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Bachelor's Degree in Economics

Demographics and the Natural Rate of Interest in Advanced Economies

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Abstract

This final degree project aims to investigate the potential negative impact of current demographic changes on the natural interest rate in the euro area and its effects on pension systems, agent allocation, and policymakers' effectiveness. To accomplish this, I replicate a macroeconomic Overlapping Generations Model proposed by Bielecki et al. (2020)[3]. This model will comprehensively analyse the intersection of population ageing and the natural interest rate in advanced economies, an increasingly relevant topic in contemporary academic research.

The project's main results illustrate the negative correlation between population ageing and natural interest rates and suggest that this process began almost forty years ago. Moreover, the increasing dependency ratios in the future highlight the critical situation of pension systems, even under an optimistic migration scenario. Finally, this study examines the effects of agents' risk aversion (σ) and substitution elasticity (ε) parameters on the model economy.

Keywords: Demographics, Natural interest rate, Macroeconomics, Monetary Theory, Population ageing, Migration, Pension systems, Life-cycle models.

Epítom

Aquest projecte de fi de grau pretén estudiar la possible pressió negativa que els actuals canvis demogràfics estan aplicant al tipus d'interès natural en la zona de l'euro. Així com els seus efectes sobre els sistemes de pensions, l'assignació d'agents i l'eficàcia dels responsables polítics.

La principal font d'informació ha sigut l'article "Demografia i el tipus d'interès natural en la zona de l'euro" de Bielecki, Brzoza-Brzezina, i Kolasa[3], el qual ha sigut estudiat, replicat i s'han realitzat diverses extensions; juntament amb diversos altres articles correlacionats. A aquest efecte, s'estudia un model macroeconòmic de generació superposada, que ajuda a proporcionar una anàlisi exhaustiva de la intersecció de l'envelliment de la població i el tipus natural d'interès en les economies avançades, que és un tema cada vegada més rellevant en l'àmbit de la recerca acadèmica contemporània.

Els principals resultats de l'estudi en qüestió confirmen la hipòtesi inicial, l'existència d'una correlació negativa entre l'envelliment de la població i els tipus d'interès natural i que aquest procés té lloc des de fa més de quaranta anys i explica més de dos terços de la caiguda dels tipus d'interès. A més a més, es posa en rellevància la situació crítica dels sistemes públics de pensions actuals, que ni tan sols en els escenaris més optimistes quant a migració poden resoldre els futurs grans ràtios de dependència en les economies més avançades. Es troba, doncs, que l'única manera d'alleujar aquesta situació seria l'augment de les edats de jubilació. Finalment, s'estudia l'efecte que els

paràmetres sobre l'aversió al risc dels agents (σ) i l'elasticitat de substitució (ε) tenen sobre el model.

Paraules Clau: Demografia, Tipus d'interès natural, Macroeconomia, Teoria Monetària, Envelliment de la població, Migració, Sistemes de pensions, Models de vida cíclica.

Contents

1 Introduction

Since Knut Wicksell introduced the idea of the natural rate of interest in 1898, it has played a central role in macroeconomic and monetary theory. In recent years, especially after the 2008 financial crisis, real interest rates in many developed economies have been in negative territory. Nominal interest rates have hovered around zero, and inflation rates have remained stable in close-to-zero terms. Furthermore, this trend has continued even after central banks normalized their policies. These circumstances suggest that the forces behind this trend, which began almost two decades ago, are other than accommodating monetary policies.

A simultaneous event has occurred in all major industrialized countries: population ageing. This process is driven by falling mortality and birth rates, which reduce population growth rates and, in turn, affect the overall economy, especially pension systems.

The intersection of both phenomena has captured my attention as a potential research topic and how both could be correlated. This trend has been an increasingly relevant topic in contemporary academic research for almost a decade. Therefore, my final degree project aims to study the possibility that demographic changes may be an explanatory factor, its consequences, and the channels that connect both trends.

In addition to the previous points, macroeconomics subjects have captivated my attention more than any others since the last years of my economics degree, making my final degree project topic a natural result.

