High-Energy Gravitational Waves Emission from a Thermal Source

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Abstract: We study the production of gravitational waves by a thermalized plasma of $\mathcal{N} = 4$ supersymmetric Yang Mills matter in the high number of colors, high coupling limits. This process is governed by a particular light-like thermal correlator of the Energy-Momentum tensor that we study via the gauge/gravity duality. We focus on the high-energy limit, $\mathbf{k} \gg T$, and via a WKB approximation we drive an analytical solution for the energy density of emitted gravitational waves from a thermal source, obtaining a non-trivial dependency $\frac{d\rho_{GW}}{dtd^3\mathbf{k}} \propto \omega^{4/3}T^{8/3}$; and for the Energy-Momentum tensor thermal correlator at arbitrary momentum.

I. INTRODUCTION

The study of gravitational waves (GW) has emerged as a groundbreaking field in modern physics, offering a unique window into the dynamics of the Universe. Recent measurements performed by the LIGO and VIRGO collaboration [1] have had a profound impact on our knowledge of compact objects in the cosmos. However, despite these significant advancements, an exciting frontier still awaits to be explored: the emission of high-energy GW. This kind of GW are found in the high-frequency region $f \gtrsim 30$ KHz, far beyond our current capabilities for detection. Nevertheless, new detector are being planned in order to study this range of frequencies [2].

In this work, we study the production of GW by a thermalized plasma of $\mathcal{N} = 4$ supersymmetric Yang Mills ($\mathcal{N} = 4$ SYM) matter. We focus on the large number of colors $N_c \to \infty$, large 't Hooft coupling constant $\lambda \to \infty$ limits. While we do not expect to find such regime within the Standard Mordel (SM), it is still an interesting limit to study, since we contemplate the possibility that Beyond the Standard Model (BSM) physics may be strongly coupled [3].

This limit can be addressed by employing the gauge/gravity duality, with which we compute the emission rate via the analysis of the Energy-Momentum (EM) tensor thermal correlator, obtaining an analytic expression in the high-energy limit for the energy density of emitted GW per unit of time after making use of a WKB approximation.

We find that this emission rate goes with a particular power of ω and T. In order to try to better understand this non-trivial dependency, we extend our study and also compute the EM tensor thermal correlator at arbitrary four-momentum $\mathbf{k} = (\omega, 0, 0, k)$.

II. GRAVITATIONAL WAVES FROM A THERMAL SOURCE

Gravitational waves far from its emission source may be described in the weak field approximation by a perturbation of the Minkowski flat spacetime [4],

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \ . \tag{1}$$

Removing the gauge freedom by sticking ourselves to the harmonic gauge, and expressing physical excitations in terms of the transverse traceless (TT) components of the fluctuating fields, $h^{TT}_{\mu\nu}$, the spatial Einstein equations read

$$\Box^2 h^{TT}_{ij} = -16\pi G T^{TT}_{ij} \ , \tag{2}$$

where T^{TT}_{ij} are the components of the transverse traceless part of the EM tensor. A solution in the momentum space can be found as

$$h^{TT}_{ij}(\omega, \mathbf{k}) = 16\pi G \frac{T^{TT}_{ij}(\omega, \mathbf{k})}{(\omega + i\epsilon)^2 - \mathbf{k}^2} \ , \tag{3}$$

where the $i\epsilon$-prescription selects the retarded solution [5]. After Fourier transforming the frequency, the classical energy carried away from the source by GW may be expressed in terms of h^{TT}_{ij} as [5]

$$E_{GW} = \frac{1}{32\pi G} \int \frac{d^3k}{(2\pi)^3} \left[\dot{h}^{TT}_{ij}(-\mathbf{k}, t) \dot{h}^{TT}_{ij}(\mathbf{k}, t) \right] \ , \tag{4}$$

and after averaging over an observation period τ that is long compared to the frequency of the wave, the energy carried away by GW is

$$\bar{E}_{GW} = \frac{1}{\tau} \int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} dt' \ E_{GW} \ . \tag{5}$$

