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Public overspending in higher education

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Abstract
We study the trade-off between governmental investments in pretertiary and tertiary education from an efficiency point of view. We develop a model comprising agents with different incomes and abilities, public and private schools, and public universities that select applicants based on an admission exam. Reallocating governmental resources from tertiary to pretertiary education may positively affect aggregate production and human capital if some conditions are satisfied. For instance, in an economy with a high proportion of credit-constrained students, a reallocation of expenditure toward public schools benefits many students, compensating for the negative effect of a decrease in public university investments. We also quantitatively investigate the optimal allocation of public investment between pretertiary and tertiary education, and we find that a 10% increase in productivity of public investments in pretertiary education could increase the optimal GDP between 2.1% and 3%.

KEYWORDS
educational stages, investments in education, public education

JEL CLASSIFICATION
I24, I25, I28

1 INTRODUCTION

The public education system of many developing countries is characterized by low-quality schools and elite universities that select the highest-ranked applicants, frequently benefiting students from wealthy families that can afford private schools. The Organisation for Economic Co-operation and Development (OECD) documents that countries such as Brazil, Costa Rica, and Mexico make low investments per-student in pretertiary education but high expenditures on tertiary education and asserts that their governments should shift public spending from tertiary to pretertiary education to raise progressivity and efficiency (OECD, 2018a, 2018b, 2019).

In this paper, we study the trade-off between public investments in schools and universities from an efficiency point of view. Using a model that features heterogeneous agents concerning income and ability, public and private schools, and public universities with a limited number of vacancies. We investigate the effects of reallocating governmental resources from tertiary to pretertiary education on aggregate production and human capital.

In the model, public schools are tuition-free, while private schools charge a price for their services and provide a higher human capital return. To study in a public university, an agent needs to apply for admission. There is a limited number of vacancies in public universities, and only applicants with the highest grades in an admission exam are selected.[1] An applicant's score is a stochastic function of her pretertiary human capital. A cutoff score for the exam is determined in equilibrium to make the mass of admitted applicants equal to the mass of vacancies. There are credit constraints so that high-ability poor agents cannot anticipate their future earnings to pay for a better education. The human capital return of attending public schools and public universities depends on the amount of per-student governmental spending in pretertiary and tertiary education.

We analytically study an equilibrium in which public schools are attended by low-income students, wealthy agents study in private schools, and high-ability agents apply to public universities as well as low-ability but rich agents. This equilibrium features inequality in access to public higher education because wealthy applicants have an advantage in admission to university due to their better pretertiary education.

If the government reallocates resources from tertiary to pretertiary education, low-income students would be benefited because they would obtain a better quality education and have higher odds of being admitted to a public university. On the other hand, university students would be harmed because the return to higher education decreases. We use this environment to investigate conditions under which reallocating governmental resources from tertiary to pretertiary education results in an increase in aggregate production and human capital, a scenario we call "public overspending in higher education."

The well-established literature reports that earlier stages of education have a higher importance in an individual's human capital formation (Cunha et al., 2010; Delalibera & Ferreira, 2019). Our model is consistent with this mechanism: the return to public educational expenditures at a given level of education is proportional to this stage's importance to human capital formation. When pretertiary education has high importance, the conditions for public overspending in tertiary

[1] We do not model private universities. Thus, the distribution of students between private and public universities could be thought of as constant in this environment.

education in the model are looser. Moreover, our model also encompasses credit constraint, an essential aspect of the educational environment (Lochner & Monge-Naranjo, 2011). We find that if the proportion of credit-constrained students is high, reallocating public expenditures towards pretertiary education alleviates this friction and improves resource allocation, as measured by the aggregate human capital of individuals.

We are interested in analyzing the trade-offs between public investments in pretertiary and tertiary education and their effects on GDP, considering the total public expenditure on education as constant. Thus, we do not explore the endogenous determination of total public educational expenditures.[2] We also quantitatively analyze the optimal policy in terms of investment in pretertiary or tertiary education. Our results suggest that optimal public investments should be relatively higher in pretertiary. Besides that, when the productivity of public investments in schools (quality of education) decreases, the government needs to invest relatively more in public schools in an optimal allocation. Furthermore, our result indicates that an increase of 10% in productivity of public investments in pretertiary education could increase the optimal GDP between 2.1% and 3%.[3]

This paper is related to the literature that studies the trade-offs between public investments in pretertiary and tertiary education (Arcalean & Schiopu, 2010; Blankenau, 2005; Blankenau et al., 2007; Brotherhood & Delalibera, 2020; Caucutt & Lochner, 2020; Driskill & Horowitz, 2002; Restuccia & Urrutia, 2004; Su, 2004, 2006; Sarid, 2016). We contribute to this literature by developing a model that delivers analytical results and a clear interpretation of several relevant mechanisms. In particular, when we analyze the effects of reallocating public resources from tertiary to nontertiary education, the model allows us to distinguish all different effects on GDP and compare them with each other in a highly tractable way.

Our paper is closely related to three articles: Caucutt and Lochner (2020), Herskovic and Ramos (2017), and Brotherhood and Delalibera (2020). Caucutt and Lochner (2020) develop a dynastic human capital investment framework to study the role of some economic mechanisms in determining the human capital investments in children at different ages. Their model also accounts that later investments are built on earlier investments. We differ from them as we consider a general equilibrium model with analytical results. Another difference is the education levels: they consider early education as primary education and late education as secondary and tertiary.

Herskovic and Ramos (2017) develop an overlapping generations model in which parents choose to invest in the education of their child and if she attends private or public school or college. Brotherhood and Delalibera (2020) construct a general equilibrium model featuring heterogeneous agents, basic and higher education, public and private educational institutions, credit frictions, and complementarity between human capital inputs. Both of them solve numerically the stationary equilibrium and calibrate the model to Brazilian data. Herskovic and Ramos (2017) implement quantitative exercises considering quotas in college admissions for two types of preferentially treated students: public school and low-income students. Brotherhood and Delalibera (2020) consider additional elements such as complementarity between human capital inputs and find that an optimal utilitarian policy that allocates per-student public expenditures equally between basic and higher education, benefits almost all households, and reduces income

[2] For papers papers on the endogenous determination of public educational expenditures, including its political aspects, see Epple and Romano (1998), Glomm and Ravikumar (1998), Su (2006), and Rauh (2017).

[3] We highlight that this result is under the assumption that the distribution of students across public and private universities is constant.

inequality. We build our model considering some of their mechanism, but we choose an analytical approach, and we can use closed forms in a more general framework to analyze how different parameters values can generate public overspending in college. We can also characterize sufficient conditions under which overspending in public universities can arise.

There is literature defending that public resources in developing countries should be maintained or even increased for higher education. Birdsall (1996) is a good exemplar. He argues that investments in higher education, such as research and postgraduate training in science and technology, may have a higher social return than primary or secondary investments. However, Birdsall (1996) admits that an individual gets admitted to a higher educational level only if he has access to good quality primary and secondary schools. In our paper, if the low-income families are credit-constrained their children do not even have the chance to apply to university, which may generate a talent's misallocation.

