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GENDERED EFFECTS OF THE MINIMUM WAGE

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JEL Codes: J08, J31, J16, E24, E64.

Keywords: Minimum wage, gender gaps, equilibrium job posting, hour requirement.

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Gendered Effects of the Minimum Wage[*]

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1 Introduction

The minimum wage is a policy tool aiming at reducing income inequality; it can also address the gender income gap. The gender income gap encompasses gender disparities in employment, working hours, and wages, and the implementation of a minimum wage has the potential to impact all three aspects. Women are more likely to work in low-hour jobs, often due to caregiving responsibilities. Low-hour jobs are associated with lower hourly pay, an empirical phenomenon known as the part-time penalty (Aaronson and French, 2004). Consequently, women, who are more concentrated in low-hour jobs, are particularly susceptible to the impact of minimum wages that establish a floor on the hourly wage.

While previous research has primarily focused on the effect of the minimum wage on the gender wage gap (DiNardo et al., 1996; Caliendo and Wittbrodt, 2022), a comprehensive understanding of its impact on the gender income gap requires consideration of the gender employment and hours gap as well. The minimum wage can result in either job losses for women in low-wage and low-hour jobs or encourage women to transition to higher-hour jobs with better compensation. The overall impact of the minimum wage on the gender employment and hours gap hinges on firms' response to this policy. In this paper, we utilize an equilibrium job search model with heterogeneous workers and firms to address the following questions: How does the minimum wage affect the gender income gap, and how does its impact vary across different demographic groups? How do firms adjust their job-posting strategies in response to minimum wages?

To shed light on these questions, we examine the impact of the introduction of the first federal minimum wage in Germany in 2015. We begin by conducting empirical analyses to estimate the policy's effects by leveraging regional variations in the impact of the minimum wage. Our findings reveal a significant negative impact of the minimum wage on female marginal employment, which is a type of employment associated with low weekly hours and low pay. We also observe a positive effect of the minimum wage on the transition of female workers from marginal employment to part-time employment. These results suggest that the policy may contribute to higher incomes for women and potentially lead to a reduction in the gender income gap.

To quantify the effect of the minimum wage on the gender income gap, we develop an equilibrium job search model that incorporates household demographic heterogeneity and firms' labor demand response to the minimum wage. This model builds upon existing literature on equilibrium job-posting models (Burdett and Mortensen, 1998; Bontemps et al., 2000; Meghir et al., 2015; Conti et al., 2018; Fang and Shephard, 2019; Aizawa and Fang, 2020). Our model is novel in the following aspects. First, we consider different levels of hours corresponding to marginal, part-time and full-time employment and allow firms optimize their hour requirements based on factors including profit per job, labor supply, and job-posting costs associated with each hours-type. Second, we introduce rich demographic heterogeneity into our model, including variables such as gender, children, marital status, and spousal income. Empirically, demographic factors are important in determining employment and hours outcomes. In our model, we allow for worker heterogeneity in preference and productivity based on these factors. While we do not explicitly model joint family decisions, we account for the effect of spousal incomes and joint taxation on labor supply decisions. Lastly, we assume that firms are prohibited from making wage offers contingent on demographic characteristics due to anti-discrimination laws, such as Germany's Equal Treatment Act. Instead, firms consider expected level of labor supply when deciding which job offers to post and are willing to employ any worker who accepts their offer.

To estimate the model, we rely on data from the German Socio-Economic Panel (SOEP), a longitudinal dataset with detailed information on demographic characteristics and employment outcomes, including hours of work, for a representative sample of German households. The unit of observation is the household, which allows us to observe labor incomes of spouses and the age of children.

Our estimated model matches key features in the data prior to the introduction of the minimum wage (2013-2014). It closely replicates the observed patterns that women are less likely employed, more likely to work in marginal or part-time jobs, and receive lower hourly wages in full-time jobs. These gender gaps derive from differences in preferences for working hours and in productivity. The model also matches the variation in employment and hours by the age of children, and the positive correlation between hourly wages and hours of employment. This correlation is explained by two features in the model. Firstly, offering a full-time job is associated with higher hourly productivity compared to marginal or part-time jobs for a given firm. This feature can be rationalized by a sunk management cost associated with each position. Secondly, there is production complementarity between firm productivity and working hours, resulting in firms with higher productivity setting higher hour requirements.

To account for the widespread non-compliance with the minimum wage, as documented in previous studies (Burauel et al., 2017, 2020; Bossler et al., 2022), we introduce a penalty that reduces the job contact rate for jobs with an hourly wage below the minimum wage. This means that firms can post jobs that are not compliant with the minimum wage, but the rate at which workers contact non-compliant job offers is reduced. We calibrate the magnitude of the penalty to match the rate of non-compliance observed in the data from SOEP.

We analyze the impact of three different minimum wage levels: the initial level of 8.5 € per hour in 2015, the 2022 level of 10 € (adjusted for inflation), and a potential level of 11 €. We find that the minimum wage affects employment and hours differently for men and women. While it has little effect on male non-employment, it progressively reduces female non-employment. Both genders experience a reallocation towards jobs with longer hours, but the effect on women is stronger.[1] The stronger employment and

[1] Dustmann et al. (2021) find that the German minimum wage leads to a reallocation of workers toward larger and higher-paying firms. We complement their paper by showing that the minimum wage also leads to reallocation to workers from marginal employment to jobs with longer hours.

reallocation effect on women can be attributed to their higher likelihood of working in jobs with lower hours and lower hourly wages prior to the introduction of the minimum wage. These gender asymmetries have implications for equilibrium gender gaps. At 8.5 €, the minimum wage moderately closes the gender income gap by 0.9 percentage points, primarily through the gender wage gap. At 11 €, the minimum wage reduces the gender income gap by 2.5 percentage points, with 24.8% of this reduction attributed to the employment channel and 5.7% attributed to the hours channel.

Firm response plays a crucial role in shaping the equilibrium effects of the minimum wage. In our model, firms can adjust wages and hour requirements of posted jobs or choose to be inactive by not posting any jobs. A penalty on non-compliant jobs induces firms to raise wages, particularly for job offers with lower hour requirements as they are more impacted. We find that increasing the minimum wage has a minimal impact on firm inactivity but leads to a shift towards job offers with full-time requirement. Firms' ability to post non-compliant jobs and adjust hour requirements enables them to maintain profitability, which helps explain the limited firm response in the activity margin. The shift in the hour requirement distribution can be attributed to the response of low-productivity firms. When these firms become inactive, there is a disproportionate reduction in jobs with low hours due to the positive sorting between firm productivity and hour requirements. In addition, low-productivity firms that remain active tend to increase the hour requirements to full-time in response to the minimum wage to take advantage of the higher hourly productivity.

To quantify the role of firm adjustments in shaping equilibrium outcomes, we compare the benchmark equilibrium with a counterfactual economy in which firms do not adjust their job-posting decisions. Our findings indicate that firm adjustments dampen the reallocation effect, particularly for women. By increasing offered wages in low-hour jobs, firms make these positions relatively more attractive and easier to find as the non-compliance penalty associated with them is reduced.

We also examine the heterogeneity in the impact of the minimum wage and find that the policy has more pronounced and more varied impact on women compared to men, with children and spousal income playing a significant role in this heterogeneity. Among women, those without children are least affected by the minimum wage, experiencing similar effects as men without children. Among women with children, the reallocation effect towards jobs with longer hours is more prominent for those with older children or without high spousal incomes.

Related Literature Our research adds to the existing literature on the impact of minimum wages on gender inequality. Previous studies such as DiNardo et al. (1996) and Autor et al. (2016) have demonstrated that minimum wages have a stronger effect on reducing wage inequality for women in the United States compared to men. Bargain et al. (2019) find similar results for Ireland, showing that the minimum wage significantly reduces the gender wage gap. Caliendo and Wittbrodt (2022) also find similar results for

Germany. However, these studies primarily focus on the gender wage gap, while our study examines the gender income gap by considering employment and hours worked alongside wage differentials. Our findings highlight that the minimum wage can reduce the gender employment and hours gaps, thereby contributing to the overall reduction of the gender income gap.

There is a growing literature that examines firm responses to minimum wages. A number of studies empirically estimate effects of the minimum wage on businesses, including pass-through on prices, substitution between capital and labor, and business entry and exit (see, for example, Aaronson et al., 2008; Harasztosi and Lindner, 2019; Jardim and Inwegen, 2019; Adamopoulou et al., 2022). Moreover, several recent works have employed quantitative equilibrium models to analyze firm responses to minimum wages. For instance, Berger et al. (2022) consider oligopsonistic labor markets and focus on the welfare effects of the minimum wage. Hurst et al. (2022) develop a general equilibrium model with monopsonistic labor market competition and firms that face putty-clay frictions in adjusting firm technology. We contribute to this literature by studying heterogeneous firms' response to the minimum wage in a frictional labor market and examining the potential role of hour requirement as a margin for firm adjustment, in addition to wages and job-posting participation.

This paper is most closely related to the literature that examines the effects of minimum wage policies using equilibrium job search models. Flinn (2006) provides seminal work demonstrating the potential for the minimum wage to increase employment and welfare within a search model featuring Nash bargaining. Engbom and Moser (2021) utilize an equilibrium job-posting model to investigate the spillover effects of the Brazilian minimum wage and find substantial impacts. Papers by Bloemer et al. (2018) and Drechsel-Grau (2022) are particularly relevant to our study as they specifically consider the effects of the German minimum wage. Bloemer et al. (2018) analyze the impact of the minimum wage on full-time employment and identify heterogenous effects in different labor market segments, driven by heterogeneity in firm productivity. Drechsel-Grau (2022) examines the short- and long-run effects of the minimum wage in a rich search-and-matching model that incorporates different hours of employment. Unlike our approach of deriving an equilibrium wage and hours distribution, he assumes that wages are determined by the higher of a fixed share of the match output and the minimum wage. We contribute to this literature by examining the effects of minimum wage policies on gender gaps, while explicitly considering the equilibrium wage and hours distribution.

This paper falls within the literature that examines gender gaps using dynamic equilibrium models. Morchio and Moser (2018) develop a model where firms make differential choices regarding amenities and recruitment strategies for men and women. Amano-Patino et al. (2020) consider gender differences in career interruptions due to fertility and explore their implications for human capital accumulation. Bartolucci (2013) investigates a search model with wage bargaining and finds that gender differences in productivity, rather than bargaining power, largely account for the gender wage gap. Our paper contributes to this literature by emphasizing the significant gender gap in hours worked, in addition to employment and wages.

Moreover, we analyze the impact of the minimum wage, an important policy tool for reducing inequality, on equilibrium gender gaps.

Organization The remainder of the paper is organized as follows. In Section 2, we document the gender gap in employment and hours in Germany, explore the factors driving the gender gap, and empirically analyze the of the gender-specific introduction of the first federal minimum wage in Germany. In Section 3, we provide a detailed description of the equilibrium job search model that we utilize to analyze the effects of minimum wages on the gender income gap. In Section 4, we outline our estimation strategy and present the results of our estimation. In Section 5, we incorporate minimum wages into our estimated model and present our findings. Finally, in Section 6, we conclude the paper.

2 Empirical Motivation

2.1 Gender asymmetry in hours of work

In Germany, despite a relatively small gender gap in employment, there is a substantial gender difference in hours of work. Women are less than half as likely than men to hold a full-time job, but six times more likely to be part-time employed and over three times more likely to be marginally employed (Table 1). Marginal employment is associated with mini-jobs, which are job contracts with monthly pay below 450 €, making them eligible for tax exemptions. Typically, marginal employment involves working for around 11 hours per week based on data from the German Socio-Economic Panel.

To explore the factors driving the gender asymmetry in employment and hours, we analyze the marginal effects of various demographic variables on the probability of different types of employment and non-employment, as shown in Tables 2 and 3. For both men and women, lower education levels are associated with a lower probability of employment and lower working hours. Marriage is linked to a lower non-employment rate for both genders, but interestingly, it is associated with a higher full-time employment rate for men, while a higher marginal employment rate for women. Additionally, spousal income plays a significant role in determining employment outcomes, with a high-income spouse associated with a lower likelihood of employment, particularly in full-time jobs, for both men and women. The most striking gender asymmetry is observed in the effect of children, where young children have little impact on the employment outcomes of men, but significantly decrease the likelihood of employment for women, particularly in full-time jobs.

Table 1: Employment Distribution By Gender

	Men	Women
Full-time	0.811	0.393
Part-time	0.052	0.302
Marginal emp.	0.026	0.085
Non-emp.	0.112	0.220

Source: SOEP. The sample contains 25-55 year old individuals between 2006 and 2017.

Table 2: Marginal effects on employment probability: Men

	(1) Full-time	(2) Part-time	(3) Marginal emp.	(4) Non-emp.
Age	0.131***	-0.0144***	-0.00868***	-0.108***
	(0.00293)	(0.000924)	(0.000917)	(0.00271)
Married, high-income spouse	0.144***	0.0387***	0.00133	-0.184***
	(0.0164)	(0.00803)	(0.00723)	(0.0161)
Married, low-income spouse	0.240***	0.00588	0.000132	-0.246***
	(0.0155)	(0.00782)	(0.00615)	(0.0145)
Unmarried	0	0	0	0
	(.)	(.)	(.)	(.)
N. kids under 6	0.0206	-0.00310	-0.0268***	0.00931
	(0.0123)	(0.00468)	(0.00648)	(0.0128)
N. kids	-0.00284	-0.000375	0.000418	0.00280
	(0.00643)	(0.00266)	(0.00260)	(0.00648)
Edu: 9 years or less	0	0	0	0
	(.)	(.)	(.)	(.)
Edu: 10-12 years	0.233***	-0.0207***	-0.0291*	-0.183***
	(0.0160)	(0.00546)	(0.0118)	(0.0187)
Edu: over 12 years	0.258***	-0.000933	-0.00334	-0.254***
	(0.0171)	(0.00664)	(0.0123)	(0.0197)
Observations	132207	132207	132207	132207
Pseudo R^2	1.000	1.000	1.000	1.000

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: SOEP. The sample contains 25-55 year old individuals between 2006 and 2017. The marginal effects are estimated in a multinomial logit regression model controlling for year and federal state fixed effects.

Table 3: Marginal effects on employment probability: Women

	(1) Full-time	(2) Part-time	(3) Marginal emp.	(4) Non-emp.
Age	0.0385***	0.0333***	0.00292**	-0.0747***
	(0.00124)	(0.00135)	(0.00103)	(0.00203)
Married, high-inc. spouse	-0.0490***	0.0646***	0.0506***	-0.0662***
	(0.00446)	(0.00836)	(0.00767)	(0.0128)
Married, low-inc. spouse	-0.00340	0.0526***	0.0579***	-0.107***
	(0.00598)	(0.00802)	(0.00847)	(0.0126)
Unmarried	0	0	0	0
	(.)	(.)	(.)	(.)
N. kids under 6	-0.127***	-0.0922***	-0.0567***	0.275***
	(0.00719)	(0.00544)	(0.00512)	(0.00936)
N. kids	-0.0422***	0.00473	0.0102***	0.0273***
	(0.00265)	(0.00293)	(0.00245)	(0.00502)
Edu: 9 years or less	0	0	0	0
	(.)	(.)	(.)	(.)
Edu: 10-12 years	0.0549***	0.0740***	0.0141	-0.143***
	(0.00525)	(0.00745)	(0.00737)	(0.0129)
Edu: over 12 years	0.106***	0.0905***	0.0262**	-0.223***
	(0.00778)	(0.00856)	(0.00858)	(0.0148)
Observations	151809	151809	151809	151809
Pseudo R^2	1.000	1.000	1.000	1.000

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: SOEP. The sample contains 25-55 year old individuals between 2006 and 2017. The marginal effects are estimated in a multinomial logit regression model controlling for year and federal state fixed effects.