In order to connect this with section III, it is convenient to express the production rate of GW in terms of a particular thermal correlator of the EM tensor. Following [5], we can write the following expression:

$$\frac{d\rho_{GW}}{dtd^3\mathbf{k}} = \frac{4\pi G}{(2\pi)^3} \int d^4x \, e^{i(\omega t - \mathbf{k}\mathbf{x})} \left\langle \frac{1}{2} \{ T^{TT}_{ij}(\mathbf{x}, t), T^{TT}_{ij}(0,0) \} \right\rangle \ , \tag{6}$$

where $\frac{d\rho_{GW}}{dtd^3\mathbf{k}}$ is the energy density ρ_{GW} of emitted GW per unit of time. Finally, as argued in [5], in the case we are studying we can write (6) in terms of a particular Wightman function as follows:

$$\frac{d\rho_{GW}}{dtd^3\mathbf{k}} = \frac{4\pi G}{(2\pi)^3} \Lambda_{ijmn} \int d^4x \ e^{i(\omega t - \mathbf{k}\mathbf{x})} \left\langle T^{ij}(0,0) T^{mn}(\mathbf{x}, t) \right\rangle \ , \tag{7}$$

where Λ_{ijmn} is a projector such that

$$T_{ij}^{TT}(k^\mu) = \Lambda_{ijmn} T_{mn}(k^\mu) \ , \tag{8}$$

and $k^\mu = (\omega, \mathbf{k})$ is the four-momentum of the wave.

This derivation has been performed in the weak-field approximation, and no assumptions on the dynamics of the plasma have been made, other than some general symmetries such as spacetime invariance.

III. HOLOGRAPHIC COMPUTATION OF THE EMISSION RATE

In order to obtain the emission rate (7), we need a framework in which to calculate the two-point correlation function of the EM tensor. The gauge/gravity duality will provide us this framework.

The gauge/gravity, AdS/CFT conjecture, or holography, is a relation between a d-dimensional conformal quantum fields theory and a $(d+1)$-dimensional string or gravity theory [6]. The original example of this duality establishes a relation between $\mathcal{N} = 4$ $SU(N_c)$ SYM theory in the large number of colors $N_c \to \infty$ and the large 't Hooft coupling $\lambda \to \infty$ limits, for $\lambda \equiv g_{YM}^2 N_c$, and type IIB supergravity on an $\mathrm{AdS}_5 \times \mathrm{S}^5$ background, whose metric, in the zero temperature case, is given by

$$ds^2 = ds^2_{AdS_5} + R^2 d\Omega_5^2 \ , \tag{9}$$

where $ds^2_{AdS_5}$ is the pure AdS metric, R is the radius of AdS and $d\Omega_5^2$ is the metric of a unit five-sphere. In this study it will suffice to restrict ourselves to the AdS part of the background, and perturbations of the unit five-sphere part of the background will not be considered [5]. For a thermal system, the above equivalence can be generalized by replacing the pure AdS metric by that of a black brane in AdS_5

$$ds^2 = \tfrac{(\pi T R)^2}{u}(-f(u)dt^2 + d\mathbf{x}^2) + \tfrac{R^2}{4u^2 f(u)} du^2 \ , \tag{10}$$

where $f(u) = 1 - u^2$, $u = r_0^2/r^2$, and r_0 represents the position of the event horizon. In this case, the temperature of the dual field theory is the Hawking temperature of the black brane, $T = r_0/(\pi R^2)$.

According to the gauge/gravity duality, to each possible source $\phi(x)$ for each possible local, gauge-invariant operator $\mathcal{O}(x)$ in the quantum field theory, there must correspond a dual bulk field $\Phi(x, u)$ (and vice-versa) such that its value at the AdS boundary (∂AdS) may be identified with the source [6]. This relation between bulk fields and operators allows us to calculate the n-point correlation function of local operators in the gauge theory in terms of a gravity description.