The rest of this paper is organized as follows. Section 2 presents the model through subsections. Section 2.1 describes the preliminaries of the model, and Section 2.2 presents the choices of the agents. Section 2.3 and 2.4 introduces the government and defines the equilibrium, respectively. Section 3 characterizes agents' choice, and subsection 3.1 studies the existence of public overspending in higher education. In Section 4, we implement a quantitative analysis to evaluate optimal allocation in this environment. Section 5 presents concluding comments.

2 | THE MODEL

There is a unitary mass of agents and two time periods. In the first period, all agents acquire pretertiary education. There are both private and public schools. Private schools provide a higher human capital return than public schools, but charge a positive price, while public schools are tuition-free. In the second period, agents can choose to apply for public university. There is a limited number of vacancies in public universities, and only applicants with the highest grades in the admission exam are selected. An applicant's score is a stochastic function of her pretertiary human capital, and a cutoff score for the exam is determined in equilibrium to make the mass of admitted applicants equal to the mass of vacancies. All agents work in period two. An agent's labor earning is equal to her human capital.

2.1 | Preliminaries

Each agent is characterized by a pair of variables (w, π), where w denotes income endowment in the first period, and π is the agent's innate ability. There are two levels of income in the first period, $w_H > w_L > 0$, and two levels of innate abilities, $\pi_H > \pi_L > 0$. Denote by μ_{ij} the exogenous mass of agents of type (w_i, π_j), for $i, j \in \{L, H\}$.

Acquired ability, $\hat{\pi}$, is the human capital that an agent has immediately after completing pretertiary education. For an agent with innate ability π, acquired ability is given by

$$\hat{\pi} = \begin{cases} a_0\pi & \text{if agent studies in public school,} \\ a_1\pi & \text{if agent studies in private school.} \end{cases} \tag{1}$$

a_0 and a_1 reflect the human capital return of attending public and private schools, respectively, with $a_1 > a_0 > 0$. The final human capital of an agent with acquired ability $\hat{\pi}$ is

$$h = \begin{cases} \hat{\pi} & \text{if agent does not attend university,} \\ a_2\hat{\pi} & \text{if agent studies in public university,} \end{cases} \tag{2}$$

where $a_2 > 1$ denotes the human capital return of university education.

There is a representative public university with a limited mass of vacancies, $\lambda \in (0, 1)$. An applicant is admitted to public university if her exam score is greater than or equal to a nonnegative exam score cutoff, π^*, which is determined in equilibrium. The exam score of an applicant with acquired ability $\hat{\pi}$ is given by $\varepsilon\hat{\pi}$, where ε is an independently and identically distributed standard uniform random variable. Using properties of the uniform distribution, the probability of an applicant with acquired ability $\hat{\pi}$ being admitted is[4]

$$p(\hat{\pi}|\pi^*) \equiv \Pr(\varepsilon\hat{\pi} \geq \pi^*) = \Pr\left(\varepsilon \geq \frac{\pi^*}{\hat{\pi}}\right) = \begin{cases} 0 & \text{if } 1 < \frac{\pi^*}{\hat{\pi}}, \\ 1 - \frac{\pi^*}{\hat{\pi}} & \text{if } 0 \leq \frac{\pi^*}{\hat{\pi}} \leq 1. \end{cases} \tag{3}$$

In our model, we abstract for the possibility that the effort exerted by students can affect significantly the exam results and the probability of getting admitted to the university. This aspect could be added to our framework through a different admission probability function and a disutility cost of effort. For example, the p function could be modified to be $p(\hat{\pi}, e|\pi^*) = \Pr(\varepsilon\hat{\pi}e^{\psi} \geq \pi^*)$, where e is effort, $\psi \in (0, 1)$ is a concavity parameter, and the disutility cost could be a convex function, $C(e) = e^{\phi}$, $\phi > 1$. In this case, ability could be positively correlated to effort. In this environment, we would have more high-ability students at the university. If we add this ingredient to our model, we would Loose tractability, simplicity, and the closed form solutions that we have, without gaining much in terms of results.

MacLeod and Urquiola (2015) and Silva (2020) follow this effort approach. Both of them add test preparation to their model in a similar way: supposing that the performance on the test depends on the effort (test preparation), and this effort generates a disutility, which is an increasing and convex function. As we said before, we would follow a similar approach if we decided that the probability of getting admitted to the university depends on effort or test preparation. Although they have similar hypotheses in test results depending on effort, they present different frameworks. MacLeod and Urquiola (2015) analyze how competition for good reputation and signaling has consequences on students' effort. They show that it increases unproductive test preparation as individuals try to gain admission to selective schools, reducing study effort after admission. Silva (2020) is interested in analyzing the nature of the admission requirements to the university, testing whether the inclusion of a general admission exam generates a better pool of admitted students.

[4] Note that as the applicant knows the probability to be admitted, she knows the exam score cutoff but she does not know her score in exam before to apply.

2.2 | Agents' choices

In the first period, an agent chooses between public and private basic education. If she decides to study at a private school, she must pay a price $q > 0$, which is exogenous. The budget constraint of an agent with income w in the first period is

$$c_1 + qs_1 = w, \tag{4}$$

where c_1 denotes consumption in period one and s_1 is a dummy variable that indicates whether the agent chooses to study in a private school. Note that there are credit constraints: a (w_L, π_H) agent has the same funds as a (w_L, π_L) agent to invest in education, although the former has higher expected future earnings than the latter.

If an agent decides to apply to public university in the second period, she incurs a utility cost $v > 0$, which is exogenous and represents the applicant's required effort to prepare for exams.[5] Denote by s_2 a dummy variable that indicates whether the agent applies to public university.[6] In the second period, the agent consumes her labor earnings, which are given by her final human capital,

$$c_2 = h. \tag{5}$$

We abstract from differences between public and private tertiary education. For tractability, we consider in our model that only the government can provide university services with a limited number of vacancies. Therefore, there are no private universities in our economy, which implies that the distribution of students between private and public universities is constant. Since we are interested in analyzing the effects of public overspending in college, this abstraction does not change our main results.[7] Furthermore, note that although we have only two periods (pretertiary and tertiary), there is a perfect credit market within each period.

An agent takes the cutoff grade π^* as given and chooses $s = (s_1, s_2) \in \{0,1\} \times \{0,1\} \equiv S$ to maximize expected lifetime utility. The utility function is linear, and there is no time discounting. The problem of an agent is

$$\begin{gathered} \max_{s \in S} u(c_1) + \mathbb{E}[u(c_2)|s] - v s_2 \\ \text{subject to} \\ (1), (2), (3), (5), (5),\ c_1 \geq 0,\ c_2 \geq 0, \end{gathered} \tag{6}$$

where the expectation is taken over shocks that determine an applicant's exam score.

2.3 | Government

We define public overspending in higher education as the situation where reallocating public expenditures from tertiary towards pretertiary education, keeping total educational expendi-

[5] The university entrance exam is not mandatory to finish pretertiary in our model. It is in line with some developing countries, such as Brazil. However, our model encompasses the case where all agents apply to university.

[6] Once the agent decides to apply, she incurs in the utility cost $v > 0$, before observing the admission result.

[7] See Section 3.1 for more details.

tures fixed, leads to higher aggregate production and human capital. The human capital returns of attending public school and university are increasing and linear functions of government spending per student in school, g_0, and university, g_2, respectively:

$$a_0 = \alpha_0 g_0, \qquad a_2 = 1 + \alpha_2 g_2, \tag{7}$$

with slope parameters $\alpha_0, \alpha_2 > 0$.