2.2 The German minimum wage and its effects on women

In 2015, Germany implemented its first federal minimum wage of 8.5 € per hour, with near-universal coverage and few exemptions, mainly for workers below the age of 18 and apprentices. At the time of its introduction, a significant proportion of employment in Germany was paid less than the minimum wage, including 9% of full-time jobs, 15% of part-time jobs, and 62% of marginal-employment jobs (Burauel et al., 2017). The introduction of the minimum wage in Germany has been associated with a reduction in marginal employment. According to Vom Berge and Weber (2017), the introduction of the German minimum wage resulted in a doubling of the number of minijobs that were converted into regular employment, including part-time or full-time jobs subject to social security contributions. Other studies, such as Caliendo et al. (2018) and Bonin et al. (2020), have found that the minimum wage has a moderate effect on employment, but a negative effect on marginal employment. Indeed, while these studies provide insights into the overall employment effects of the minimum wage, they do not specifically examine gender differences in these effects.

Effects on employment and hours To examine the gender-specific employment effects of the minimum wage, we adopt a methodology similar to that employed by Caliendo et al. (2018), which utilizes regional differences in the bite of the minimum wage across labor markets in Germany. The bite of the minimum wage is measured by the Kaitz index, which is the ratio between the minimum wage and the median wage in a given region.[2] We estimate the following regression model:

$$Y_{j,t} = \gamma_t Bite_j + \alpha_j + \beta X_{j,t} + \theta_t + v_{j,t}, \tag{1}$$

where $Y_{j,t}$ represents the labor market outcome variable of interest, and $Bite_j$ is the bite of the minimum wage (Kaitz index) for region j. The model includes region-specific fixed effects α_j, time-varying controls $X_{j,t}$, including regional unemployment rate and income per capita, time fixed effects θ_t, and a region-time specific error term $v_{j,t}$. We estimate the regression based on German administrative data at the regional level. There are 257 regions ("Arbeitsmarktregion") in our dataset.

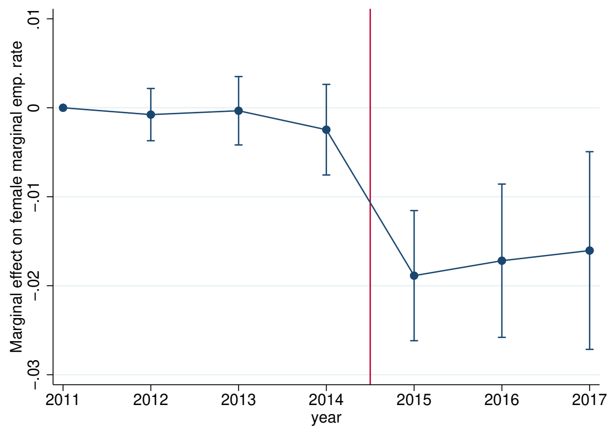
The parameter of primary interest in this regression model is γ_t, which represents the effect of the bite of the minimum wage on the labor market outcome, with γ_{2011} normalized to zero. If γ_t is not significantly different from zero before the introduction of the minimum wage but becomes significantly different from zero after the introduction, then the change in the parameter γ_t can be interpreted as the causal effect of the minimum wage on the labor market outcome being studied.

Figure 1 shows that the minimum wage has a significant and negative effect on the female marginal employment rate. Specifically, a 1 percentage point increase in the bite of the minimum wage leads to an

[2] The bite variable is time invariant in the regression model; it is calculated based on data from 2018.

average drop of 0.016 in the female marginal employment rate during the period of 2015-2017. [3]

Figure 1: Estimated effects of the bite of minimum wage on labor market outcomes (γ_τ)



Source: Regional data accessed on INKAR.de. Vertical bars represent 95% confidence interval. Regions are defined as labor market regions ("Arbeitsmarktregion"). In our dataset, there are 257 regions over the period from 2011 to 2017. Marginal Employment excludes individuals who hold such contracts as secondary jobs. The regressions are weighted by region population and estimated with robust standard errors. See Eq. 1.

Effects on labor force transition Does the reduction in female marginal employment lead to an increase in non-employment or employment in other types of jobs? To answer this question, we estimate the effect of the minimum wage on the female labor force transition rate from marginal employment. We label labor market regions ("Arbeitsmarktregion") as high-impact if the Kaitz-index, or the minimum wage to median wage ratio, is above 0.45. We estimate a multinomial logit regression of the labor force state a year later, conditional on being marginally employed in the current year. The dependent variable is the log odds of transitioning to the labor force state l_1 in year $t+1$ conditional on being in the labor force state l_0 in year t. That is,

$$\eta_{i,t,l_0,l_1} = \ln \frac{\Pr(y_{i,t} = l_0, y_{i,t+1} = l_1)}{\Pr(y_{i,t} = l_0, y_{i,t+1} = l_0)},$$

where $y_{i,t}$ is individual i's labor force state in year t. The regression equation is

$$\eta_{i,t,l_0,l_1} = \gamma_{l_0,l_1} \boldsymbol{I}_{\{t \geq 2015\}} B_{i,t} + \alpha_{s_{i,t},l_0,l_1} + \beta_{l_0,l_1} X_{i,t} + \theta_{t,l_0,l_1} + \epsilon_{i,t,l_0,l_1}, \tag{2}$$

[3] We present results for other labor market outcomes in Figure 10 in Appendix A. Our analysis is inclusive regarding the effect of the minimum wage on both genders' part-time and overall employment rates. Note that variation in the bite of the minimum wage also captures the variation in the median wage of the regional labor market, with a high bite indicating a low median wage. The bite variable appears to affect these employment outcomes even before the introduction of the minimum wage in 2015, which suggests that there may be underlying trends in the differences between low-wage and high-wage regions that predate the implementation of the minimum wage policy.

where $\boldsymbol{I}_{\{t \geq 2015\}}$ is an indicator for the presence of a minimum wage and $B_{i,t}$ is an indicator that individual i is in a high-bite region in year t. $\alpha_{s_{i,t},l_0,l_1}$ is the federal state fixed effect where $s_{i,t}$ is the federal state of individual i in year t. θ_{t,l_0,l_1} is the year fixed effect. $X_{i,t}$ are time-varying individual characteristics including age, education, marital state, spousal income, and children.

The coefficient of primary interest is γ_{l_0,l_1}, which measures the causal effect of the minimum wage on the labor force transition from l_0 to l_1. Table 4 shows estimation results for the female labor force transition from marginal employment, with each column corresponding to a destination state (the base is marginal employment). The estimated value of γ_{l_0,l_1} (see row "Min. wage x high bite") is significantly positive for the transition to part-time, suggesting that the minimum wage has a positive effect on the female labor market transition from marginal employment to part-time employment. [4]

The empirical evidence presented in this section shows that the minimum wage results in a reallocation of women toward jobs with longer hours, which can potentially translate into higher female labor incomes and a smaller gender gap. However, we cannot rule out that those impacted by the minimum wage may drop out of the labor force. We next turn to an equilibrium job search model to quantify the effects of the minimum wage in labor market equilibrium on different demographic groups and examine the role of firms' labor demand response to the minimum wage.

[4]We also estimate the regression (Eq. 2) for other labor force transitions and for both genders. Tables 10 and 11 in Appendix A show the estimated γ_{l_0,l_1} for men and women, respectively. Apart from the coefficient for the female transition rate from marginal employment to part-time employment, the rest of the coefficients are largely statistically insignificant.

Table 4: Estimated effects on the labor force transition rate from marginal employment, women

	To full-time	To part-time	To non-emp.
High bite	-0.440	-0.136	0.00724
	(0.289)	(0.165)	(0.185)
Min. wage x high bite	0.584	0.565*	-0.346
	(0.510)	(0.260)	(0.319)
N. kids under 6	0.0306	0.219*	0.247*
	(0.216)	(0.0918)	(0.119)
N. kids	-0.409**	-0.00290	-0.169*
	(0.127)	(0.0499)	(0.0714)
Age	-0.0608	0.113	0.0134
	(0.119)	(0.0734)	(0.0735)
Age sq.	0.000385	-0.00131	-0.000398
	(0.00151)	(0.000896)	(0.000902)
Married, high-inc. spouse	0	0	0
	(.)	(.)	(.)
Married, low-inc. spouse	0.141	0.0518	0.112
	(0.388)	(0.143)	(0.178)
Unmarried	0.957***	0.458***	0.756***
	(0.277)	(0.130)	(0.168)
Edu.: up to 9 years	0	0	0
	(.)	(.)	(.)
Edu.: 10-12 years	-0.341	-0.213	-0.407*
	(0.356)	(0.162)	(0.181)
Edu.: over 12 years	0.245	-0.132	-0.463*
	(0.380)	(0.189)	(0.224)
Constant	-0.505	-3.571*	-0.763
	(2.274)	(1.507)	(1.540)
Observations	6920		
Pseudo R^2	1.000		

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: SOEP. The sample contains 25-55 year old individuals between 2006 and 2019. The table shows estimated coefficients of the multinomial logit regression of the annual labor force transition rate from marginal employment. The regression controls for year and federal state fixed effects.

3 Model

We consider an equilibrium job-posting model populated with heterogeneous workers and firms based on Burdett and Mortensen (1998) and Bontemps et al. (2000), in which workers conduct random search for job offers and make job acceptance decisions and firms take the labor supply function as given and make job-posting decisions. Building on Meghir et al. (2015), Conti et al. (2018), Fang and Shephard (2019) and Aizawa and Fang (2020), we consider a multi-sector labor market structure, where a sector corresponds to a specific hour requirement. That is, we assume that firms choose not only the wage they post, but also the hour requirement, which can be marginal, part-time, or full-time.

3.1 Environment

There is a continuum of workers that differ in gender j and their time-invariant demographic type $x \in \mathcal{X}$. The demographic type includes being single or married with various levels of spousal income, and men and women potentially draw x from different distributions. Marital status and spousal income are important determinants of employment and hours outcomes (Sec. 2.1). In the model, married workers pool their incomes with the spouse and are subject to joint income taxation. As a result, workers with different marital statuses and spousal incomes make different labor market decisions.[5] Workers also differ in the stochastic child state k. There are three child states: chidless ($k = 1$), having at least one young child $k = 2$, and having only grown children ($k = 3$). The transition matrix across child states is exogenous and depends on the worker's gender and demographic type. The transition rate from any state $k \in \{1, 2\}$ to $k + 1$ is $\phi_{j,x}(k)$. There is no transition from k to $k' < k$ or $k' > k + 1$. Workers derive utility from consumption and leisure, exhibit risk aversion, and face a discount rate ρ. They can be either employed (e) or non-employed (n). At rate ρ_d, a worker in any child and employment state exits the model exogenously. Upon exiting, the worker is replaced by a new worker with the same gender and demographic type but is non-employed and childless.

There is a continuum of firms that differ in productivity level p. Firms operate a production technology with constant returns to scale in the measure of workers. There are three types of jobs by the hour requirement: marginal, part-time, and full-time. That is, $h \in \{h_{ME}, h_{PT}, h_{FT}\} \equiv \mathcal{H}$ with $h_{ME} < h_{PT} < h_{FT}$.

Firms make job-posting decisions by choosing an hour requirement and an hourly wage per unit of labor efficiency. We use Δ^h to denote the measure of job offers with hours-requirement h, and use $F_h(w)$ to denote the hourly wage distribution of job offers with hour requirement h. Together, Δ^h and $F_h(w)$ constitute the offer distribution, which is determined in equilibrium. Workers contact job offers at rate

[5] We implicitly assume that the spousal income distribution is invariant to minimum wage changes. Compared to the direct effect of the minimum wage, we believe that the indirect effect of the minimum wage due to changes in spousal income is quantitatively less significant.

$\lambda \sum_{h\in\mathcal{H}} \Delta^h$, where λ is the baseline contact rate.[6] In the absence of any minimum wage policy, we have $\sum_{h\in\mathcal{H}} \Delta^h = 1$. As we explain in Sec. 3.7, when we implement the minimum wage policy in the model, we allow firms to choose to become inactive such that $\sum_{h\in\mathcal{H}} \Delta^h$ can become smaller than 1, reducing the contact rate for workers.[7]

We allow men and women to have different levels of labor efficiency, and the difference can depend on the hour requirement of the job. This feature is important to replicate the observed gender wage gaps conditional on the hour-type.[8] Specifically, an employed worker in job (w, h) receives gross labor income $wa_j(h)h$, where $a_j(h)$ is the labor efficiency of a worker of gender j in a job with an hour requirement h. Without loss of generality, we normalize the labor efficiency for men to one (i.e., $a_{j=1}(h) = 1$ for $h \in \mathcal{H}$) and allow that for women to take on any positive value (i.e., $a_{j=2}(h) \geq 0$ for $h \in \mathcal{H}$).[9]

Employed workers may leave their jobs for three reasons. First, workers lose their jobs at an exogenous rate $\delta(h)$, which can be different for different hour types to capture potential differences in job security. Second, workers may leave for a new job if they accept a job offer. Lastly, workers can choose to quit their job, which may happen if the child state changes.

We assume a one-time disutility $\mu(j, k, h)$ that incurs when a worker accepts a job offer or decides to remain employed upon a child state change. The disutility captures adjustment costs associated with starting a new job or accommodating the arrival of children as working parents.

The flow utility of gender j can be written as

$$u^j\left(c, h, k\right) = \ln c + \psi_k^j \frac{\left(\bar{h} - h\right)^{1-\gamma_k^j}}{1 - \gamma_k^j}, \tag{3}$$

where ψ_k^j and γ_k^j are parameters specific to gender and the child state. We assume no saving or borrowing technology. If the worker is employed, consumption is $c = \mathcal{N}^e(y; j, k, x)$, where $\mathcal{N}^e$ gives the disposable income for the employed worker given gross labor income y. For non-employed workers, the consumption is $c = \mathcal{N}^n\left(j, k, x\right)$. Since consumption depends on x, so do all endogenous objects, including workers' value functions, decisions, and the distribution. In the rest of the paper, we omit x whenever it does not cause confusion.

[6]Although we assume that the contact rate is the same regardless of gender or employment status, different workers make different job-acceptance decisions, resulting in different job-finding rates.

[7]We implicitly assume the elasticity of job matches with respect to posted offers to be 1 and that there is no congestion externality on the firm side of the labor market. We believe that such congestion externality is of secondary importance for our quantitative results. In Sec. 5.2, we show that the inactivity response is very small.

[8]Although men and women can potentially sort into different firms as they self-select into jobs with different working hours, this is not enough to replicate the observed gender wage gap conditional on hours.