In holographic AdS/CFT duality, fluctuations of the metric $h_{\mu\nu}$[13] couple to the EM tensor. Therefore, the EM tensor two-point correlation function can be determined by a perturbation of the form [5, 7]

$$h_{\mu\nu}(t, \mathbf{x}, u) = h_{\mu\nu}(u) e^{-i\omega t + ikz} \ , \tag{11}$$

where, without a loss of generality, we have assumed these fluctuations to propagate along the z-axis, with momentum $\mathbf{k} = (0, 0, k)$.

These metric perturbations can be classified according to their transformation properties under rotations in the xy-plane. Accordingly, in a conformal quantum field theory, such as $\mathcal{N} = 4$ SYM theory, the retarded two-point correlation function of the EM tensor in a thermal rotation-invariant state is given by

$$\begin{aligned} G^R_{\mu\nu\alpha\beta}(k) &= -i \int d^4x e^{i(kt - \mathbf{kx})} \left\langle \theta(t)[T_{\mu\nu}(0,0), T_{\alpha\beta}(\mathbf{x}, t)] \right\rangle \\ &= S_{\mu\nu\alpha\beta} G_1(k) + Q_{\mu\nu\alpha\beta} G_2(k) + L_{\mu\nu\alpha\beta} G_3(k) \ , \end{aligned} \tag{12}$$

where $Q_{\mu\nu\alpha\beta}$, $S_{\mu\nu\alpha\beta}$, $L_{\mu\nu\alpha\beta}$ are orthogonal projectors onto the sound (spin 0), shear (spin 1) and tensor channel (spin 2) respectively, providing three different Lorentz index structures, and G_1, G_2, G_3 three scalar functions that determine the shape of the correlator. For a more detailed derivation, and an extensive discussion on the different symmetry channels, see [5, 7].

However, the energy production rate (7) we are interested in does not rely on the retarded two-point function, but rather on the spatial components of the Wightman function. Nonetheless, following [5], and for a three-momentum in the z-direction $\mathbf{k} = (0, 0, k)$, this retarded correlator can be determined using the Kubo-Martin-Schwinger (KMS) relations, finally obtaining:

$$\frac{d\rho_{GW}}{dt d^3\mathbf{k}} = \frac{-16\pi G n_B}{(2\pi)^3} \mathrm{Im} G_3 \ , \tag{13}$$

where $n_B = 1/(exp(k/T) - 1)$.

To compute G_3 we take a perturbation of the form of (11) over the black brane metric (10). To linear order in $h_{\mu\nu}$ the Einstein equations are:

$$\mathcal{R}^{(1)} = -\frac{4}{R^2} h_{\mu\nu} \ , \tag{14}$$

with $\mathcal{R}^{(1)}$ the Ricci tensor to linear order in h. Restricting ourselves to the gauge-invariant variable in the tensor channel [5, 7] $Z(u) = h^x_y(u)$, and introducing it in (14), we obtain the following second order differential equation for Z:

$$Z'' - \frac{1+u^2}{uf} Z' + \frac{\mathfrak{w}^2 - \kappa^2 f}{uf^2} Z = 0 \ , \tag{15}$$

where $\mathfrak{w} = \omega/(2\pi T)$ and $\kappa = k/(2\pi T)$.

We express the solution obeying the incoming wave boundary condition at the horizon (since classically a black hole does not radiate) in terms of two local solutions at the boundary as

$$Z(u) = \mathcal{A}\phi_1(u) + \mathcal{B}\phi_2(u) \ , \tag{16}$$

where ϕ_1 and ϕ_2 are of the form $\phi_1(u) = (1 + ...)$ and $\phi_2(u) = (u^2 + ...)$ [7]. Then, G_3 is given by [7]

$$G_3(\mathfrak{w}, \kappa) = -\pi^2 N_c^2 T^4 \frac{\mathcal{B}(\mathfrak{w}, \kappa)}{\mathcal{A}(\mathfrak{w}, \kappa)} \ , \tag{17}$$

where $\mathcal{A}$ and $\mathcal{B}$ are the connection (or matching) coefficients of the ODE (15).

IV. THE WKB APPROXIMATION

Equation (15) it has no analytical solution. Although it has been solved numerically before in [5], in this work we will employ the Wentzel–Kramers–Brillouin (WKB) approximation to find an analytical solution in the high-energy limit.

