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Impact of the entrance of a new country into the European Union in the voting power in the Council of the EU

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Abstract

Power indexes are key indicators in game theory and allow us to measure the voting power of different agents in a voting system, also called a weighted majority game. If we look at the theory of cooperative games, we can find different indices that try to measure the voting power of the coalitions that can be formed between the various agents or players. In our case, we have focused on the study of the Shapley-Shubik index.

The objective of this work is to analyze the impact that the entry of new countries into the European Union would cause on the voting power within the Council of the European Union of the different member states. To do this, we analyzed the variations in the Shalpey-Shubik index of the EU member states as a result of the entry of new countries in four different scenarios.

Resum

Els índexs de poder són indicadors clau dins la teoria de jocs i ens permeten mesurar el poder de vot dels diferents agents en un sistema de votació, també anomenat joc de majoria ponderada. Si ens fixem en la teoria de jocs cooperatius, podem trobar diferents índexs que intenten mesurar el poder de vot de les coalicions que es poden formar entre els diversos agents o jugadors. En el nostre cas, ens hem centrat en l'estudi de l'índex de Shapley-Shubik.

L'objectiu d'aquest treball és analitzar l'impacte que causaria l'entrada de nous països a la Unió Europea en el poder de vot dins el Consell de la Unió Europea dels diferents estats membres. Per fer-ho, hem analitzat les variacions en l'índex de Shalpey-Shubik dels estats membres de la UE com a resultat de l'entrada de nous països en quatre escenaris diferents.

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Contents

Introduction

The European Union is a unique economic union between countries. It is the largest economic integration between states without actually constituting a country. Its structure and operation emanates from the treaties of the European Union, where the countries agreed to create a single market with free movement of goods, services, people and capital; apply a common foreign trade policy; a single monetary policy; and even the use of the same currency for some of the member states.

The European Union has an institutional framework aimed at promoting and defending its values, objectives and interests, as well as the interests of its citizens and that of all its member states. Within this institutional framework we find four main decision-making institutions which lead the EU's administration. These institutions collectively provide the EU with policy direction and play different roles in the law-making process: the European Parliament, the European Council, the Council of the European Union and the European Commission. The governments of the EU member states are represented in the Council of the European Union, an institution in which they participate in the decision-making process.

The objective of this work is to study the voting power that the governments of the different member states possess in the framework of a qualified majority voting process in the Council of the European Union. More specifically, a study is carried out on the impact that the entry of a new country in the EU would have on the voting power of the member states. For this, different scenarios are analyzed in which the entry of certain combinations of countries that are currently in the process of accession to the European Union is simulated.

To analyze the voting power of countries, the Shapley-Shubik power index is used. This is one of the most widely used power indices in weighted majority game theory. To calculate it, a computational model has been built using programming in C language. This index allows us to represent the voting power of the different member states of the EU in a simple and direct way, since it assigns to each one of the countries a percentage that is equivalent to to their ability to influence the vote.

The Shapley-Shubik index can be a very useful tool for the governments of the different EU member states that allows them to better understand the influence they exert on the decision-making process of the European Union, as well as the possible consequences derived from the acceptance of new member states.

About this work

In the first chapter of the work, a brief introduction to game theory is made and the mathematical bases are established to understand the theory related to power indices. At the end of the chapter, the voting games and the Shpalye-Shubik index are presented.

The second chapter is divided into two large blocks. The first explains in detail the decision-making process within the Council of the European Union as well as the operation of the institution's voting system. The second block explains the necessary conditions to become part of the European Union and presents the seven candidate countries that are currently in the process of joining the EU.

The third chapter begins with the description of the computational model that has been used to carry out the calculations related to the Shapley-Shubik index. Next, the results obtained in the different case studies are analyzed.

Finally, the fourth chapter presents the conclusions that have been drawn from the Shapley-Shubik indices obtained in each of the case studies.

Chapter 1

Game Theory

In this chapter, we will introduce the basic notions of game theory and we will focus on cooperative games, more specifically, the so-called Transferable Utility Games (TU-games). We will study the most important concepts: the core (set-valued solution concept) and the Shapley value (allocation rule). Inside the TU-games, we will give the definition of simple games, which are the ones that we will use to describe the interactive decision situation produced during a voting process in the Council of the European Union. Finally, we will describe the Shapley-Shubik index, which is the power index used to valuate the voting power of the UE members. To carry out the writing of this chapter, the references that have been used are J. González-Díaz, I.García-Jurado and M. G. Fiestras-Janeiro [4], and A. E. Roth [5].

1.1 Introduction to Game Theory

Game theory is the mathematical theory of interactive decision situations. These situations are characterized by the following elements:

i) There is a group of agents.

ii) Each agent has to make a decision.

iii) An outcome results as a function of the decisions of all agents.

iv) Each agent has his own preferences on the set of possible outcomes.

Notation:

Game: the interactive decision situation itself.

Player: equivalent to agent.

Strategy: each of the available decisions for players.

We will start talking about mathematical decision theory for problems with one *decision maker*. This is the basis to develop game theory, where several decision makers interact. In a decision problem there is a *decision maker* who has to choose one or more *alternatives* out of a set A. The decision maker has *preferences* over A, which are usually modeled through a binary relation $\succeq$ in $A \times A$. For each pair $a, b \in A$, $a \succeq b$ is interpreted as "the decision maker either prefers a over b or is indifferent between a and b".

Definition 1.1. A *decision problem* is a pair $(A, \succeq)$, where A is a set of alternatives and $\succeq$ is a weak preference over A. A *weak preference* over A is a complete and transitive binary relation over a set A.

A binary relation $\succeq$ is *complete* if for each pair $a, b \in A, a \succeq b$ or $b \succeq a$ (or both). A binary relation $\succeq$ is *transitive* if for each triple $a, b, c \in A$, if $a \succeq b$ and $b \succeq c$, then $a \succeq c$.

It is complex to deal with preference relations, but there are some commonly used alternative representations that can help us.

Definition 1.2. Let $(A, \succeq)$ be a decision problem. A *utility function* representing $\succeq$ is a funtion $u : A \to \mathbb{R}$ satisfying, for each pair $a, b \in A$, that $a \succeq b$ if and only if $u(a) \geq u(b)$.

Definition 1.3. A convex decision problem is a decision problem $(X, \succeq)$ such that X is a convex subset of a finite dimensional real vector space.

Definition 1.4. Let $(X, \succeq)$ be a convex decision problem. A *linear utility function* representing $\succeq$ is a function $\bar{u} : X \to \mathbb{R}$ satisfying, for each pair $x, y \in X$, the following two conditions:

i) $x \succeq y \Leftrightarrow \bar{u}(x) \geq \bar{u}(y)$, *i.e.*, $\bar{u}$ represents $\succeq$.

ii) For each $t \in [0, 1], \bar{u}(tx + (1 - t)y) = t\bar{u}(x) + (1 - t)\bar{u}(y)$.

The benefit of using a linear utility function is that not only reveals the relative order of each pair of alternatives, but it also tells us how big is the difference between them.

Now, we introduce the concept of *Strategic Game*, which is a static model that describes interactive situations among several players. We will implicitly assume that each player is rational, which means that he tries to maximize his own payoff. We denote the set of players of a game by $N := \{1, ..., n\}$.

Definition 1.5. A n-player *strategic game* with set of players N is a pair $G := (A, u)$ whose elements are the following:

- **Sets of strategies:** For each $i \in N, A_i$ is the nonempty set of strategies of player i and $A := \prod_{i=1}^{n} A_i$ is the set of strategy profiles.
- **Payoff functions:** For each $i \in N, u_i : A \to \mathbb{R}$ is the payoff function of player i and $u := \cup_{i=1}^{n} u_i; u_i$ assigns, to each strategy profile $a \in A$, the payoff that player i gets if a is played.

Example 1.1. Prisoner's dilemma

This is one of the most typical examples in game theory. The situation is the following: two criminals have been arrested for committing a robbery and are suspected of murder. It is known that they are guilty of the robbery but there is no evidence of the murder. The police imprison them in separate cells and question them about the murder. If neither confesses to the crime, they will both go to prison for the robbery. If the two confess to the crime, they will spend 10 years in prison each. If one confesses the crime and the other does not, the confessor will not go to prison while the one who has decided to remain silent will spend 15 years in prison.

The prisoner's dilemma game is a strategic game (A, u) in which

- $A_1 = A_2 = \{NoConfession(NC), Confession(C)\}$
- $u_1 : A_1 \times A_2 \to \{-15, -10, -1, 0\} \subset \mathbb{R}$ defined as $u_1(NC, NC) = -1, u_1(NC, C) = -15, u_1(C, NC) = 0, u_1(C, C) = -10$
- $u_2 : A_1 \times A_2 \to \{-15, -10, -1, 0\} \subset \mathbb{R}$ defined as $u_1(NC, NC) = -1, u_1(NC, C) = 0, u_1(C, NC) = -15, u_1(C, C) = -10$

The most commonly used representation for this game is

	NC	C
NC	-1, -1	-15, 0
C	0, -15	-10, -10

To finish this section, we will propose solution concepts that aim to describe how rational agents should behave. The most important solution concept for strategic games is *Nash Equilibrium*, which is simply a strategy profile such that no player gains when unilaterally deviating from it; *i.e.*, The Nash equilibrium concept searches for "rest points" of the interactive situation described by the strategic game. Let us develop the Nash equilibrium concept in a formal way. Given a game $G = (A, u)$ and a strategic profile $a \in A$, let $(a_{-i}, \hat{a}_i)$ denote the profile $(a_1, ..., a_{i-1}, \hat{a}_i, a_{i+1}, ..., a_n)$.

Definition 1.6. Let $G = (A, u)$ be a strategic game. A *Nash equilibrium* of G is a strategy profile $a^* \in A$ such that, for each $i \in N$ and each $\hat{a}_i \in A_i$,

$$u_i(a^*) \geq u_i(a^*_{-i}, \hat{a}_i)$$

Example 1.2. Going back to the prisoner's dilemma example, the "cooperative" outcome, (-1, -1), is quite good for both players, it is almost all they can get in the game. However, for each player, the strategy C leads to a strictly higher payoff than NC, regardless of the strategy chosen by the other player. Hence, a rational decision maker should always play C. Thus, if both players behave rationally, they get payoffs (-10, -10), which are much worse than the payoffs in the "cooperative" outcome. So, the only Nash equilibrium of the prisoner's dilemma is $a^* = (C, C)$. Moreover, this is the rational behavior in a non-cooperative environment.

1.2 Cooperative Games

As we have already seen in the prisoner's dilemma example, in non-cooperative models the main focus is on strategic aspects of the interaction among the players. All players make their decisions looking for its maximum benefit. An equilibrium situation is obtained when no player has incentives to change its strategy.

On the other hand, in cooperative games, players can agree with the aim of reaching an optimal result for society. The main issue is how to share the benefits arising from cooperation. Now players are worried about notions like fairness and equity instead of being centered in issues like self-enforceability. An important concept in this field are the so-called *coalitions*, which are the different group of players that are formed, and the set of outcomes that they can obtain regardless of what the players outside the coalition do.

Notation: Let $N := \{1, ..., n\}$ be the set of players, then a subset $S \subset N$ is called a *coalition* and $|S|$ denotes the number of players in S.

The most general class of cooperative games are the so-called *nontransferable utility games* or *NTU-game.* The main characteristic of these kind of games is that, even if binding agreements can be agreed between players, the utility generated cannot be freely transferred between them.

Definition 1.7. Given $S \subset N$ and a set $A \subset \mathbb{R}^S$, we say that A is *comprehensive* if, for each pair $x, y \in \mathbb{R}^S$ such that $x \in A$ and $y \leq x$, we have that $y \in A$. Moreover, the *comprehensive hull* of a set A is the smallest comprehensive set containing A.

Definition 1.8. An n-player *nontransferable utility game* (*NTU-game*) is a pair (N, V), where N is the set of players and V is a function that assigns, to each coalition $S \subset N$, a set $V(S) \subset \mathbb{R}^S$. By convention, $V(\emptyset) := \{0\}$. Moreover, for each $S \subset N, S \neq \emptyset$:

i) $V(S)$ is a nonempty and closed subset of $\mathbb{R}^S$.

ii) $V(S)$ is *comprehensive.* Moreover, for each $i \in N, V(\{i\}) \neq \mathbb{R}$, *i.e.*, there is $v_i \in \mathbb{R}$ such that $V(\{i\}) = (-\infty, v_i]$.

iii) The set $V(S) \cap \{y \in \mathbb{R}^S : \text{for each } i \in S, y_i \geq v_i\}$ is bounded.

Hence, an NTU-game is a "simplification" in which, for each $S \subset N$ and each $x \in V(S)$, there is an outcome $r \in R^S$ such that, for each $i \in S, x_i = U_i^S(r)$, where $\{U_i^S\}_{i \in S}$ are the utility functions of the players, which represent their preferences on R^S.

Definition 1.9. Let (N, V) be an NTU-game. Then, the vectors in $\mathbb{R}^N$ are called *allocations.* An allocation $x \in \mathbb{R}^N$ is *feasible* if there is a partition $\{S_1, ..., S_k\}$ of N satisfying that, for each $l \in \{1, ..., k\}$, there is $y \in V(S_l)$ such that, for each $i \in S_l, y_i = x_i$.

Finally, we move to the most widely studied class of cooperative games and the ones that we will use for our study: the *transferable utility games* (*TU-games*). The situation is very similar to the one described before: coalitions can be formed among players and they can enforce certain allocations (through binding agreements); but now, the key point is to decide how the benefits generated by the cooperation of players can be shared among them.

The main difference between TU-games and NTU-games is that, given a coalition S and an allocation $x \in V(S) \subset \mathbb{R}^S$ that players in S can enforce, all the allocations that can be obtained form x by transfers of utility among the players in S, also belong to $V(S)$. Hence, $V(S)$ can be characterized by a single number, denoted by $v(S)$ and called the *worth* of coalition S, given by $\max_{x \in V(S)} \sum_{i \in S} x_i$.

Definition 1.10. A TU-game is a pair (N, v), where N is the set of players and $v : 2^N \rightarrow \mathbb{R}$ is the *characteristic function* of the game. By convention, $v(\emptyset) := 0$.

Notation: When no confusion arises, we denote the game (N, v) by v and, $v(\{i\})$ and $v(\{i, j\})$ by $v(i)$ and $v(i, j)$, respectively.

A TU-game v can be seen as an NTU-game (N, V) by defining, for each nonempty coalition $S \subset N, V(S) := \{y \in \mathbb{R}^S : \sum_{i \in S} y_i \leq v(S)\}$. In general, we interpret $v(S)$, the worth of coalition S, as the benefit that S can generate. Let G^N be the class of TU-games with n players.

Definition 1.11. Let $v \in G^N$ and let $S \subset N$. The restriction of (N, v) to the coalition S is the TU-game (S, v_s), where, for each $T \subset S, v_s(T) := v(T)$.

Definition 1.12. A TU-game $v \in G^N$ is *superadditive* if, for each pair $S, T \subset N$, with $S \cap T = \emptyset$,

$$v(S \cup T) \geq v(S) + v(T)$$

Definition 1.13. A TU-game is *monotonic* if, for each pair $S, T \subset N$ with $S \subset T$, we have $v(S) \leq v(T)$.

We denote by SG^N the set of n-player superadditive TU-games. It is a very important class of TU-games since players have real incentives for cooperation, that is, the union of any pair of disjoint coalitions of players never diminishes the total benefits. Hence, when dealing with superadditive games, it is natural to assume that the *grand coalition*, which is the coalition of all players N, will form and so, the question is how to allocate $v(N)$ among the players.

1.3 The Core of a TU-game

In this section we will study the most important concept dealing with stability: the *core*. Before defining this concept, we are going to introduce some properties of the allocations associated with a TU-game.

Properties: Let $v \in G^N$ and let $x \in \mathbb{R}^N$ be an allocation.

- x is *efficient* if $\sum_{i \in N} x_i = v(N)$
- x is *individually rational* if, for each $i \in N, x_i \geq v(i)$, that is, no player gets less than what he can get by himself.

Definition 1.14. Let $v \in G^N$. The *set of imputations* of $v, I(v)$, is defined by

$$I(v) := \{x \in \mathbb{R}^N : \sum_{i \in N} x_i = v(N) \quad \text{and, for each} \quad i \in N, \quad x_i \geq v(i)\}$$

The *set of imputations* of a TU-game consists of all the efficient and individually rational allocations.

In the same way that we have defined a player with rational behavior as one who seeks the maximum benefit, the same happens in the case of rational coalitions. Then, we introduce a new property:

- x is *coalitionally rational* if, for each $S \in N, \sum_{i \in S} x_i \geq v(S)$, that is, no coalition gets less than what it can get by itself.

Definition 1.15. Let $v \in G^N$. The *core* of $v, C(v)$, is defined by

$$C(v) := \{x \in I(v) : \quad \text{for each} \quad S \subset N, \quad \sum_{i \in S} x_i \geq v(S)\}$$

The *core* is the set of efficient and coalitionally rational allocations. The elements of $C(v)$ are usually called *core allocations*.