[9]We allow hourly productivity to differ in the hour type. This is

3.2 Worker value functions

The Bellman equation for a non-employed worker of gender j is as follows

$$\begin{aligned} \mathcal{D}^n_j V^n_j(k) &= u^j\left(\mathcal{N}^n\left(j,k\right),0,k\right) + \phi_j(k) V^n_j(k') \\ &+\lambda \sum_{h\in\mathcal{H}} \int \Delta^h \max\left\{V^e_j\left(\left\{w,h\right\},k\right) - \mu(j,k,h), V^n_j\left(k\right)\right\} dF_h(w), \end{aligned} \tag{4}$$

where $k' = \min\{k+1, 3\}$ and $\mathcal{D}^n_j = \rho + \rho_d + \phi_j(k) + \lambda$. A non-employed worker with child state k experiences a child state transition at rate $\phi_j(k)$, in which case the child state becomes k'. The worker receives a job offer at rate λ, and the offer is drawn from the offer distribution $\{\Delta^h, F_h(w)\}$. Upon receiving an offer, the worker decides whether to accept it. The optimal job acceptance decision of a non-employed worker is denoted by $\Omega^n_j\left(\left\{w,h\right\};k\right)$ as follows:

$$\Omega^n_j\left(\left\{w,h\right\};k\right) = \begin{cases} 1 & \text{if } V^e_j\left(\left\{w,h\right\},k\right) - \mu(j,k,h) > V^n_j\left(k\right), \\ 0 & \text{else.} \end{cases} \tag{5}$$

The Bellman equation for an employed worker of gender j is as follows:

$$\begin{aligned} \mathcal{D}^e_j V^e_j\left(\left\{w,h\right\},k\right) &= u^j\left(\mathcal{N}^e(wa_j(h)h; j,k), h, k\right) + \delta(h) V^n_j(k) \\ &+\phi_j(k) \max\left\{V^e_j\left(\left\{w,h\right\},k'\right) - \mu(j,k,h), V^n_j(k')\right\} \\ &+\lambda \sum_{h'\in\mathcal{H}} \int \Delta^{h'} \max\left\{V^e_j\left(\left\{w,h\right\},k\right), V^e_j\left(\left\{w',h'\right\},k\right) - \mu(j,k,h)\right\} dF_{h'}(w'), \end{aligned} \tag{6}$$

where $k' = \min\{k+1, 3\}$ and $\mathcal{D}^e_j = \rho + \rho_d + \delta(h) + \phi_j(k) + \lambda$. The employed worker becomes non-employed exogenously at rate $\delta(h)$. At rate $\phi_j(k)$, the worker experiences a child state transition and decides to remain employed or quit. Eq. 5 also describes the optimal decision for remaining employed upon a child state transition. Finally, at rate λ, the worker draws a new job offer (w', h') and chooses to either accept it or reject it by remaining with the incumbent employer. The optimal job acceptance decision of a worker employed in job (w, h) who receives an offer (w', h') is

$$\Omega^e_j\left(\left\{w',h'\right\},\left\{w,h\right\};k\right) = \begin{cases} 1 & \text{if } V^e_j\left(\left\{w',h'\right\},k\right) - \mu(j,k,h) > V^e_j\left(\left\{w,h\right\},k\right), \\ 0 & \text{else.} \end{cases} \tag{7}$$

In solving the model numerically and estimating the model, we assume a preference shock associated with the decisions of job acceptance and remaining employed in the case of a change in the child state. The shock helps to smooth the labor supply function and we describe it in Appendix B.2.

3.3 Stationary distribution

In steady-state equilibrium, the distribution of workers is stationary. Let $g^{j,e}(w,h,k,x)$ denote the measure of gender j workers employed in job (w,h) with child state k, and let $g^{j,n}(k,x)$ denote the measure of gender j non-employed workers with child state k and demographic type x.

We solve for the stationary distribution by equating inflows to outflows in and out of each worker state. First, consider non-employment with child state k. Inflow into this state involves a change in the child state, entry of a new worker (only if $k=1$), or exogenous job separation:

$$\underbrace{\phi_j(k-1,x)\left[g^{j,n}\left(k-1,x\right)+\sum_{h'\in\mathcal{H}}\int\left[1-\Omega_j^n(\left\{w',h'\right\},k,x)\right]g^{j,e}\left(\left\{w',h'\right\},k-1,x\right)dw'\right]}_{\text{change in children state}}$$
$$+\underbrace{\rho_d N_j(x)\mathbf{1}_{\{k=1\}}}_{\text{new worker entrance}}+\underbrace{\sum_{h'\in\mathcal{H}}\delta(h')\int g^{j,e}\left(\left\{w',h'\right\},k,x\right)dw'}_{\text{exogenous job separation}},$$

where $N_j(x)$ is the measure of gender j workers with demographic type x, with $\sum_{j\in\{1,2\},x\in\mathcal{X}}N_j(x)=1$. To keep the distribution stationary, the measure of new workers entering the model must be equal to the measure of workers exiting the model.

Outflow from non-employment involves a change in the child state, model exit, or job finding:

$$g^{j,n}\left(k,x\right)\left[\underbrace{\phi_{j,x}(k)}_{\text{children state change}}+\underbrace{\rho_d}_{\text{exit}}+\underbrace{\lambda\sum_{h'\in\mathcal{H}}\Delta^{h'}\int\Omega_j^n\left(\left(w',h'\right);k,x\right)f_{h'}\left(w'\right)dw'}_{\text{job finding}}\right]$$

Then, consider employment with job (w,h) and child state k and demographic type x. Inflow into this state involves a change in the child state or job finding from non-employment and employment:

$$\underbrace{\phi_{j,x}(k-1)\left[\Omega_j^n(\{w,h\},k,x)g^{j,e}\left(\{w,h\},k-1,x\right)\right]}_{\text{change in child state}}+\underbrace{\Delta^h f_h\left(w\right)g^{j,n}\left(k,x\right)\lambda\Omega_j^n\left(\{w,h\};k,x\right)}_{\text{job finding from non-employment}}$$
$$+\underbrace{\Delta^h f_h\left(w\right)\lambda\sum_{h'\in\mathcal{H}}\int g^{j,e}\left(\{w',h'\},k,x\right)\Omega_j^e\left(\{w,h\},\{w',h'\};k,x\right)dw'}_{\text{job finding from employment}},$$

and outflow from this state involves a change in the child state, model exit, exogenous job separation, and job finding:

$$g^{j,e}\left(\{w,h\},k,x\right)\left[\underbrace{\phi_{j,x}(k)}_{\text{children state change}}+\underbrace{\rho_d}_{\text{exit}}+\underbrace{\delta(h)}_{\text{exogenous job separation}}\right.$$

$$\left.+\underbrace{\lambda\sum_{h'\in\mathcal{H}}\int\Delta^{h'}f_{h'}(w')\Omega_j^e\left(\{w',h'\},\{w,h\};k,x\right)dw'}_{\text{job finding}}\right]$$

3.4 Labor supply

The stationary distribution allows us to compute the labor supply function $l\left(w,h\right)$, which is the expected firm size for a firm that posts job (w,h). The labor supply is in terms of efficiency units of labor and is the ratio between the efficiency units of labor employed in job (w,h) and the measure of firms posting the job. Specifically, the labor supply is computed as follows:

$$l\left(w,h\right) = \sum_{j=1}^{2}\sum_{k=1}^{3}\sum_{x\in\mathcal{X}}\frac{a_j(h)g^{j,e}\left(w,h,k,x\right)}{\Delta^h f_h\left(w\right)}. \tag{8}$$

3.5 Firms

Firms are risk neutral and exogenously heterogeneous in productivity p, which is drawn from the distribution $\Gamma\left(\cdot\right)$ with the corresponding PDF $\gamma\left(\cdot\right)$ on the support $\left[\underline{p},\bar{p}\right]\subset(0,\infty)$. Firms also face heterogeneous job-posting costs, which we will explain later.

The flow output per job is given by the following production function

$$y\left(p,h\right)=\theta_h ph, \tag{9}$$

where θ_h is the hourly productivity shifter. We allow hourly labor productivity to differ between jobs with different hour requirements to reflect potential sunk management costs.[10]

[10]That is, given the same amount of working hours, employing fewer workers who each works longer hours can likely result in a lower management cost.

3.5.1 Firm strategy

Firms take as given the labor supply function (Eq. 8) and job-posting costs; they choose (w, h) to maximize the flow profit net of the job-posting cost. We can break down the firm problem into two steps. In the first step, firms choose the optimal hour requirement h. In the second step, firms choose the optimal hourly wages conditional on h. We solve the firms' problem backward from the second step.

Given productivity p and hour requirement h, the expected flow profit of posting w is

$$\pi_h(w;p) = [y(p,h) - wh]\, l(w,h)\,. \tag{10}$$

We can solve for $w_h(p)$, the optimal wage strategy, by maximizing Eq. 10 with respect to w. The maximized expected flow profit can be written as

$$\pi_h(p) = [y(p,h) - w_h(p)\,h]\, l(w_h(p), h)\,. \tag{11}$$

Knowing the expected profit of posting jobs with each hour requirement h, firms choose h that maximizes the expected profit net of job posting costs. Let $h(p, \boldsymbol{\varepsilon}) \in \mathcal{H}$ be the optimal choice of job-posting for a firm with productivity p and job posting costs $\boldsymbol{\varepsilon}$. We have

$$h(p, \boldsymbol{\varepsilon}) = \arg\max_{h\in\mathcal{H}} \{\pi_h(p) - (\bar{\epsilon}_h - \varepsilon_h)\}\,, \tag{12}$$

where $\bar{\epsilon}_h$ is the common component of the job-posting cost and $\boldsymbol{\varepsilon} \equiv \{\varepsilon_h, h \in \mathcal{H}\}$ is the idiosyncratic component. The idiosyncratic job-posting cost implies that the hour-requirement distribution conditional on p is non-degenerate. We assume that $\boldsymbol{\varepsilon}$ follows the type-I extreme value distribution with scale parameter σ_ε. The job posting choice follows a logit model. That is, conditional on p, the proportion of firms choosing hour requirement h is given by $\Delta(h;p)$, which can be expressed as:

$$\Delta(h;p) = \frac{\exp[(\pi_h(p) - \bar{\epsilon}_h)/\sigma_\varepsilon]}{\sum_{h'\in\mathcal{H}} \exp[(\pi_{h'}(p) - \bar{\epsilon}_h)/\sigma_\varepsilon]}. \tag{13}$$

We can construct the offer distribution $\{F_h(w), \Delta^h\}_{h\in\mathcal{H}}$ using the optimal wage strategy $w_h(p)$ and hour requirement choice probability $\Delta(h;p)$ as follows. For each $h \in \mathcal{H}$,

$$F_h(w) = \frac{\int_{\underline{p}}^{(w_h)^{-1}(w)} \Delta(h;p)\, d\Gamma(p)}{\Delta^h}, \tag{14}$$

$$\Delta^h = \int_{\underline{p}}^{\bar{p}} \Delta(h;p)\, d\Gamma(p)\,, \tag{15}$$

where $(w_h)^{-1}(\cdot)$ is the inverse function of the wage strategy $w_h(p)$. In equilibrium without any minimum wage, we have that $\sum_{h\in\mathcal{H}} \Delta^h = 1$.

3.5.2 Correlation between posted wages and hours

Our model can generate the observed part-time penalty via two potential mechanisms that give rise to a positive correlation between the hour requirement and posted wages.[11] First, if the estimated hourly productivity shifter θ_h is higher for full-time jobs than for marginal or part-time jobs, firms in equilibrium will likely post higher wages if they choose full-time requirement. Second, Eq. 9 implies production complementarity between productivity p and hour requirement h. In other words, the hourly productivity increases in hours more for high-p firms than low-p firms. The complementarity can lead to positive sorting between p and h, resulting in a pattern that more productive firms are more likely to post jobs with longer hours. Since more productive firms also post higher hourly wages (Burdett and Mortensen, 1998), the complementarity can also lead to a positive correlation between hourly wages and hours.

3.6 Steady state equilibrium

Definition 1. A steady state equilibrium is defined by the offer distribution $\{F_h(\cdot), \Delta^h\}_{h\in\mathcal{H}}$ such that

i. Given the offer distribution, workers make optimal job acceptance and quitting decitions.

ii. The distribution of workers is stationary.

iii. Given the distribution of workers, firms make optimal hour requirement choice and wage policy $w_h(p)$ and $\Delta(h;p)$.

iv. The wage distribution over job offers with hour requirement h is given by Eq. 14 and the measure of offers with hour requirement h is given by Eq. 15.

The equilibrium can be solved numerically by iterating on $\{F_h(\cdot), \Delta^h\}_{h\in\mathcal{H}}$. Details of the numerical solution are given in Appendix B.1.

[11]The empirical phenomenon of a part-time penalty is documented by Aaronson and French (2004), which states that longer hours of work leads to higher hourly wages. Erosa et al. (2016) assume a positive relationship between hourly wage and hours in the offer distribution of their theory of labor supply. Devicienti et al. (2017) show that a more significant fraction of part-time workers in firms is linked to lower firm productivity. See also Calvo et al. (2021).

3.7 Implementing minimum wages

Using data from the German Socio-Economic Panel (SOEP), Burauel et al. (2017) and Burauel et al. (2020) document the prevalence of non-compliance with the German minimum wage legislation. Their findings reveal that in 2016, one year after the introduction of the federal minimum wage, approximately 1.8 million eligible employees were still receiving hourly wages below the 8.5 € threshold.[12] In addition, based on administrative data, Bossler et al. (2022) confirm the existence of widespread non-compliance. They attribute this non-compliance to a lack of resources allocated for the enforcement of the German minimum wage.[13]

The association between low-hour jobs and low hourly wages makes low-hour jobs more susceptible to non-compliance issues following the implementation of the minimum wage. Burauel et al. (2017) find that in 2016, over 40% of marginal employment jobs were non-compliant with the minimum wage, whereas less than 10% of full-time jobs were non-compliant.

To account for the prevalence of non-compliance and the disparity across job types, we incorporate the minimum wage into our model with a penalty mechanism. We assume that firms have the ability to post job offers with wages below the minimum wage. However, workers encounter such offers at a reduced contact rate, reflecting the notion that firms may not be able to openly recruit for non-compliant positions through standard advertising channels. Specifically, we assume that the job contact rate is multiplied by a factor $\max\{0, 1 - \kappa(wa_j(h); w_{min})\} \leq 1$ where

$$\kappa(wa_j(h); w_{min}) = \begin{cases} \kappa_0 \left(w_{min} - wa_j(h)\right)^2 & \text{if } wa_j(h) < w_{min}, \\ 0 & \text{else.} \end{cases} \tag{16}$$

We calibrate the penalty parameter κ_0 to match the observed change in the share of full-time jobs paying less than 8.5 € after the introduction of the minimum wage.

When we impose the minimum wage in the model, we allow firms to become inactive by not posting any jobs. Inactivity is associated with zero profit. We modify the firm problem as follows. Before making the optimal job-posting decisions, each firm draws preference shocks $\boldsymbol{\varepsilon}_{act} = \{\varepsilon_{act}, \varepsilon_{inact}\}$ from a type-I extreme value distribution with scale parameter σ_ε and makes the decision of whether to be active:

$$\max\{\varepsilon_{inact}, \bar{\pi}(p) + \varepsilon_{act}\}.$$

[12] Even after accounting for potential measurement errors, the level of non-compliance remains significant (Burauel et al., 2017)

[13] According to Bossler et al. (2022), the German government did not allocate additional personnel to the relevant enforcement agency when the federal minimum wage was introduced in 2015. Furthermore, in 660 out of 733 industries, employers are not legally obliged to record the working hours of their employees, making it challenging for the enforcement agency to provide evidence of non-compliance. The enforcement of the minimum wage relies on random audits; however, the actual audit rate is reported to be less than 5% even in low-wage industries where the obligation to record hours exists.

The value associated with inactivity is ε_{inact}, whereas the value associated with activity is $\bar{\pi}(p) + \varepsilon_{act}$, with $\bar{\pi}(p)$ being the expected profit from posting a job net of job posting costs for a firm with productivity p. We can write $\bar{\pi}(p)$ as follows.

$$\bar{\pi}(p) \equiv \mathbb{E}_{\boldsymbol{\varepsilon}} \max_{h\in\mathcal{H}} \left\{\pi_h\left(p\right) - \left(\bar{\epsilon}_h - \varepsilon_h\right)\right\},$$

where $\boldsymbol{\varepsilon}$ is defined in Sec. 3.5.1 as the vector of job-posting costs. The probability that a firm with productivity p chooses to be active is

$$\Delta^{act}\left(p\right) = \frac{\exp\left[\bar{\pi}(p)/\sigma_{\varepsilon}\right]}{1 + \exp\left[\bar{\pi}(p)/\sigma_{\varepsilon}\right]}. \tag{17}$$

Active firms proceed to make the hour requirement choice and hourly wage choice as described in Section 3.5. Eq. 13 becomes

$$\Delta\left(h;p\right) = \Delta^{act}(p)\ \frac{\exp\left[\left(\pi_h\left(p\right) - \bar{\epsilon}_h\right)/\sigma_{\varepsilon}\right]}{\sum_{h'\in\mathcal{H}} \exp\left[\left(\pi_{h'}\left(p\right) - \bar{\epsilon}_h\right)/\sigma_{\varepsilon}\right]}\ , \tag{18}$$

which gives the fraction of productivity-p firms choosing hour requirement h.