The WKB theory is a method for approximating the solution of a differential equation whose highest derivative is multiplied by a small parameter ϵ. A well known example in physics in which it is used, easily found in many quantum mechanics text books (for example [8, 9]), is to solve the Schrödinger equation.

For a Schrödinger-like equation

$$-\hbar^2\psi''(x) = 2m[E - V(x)]\psi(x) = F(x)\psi(x) \; , \qquad (18)$$

where $\hbar$ is a small parameter and $F(x)$ a potential this, this method consists in proposing a solution of the form

$$\psi(x) = e^{\frac{iQ(x)}{\hbar}} \; , \qquad (19)$$

and then expand $Q(x)$ as a power series in $\hbar$,

$$Q(x) = Q_0(x) + \hbar Q_1(x) + \hbar^2 Q_2(x) + \dots \; . \qquad (20)$$

Introducing (19) into (18), and equating terms with the same $\hbar$ power, we obtain the following system of equations:

$$[Q_0'(x)]^2 = F(x) \; , \qquad (21)$$
$$-iQ_0''(x) + 2Q_0'(x)Q_1'(x) = 0 \; , \qquad (22)$$
$$-iQ_1''(x) + [Q_1'(x)]^2 + 2Q_0'(x)Q_2'(x) = 0 \; , \qquad (23)$$
$$\dots \; , \qquad (24)$$

which is totally analogous to (18). From here, integrating these equations we obtain

$$Q_0(x) = \pm\int_X^x \sqrt{F(x)}dx + C_1 \; , \qquad (25)$$
$$Q_1(x) = \frac{i}{2}\ln|\sqrt{F(x)}| + C_2 \; , \qquad (26)$$
$$\dots \; , \qquad (27)$$

where X is a point in between the turning points of the potential x_a, x_b, namely $X \in [x_a, x_b]$, and C_1, C_2 are two integration constants.

The WKB approximation consists on taking $Q(x) \simeq Q_0(x) + \hbar Q_1(x)$, thus for this approximation to be valid we require the series to be rapidly convergent. This is true when [9]

$$\frac{\lambda}{4\pi}|V'(x)| << |E - V(x)| \; , \qquad (28)$$

where λ is the local wavelength, and it is defined as $\lambda \equiv 2\pi\hbar/\sqrt{|F(x)|}$.

Therefore, the WKB solution is valid when the relative variation of the potential energy over a wavelength is small, or, equivalently, when the potential varies slowly compared to the frequency of the solution. Namely, far from the turning points. Near these points the solution has to be studied more carefully and then matched to the WKB solution.

Defining the analogous momentum and action as $p(x) \equiv \sqrt{F(x)}/\hbar$ and $S(x) \equiv \int_X^x p(x)dx$, and A', B', A, B being constants, the WKB solution is:

$$\psi(x) = e^{\frac{i}{\hbar}(Q_0(x)+\hbar Q_1(x))} = \frac{A'}{\sqrt{p(x)}}e^{iS(x)} + \frac{B'}{\sqrt{p(x)}}e^{-iS(x)} = \frac{A}{\sqrt{p(x)}}\sin(S(x)+C_1) + \frac{B}{\sqrt{p(x)}}\cos(S(x)+C_1) \; . \qquad (29)$$

V. WKB COMPUTATION OF THE EMISSION RATE

In the high-energy limit, when $\mathfrak{w}$ is large, we may use the WKB approximation to solve equation (15) away from its singular points $u = 0$ and $u = 1$. Introducing the following variable change in (15),

$$\psi(u) \equiv \sqrt{\frac{1-u^2}{u}}Z(u) \; , \qquad (30)$$

and for a light-like momentum ($\omega = k$), we get a Schrödinger-like equation

$$\frac{-1}{\mathfrak{w}^2}\frac{d^2\psi(u)}{du^2} = \frac{1}{4u^2(1-u^2)^2}\left(\frac{-3+6u^2+u^4}{\mathfrak{w}^2} + 4u^3\right)\psi(u) \; . \qquad (31)$$