It is easy to see that the core is always a subset of the set of imputations. In the set of imputations, $I(v)$, no player will individually block an allocation since he cannot get anything better by himself, while in core allocations, $C(v)$, the same happens with coalitions, they have no incentives to block any of them. That is the reason why we say these allocations are related to stability. Now, we will give a slightly different definition of stability:

Definition 1.16. Let $v \in G^N$. Let $S \subset N, S \neq \emptyset$, and let $x, y \in I(v)$. We say that y *dominates x through S* if

i) for each $i \in S, y_i > x_i$, and

ii) $\sum_{i \in S} y_i \leq v(S)$

We say that y *dominates* x if there is a nonempty coalition $S \subset N$ such that y dominates x through S. Finally, x is an *undominated* imputation of v if there is no $y \in I(v)$ such that y dominates x.

Observe that y dominates x through S if there are players in S that prefer the allocation y than the allocation x, that is, y is a reasonable claim for S. Moreover, if y dominates x, then there are coalitions willing and able to block x. Hence, a stable allocation should be undominated, and that is the case for core allocations.

Proposition 1.1. *Let $v \in G^N$. Then,*

i) If $x \in C(v)$, x is undominated.

ii) If $v \in SG^N, C(v) = \{x \in I(v) : x \text{ is undominated}\}$.

Proof.

i) Let $x \in C(v)$ and suppose there is $y \in I(v)$ and $S \subset N, S \neq \emptyset$, such that y dominates x through S. Then $v(S) \geq \sum_{i \in S} y_i > \sum_{i \in S} x_i \geq v(S)$, which is a contradiction.

ii) Let $x \in I(v) \setminus C(v)$. Then, there is $S \subset N$ such that $\sum_{i \in S} x_i < v(S)$. Let $y \in \mathbb{R}^N$ be defined, for each $i \in N$, by

$$y_i := \begin{cases} x_i + \frac{v(S) - \sum_{j \in S} x_j}{|S|} & i \in S \\ v(i) + \frac{v(N) - v(S) - \sum_{j \in N \setminus S} v(j)}{|N \setminus S|} & i \notin S \end{cases}$$

Since v is superadditive, $v(N) - v(S) - \sum_{j \in N \setminus S} v(j) \geq 0$ and, hence, $y \in I(v)$. Therefore, y dominates x through S.

□

Both the set of imputations and the core of a TU-game are bounded sets. Finally, we introduce one last class of games.

Definition 1.17. A TU-game $v \in G^N$ is a *simple game* if

i) It is monotonic.

ii) For each $S \subset N, v(S) \in \{0,1\}$.

iii) $v(N) = 1$.

We denote the class of simple games with n players by S^N. Observe that, to characterize a simple game $v \in S^N$, it is enough to specify the collection W of its *winning coalitions* $W := \{S \subset N : v(S) = 1\}$ or, equivalently, the collection W^m of its *minimal winning coalitions* $W^m := \{S \in W :$ for each $T \in W$, if $T \subset S$, then $T = S\}$.

Definition 1.18. Let $v \in S^N$. Then, a player $i \in N$ is a *veto player* in v if, for any $S \subseteq N$, $v(S \setminus \{i\}) = 0$.

Proposition 1.2. *Let $v \in S^N$ be a simple game. Then $C(v) \neq \emptyset \Leftrightarrow$ there is at least one veto player in v. Moreover, if $C(v) \neq \emptyset$, then*

$$C(v) = \{x \in I(v) : \quad \text{for each nonveto player} \quad i \in N, \quad x_i = 0\}$$

Proof. Let $v \in S^N$. Let $x \in C(v)$ and let A be the set of veto players. Suppose that $A = \emptyset$. Then, for each $i \in N, v(N \setminus \{i\}) = 1$ and, hence,

$$0 = v(N) - v(N \setminus \{i\}) \geq \sum_{j \in N} x_j - \sum_{j \in N \setminus \{i\}} x_j \geq 0$$

which is incompatible with the efficiency of x.

□

1.4 The Shapley Value

In the previous section we studied the core of a TU-game, which is the most important set-valued solution concept for TU-games. Now, we present the most important allocation rule: the *Shapley value*. First, we will give a formal definition of what an allocation rule is.

Definition 1.19. An *allocation rule* for n-player TU-game is a map $\varphi : G^N \to \mathbb{R}^N$.

Shapley gave some properties that an allocation rule should satisfy and proved that they characterize a unique allocation rule. First, we introduce two new concepts.

Definition 1.20. Let $v \in G^N$.

i) A player $i \in N$ is a *null player* if, for each $S \subset N, v(S \cup \{i\}) - v(S) = 0$.

ii) Two players i and j are *symmetric* if, for each coalition $S \subset N \setminus \{i,j\}, v(S \cup \{i\}) = v(S \cup \{j\})$.

Properties: Let φ be an allocation rule.

- **Efficiency (EFF):** The allocation rule φ satisfies EFF if, for each $v \in G^N$, $\sum_{i \in N} \varphi_i(v) = v(N)$. Hence, EFF requires that φ allocates the total worth of the grand coalition, $v(N)$, among the players.

- **Null Player (NPP):** The allocation rule φ satisfies NPP if, for each $v \in G^N$ and each null player $i \in N, \varphi_i(v) = 0$. Hence, NPP says that players that contribute zero to every coalition should receive nothing.
- **Symmetry (SYM):** The allocation rule φ satisfies SYM if, for each $v \in G^N$ and each pair $i, j \in N$ of symmetric players, $\varphi_i(v) = \varphi_j(v)$. Hence, SYM asks φ to treat equal players equally.
- **Additivity (ADD):** The allocation rule φ satisfies ADD if, for each pair $v, w \in G^N, \varphi(v + w) = \varphi(v) + \varphi(w)$.

Definition 1.21. The *Shapley value*, Φ, is defined, for each $v \in G^N$ and each $i \in N$, by

$$\Phi_i(v) := \sum_{S \subset N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{i\}) - v(S)) \tag{1.1}$$

Therefore, in the Shapley value, each player gets a weighted average of the contributions he makes to the different coalitions. Actually, the formula in Eq. (1.1) can be interpreted as follows: the grand coalition is to be formed inside a room, but the players have to enter the room sequentially, one at a time. When a player i enters, he gets his contribution to the coalition of the players that are already inside. The order of the players is decided randomly, with all the $n!$ possible ordenings being equally likely. It is easy to see that the Shapley value assigns to each player his expected value under this random ordered entry process.

The above discussion suggest an alternative definition of the Shapley value, based on the so-called *vectors of marginal contributions*. Let $\prod(N)$ denote the set of all permutations of the elements in N and, for each $\pi \in \prod(N)$, let $P^\pi(i)$ denote the set of predecessors of i under the ordering given by π, *i.e.*, $j \in P^\pi(i)$ if and only if $\pi(j) < \pi(i)$.

Definition 1.22. Let $v \in G^N$ be a TU-game. Let $\pi \in \prod(N)$. The *vector of marginal contributions* associated with π; $m^\pi(v) \in \mathbb{R}^N$, is defined, for each $i \in N$, by $m_i^\pi(v) := v(P^\pi(i) \cup \{i\}) - v(P^\pi(i))$.

The convex hull of the set of vectors of marginal contributions is commonly known as the *Weber set*, formally introduced as a solution concept by Weber (1988). It is clear from the random order story that the formula of the Shapley value is equivalent to

$$\Phi_i(v) := \frac{1}{n!} \sum_{\pi \in \prod(N)} m_i^\pi(v) \tag{1.2}$$

Definition 1.23. Inside the class G^N, given $S \subset N$, the *unanimity game of coalition* S, w^S, is defined as follows. For each $T \subset N, v(T) := 1$ if $S \subset T$ and $v(T) := 0$ otherwise.

Now, we are able to present the most classic characterization of the Shapley value.

Theorem 1.1. *The Shapley value is the unique allocation rule in G^N that satisfies EFF, NPP, SYM, and ADD.*

Proof. First, it is clear that Φ satisfies both NPP and ADD. Then, as each vector of marginal contributions is an efficient allocation, from Eq. (1.2), it is clear that Φ satisfies EFF. Also, SYM can be easly derived from Eq. (1.2).

Now, let φ be an allocation rule satisfing EFF, NPP, SYM and ADD. Recall that each $v \in G^N$ can be seen as the vector $\{v(S)\}_{S\in 2^N\setminus\{\emptyset\}} \in \mathbb{R}^{2^n-1}$. Then, G^N can be identified with a $2^n - 1$ dimensional vector space. Now, we show that the unanimity games $\mathcal{U}(N) := \{w^S : S \in 2^N\{\emptyset\}\}$ are a basis of such a vector sapce, *i.e.*, we show that $\mathcal{U}(N)$ is a set of linearly independent vectors. Let $\{\alpha_S\}_{S\in 2^N\setminus\{\emptyset\}} \subset \mathbb{R}$ be such that $\sum_{S\in 2^N\setminus\{\emptyset\}} \alpha_S w^S = 0$ and suppose that there is $T \in 2^N \setminus \{\emptyset\}$ with $\alpha_T \neq 0$. We can assume, without loss of generality, that there is no $\hat{T} \subsetneq T$, such that $\alpha_{\hat{T}} \neq 0$. Then, $0 = \sum_{S\in 2^N\setminus\{\emptyset\}} \alpha_S w^S(T) = \alpha_T \neq 0$ and we have a contradiction. Sine φ satisfies EFF, NPP, and SYM, we have that, for each $i \in N$, each $\emptyset \neq S \subset N$, and each $\alpha_S \in \mathbb{R}$,

$$\varphi_i(\alpha_S w^S) = \begin{cases} \frac{\alpha_S}{|S|} & i \in S \\ 0 & otherwise \end{cases} \tag{1.3}$$

Then, if φ also satisfies ADD, φ is uniquely determined, because $\mathcal{U}(N)$ is a basis of G^N.

□

1.5 Voting Games: Power Indices (Shapley-Shubik index)

We have already introduced the class of simple games, S^N. These games are usually used to model voting situations or, more specifically, to measure the power of the different players that vote. That is why, when restricting attention to simple games, the allocation rules are commonly referred to as *power indices*. Suppose that the set N represents the members of a committee and they whether to approve some action or not. Now, the fact that a coalition S is such that $v(S) = 1$ means that, if players in S vote in favor of the action, it will be undertaken. In this context, power indices can be interpreted as the *a priori* ability of the players to change the outcome.

Sometimes in voting games different players have different weights. This situation can be represented by a special class of simple games: the *weighted majority games*.

Definition 1.24. A simple game $v \in S^N$ is a *weighted majority game* if there are a *quota*, $q > 0$, and a system of nonnegative weights, $p_i, ..., p_n$ such that

$$v(S) = 1 \Longleftrightarrow \sum_{i\in S} p_i \geq q$$

The restriction of the Shapley value to the class of simple games is known as the *Shapley-Shubik index*, which therefore is also denoted by Φ. This is the most commonly used index to calculate the *p-power* of a player, which measures the influence or control that a player has over the outcomes of the game. In the context of budgetary decisions, for example, the index shows a voter's influence on spending the money.

Although we have already given a characterization of the Shapley value for the class of TU-games with n players, G^N, this characterization is not valid when restricting attention to S^N. The reason is that the sum of simple games is never a simple game and, therefore, the additivity axiom is senseless in S^N. The characterization of Shapley-Shubik index replacing ADD is what is called the *transfer property* (TF). Before introducing this new property, we have to give three new definitions.

Definition 1.25. Let $v, \hat{v} \in G^N$. The *maximum game of* v *and* $\hat{v}$, $v \vee \hat{v}$, is defined, for each $S \subset N$, by $(v \vee \hat{v})(S) := \max\{v(S), \hat{v}(S)\}$. Analogously, the *minimum game of* v *and* $\hat{v}$, $v \wedge \hat{v}$, is defined, for each $S \subset N$, by $(v \wedge \hat{v})(S) := \min\{v(S), \hat{v}(S)\}$.

The TF property relates the power indices assigned to the maximum and the minimum games of two simple games. Let $v, \hat{v} \in S^N$. The set of winning coalitions of the simple game $v \vee \hat{v}$ is the union of the sets of winning coalitions of v and $\hat{v}$. In the simple game $v \wedge \hat{v}$, the set of winning coalitions is the intersection of the set of winning coalitions of v and $\hat{v}$.

Definition 1.26. Inside the class G^N, given $S \subset N$, the *unanimity game of coalition* S, w^S, is defined as follows. For each $T \subset N, v(T) := 1$ if $S \subset T$ and $v(T) := 0$ otherwise.

Given a simple game $v \in S^N$ with minimal winning coalition set $W^m = \{S_1, ..., S_k\}$, then $v = w^{S_1} \vee w^{S_2} \vee ... \vee w^{S_k}$. We now present the transfer property.

- **Transfer (TF):** A power index φ satisfies TF if, for each pair $v, \hat{v} \in S^N, \varphi(v \vee \hat{v}) + \varphi(v \wedge \hat{v}) = \varphi(v) + \varphi(\hat{v})$.

Theorem 1.2. *The Shapley-Shubik index is the unique allocation rule in* S^N *that satisfies EFF, NPP, SYM and TF.*

Proof. From Theorem 1.1, the Shapley-Shubik index satisfies EFF, NPP and SYM. Moreover, given two simple games v and $\hat{v}, v + \hat{v} = v \vee \hat{v} + v \wedge \hat{v}$ and, hence, since the Shapley value satisfies ADD, the Shapley-Shubik index satisfies TF.

We prove the uniqueness by induction on the number of minimal winning coalitions, *i.e.*, on $|W^m|$. Let φ be a power index satisfying EFF, NPP, SYM and TF. Let v be a simple game. If $|W^m| = 1$, then v is the unanimity game for some coalition $S, i.e., v = w^S$. By EFF, NPP and SYM $\varphi(v) = \Phi(v)$. Assume that, for each simple game v such that $|W^m| \leq k-1, \varphi(v) = \Phi(V)$. Let v be a simple game $|W^m| = k, i.e., W^m = \{S_1, ..., S_k\}$ and

$$v = w^{S_1} \vee w^{S_2} \vee ... \vee w^{S_k}$$

Let $\hat{v} = w^{S_2} \vee ... \vee w^{S_k}$. Then, $w^{S_1} \vee \hat{v} = v$ and

$$w^{S_1} \wedge \hat{v} = \vee_{T \in W^m \setminus \{S_1\}} w^{T \cup S_1}$$

The number of minimal winning coalitions of each of the simple games $w^{S_1}, \hat{v}$ and $w^{S_1} \wedge \hat{v}$ is smaller than k. Then, by the induction hypothesis, $\varphi(w^{S_1}) = \Phi(w^{S_1}), \varphi(\hat{v}) = \Phi(\hat{v})$, and $\varphi(w^{S_1} \wedge \hat{v}) = \Phi(w^{S_1} \wedge \hat{v})$. By TF, $\varphi(w^{S_1} \vee \hat{v}) + \varphi(w^{S_1} \wedge \hat{v}) = \varphi(w^{S_1}) + \varphi(\hat{v})$. Hence, since $w^{S_1} |lor \hat{v} = v$,

$$\begin{aligned}\varphi(v) = \varphi(w^{S_1}) + \varphi(\hat{v}) - \varphi(w^{S_1} \wedge \hat{v}) &= \Phi(w^{S_1}) + \Phi(\hat{v}) - \Phi(w^{S_1} \wedge \hat{v}) \\ &= \Phi(w^{S_1} + \hat{v} - w^{S_1} \wedge \hat{v}) = \Phi(v)\end{aligned}$$

□

Coming back to a previous example, voters arrive to the room randomly; if and when a coalition turns, winning of the full credit is given to the last arriving, the so-called *pivotal player*. Now, we are able to give a formula to calculate the Shapley-Shubik index. Let $i \in N$, we denote by $\mu_i(S)$ the set of coalitions $S \subset N$, with $i \notin S$, such that $S \cup \{i\}$ is a winning coalition, the, if N is the set of players,

$$\Phi_i(v) = \sum_{S \subseteq \mu_i(S)} \frac{|S|!(n - |S| - 1)!}{n!}$$

Chapter 2

The Council of the European Union

In this chapter we will talk about the European Union (EU) and its most important institutions. We will focus on the Council of the European Union, which is one of the main decision-making bodies of the EU. We will see in detail the decision-making process within the EU and how the voting system works within the Council of the EU. Finally, we will talk about the process that a country has to go through to enter the EU and which are the candidate countries at the moment.

2.1 Introduction to the European Union

The European Union (EU) is a supranational political and economic union. It is formed by 27 member states with a total population of about 447 million and its total area is about 4,233 million km^2. EU's constitutional basis is set by the Treaties of the European Union, which are a set of international treaties between the EU member states.

Figure 2.1: Map of the countries that are part of the European Union.

The EU operates through a hybrid system of supranational and intergovernmental decision-making, and according to two principles:

- **Principle of Conferral:** it says that the EU should act only within the limits of the competences conferred on it by the treaties.
- **Principle of Subsidiarity:** it says that EU should act only where an objective cannot be sufficient achieved by the member states acting alone.