The measure of job offers with hour requirement h becomes

$$\Delta^h = \int_{\underline{p}}^{\bar{p}} \Delta\left(h;p\right) d\Gamma\left(p\right), \tag{19}$$

and the total measure of job offers is

$$\Delta^{act} = \sum_{h\in\mathcal{H}} \Delta^h \leq 1. \tag{20}$$

The rate that workers contact a job offer (w,h) is

$$\lambda \max\{0, 1 - \kappa(wa_j(h); w_{min})\}\Delta^h f_h(w).$$

4 Estimation strategy and model fit

We estimate the model to match the German labor market prior to the introduction of the first federal minimum wage in 2015. Some model parameters can be externally calibrated to values computed directly from the data, while others are internally estimated via a two-step method. One unit of time corresponds to a year.

4.1 Data

Our data source is the Core Study of the German Socio-Economic Panel (SOEP). SOEP is a representative household survey conducted annually with topics including household composition, education, employment biographies and earnings. We estimate the model based on the time period 2013-2014 and a sample of civilians between ages 25 and 54 who are not in school or retirement. Since the minimum wage mainly affects workers without a tertiary education, we consider only those with up to 12 years of education. We further exclude individuals who have never worked from our sample because our model concerns only individuals that are active or potential labor market participants. In the SOEP data, we not only observe the type of employment (e.g. full-time, part-time, or marginal employment) but also the weekly hours of work. We focus on dependent employment in the private sector (i.e., we exclude civil service and self-employment) and use contractual hours to compute hourly wages. "Singles" in our data refer to all individuals who are not married, including cohabiting couples, because unmarried people are treated like singles for tax purposes.

4.2 External calibration

Table 5 shows the values of externally determined parameters. We assume the discount rate ρ to be 4%. ρ_d is the rate at which workers exit the model. Since we consider workers between ages 25 and 54, we set the exit rate to be $1/30$. The utility of leisure in Eq. 3 is expressed in terms of weekly hours. We assume that $\bar{h}$, the maximum hours per week, is 80. The hours of each type of employment corresponds to the median contractual hours observed in our data. The median weekly hours of a full-time job is 40, while the weekly hours of a part-time and marginal-employment job are 25 and 11 hours, respectively.

Child state transition rates The state of "young children" refers to having at least one child age 6 or younger. We choose this age cut-off because mothers' labor supply is most affected in the first 6 years after childbirth (Keller and Kahle, 2018; Turon, 2022). We estimate the child state transition rates ($\phi_{j,x}(k)$) directly from the observed yearly child state transition probabilities in SOEP. We assume different transition processes for single men, single women, and married couples. Couples have the highest transition rate to the state of young children. They have the lowest transition rate out of the state of young children because they have more children on average compared to singles.

Marital and spousal income distribution We assume six demographic types. $x = 1$ indicates that the worker is single, and $x > 1$ indicates that the worker is married. We approximate the spousal income distribution with a five-point discrete distribution. Spousal income levels $\{\mathcal{X}(x)\}_{x=2,3,\ldots,6}$ are the 10th, 30th, 50th, 70th, and 90th percentiles of the empirical distribution of the annual income of married individuals in our sample (regardless of gender), where the income includes gross labor incomes and unemployment benefits. We group husbands and wives into income bins, with each bin corresponding to an income quintile among married people, and calculate the fraction of husbands or wives in each income bin.

Figure 2 shows the distribution of x, which derives from the distribution over marital status and spousal income bins. For both genders, the probability of having $x = 1$ is equal to 43%, the fraction of singles in our sample. The probability of having an $x > 1$ equals the probability of being married times the probability of having a spouse in the income bin $x - 1$. For example, for women, the probability of drawing $x = 2$ is equal to the probability of being married times the probability of having a husband in the lowest (first) income bin. We can see that married men are more likely to have a low-income wife while married women are more likely to have a high-income husband.

Tax and benefits Single workers ($x = 1$) consume their own after-tax income, and married workers ($x > 1$) pool their income with their spouse and consume half of the household after-tax income. We parametrize the income tax function such that the net-of-tax income is log-linear in gross income (Heathcote et al., 2017). The net-of-tax function for employed workers is

$$\mathcal{N}^e(y; j, k, x) = \begin{cases} \tau_{0,k,x} y^{1-\tau_{1,k,x}} & \text{if } x = 1, \\ \frac{1}{2}\tau_{0,k,x}(y + \mathcal{X}(x))^{1-\tau_{1,k,x}} & \text{else,} \end{cases}$$

where $\mathcal{X}(x)$ is the spousal income of a married worker with type x. The net-of-tax function for non-employed workers is

$$\mathcal{N}^n(j, k, x) = \begin{cases} b_{j,x} + b_j^k & \text{if } x = 1, \\ \frac{1}{2}\left[b_{j,x} + b_j^k + \tau_{0,k,x}(\mathcal{X}(x))^{1-\tau_{1,k,x}}\right] & \text{else,} \end{cases}$$

where $b_{j,x}$ is non-employment benefit and b_j^k is parental benefit. Non-employment and parental benefits are not subject to the income tax.

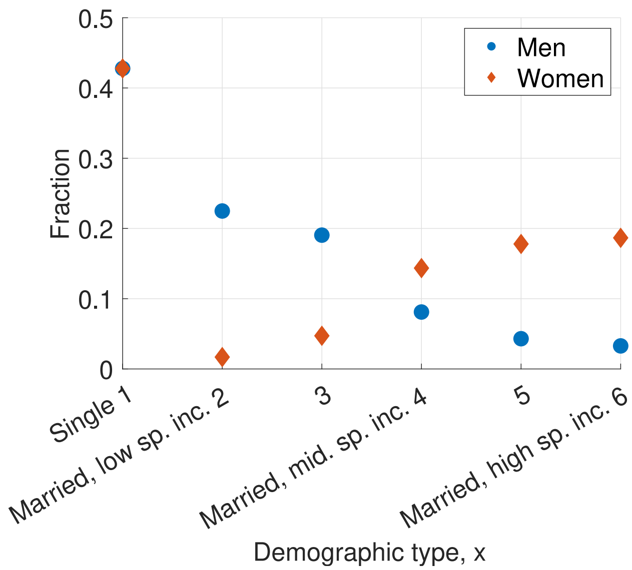
We estimate the parameters of the income tax function using data from SOEP. In particular, we estimate an OLS regression of log annual household post-government income ($\log y^{post}$) on log annual household pre-government income ($\log y^{pre}$):

$$\log y^{post} = \log \tau_{0,k,x} + (1 - \tau_{1,k,x}) \log y^{pre}.$$

The post-government income accounts for all household taxes and transfers except unemployment benefits and parental benefits. We estimate the regression separately for singles and couples of each child state.

$b_{j,x}$ is calibrated to the average unemployment benefit for each gender and marital status. We assume that only workers with young children receive parental benefit b_j^k, which is calibrated to the average data counterpart for each gender.

Figure 2: Distribution of the demographic type x



Note: The figure shows the frequency distribution of the demographic type x. $x = 1$ corresponds to the state of being single. $x > 1$ corresponds to the state of being married to a spouse in income bin $x - 1$. For example, women with $x = 2$ are married to husbands in the lowest (first) income bin. See Table 5 for the spousal income levels.

4.3 Two-step estimation

The remaining parameters are estimated jointly via a two-step estimation procedure. The two-step procedure has the advantage of avoiding solving the model equilibrium repeatedly, which is highly costly computationally.[14] In the first step, we estimate parameters governing households' labor supply. We parametrize the wage offer distributions $\{F_h\}_{h \in \mathcal{H}}$ with Beta distributions and estimate the parameters of the offer distributions, preference parameters, and labor market frictions to match a set of moments. In the second step, we take as given labor supply from the first step and estimate parameters governing firm decisions, including parameters of the production function and the distribution of job-posting costs, and the firm productivity distribution.

[14]See Bontemps et al. (2000), Meghir et al. (2015) and Fang and Shephard (2019).

Table 5: External parameters

Parameter	Value
Discount rate, ρ	0.040
Exit rate, ρ_d	0.033
Child transition rate, $\phi_{j,x}(k)$:	
No children to young children, single men	0.012
No children to young children, single women	0.044
No children to young children, married	0.069
Young children to grown children, single men	0.224
Young children to grown children, single men	0.198
Young children to grown children, married	0.149
Hours grid, $\mathcal{H}$:	
Weekly hours of ME	11
Weekly hours of PT	25
Weekly hours of FT	40
Spousal income grid (monthly), $\mathcal{X}$:	
$x = 2$ (married, lowest spousal income)	409.2
$x = 3$	1341.7
$x = 4$	2200.0
$x = 5$	2950.0
$x = 6$ (married, highest spousal income)	4308.3
Social security tax rate, τ_s	0.093
Tax function parameters, τ_0, τ_1 :	
Single, no children	3.8, 0.14
Single, young children	156.0, 0.50
Single, grown children	133.0, 0.48
Married, no children	4.8, 0.16
Married, young children	101.5, 0.44
Married, grown children	59.1, 0.39
Non-employment benefit (monthly), $b_{j,x}$:	
Men, single	849.5
Women, single	772.1
Men, married	697.1
Women, married	209.9
Parental benefit (monthly), b_j^k :	
Men	13.9
Women	102.0

Note: See Sec. 4.2 for details. Monetary values are in 2013 Euros.

4.3.1 Step 1: Estimation of supply-side parameters

We estimate supply side parameters to match moments of the observed distribution over wage and hours and labor force transition rates. We compute the model counterpart of the moments of the wage and hours distribution from the stationary distribution, which we obtain by iterating on the distribution functions $g^{j,e}$ and $g^{j,n}$. To compute the model counterpart of the labor force transition rates, we simulate a panel of workers based on our model. While the supply-side parameters are jointly estimated, we discuss how each parameter can be identified below.

For each $h \in \mathcal{H}$, we parametrize the wage offer distributions F_h with a General Beta distribution with parameters $\{\alpha_{F_h}, \beta_{F_h}\}$ and support $[\underline{w}_h, \overline{w}_h]$. We set the support for each h to be sufficiently wide such that we can replicate the observed hourly wage distribution. The parameters $\{\alpha_{F_h}, \beta_{F_h}\}$ determine the shape of the wage offer distribution and influence the observed hourly wage distribution.

Indeed, labor efficiency $a_j(h)$ also influences the hourly wage distribution. Recall that we normalize labor efficiency of men $a_1(h) = 1$ for all $h \in \mathcal{H}$. Labor efficiency of women $a_2(h)$ captures the gender difference in hourly labor productivity conditional on hours h. It determines the within-firm gender wage gap conditional on h, which in turn influences the overall gender wage gap conditional on h. In the estimation, we target the 25th, 50th, and 75th percentiles of the hourly wage distribution for each hours-type and each gender.

In estimation, there is no minimum wage policy and we do not allow firms to be inactive. Thus, we have $\sum_{h \in \mathcal{H}} \Delta^h = 1$, and $\{\Delta^h\}_{h \in \mathcal{H}}$ can be identified from the observed hours distribution. The offer arrival rate λ is identified from the employment rate, and the exogenous job separation rates $\{\delta(h)\}_{h \in \mathcal{H}}$ are identified from the transition rate from each type of employment to non-employment.[15]

Finally, we turn to preference parameters. The average ψ cannot be separately identified from the offer arrival rate λ because both a higher average ψ and a lower λ lead to a lower employment rate. Without loss of generality, we normalize $\psi_{k=1}^{j=1} = 1$. Moreover, we set $\gamma_{k=1}^{j=1} = 2$, corresponding to an intertemporal elasticity of labor supply of 0.5 when working full-time.[16] The rest of preference parameters, including ψ_k^j and γ_k^j for men with $k = 2, 3$ and women with $k = 1, 2, 3$, are estimated to match the hours distribution by gender and child state. In the data, women with young and grown children have a non-monotone employment pattern over hours, with the part-time rate being substantially higher than both marginal employment and full-time rates. To replicate this feature, we allow the disutility $\mu(j, k, h = FT)$ to be positive for women with young or grown children. We set the distaste to zero in all other cases since allowing the parameter to vary does not significantly improve the fit of the model.

[15] The exogenous separation rates ($\{\delta(h)\}_{h \in \mathcal{H}}$) cannot be externally calibrated from the observed job separation rates because workers can quit their jobs upon a child state transition in our model.

[16] We borrow the value from Erosa et al. (2016), who have a similar utility function specification as ours. Chetty et al. (2011) find the Frisch labor supply elasticity to be 0.54 in their meta-study. We do not estimate γ_1^1 internally because we cannot separately identify it from $\{\Delta^h, h \in \mathcal{H}\}$ without exogenous variations that affect their labor supply.

4.3.2 Step 2: Estimation of demand-side parameters

Demand-side parameters to be estimated include the hourly productivity shifters $\{\theta_h\}_{h\in\mathcal{H}}$, the firm productivity distribution and parameters of firm fixed cost distributions $(\sigma_\varepsilon, \bar{\epsilon}_{ME}, \bar{\epsilon}_{PT})$.

Since a shift in the average firm productivity p is observationally equivalent to a shift in the average θ_h, we normalize $\theta_{FT} = 1$ and estimate θ_{ME}, θ_{PT}. Moreover, we set the average job-posting cost of full-time jobs $\bar{\epsilon}_{FT}$ to zero and estimate $\bar{\epsilon}_{ME}$ and $\bar{\epsilon}_{PT}$, which capture the difference in job-posting cost between different hours-types.[17]

To estimate the demand-side parameters, we follow Fang and Shephard (2019) and construct a theoretical hour requirement choice function $\Delta(h;p)$ and an observed one $\widehat{\Delta}(h;p)$. We find parameters that minimize the distance between the two. The theoretical choice is derived from firms' optimal job-posting decisions given estimated labor supply from step 1, while the observed choice is the one that is consistent with the estimated offer distribution from step 1. In other words, we look for demand-side parameters such that, when firms make optimal job-posting decisions, the resulting offer distribution is the one uncovered in step 1.

We construct the two versions of the hour requirement choice function as follows. First, we compute labor supply $l(w,h)$ using Eq. 8 given the stationary distributions and the estimated offer distribution from Step 1. This allows us to compute the inverse wage function for each $h \in \mathcal{H}$ using the first order condition of firms' wage-posting problem.

$$p \equiv w_h^{-1}(w) = \frac{l(w,h) + w\frac{\partial l(w,h)}{\partial w}}{\theta_h \frac{\partial l(w,h)}{\partial w}} \tag{21}$$

We verify in numerical solutions that the inverse wage function is monotonically increasing. Given the wage function, we can compute the expected profit associated with posting jobs with each hour requirement h using Eq. 11. Then, we can compute the "theoretical" choice function $\Delta(h;p)$ using Eq. 13. We refer to this as the theoretical choice because the choice is optimal given the labor supply function $l(w,h)$ and demand-side parameters.

Next, we construct the "observed" choice function $\widehat{\Delta}(h;p)$. To do so, we note that the offer distribution that derives from $\widehat{\Delta}(h;p)$ is

$$F_h(w_h(p)) = \frac{\int_{\underline{p}}^{p} \widehat{\Delta}(h;x)\, d\Gamma(x)}{\Delta^h}. \tag{22}$$

[17] In our model, the job-posting cost is a flow cost for each job opening. The data counterpart is the recruitment cost divided by the expected duration of the filled position. Using the German Job Vacancy Survey 2014 and 2015, Carbonero and Gartner (2022) find the average cost of recruitment per job opening (full-time or part-time) is 1576 €. The average job tenure is 10.5 years in Germany in 2014 (OECD.stat). Thus, the flow recruitment cost is only 150 € per year of job tenure, a negligible amount compared to the expected profit from a position.

