Reviewing and delving into the causes of the Hubble tension

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Abstract: This paper explores different techniques for measuring the Hubble-Lemaître constant. After reviewing how the measurements of this important cosmological parametre have evolved through history, we present and compare the fundamental methods, both model-independent, based on standard candles/sirens and standard rulers, and model-dependent, based on the cosmic microwave background, where a discrepancy between the two groups of results has been recently named Hubble tension. Sources of systematic uncertainty as well as possible modifications to the ΛCDM model are discussed in order to mitigate this tension.

I. INTRODUCTION

The discovery of the accelerated expansion of the universe is one of the major achievements in science in the 20th century. It allowed to formulate a new model for the universe, ΛCDM, as well as to determine its precise age.

This model is based on general relativity, developed by Albert Einstein. He was also the first to apply this new theory of gravitation to the universe, but mistakenly adopted the Newtonian view that the solution had to be static, so he had to introduce a parametre (Λ), also known as cosmological constant, in his field equations to make them compatible with this assumption. However, in the early 1920s Alexander Friedmann introduced a set of stable solutions for the Einstein field equations, later known as Friedmann equations, which implied a non-static cosmos, bringing a groundbreaking progress in cosmology.

Georges Lemaître was the first to connect the expansion of the universe with the receding velocities of extragalactic objects [1]. It was in accordance with his non-stationary solutions from Einstein's equations with non-zero cosmological constant, which were very similar to the ones announced by Friedmann.
In spite of the fact that Lemaître was the first to interpret the recessional velocities due to the universe expansion, his work did not automatically spread among the scientific community; hence, when the astronomer Edwin Hubble delivered in 1929 empirical evidence, he took all the credit for the discovery [2].
Even though Lemaître's work was published later, it has been proved that his findings were set independently of Hubble's; thus, in an attempt to give recognition to Lemaître's contribution to the development of cosmology, the International Astronomical Union renamed the constant as Hubble-Lemaître constant (H_0) in 2018.

In this work we will study different methodologies for determining H_0, focusing on the use of standard candles/sirens and standard rulers (Sec. III) and their main sources of systematic uncertainties, as well as on the power spectrum of temperature fluctuations in the cosmic microwave background (CMB). Furthermore, in order to explain possible ways to mitigate the tension, we will mention some relevant modifications to the ΛCDM model proposed by experts in the field.

This work is organised as follows. Section II gives a historical introduction of the measurements of H_0. In Section III, we describe the origin of the Hubble tension and the most important techniques to find H_0, both model independent (Cepheids, Tip of Red Giant Branch stars and Gravitational Waves) and model dependent (CMB), delving into the possible causes of systematic uncertainties. In Section IV some explanations of the tension are mentioned. A proposed brief summary of this work is presented in Section V.

II. THE ORIGIN OF THE HUBBLE TENSION

In 1929, Edwin Hubble observations revealed a linear correlation between the distance to 24 galaxies and their recessional velocities. To determine the distance to these galaxies, Hubble used the standard-candle method based on distances to Cepheids (see Sec. III.A.). Hubble computed the distances from the period-luminosity (PL) relation for Cepheids (lately recognised as (Henrietta S.) Leavitt Law using galaxies of the Virgo cluster combined with radial velocity values he and others had measured [3]. The constant slope of the radial velocity-distance (H_0) sets the cosmic distance scale for the present universe:

$$v = H_0 d. \tag{1}$$

Recessional velocities can be related to redshift through the Doppler effect:

$$z = \frac{\lambda_o - \lambda_e}{\lambda_e} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} - 1 = \sqrt{\frac{(1+\frac{v}{c})^2}{1-(\frac{v}{c})^2}} - 1 \approx \frac{v}{c} \tag{2}$$

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when velocities are non-relativistic. Hence,

$$cz = v = H_0 d. \qquad (3)$$

Due to the large uncertainties affecting the measurements of the distances and to the misidentification of certain Cepheids, such as confounding HII regions with these bright stars, H_0 was greatly overestimated at first, resulting on an underestimation of the age of the universe by an order of magnitude (see FIG. 1).

In the 50s, Walter Baade realised that there were two types of Cepheids, which reduced the value of H_0 by a factor of nearly two.