A high α_0 may represent an efficient pretertiary public educational system, but also an environment where pretertiary education is highly important for an individual's human capital formation. In an environment where low-income families, who send their children to public schools, do not use their private funds to complement governmental educational investments,[8] an increase in public investments will not be followed by a crowding-out of private investments. In this case, α_0 is proportional to the importance of pretertiary educational investments to an individual's human capital formation (Cunha et al., 2010).

Let G be the government's total educational spending. As w_L and w_H can be interpreted as after-tax income, the total government expenditure is obtained from taxes in the first period. Total public spending is the sum of educational expenditures per student times the mass of students for each level of public education. Using agents' optimal choices,

$$G = g_0 \sum_{i,j \in \{L,H\}} \mu_{i,j} s_1^{i,j} + g_2 \lambda. \tag{8}$$

Equations (7) and (8) link a_2 and a_0. First, invert the first equation in (7) to get $g_0(a_0)$. Second, substitute this in (8) and isolate g_2 to get $g_2(a_0)$. Finally, substituting this into the second equation in (7) gives us

$$a_2(a_0) = 1 + \alpha_2 \left\{ \frac{1}{\lambda} \left[G - \left(\frac{a_0}{\alpha_0} \right) (\sum_{i,j \in \{L,H\}} \mu_{i,j} s_1^{i,j}) \right] \right\}. \tag{9}$$

Note that it describes how a_2 is determined as a function of a_0, assuming that total government expenditures G are fixed. If the government increases a_0 through higher expenditures per student in public schools, g_0, then g_2 must decrease for G to remain constant, leading to a smaller a_2. Another way to see this same problem of overspending is to focus on the variables that the government controls directly: spending per pupil in pretertiary and tertiary education (g_0, g_2) and explaining the slope of the budget constraint in (g_0, g_2) space, where it measures the cost of one more unit of g_2 expressed in terms of given up units of g_0. Taking G as fixed and differentiating (8) totally with respect to g_2 and g_0, we have

$$\frac{\partial g_2}{\partial g_0} = -\frac{\sum_{i,j \in \{L,H\}} \mu_{i,j} s_1^{i,j}}{\lambda}. \tag{10}$$

This framework allows us to think more clearly about the OECD's policy recommendation that we mention in the introduction. A large amount of public resources devoted to tertiary education

[8] We do not include this mechanism in the model for simplicity and tractability.

generates a relatively high a_2 and low a_0. In this case, a reallocation of expenditures to public schools should generate gains that compensate for the loss of quality in universities.

Also, if the importance of pretertiary education in human capital formation is significantly larger than that of tertiary education (high α_0 or low α_2), a reallocation of public spending from universities to schools should lead to a relatively small loss of the returns to higher education.

Finally, Equation (10) also conveys a simple mechanism that is nonetheless important for thinking about public expenditures across educational stages. Consider that there are either many public school students or a low quantity of vacancies in a public university. Then, when switching expenses from tertiary to pretertiary education, one monetary unit of investment made for a university student needs to be split among several public school students, resulting in a small increase in pretertiary education quality per student.

2.4 | Equilibrium

Definition 1. An equilibrium is given by agents' choices, a university grade point cutoff π^*, and government expenditure such that

1. Agents maximize expected lifetime utility taking π^* as given,
2. The mass of public university students is less than or equal to the mass of public university vacancies, with equality if $\pi^* > 0$,
3. The government budget represented by (8) is balanced.[9]

Equilibrium condition 1 states that if $\pi^* > 0$, then the mass of students in university must be exactly equal to the mass of vacancies. Additionally, we can also have the case in which there are empty vacancies and the exam cutoff score is zero in equilibrium.

3 | AN ANALYTICAL INVESTIGATION

We analyze an equilibrium where low-income agents study in public schools, high-income agents study in private schools, low-ability and poor agents do not apply to public university, but high-ability and poor agents as well as rich agents apply. The next proposition shows conditions under which the model generates such optimal choices.[10]

Proposition 1. *Denote by s_{ij} the choice of a type-(w_i, π_j) agent. If the following conditions hold:*

1. $w_L < q < w_H$,
2. $v < (1 - \frac{\pi^*}{a_1 \pi_L})(a_2 - 1) a_1 \pi_L$,
3. $a_1 \pi_L < a_0 \pi_H$,
4. $v > \pi_L a_0 (a_2 - 1)$,
5. $q + v < \pi_L (a_1 - a_0)$.

[9] We can see in Equation (8) that given a ratio of public investments between pretertiary and tertiary (i.e., definig g_0/g_2 as parameter), the general equilibrium variable for public investment is either g_0 or g_2.

[10] Note that we could adjust parameters to allow a different set of choices. For instance, there are restrictions where all agents apply to university or only high-ability agents apply.

then $s_{LL} = (0,0)$, $s_{LH} = (0,1)$, $s_{HL} = (1,1)$, *and* $s_{HH} = (1,1)$.

Proof. See Appendix A. □

Next, we interpret conditions in Proposition 1. Condition 1 says that poor agents do not have funds to pay for private school, which leads them to choose public schools. Condition 2 induces (w_H, π_L) agents to apply for university, defining an upper bound on the cost to apply. This condition is stronger than the one that holds for (w_L, π_H).[11] Condition 3 implies that all high-ability agents apply. Condition 4 defines a lower bound on v to make (w_L, π_L) agents decide not to apply. The fifth condition implies that a (w_H, π_L) agent prefers $(1,1)$ over $(0,0)$ even if she is not admitted to the university. Furthermore, this condition defines an upper bound for the costs of studying in a private school and applying to the university, allowing all wealthy agents to choose private schools.

If the conditions stated in Proposition 1 hold, high-ability agents and low-ability but rich agents apply to college, while low-ability and poor agents do not. Suppose that the mass of applicants is greater than or equal to the mass of vacancies, $\mu_{LH} + \mu_{HL} + \mu_{HH} \geq \lambda$, so that there are no empty vacancies in equilibrium. The condition for college market clearing in this case is

$$\mu_{LH}\ p(a_0\pi_H|\pi^*) + \mu_{HL}\ p(a_1\pi_L|\pi^*) + \mu_{HH}\ p(a_1\pi_H|\pi^*) = \lambda. \tag{11}$$

We look for an equilibrium in which both types of high-ability individuals have positive probabilities of being admitted as well as the low-ability but rich agents. From Equation (3), for this to happen we must have $0 < \pi^*/(a_0\pi_H) < 1$ and $0 < \pi^*/(a_1\pi_L) < 1$. Assuming that this is true and substituting (3) in (11), we can solve for π^*:

$$\pi^* = \frac{a_0 a_1 \pi_L \pi_H(\mu_{LH} + \mu_{HL} + \mu_{HH} - \lambda)}{\mu_{LH} a_1 \pi_L + \mu_{HL} a_0 \pi_H + \mu_{HH} a_0 \pi_L}. \tag{12}$$

With this closed form solution for π^*, it is straightforward to verify that a necessary and sufficient condition for $0 < \pi^*/(a_1\pi_L) < 1$ is $\lambda > \mu_{LH}(1 - \frac{a_1\pi_L}{a_0\pi_H}) + \mu_{HH}(1 - \frac{\pi_L}{\pi_H})$. If $a_0\pi_H > a_1\pi_L$, then $0 < \pi^*/(a_1\pi_L) < 1$ guarantees that (w_L, π_H) individuals have a positive probability of being admitted, because the lower bound for λ is a stronger condition than the one implied by high-ability agents being admitted to the university. It also implies that (w_H, π_H) agents have a positive probability of being admitted because they have higher acquired ability.