Differentiating Eq. 22 with respect to p and rearranging, we have

$$\widehat{\Delta}(h;p) = \frac{\Delta^h f_h(w_h(p))\, w_h'(p)}{\gamma(p)}. \tag{23}$$

To obtain $\gamma(p)$, the probability density of the firm productivity distribution, we sum both sides of Eq. 23 over h and use the fact that $\sum_{h\in\mathcal{H}} \widehat{\Delta}(h;p) = 1$. This gives

$$\gamma(p) = \sum_{h\in\mathcal{H}} \Delta^h f_h(w_h(p))\, w_h'(p). \tag{24}$$

4.4 Parameters and model fit

Table 6 shows the parameter estimates. On the labor supply-side, the estimated offer distribution indicates that 85% of job offers are full-time while only 3% are marginal-employment. The exogenous job separation rate of marginal employment is 15 times as high as a full-time job and 5 times as high as a part-time job. The estimated γ_k^j for women is significantly higher than for men. Among women, those with children have a higher ψ_k^j, the scale parameter on the utility from leisure, and a substantial distaste for working full-time.

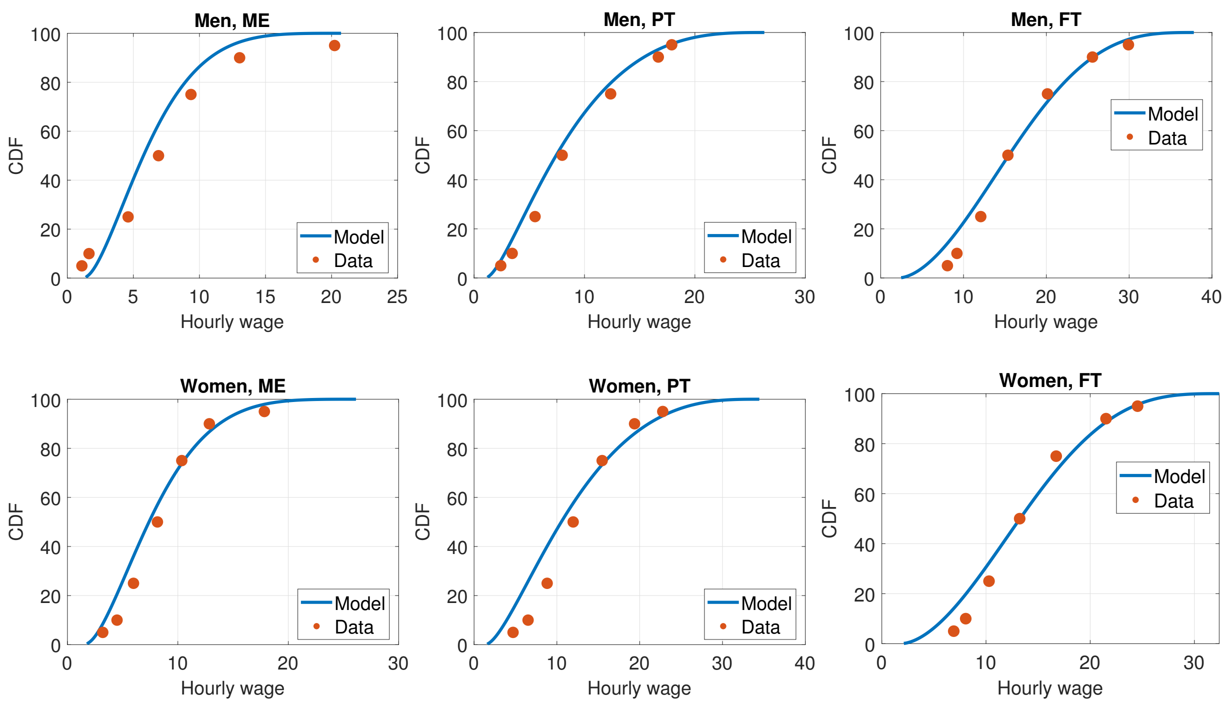
Table 7 shows the fit of targeted moments. The heterogeneity in preferences allows us to match several key employment pattern across demographic groups. Women, especially those with young children, have a lower employment rate than men, where employment rate is the sum of marginal, part-time, and full-time employment rates. Among women, those with children are more likely to hold a marginal-employment or part-time job and less likely to hold a full-time job than those without children.

Our model also performs well in matching the observed hourly wage distribution and the pattern in job separation probabilities. Fig. 3 shows the fit of the observed hourly wage distribution. We replicate the gender hourly-wage gap in each type of jobs: while the female hourly wage distribution in full-time jobs is first-order dominated by the male distribution, the reverse is true for marginal-employment and part-time jobs.

The gender differences in the estimated labor efficiency ($a_j(h)$) are crucial for matching the gender hourly-wage gaps. Our estimation reveals that women are more efficient in marginal-employment and part-time jobs than men, but less efficient in full-time jobs. Without the gender difference in labor efficiency, women, who have a stronger dislike for working full-time compared to men, would set higher reservation wages, resulting in counterfactually higher hourly wages than men. The estimated $a_j(h)$ for women in full-time jobs is less than 1 (the value for men). As a result, women make lower hourly wages in full-time jobs despite their time preference.

On the demand-side, we find that the hourly labor productivity in marginal employment is about 65% of that in full-time jobs. There is also non-monotonicity: the hourly labor productivity in part-time jobs is lower than that in marginal jobs. The low hourly productivity in part-time jobs is identified from the fact that, even though labor supply to part-time jobs is high, both the share of job offers that are part-time and wages of part-time jobs are low. Finally, we find that the job-posting cost is decreasing in hours. Even though the total recruitment cost for a marginal-employment or part-time position may be lower than a full-time position, the expected job-tenure also increases in hours. Our estimate of the job-posting cost suggests that the average recruitment cost per year of job tenure is the lowest among full-time jobs.

Figure 3: Model fit of hourly wage distributions



Note: Red circles represent the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the empirical hourly distribution based on SOEP. Of these, the 25th, 50th, and 75th wage percentiles are targeted in the estimation. Solid lines represent the hourly wage distributions in the estimated model.

Table 6: Estimated parameters

Parameter		value
Supply side:		
Offer distrib. parameter, α_{F_h}	ME	1.461
	PT	1.615
	FT	2.089
Offer distrib. parameter, β_{F_h}	ME	3.806
	PT	3.466
	FT	3.310
Offer distrib. parameter, Δ^h	ME	0.033
	PT	0.120
	FT	0.847
Contact rate, λ		2.200
Exog. separation rate, $\delta(h)$	ME	0.150
	PT	0.030
	FT	0.010
Scale parameter in utility function,ψ_k^j	male, young children	0.057
	male, grown children	0.363
	female, no children	0.847
	female, young children	1.814
	female, grown children	1.675
Exponent parameter in utility function,γ_k^j	male, young children	2.636
	male, grown children	1.159
	female, no children	4.665
	female, young children	5.044
	female, grown children	4.465
Distaste, $\mu_\alpha(j, k, FT)$	female, young children	0.797
	female, grown children	0.668
Labor efficiency, $a_j(h)$	female, ME	1.261
	female, PT	1.310
	female, FT	0.856
Demand side:		
Production function param., θ_h	ME	0.649
	PT	0.386
Job-posting cost (monthly), $\bar{\epsilon}_h$	ME	2477.5
	PT	913.5
Job-posting cost (monthly), σ_ε		1299.1
Minimum wage implementation:		
Non-compliance penalty, κ_0		0.007

Table 7: Model fit

Moment	**Men**		**Women**	
	Data	Model	Data	Model
Employment distribution:				
No children				
ME	0.031	0.024	0.061	0.040
PT	0.072	0.096	0.213	0.152
FT	0.756	0.718	0.608	0.588
Young children				
ME	0.028	0.024	0.153	0.128
PT	0.055	0.103	0.318	0.322
FT	0.809	0.842	0.111	0.106
Grown children				
ME	0.023	0.024	0.129	0.127
PT	0.027	0.105	0.417	0.330
FT	0.857	0.849	0.288	0.271
Hourly wage:				
ME				
P25	4.603	3.834	5.984	4.923
P50	6.904	5.763	8.151	7.389
P75	9.359	8.245	10.356	10.539
PT				
P25	5.523	4.487	8.826	6.396
P50	7.978	7.444	11.967	10.534
P75	12.370	11.424	15.465	15.902
FT				
P25	12.082	10.519	10.289	9.016
P50	15.362	15.473	13.233	13.263
P75	20.137	20.925	16.737	17.934
Job separation prob. (monthly, both genders):				
ME-to-NE	0.022	0.019		
PT-to-NE	0.007	0.004		
FT-to-NE	0.006	0.003		

5 Equilibrium effects of the minimum wage

In this section, we examine the effects of minimum wage policies using our estimated model. The German federal minimum wage was first introduced in 2015 at 8.5 € per hour. In 2022, the minimum wage reaches 10 € (after adjusting for inflation). As described in Sec. 3.7, we implement the minimum wage in our model via a penalty on the rate that workers contact non-compliant jobs. We calibrate the penalty parameter κ_0 (in Eq. 16) to match the drop in the share of full-time jobs with an hourly wage below 8.5 € (bottom panel, Table 6). Using our SOEP sample, we find a 3.93 percentage point drop in this share from our baseline period (2013-2014) to 2016. In the model, the minimum wage of 8.5 € results in a 4.04 percentage point drop.

5.1 Gendered effects

In response to the minimum wage, firms in our model adapt their job-posting decisions by making adjustments to wages, hour requirements, and their participation in job-posting, while workers adjust their decisions regarding job acceptance and quitting. Theoretical predictions regarding the impact of the minimum wage on employment rates are ambiguous. On one hand, the minimum wage can lead to a reduction in employment due to the penalty imposed on non-compliant jobs and the potential response of firms to become inactive. On the other hand, the minimum wage can have a positive effect on employment. The policy may induce firms to increase their posted wages, leading workers to accept a larger share of job offers. Additionally, workers may be incentivized to work in jobs with longer hours, as lower-hour jobs are more affected by the minimum wage, and firms may choose to raise hour requirements. Since jobs with longer hours have lower job separation rates, this shift towards longer hour types can contribute to an overall increase in the employment rate.

Gender asymmetric effects on employment and hours We examine the impact of three distinct minimum wage levels in our estimated model: the initial minimum wage of 8.5 €, the minimum wage in 2022 of 10 €, and a potential minimum wage of 11 €. Fig. 4 shows the effects of these minimum wage levels on the employment and non-employment rates, disaggregated by gender. Our findings reveal that the minimum wage has a limited effect on the non-employment rate of men, whereas it progressively reduces the non-employment rate of women. Additionally, the minimum wage reallocates both men and women toward jobs with longer hours. However, gender differences emerge in this reallocation pattern. Specifically, the minimum wage decreases both marginal and part-time employment rates for men while increasing their full-time employment rate. In contrast, the minimum wage primarily reduces the marginal-employment rate for women while predominantly increasing their part-time employment rate. Notably, at the higher minimum wage levels of 10 € and 11 €, we observe an additional increase in the full-time employment rate of women.

The gender asymmetry in the effect of the minimum wage stems from the fact that women are more likely to work in jobs with low hours and low hourly wages before the implementation of the minimum wage, making them more susceptible to the impact of the minimum wage. This, in turn, is driven by gender differences in preferences for working hours. For women with children, the distaste for full-time employment further shapes the reallocation effect, limiting the impact to mainly reallocating from marginal to part-time jobs.

Gender gaps The gender income gap, which represents disparities in gross labor earnings between men and women, can be attributed to three distinct gender gaps: employment, working hours (conditional on employment), and hourly wages. Given the gender-asymmetric effects of the minimum wage on employment and hours presented above, there is potential for the minimum wage to reduce the gender employment and hours gaps. To analyze the impact of the minimum wage on gender disparities, Table 8 shows gender gaps in economies with no minimum wage and with different minimum wage levels. We define a gender gap in a specific variable as the ratio between the corresponding value for females and that for males, with a value of 1 indicating gender equity.

Without a minimum wage, our model shows that women earn 36.0% of the gross labor income of men on average (as shown in panel A column 1 of Table 8).[18] The introduction of a minimum wage set at 8.5 € slightly increases this fraction to 36.9% (panel A column 2), a modest reduction in the gender income gap. Most of this reduction (90.9%) is due to the narrower gender wage gap (panel B column 2) and it is because women, who tend to be more likely to hold marginal employment jobs and earn lower hourly wages, benefit more from increases in posted wages.[19]

The minimum wage set at 10 € and 11 € (panel A columns 3 and 4) leads to significant reductions in also the gender employment and hours gaps. The disproportionate reallocation of women toward jobs with longer hours and lower separation rates explains the reduction in these gender gaps, which contributes to a further reduction in the gender income gap. When the minimum wage is set to 11 €, the gender income gap reduces to 0.385, with 24.8% of the reduction coming from the employment channel and 5.7% from the hours channel (panel B column 4).

[18]The gender income gap is unconditional on employment or hours and does not account for non-labor incomes such as non-employment benefits.

[19]We present details of the decomposition in Appendix C.

Figure 4: Effects on Employment Rates

(a) Men

(b) Women

Note: Bars represent equilibrium effects of minimum wages on rates of marginal employment (ME), part-time employment (PT), full-time employment (FT) and non-employment (NE).

Table 8: Effects on Gender Gaps

	(1)	(2)	(3)	(4)
Min. wage =	none	8.5	10	11
A. Gender gaps				
Gender income gap	0.360	0.369	0.378	0.385
Gender employment gap	0.626	0.627	0.632	0.636
Gender hours gap	0.717	0.717	0.719	0.720
Gender wage gap	0.802	0.820	0.831	0.841
B. Decomposition of minimum wage effects				
Gender income gap (%)	-	100	100	100
Gender employment gap (%)	-	8.0	21.2	24.8
Gender hours gap (%)	-	1.1	4.5	5.7
Gender wage gap (%)	-	90.9	74.4	69.6

Note: *Panel A* shows gender gaps in the benchmark (no minimum wage) economy and in economies with minimum wages. The gender gap in a variable is the ratio between the corresponding value for females and that for males, with a value of 1 indicating gender equity. Income refers to gross labor income, excluding non-employment benefits, and wage refers to hourly wage.
Panel B shows the decomposition of minimum wage effects on the gender income gap into employment, hours, and wage channels. See Appendix C for details of the decomposition.

5.2 Firm response to minimum wages

We use our model to analyze how firms adjust to minimum wages and the their impact on equilibrium outcomes. In response to the implementation of a minimum wage, firms have three main dimensions of adjustment: adjusting the wage rate, changing the hour requirement, and deciding whether to participate in job-posting or become inactive.

Firm adjustments Let us begin by examining the wage decision. The penalty imposed on the contact rate for non-compliant jobs restricts the available labor supply for such positions, compelling firms to increase the wages they offer. To illustrate this, we present Fig. 5, which compares the equilibrium distribution of offered wages between the benchmark economy (without a minimum wage) and the economy with an 8.5 € minimum wage.[20] Notably, hourly wages of marginal-employment job offers increase the most because these job offers are associated with the lowest hourly wages prior to the minimum wage policy. Specifically, the minimum wage policy leads to wage increases in 78% of marginal-employment job offers on the left tail of the wage distribution. For part-time and full-time job offers, the corresponding percentages are 63% and 26%, respectively.