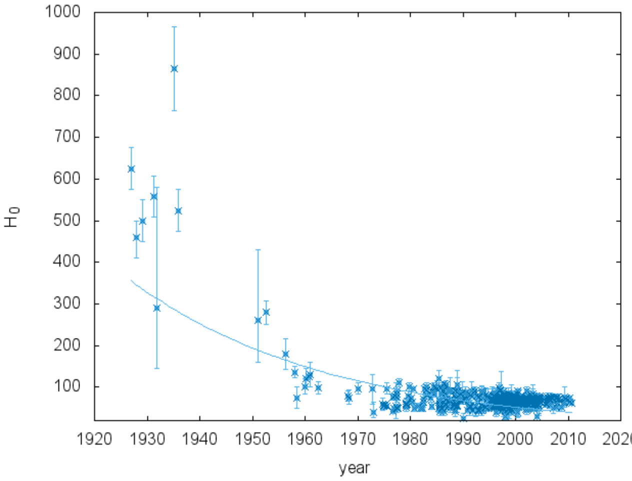

FIG. 1: The median of H_0 values multiplied by a factor of 10^{-24} up to 2010 shows an exponential decrement of equation $H_0(\text{year}) = 2.927e^{-0.026\cdot\text{year}}$.

In the mid 1970s H_0 measurements ranged between 50 and 100 km s^{-1}Mpc^{-1}. Allan R. Sandage, Gustav A. Tammann and collaborators held that H_0 hovered around 55 km s^{-1} Mpc^{-1}. On the other hand, Sidney van den Bergh, Gerard de Vaucouleurs and other astronomers using similar methodologies claimed that H_0 was near 100 km s^{-1} Mpc^{-1} (see FIG. 2).

For several decades the astronomical community remained divided between supporters of one or another set of values until the Hubble Space Telescope (HST) was launched in the 1990s. Its new accurate measurements, which had a global precision under 10%, resolved distances to Cepheids up to 20 Mpc, which enabled a precise distance to the Virgo cluster, and consequently calibrations to secondary (further) distance indicators such as the Tully-Fisher relation and SNIa were tightened. This started the era of precision cosmology, setting the value of H_0 around 70 km s^{-1} Mpc^{-1}.

Nevertheless, stellar astrophysicists dated certain stars higher than the age of the universe, implying that H_0 had to be presumably still underestimated. In 1998, evidence of acceleration of the universe was seen throughout the study of SNIa, which eliminated such discrepancies and introduced a substantial change in the cosmological model. This era was the birth of concordance cosmology.

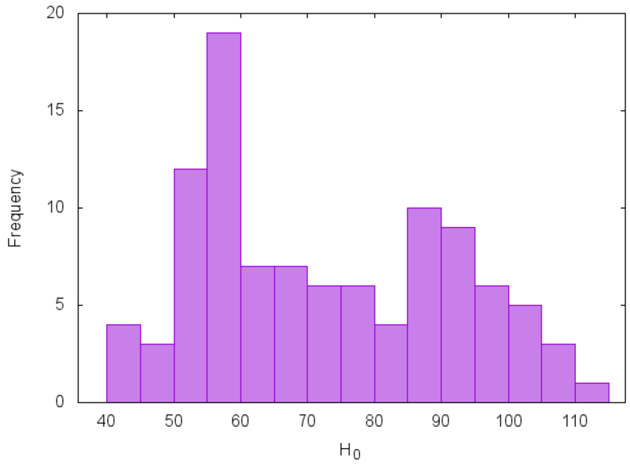

FIG. 2: Histogram of H_0 measurements between 1974 and 1990. Two peaks arise: one at 55 km s^{-1} Mpc^{-1} from Sandage and collaborators and another around 90 km s^{-1}Mpc^{-1} from van den Bergh et al.