3.1 | Public overspending in higher education

As we said before, we define public overspending in higher education as the case where reallocating public expenditures from tertiary towards pretertiary education, keeping total educational expenditures fixed, leads to higher aggregate production and human capital. And now, we analyze carefully under which conditions this situation can arise. For this, we need to define the total output. First, denote by p_{LH}, p_{HL}, and p_{HH} the equilibrium probabilities of high-ability agents and low-ability but rich students being admitted to university.[12] GDP in the second period is given by

[11] See Equation (A.3) in Appendix A.

[12] Which are given by $p_{LH} = 1 - \pi^*/(a_0\pi_H)$, $p_{HL} = 1 - \pi^*/(a_1\pi_L)$, and $p_{HH} = 1 - \pi^*/(a_1\pi_H)$.

the sum of the expected human capital of all agents weighted by their masses:

$$\begin{aligned} Y = \mu_{LL} a_0 \pi_L \; &+ \mu_{LH} p_{LH} a_0 a_2 \pi_H + \mu_{LH}(1 - p_{LH}) a_0 \pi_H \\ &+ \mu_{HL} a_1 a_2 p_{HL} \pi_L + \mu_{HL}(1 - p_{HL}) a_1 \pi_L \\ &+ \mu_{HH} p_{HH} a_1 a_2 \pi_H + \mu_{HH}(1 - p_{HH}) a_1 \pi_H . \end{aligned} \tag{13}$$

We can use the GDP equation (13) to gain some insights into how aggregate output reacts to an increase in the quality of public schools and a corresponding decrease in the quality of universities. A variation in a_0 and a_2 affects (i) the productivity that individuals gain from education (i.e., terms a_0 and a_2 themselves) and (ii) the distribution of agents who enter university (i.e., terms p_{LH}, p_{HL}, and p_{HH}). First, a higher a_0 increases the human capital of low-income agents, all else equal, but a lower a_2 decreases the human capital of high-ability agents who are admitted to university. Second, such variation in a_0 and a_2 increases the probability of low-income agents entering university, p_{LH}, since their acquired ability depends positively on a_0. At the same time, it decreases p_{HH} and p_{HL} because the acquired ability of (ω_H, π_H) and (ω_H, π_L) agents is kept constant at $a_1\pi_H$ and $a_1\pi_L$, respectively.

The previous discussion shows that there are some mechanisms through which a higher a_0 and lower a_2 affect GDP positively, while there are others that produce the opposite effect. The next proposition shows conditions under which the positive effects prevail over the negative effects.

Proposition 2. *If the following conditions hold, there is overspending in public universities, i.e.,* $\partial Y/\partial g_0 > 0$[13]*:*

1. $\dfrac{a_0}{a_1} > (\dfrac{\mu_{HH}\pi_H + \mu_{HL}\pi_L}{\mu_{HH} + \mu_{HL}})(\dfrac{\mu_{HL}\pi_H + \mu_{HH}\pi_L}{\mu_{HH} + \mu_{HL}})^{-1}$,
2. $\mu_{LL} > (-\frac{\partial a_2}{\partial g_0})(\frac{\pi_H}{\pi_L})[\dfrac{\mu_{LH}a_0 + (\mu_{HH} + \mu_{HL})a_1}{\mu_{LH} + \mu_{HH} + \mu_{HL}}]$.

Proof. See Appendix B. □

The two conditions in the proposition above are obtained by requiring some specific positive effects of substituting a_2 for a_0 on aggregate production to be larger than the negative effects. First, when shifting public expenditures towards pretertiary, more underprivileged agents will be admitted to the university. Since private schools have better quality, this substitution of admitted students will lead to a negative effect on GDP because the final human capital of newly admitted low-income students is lower than the one that wealthy students would have in case they were admitted.[14] The first condition in Proposition 2 is implied by requiring this negative effect to be outweighed by the human capital gain due to a higher a_0 for low-income applicants who are not admitted to university.

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[13] The subset of the parameter space for which the conditions in Propositions 1 and 2 hold is nonempty. For example, all conditions hold for the following parameter values: $\omega_L = 0.2$, $\omega_H = 0.75$, $\pi_L = 0.85$, $\pi_H = 25$, $\mu_{LL} = 0.4$, $\mu_{LH} = 0.25$, $\mu_{HL} = 0.34$, $\mu_{HH} = 0.01$, $\lambda = 0.2$, $a_0 = 0.2$, $a_1 = 3$, $a_2 = 2$, $\partial a_2/\partial g_0 = -0.0033$, $q = 0.5$, and $\upsilon = 0.3$.

[14] The existence of private universities would alleviate this negative effect of substituting university entrants, becausenon-admitted wealthy applicants could decide to study in private universities, mitigating their human capital loss compared to the case with no private universities. Therefore, including private universities in this model would loose the conditions for the existence of public overspending in higher education.

Condition 1 requires that the relative quality of public schools, compared to that of private schools, must be greater than a measure of the equality of innate abilities among high-income families. This equality index measures the relative distance between the average innate ability of high-income households compared to the average ability in a counterfactual scenario where the innate abilities of low- and high-ability wealthy families are switched. For example, suppose that the average innate ability of high-income households when innate abilities are switched increases by a large magnitude. Then, the right-hand side of condition 1 is smaller. It can happen because π_H/π_L is high and/or μ_{LH}/μ_{HH} is high, which indicate large innate ability inequality among high-income families.[15]

The second condition is obtained by requiring that the GDP gain due to higher a_0 for (w_L, π_L) agents prevails over the GDP loss driven by lower a_2 for agents admitted in university. This condition is satisfied when a combination of the following holds: (i) μ_{LL} is high, (ii) $\partial a_2/\partial g_0$ is close to zero, (iii) π_H/π_L is low, (iv) the mean pretertiary education quality of high-ability students and low-ability but rich students is sufficiently low.

First, the higher μ_{LL} is, the larger is the number of individuals who benefit from higher g_0. Second, the closer $\partial a_2/\partial g_0$ is to zero, the smaller is the decrease in the quality of public university. Third, the GDP gain (loss) due to higher g_0 (lower a_2) is amplified by the innate ability of affected individuals. Since (w_L, π_L) agents study in public school and high-ability agents apply to university, π_H/π_L needs to be sufficiently low so that this amplification effect does not become sizable. Fourth, similar to the previous point, pretertiary education quality amplifies the absolute human capital decrease driven by lower a_2. Therefore, one of the negative forces produced by lower a_2 is proportional to the pretertiary education quality of admitted students, which is correlated with the basic education quality of high-ability students.