Constitutionally, the EU bears some resemblance to both a confederation and a federation, but has not formally defined itself as either. It does not have a formal constitution, but its status is defined by the *Treaty of European Union* and the *Treaty on the Functioning of the European Union*. It is more integrated than a confederation of states because the general level of government widely employs qualified majority voting in some decision-making among the member states, but it is less integrated than a federal state because it is not a state in its own right. This is reflected in the fact that member states remain the control over the Treaties, retaining control over the allocation of competences to the union through constitutional change.

EU policy is in general promulgated by EU directives, which are then implemented in the domestic legislation of the member states, and EU regulations, which are immediately enforceable in all member states. Under the principle of supremacy, national courts are required to enforce the treaties that their member states have ratified. The EU legislation is divided into two parts: Primary law (which is formed by the Treaties of the European Union) and Secondary law (which is formed by regulations, directives and decisions).

The most important EU institutions are:

- **European Council:** Is where the heads of state or government of the EU countries meet to define the general political direction and priorities of the European Union.
- **Council of the European Union:** Represents the governments of EU countries. National ministers from each government meet in the Council of the EU to adopt laws and coordinate policies.
- **European Commission:** Represents the common interests of the EU an is the EU's main executive body. It uses its *right of initiative* to put forward proposals for new laws, which are scrutinised and adopted by the European Parliament and the Council of the EU. It also manages EU policies and the EU's budget and ensures that countries apply EU law correctly.
- **European Parliament:** Represents the citizens of EU countries ans is directly elected by them. It takes decisions on European laws jointly with the Council of the EU. It also approves the EU budget.
- **Court of Justice of the European Union:** Ensures that EU law is followed, and that the Treaties are correctly interpreted and applied.
- **European Court of Auditors:** Audits and supervises the accounts and budgets of EU institutions.
- **European Central Bank:** It is responsible for keeping price stable in the euro area and for the monetary and exchange rate policy in the Eurozone.

2.2 Council of the European Union and its voting system

In the Council of the European Union, informally known as the Council, government ministers from each EU country meet to discuss, amend and adopt laws, and coordinate policies. The ministers have the authority to commit their governments to the actions agreed on in the meetings. Together with the European Parliament, the Council is the main decision-making body of the EU.

Council of the
European Union

Figure 2.2: Council of the EU logo (left) and room where Council of EU meetings take place (right).

Council meetings take place in Brussels, except for three months (April, June and October) when they are held in Luxembourg. There are no fixed members of the EU Council, instead, the Council meets in 10 different configurations, each corresponding to the policy area being discussed. Depending on the configuration, each country sends their minister responsible for that policy area. European Commissioners responsible for the areas concerned are also invited to Council meetings. This is the list of the Council's 10 configurations:

- Agriculture and fisheries
- Competitiveness
- Economic and financial affairs
- Environment
- Employment, social policy, health and consumer affairs
- Education, youth, culture and sport
- Foreign affairs
- General affairs
- Justice and home affairs
- Transport, telecommunications and energy

There is no hierarchy among the Council configurations, although the General Affairs Council has a special coordination role and is responsible for institutional, administrative and horizontal matters. The Foreign Affairs Council also has a special remit. Any of the Council's 10 configuration can adopt an act that falls under the remit of another configuration. Therefore, with any legislative act, the Council adopts no mention is made of the configuration. Meeting are chaired by the minister of the member state holding the 6-month Council presidency. The exception is the Foreign Affairs Council, which is usually chaired by the High Representative of the Union for Foreign Affairs and Security Policy.

2.2.1 Decision-Making Process

The Council of the European Union is an EU essential decision-maker. In most cases, the Council decides together with the European Parliament through the *ordinary legislative procedure*, also known as *codecision*. Codecision is used for policy areas where the EU has exclusive or shared competence with the member states. In these cases, the Council legislates on the basis of proposals submitted by the European Commission. In a number of very specific areas, the Council takes decisions using special legislative procedures (the consent procedure and the consultation procedure) where the role of the Parliament is limited. Under certain conditions the power to adopt delegated and implementing acts may be conferred on the European Commission.

In the Council there are more than 150 working parties and committees (known as *preparatory bodies*) that help prepare the work of ministers who examine proposals in the different Council configurations. These working parties and committees are formed by officials from all the member states. Once a Commission proposal has been received by the Council, the text is examined simultaneously by the Council ans the Parliament (this examination is known as a *reading*). There can be up to three readings before they agree on or reject a legislative proposal.

The Council may sometimes adopt a political agreement pending first reading position of the Parliament, also known as a *general approach*. A general approach agreed in the Council can help to speed up the legislative procedure and even facilitate an agreement between the two institutions, as it gives the Parliament an indication of the Council's position prior to their first reading opinion. The Council's final position, however, cannot be adopted until the Parliament has delivered its own first reading opinion.

At each reading the proposal passes through three levels at the Council:

- **The working party:** The presidency of the Council, with the assistance of the General Secretariat, identifies and convenes the appropriate working party to handle a proposal. A working party begins with a general examination of the proposal, and then makes a line-by-line scrutiny of it. There is no formal time limit for a working party to complete its work; the time taken depends on the nature of the proposal. There is also no obligation for the working party to present an agreement, but the outcome of their discussions is presented to *Coreper*.
- **Permanent Representatives Committee (Coreper)** The Coreper is a EU body inside the Council where each member state of the EU has a permanent representative and a deputy permanent representative with the rank of EU ambassador. It is divided into two main committees:
  - Coreper I: consists of deputy heads of mission and delays largely with social and economic issues.

  - Coreper II: consists of heads of mission and deals largely with political, financial and foreign policy issues.

  Coreper treatment of the proposal depends on the level of agreement reached at working party level. If agreement can be reached without discussion, items appear on Part I of the Coreper agenda. If further discussion is needed within Coreper because agreement has not been reached in the working party on certain aspects of a proposal, items are listed in Part II of the Coreper agenda. In this case, Coreper can: try to negotiate a settlement itself; refer the proposal back to the working party, perhaps with suggestions for a compromise; pass the matter up to the Council. Most proposals feature on the agenda of Coreper several times, as they try to resolve differences that the working party has not overcome.

- **Council configuration:** This last part of the decision-making process has two main paths to decide on the proposal, depending again on the level of agreement reached before:
  - **'A' item on the Council agenda:** When Coreper has been able to finalise discussions on a proposal, meaning that agreement is expected without debate. As a rule, around two-thirds of the items on a Council agenda will be for adoption as 'A' items. Discussion on these items can nevertheless be re-opened if one or more member states so request.
  - **'B' section of the Council agenda:** It includes proposals: (*i*) left over from previous Council meetings; (*ii*) upon which no agreement was reached in Coreper or at working party level; (*iii*) that are too politically sensitive to be settled at a lower level

The results of Council votes are automatically made public when the Council acts in its capacity as legislator.

2.2.2 Voting System

Depending on the issue under discussion, the Council of the EU takes its decisions by:

- **Simple majority:** 14 member states vote in favour. The Council takes decisions by simple majority:
  - in procedural matters, such as the adoption of its own rules of procedure and organisation of its secretariat general, the adoption of the rules governing the committees foreseen in the Treaties.
  - to request the Commission to undertake studies or submit proposals.
- **Qualified majority:** 55% of member states (in practice this means 15 out of 27), representing at least 65% of the EU population, vote in favour. This majority is required when the Council votes on a proposal by the Commission or the High Representative of the Union for Foreign Affairs and Security Policy. This procedure is also known as the *double majority* rule. However, a blocking minority can be formed when at least four Council members representing more than 35% of the EU population vote against the proposal. Special cases:

  - When not all Council members participate in the vote, for example due to an opt-out in certain policy areas, a decision is adopted if 55% of the participating Council members, representing at least 65% of the population of the participating member states, vote in favour.
  - When the Council votes on a proposal not coming from the Commission or the high representative a decision is adopted if, the so-called *reinforced qualified majority* is reached: at least 72% of Council members vote in favour and they represent at least 65% of the EU population.

  As for abstentions, under qualified majority voting, they count as votes against. Abstention is not the same as not participating in the vote. Any member can abstain at any time.

  Qualified majority is the most widely used voting method in the Council. It's used when the Council takes decisions during the ordinary legislative procedure, also known as co-decision. About 80% of all EU legislation is adopted with this procedure.

- **Unanimous vote:** All member states vote in favour. The Council has to vote unanimously on a number of matters which the member states consider to be sensitive. For example:
  - Common foreign and security policy (with the exception of certain clearly defined cases which require qualified majority, e.g. appointment of a special representative).
  - Citizenship (the granting of new rights to EU citizens).
  - EU membership.
  - Harmonisation of national legislation on indirect taxation.
  - EU finances (own resources, the multiannual financial framework).
  - Certain provisions in the field of justice and home affairs (the European prosecutor, family law, operational police cooperation, etc.).
  - Harmonisation of national legislation in the field of social security and social protection.

  In addition, the Council is required to vote unanimously to diverge from the Commission proposal when the Commission is unable to agree to the amendments made to its proposal. This rule does not apply to acts that need to be adopted by the Council on a Commission recommendation, for example, acts in the area of economic coordination. Under unanimous voting, abstention does not prevent a decision from being taken.

The Council can vote only if a majority of its members is present. A member of the Council may only act on the behalf of one other member. The Council can vote on a legislative act 8 weeks after the draft act has been sent to national parliaments for their examination. The national parliaments have to decide whether the draft legislation complies with the principle of subsidiarity. Earlier voting is only possible in special urgent cases. Voting is initiated by the President of the Council. A member of the Council or the Commission can also initiate the voting procedure, but a majority of the Council's members have to approve this initiative.

The results of Council votes are automatically made public when the Council acts in its capacity as legislator. If a member wants to add an explanatory note to the vote, this note will also be made public, if a legal act is adopted. In other cases, when explanations of votes are not automatically published, it can be made public on the request of the author. Where the Council is not acting as legislator, it is also possible for the results of votes and explanations of vote to be made public by a unanimous Council decision. The Council and Commission members may make statements and request that they be included in the Council minutes. Such statements have no legal effect and are regarded as a political instrument intended to facilitate decision-making.

2.3 Countries in the process of joining the EU

Currently there are 27 member states that form part of the European Union. Here is a list of these countries and their year of entry into the EU:

- **1958:** Germany, Belgium, France, Italy, Luxembourg, Netherlands
- **1973:** Denmark, Ireland
- **1981:** Greece
- **1986:** Spain, Portugal
- **1995:** Austria, Finland, Sweden
- **2004:** Czechia, Cyprus, Slovakia, Slovenia, Estonia, Hungary, Latvia, Lithuania, Malta, Poland
- **2007:** Bulgaria, Romania
- **2013:** Croatia

2.3.1 Conditions for membership and joining process

The EU operates comprehensive approval procedures that ensure new members are admitted only when they can demonstrate they will be able to play their part fully as members, namely by:

- Complying with all the EU's standards and rules.
- Having the consent of the EU institutions and EU member states.
- Having the consent of their citizens, as expressed through approval in their national parliaments or by referendum.

The Treaty on the European Union states that any European country may apply for membership if it respects the democratic values of the EU and is committed to promoting them. The first step is for the country to meet the key criteria for accession. These were mainly defined at the European Council in Copenhagen in 1993 and are hence referred to as *Copenhagen criteria*:

- Stable institutions guaranteeing democracy, the rule of law, human rights and respect for and protection of minorities.
- A functioning market economy and the capacity to cope with competition and market forces in the EU.
- The ability to take on and implement effectively the obligations of membership, including adherence to the aims of political, economic and monetary union.

In the case of the countries of the Western Balkans additional conditions for membership, were set out in the so-called *Stabilisation and Association process*, mostly relating to regional cooperation and good neighbourly relations.

When a candidate country accomplish the Copenhagen Criteria, a negotiation process between the country and the EU starts. They negotiate the conditions and timing of the candidate's adoption, implementation and enforcement of all current EU rules (known as *acquis*). These rules are divided into different policy fields, such as transport, energy, environment, etc. each of which is negotiated separately. Other issues discussed are:

- **Financial arrangements:** such as how much the new member is likely to pay into and receive from the EU budget.
- **Transitional arrangements:** sometimes certain rules are phased in gradually, to give the new member or existing members time to adapt.

Throughout the negotiations, the Commission monitors the candidate's progress in applying EU legislation and meeting its other commitments, including any benchmark requirements. This gives the candidate additional guidance as it assumes the responsibilities of membership, as well as an to current members that the candidate is meeting the conditions for joining. The Commission also keeps the EU Council and European Parliament informed throughout the process, through regular reports, *Communication*, and clarifications on conditions for further progress.

2.3.2 Candidate Countries

When a country wants to join the EU but does not yet meet the Copenhagen criteria, it is referred to as *potential candidate*. This is the case of Bosnia and Herzegovina, Georgia and Kosovo. The *candidates countries* to be part of the European Union are the countries which have already acquired the Copenhagen criteria and are in process of *transposing* (integrating) EU legislation into national law. These countries are:

Albania

With a population of 2.793.592 inhabitants, Albania (along with other Western Balkans countries) was identified as a potential candidate for EU membership during the *Thessaloniki European Council* summit in June 2013.

In 2009, Albania submitted its formal application for EU membership. In 2010, the Commission assessed that before accession negotiations could be formally opened, Albania still had to achieve a necessary degree of compliance with the membership criteria. In

Figure 2.3: Map of the candidate countries to join the EU.

October 2012, Commission recommended that Albania be granted EU candidate status, subject to completion of key measures in the areas of judicial and public administration reform and revision of the parliamentary rules of procedures.

In June 2014, Albania was awarded candidate status by the EU. In April 2018, the Commission issued an unconditional recommendation to open accession negotiations. In its June 2018 Conclusions, the Council set out the path towards opening accession negotiations in June 2019, depending on progress made in key areas such as the judiciary, fight against corruption and organised crime, intelligence services and public administration. The Commission reiterated the recommendation to open accession talks in the Enlargement Package adopted in May 2019. In its June 2019 Conclusions, the Council took good note of the Commission's recommendation. In March 2020 the members of the European Council endorsed the General Affairs Council's decision to open accession negotiations with Albania and in July 2020 the draft negotiating framework were presented to the Member States. In July 2022, the Intergovernmental Conference on accession negotiations was held with Albania. The Commission started the screening process.

Moldova

With a population of 2.597.100 inhabitants, Moldova cooperates with the EU in the framework of the *European Neighbourhood Policy* and its eastern regional dimension, the *Eastern Partnership*. The key goal is to bring Moldova closer to the EU.

On 3 March 2022, the Republic of Moldova presented its applications for EU membership. On 17 June 2022, the European Commission presented its Opinions on the application for EU membership submitted by Ukraine, Georgia and the Republic of Moldova. Based on the Commission's opinion on the country's application for EU membership, Moldova was given a European perspective and granted candidate status on 23 June 2022 by unanimous agreement between the leaders of all 27 EU Member States. Candidate status was granted on the understanding that Moldova take some key steps. The Commission will monitor their progress in fulfilling these steps and report on them, as part of its regular enlargement package.

Montenegro

Montenegro is a country located in the Balkan peninsula (South-East Europe) that has 617.683 inhabitants. In 2006 Montenegro's parliament declared independence from the

State Union of Serbia and Montenegro. In 2008, the new country applied for EU membership. In 2010, the Commission issued a favourable opinion on Montenegro's application, identifying 7 key priorities that would need to be addressed for negotiations to begin, and the Council granted it candidate status. In December 2011, the Council launched the accession process with a view to opening negotiations in June 2012. The accession negotiations with Montenegro started on 29 June 2012. After eight years of accession negotiations all the 33 screened chapters have been opened, of which 3 are provisionally closed.

North Macedonia

With a population of 1.836.713 inhabitants, North Macedonia (along with other Western Balkans countries) was identified as a potential candidate for EU membership during the *Thessaloniki European Council* summit in June 2013. Its Stabilisation and Association Agreement, the first in the region, is in force since 2004. It applied for EU membership in March 2004 and the Council decided in December 2005 to grant the country candidate status.

Since October 2009, the Commission has continuously recommended to open accession negotiations with North Macedonia. In 2015 and 2016, the recommendation was made conditional on the continued implementation of the Pržino agreement and substantial progress in the implementation of the *Urgent Reform Priorities*. In light of the progress achieved, the Commission repeated its unconditional recommendation to open accession negotiations in April 2018. In light of the significant progress achieved and the conditions set unanimously by the Council in June 2018 having been met, the Commission recommended in May 2019 to open accession negotiations with North Macedonia. In March 2020, the General Affairs Council decided to open accession negotiations with North Macedonia and endorsed the Commission Communication on a revised methodology *Enhancing the accession process - A credible EU perspective for the Western Balkans* of February 2020. The decision was endorsed by members of the European Council. In July 2020 the draft negotiating framework was presented to the Member States. In July 2022, the Intergovernmental Conference on accession negotiations was held with North Macedonia. The Commission started the screening process.

Serbia

With a population of 6.797.105 inhabitants, Serbia (along with other Western Balkans countries) was identified as a potential candidate for EU membership during the *Thessaloniki European Council* summit in June 2013.