Identifying $1/\mathfrak{w}$ with $\hbar$, and the function of u that multiplies $\psi(u)$ in the R.H.S. in (31) with a potential $F(x)$, following section IV we may write a WKB solution of the form (29), where

$$p(u) = \frac{\mathfrak{w}\sqrt{u}}{1-u^2} \; , \qquad (32)$$

$$S(u) = \int_0^u p(u')du' = \mathfrak{w}(\tanh^{-1}(\sqrt{u}) - \tan^{-1}(\sqrt{u})) \; , \qquad (33)$$

in the high-energy limit. Near the turning points this solution is not valid and a more detailed study is needed. Near the boundary, when $u \to 0$, we expand the potential and obtain the following equation

$$\frac{d^2\psi(u)}{du^2} + \left(-\frac{3}{4u^2} + \mathfrak{w}^2 u\right)\psi(u) = 0 \; , \qquad (34)$$

which has an analytical solution

$$\psi(u) = C_1\frac{\mathrm{Ai}'((-1)^{1/3}\mathfrak{w}^{2/3}u)}{\sqrt{u}} + C_2\frac{\mathrm{Bi}'((-1)^{1/3}\mathfrak{w}^{2/3}u)}{\sqrt{u}} \; . \qquad (35)$$

Taking the determination $(-1)^{1/3} = e^{-i\pi/3}$ and setting $C_2 = 0$, using the asymptotic expansions of the derivatives of the Airy functions we obtain the matching formula

$$\frac{\mathrm{Ai}'(e^{-i\pi/3}\mathfrak{w}^{2/3}u)}{\sqrt{u}} \to \frac{\frac{i-1}{4}\left(i+\sqrt{3}\right)\mathfrak{w}^{2/3}}{\sqrt{p(u)}\sqrt{2\pi}} e^{iS(u)} \ , \quad (36)$$

where in obtaining this formula we have used that $p(u) \simeq \mathfrak{w}\sqrt{u}$ and $S(u) \simeq 2/3\mathfrak{w}u^{3/2}$ when $u \to 0$.

Near the horizon, when $u \to 1$, we expand the potential and obtain the following equation

$$\frac{d^2\psi(u)}{du^2} + \frac{1}{4(u-1)^2}(1+\mathfrak{w}^2)\psi(u) = 0 \ , \quad (37)$$

which has an analytical solution

$$\psi(u) = C_1'(1-u)^{\frac{1}{2}-i\frac{\mathfrak{w}}{2}} + C_2'(1-u)^{\frac{1}{2}+i\frac{\mathfrak{w}}{2}} \ . \quad (38)$$

Taking the in-falling boundary condition we set $C_2' = 0$, and then we see that

$$\frac{\frac{i-1}{4}\left(i+\sqrt{3}\right)\mathfrak{w}^{2/3}}{\sqrt{p(u)}\sqrt{2\pi}} e^{iS(u)} \xrightarrow{u\to 1} C(1-u)^{\frac{1}{2}-i\frac{\mathfrak{w}}{2}} \ , \quad (39)$$

where we have used that $S(u) \simeq -\mathfrak{w}/2\ln(-1+u)$ and $p(u) \simeq \mathfrak{w}/(2(1-u))$ when $u \to 1$, and C is a constant irrelevant to what follows. Putting together the connection formulas (36), (39), and undoing the variable change (30), we find that

$$\mathrm{Ai}'(e^{-i\pi/3}\mathfrak{w}^{2/3}u) \xrightarrow{u\to 1} C(1-u)^{-i\frac{\mathfrak{w}}{2}} \ . \quad (40)$$

Up to a normalization constant this is the physical solution. Expanding $\mathrm{Ai}'(e^{-i\pi/3}\mathfrak{w}^{2/3}u)$ in a power series of u near $u = 0$, we find that the asymptotic behavior of the physical solution $Z(u)$ near the boundary is

$$Z(u) = \frac{-1}{3^{1/3}\Gamma(\frac{1}{3})} + \frac{\mathfrak{w}^{4/3}e^{-i2\pi/3}u^2}{2\cdot 3^{2/3}\Gamma(\frac{2}{3})} + \ldots \ . \quad (41)$$