4 | OPTIMAL ALLOCATION: A QUANTITATIVE APPROACH

This section carries out a quantitative exercise to assess the sensitiveness of an optimal allocation of government resources between basic and tertiary education. We use the parameter values given in Table 1 for the simulation. The optimal allocation would be given by a ratio of g_0 over g_2 equal to 2.22; that is, the government should spend relatively more on pretertiary education in the optimal allocation. This optimal allocation could differ depending on the parameter values presented in Table 1. Thus, we present the optimal policy's sensitivity to the model parameters at the end of this section.

In Figure 1, we increased the mass of students with high ability but low income ($\mu_{L,H}$), reducing the mass of those who have high income and low ability ($\mu_{H,L}$). According to the results, this would reduce spending on pretertiary education vis-a-vis tertiary education. Since the number of students with high ability is higher, their share entering tertiary education will also be higher. Therefore, from the central planner's point of view, investing relatively more in tertiary education helps high-ability agents even more and leads to an increase in GDP. However, this effect becomes

[15] Since innate ability is a binary variable, a comparison between the equality measure on the right-hand side of condition 1 and the Gini index can be made. Remember that the Gini index would compare the distribution of innate abilities to the counterfactual situation where all individuals have the same average ability. Therefore, the counterfactual associated with the equality measure in condition 1 (switching abilities between two groups) is the symmetrically opposite case of the benchmark scenario, with the reference midpoint being the counterfactual associated with the Gini exercise (equalizing abilities across the two groups).

TABLE 1 Base case parameters

Parameter description	Parameter	Value
Low wage	w_L	1
High wage	w_H	2
Low ability	π_L	1
High ability	π_H	2
Mass of low income and low ability	μ_{LL}	0.5
Mass of low income and high ability	μ_{LH}	0.3
Mass of high income and low ability	μ_{HL}	0.1
Mass of high income and high ability	μ_{HH}	0.1
Vacancies in tertiary education	λ	0.1
Productivity of public investment in pretertiary	α_0	0.5
Productivity of public investment in tertiary	α_2	0.2
Productivity of pretertiary private education	a_1	1
Cost of pretertiary private education	q	0.5
Utility cost to apply to tertiary	v	0.01
Government budget	G	1

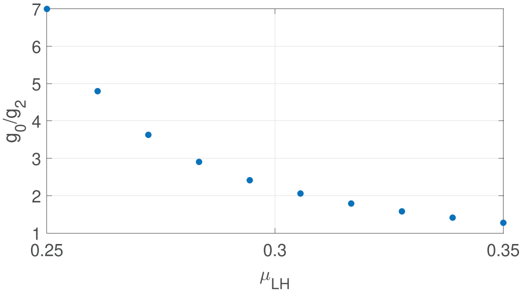


FIGURE 1 Ratio of government expenditure on pretertiary to tertiary versus the mass of low income and high-ability agents [Colour figure can be viewed at wileyonlinelibrary.com]

smaller because the reduction in spending on pretertiary education negatively affects GDP via human capital decreases of students with low income and low ability (w_L, π_L). Moreover, due to the limited number of vacancies, some high-ability individuals who have low income will not enter university, which also requires that pretertiary education has a minimum level of quality to avoid a fall in GDP.

Figure 2a–c shows the changes in optimal allocation according to the educational quality parameters. Figure 2a,b presents the simulation for α_0 and α_2, which indicate the productivity of public spending in pretertiary and tertiary education, respectively. When the productivity of the expenditure on pretertiary education is higher, the probability of individuals with low income and high ability (w_L, π_H) entering tertiary education is more elevated. Therefore, the government can reallocate spending from pretertiary to tertiary education to increase GDP. In fact, the acquired ability is a function of innate ability, α_0, and g_0. Then, (α_0, g_0) need to be jointly considered when analyzing the impact on GDP. Thus, when we increase α_0, the human capital associated with

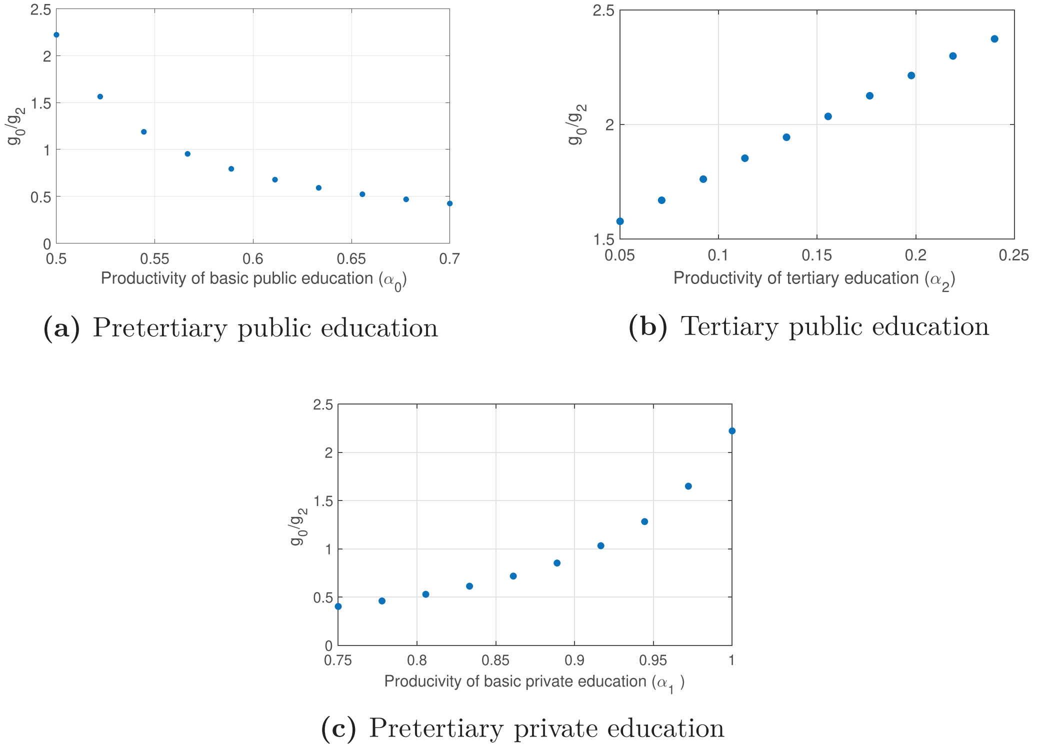


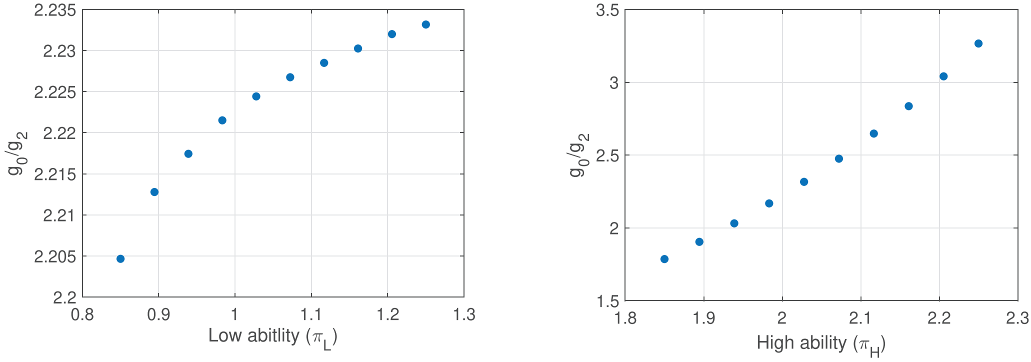
FIGURE 2 Ratio of government expenditure on pretertiary to tertiary versus the productivity of educational investments [Colour figure can be viewed at wileyonlinelibrary.com]

higher education also needs to grow (see Equation 2). It is analogous to the productivity of tertiary education α_2 (Figure 2b), which in turn increases g_0 vis-a-via g_2.