Regarding the project's primary purpose, I aim to study the possible correlation between demographic changes and the natural interest rate in the euro area. For that purpose, I replicate a macroeconomic Overlapping Generations Model that incorporates fertility, productivity, and mortality rates data from the Household Finance and Consumption Survey (HFCS) that closely matches reality. The model also considers cross-border capital flows and the dependence of the natural interest rate (NRI) on the real interest rate in the rest of the world. Finally, I analyze the possible effects of pension system reforms on the NRI and discuss the potential role of increased migration in mitigating the fall in the natural interest rate.

This model will allow me to study the consequences of the negative pressure that demographics apply to the natural interest rate while observing their effects on pension systems, agent allocation, and policymakers' effectiveness.

Finally, two extensions, called "Cases of Study" of the baseline model, were completed: the effects that the parameters of agents' risk aversion (σ) and substitution elasticity (ε) have on the model economy were studied.

The main findings suggest that demographic trends account for over two-thirds of a decline of 2 percentage points in the projected NRI from 1985 to 2030. Also, because of the globalization process and the open economy setup implemented in almost all countries, even if the country does not have explicitly immediate demographic issues, it will suffer the consequences of those already having them. Those will come through increasing the allocation of savings to foreign assets, starting from the worsened ones (Europe) to the rest of the world, thus, decreasing the NRI. Two nuances could positively impact the so-called negative trend: migration, but in the long-term and with marginal effectivity, and pension systems reforms, mainly through constant increasings in the effective retirement age. Also, it is worth mentioning that because of the European sample selected for the data, the model fits better when increasing risk-averse values are computed. Finally, any alteration of the production function tends to dispel the data, mismatching the results.

The rest of this paper is structured as follows. In Section 2, I present some of the most recent research papers related to this topic to provide a theoretical basis for this project. In Section 3, the model is described, and the general equilibrium is characterized, along with the main assumptions, characteristics of the agents, and calibration parameters. Section 4 presents the main results obtained under the baseline model and assesses the effects of several scenarios by considering several alternative parameter values. Finally, Section 5 provides the main conclusions of the overall research.

2 Literature Review

The rest of this chapter will focus on understanding the Natural Rate of Interest, which is crucial for policymakers to set appropriate monetary policies, as deviations from it can lead to imbalances such as inflation, recessions, or both. Specifically, this chapter will review the current literature on its definition and its potential implications as an explanatory factor for demographic factors. Furthermore, the behaviour and routes of this relationship within the Euro region will also be covered.

2.1 The concept of Natural Interest Rate

The Natural Interest Rate has become a widely studied concept in macroeconomics, especially after central banks of major economies have struggled to manage their balance sheets under extraordinary policy rates in the last decade. It represents the theoretical equilibrium interest rate that would prevail in an economy with full employment and stable inflation, given its structural characteristics, id est, an interest rate that neither stimulates nor contracts the economy.

An increasing number of papers have attempted to compute the NRI in recent years. One of the most cited is by King and Low (2014)[13]. In their paper, King and Low focus on the real rate offered by the ten-year real bond yields, since those offer complete insurance against inflation in order to avoid paying a risk premium for investors' uncertainty about the course of inflation. Authors state that there are two major ways of measuring it: ex-ante rate, which subtracts the expected rate of inflation from the actual nominal rate, and ex-post rate, which relies on estimations based on past observations of inflation and output. The authors also recommend applying a GDP-weighted average when measuring, to avoid disproportionate impacts on the risk premium if terms are expressed in real terms.

Another methodology is used by Holston et al. (2017) [9]. While many papers have studied the factors affecting the natural rate of interest, this one focuses on its long-run measurement. In this case, the NRI is modelled on a New Keynesian framework of a Phillips curve built on a dynamic stochastic general-equilibrium model (DSGE) that varies over time in response to shifts in preferences and the growth rate of the natural rate of output.

In conclusion, while both methodologies differ in their estimation methods, they reveal similar main findings. Firstly, there is statistical evidence of a downward trend in estimated natural rates of interest for all countries, with a significant decrease after the 2008 Crisis. In the Euro area, it has gone from a 4% in 1985 to a -0,5% in 2013. The primary reasons for this trend are a substantial decline in the trend of GDP growth in advanced economies, with potential contributions from demographic factors. Secondly, although estimates of the natural rate of interest are highly imprecise, they are relatively more accurate for the United States due to the safe asset effect of the dollar. Lastly, there is evidence of the interdependence of natural rates of interest across advanced economies, implying a global factor that influences asset prices across countries.