Identifying this expression with (16) we obtain the matching coefficients

$$\mathcal{A}(\mathfrak{w},\kappa) = \frac{-1}{3^{1/3}\Gamma\left(\frac{1}{3}\right)} \ , \qquad \mathcal{B}(\mathfrak{w},\kappa) = \frac{\mathfrak{w}^{4/3}e^{-i2\pi/3}}{2\cdot 3^{2/3}\Gamma\left(\frac{2}{3}\right)} \ , \quad (42)$$

and putting together the equations (17), (13), we obtain an expression for the energy density of emitted GW from a thermal source per unit of time

$$\frac{d\rho_{GW}}{dtd^3\mathbf{k}} = \frac{3^{1/6}n_B G N_c^2 \Gamma\left(\frac{1}{3}\right)\omega^{4/3}T^{8/3}}{4\cdot 2^{1/3}\pi^{4/3}\Gamma\left(\frac{2}{3}\right)} \ . \quad (43)$$

We observe that the emission rate obtained has a non-trivial T dependency, with a particular fractional power dependency on ω and T. In the high-energy limit, when the wave fluctuations are much greater than the thermal fluctuations, we could have expected the result found to be the same as the one for a conformal theory in the vacuum, and therefore to have only a dependency on ω^4, but it does not. Nonetheless, note that when $T \to 0$ there is no emission, as expected in the vacuum.

In order to clarify this result, we will study how this power dependency on ω changes for different types of momentum.

VI. THERMAL CORRELATOR AT ARBITRARY MOMENTUM

Although the energy density spectrum in (6) is controlled by a light-like momentum two point correlation function, in order to better understand the dependency in ω and T obtained in (43), we may also study the two point correlation function of the EM tensor at arbitrary four-momentum $\mathbf{k} = (\omega, 0, 0, k)$.

Parametrizing k as $\alpha\omega$, and following the same steps as in section V, we obtain the following equation

$$\frac{-1}{\mathfrak{w}^2}\frac{d^2\psi(u)}{du^2} = \frac{1}{4u^2(1-u^2)^2} \left(\frac{-3+6u^2+u^4}{\mathfrak{w}^2} + 4u(1-\alpha^2+u^2\alpha^2)\right)\psi(u) \ , \quad (44)$$

after the variable change (30). The WKB solution far from the singular points is (29), where

$$p(u) = \mathfrak{w}\sqrt{\frac{1-\alpha^2+u^2\alpha^2}{u(1-u^2)^2}} \ , \quad (45)$$

$$S(u) = \int_0^u p(u')du' \ , \quad (46)$$

in the high-energy limit. Analogous to what we did in section V, near the boundary the equation (44) becomes

$$\frac{d^2\psi(u)}{du^2} + \left(-\frac{3}{4u^2} + \frac{\mathfrak{w}^2(1-\alpha^2)}{u}\right)\psi(u) = 0 \ . \quad (47)$$

We can write an analytical solution to this equation as

$$\psi(u) = \mathfrak{w}\sqrt{u(-1+\alpha^2)} \left(c_1 Y_2\left(2\mathfrak{w}\sqrt{u}\sqrt{1-\alpha^2}\right) + c_2 J_2\left(2\mathfrak{w}\sqrt{u}\sqrt{1-\alpha^2}\right)\right) \ , \quad (48)$$

where c_1, c_2 are constants, and J_2, Y_2 are Bessel functions of the first and second kind, respectively. Using the asymptotic expansions of the Bessel functions [10] we obtain the matching formulas

$$\mathfrak{w}\sqrt{u(-1+\alpha^2)}Y_2\left(2\mathfrak{w}\sqrt{u}\sqrt{1-\alpha^2}\right) \to \frac{\mathfrak{w}\sqrt{1-\alpha^2}}{\sqrt{\pi p(u)}}\sin\left(S(u) - \frac{5\pi}{4}\right) \ , \quad (49)$$