For Figure 2c, an increase of α_1 reduces the likelihood of type-(w_L, π_H) students entering tertiary education because the threshold π^* will be higher. In this case, the government needs to increase spending on pretertiary education so that high-ability people can enter university and thus increase GDP.

Figure 3 shows the impact of changes in the ability's parameter. In both cases, the optimal allocation implies increasing spending on pretertiary education. Nevertheless, it is important to note that when π_L increases, the relative expenditures on pretertiary education are lower than in the case of π_H. This effect is because the share of high-ability students entering university is higher than that with low ability. Thus, a rise in π_L and its increase in pretertiary expenditure (and decrease in tertiary) negatively affects high-ability students who join tertiary education. On the other hand, when it increases π_H most students who join the university have already benefited, and the government has more room to allocate resources in pretertiary.

Figure 4 shows the results of an increase in vacancies in tertiary education. With this increase, the government needs to invest more in pretertiary education to increase GDP. Indeed, the increase in vacancies in the university increases the probability of both poor and wealthy students entering university. Thus, as the poor have the largest mass in the benchmark, the increase in investment in pretertiary education leads to an increase in the human capital of those who are successful in joining university and students who did not enter but now have the higher acquired ability.



(a) Low ability (b) High ability

FIGURE 3 Ratio of government expenditure on pretertiary to tertiary versus the agents' abilities [Colour figure can be viewed at wileyonlinelibrary.com]

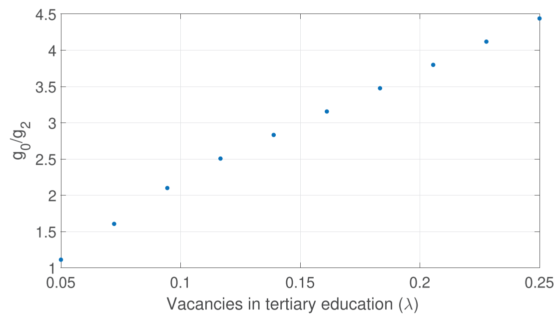


FIGURE 4 Ratio of government expenditure on pretertiary to tertiary versus vacancies in tertiary education [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 2 Impact on GDP and on public optimal allocation from a 10% increase in productivity parameters

GDP variation (%)						
Base case changing only:						
	Base case	$\boldsymbol{w_L = 1.5}$	$\boldsymbol{\pi_L = 1.5}$	$\boldsymbol{\lambda = 0.05}$	$\boldsymbol{\mu_{LL} = \mu_{LH} = \mu_{HL} = 0.25}$	$\boldsymbol{a_1 = 0.75}$
10% increase in α_0	2.9	2.9	2.9	3.0	2.1	2.1
10% increase in α_2	0.2	0.2	0.2	0.2	0.5	0.9
$\boldsymbol{g_0/g_2}$ **variation (%)**						
Base case changing only:						
	Base case	$\boldsymbol{w_L = 1.5}$	$\boldsymbol{\pi_L = 1.5}$	$\boldsymbol{\lambda = 0.05}$	$\boldsymbol{\mu_{LL} = \mu_{LH} = \mu_{HL} = 0.25}$	$\boldsymbol{a_1 = 0.75}$
10% increase in α_0	−49.6	−49.6	−49.8	−47.2	−30.0	−21.8
10% increase in α_2	3.7	3.6	3.8	3.7	2.5	2.7

To evaluate the elasticity of GDP and the optimum allocation regarding pretertiary education (α_0) and tertiary education (α_2), we performed a sensitivity analysis increasing each parameter by 10%. We evaluated its impact on GDP and the optimal allocation of g_0/g_2. In addition, we perform this exercise for different parameter values, as shown in Table 2.

In the first column (base case), we are comparing how much the GDP or the optimal allocation g_0/g_2 change when we increase α_0 or α_2 by 10% compared to the baseline economy whose parameters are those presented in Table 1. In the following column, we do a similar exercise as before, but we compute how much the GDP increases or the ratio g_0/g_2 decreases in percentage terms if we increase either α_0 or α_2 by 10% compared now to an economy whose parameters are those of the base case except for w_L (which is 1.5 instead of 1). The exercise is analogous to the other columns.

According to the results in Table 2, a 10% increase in the productivity parameter of pretertiary education and tertiary education could increase GDP between 2.1% and 3% and between 0.2% and 0.9%, respectively, compared to the baseline economy or to the economy whose values for the parameters presented in each column (w_L, π_L, λ, etc) has changed. For the g_0/g_2 ratio, a 10% increase in α_0 could reduce optimal allocation ranging between 21.8% and 49.8%, while a 10% increase in α_2 could increase the g_0/g_2 ratio between 2.5% and 3.8%. Note that pretertiary education productivity has a greater impact on GDP. The explanation is direct because the mass of students in pretertiary public education is greater than in higher education. This result is robust even if we divide equally the masses among the four possibilities of the ordered pair (w, π). In addition, increases in α_0 lead to a g_0/g_2 reduction, while increases in α_2 lead to g_0/g_2 increase, as we discussed earlier. In this way, the model is well behaved even when we change the values of the parameters.

5 | CONCLUSION

We develop a model in which underprivileged students attend low-quality public schools, and wealthy agents obtain better education through the private system. This fact generates inequality in admission exams to public higher education, leading to a situation in which public resources may be directed to rich agents.

The model rationalizes the intuition that shifting public resources from tertiary to pretertiary education may positively affect aggregate production and human capital. In this framework, the government distributive role arises, and when there are a high proportion of credit-constrained students, a reallocation of expenditure towards public schools benefits many students, compensating for the negative effect of a decrease in public university investments. Furthermore, this mechanism is powered if pretertiary education is more important to human capital formation than tertiary education.

The optimal policy analysis regarding investment in pretertiary or tertiary education suggests that optimal public investments should be relatively higher in pretertiary. Besides that, when the productivity of public investments in schools (quality of education) decreases, the government needs to invest relatively more in public schools in an optimal allocation.

A natural extension of this research is to incorporate a mechanism for the government or society to determine the allocation of the educational budget between spending per pupil in public schools and universities. Thus, comparing, for example, a majority vote allocation against the output-maximizing optimal one could bring additional insights to answer why developing countries tend to overspend on higher education.

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SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

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APPENDIX A: PROOF OF PROPOSITION 1

First, we take as given that equilibrium π^* is the one implied by the case where high-ability agents apply to university and low-ability but rich agents apply as well. Then, we show that the hypothesis in this proposition implies that those are indeed agents' optimal choices.

If high-ability agents apply and low-ability poor agents do not, then

$$\pi^* = \frac{a_0 a_1 \pi_H \pi_L(\mu_{LH} + \mu_{HL} + \mu_{HH} - \lambda)}{a_1\pi_L\mu_{LH} + a_0\pi_H\mu_{HL} + a_0\pi_L\mu_{HH}}. \tag{A.1}$$

Denote by $V_{ij}(s)$ the value function of a type-(w_i, π_j) individual choosing s. We proceed by studying optimal choices for each type of agent.

