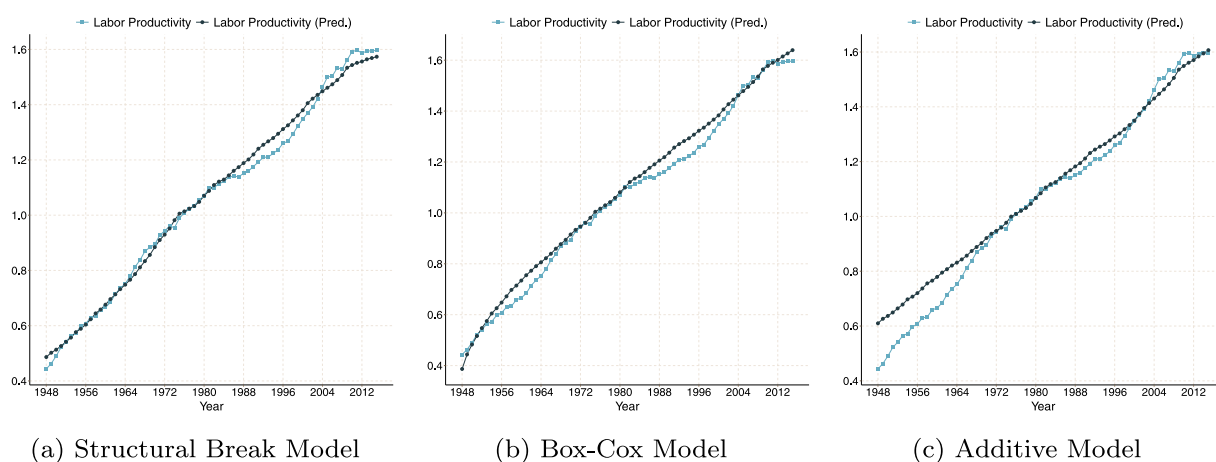


Fig. 3. Predicted and Actual Labor Productivity.

rates, a similar model with a structural break, a Box-Cox specification that allows for time-varying growth rates, and an additive model. I find that the elasticity of substitution is below one in all cases and provide supporting evidence that the growth rates of factor-augmenting technical change are not constant. In particular, I find labor-augmenting technical change to be the main driver of growth, although all models suggest growth rates are decelerating. Capital-augmenting technical change is non-negligible but vanishes quickly.

I use the estimates to predict the dynamics of labor productivity and find that the models with non-constant growth rates perform well. In sum, the paper provides more evidence in favor of non-constant growth rates of technical change.

**Data availability**

Data will be made available on request.

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**References**

Acemoglu, D., 2003. Labor- and capital-augmenting technical change. *J. Eur. Econom. Assoc.* 1 (1), 1–37.

Baqae, D.R., Farhi, E., 2019. Productivity and misallocation in general equilibrium. *Q. J. Econ.* 135 (1), 105–163.  
 Box, G.E., Cox, D.R., 1964. An analysis of transformations. *J. R. Stat. Soc. Ser. B Stat. Methodol.* 211–252.  
 Byrne, D.M., Fernald, J.G., Reinsdorf, M.B., 2016. Does the United States have a productivity slowdown or a measurement problem? *Brook. Pap. Econ. Act.* 2016 (1), 109–182.  
 de La Grandville, O., 1989. In quest of the slusky diamond. *Am. Econ. Rev.* 79 (3), 468–481.  
 Fernald, J., 2014. A Quarterly, Utilization-Adjusted Series on Total Factor Productivity. Working Paper Series 2012–19, Federal Reserve Bank of San Francisco.  
 Gordon, R.J., 2010. Revisiting U. S. Productivity Growth over the Past Century with a View of the Future. Working Paper Series 15834, National Bureau of Economic Research.  
 Herrendorf, B., Herrington, C., Valentinyi, Á., 2015. Sectoral technology and structural transformation. *Am. Econ. J.: Macroecon.* 7 (4), 104–133.  
 Jiang, W., León-Ledesma, M., 2018. Variable markups and capital-labor substitution. *Econom. Lett.* 171, 34–36.  
 Klump, R., McAdam, P., Willman, A., 2007. Factor substitution and factor-augmenting technical progress in the United States: A normalized supply-side system approach. *Rev. Econ. Stat.* 89 (1), 183–192.  
 Kmenta, J., 1967. On estimation of the CES production function. *Internat. Econom. Rev.* 8 (2), 180–189.  
 Koh, D., Santaella-Llopis, R., Zheng, Y., 2020. Labor share decline and intellectual property products capital. *Econometrica* 88 (6), 2609–2628.  
 León-Ledesma, M.A., McAdam, P., Willman, A., 2010. Identifying the elasticity of substitution with biased technical change. *Am. Econ. Rev.* 100 (4), 1330–1357.  
 Mućk, J., 2017. Elasticity of Substitution Between Labor and Capital: Robust Evidence from Developed Economies. Working Paper Series 271, Narodowy Bank Polski.  
 Philippon, T., 2022. Additive Growth. Working Paper Series 29950, National Bureau of Economic Research.