$$\mathfrak{w}\sqrt{u(-1+\alpha^2)}J_2\left(2\mathfrak{w}\sqrt{u}\sqrt{1-\alpha^2}\right) \rightarrow \frac{\mathfrak{w}\sqrt{1-\alpha^2}}{\sqrt{\pi p(u)}}\cos\left(S(u)-\frac{5\pi}{4}\right) , \quad (50)$$

where we have used that $p(u) \simeq \sqrt{1-\alpha^2}\mathfrak{w}/\sqrt{u}$ and $S(u) \simeq 2\mathfrak{w}\sqrt{u}\sqrt{1-\alpha^2}$ when $u \to 0$. Near the horizon we obtain the equation (37), thus the solution is (38). After taking the in-falling boundary condition, we can see that

$$\frac{\mathfrak{w}\sqrt{1-\alpha^2}}{\sqrt{\pi p(u)}}\left[\cos\left(S(u)-\frac{5\pi}{4}\right)+i\sin\left(S(u)-\frac{5\pi}{4}\right)\right] \xrightarrow{u\to 1} C(1-u)^{\frac{1}{2}-i\frac{\mathfrak{w}}{2}} , \quad (51)$$

where we have used that $p(u) \simeq \frac{\mathfrak{w}}{2(1-u)}$ and $S(u) \simeq -\frac{\mathfrak{w}}{2}\ln(1-u)$ when $u \to 1$.

Putting together the connection formulas (49), (50), (51), undoing the variable change (30), we find that, up to a normalization constant, we obtain the physical solution

$$u\left(J_2\left(2\mathfrak{w}\sqrt{u-u\alpha^2}\right)+iY_2\left(2\mathfrak{w}\sqrt{u-u\alpha^2}\right)\right) \xrightarrow{u\to 1} C(1-u)^{-i\frac{\mathfrak{w}}{2}} . \quad (52)$$

Note that $uY_2 \propto (1+...)$ and $uJ_2 \propto (u^2+...)$ when $u \to 0$. Thus, expanding Y_2 and J_2 in power series of u near the boundary, the physical solution becomes

$$Z(u) = \frac{i}{\mathfrak{w}^2\pi(-1+\alpha^2)} - \frac{1}{2}(\mathfrak{w}^2(-1+\alpha^2))u^2 + ... , \quad (53)$$

and identifying this expression with (16) we obtain the matching coefficients

$$\mathcal{A}(\mathfrak{w},\kappa) = \frac{i}{\mathfrak{w}^2\pi(-1+\alpha^2)} , \quad \mathcal{B}(\mathfrak{w},\kappa) = -\frac{1}{2}(\mathfrak{w}^2(-1+\alpha^2)) . \quad (54)$$

Using (17), we find that

$$G_3(\mathfrak{w},\kappa) = \frac{-iN_c^2\omega^4(-1+\alpha^2)^2}{32\pi} . \quad (55)$$

Note that for $\alpha = 0$, we recover the results obtained in [11]. This thermal correlator does have the form we could expect for a conformal theory in the vacuum.

VII. CONCLUSIONS

In this study we have obtained an analytical expression for the energy density of emitted GW from a thermal source per unit of time, which has a particular non-trivial dependency on ω and T (see equation (43)). When we try to better understand this dependency by studying the EM tensor thermal correlation function at arbitrary momentum, we find that it fades out for $\alpha = 1$, making $\alpha = 1$ a special case. It would be interesting to continue this study by doing further research in trying to understand this discontinuity.

We also note that the result obtained in this study (equation (43)) differs from the one obtained via perturbative analysis in [12], according to which $\frac{d\rho_{GW}}{dtd^3\mathbf{k}} \propto \omega$. Given this discrepancy, the analysis of future data could serve as an indication of the presence of BSM strongly coupled physics.

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[13] Here $h_{\mu\nu}$ correspond to a perturbation of the black brane metric, and it will be used a calculation tool. It should not be confused with the perturbations of the Minkowski metric in the weak field approximation for GW.