(w_L, π_L) agents. First, since $w_L < q$, consumption nonnegativeness implies that this agent cannot study in a private school. Second, note that a sufficient condition for this agent to prefer $(0, 0)$ over $(0, 1)$ is that the utility of choosing $(0, 0)$ is greater than choosing $(0, 1)$ and being admitted for sure:

$$\begin{gathered} V_{LL}(0,0) > V_{LL}(0,1) \\ \Longleftarrow w_L + a_0\pi_L > w_L + a_0a_2\pi_L - \upsilon \iff \upsilon > a_0\pi_L(a_2 - 1). \end{gathered} \tag{A.2}$$

(w_L, π_H) agents. As in the previous case, this agent cannot study in a private school. Now note that

$$\begin{gathered} V_{LH}(0,1) > V_{LH}(0,0) \\ \iff w_L + \left(1 - \frac{\pi^*}{a_0\pi_H}\right)a_0a_2\pi_H + \left(\frac{\pi^*}{a_0\pi_H}\right)a_0\pi_H - \upsilon > w_L + a_0\pi_H \\ \iff \upsilon < \left(1 - \frac{\pi^*}{a_0\pi_H}\right)a_0(a_2 - 1)\pi_H. \end{gathered} \tag{A.3}$$

As we are going to show below, if $a_1\pi_L < a_0\pi_H$ then the condition above is weaker than the condition derived for the (w_H, π_L) agents.

Using (A.1), the last expression can be written as

$$\upsilon < \left(1 - \frac{a_1\pi_L(\mu_{LH} + \mu_{HL} + \mu_{HH} - \lambda)}{a_1\pi_L\mu_{LH} + a_0\pi_H\mu_{HL} + a_0\pi_L\mu_{HH}}\right)a_0(a_2 - 1)\pi_H. \tag{A.4}$$

(w_H, π_L) agents. First, if we consider that the agent is admitted to university if she applies, then her utility would be greater than not studying at all, i.e., if she chooses $s_{HL} = (0,0)$

$$\begin{aligned} &V_{HL}(1,1) > V_{HL}(0,0) \\ \Longleftarrow\ & w_H - q + a_1 a_2 \pi_L \left(1 - \frac{\pi^*}{a_1 \pi_L}\right) + \left(\frac{\pi^*}{a_1 \pi_L}\right) a_1 \pi_L - v > w_H + a_0 \pi_L \\ \Longleftrightarrow\ & q + v < \pi_L \left\{ a_1 \left[a_2 \left(1 - \frac{\pi^*}{a_1 \pi_L}\right) + \frac{\pi^*}{a_1 \pi_L}\right] - a_0 \right\}. \end{aligned} \tag{A.5}$$

Since $\pi^*/(a_1\pi_L) < 1$, (A.5) is weaker than the one derived below (A.6). Another condition would be that this agent's utility in the case where $s_{HL} = (1,1)$ and she is not admitted to university is greater than the utility if choosing $(0,0)$

$$V_{HL}(1,1) > V_{HL}(0,0) \iff w_H - q + a_1\pi_L - v > w_H + a_0\pi_L \iff q + v < (a_1 - a_0)\pi_L. \tag{A.6}$$

This condition is stronger than the condition derived for the (w_H, π_H), as we are going to see below.

Third, we have that

$$\begin{aligned} &V_{HL}(1,1) > V_{HL}(1,0) \\ \Longleftarrow\ & w_H - q + a_1 a_2 \pi_L \left(1 - \frac{\pi^*}{a_1 \pi_L}\right) + \left(\frac{\pi^*}{a_1 \pi_L}\right) a_1 \pi_L - v > w_H - q + a_1 \pi_L \\ \Longleftrightarrow\ & v < a_1 (a_2 - 1)\pi_L \left(1 - \frac{\pi^*}{a_1 \pi_L}\right). \end{aligned} \tag{A.7}$$

And as we said previously, if $a_1\pi_L < a_0\pi_H$ then this condition is stronger than the one for agent (w_L, π_H). And using (A.1), we have

$$v < \left(1 - \frac{a_0 \pi_H (\mu_{LH} + \mu_{HL} + \mu_{HH} - \lambda)}{a_1 \pi_L \mu_{LH} + a_0 \pi_H \mu_{HL} + a_0 \pi_L \mu_{HH}}\right) a_1 (a_2 - 1) \pi_L. \tag{A.8}$$

(w_H, π_H) agents. First,

$$\begin{aligned} &V_{HH}(1,1) > V_{HH}(0,1) \\ \Longleftrightarrow\ & w_H - q + \left(1 - \frac{\pi^*}{a_1 \pi_H}\right) a_2 a_1 \pi_H + \left(\frac{\pi^*}{a_1 \pi_H}\right) a_1 \pi_H - v > \\ > & w_H + \left(1 - \frac{\pi^*}{a_0 \pi_H}\right) a_2 a_0 \pi_H + \left(\frac{\pi^*}{a_0 \pi_H}\right) a_0 \pi_H - v \\ \Longleftrightarrow\ & q < (a_1 - a_0) a_2 \pi_H. \end{aligned} \tag{A.9}$$

Note that Equation (A.9) is a weaker condition than Equation (A.6) because

$$q < q + v < (a_1 - a_0)\pi_L < (a_1 - a_0) a_2 \pi_H.$$

Second, a sufficient condition for $V_{HH}(1,1) > V_{HH}(0,0)$ is that this agent's utility in the case where $s = (1,1)$ and she is not admitted to university is greater than her utility if choosing $(0,0)$. That is,

$$\begin{aligned} & V_{HH}(1,1) > V_{HH}(0,0) \\ \Longleftarrow\ & w_H - q + a_1\pi_H - v > w_H + a_0\pi_H \\ \Longleftrightarrow\ & q + v < (a_1 - a_0)\pi_H. \end{aligned} \tag{A.10}$$

Finally,

$$\begin{aligned} & V_{HH}(1,1) > V_{HH}(1,0) \\ \Longleftarrow\ & w_H - q + \left(1 - \frac{\pi^*}{a_1\pi_H}\right)a_2a_1\pi_H + \left(\frac{\pi^*}{a_1\pi_H}\right)a_1\pi_H - v > w_H - q + a_1\pi_H \\ \Longleftrightarrow\ & v < \left(1 - \frac{\pi^*}{a_1\pi_H}\right)a_1(a_2 - 1)\pi_H. \end{aligned} \tag{A.11}$$

APPENDIX B: PROOF OF PROPOSITION 2

The probabilities of a low- and a high-income agent entering university are given by

$$p_{LH} = 1 - a_1\pi_L\left(\frac{\mu_{LH} + \mu_{HL} + \mu_{HH} - \lambda}{a_1\pi_L\mu_{LH} + a_0\pi_H\mu_{HL} + a_0\pi_L\mu_{HH}}\right) \tag{B.1}$$

$$p_{HL} = 1 - a_0\pi_H\left(\frac{\mu_{LH} + \mu_{HL} + \mu_{HH} - \lambda}{a_1\pi_L\mu_{LH} + a_0\pi_H\mu_{HL} + a_0\pi_L\mu_{HH}}\right), \tag{B.2}$$

$$p_{HH} = 1 - a_0\pi_L\left(\frac{\mu_{LH} + \mu_{HL} + \mu_{HH} - \lambda}{a_1\pi_L\mu_{LH} + a_0\pi_H\mu_{HL} + a_0\pi_L\mu_{HH}}\right). \tag{B.3}$$