III. LEADING METHODS TO FIND H_0

At the end of the 20th century and early years of the 21st, the objectives were focused on reducing uncertainties as well as discovering alternative ways to determine H_0 without resorting to the cosmic distance ladder [5]. In addition to the direct techniques used until then, new indirect methods for measuring this parametre were introduced. For instance, the observed polarization and temperature fluctuations spectra of CMB are dependent on H_0, as well as other cosmological parametres. Observations of the CMB were collected from different surveys such as NASA's WMAP and especially ESA Planck Satellite, whose computed H_0 value in 2013 was $H_0 = 67.3 \pm 1.2$ km s^{-1} Mpc^{-1} assuming a flat ΛCDM cosmology [4].

Uncertainties in the value of H_0 were progressively decreasing. Surprisingly, although theoretically the measurements from different techniques were expected to converge, a dichotomy of the groups of results emerged. Currently, small error bars share an inconsistency of more than 3σ between the direct and indirect methods for measuring H_0 that reminds of the tension between measurements à la Sandage and à la de Vaucouleurs. To understand the origin of this new tension it is essential to grasp how the different methods of measuring H_0 work.

Among the myriad of techniques that exist for the distance determination, we are going to focus in this paper on the most representative of the direct measurements, the Cepheids-SNIa-based distance ladder, that relies on

EM information from one primary (Cepheid) and one secondary (SNIa) standard candle in the late ($z < 2$) universe, as well as on an the indirect method that uses EM information from the early ($z > 1000$) universe. We will also discuss two rather new tecniques that are producing intermediate results but may carry somewhat larger errors: a new distance ladder whose first step is the Tip of the Red Giant Branch (TRGB) and the recent detection of the gravitational waves (GW) produced during the final stages of the merger of a binary system of NS and/or BHs. Systematic errors will be discussed in order to briefly illustrate the accuracy of the methods.

A. The Cepheid-SNIa distance ladder

Cepheids are massive luminous radially pulsating stars. They are a relatively abundant type of short-period variable stars that can be found especially in spiral galaxies. Even though those from the Large Magellanic Cloud have historically been test beds for calibrations of the PL relation, currently research exploit Cepheid variables in a variety of clusters.
Their pulsating periods range from 2 to 100 days and their intrinsic brightness from -2 to -6 mag. As both variables have a significant direct correlation (Leavitt Law), these stars have become very useful for measuring galactic and extragalatic distances. From the Stefan-Boltzmann's law

$$L = 4\pi R^2 \sigma T_e^4, \tag{4}$$

where R is the stellar radius, T_e is the effective temperature and L is the bolometric luminosity. If we express the latter in magnitudes, then

$$\begin{aligned} M_{bol} = &- 2.5\log_{10}\frac{L}{L_0} \implies \\ M_{bol} &= -5\log_{10} R - 10\log_{10} T_e + C, \end{aligned} \tag{5}$$

where L_0 is a zero point luminosity and C is a constant.

The magnitude of the value of $\log T_e$ can be obtained from the intrinsic colour. Furthermore, by considering Cepheids as thermodynamic heat engines, a relation between their average density and period of pulsation can be derived (cf. Eddington, 1917)

$$P\rho^{1/2} = P\Big(\frac{m}{4/3\pi R^3}\Big)^{1/2} = K, \tag{6}$$

where ρ the density and K a constant. Assuming that the mass is mainly radius-dependent, then the radius can be obtained from P. We can replace this equation into eq. (5) and after considering that $BC = M_{bol} - M_V$ infer

$$M_V = \alpha\log_{10} P + \beta(B-V)_0 + \gamma, \tag{7}$$

where α, β, γ are constants. This equation sets a calibration for all Cepheids. Thus, given the period of the pulsation and the colour of a Cepheid, the absolute magnitude can be computed. Finally, considering the magnitude-distance equation the distance to the star is found.

In order to calibrate eq. (7), Cepheids have to be identified and its luminosity selected among fainter stars from the background. Measurements from space are preferred for this task.

Other notable sources of systematic uncertainties are the determination of zero-points, metallicity effects, which have a direct impact on the colours and the periods of the pulsations, and reddening, which may result in stars to appear redder and therefore cooler than in reality [6].

Cepheids' distances can be used to calibrate SNIa luminosities that occur in the same environment. Therefore, these extragalactic objects can be used as standard candles to compute distances at significantly larger scales.

According to recent publications, all H_0 values determined through this method are above 73 km s^{-1}Mpc^{-1}, and uncertainties of the order of 2% [6].