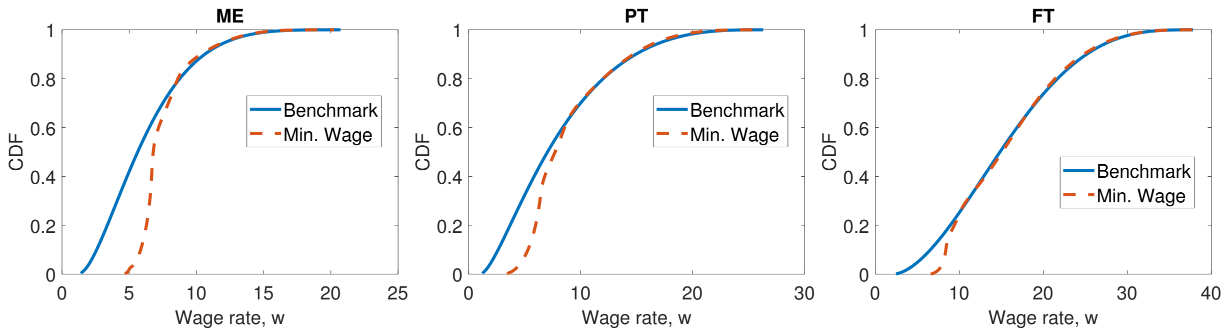
Next, we delve into the decisions regarding firm inactivity. Figures 6a and 6b show that low-productivity firms are more likely to become inactive after the implementation of the minimum wage. This tendency arises from the fact that these firms typically offer jobs with lower wages, making them more susceptible to the impact of the minimum wage. However, as the minimum wage is raised from 8.5 € to 11 €, we observe minimal changes in the activity decisions. As illustrated in Fig. 7a, while the implementation of an 8.5 € minimum wage results in a 0.90% decrease in the total share of active firms, this figure slightly increases to 0.99% with the implementation of an 11 € minimum wage. The lack of significant firm response in the activity margin can be understood by considering two factors. Firstly, in our model, minimum wage regulations can be violated, subject to the non-compliance penalty. The option not to comply allows low-productivity firms to sustain profitability and remain active. Secondly, firms have the option to adjust their hour requirement. Given that full-time jobs exhibit higher hourly labor productivity compared to marginal or part-time jobs, firms may find it more profitable to increase the hour requirement rather than opting for inactivity.

Focusing on the decision regarding hour requirements, Figures 6c to 6h present the distribution of hour requirements conditioned on firm productivity and activity status. It is evident that low-productivity firms tend to decrease the probability of posting marginal and part-time jobs, while simultaneously increasing the likelihood of offering full-time jobs. As the minimum wage rises from 8.5 € to 11 €, we observe a more pronounced upward shift in the hour requirement among low-productivity firms.

[20]Fig. 11 in Appendix D shows the effect of the 11 € minimum wage on the distribution of offered wages. The effect is larger but qualitatively similar to that of the 8.5 € minimum wage.

Overall, imposing higher minimum wages results in more substantial reductions in the total measure of marginal-employment and part-time job offers, while the decrease in the total measure of full-time offers become milder (Fig. 7b). This shift in the offer distribution toward full-time jobs can be attributed to the response of low-productivity firms. Firms with lower productivity are more likely to post marginal-employment and part-time jobs. When low-productivity firms become inactive, there is a disproportionate reduction in the measure of such job offers. The decision of active low-productivity firms to raise their hour requirement to full-time further contributes to the overall shift in the offer distribution.

Figure 5: Wage offer distributions

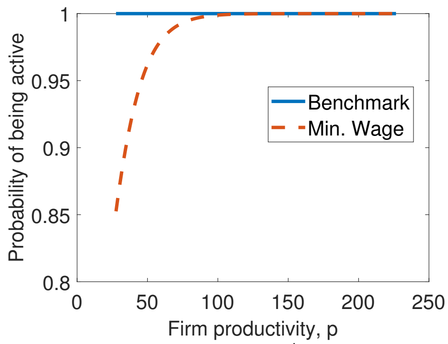


Note: The figure shows the offer distribution $F_h(w)$ in the benchmark equilibrium without any minimum wage (solid line) and that in the equilibrium with a minimum wage set at 8.5 € (dashed line).

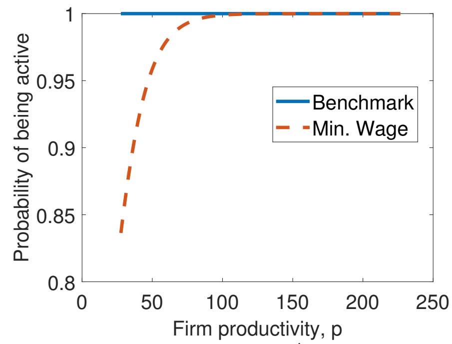
Impact of firm adjustments on equilibrium outcomes Firm adjustments play a crucial role in determining the equilibrium effects of the minimum wage, and its impact on the distribution of employment is theoretically ambiguous. On one hand, the minimum wage leads to disproportionate wage increases for low-hour job offers, making marginal- and part-time jobs more attractive to workers and potentially increasing their acceptance rates. On the other hand, the minimum wage induces a shift in the offer distribution towards full-time jobs, which can result in a higher equilibrium full-time employment rate compared to other types of employment.

To assess the role of firm adjustments, we conduct a counterfactual analysis where we restrict firms from making job-posting adjustments in response to the minimum wage. In this scenario, the offer distribution remains the same as in the benchmark economy without a minimum wage. Fig. 8 compares the employment distribution effects of implementing an 8.5 € minimum wage in the equilibrium model to those in the counterfactual environment. Notably, the minimum wage has amplified effects on employment and hours in the absence of firm adjustments. The reallocation of women towards jobs with longer hours is more pronounced when firm adjustments are not allowed than in equilibrium. Without firm adjustments, the minimum wage can more substantially reduce the gender income gap by closing gender gaps in employment, hours, and wages (Table 9).

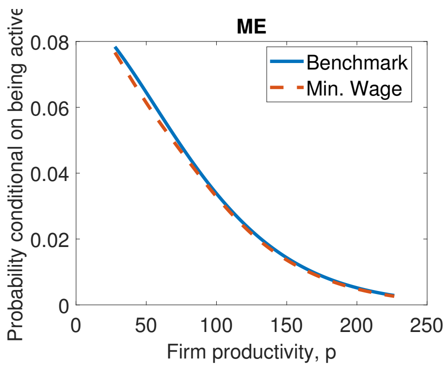
Figure 6: Effects on the share of active firms and hour requirement distribution



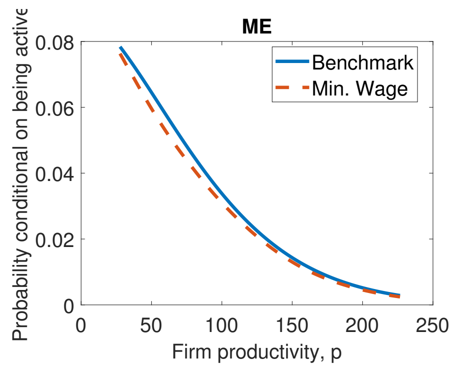
(a) Share of active firms $\Delta^{act}(p)$, min. wage 8.5 €



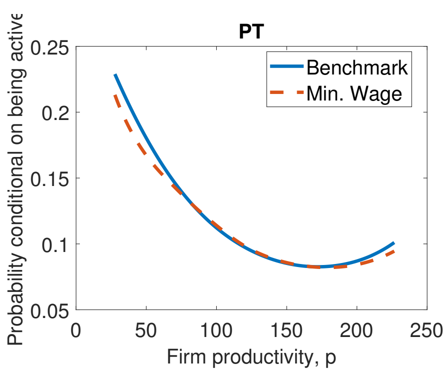
(b) Share of active firms $\Delta^{act}(p)$, min. wage 11 €



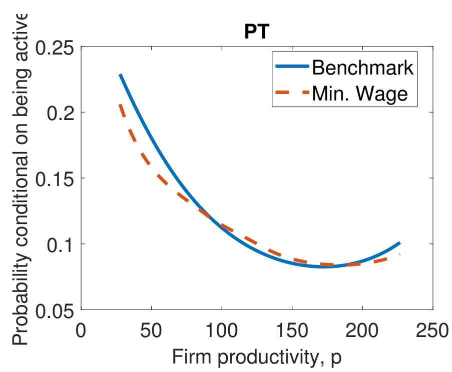
(c) Share posting ME jobs cond. on active $\frac{\Delta(ME;p)}{\Delta^{act}(p)}$, min. wage 8.5 €



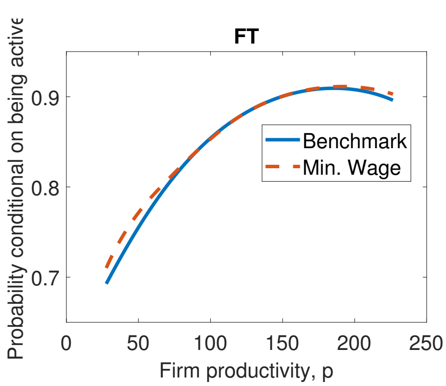
(d) Share posting ME jobs cond. on active $\frac{\Delta(ME;p)}{\Delta^{act}(p)}$, min. wage 11 €



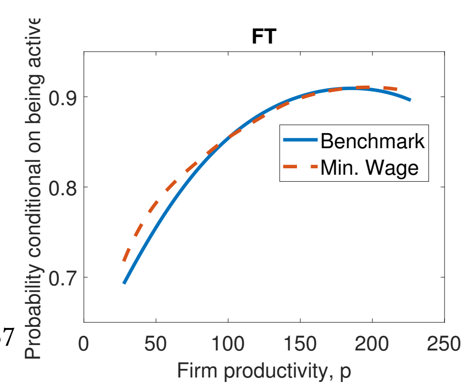
(e) Share posting PT jobs cond. on active $\frac{\Delta(PT;p)}{\Delta^{act}(p)}$, min. wage 8.5 €



(f) Share posting PT jobs cond. on active $\frac{\Delta(PT;p)}{\Delta^{act}(p)}$, min. wage 11 €

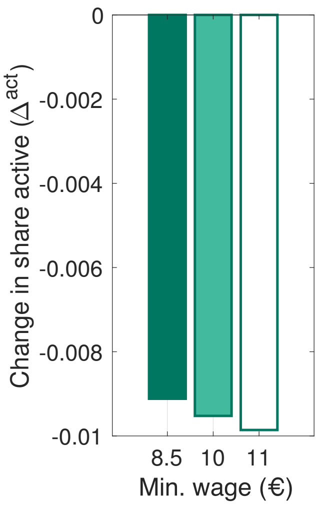


(g) Share posting FT jobs cond. on active $\frac{\Delta(FT;p)}{\Delta^{act}(p)}$, min. wage 8.5 €

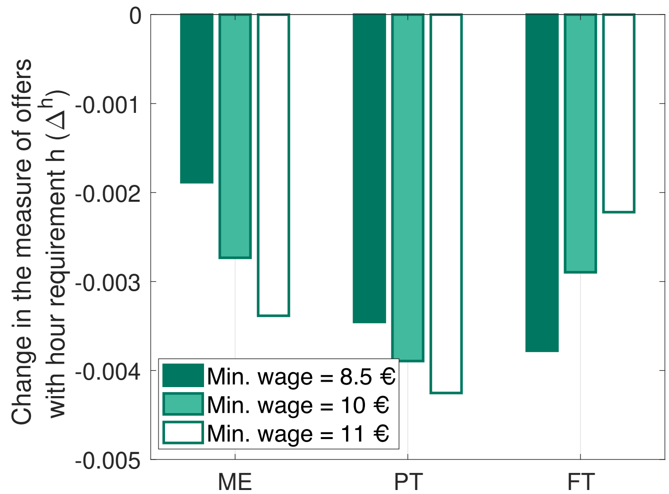


(h) Share posting FT jobs cond. on active $\frac{\Delta(FT;p)}{\Delta^{act}(p)}$, min. wage 11 €

Figure 7: Minimum wage effects on the share of active firms and the measure of job offers by hour requirement

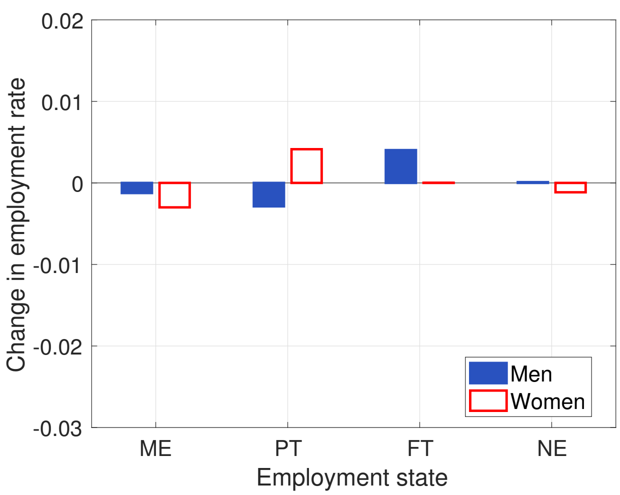


(a) Share of active firms, Δ^{act}

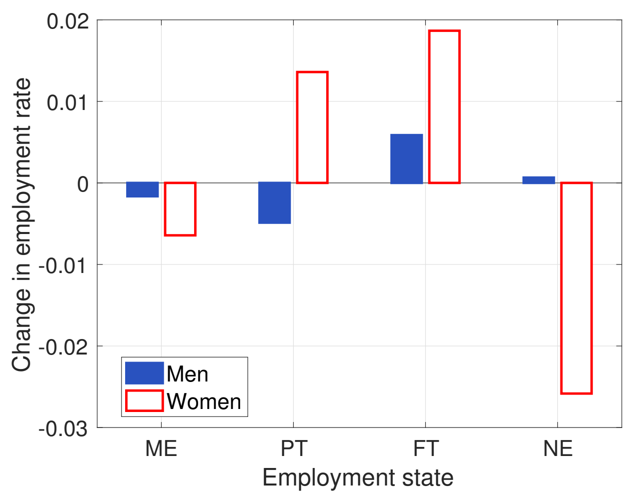


(b) Measure of job offers by hour requirement, Δ^h

Figure 8: Effects of the minimum wage on employment distribution, benchmark vs. counterfactual



(a) Equilibrium effects



(b) Counterfactual effects

Note: The figures show effects of imposing a minimum wage of 8.5 €. Counterfactual effects refers to minimum wage effects in the environment in which the offer distribution is set to that in the economy without any minimum wage.

Table 9: Counterfactual effects on gender gaps

	(1)	(2)
Min. wage =	none	8.5 €
Gender income gap	0.360	0.391
Gender employment gap	0.626	0.653
Gender hours gap	0.717	0.728
Gender wage gap	0.802	0.821

Note: Gender gap is measured by the ratio between men and women. A value of 1 indicates gender equity. Column (2) shows gender gaps in the counterfactual environment in which the offer distribution is set to that in the economy without any minimum wage.

5.3 Heterogenous effects on women

In this subsection, we analyze the equilibrium effects of minimum wages on different demographic groups. Fig. 9a illustrates the impact of the 8.5 € minimum wage on men, which shows limited variation across different child states, marital statuses, and spousal incomes. Regardless of these factors, the minimum wage leads to a slight reduction in marginal and part-time employment rates and a slight increase in full-time employment rates among men. The non-employment rate experiences minimal changes. The homogeneity of the effect on men persists even at higher levels of the minimum wage (Fig. 13a in Appendix D).

In contrast, the impact of the minimum wage on women exhibits significant variation depending on their child status, marital status, and spousal income, as depicted in Fig. 9b. First, consider women without children. This group of women has the highest employment rate and the longest working hours prior to the implementation of the minimum wage (Fig. 12b in Appendix D). The effect of the minimum wage on this group is relatively small compared to other women and is qualitatively similar to the impact experienced by childless men.