In 2008, a European partnership for Serbia was adopted, setting out priorities for the country's membership application, and in 2009 Serbia formally applied. In March 2012 Serbia was granted EU candidate status. In September 2013 a Stabilisation and Association Agreement between the EU and Serbia entered into force. In line with the decision of the European Council in June 2013 to open accession negotiations with Serbia, the Council adopted in December 2013 the negotiating framework and agreed to hold the 1st Intergovernmental Conference with Serbia in January 2014. On 21 January 2014, the 1st Intergovernmental Conference took place, signaling the formal start of Serbia's accession negotiations. So far, 22 out of 35 chapters have been opened, two of which are provisionally closed.

Turkey

The Republic of Turkey is a transcontinental country mainly located on the Anatolian Peninsula in Western Asia, with a small portion om the Balkan Peninsula in Southeast Europe that has 84.680.273 inhabitants. In 1987, Turkey applied to join what was then the European Economic Community, and in 1999 it was declared eligible to join the EU. Turkey's involvement with European integration dates back to 1959 and includes the *Ankara Association Agreement* (1963) for the progressive establishment of a Customs Union (ultimately set up in 1995).

Accession negotiations started in 2005, but until Turkey agrees to apply the Additional Protocol of the Ankara Association Agreement to Cyprus, eight negotiation chapters will not be opened and no chapter will be provisionally closed. In 2018, due to continuing backsliding in reforms in the key areas of the enlargement strategy, in particular in the functioning of the democratic system, respect for fundamental rights and independence of the judiciary, the Council decided that accession negotiations were at a standstill. Turkey's economy is facing several challenges, such as high unemployment, depreciation of its currency and record-high inflation. Strong economic volatility has undermined the business environment and overreliance on external financing has created vulnerabilities.

Turkey has seen an unprecedented influx of people seeking refuge from Syria which has exceeded 3.6 million to date. Furthermore, the increasing political instability across its eastern border has brought around 330.000 asylum-seekers and refugees from other countries including Iraq, Afghanistan, Iran and Somalia to Turkey. Overall, Turkey is hosting one of the largest refugee populations worldwide and has already spent significant financial resources on addressing this crisis. The EU and Turkey confirmed their shared commitment to end irregular migration from Turkey to the EU in their *joint statement of 18 March 2016*.

Turkey is eligible for EU financial support through the Instrument for Pre-accession Assistance (*IPA III 2021-2027*) and also from the *European Fund for Sustainable Development Plus* (EFSD+). To support public and private investments in the priority areas of the EFSD+ (Green Deal, Global Gateways and Decent Jobs), the Turkey Investment Platform was established in 2022.

Ukraine

With a population of 41.167.336 inhabitants, Ukraine sent its application for EU membership on 28 February 2022. On 17 June 2022, the European Commission presented its Opinions on the application for EU membership submitted by Ukraine, Georgia and the Republic of Moldova. Based on the Commission's opinion on the country's application for EU membership, Ukraine was given a European perspective and granted candidate status on 23 June 2022 by unanimous agreement between the leaders of all 27 EU Member. Candidate status was granted on the understanding that Ukraine take some key steps. The Commission will monitor their progress in fulfilling these steps and report on them, as part of its regular enlargement package.

Almost eight years after the beginning of the conflict in eastern Ukraine and the illegal annexation of Crimea, Russia launched on 24 February 2022 a large-scale military invasion of the whole country. Over 8 million people have fled to neighbouring countries and there are 6.6 million internal displaced persons within the country. The European

Union has reacted adopting four sets of unprecedented sanctions against Putin's regime, and its collaborator, the Lukashenko regime. The European Commission (Support Group for Ukraine, SGUA) together with the EU Delegation to Ukraine have been working relentlessly to coordinate support for Ukraine and rapidly mobilise emergency assistance to the country (apart from humanitarian aid and aid through the Union Civil Protection Mechanism).

Chapter 3

Study on the impact on the EU member states' power indexes due to the entrance of a new country

In the third and last chapter we will proceed to carry out a study of four cases in which we will analyze how the voting power of the different EU countries in the qualified majority voting process in the Council of the EU varies depending on the impact produced by the entry of one or more countries according to Shapley-Shubik. First we will talk about the computational model developed to calculate the Shapley-Shubik index and then we will analyze the results obtained in each case and we will draw conclusions from each of the scenarios.

3.1 Computational Model

3.1.1 Initial Approximations

In order to elaborate a computational model that would allow us to obtain the value of the Shapley-Shubik index in a voting game with so many players, we have started from the theoretical base developed in the first chapter of this thesis and the knowledge in programming with the C language.

The initial idea was to create a program that would calculate all the possible permutations of a vector of n components, where n varies between 27 and 34 (the number of countries to study in the different cases). Then, the different coalitions obtained by adding each of the countries and their populations in the order of the permutation would be analyzed, the first winning coalition would be found and a point would be added to the last country to enter, as explained in chapter 1. Finally, using the formula for calculating the Shapley-Shubik index, that is, dividing the score obtained by each country by $n!$, the value of the Shapley-Shubik index for each of the countries would be obtained. After finding an algorithm with which to calculate all possible permutations, we realized that it would take years to run the program. The number of possible permutations of a vector of n components is equal to $n!$, so it is a factorial increase, which is the largest of all. The execution time of the program would also increase in a factorial way, so it was not feasible to apply a model of this type.

Next, it was decided to try to limit the number of permutations to calculate. The idea was that when a winning coalition of players was found, there was no need to calculate the permutations resulting from permuting the elements of the vector behind the country that makes the coalition winning. Let's see an example to understand it better:

Example 3.1. Let's say we have two vectors, one with the populations of the 27 EU countries ordered from highest to lowest and another with the respective names of each country:

(83.294.632, 64.756.583, 58.870.762, 47.519.627, 41.026.067, 19.892.811, 17.618.298, 11.686.140, 10.612.086, 10.495.295, 10.341.277, 10.156.239, 8.958.960, 6.687.716, 5.910.913, 5.795.199, 5.545.474, 5.056.934, 4.008.616, 2.718.351, 2.119.674, 1.830.211, 1.322.765, 1.260.138, 654.767, 535.064)

(Germany, France, Italy, Spain, Poland, Romania, Netherlands, Belgium, Sweden, Czech Republic, Greece, Portugal, Hungary, Austria, Bulgaria, Denmark, Slovakia, Finland, Ireland, Croatia, Lithuania, Slovenia, Latvia, Estonia, Cyprus, Luxembourg, Malt)

It is reasonable to think that in this arrangement of the elements of the population vector, the winning coalition will be obtained when the country that occupies the 15th position (Bulgaria) enters the coalition, since with it it would reach the 55% of the member states required to obtain a qualified majority. On the other hand, as the populations are ordered from highest to lowest, the 65% of the EU population required is widely exceeded. For this order of the population vector, Bulgaria would get the point.

Let's note that even if we change the order of the populations of the countries that are behind Bulgaria, this country would always take the point, since with the first 15 countries we already obtain a winning coalition and the rest of the populations do not play any role. In fact, let's note that the same thing happens if we change the order of the countries that are ahead of Bulgaria. Here is an example of what we are talking about:

(83.294.632, 64.756.583, 58.870.762, 47.519.627, 41.026.067, 19.892.811, 17.618.298, 11.686.140, 10.612.086, 10.495.295, 10.341.277, 10.156.239, 8.958.960, 6.687.716, 5.910.913, 5.795.199, 5.545.474, 5.056.934, 4.008.616, 2.718.351, 2.119.674, 1.830.211, 1.322.765, 1.260.138, 535.064, 654.767)

(Germany, France, Italy, Spain, Poland, Romania, Netherlands, Belgium, Sweden, Czech Republic, Greece, Portugal, Hungary, Austria, Bulgaria, Denmark, Slovakia, Finland, Ireland, Croatia, Lithuania, Slovenia, Latvia, Estonia, Cyprus, Malt, Luxembourg)

In this case, where the populations of Malta and Luxembourg are swapped, Bulgaria would still be the one to take the point. Following this same logic, it is unnecessary to calculate the 12! permutations resulting from changing the order of the last 12 countries of the vector, since Bulgaria would always take the point. In this way we could award 12! points directly to Bulgaria and save us a lot of calculating permutations.

As we have observed in the example, with this method we could save ourselves from calculating a large number of permutations. However, once the algorithm of the program was adjusted, the time required to execute the program was still too long and it was not feasible to calculate the Shapley-Shubik index using this method.

Another idea that was considered was to use the transfer property (TF) of the Shapley-Shubik index that is explained in chapter 1. Let's do a little reminder:

- **Transfer (TF):** A power index φ satisfies TF if, for each pair $v, \hat{v} \in S^N, \varphi(v \vee \hat{v}) +$

$\varphi(v \wedge \hat{v}) = \varphi(v) + \varphi(\hat{v})$.

Where $v \vee \hat{v}$ is the *maximum game of* v *and* $\hat{v}$, defined, for each $S \subset N$, by $(v \vee \hat{v})(S) := \max\{v(S), \hat{v}(S)\}$. Analogously, $v \wedge \hat{v}$ is the *minimum game of* v *and* $\hat{v}$, defined, for each $S \subset N$, by $(v \wedge \hat{v})(S) := \min\{v(S), \hat{v}(S)\}$.

Rearranging the formula, we get:

$$\varphi(v \wedge \hat{v}) = \varphi(v) + \varphi(\hat{v}) - \varphi(v \vee \hat{v})$$

We must bear in mind that the qualified majority voting game we are dealing with is a double majority game. In other words, a coalition is winning when both quotas are met, the number of population and the number of countries. The idea was to adjust the program to calculate $\varphi(v \vee \hat{v})$, which would further limit the calculation of permutations since in this case the winning coalitions are those that meet either of the two conditions: represent 55% of the countries or 65% of the the population. On the other hand, $\varphi(v)$ and $\varphi(\hat{v})$ would be obtained from programs found on the internet that allow to calculate the voting power of weighted majority games. Then we would only have to apply the formula to obtain $\varphi(v \wedge \hat{v})$, which is equivalent to the Shapley-Shubik index. The idea was discarded given the complexity and the number of steps required to calculate the index. Also, because the intention was to make a program ourselves, not to use other people's programs.

3.1.2 Calculation of the Shapley-Shubik indices based on sampling

Finally, it was decided to use a model for calculating the Shapley-Shublic index by sampling. In Castro et al. [1] a polynomial calculation of the Shapley value based on sampling is explained. As we have seen, exact calculations of Shapley-Shubik indices are very difficult to obtain, since, in general, with a large number of players, the complexity of the calculation is exponential. Sampling (see Cochran [2]) is a process or method of drawing a representative group of individuals or cases from a particular population. Another example would be Fatima et al. [3], where a randomized polynomial method for determining the approximate Shapley value is presented for voting games.

In our case, we have decided to calculate an approximation of the Shapley-Shubik index using a random selection of 10.000.000 permutations of the population vector. The algorithm (Annex 1) works as follows:

i) The data of the countries and their respective populations are read from a file (Annex 2) and inserted into vectors.

ii) A permutation of the population vector is randomly chosen.

iii) The permutation is sent to a function that calculates the winning coalition and awards a point to the last country to enter.

iv) Steps 2 and 3 are repeated 10.000.000 times.

v) Divide each of the elements of the country point vector by 10.000.000 to obtain the approximate value of the Shapley-Shubik index for each country.

vi) The results are entered in an output file (Annex 3).

As it is a random selection of permutations, the results obtained each time the program is executed are different. However, it is a fairly good approximation since the dispersion between different results is bounded by $\pm 0,0001$. Below is a table that compares the results obtained in five different executions for the same data (population of UE27 in 2023):

Country	Population	Result 1	Result 2	Result 3	Result 4	Result 5
Austria	8.958.960	0,02347	0,02347	0,02346	0,02346	0,02348
Belgium	11.686.140	0,02751	0,02753	0,02751	0,02757	0,02747
Bulgaria	6.687.716	0,02028	0,02030	0,02020	0,02026	0,02028
Croatia	4.008.616	0,01648	0,01647	0,01642	0,01649	0,01649
Cyprus	1.260.138	0,01254	0,01252	0,01249	0,01252	0,01250
Czech Republic	10.495.295	0,02580	0,02585	0,02582	0,02585	0,02575
Denmark	5.910.913	0,01916	0,01915	0,01919	0,01911	0,01910
Estonia	1.322.765	0,01256	0,01258	0,01254	0,01254	0,01260
Finland	5.545.474	0,01869	0,01867	0,01869	0,01868	0,01857
France	64.756.583	0,11852	0,11861	0,11865	0,11864	0,11862
Germany	83.294.632	0,16618	0,16615	0,16608	0,16608	0,16609
Greece	10.341.277	0,02561	0,02559	0,02567	0,02564	0,02565
Hungary	10.156.239	0,02542	0,02544	0,02539	0,02535	0,02545
Ireland	5.056.934	0,01797	0,01799	0,01797	0,01796	0,01796
Italy	58.870.762	0,10593	0,10602	0,10593	0,10594	0,10592
Latvia	1.830.211	0,01324	0,01321	0,01328	0,01328	0,01335
Lithuania	2.718.351	0,01449	0,01452	0,01453	0,01455	0,01454
Luxembourg	654.767	0,01165	0,01170	0,01175	0,01166	0,01169
Malt	535.064	0,01148	0,01150	0,01151	0,01143	0,01152
Netherlands	17.618.298	0,03630	0,03627	0,03621	0,03624	0,03631
Poland	41.026.067	0,06932	0,06927	0,06932	0,06936	0,06926
Portugal	10.247.604	0,02558	0,02554	0,02561	0,02557	0,02552
Romania	19.892.811	0,03956	0,03952	0,03959	0,03949	0,03947
Slovakia	5.795.199	0,01900	0,01897	0,01902	0,01900	0,01897
Slovenia	2.119.674	0,01370	0,01361	0,01365	0,01372	0,01381
Spain	47.519.627	0,08363	0,08353	0,08358	0,08359	0,08367
Sweden	10.612.086	0,02593	0,02602	0,02597	0,02603	0,02594
TOTAL	448.922.203	1,0000	1,0000	1,0000	1,0000	1,0000

Figure 3.1: Comparison of the Shapley-Shubik indices obtained with the computational model in five different runs using the same population data.

In conclusion, the calculation of the Shapley-Shubik indices by sampling approximation is a practice studied and validated by several researchers and the results obtained with our computational model also demonstrate this. This is why we believe that it is a good calculation method to carry out our study.

3.2 Cases of study

Next we will proceed to study four different scenarios using the computational model described above. For this, we have used the following population data:

Country	2023	2024	2025	2026	2027	2030	2035	2040	2045	2050
Austria	8.958.960	8.977.139	8.994.123	9.009.822	9.023.827	9.054.576	9.068.346	9.047.039	8.999.967	8.924.189
Belgium	11.686.140	11.715.774	11.744.520	11.772.170	11.798.878	11.873.460	11.978.612	12.056.739	12.097.719	12.090.657
Bulgaria	6.687.716	6.618.615	6.565.190	6.511.162	6.456.605	6.290.166	6.008.012	5.727.005	5.453.349	5.187.392
Croatia	4.008.616	3.986.626	3.964.391	3.941.961	3.919.328	3.850.140	3.729.368	3.600.991	3.467.670	3.333.424
Cyprus	1.260.138	1.268.467	1.276.508	1.284.246	1.291.644	1.311.783	1.339.204	1.360.553	1.377.817	1.391.784
Czech Republic	10.495.295	10.503.733	10.509.965	10.514.272	10.516.793	10.515.198	10.501.506	10.504.313	10.535.748	10.577.126
Denmark	5.910.913	5.939.695	5.968.466	5.996.995	6.024.987	6.104.473	6.217.243	6.307.040	6.380.629	6.445.109
Estonia	1.322.765	1.319.041	1.314.899	1.310.378	1.305.538	1.289.441	1.259.966	1.230.997	1.202.180	1.171.695
Finland	5.545.474	5.549.885	5.553.985	5.557.511	5.560.463	5.565.474	5.558.657	5.536.211	5.502.517	5.460.917
France	64.756.583	64.881.830	65.003.383	65.121.251	65.232.614	65.543.452	65.962.122	66.150.546	66.099.099	65.827.072
Germany	83.294.632	83.252.474	83.199.069	83.134.876	83.058.838	82.762.675	82.073.884	81.201.103	80.157.776	78.932.227
Greece	10.341.277	10.302.720	10.263.297	10.223.279	10.182.866	10.059.703	9.852.471	9.640.503	9.408.564	9.144.951
Hungary	10.156.239	9.994.992	9.870.869	9.789.542	9.741.315	9.642.911	9.458.298	9.245.460	9.026.702	8.817.395
Ireland	5.056.934	5.089.478	5.120.867	5.151.398	5.181.143	5.266.880	5.403.353	5.531.181	5.642.439	5.724.685
Italy	58.870.762	58.697.744	58.518.843	58.334.489	58.145.106	57.544.258	56.462.485	55.258.471	53.878.464	52.250.483
Latvia	1.830.211	1.810.240	1.790.796	1.771.916	1.753.618	1.701.337	1.622.114	1.554.124	1.492.555	1.433.728
Lithuania	2.718.351	2.692.798	2.668.445	2.645.221	2.622.933	2.558.928	2.456.932	2.360.589	2.271.313	2.187.550
Luxembourg	654.767	661.594	668.099	674.296	680.225	697.085	722.516	744.883	764.686	781.910
Malt	535.064	536.740	538.296	539.715	540.955	543.425	542.874	537.574	530.247	522.737
Netherlands	17.618.298	17.671.125	17.722.333	17.771.603	17.818.838	17.943.802	18.075.641	18.096.391	18.027.048	17.897.025
Poland	41.026.067	40.221.725	39.616.729	39.242.577	39.047.654	38.700.518	37.971.541	37.043.145	36.001.210	34.932.339
Portugal	10.247.604	10.223.348	10.198.115	10.172.106	10.145.529	10.062.183	9.910.154	9.730.312	9.513.430	9.261.305
Romania	19.892.811	19.618.995	19.424.332	19.291.945	19.208.482	19.023.383	18.680.470	18.295.602	17.882.022	17.457.212
Slovakia	5.795.199	5.702.832	5.635.034	5.595.797	5.578.672	5.555.114	5.491.597	5.400.487	5.296.170	5.186.967
Slovenia	2.119.674	2.118.964	2.117.761	2.116.129	2.114.108	2.105.945	2.087.469	2.064.752	2.037.161	2.003.258
Spain	47.519.627	47.473.373	47.420.024	47.360.679	47.296.155	47.067.573	46.629.611	46.047.876	45.269.679	44.219.565
Sweden	10.612.086	10.673.669	10.733.866	10.792.212	10.848.755	11.007.227	11.238.751	11.458.122	11.683.489	11.902.033
Albania	2.832.439	2.826.020	2.821.626	2.816.694	2.811.110	2.789.598	2.736.079	2.660.261	2.565.419	2.456.472
Moldova	3.435.931	3.329.865	3.254.966	3.211.875	3.193.756	3.174.727	3.133.020	3.081.524	3.034.910	2.997.532
Montenegro	626.484	626.102	625.622	625.039	624.343	621.696	615.426	607.200	597.396	586.385
North Macedonia	2.085.679	2.082.705	2.082.424	2.081.572	2.080.144	2.072.255	2.047.454	2.010.114	1.963.152	1.909.308
Serbia	7.149.076	7.097.027	7.056.384	7.014.413	6.971.013	6.832.604	6.581.784	6.315.060	6.044.879	5.777.805
Turkey	85.816.199	86.260.416	86.696.476	87.132.332	87.569.006	88.879.697	91.121.384	93.057.499	94.679.254	95.829.257
Ukraine	36.744.633	37.937.820	38.759.075	39.093.309	39.045.382	38.295.428	36.959.759	35.656.113	34.310.784	32.867.719
TOTAL	**587.612.644**	**587.663.571**	**587.698.778**	**587.602.782**	**587.390.623**	**586.307.115**	**583.498.103**	**579.119.780**	**573.195.444**	**565.489.213**

Figure 3.2: Estimated population for the next few years of the EU27 countries and the candidate countries to join the EU. (Source: www.populationpyramid.net)

3.2.1 Previous considerations

In order to approve a proposal in the EU council through a qualified majority voting process, 55% of the member states, representing at least 65% of the EU population have to vote in favour. So, it is reasonable to think that the voting power of the countries will depend on their volume of population, with the countries with the largest population having more voting power and the less populated countries having less voting power. Indeed, if we look at the year 2023, ordering the countries based on their population volume (from smallest to largest) and based on their Shapley-Shubik index (from smallest to largest), we observe that the ordering that we obtain in both cases is identical.

It is for this reason that the first thing we will do to analyze the results in each case study is to add the new countries to the list of countries ordered by volume of population and the list of countries ordered by their power index and observe if it continues to be fulfilled the rule of the more population the more voting power. On the other hand, to continue with the analysis of the results, we will also identify which are the countries whose voting power is most affected by the entry of new countries, as well as the countries whose voting power is least affected. Our hypothesis is that the countries with the largest population, that is, those with a greater voting power, are the ones that will be most affected, while the countries with a lower population size and lower power indexes will not be as affected.

3.2.2 Case 1: Turkey

In this first case that we are going to study, we will analyze the impact that Turkey's entry into the European Union would have. It is the first country to apply to enter the European Union from the group of candidates that are currently in the entry process. In addition, it is the largest both in extension and in number of inhabitants. For these reasons, it has been decided to treat it as a separate case and on an individual basis.

Given the large volume of Turkey's population, it is reasonable to think that its entry could cause a great impact. In fact, its population is greater than that of any of the EU countries. Below is a table comparing the Shapley-Shubik indices before and after Turkey's entry.

	Before the entry of Turkey						After the entry of Turkey					
Country	**2023**	**2025**	**2027**	**2030**	**2040**	**2050**	**2023**	**2025**	**2027**	**2030**	**2040**	**2050**
Malt	1,15%	1,16%	1,16%	1,16%	1,17%	1,17%	1,23%	1,24%	1,24%	1,24%	1,26%	1,28%
Luxembourg	1,17%	1,17%	1,18%	1,19%	1,20%	1,21%	1,25%	1,25%	1,25%	1,26%	1,28%	1,31%
Estonia	1,26%	1,25%	1,27%	1,27%	1,27%	1,26%	1,32%	1,33%	1,33%	1,32%	1,34%	1,34%
Bulgaria	2,03%	2,01%	2,01%	1,98%	1,93%	1,86%	1,94%	1,93%	1,91%	1,90%	1,84%	1,80%
Austria	2,35%	2,37%	2,38%	2,39%	2,39%	2,41%	2,20%	2,21%	2,21%	2,22%	2,22%	2,22%
Hungary	2,54%	2,49%	2,48%	2,47%	2,42%	2,38%	2,34%	2,30%	2,29%	2,28%	2,25%	2,21%
Italy	10,59%	10,60%	10,57%	10,47%	10,24%	9,96%	8,49%	8,45%	8,39%	8,30%	8,00%	7,64%
France	11,85%	11,96%	12,03%	12,18%	12,59%	12,94%	9,40%	9,45%	9,48%	9,52%	9,64%	9,75%
Germany	16,62%	16,70%	16,67%	16,65%	16,64%	16,59%	12,61%	12,64%	12,61%	12,53%	12,21%	11,94%
Turkey	-	-	-	-	-	-	13,13%	13,34%	13,51%	13,78%	14,75%	15,67%

Figure 3.3: Extract from the results of the Shapley-Shubik indices in case 1: Turkey.

As we have said before, of the 28 countries that would form the EU in the event that Turkey would become a part, this country would be the most populous. Indeed, it is easy to see that it would also be the country with the highest voting power, obtaining between 13.13% and 15.67% of voting power in the Council of the EU for the period between 2023 and 2050. We can then affirm that the rule of the more population, the more voting power, continues to be fulfilled.

On the other hand, if we analyze the impact that Turkey's entry into the EU has on the voting power of the other countries, we observe that the countries with the largest population are the most affected, that is, Germany, France and Italy. However, countries with a smaller population are even benefited by Turkey's entry, gaining up to 0.1% of voting power, in the case of Malt or Luxembourg.

Country	2023	2024	2025	2026	2027	2030	2035	2040	2045	2050
Malt	0,09%	0,09%	0,08%	0,08%	0,08%	0,08%	0,09%	0,09%	0,10%	0,10%
Luxembourg	0,09%	0,08%	0,08%	0,08%	0,08%	0,07%	0,08%	0,08%	0,09%	0,10%
Estonia	0,07%	0,07%	0,07%	0,07%	0,06%	0,05%	0,08%	0,07%	0,06%	0,08%
Italy	-2,11%	-2,15%	-2,15%	-2,16%	-2,17%	-2,18%	-2,21%	-2,24%	-2,30%	-2,32%
France	-2,45%	-2,49%	-2,51%	-2,56%	-2,56%	-2,65%	-2,77%	-2,95%	-3,03%	-3,19%
Germany	-4,01%	-4,02%	-4,06%	-4,06%	-4,07%	-4,12%	-4,32%	-4,43%	-4,52%	-4,65%

Figure 3.4: Annual variation (in %) in the voting power of the countries that have benefited the most (above) and those that have suffered the most (below) from Turkey's entry into the EU.

3.2.3 Case 2: Ukraine and Moldova

The second case study analyzes the impact that would cause the entry of Ukraine and Moldova at the same time. Both countries applied to the entry process at the beginning of 2022, partly due to the war between Russia and Ukraine. These are countries that share a large part of their borders and that have followed practically the same route to enter the EU. For all these reasons it has been decided to study the entry of both countries as a single case.

Ukraine is a country with a population higher than the average of the EU countries, while Moldova would be in position number eight in the list of countries with the least population in the EU. The following table shows the results obtained in this case:

	Before the entry of Moldova and Ukraine						After the entry of Moldova and Ukraine					
Country	2023	2025	2027	2030	2040	2050	2023	2025	2027	2030	2040	2050
Malt	1,15%	1,16%	1,16%	1,16%	1,17%	1,17%	0,99%	0,99%	0,99%	0,99%	0,99%	0,99%
Luxembourg	1,17%	1,17%	1,18%	1,19%	1,20%	1,21%	1,00%	1,01%	1,01%	1,01%	1,02%	1,03%
Estonia	1,26%	1,25%	1,27%	1,27%	1,27%	1,26%	1,09%	1,10%	1,10%	1,09%	1,09%	1,09%
Bulgaria	2,03%	2,01%	2,01%	1,98%	1,93%	1,86%	1,82%	1,81%	1,79%	1,77%	1,72%	1,66%
Austria	2,35%	2,37%	2,38%	2,39%	2,39%	2,41%	2,14%	2,15%	2,14%	2,15%	2,19%	2,21%
Hungary	2,54%	2,49%	2,48%	2,47%	2,42%	2,38%	2,30%	2,27%	2,24%	2,24%	2,21%	2,19%
Poland	6,93%	6,71%	6,63%	6,61%	6,45%	6,32%	6,86%	6,65%	6,56%	6,55%	6,39%	6,24%
Spain	8,36%	8,44%	8,46%	8,45%	8,44%	8,39%	7,96%	7,95%	7,96%	7,97%	7,96%	7,91%
Italy	10,59%	10,60%	10,57%	10,47%	10,24%	9,96%	9,96%	9,90%	9,87%	9,80%	9,62%	9,38%
France	11,85%	11,96%	12,03%	12,18%	12,59%	12,94%	11,10%	11,16%	11,24%	11,35%	11,79%	12,19%
Germany	16,62%	16,70%	16,67%	16,65%	16,64%	16,59%	15,39%	15,38%	15,42%	15,40%	15,42%	15,53%
Moldova	-	-	-	-	-	-	1,38%	1,36%	1,35%	1,35%	1,35%	1,34%
Ukraine	-	-	-	-	-	-	6,21%	6,52%	6,59%	6,50%	6,18%	5,91%

Figure 3.5: Extract from the results of the Shapley-Shubik indices in case 2: Ukraine and Moldova.

In this scenario, Ukraine occupies position 6th in the list of most populous countries, while Moldova occupies position 22nd. If we look at the list of countries ordered according to their voting power, both Ukraine and Moldova continue to occupy the same positions, 6th and 22nd, respectively. We affirm again that the rule of the more population, the more voting power is fulfilled.

If we look now at the countries that are most affected by the entry of Ukraine and Moldova into the EU, we see again that the countries with a higher population volume are the ones that are most affected. On the other hand, if we look at the countries least affected by the entry of Ukraine and Moldova into the EU, we observe something surprising: Poland is the least affected country, however, it is the 5th most populous country in the EU. It is also the country with the volume of population most similar to that of Ukraine, which could explain the fact that it is the least affected country. Within the group of least affected countries, after Poland, we would find the countries with the lowest population volume in the EU, which is in line with our hypothesis.

Country	2023	2024	2025	2026	2027	2030	2035	2040	2045	2050
Poland	-0,07%	-0,06%	-0,06%	-0,05%	-0,07%	-0,06%	-0,05%	-0,06%	-0,08%	-0,08%
Malt	-0,16%	-0,16%	-0,17%	-0,17%	-0,18%	-0,17%	-0,18%	-0,17%	-0,17%	-0,18%
Estonia	-0,17%	-0,17%	-0,16%	-0,17%	-0,17%	-0,18%	-0,18%	-0,19%	-0,19%	-0,18%
Italy	-0,64%	-0,63%	-0,70%	-0,68%	-0,70%	-0,68%	-0,65%	-0,62%	-0,63%	-0,59%
France	-0,75%	-0,76%	-0,80%	-0,81%	-0,80%	-0,82%	-0,77%	-0,81%	-0,74%	-0,75%
Germany	-1,23%	-1,19%	-1,32%	-1,31%	-1,25%	-1,25%	-1,23%	-1,22%	-1,09%	-1,06%

Figure 3.6: Annual variation (in %) in the voting power of the countries that have suffered the least (above) and those that have suffered the most (below) the entry of Ukraine and Moldova into the EU.

3.2.4 Case 3: Western Balkans countries

The third case study analyzes the case in which the four countries of the Western Balkans enter the EU. Three of the four countries (Albania, North Macedonia and Serbia) were identified as potential candidates for EU membership during the *Thessaloniki European Council* summit in June 2013. The fourth country, Montenegro, became independent from the State Union of Serbia and Montenegro in 2006, and applied for EU membership in 2008. In fact, North Macedonia, Montenegro and Serbia were part of Yugoslavia, a country that dissolved in February 2003. Due to all these reasons, and their geographical proximity, it has been decided to do the analysis of the four countries jointly.

This group of countries, with the exception of Serbia, have a rather small volume of population. Albania, Montenegro and North Macedonia are among the 10 countries with the smallest population, occupying positions 10th, 2nd and 7th respectively. On the other hand, Serbia occupies the 15th position if we look at the most populous countries, even so it is quite far from the countries with the most population in the EU.

	Before the entry of Western Balkans Countries						After the entry of Western Balkans Countries					
Country	2023	2025	2027	2030	2040	2050	2023	2025	2027	2030	2040	2050
Malt	1,15%	1,16%	1,16%	1,16%	1,17%	1,17%	1,21%	1,22%	1,23%	1,22%	1,23%	1,24%
Luxembourg	1,17%	1,17%	1,18%	1,19%	1,20%	1,21%	1,23%	1,24%	1,25%	1,25%	1,25%	1,27%
Estonia	1,26%	1,25%	1,27%	1,27%	1,27%	1,26%	1,31%	1,32%	1,32%	1,32%	1,32%	1,32%
Bulgaria	2,03%	2,01%	2,01%	1,98%	1,93%	1,86%	1,98%	1,98%	1,95%	1,95%	1,88%	1,83%
Austria	2,35%	2,37%	2,38%	2,39%	2,39%	2,41%	2,28%	2,28%	2,28%	2,30%	2,31%	2,32%
Hungary	2,54%	2,49%	2,48%	2,47%	2,42%	2,38%	2,43%	2,40%	2,37%	2,37%	2,34%	2,32%
Italy	10,59%	10,60%	10,57%	10,47%	10,24%	9,96%	9,61%	9,61%	9,58%	9,49%	9,31%	9,06%
France	11,85%	11,96%	12,03%	12,18%	12,59%	12,94%	10,70%	10,82%	10,89%	11,01%	11,38%	11,71%
Germany	16,62%	16,70%	16,67%	16,65%	16,64%	16,59%	15,05%	15,07%	15,13%	15,09%	15,06%	15,07%
Albania	-	-	-	-	-	-	1,50%	1,51%	1,51%	1,51%	1,49%	1,48%
Montenegro	-	-	-	-	-	-	1,23%	1,23%	1,24%	1,23%	1,24%	1,24%
North Macedonia	-	-	-	-	-	-	1,40%	1,42%	1,41%	1,42%	1,41%	1,41%
Serbia	-	-	-	-	-	-	2,04%	2,04%	2,02%	2,01%	1,95%	1,91%

Figure 3.7: Extract from the results of the Shapley-Shubik indices in case 3: Western Balkans Countries.

In this case, the rule of the more population, the more voting power is also fulfilled. The four countries occupy the same position if we order the countries based on their volume of population and based on their power index. Montenegro occupies the 30th position, North Macedonia the 25th, Albania the 22nd and Serbia the 15th.

As in case 1, with the entry of the Western Balkans countries into the EU there are also countries whose power index increases. Again, they are the countries whose volume of population is smaller (Malt, Luxembourg, Estonia, among others). On the other hand, the countries that suffer the most are still, as in the two previous cases, the countries with the largest population.