B. CMB

According to the ΛCDM model, the early universe had a hot early stage where photons and baryons were coupled in a dense plasma. Over time, adiabatic cooling caused a separation of those particles and photons started to travel freely through space. This stage is known as recombination.
However, due to the expansion of the universe, this travelling photons have redshifted and consequently the temperature of the photons has decreased ever since. For instance, the present detections determine that the mean temperature of the photon fluid is slightly less than 3 K [4].

A further hypothesis is that the formation of galaxies and clusters had to come from small fluctuations of the stability generated by the photon-baryon fluid that propagated at the relativistic speed of sound. These anisotropies are reflected in the radiation as they froze during recombination at different oscillating phases.

Satellites, such as Cosmic Background Satellite (COBE), Boomerang, Wilkinson Microwave Anisotropy Probe (WMAP) and Planck, where launched for measuring the temperature fluctuations. As a result, a pattern of anisotropies was traced, where peaks at different angular scales arise and from which abundant cosmic information can be obtained by fitting a model in the measurements.

Model-dependent techniques estimate values of H_0 lower than 68 km s^{-1} Mpc^{-1} and are more than 3σ apart from model-independent measurements [4].

C. The TRGB-SNIa distance ladder

The TRGB is an alternative technique that arises from the existent tension of the H_0 results when using CMB or Cepheids. It aims to solve the dichotomy providing a new and independent distance ladder calibration. In this case the primary standard candles are the low-mass red giants at the moment helium starts to burn on the core; that is when the luminosity function of the Red Giant Branch (RGB) exhibits a discontinuity.

Systematic uncertainties are analogous to the ones described for Cepheids in Sect.III. Some advantages include that RGB stars are located in all types of galaxies. Furthermore, those located in halo galaxies experience little reddening as well as they are quite isolated or not surrounded by brighter stars. Also, observations are usually conducted in I-band, which is little affected by metallicity. Finally, as measurements involve a shorter amount of time, this technique is more efficient.
On the other hand, RGB stars are usually fainter than most of the Cepheids, so increasing the data base can become challenging, which is essential for reducing uncertainties.

Overall, this procedure exhibits a similar precision than the PL relation for Cepheids, and so does the accuracy [7]. Furthermore, although the TRGB values of H_0 are slightly lower, they are in agreement with the ones inferred from Cepheids, which implies that both techniques can be supported with one another.

D. GW

In recent years the analysis of GW has become a brand new technique to determine H_0. Although this method had already been suggested in 1986 by Bernard Schutz, it was firstly executed in 2017, when LIGO-Virgo interferometres unprecedentedly detected the pulse of GW from the merger of two neutron stars.

Drawing a parallelism with standard candles, GW deals with standard sirens, which are objects whose GW can be detected, such as mergers of compact neutron stars (NS) and/or black hole (BH) binary. The GW signal is responsible for the distance computation while in principle an EM signal from the host galaxy is required to compute the recessing velocity. Once we have both components, H_0 can be found.

The use of GW to measure H_0 has many advantages. Although certain stars (e.g., Cepheids, Red Giants, SNIa, ...) provide decent standard candles, it is required to effectively leapfrog some intermediate distance measurements of objects located at shorter distance ranges to compute a proper distance measurement for their calibration. In this way, errors and uncertainty can creep in at many points in the calculations. In contrast, distance to the source of a GW signal can be determined directly.

However, there are still numerous uncertainties regarding this technique that lead to larger errors (see FIG. 3). The unknown orientation of NS and BH as well as that of their merge with respect to the received signal bring distance uncertainty. Also, the detected EM signal may be considered to arise from different host galaxies. Thus, a study of relative arrival times of the signal to multiple detectors must be performed in order to accurately determine the location [8]. There are also limitations in the detectors resolution of faint signals and those with very low frequencies. In fact, although GW from higher mass systems, such as BH binaries, are more likely to be detected, the EM signal is mostly too faint to be considered.

Alternatively, mathematical tools, such as bayesian statistics, can be introduced in the computation of H_0 that do not require EM information [9].