Next, let's consider women with young children. This group of women tends to have the lowest level of employment, with a higher likelihood of being non-employed and a lower likelihood of being employed full-time (Fig. 12b in Appendix D). The impact of the minimum wage on this group is heavily influenced by marital status and spousal income. For women who are single or married to a low-income spouse, the implementation of the 8.5 € minimum wage results in lower non-employment and marginal employment rates, as well as a higher part-time employment rate. In other words, these women experience an "upward" reallocation in terms of working hours. However, for women who are married to a high-income spouse, the minimum wage leads to a decrease in the marginal employment rate and an increase in the non-employment rate. As for women with grown children, the effects of the 8.5 € minimum wage are similar to those experienced by women with young children, but quantitatively weaker.

The significance of spousal income in determining the impact of the minimum wage can be attributed to the following factors. First, due to the progressive nature of income tax and joint filing of taxes by couples, higher spousal income implies a higher marginal tax rate, which disincentivizes women from accepting jobs with higher hours. Second, higher spousal incomes reduce the willingness to accept jobs with long hours due to the income effect resulting from household income-pooling. Consequently, women with higher spousal incomes are less likely to reallocate to jobs with longer hours.

The implementation of the 11 € minimum wage further amplifies the upward reallocation for all women with both young and grown children (Fig. 13b in Appendix D). Interestingly, for women with grown children and a high-income spouse, increasing the minimum wage has a particularly significant positive effect on the full-time employment rate. It is worth noting that these women exhibit a relatively smaller aversion to full-time jobs compared to women with young children (as indicated by $\mu(j,k,FT)$ in Table 6), which helps explain their more responsive labor supply when faced with increases in the minimum wage.

6 Conclusion

Women are disproportionately affected by the minimum wage due to their higher likelihood of working in low-hour jobs with lower hourly wages. This study quantifies the impact of the minimum wage on the gender income gap by considering employment, hours, and wage differentials.

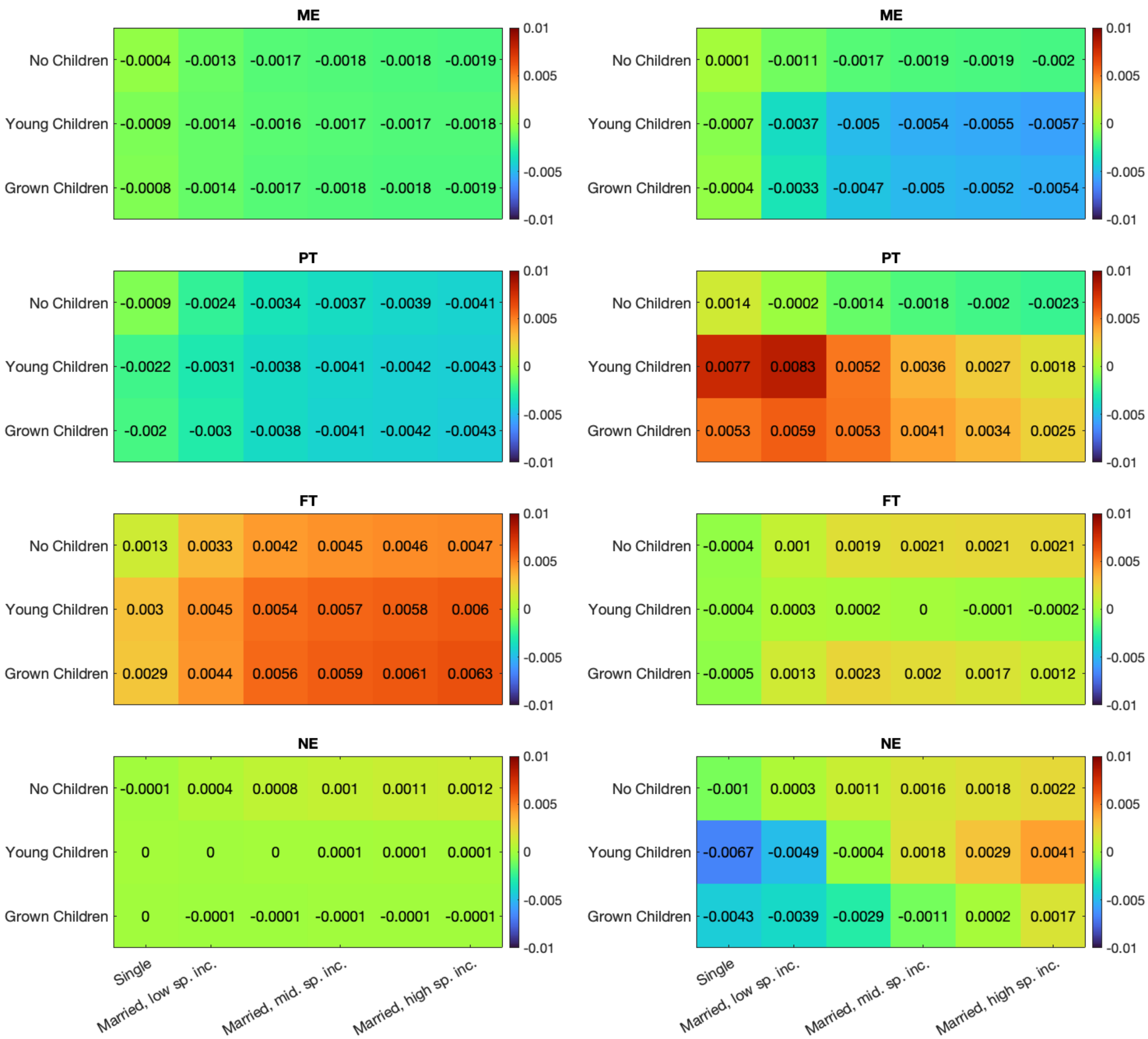
Our empirical analysis of the first German federal minimum wage reveals a negative impact on female marginal employment and a positive effect on the transition from marginal to part-time employment. These findings suggest that the policy may contribute to higher incomes for women and potentially reduce the gender income gap.

To examine the effects of the minimum wage, we develop an equilibrium job-posting model that incorporates household demographic heterogeneity and firm productivity heterogeneity. A key feature of our model is that firms can determine the hour requirements of jobs, which can be marginal, part-time, or full-time. The model successfully matches important features in the data, such as gender gaps in employment, hours and wage, and the correlation between hours and hourly wages.

To address non-compliance with the minimum wage, we introduce a penalty that reduces the job contact rate for jobs offering wages below the minimum. This penalty is calibrated to observed non-compliance rates. We analyze the impact of three different minimum wage levels: the initial level of 8.5 € per hour in 2015, the 2022 level of 10 € (adjusted for inflation), and a potential level of 11 €.

The minimum wage affects employment and hours differently for men and women. While it has a limited effect on male non-employment, it progressively reduces female non-employment. Both genders experience a reallocation towards jobs with longer hours, but the effect is more pronounced for women due to

Figure 9: Effect of the 8.5 € minimum wage on the employment distribution by gender



(a) Men

(b) Women

Note: The effect of the minimum wage is measured by the employment (or, non-employment) rate in the minimum-wage equilibrium minus that in benchmark equilibrium.

their higher concentration in low-hour, low-wage jobs. These gender asymmetries contribute to narrowing the gender income gap, primarily through the gender wage gap at the 8.5 € level. Higher minimum wage levels additionally reduce the gender employment and hours gap.

We also analyze firms' response to the minimum wage and their role in shaping equilibrium outcomes. The penalty on non-compliant jobs encourages firms to increase wages, particularly for jobs with lower hour requirements. While the minimum wage has only a modest impact on firm inactivity, it leads to a shift towards full-time job offers. Overall, firm adjustments dampen the reallocation effect of the minimum wage.

Finally, we highlight the heterogeneity of the minimum wage's impact among women, with children and spousal income playing a significant role. Women without children are less affected, experiencing effects similar to men without children. Among women with children, those with older children or without high spousal incomes experience a more prominent reallocation effect towards longer-hour jobs.

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Appendix

A Empirical evidence: additional tables and figures

Table 10: Estimated effect of the minimum wage on transition rates, men.

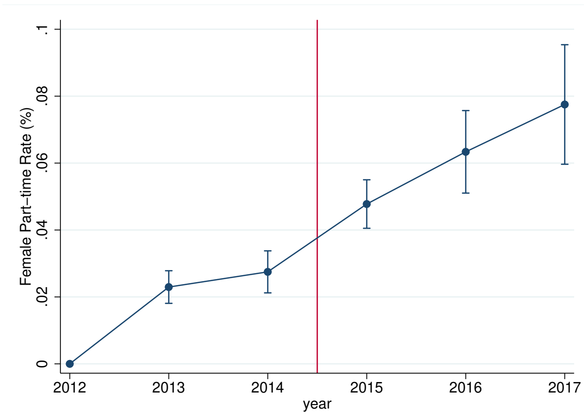
	(1) To full-time	(2) To part-time	(3) To marginal emp.	(4) To non-emp.
From full-time	0	0.608*	-0.790	-0.0303
	(.)	(0.284)	(0.718)	(0.242)
From part-time	-0.177	0	0.390	-0.0664
	(0.379)	(.)	(0.658)	(0.481)
From marginal emp.	-0.0659	0.513	0	0.154
	(0.622)	(0.597)	(.)	(0.599)
From non-emp.	0.163	-0.327	0.340	0
	(0.261)	(0.436)	(0.386)	(.)

Standard errors in parentheses

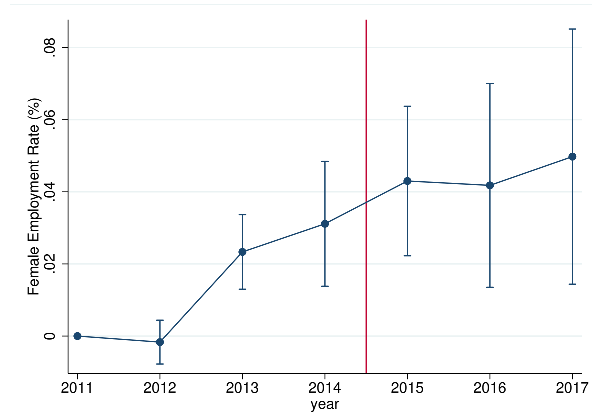
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: SOEP. The sample contains 25-55 year old individuals between 2006 and 2019. The table shows estimated value of γ_{l_0,l_1} of the multinomial logit regression of the annual labor force transition rate (Eq. 2). The regression controls for age, education, marital state, spousal income, children, and year and federal state fixed effects.

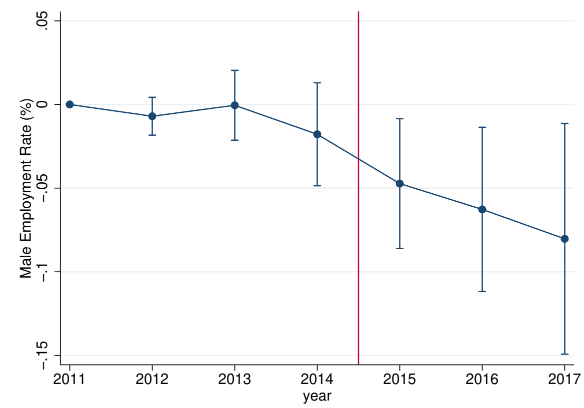
Figure 10: Estimated effects of the bite of minimum wage on labor market outcomes (γ_τ)



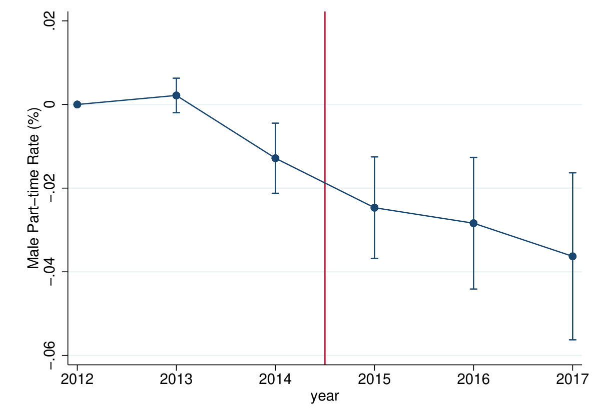
(a) Female part-time employment rate



(b) Female employment rate



(c) Male employment rate



(d) Male part-time employment rate

Source: Regional data accessed on INKAR.de. Vertical bars represent 95% confidence interval. Regions are defined as labor market regions ("Arbeitsmarktregion"). In our dataset, there are 257 regions over the period from 2011 to 2017. Marginal Employment excludes individuals who hold such contracts as secondary jobs. The regressions are weighed by region population and estimated with robust standard errors. See Eq. 1.

Table 11: Estimated effect of the minimum wage on transition rates, women.

	(1) To full-time	(2) To part-time	(3) To marginal emp.	(4) To non-emp.
From full-time	0	-0.0744	-0.444	0.0373
	(.)	(0.192)	(0.591)	(0.223)
From part-time	0.210	0	0.185	-0.137
	(0.176)	(.)	(0.261)	(0.235)
From marginal emp.	0.584	0.565*	0	-0.346
	(0.510)	(0.260)	(.)	(0.319)
From non-emp.	-0.300	-0.159	-0.346	0
	(0.287)	(0.187)	(0.230)	(.)

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Source: SOEP. The sample contains 25-55 year old individuals between 2006 and 2019. The table shows estimated value of γ_{l_0,l_1} of the multinomial logit regression of the annual labor force transition rate (Eq. 2). The regression controls for age, education, marital state, spousal income, children, and year and federal state fixed effects.

B Numerical appendix

B.1 Numerical solution

In this section, we briefly explain the numerical algorithm to solve the equilibrium model laid out in Sec. 3.

1. Discretize firm productivity p and denote the p-grid by $\mathbf{p} = \left(p_1, p_2, ..., p_{N_p}\right)$. For each grid point p_n, provide an initial guess of $w_{h,n} = w_h\left(p_n\right)$ and the hour requirement choice probability $\Delta_{h,n} = \Delta\left(h; p_n\right)$ for each $h \in \mathcal{H}$. We refer to $\mathbf{w}_h \equiv \left(w_{h,1}, w_{h,2}, ..., w_{h,N_p}\right)$ as the derived wage grid.

2. Construct the offer distributions Δ^h using Eq. 15 and $F_h\left(w\right)$ using Eq. 14. Note that $F_h\left(w\right)$ is only defined on the derived wage grid $\mathbf{w}_h$.

3. Given Δ^h and $F_h\left(w\right)$, solve workers' problems by value function iteration. We can derive worker decisions (Ω's). Given worker decisions, solve for the steady state distributions $g^{j,e}$ and $g^{j,n}$ by iterating equal-flow equations. Note that the value functions, worker decisions and $g^{j,e}$ are defined on the derived w-grids.

4. Given the distribution of workers $g^{j,e}$ and the offer distributions Δ^h and $F_h\left(w\right)$, calculate labor supply $l\left(w, h\right)$ using Eq. 8.

5. Given labor supply $l\left(w, h\right)$, solve for firms' job-posting decision (Eq. 10). To do so, we first numerically

solve for $\pi_h(p_1)$ for each $h \in \mathcal{H}$, the optimal profit of the least productive firm on $\mathbf{p}$ given h. Then, we compute profits and posted wages of $p > p_1$ using the envelope condition, which we explain below.

To solve the problem of firm p_1, construct a wage sub-grid for each $h \in \mathcal{H}$. The sub-grid should include points around the candidate (initial guess or wage from the previous iteration) $w_h(p_1)$, i.e.

$$\underline{\mathcal{W}} = \{w_h(p_1) - N_{sg} \cdot \epsilon, w_h(p_1) - (N_{sg} - 1)\epsilon, ..., w_h(p_1), w_h(p_1) + \epsilon, ..., w_h(p_1) + N_{sg} \cdot \epsilon\},$$

where $\epsilon > 0$ denotes the spacing between grid points. For each point on the sub-grid, perform value function iteration with $w_h(p_1)$ replaced by the point on the sub-grid and leave the rest of the wage grid unchanged. Then, compute steady state distributions and labor supply $l(w, h)$. An update of the wage for firm p_1 is obtained by considering which $w \in \underline{\mathcal{W}}$ maximizes flow profits, i.e.

$$w_h^+(p_1) = \arg\max\{\pi_h(w; p_1) : w \in \underline{\mathcal{W}}\}$$

and the maximized flow profit is $\pi_h(p_1)$ for each h.