Country	2023	2024	2025	2026	2027	2030	2035	2040	2045	2050
Malt	0,06%	0,07%	0,06%	0,06%	0,06%	0,06%	0,06%	0,07%	0,07%	0,07%
Luxembourg	0,07%	0,07%	0,07%	0,07%	0,07%	0,06%	0,06%	0,05%	0,06%	0,06%
Estonia	0,06%	0,06%	0,06%	0,06%	0,05%	0,05%	0,05%	0,05%	0,05%	0,06%
Italy	-0,99%	-1,00%	-0,99%	-0,97%	-0,99%	-0,98%	-0,96%	-0,94%	-0,94%	-0,91%
France	-1,15%	-1,17%	-1,14%	-1,17%	-1,14%	-1,17%	-1,17%	-1,21%	-1,15%	-1,23%
Germany	-1,57%	-1,60%	-1,63%	-1,56%	-1,54%	-1,56%	-1,57%	-1,58%	-1,57%	-1,51%

Figure 3.8: Annual variation (in %) in the voting power of the countries that have benefited the most (above) and those that have suffered the most (below) the entry of Western Balkans countries into the EU.

3.2.5 Case 4: Extreme case

Finally, in this last case we have decided to apply the extreme assumption in which all candidate countries to join the EU succeed. Obviously this is an unlikely assumption in the short term, but since they are all in the process of joining the EU, we believe it is relevant to analyze what impact it would have on the Shapley-Shubik index.

These are countries with very different population volumes, so it is interesting to analyze how they will affect the voting power of small and large countries. The table of results obtained is shown below:

	Before the entry of Candidate Countries						After the entry of Candidate Countries					
Country	**2023**	**2025**	**2027**	**2030**	**2040**	**2050**	**2023**	**2025**	**2027**	**2030**	**2040**	**2050**
Malt	1,15%	1,16%	1,16%	1,16%	1,17%	1,17%	0,86%	0,86%	0,86%	0,87%	0,88%	0,89%
Luxembourg	1,17%	1,17%	1,18%	1,19%	1,20%	1,21%	0,87%	0,88%	0,88%	0,88%	0,90%	0,92%
Estonia	1,26%	1,25%	1,27%	1,27%	1,27%	1,26%	0,95%	0,95%	0,95%	0,95%	0,96%	0,97%
Denmark	1,92%	1,93%	1,95%	1,97%	2,00%	2,05%	1,46%	1,47%	1,47%	1,48%	1,52%	1,57%
Bulgaria	2,03%	2,01%	2,01%	1,98%	1,93%	1,86%	1,54%	1,54%	1,51%	1,50%	1,46%	1,42%
Austria	2,35%	2,37%	2,38%	2,39%	2,39%	2,41%	1,80%	1,81%	1,80%	1,82%	1,84%	1,85%
Italy	10,59%	10,60%	10,57%	10,47%	10,24%	9,96%	8,18%	8,15%	8,05%	7,98%	7,66%	7,37%
France	11,85%	11,96%	12,03%	12,18%	12,59%	12,94%	9,06%	9,15%	9,14%	9,18%	9,29%	9,37%
Germany	16,62%	16,70%	16,67%	16,65%	16,64%	16,59%	12,07%	12,11%	11,99%	11,94%	11,70%	11,52%
Albania	-	-	-	-	-	-	1,11%	1,11%	1,11%	1,12%	1,11%	1,11%
Moldova	-	-	-	-	-	-	1,18%	1,16%	1,16%	1,15%	1,16%	1,17%
Montenegro	-	-	-	-	-	-	0,87%	0,87%	0,87%	0,88%	0,89%	0,90%
North Macedonia	-	-	-	-	-	-	1,03%	1,03%	1,03%	1,03%	1,04%	1,05%
Serbia	-	-	-	-	-	-	1,59%	1,59%	1,57%	1,57%	1,53%	1,49%
Turkey	-	-	-	-	-	-	12,55%	12,61%	12,83%	13,06%	13,99%	14,89%
Ukraine	-	-	-	-	-	-	5,23%	5,23%	5,50%	5,40%	5,07%	4,76%

Figure 3.9: Extrat from the results of the Shapley-Shubik indices in case 4: Extreme case.

In this last scenario, the rule of the more population, the more voting power continues to be fulfilled. In this case, Turkey continues to occupy the 1st position, Ukraine the 7th, Serbia the 17th, Moldova the 24th, Albania the 25th, North Macedonia the 28th and Montenegro the 33rd.

Finally, as in 2 of the previous case studies, the countries that are most affected are those with the largest population, while the countries that are least affected are those with the smallest population. In this case, as in the second, there are no countries that increase their voting power.

Country	**2023**	**2024**	**2025**	**2026**	**2027**	**2030**	**2035**	**2040**	**2045**	**2050**
Malt	-0,29%	-0,30%	-0,30%	-0,30%	-0,30%	-0,29%	-0,29%	-0,28%	-0,28%	-0,28%
Luxembourg	-0,30%	-0,30%	-0,30%	-0,30%	-0,30%	-0,31%	-0,30%	-0,30%	-0,28%	-0,29%
Estonia	-0,31%	-0,32%	-0,31%	-0,32%	-0,32%	-0,32%	-0,32%	-0,32%	-0,31%	-0,30%
Italy	-2,41%	-2,45%	-2,45%	-2,47%	-2,52%	-2,49%	-2,52%	-2,58%	-2,61%	-2,59%
France	-2,79%	-2,82%	-2,81%	-2,92%	-2,90%	-3,00%	-3,13%	-3,31%	-3,39%	-3,57%
Germany	-4,54%	-4,61%	-4,59%	-4,66%	-4,68%	-4,71%	-4,83%	-4,94%	-5,02%	-5,07%

Figure 3.10: Annual variation (in %) in the voting power of the countries that have suffered the least (above) and those that have suffered the most (below) the entry of all candidate countries into the EU.

Chapter 4

Conclusions

As expected, the voting power of the EU Council countries is directly related to the volume of population of each country. In the four case studies, the results show that the larger the volume of a country's population, the higher the Shapley-Shubik index obtained.

We also observe that the countries with a greater volume of population and, therefore, with greater voting power, are always the ones that suffer the most from the entry of new member states into the EU. This is the case of Germany, France and Italy. Let's look at the following table:

Cases	Nº of new countries	Amount of new population	Germany % change	France % change	Italy % change
Case 1	1	85.816.199	-4,01%	-2,45%	-2,11%
Case 2	2	40.180.564	-1,23%	-0,75%	-0,64%
Case 3	4	12.693.678	-1,57%	-1,15%	-0,99%
Case 4	7	138.690.441	-4,54%	-2,79%	-2,41%

Figure 4.1: Variation (in %) of 2023 in the voting power of the EU member states with the largest population.

In case 4 is when countries with the largest volume of population lose more voting power, which is quite reasonable considering that it is when a greater number of countries and a greater volume of the population enter. The next case in which they lose more voting power is case 1 where, despite the fact that only Turkey enters, it is the case with the second largest volume of incoming population. Next we would find case 3, which, although it is the case in which the smallest volume of population enters, is the second case with the largest number of incoming countries. Finally, in case 2 is when less voting power is lost since the volume of incoming population is not very high and only two new countries enter. Clearly, the number of new countries and the volume of incoming population are the key factors affecting the decline in voting power of the most populous EU countries.

On the other hand, the results show that the EU member states with the smallest population volume are the ones that are least affected by the entry of new countries into the EU, even increasing their voting power in some cases.

Cases	Nº of new countries	Amount of new population	Malt % change	Luxembourg % change	Estonia % change
Case 1	1	85.816.199	0,09%	0,09%	0,07%
Case 2	2	40.180.564	-0,16%	-0,16%	-0,17%
Case 3	4	12.693.678	0,06%	0,07%	0,06%
Case 4	7	138.690.441	-0,29%	-0,30%	-0,31%

Figure 4.2: Variation (in %) of 2023 in the voting power of the EU member states with the smallest population.

In case 1 is when the least populated countries of the EU gain more voting power due to the impact caused by the entry of Turkey. As it is the entry of a single country but with a large volume of population, in the qualified majority voting process the premise that 55% of the countries have to vote in favor gains more weight compared to the premise of 65% referring to the population. That is why countries with smaller populations increase their voting power. In case 3, the countries with smaller population volumes also increase their voting power, for exactly the same reason as before, but this time due to the entry of a large number of countries with small population volumes. In both case 2 and case 3, all EU countries lose voting power, including those with a smaller population. In these two cases several countries with different population volumes enter, so the decrease in voting power is reasonable. In addition, in case 4 is when a greater number of countries enter and with highly varied population volumes, so the decrease in voting power is greater, as happens with countries with a larger population volume.

Finally, we must highlight the results obtained by Poland in Case 2: Ukraine and Moldova. It is the EU country least affected by the entry of these two countries, despite having a fairly high volume of population, it is the fifth most populous country. To explain such a surprising result, we must note that Poland is the country with a population volume closest to that of Ukraine, reaching a population volume practically identical in the estimate of 2027. Since the population volume of Poland is slightly larger than that of Ukraine, its voting power could not be greatly diminished, since in that case the rule of the greater the volume of the population, the greater the voting power would not be fulfilled.

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Appendix A

C Language

A.1 Input file example

```
27

Austria          8958960
Belgium         11686140
Bulgaria         6687716
Croatia          4008616
Cyprus           1260138
Czech_Republic  10495295
Denmark          5910913
Estonia          1322765
Finland          5545474
France          64756583
Germany         83294632
Greece          10341277
Hungary         10156239
Ireland          5056934
Italy           58870762
Latvia           1830211
Lithuania        2718351
Luxembourg        654767
Malt              535064
Netherlands     17618298
Poland          41026067
Portugal        10247604
Romania         19892811
Slovakia         5795199
Slovenia         2119674
Spain           47519627
Sweden          10612086
```

A.2 Program in C language

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
#define P 0.65
#define Q 1+55*n/100
#define M 10000000
#define N 40

int guanyador(int n, int v[], double p[]);

int main(void) {
    int n, v[N], i, j, k, l, m;
    double p[N], s[N], aux, ptotal;
    clock_t t0, t;
    char nom1[20], nom2[20], c[N][N];
    FILE *fin, *fout;

    printf("Dona'm el nome del fitxer de dades.\n");
    scanf("%s", nom1);
    fin = fopen(nom1, "r");
    if (fin==NULL) {
        printf("Problemes amb el fitxer de dades.\n");
        exit(1);
    }
    printf("Dona'm el nome del fitxer de resultats.\n");
    scanf("%s", nom2);
    fout = fopen(nom2, "w");
    if (fout==NULL) {
        printf("Problemes amb el fitxer de resultats.\n");
        exit(1);
    }
    fscanf(fin, "%d", &n);
    for (i=0; i<n; i++) {
        fscanf(fin, " %s", c[i]);
        fscanf(fin, " %lf", &p[i]);
    }
    ptotal = 0;
    for (i=0; i<n; i++)
    ptotal = ptotal+p[i];
    fprintf(fout, "Suma poblacio total = %+12.0f \n\n", ptotal);
    for (i=0; i<n; i++) {
        p[i] = p[i]/ptotal;
    }
    t0 =clock();
    for (i=0; i<n; i++) {
        v[i] = i;
        s[i] = 0.;
```

```c
    }
    srand((unsigned)time(NULL));
    for (i=0; i<M; i++) {
        for (j=0; j<n; j++) {
            k = rand()%n;
            l = rand()%n;
            m = v[k];
            v[k] = v[l];
            v[l] = m;
        }
        j = guanyador(n, v, p);
        s[v[j]] = s[v[j]]+1;
    }
    aux = 0;
    for (i=0; i<n; i++) {
        aux = aux+s[i];
    }
    fprintf(fout, "Total permutacions realitzades = %+15.0lf \n\n", aux);
    fprintf(fout, "Paisos Poblacio Shapley-Shubik\n");
    for (i=0; i<n; i++) {
        s[i] = s[i]/aux;
        p[i] = p[i]*ptotal;
        fprintf(fout, "%2d ", i);
        fprintf(fout, " %-15s", c[i]);
        fprintf(fout, "%+10.0lf %+.5lf \n", p[i], s[i]);
    }
    t = clock();
    fprintf(fout, "\nNumero de paisos = %d, Temps = %.2le \n",
    n, (double)(t-t0)/CLOCKS_PER_SEC);

    return 0;
}

int guanyador(int n, int v[], double p[]) {
    int k = 0;
    double suma = p[v[0]];
    while (suma < P || k < Q) {
        k = k+1;
        suma = suma+p[v[k]];
    }
    return k;
}
```

A.3 Output file example

```
Suma poblacio total =   +448922203

Total permutacions realitzades =       +10000000

     Paisos              Poblacio   Shapley-Shubik
 0   Austria             +8958960         +0.02348
 1   Belgium            +11686140         +0.02747
 2   Bulgaria            +6687716         +0.02028
 3   Croatia             +4008616         +0.01649
 4   Cyprus              +1260138         +0.01250
 5   Czech_Republic     +10495295         +0.02574
 6   Denmark             +5910913         +0.01922
 7   Estonia             +1322765         +0.01260
 8   Finland             +5545474         +0.01857
 9   France             +64756583         +0.11862
10   Germany            +83294632         +0.16607
11   Greece             +10341277         +0.02565
12   Hungary            +10156239         +0.02545
13   Ireland             +5056934         +0.01796
14   Italy              +58870762         +0.10592
15   Latvia              +1830211         +0.01335
16   Lithuania           +2718351         +0.01454
17   Luxembourg           +654767         +0.01169
18   Malt                 +535064         +0.01152
19   Netherlands        +17618298         +0.03631
20   Poland             +41026067         +0.06919
21   Portugal           +10247604         +0.02552
22   Romania            +19892811         +0.03947
23   Slovakia            +5795199         +0.01897
24   Slovenia            +2119674         +0.01381
25   Spain              +47519627         +0.08367
26   Sweden             +10612086         +0.02594