Define

$$\delta \equiv a_1\pi_L\mu_{LH} + a_0\pi_H\mu_{HL} + a_0\pi_L\mu_{HH}, \quad \psi \equiv \frac{\mu_{LH} + \mu_{HL} + \mu_{HH} - \lambda}{\delta}. \tag{B.4}$$

Note that

$$\frac{\partial\psi}{\partial a_0} = -\frac{\psi}{\delta}(\pi_H\mu_{HL} + \pi_L\mu_{HH}). \tag{B.5}$$

We can write

$$p_{LH} = 1 - a_1\pi_L\psi, \quad p_{HL} = 1 - a_0\pi_H\psi \quad \text{and} \quad p_{HH} = 1 - a_0\pi_L\psi. \tag{B.6}$$

Using (B.5) and (B.6),

$$p'_{LH} \equiv \frac{\partial p_{LH}}{\partial a_0} = a_1\pi_L\frac{\psi}{\delta}(\pi_H\mu_{HL} + \pi_L\mu_{HH}); \tag{B.7}$$

$$p'_{HL} \equiv \frac{\partial p_{HL}}{\partial a_0} = \pi_H \psi \left[\frac{a_0}{\delta} (\pi_H \mu_{HL} + \pi_L \mu_{HH}) - 1 \right]; \tag{B.8}$$

and

$$p'_{HH} \equiv \frac{\partial p_{HH}}{\partial a_0} = \pi_L \psi \left[\frac{a_0}{\delta} (\pi_H \mu_{HL} + \pi_L \mu_{HH}) - 1 \right]. \tag{B.9}$$

The derivative of GDP with respect to g_0 is given by

$$\frac{\partial Y}{\partial g_0} = \frac{\partial Y}{\partial a_0} \frac{\partial a_0}{\partial g_0} = \alpha_0 \frac{\partial Y}{\partial a_0} \tag{B.10}$$

Since $\alpha_0 > 0$, we just need to analyze the expression for $\partial Y / \partial a_0$. Therefore, the derivative of GDP with respect to a_0 can be written as

$$\begin{aligned} \frac{\partial Y}{\partial a_0} = & \underbrace{\mu_{LL}\pi_L}_{\equiv T_1} + \underbrace{\mu_{LH} p'_{LH} a_0 a_2 \pi_H}_{\equiv T_2} + \underbrace{\mu_{LH} p_{LH} a_2 \pi_H}_{\equiv T_3} + \underbrace{\mu_{LH} p_{LH} a_0 a'_2 \pi_H}_{\equiv T_4} \\ & - \underbrace{\mu_{LH} p'_{LH} a_0 \pi_H}_{\equiv T_5} + \underbrace{\mu_{LH}(1 - p_{LH})\pi_H}_{\equiv T_6} + \underbrace{\mu_{HH} p'_{HH} a_1 a_2 \pi_H}_{\equiv T_7} + \underbrace{\mu_{HH} p_{HH} a_1 a'_2 \pi_H}_{\equiv T_8} \\ & - \underbrace{\mu_{HH} p'_{HH} a_1 \pi_H}_{\equiv T_9} + \underbrace{\mu_{HL} p'_{HL} a_1 a_2 \pi_L}_{\equiv T_{10}} + \underbrace{\mu_{HL} p_{HL} a_1 a'_2 \pi_H}_{\equiv T_{11}} - \underbrace{\mu_{HL} p'_{HL} a_1 \pi_L}_{\equiv T_{12}}, \end{aligned} \tag{B.11}$$

where $a'_2 \equiv \partial a_2 / \partial a_0$.

Using (B.7), (B.8), (B.9), and (B.11), after some algebra, we get

$$\begin{aligned} & T_2 + T_5 + T_7 + T_9 + T_{10} + T_{12} = \\ & \quad = (a_2 - 1)\frac{\psi}{\delta} a_1 \pi_L \pi_H [\mu_{LH}\mu_{HL}(a_0\pi_H - a_1\pi_l) + \mu_{LH}\mu_{HH}(a_0\pi_L - a_1\pi_H)]. \end{aligned}$$

Since $a_2 > 1$ and ψ, δ, a_1, π_L , and π_H are all positive, $T_2 + T_5 + T_7 + T_9 + T_{10} + T_{12} > 0$ if and only if the following condition is satisfied:

$$\frac{\mu_{HL}}{\mu_{HH}} > \frac{a_1\pi_H - a_0\pi_L}{a_0\pi_H - a_1\pi_L}. \tag{B.12}$$

Now, apart from $T_2 + T_5 + T_7 + T_9 + T_{10} + T_{12}$, the only remaining negative terms in (B.11) are T_4, T_8, and $T11$. Note that

$$T_1 > -(T_4 + T_8 + T_{11}) \iff \tag{B.13}$$

$$\iff \mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right) [\pi_H(\mu_{LH} p_{LH} a_0 + \mu_{HH} p_{HH} a_1) + \pi_L \mu_{HL} p_{HL} a_1] \tag{B.14}$$

$$\Longleftarrow \mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right)[\pi_H(\mu_{LH}p_{HH}a_0 + \mu_{HH}p_{HH}a_1) + \pi_L\mu_{HL}p_{HH}a_1] \tag{B.15}$$

$$\Longleftarrow p_{HH}\mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right)p_{HH}[\pi_H(\mu_{LH}a_0 + \mu_{HH}a_1) + \pi_L\mu_{HL}a_1] \tag{B.16}$$

$$\Longleftrightarrow \mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right)[\pi_H(\mu_{LH}a_0 + \mu_{HH}a_1) + \pi_L\mu_{HL}a_1] \tag{B.17}$$

$$\Longleftarrow \mu_{LL}\pi_L > \left(-\frac{\partial a_2}{\partial a_0}\right)[\pi_H(\mu_{LH}a_0 + \mu_{HH}a_1) + \pi_H\mu_{HL}a_1] \tag{B.18}$$

$$\Longleftarrow \mu_{LL} > \left(-\frac{\partial a_2}{\partial a_0}\right)\frac{\pi_H}{\pi_L}[\mu_{LH}a_0 + (\mu_{HH} + \mu_{HL})a_1] \tag{B.19}$$

$$\Longleftarrow \mu_{LL} > \left(-\frac{\partial a_2}{\partial a_0}\right)\frac{\pi_H}{\pi_L}\left[\frac{\mu_{LH}a_0 + (\mu_{HH} + \mu_{HL})a_1}{\mu_{LH} + \mu_{HH} + \mu_{HL}}\right] \tag{B.20}$$

where we use $p_{HH} > p_{LL}$ in (B.15), $0 < p_{HH} < 1$ in (B.16), $\pi_H > \pi_L$ in (B.18), $\pi_L > 0$ in (B.19), and $\mu_{LH} + \mu_{HH} + \mu_{HL} < 1$ in (B.20).