Once we solved the firm problem for p_1, we can compute $w_h(p)$ for each $p > p_1$ on $\mathbf{p}$ starting from p_2. This step uses the Envelope Theorem to derive a wage equation. Eq. 11 can be rewritten as

$$w_h(p)h = y(p,h) - \frac{\pi_h(p)}{l(w_h(p), h)}. \tag{25}$$

Applying the Envelope Theorem, we have

$$\begin{aligned}\frac{d\pi_h(p)}{dp} &= \frac{\partial \pi_h(w_h(p); p)}{\partial p} \\ &= \frac{\partial y(p,h)}{\partial p} l(w_h(p), h) \\ &= \theta_h h l(w_h(p), h)\end{aligned} \tag{26}$$

$\pi_h(p)$ can be expressed in terms of $\pi_h(p_1)$ such that

$$\begin{aligned}\pi_h(p) &= \pi_h(p_1) + \int_{p_1}^{p} \frac{d\pi_h(x)}{dx} dx \\ &= \pi_h(p_1) + \int_{p_1}^{p} \theta_h h l(w_h(x), h)\, dx\end{aligned} \tag{27}$$

Using Eqs. 26 and 27, we can rewrite Eq. 25 as

$$w_h^+(p)h = y(p,h) - \frac{\left[\pi_h(p_1) + \int_{p_1}^{p} \theta_h h l(w_h(x), h)\, dx\right]}{l(w_h(p), h)} \tag{28}$$

and the $+$ superscript indicates the updated value.

Given $\pi_h(p)$ from Eq. 27, the hour requirement choice probability function $\Delta(h;p)$ can be updated using Eq. 13.

6. Repeat steps 2-5 until $w_h(p)$ and $\Delta(h;p)$ converge.

B.2 Preference shock

In solving the model numerically, we assume that, when facing the decision to accept a job offer or to quit their job, the worker draws a one-time idiosyncratic taste shock $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2\}$ from the type-I extreme value distribution with scale parameter σ_α. The taste shock allows us to smooth the labor supply function; we set σ_α to be small such that the shock does not change the model solution.

Given the taste shock, the value functions and decisions in Sec. 3.2 can be modified as follows. The Bellman equation for the value of non-employment (Eq. 4) becomes

$$\begin{aligned}\mathcal{D}_j^n V_j^n(k) &= u^j\left(\mathcal{N}^n(j,k),0,k\right) + \phi_j(k)V_j^n(k') \\ &+\lambda\sum_{h\in\mathcal{H}}\int \Delta^h \mathbb{E}_{\boldsymbol{\alpha}} \max\left\{V_j^e\left(\{w,h\},k\right) - \mu(j,k,h) + \alpha_1, V_j^n(k) + \alpha_2\right\} dF_h(w).\end{aligned}$$

The Bellman equation for the value of employment (Eq. 6) becomes

$$\begin{aligned}\mathcal{D}_j^e V_j^e\left(\{w,h\},k\right) &= u^j\left(\mathcal{N}^e(wa_j(h)h;j,k),h,k\right) + \delta(h)V_j^n(k) \\ &+\phi_j(k)\mathbb{E}_{\boldsymbol{\alpha}} \max\left\{V_j^e\left(\{w,h\},k'\right) - \mu(j,k,h) + \alpha_1, V_j^n(k') + \alpha_2\right\} \\ &+\lambda\sum_{h'\in\mathcal{H}}\int \Delta^{h'} \mathbb{E}_{\boldsymbol{\alpha}} \max\left\{V_j^e\left(\{w,h\},k\right) + \alpha_1, V_j^e\left(\{w',h'\},k\right) - \mu(j,k,h') + \alpha_2\right\} dF_{h'}(w').\end{aligned}$$

The probability of accepting job offer (w',h') from non-employment (Eq. 5) becomes

$$\Omega_j^n\left(\{w',h'\};k\right) = \frac{\exp\left(\dfrac{V_j^e(\{w',h'\},k) - \mu(j,k,h')}{\sigma_\alpha}\right)}{\exp\left(\dfrac{V_j^e(\{w',h'\},k) - \mu(j,k,h')}{\sigma_\alpha}\right) + \exp\left(\dfrac{V_j^n(k)}{\sigma_\alpha}\right)}.$$

The probability that an employed worker with job (w, h) accepts a job offer (w', h') (Eq. 7) becomes

$$\Omega_j^e\left(\{w', h'\}, \{w, h\}; k\right) = \frac{\exp\left(\frac{V_j^e(\{w', h'\}, k) - \mu(j, k, h')}{\sigma_\alpha}\right)}{\exp\left(\frac{V_j^e(\{w', h'\}, k) - \mu(j, k, h')}{\sigma_\alpha}\right) + \exp\left(\frac{V_j^e(\{w, h\}, k)}{\sigma_\alpha}\right)}.$$

C Decomposition of minimum wage effects on the gender income gap

Let G denote gender gap, defined as the ratio between the female value and the male value. The gender income gap (G_i) is the product of the gender employment gap (G_e), gender hours gap (G_h) and gender wage gap (G_w). Taking logs, we have

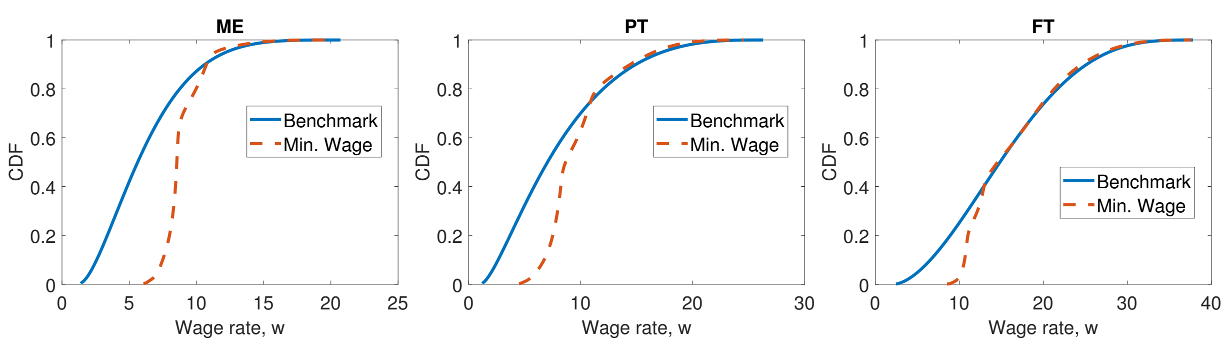
$$\log G_i = \log G_e + \log G_h + \log G_w.$$

We measure the minimum wage effect on the gender income gap in log-point deviation from the benchmark (no minimum wage) economy, $\log G_i^M - \log G_i^B$, where G_i^M is the gender income gap in the minimum wage economy and G_i^B is that in the benchmark economy. We can decompose the minimum wage effect on the gender income gap into employment, hours, and wage channels such that

$$\log G_i^M - \log G_i^B = \underbrace{\left(\log G_e^M - \log G_e^B\right)}_{\text{Gender employment gap channel}} + \underbrace{\left(\log G_h^M - \log G_h^B\right)}_{\text{Gender hours gap channel}} + \underbrace{\left(\log G_w^M - \log G_w^B\right)}_{\text{Gender wage gap channel}}.$$

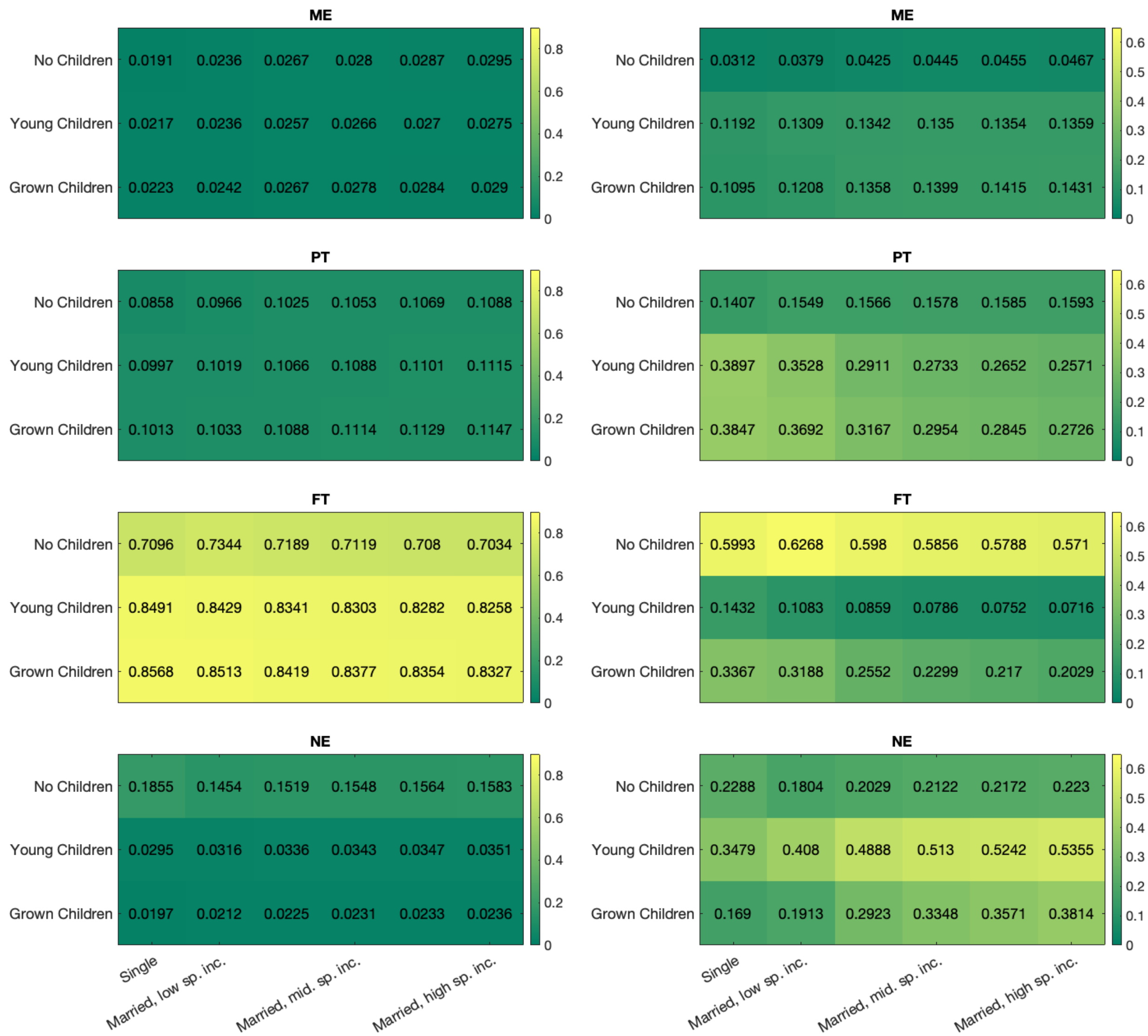
D Results: additional tables and figures

Figure 11: Wage offer distributions, 11 € minimum wage



Note: The figure shows the offer distribution $F_h(w)$ in the benchmark equilibrium without any minimum wage (solid line) and that in the equilibrium with a minimum wage set at 11 € (dashed line).

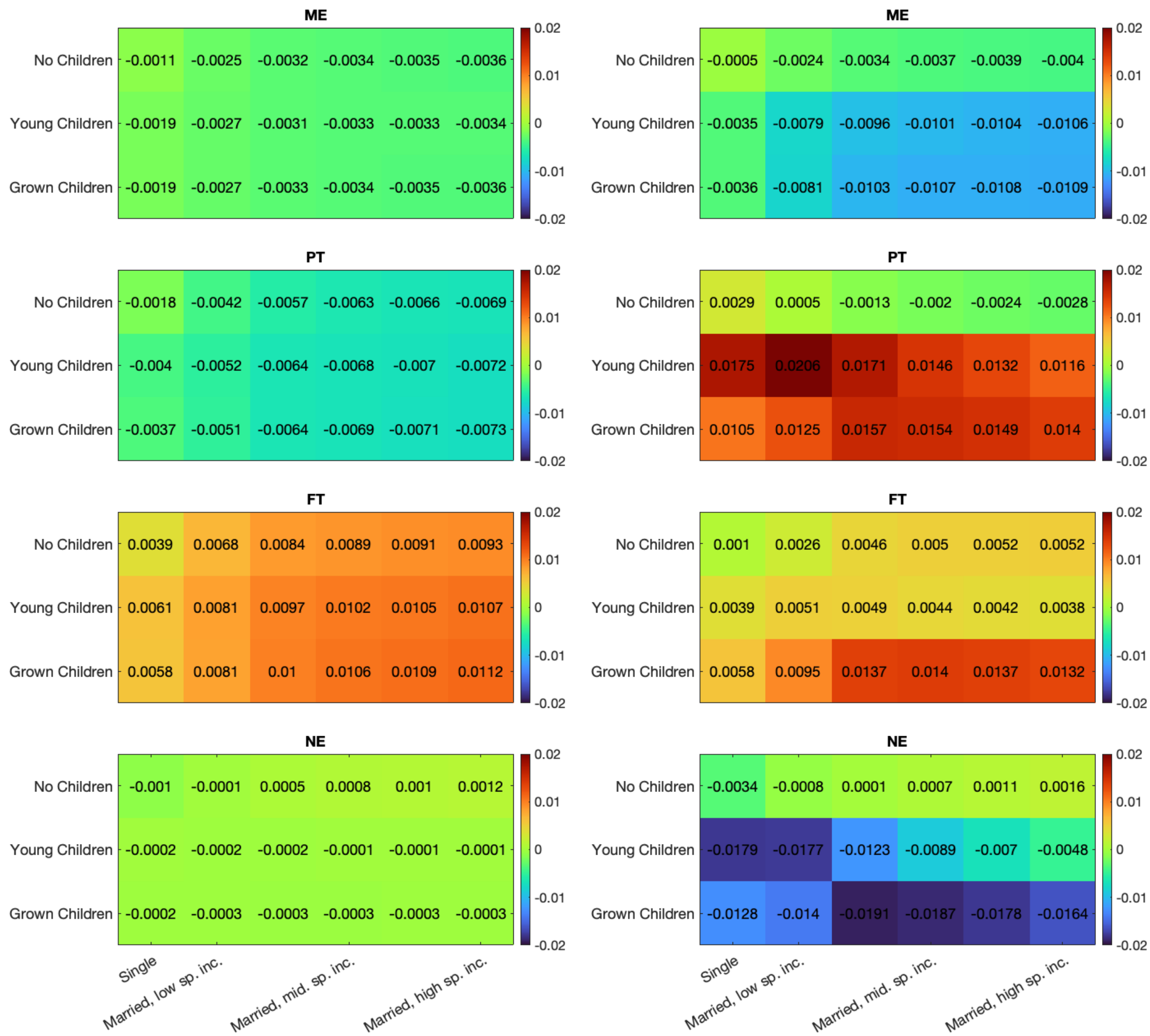
Figure 12: Equilibrium employment distribution by demographic types



(a) Men   **(b)** Women

Note: The employment distribution is the one in our equilibrium model without minimum wages.

Figure 13: Effect of the 11 € minimum wage on the employment distribution by gender



(a) Men **(b)** Women

Note: The effect of the minimum wage is measured by the employment (or, non-employment) rate in the minimum-wage equilibrium minus that in benchmark equilibrium.