Numero de paisos = 27, Temps = 1.67e+01
```

Appendix B

Results tables

B.1 Population table

Country	2023	2024	2025	2026	2027	2030	2035	2040	2045	2050
Austria	8.958.960	8.977.139	8.994.123	9.009.822	9.023.827	9.054.576	9.068.346	9.047.039	8.999.967	8.924.189
Belgium	11.686.140	11.715.774	11.744.520	11.772.170	11.798.878	11.873.460	11.978.612	12.056.739	12.097.719	12.090.657
Bulgaria	6.687.716	6.618.615	6.565.190	6.511.162	6.456.605	6.290.166	6.008.012	5.727.005	5.453.349	5.187.392
Croatia	4.008.616	3.986.626	3.964.391	3.941.961	3.919.328	3.850.140	3.729.368	3.600.991	3.467.670	3.333.424
Cyprus	1.260.138	1.268.467	1.276.508	1.284.246	1.291.644	1.311.783	1.339.204	1.360.553	1.377.817	1.391.784
Czech Republic	10.495.295	10.503.733	10.509.965	10.514.272	10.516.793	10.515.198	10.501.506	10.504.313	10.535.748	10.577.126
Denmark	5.910.913	5.939.695	5.968.466	5.996.995	6.024.987	6.104.473	6.217.243	6.307.040	6.380.629	6.445.109
Estonia	1.322.765	1.319.041	1.314.899	1.310.378	1.305.538	1.289.441	1.259.966	1.230.997	1.202.180	1.171.695
Finland	5.545.474	5.549.885	5.553.985	5.557.511	5.560.463	5.565.474	5.558.657	5.536.211	5.502.517	5.460.917
France	64.756.583	64.881.830	65.003.383	65.121.251	65.232.614	65.543.452	65.962.122	66.150.546	66.099.099	65.827.072
Germany	83.294.632	83.252.474	83.199.069	83.134.876	83.058.838	82.762.675	82.073.884	81.201.103	80.157.776	78.932.227
Greece	10.341.277	10.302.720	10.263.297	10.223.279	10.182.866	10.059.703	9.852.471	9.640.503	9.408.564	9.144.951
Hungary	10.156.239	9.994.992	9.870.869	9.789.542	9.741.315	9.642.911	9.458.298	9.245.460	9.026.702	8.817.395
Ireland	5.056.934	5.089.478	5.120.867	5.151.398	5.181.143	5.266.880	5.403.353	5.531.181	5.642.439	5.724.685
Italy	58.870.762	58.697.744	58.518.843	58.334.489	58.145.106	57.544.258	56.462.485	55.258.471	53.878.464	52.250.483
Latvia	1.830.211	1.810.240	1.790.796	1.771.916	1.753.618	1.701.337	1.622.114	1.554.124	1.492.555	1.433.728
Lithuania	2.718.351	2.692.798	2.668.445	2.645.221	2.622.933	2.558.928	2.456.932	2.360.589	2.271.313	2.187.550
Luxembourg	654.767	661.594	668.099	674.296	680.225	697.085	722.516	744.883	764.686	781.910
Malt	535.064	536.740	538.296	539.715	540.955	543.425	542.874	537.574	530.247	522.737
Netherlands	17.618.298	17.671.125	17.722.333	17.771.603	17.818.838	17.943.802	18.075.641	18.096.391	18.027.048	17.897.025
Poland	41.026.067	40.221.725	39.616.729	39.242.577	39.047.654	38.700.518	37.971.541	37.043.145	36.001.210	34.932.339
Portugal	10.247.604	10.223.348	10.198.115	10.172.106	10.145.529	10.062.183	9.910.154	9.730.312	9.513.430	9.261.305
Romania	19.892.811	19.618.995	19.424.332	19.291.945	19.208.482	19.023.383	18.680.470	18.295.602	17.882.022	17.457.212
Slovakia	5.795.199	5.702.832	5.635.034	5.595.797	5.578.672	5.555.114	5.491.597	5.400.487	5.296.170	5.186.967
Slovenia	2.119.674	2.118.964	2.117.761	2.116.129	2.114.108	2.105.945	2.087.469	2.064.752	2.037.161	2.003.258
Spain	47.519.627	47.473.373	47.420.024	47.360.679	47.296.155	47.067.573	46.629.611	46.047.876	45.269.679	44.219.565
Sweden	10.612.086	10.673.669	10.733.866	10.792.212	10.848.755	11.007.227	11.238.751	11.458.122	11.683.489	11.902.033
Albania	2.832.439	2.826.020	2.821.626	2.816.694	2.811.110	2.789.598	2.736.079	2.660.261	2.565.419	2.456.472
Moldova	3.435.931	3.329.865	3.254.966	3.211.875	3.193.756	3.174.727	3.133.020	3.081.524	3.034.910	2.997.532
Montenegro	626.484	626.102	625.622	625.039	624.343	621.696	615.426	607.200	597.396	586.385
North Macedonia	2.085.679	2.082.705	2.082.424	2.081.572	2.080.144	2.072.255	2.047.454	2.010.114	1.963.152	1.909.308
Serbia	7.149.076	7.097.027	7.056.384	7.014.413	6.971.013	6.832.604	6.581.784	6.315.060	6.044.879	5.777.805
Turkey	85.816.199	86.260.416	86.696.476	87.132.332	87.569.006	88.879.697	91.121.384	93.057.499	94.679.254	95.829.257
Ukraine	36.744.633	37.937.820	38.759.075	39.093.309	39.045.382	38.295.428	36.959.759	35.656.113	34.310.784	32.867.719
TOTAL	**587.612.644**	**587.663.571**	**587.698.778**	**587.602.782**	**587.390.623**	**586.307.115**	**583.498.103**	**579.119.780**	**573.195.444**	**565.489.213**

Figure B.1: Estimated population for the next few years of the EU27 countries and the candidate countries to join the EU.

B.2 Results table Case 1: Turkey

	Before the entry of Turkey										After the entry of Turkey									
Country	**2023**	**2024**	**2025**	**2026**	**2027**	**2030**	**2035**	**2040**	**2045**	**2050**	**2023**	**2024**	**2025**	**2026**	**2027**	**2030**	**2035**	**2040**	**2045**	**2050**
Austria	0,0235	0,0236	0,0237	0,0238	0,0238	0,0239	0,0239	0,0239	0,0241	0,0241	0,0220	0,0220	0,0221	0,0220	0,0221	0,0222	0,0222	0,0222	0,0223	0,0222
Belgium	0,0275	0,0275	0,0276	0,0276	0,0276	0,0278	0,0280	0,0282	0,0286	0,0288	0,0253	0,0253	0,0254	0,0254	0,0254	0,0256	0,0256	0,0257	0,0257	0,0258
Bulgaria	0,0203	0,0202	0,0201	0,0201	0,0201	0,0198	0,0195	0,0193	0,0189	0,0186	0,0194	0,0194	0,0193	0,0193	0,0191	0,0190	0,0187	0,0184	0,0182	0,0180
Croatia	0,0165	0,0164	0,0164	0,0165	0,0164	0,0163	0,0161	0,0160	0,0160	0,0160	0,0163	0,0162	0,0163	0,0162	0,0162	0,0162	0,0161	0,0160	0,0160	0,0159
Cyprus	0,0125	0,0126	0,0127	0,0126	0,0126	0,0127	0,0128	0,0128	0,0129	0,0130	0,0131	0,0132	0,0132	0,0132	0,0133	0,0133	0,0134	0,0136	0,0136	0,0137
Czech Republic	0,0258	0,0259	0,0258	0,0259	0,0258	0,0259	0,0260	0,0261	0,0263	0,0265	0,0238	0,0238	0,0239	0,0239	0,0239	0,0239	0,0239	0,0240	0,0239	0,0241
Denmark	0,0192	0,0192	0,0193	0,0194	0,0195	0,0197	0,0199	0,0200	0,0203	0,0205	0,0185	0,0185	0,0185	0,0186	0,0187	0,0188	0,0189	0,0191	0,0192	0,0193
Estonia	0,0126	0,0126	0,0125	0,0126	0,0127	0,0127	0,0127	0,0127	0,0127	0,0126	0,0132	0,0133	0,0133	0,0133	0,0133	0,0132	0,0134	0,0134	0,0133	0,0134
Finland	0,0187	0,0188	0,0188	0,0188	0,0188	0,0188	0,0188	0,0190	0,0190	0,0191	0,0181	0,0181	0,0181	0,0181	0,0182	0,0182	0,0181	0,0182	0,0182	0,0182
France	0,1185	0,1192	0,1196	0,1203	0,1203	0,1218	0,1235	0,1259	0,1272	0,1294	0,0940	0,0943	0,0945	0,0947	0,0948	0,0952	0,0959	0,0964	0,0969	0,0975
Germany	0,1662	0,1666	0,1670	0,1668	0,1667	0,1665	0,1666	0,1664	0,1661	0,1659	0,1261	0,1264	0,1264	0,1261	0,1261	0,1253	0,1234	0,1221	0,1209	0,1194
Greece	0,0256	0,0256	0,0257	0,0254	0,0255	0,0253	0,0251	0,0247	0,0245	0,0244	0,0236	0,0236	0,0236	0,0236	0,0235	0,0233	0,0231	0,0229	0,0228	0,0226
Hungary	0,0254	0,0252	0,0249	0,0249	0,0248	0,0247	0,0244	0,0242	0,0241	0,0238	0,0234	0,0232	0,0230	0,0230	0,0229	0,0228	0,0227	0,0225	0,0222	0,0221
Ireland	0,0180	0,0181	0,0182	0,0182	0,0183	0,0184	0,0186	0,0190	0,0192	0,0194	0,0175	0,0176	0,0176	0,0176	0,0177	0,0178	0,0180	0,0181	0,0183	0,0186
Italy	0,1059	0,1061	0,1060	0,1057	0,1057	0,1047	0,1036	0,1024	0,1013	0,0996	0,0849	0,0846	0,0845	0,0842	0,0839	0,0830	0,0815	0,0800	0,0783	0,0764
Latvia	0,0132	0,0133	0,0133	0,0133	0,0133	0,0133	0,0132	0,0131	0,0130	0,0130	0,0138	0,0138	0,0138	0,0138	0,0138	0,0137	0,0138	0,0137	0,0137	0,0138
Lithuania	0,0145	0,0144	0,0145	0,0145	0,0145	0,0145	0,0143	0,0142	0,0142	0,0140	0,0148	0,0148	0,0147	0,0147	0,0147	0,0147	0,0147	0,0146	0,0146	0,0146
Luxembourg	0,0117	0,0117	0,0117	0,0117	0,0118	0,0119	0,0119	0,0120	0,0120	0,0121	0,0125	0,0125	0,0125	0,0126	0,0125	0,0126	0,0127	0,0128	0,0129	0,0131
Malt	0,0115	0,0115	0,0116	0,0116	0,0116	0,0116	0,0117	0,0117	0,0117	0,0117	0,0123	0,0124	0,0124	0,0124	0,0124	0,0124	0,0125	0,0126	0,0127	0,0128
Netherlands	0,0363	0,0364	0,0365	0,0363	0,0367	0,0368	0,0371	0,0372	0,0375	0,0377	0,0326	0,0327	0,0327	0,0328	0,0329	0,0329	0,0329	0,0329	0,0328	0,0326
Poland	0,0693	0,0680	0,0671	0,0665	0,0663	0,0661	0,0653	0,0645	0,0638	0,0632	0,0605	0,0594	0,0588	0,0583	0,0580	0,0574	0,0565	0,0550	0,0539	0,0530
Portugal	0,0256	0,0255	0,0254	0,0254	0,0253	0,0254	0,0251	0,0249	0,0247	0,0246	0,0236	0,0235	0,0234	0,0234	0,0235	0,0234	0,0232	0,0229	0,0228	0,0227
Romania	0,0396	0,0391	0,0388	0,0388	0,0385	0,0382	0,0380	0,0376	0,0374	0,0372	0,0354	0,0353	0,0349	0,0347	0,0347	0,0342	0,0337	0,0331	0,0326	0,0319
Slovakia	0,0190	0,0189	0,0187	0,0188	0,0188	0,0188	0,0188	0,0188	0,0187	0,0186	0,0183	0,0183	0,0182	0,0182	0,0181	0,0181	0,0181	0,0180	0,0180	0,0179
Slovenia	0,0137	0,0137	0,0137	0,0137	0,0137	0,0138	0,0138	0,0138	0,0139	0,0138	0,0141	0,0142	0,0142	0,0142	0,0142	0,0142	0,0143	0,0143	0,0142	0,0144
Spain	0,0836	0,0840	0,0844	0,0847	0,0846	0,0845	0,0844	0,0844	0,0843	0,0839	0,0676	0,0674	0,0671	0,0670	0,0668	0,0665	0,0659	0,0651	0,0645	0,0637
Sweden	0,0259	0,0261	0,0261	0,0263	0,0264	0,0264	0,0270	0,0273	0,0279	0,0286	0,0240	0,0240	0,0241	0,0242	0,0243	0,0245	0,0247	0,0250	0,0253	0,0257
Turkey	-	-	-	-	-	-	-	-	-	-	0,1313	0,1325	0,1334	0,1344	0,1351	0,1378	0,1422	0,1475	0,1523	0,1567

Figure B.2: Results of the Shapley-Shubik indices in Case 1: Turkey.

B.3 Results table Case 2: Ukraine and Moldova

	Before the entry of Moldova and Ukraine										After the entry of Moldova and Ukraine									
Country	**2023**	**2024**	**2025**	**2026**	**2027**	**2030**	**2035**	**2040**	**2045**	**2050**	**2023**	**2024**	**2025**	**2026**	**2027**	**2030**	**2035**	**2040**	**2045**	**2050**
Austria	0,0235	0,0236	0,0237	0,0238	0,0238	0,0239	0,0239	0,0239	0,0241	0,0241	0,0214	0,0213	0,0215	0,0215	0,0214	0,0215	0,0216	0,0219	0,0219	0,0221
Belgium	0,0275	0,0275	0,0276	0,0276	0,0276	0,0278	0,0280	0,0282	0,0286	0,0288	0,0253	0,0252	0,0252	0,0253	0,0253	0,0256	0,0259	0,0261	0,0265	0,0268
Bulgaria	0,0203	0,0202	0,0201	0,0201	0,0201	0,0198	0,0195	0,0193	0,0189	0,0186	0,0182	0,0181	0,0181	0,0180	0,0179	0,0177	0,0174	0,0172	0,0168	0,0166
Croatia	0,0165	0,0164	0,0164	0,0165	0,0164	0,0163	0,0161	0,0160	0,0160	0,0160	0,0145	0,0145	0,0144	0,0145	0,0145	0,0144	0,0143	0,0142	0,0140	0,0139
Cyprus	0,0125	0,0126	0,0127	0,0126	0,0126	0,0127	0,0128	0,0128	0,0129	0,0130	0,0109	0,0109	0,0109	0,0108	0,0110	0,0110	0,0110	0,0111	0,0111	0,0112
Czech Republic	0,0258	0,0259	0,0258	0,0259	0,0258	0,0259	0,0260	0,0261	0,0263	0,0265	0,0235	0,0236	0,0236	0,0236	0,0235	0,0237	0,0238	0,0239	0,0241	0,0245
Denmark	0,0192	0,0192	0,0193	0,0194	0,0195	0,0197	0,0199	0,0200	0,0203	0,0205	0,0172	0,0171	0,0172	0,0172	0,0174	0,0174	0,0177	0,0179	0,0182	0,0183
Estonia	0,0126	0,0126	0,0125	0,0126	0,0127	0,0127	0,0127	0,0127	0,0127	0,0126	0,0109	0,0109	0,0110	0,0110	0,0110	0,0109	0,0109	0,0109	0,0108	0,0109
Finland	0,0187	0,0188	0,0188	0,0188	0,0188	0,0188	0,0188	0,0190	0,0190	0,0191	0,0167	0,0166	0,0166	0,0166	0,0166	0,0167	0,0167	0,0168	0,0169	0,0170
France	0,1185	0,1192	0,1196	0,1203	0,1203	0,1218	0,1235	0,1259	0,1272	0,1294	0,1110	0,1116	0,1116	0,1122	0,1124	0,1135	0,1159	0,1179	0,1199	0,1219
Germany	0,1662	0,1666	0,1670	0,1668	0,1667	0,1665	0,1666	0,1664	0,1661	0,1659	0,1539	0,1547	0,1538	0,1536	0,1542	0,1540	0,1543	0,1542	0,1552	0,1553
Greece	0,0256	0,0256	0,0257	0,0254	0,0255	0,0253	0,0251	0,0247	0,0245	0,0244	0,0232	0,0233	0,0232	0,0232	0,0231	0,0230	0,0228	0,0226	0,0225	0,0224
Hungary	0,0254	0,0252	0,0249	0,0249	0,0248	0,0247	0,0244	0,0242	0,0241	0,0238	0,0230	0,0228	0,0227	0,0225	0,0224	0,0224	0,0222	0,0221	0,0220	0,0219
Ireland	0,0180	0,0181	0,0182	0,0182	0,0183	0,0184	0,0186	0,0190	0,0192	0,0194	0,0160	0,0160	0,0161	0,0161	0,0161	0,0162	0,0166	0,0168	0,0171	0,0174
Italy	0,1059	0,1061	0,1060	0,1057	0,1057	0,1047	0,1036	0,1024	0,1013	0,0996	0,0996	0,0998	0,0990	0,0989	0,0987	0,0980	0,0971	0,0962	0,0950	0,0938
Latvia	0,0132	0,0133	0,0133	0,0133	0,0133	0,0133	0,0132	0,0131	0,0130	0,0130	0,0116	0,0116	0,0116	0,0116	0,0115	0,0115	0,0114	0,0113	0,0112	0,0112
Lithuania	0,0145	0,0144	0,0145	0,0145	0,0145	0,0145	0,0143	0,0142	0,0142	0,0140	0,0128	0,0127	0,0128	0,0128	0,0127	0,0126	0,0126	0,0124	0,0125	0,0123