IV. SIGNIFICANCE OF THE TENSION

Up to this day, many computations of H_0 have been published after elaborating and improving a wide variety of methods. Nevertheless, the tension between model-dependent and model-independent techniques (see FIG. 3) remains unsolved. To make things worse, successive refinements of measurements appear to increase the tension even more by not only reducing the uncertainties but also increasing the divergence of the central values.

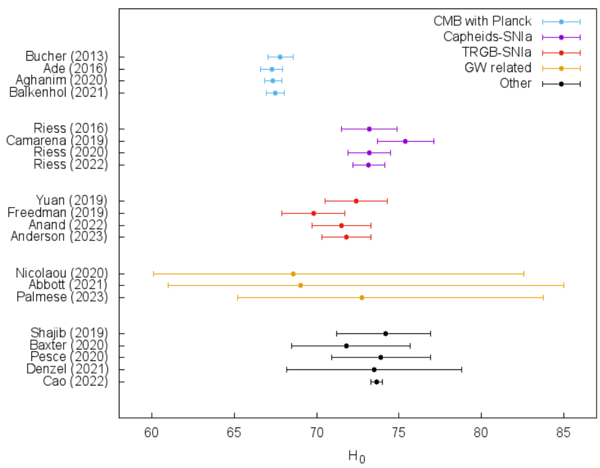

FIG. 3: H_0 tension between model-dependent and model-independent methodologies. Values are from the last decade.

Due to multiple reanalyses of the data, from Planck satellite and HST, there has been a spreading tendency among the scientific community that the discrepancy cannot (only) be caused by systematic uncertainties but by undiscovered physics beyond the ΛCDM model [10]. However, explanations and theories up to this day have only been able to mitigate the tension, without solving it. Hence, this subject is still a matter of ongoing research.

The ΛCDM model is in excellent consistency with a substantial number of cosmological measurements. Besides, these parametres carry very small uncertainties. Thus, applying changes to the present model without diminishing accuracy has become a labourous task. Possible solutions can be classified as follows: early-time or late-time modifications and changing gravity effects.

A. Early-time solutions

These can be obtained by modifying the expansion or the recombination theory, for instance, adding transient energy, named exotic early dark energy (EDE).
This alteration increases the early expansion rate (as this energy brings a negative pressure) by modifying the early stage of the universe without disrupting the late-time. This energy exhibits the same behaviour as a cosmological constant in the pre-recombination stage until it reaches a critical redshift when it becomes dynamic and the energy density fades at a higher rate than radiation and matter.

Other early-time possibilities are the existence of Dark Radiation, both with uninterrupted and interrupted propagation, neutrino self-interactions and the existence of primordial magnetic fields. Although they are beyond the scope of this project, we will remark that in general these hypothesis provide tension to large-scale data [11].

B. Late-time solutions

These include interacting Dark Energy (DE), Phantom DE and a vacuum phase transition among many others. The hypothesis needed for each solution are different from one another. For instance, Phantom DE equation of state is lower than -1 while vacuum phase transition refers to a modification of the vacuum properties in the Universe, in particular in relation to the energy density. However, they all share the characteristic that the modifications to the ΛCDM model, which accelerate the expansion of the universe, occur after recombination.

V. CONCLUSIONS

This work provides an overview of the main techniques employed to determine H_0 and shows the current tension between the model-dependent and model-independent values. It emphasises sources of systematic uncertainties associated to the measurements and discusses possible modifications of the ΛCDM model that, up to date, only have been able to mitigate the discrepancy.

It is concluded that, instead of being a handicap for the progress of cosmology, the Hubble tension offers an excellent opportunity for the discovery of new physics.

Acknowledgments

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[1] Receding velocity is the result of cosmic velocity without peculiar velocity.

[2] V. Frenkel and A. Grib, "Einstein, Friedmann, Lemaitre: Discovery of the Big Bang", *2nd Alexander Firedmann International Seminar on Gravitation and Cosmology*, 1, 1994.

[3] She discovered PL relation for Large Magellanic Cloud Cepheids. Subsequently, eq. (7) was formulated.

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VI. APPENDIX: COMPLETE LIST OF PUBLICATIONS CONSULTED DURING THE ELABORATION OF THIS WORK

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