Luxembourg	0,0117	0,0117	0,0117	0,0117	0,0118	0,0119	0,0119	0,0120	0,0120	0,0121	0,0100	0,0100	0,0101	0,0101	0,0101	0,0101	0,0101	0,0102	0,0103	0,0103
Malt	0,0115	0,0115	0,0116	0,0116	0,0116	0,0116	0,0117	0,0117	0,0117	0,0117	0,0099	0,0099	0,0099	0,0099	0,0099	0,0099	0,0099	0,0099	0,0100	0,0099
Netherlands	0,0363	0,0364	0,0365	0,0363	0,0367	0,0368	0,0371	0,0372	0,0375	0,0377	0,0337	0,0338	0,0340	0,0339	0,0340	0,0344	0,0347	0,0351	0,0354	0,0356
Poland	0,0693	0,0680	0,0671	0,0665	0,0663	0,0661	0,0653	0,0645	0,0638	0,0632	0,0686	0,0674	0,0665	0,0660	0,0656	0,0655	0,0649	0,0639	0,0630	0,0624
Portugal	0,0256	0,0255	0,0254	0,0254	0,0253	0,0254	0,0251	0,0249	0,0247	0,0246	0,0231	0,0232	0,0230	0,0231	0,0231	0,0230	0,0230	0,0229	0,0227	0,0226
Romania	0,0396	0,0391	0,0388	0,0388	0,0385	0,0382	0,0380	0,0376	0,0374	0,0372	0,0370	0,0367	0,0364	0,0361	0,0361	0,0358	0,0357	0,0354	0,0352	0,0350
Slovakia	0,0190	0,0189	0,0187	0,0188	0,0188	0,0188	0,0188	0,0188	0,0187	0,0186	0,0170	0,0168	0,0168	0,0167	0,0166	0,0167	0,0166	0,0167	0,0166	0,0166
Slovenia	0,0137	0,0137	0,0137	0,0137	0,0137	0,0138	0,0138	0,0138	0,0139	0,0138	0,0120	0,0120	0,0120	0,0120	0,0120	0,0120	0,0120	0,0121	0,0120	0,0121
Spain	0,0836	0,0840	0,0844	0,0847	0,0846	0,0845	0,0844	0,0844	0,0843	0,0839	0,0796	0,0799	0,0795	0,0797	0,0796	0,0797	0,0796	0,0796	0,0795	0,0791
Sweden	0,0259	0,0261	0,0261	0,0263	0,0264	0,0264	0,0270	0,0273	0,0279	0,0286	0,0237	0,0238	0,0238	0,0240	0,0241	0,0243	0,0248	0,0253	0,0259	0,0264
Moldova	-	-	-	-	-	-	-	-	-	-	0,0138	0,0137	0,0136	0,0135	0,0135	0,0135	0,0135	0,0135	0,0134	0,0134
Ukraine	-	-	-	-	-	-	-	-	-	-	0,0621	0,0621	0,0652	0,0658	0,0659	0,0650	0,0634	0,0618	0,0604	0,0591

Figure B.3: Results of the Shapley-Shubik indices in Case 2: Ukraine and Moldova.

B.4 Results table Case 3: Western Balkans countries

	Before the entry of Western Balkans Countries										After the entry of Western Balkans Countries									
Country	2023	2024	2025	2026	2027	2030	2035	2040	2045	2050	2023	2024	2025	2026	2027	2030	2035	2040	2045	2050
Austria	0,0235	0,0236	0,0237	0,0238	0,0238	0,0239	0,0239	0,0239	0,0241	0,0241	0,0228	0,0228	0,0228	0,0228	0,0228	0,0230	0,0230	0,0231	0,0233	0,0232
Belgium	0,0275	0,0275	0,0276	0,0276	0,0276	0,0278	0,0280	0,0282	0,0286	0,0288	0,0262	0,0263	0,0263	0,0264	0,0264	0,0267	0,0268	0,0270	0,0271	0,0274
Bulgaria	0,0203	0,0202	0,0201	0,0201	0,0201	0,0198	0,0195	0,0193	0,0189	0,0186	0,0198	0,0198	0,0198	0,0196	0,0195	0,0195	0,0190	0,0188	0,0185	0,0183
Croatia	0,0165	0,0164	0,0164	0,0165	0,0164	0,0163	0,0161	0,0160	0,0160	0,0160	0,0165	0,0164	0,0164	0,0165	0,0164	0,0163	0,0162	0,0162	0,0161	0,0159
Cyprus	0,0125	0,0126	0,0127	0,0126	0,0126	0,0127	0,0128	0,0128	0,0129	0,0130	0,0131	0,0131	0,0131	0,0131	0,0132	0,0132	0,0133	0,0133	0,0133	0,0134
Czech Republic	0,0258	0,0259	0,0258	0,0259	0,0258	0,0259	0,0260	0,0261	0,0263	0,0265	0,0247	0,0248	0,0248	0,0247	0,0247	0,0248	0,0249	0,0250	0,0252	0,0254
Denmark	0,0192	0,0192	0,0193	0,0194	0,0195	0,0197	0,0199	0,0200	0,0203	0,0205	0,0189	0,0189	0,0189	0,0190	0,0190	0,0191	0,0193	0,0195	0,0197	0,0199
Estonia	0,0126	0,0126	0,0125	0,0126	0,0127	0,0127	0,0127	0,0127	0,0127	0,0126	0,0131	0,0132	0,0132	0,0132	0,0132	0,0132	0,0132	0,0132	0,0132	0,0132
Finland	0,0187	0,0188	0,0188	0,0188	0,0188	0,0188	0,0188	0,0190	0,0190	0,0191	0,0184	0,0183	0,0184	0,0184	0,0184	0,0184	0,0185	0,0185	0,0185	0,0187
France	0,1185	0,1192	0,1196	0,1203	0,1203	0,1218	0,1235	0,1259	0,1272	0,1294	0,1070	0,1075	0,1082	0,1087	0,1089	0,1101	0,1119	0,1138	0,1157	0,1171
Germany	0,1662	0,1666	0,1670	0,1668	0,1667	0,1665	0,1666	0,1664	0,1661	0,1659	0,1505	0,1506	0,1507	0,1511	0,1513	0,1509	0,1509	0,1506	0,1505	0,1507
Greece	0,0256	0,0256	0,0257	0,0254	0,0255	0,0253	0,0251	0,0247	0,0245	0,0244	0,0244	0,0245	0,0244	0,0244	0,0243	0,0242	0,0241	0,0240	0,0237	0,0236
Hungary	0,0254	0,0252	0,0249	0,0249	0,0248	0,0247	0,0244	0,0242	0,0241	0,0238	0,0243	0,0241	0,0240	0,0239	0,0237	0,0237	0,0235	0,0234	0,0232	0,0232
Ireland	0,0180	0,0181	0,0182	0,0182	0,0183	0,0184	0,0186	0,0190	0,0192	0,0194	0,0177	0,0179	0,0178	0,0180	0,0180	0,0180	0,0183	0,0185	0,0187	0,0191
Italy	0,1059	0,1061	0,1060	0,1057	0,1057	0,1047	0,1036	0,1024	0,1013	0,0996	0,0961	0,0961	0,0961	0,0961	0,0958	0,0949	0,0940	0,0931	0,0919	0,0906
Latvia	0,0132	0,0133	0,0133	0,0133	0,0133	0,0133	0,0132	0,0131	0,0130	0,0130	0,0138	0,0138	0,0138	0,0137	0,0138	0,0137	0,0136	0,0136	0,0135	0,0135
Lithuania	0,0145	0,0144	0,0145	0,0145	0,0145	0,0145	0,0143	0,0142	0,0142	0,0140	0,0149	0,0149	0,0149	0,0148	0,0148	0,0147	0,0147	0,0146	0,0145	0,0145
Luxembourg	0,0117	0,0117	0,0117	0,0117	0,0118	0,0119	0,0119	0,0120	0,0120	0,0121	0,0123	0,0124	0,0124	0,0124	0,0125	0,0125	0,0125	0,0125	0,0126	0,0127
Malt	0,0115	0,0115	0,0116	0,0116	0,0116	0,0116	0,0117	0,0117	0,0117	0,0117	0,0121	0,0122	0,0122	0,0122	0,0123	0,0122	0,0123	0,0123	0,0123	0,0124
Netherlands	0,0363	0,0364	0,0365	0,0363	0,0367	0,0368	0,0371	0,0372	0,0375	0,0377	0,0341	0,0341	0,0341	0,0342	0,0342	0,0345	0,0346	0,0349	0,0351	0,0352
Poland	0,0693	0,0680	0,0671	0,0665	0,0663	0,0661	0,0653	0,0645	0,0638	0,0632	0,0630	0,0620	0,0612	0,0607	0,0605	0,0602	0,0596	0,0589	0,0583	0,0579
Portugal	0,0256	0,0255	0,0254	0,0254	0,0253	0,0254	0,0251	0,0249	0,0247	0,0246	0,0243	0,0244	0,0243	0,0242	0,0243	0,0242	0,0241	0,0240	0,0238	0,0236
Romania	0,0396	0,0391	0,0388	0,0388	0,0385	0,0382	0,0380	0,0376	0,0374	0,0372	0,0371	0,0367	0,0364	0,0362	0,0362	0,0359	0,0355	0,0352	0,0349	0,0345
Slovakia	0,0190	0,0189	0,0187	0,0188	0,0188	0,0188	0,0188	0,0188	0,0187	0,0186	0,0188	0,0185	0,0185	0,0185	0,0184	0,0184	0,0184	0,0183	0,0183	0,0182
Slovenia	0,0137	0,0137	0,0137	0,0137	0,0137	0,0138	0,0138	0,0138	0,0139	0,0138	0,0141	0,0142	0,0142	0,0142	0,0141	0,0143	0,0142	0,0142	0,0142	0,0142
Spain	0,0836	0,0840	0,0844	0,0847	0,0846	0,0845	0,0844	0,0844	0,0843	0,0839	0,0756	0,0760	0,0763	0,0764	0,0765	0,0765	0,0765	0,0766	0,0766	0,0761
Sweden	0,0259	0,0261	0,0261	0,0263	0,0264	0,0264	0,0270	0,0273	0,0279	0,0286	0,0249	0,0250	0,0250	0,0251	0,0252	0,0254	0,0259	0,0262	0,0267	0,0271
Albania	-	-	-	-	-	-	-	-	-	-	0,0150	0,0150	0,0151	0,0150	0,0151	0,0151	0,0151	0,0149	0,0149	0,0148
Montenegro	-	-	-	-	-	-	-	-	-	-	0,0123	0,0123	0,0123	0,0124	0,0124	0,0123	0,0124	0,0124	0,0124	0,0124
North Macedonia	-	-	-	-	-	-	-	-	-	-	0,0140	0,0141	0,0142	0,0141	0,0141	0,0142	0,0141	0,0141	0,0142	0,0141
Serbia	-	-	-	-	-	-	-	-	-	-	0,0204	0,0203	0,0204	0,0203	0,0202	0,0201	0,0198	0,0195	0,0193	0,0191

Figure B.4: Results of the Shapley-Shubik indices in Case 3: Western Balkans countries.

B.5 Results table Case 4: Extreme case

	Before the entry of Candidate Countries										After the entry of Candidate Countries									
Country	2023	2024	2025	2026	2027	2030	2035	2040	2045	2050	2023	2024	2025	2026	2027	2030	2035	2040	2045	2050
Austria	0,0235	0,0236	0,0237	0,0238	0,0238	0,0239	0,0239	0,0239	0,0241	0,0241	0,0180	0,0181	0,0181	0,0181	0,0180	0,0182	0,0182	0,0184	0,0185	0,0185
Belgium	0,0275	0,0275	0,0276	0,0276	0,0276	0,0278	0,0280	0,0282	0,0286	0,0288	0,0211	0,0211	0,0213	0,0212	0,0212	0,0215	0,0215	0,0219	0,0220	0,0222
Bulgaria	0,0203	0,0202	0,0201	0,0201	0,0201	0,0198	0,0195	0,0193	0,0189	0,0186	0,0154	0,0153	0,0154	0,0152	0,0151	0,0150	0,0148	0,0146	0,0144	0,0142
Croatia	0,0165	0,0164	0,0164	0,0165	0,0164	0,0163	0,0161	0,0160	0,0160	0,0160	0,0124	0,0124	0,0124	0,0124	0,0123	0,0123	0,0122	0,0122	0,0121	0,0121
Cyprus	0,0125	0,0126	0,0127	0,0126	0,0126	0,0127	0,0128	0,0128	0,0129	0,0130	0,0094	0,0094	0,0094	0,0094	0,0094	0,0095	0,0096	0,0097	0,0098	0,0099
Czech Republic	0,0258	0,0259	0,0258	0,0259	0,0258	0,0259	0,0260	0,0261	0,0263	0,0265	0,0198	0,0198	0,0199	0,0198	0,0198	0,0198	0,0199	0,0200	0,0202	0,0205
Denmark	0,0192	0,0192	0,0193	0,0194	0,0195	0,0197	0,0199	0,0200	0,0203	0,0205	0,0146	0,0146	0,0147	0,0146	0,0147	0,0148	0,0150	0,0152	0,0155	0,0157
Estonia	0,0126	0,0126	0,0125	0,0126	0,0127	0,0127	0,0127	0,0127	0,0127	0,0126	0,0095	0,0094	0,0095	0,0094	0,0095	0,0095	0,0095	0,0096	0,0096	0,0097
Finland	0,0187	0,0188	0,0188	0,0188	0,0188	0,0188	0,0188	0,0190	0,0190	0,0191	0,0142	0,0141	0,0142	0,0141	0,0142	0,0143	0,0144	0,0144	0,0145	0,0146
France	0,1185	0,1192	0,1196	0,1203	0,1203	0,1218	0,1235	0,1259	0,1272	0,1294	0,0906	0,0910	0,0915	0,0911	0,0914	0,0918	0,0923	0,0929	0,0933	0,0937
Germany	0,1662	0,1666	0,1670	0,1668	0,1667	0,1665	0,1666	0,1664	0,1661	0,1659	0,1207	0,1205	0,1211	0,1201	0,1199	0,1194	0,1183	0,1170	0,1160	0,1152
Greece	0,0256	0,0256	0,0257	0,0254	0,0255	0,0253	0,0251	0,0247	0,0245	0,0244	0,0196	0,0195	0,0195	0,0194	0,0194	0,0193	0,0191	0,0190	0,0190	0,0188
Hungary	0,0254	0,0252	0,0249	0,0249	0,0248	0,0247	0,0244	0,0242	0,0241	0,0238	0,0193	0,0192	0,0191	0,0189	0,0189	0,0188	0,0186	0,0186	0,0185	0,0184
Ireland	0,0180	0,0181	0,0182	0,0182	0,0183	0,0184	0,0186	0,0190	0,0192	0,0194	0,0136	0,0136	0,0137	0,0138	0,0138	0,0140	0,0141	0,0143	0,0146	0,0149
Italy	0,1059	0,1061	0,1060	0,1057	0,1057	0,1047	0,1036	0,1024	0,1013	0,0996	0,0818	0,0816	0,0815	0,0810	0,0805	0,0798	0,0784	0,0766	0,0752	0,0737
Latvia	0,0132	0,0133	0,0133	0,0133	0,0133	0,0133	0,0132	0,0131	0,0130	0,0130	0,0100	0,0100	0,0100	0,0100	0,0100	0,0099	0,0099	0,0098	0,0100	0,0100
Lithuania	0,0145	0,0144	0,0145	0,0145	0,0145	0,0145	0,0143	0,0142	0,0142	0,0140	0,0111	0,0110	0,0109	0,0109	0,0109	0,0109	0,0109	0,0108	0,0108	0,0108
Luxembourg	0,0117	0,0117	0,0117	0,0117	0,0118	0,0119	0,0119	0,0120	0,0120	0,0121	0,0087	0,0087	0,0088	0,0088	0,0088	0,0088	0,0089	0,0090	0,0092	0,0092
Malt	0,0115	0,0115	0,0116	0,0116	0,0116	0,0116	0,0117	0,0117	0,0117	0,0117	0,0086	0,0085	0,0086	0,0086	0,0086	0,0087	0,0087	0,0088	0,0089	0,0089
Netherlands	0,0363	0,0364	0,0365	0,0363	0,0367	0,0368	0,0371	0,0372	0,0375	0,0377	0,0280	0,0281	0,0283	0,0282	0,0282	0,0285	0,0287	0,0289	0,0291	0,0292
Poland	0,0693	0,0680	0,0671	0,0665	0,0663	0,0661	0,0653	0,0645	0,0638	0,0632	0,0577	0,0566	0,0559	0,0554	0,0550	0,0544	0,0535	0,0524	0,0512	0,0504
Portugal	0,0256	0,0255	0,0254	0,0254	0,0253	0,0254	0,0251	0,0249	0,0247	0,0246	0,0195	0,0195	0,0194	0,0194	0,0194	0,0192	0,0192	0,0191	0,0190	0,0189
Romania	0,0396	0,0391	0,0388	0,0388	0,0385	0,0382	0,0380	0,0376	0,0374	0,0372	0,0307	0,0304	0,0303	0,0301	0,0300	0,0297	0,0296	0,0292	0,0289	0,0286
Slovakia	0,0190	0,0189	0,0187	0,0188	0,0188	0,0188	0,0188	0,0188	0,0187	0,0186	0,0145	0,0144	0,0143	0,0142	0,0143	0,0142	0,0142	0,0142	0,0142	0,0142
Slovenia	0,0137	0,0137	0,0137	0,0137	0,0137	0,0138	0,0138	0,0138	0,0139	0,0138	0,0103	0,0103	0,0104	0,0103	0,0104	0,0104	0,0105	0,0105	0,0105	0,0106
Spain	0,0836	0,0840	0,0844	0,0847	0,0846	0,0845	0,0844	0,0844	0,0843	0,0839	0,0658	0,0657	0,0658	0,0654	0,0655	0,0650	0,0645	0,0638	0,0629	0,0619
Sweden	0,0259	0,0261	0,0261	0,0263	0,0264	0,0264	0,0270	0,0273	0,0279	0,0286	0,0199	0,0200	0,0201	0,0201	0,0202	0,0204	0,0208	0,0210	0,0215	0,0220
Albania	-	-	-	-	-	-	-	-	-	-	0,0111	0,0111	0,0111	0,0111	0,0111	0,0112	0,0112	0,0111	0,0111	0,0111
Moldova	-	-	-	-	-	-	-	-	-	-	0,0118	0,0117	0,0116	0,0116	0,0116	0,0115	0,0116	0,0116	0,0117	0,0117
Montenegro	-	-	-	-	-	-	-	-	-	-	0,0087	0,0087	0,0087	0,0087	0,0087	0,0088	0,0088	0,0089	0,0089	0,0090
North Macedonia	-	-	-	-	-	-	-	-	-	-	0,0103	0,0103	0,0103	0,0103	0,0103	0,0103	0,0103	0,0104	0,0105	0,0105
Serbia	-	-	-	-	-	-	-	-	-	-	0,0159	0,0159	0,0159	0,0158	0,0157	0,0157	0,0155	0,0153	0,0151	0,0149
Turkey	-	-	-	-	-	-	-	-	-	-	0,1255	0,1258	0,1261	0,1275	0,1283	0,1306	0,1354	0,1399	0,1442	0,1489
Ukraine	-	-	-	-	-	-	-	-	-	-	0,0523	0,0537	0,0523	0,0549	0,0550	0,0540	0,0522	0,0507	0,0492	0,0476

Figure B.5: Results of the Shapley-Shubik indices in Case 4: Extreme case.