2.2 Current state and the role of interest rates on the economy

Several studies have attempted to explain the current economic situation of advanced economies, which have been characterized by low rates of interest, the zero lower bound, and, as some economists claim, secular stagnation.

Those last ones had been led by Lawrence H. Summers (2014) [18]. In this paper, Summers points out several disturbing signs in the economy that reflect such a decline in real interest rates. First, reductions in demand for debt-financed investment, and consequently, for investment. Although, he makes two nuances concerning the reflection of the legacy of a period of excessive leverage and the fact that leading technological companies "swim in cash". Second, following Alvin Hansen's[1] ideas, Summers states that a declining rate of population growth implies a declining natural rate of interest. Third, changes in income distribution have increased inequality and the level of savings. Thus, reducing investment demand and increasing the propensity to save and, therefore, lowering the equilibrium real interest rate. Finally, there had been substantial global moves to accumulate safe assets, U.S. Treasuries, in particular, that also helped to reduce the natural real interest rate.

Other studies, for example, Rachel and Summers (2019) [16], assert that the neutral real rate would have declined several hundred basis points, but for increases in government debt and expansions in social insurance programs.

As a result, there had been a renewed interest in the secular stagnation hypothesis, which is a theory that suggests that chronic low private investment leads to low-interest rates, lower-than-desirable inflation, and sluggish economic growth. Eggertsson et al. (2019) [6] quantifies this hypothesis in a unique model that allows for constant negative interest rates to properly study the zero lower bound. Its main conclusions are that declining fertility, mortality, and productivity growth are the main factors that have contributed to the decline in real interest rates from 1970 to 2015. He also points out that government debt has tended to counterbalance these factors. Finally, he also finds out that the evolution of productivity will be a key determinant of whether real interest

[1] Alvin Hansen (1887 - 1975) was a prominent American economist who contributed significantly to the development of Keynesian economics in the mid-twentieth century. Hansen was also known for his work on business cycles, secular stagnation, and economic growth.

rates return to a more normal level of 1% and whether the zero lower bound will continue to pose a problem for business cycle stabilization.

Overall, the current economic situation of advanced economies has been the subject of much research and debate, with some economists pointing to secular stagnation as a key factor in the low natural rate of interest and sluggish economic growth. This phenomenon has been influenced by a fall in the demand for debt-financed investment, changes in income distribution, and a global trend toward accumulating secure assets. Other research, however, contends that the rise in public debt and the expansion of social insurance systems have partially offset these influences. In the next sections, I will delve deeper into this idea.

2.3 Influence of Demographics on Natural Interest Rates

Among the many papers exploring the reasons for the decreasing trend of the natural rate of interest in advanced economies, shifts in saving and investment preferences, as well as productivity slowdown, have been identified as key explanatory factors.

For instance, a recent study by Lunsford and West (2019) [15] investigated the long-run correlations between safe real rates (i.e., government bonds) and over 30 variables that may explain the phenomenon between 1890 and 2016 in the United States. They found that labour hours growth and demographic variables seemed to co-move with the safe rate, as predicted by growth models and OLG models, respectively. Moreover, they discovered a negative low-frequency correlation between productivity growth (i.e., total factor productivity growth) and real rates.

Regarding demographic movements, Gagnon et al. (2021) [7] identified two channels through which demographic changes have contributed to low real interest rates by 1.25 percentage points. Firstly, a drop in fertility rates has reduced the number of workers, increasing the capital-to-labour ratio and a drop in the real interest rates. Secondly, if baby boomers expect to live longer, they will save more, and therefore, the increase in savings will put downward pressure on the real interest rate.

Along similar lines, Carvalho et al. (2016) [4] observed that the real interest rate progressively fell from 4% in 1990 to 2.5% in 2014, and is expected to fall an additional 50 basis points over the next forty years. The main driver behind this decline is the increase in life expectancy rather than the decline in population growth. On the other hand, if the model incorporates some public pension system, the decrease starts a few years later but a more prominent decrease begins and stabilizes the real interest rate at -1% (a Secular Stagnation steady state). This occurs because, thanks to the initial pensions, agents have less incentive to save. However, as the demographic transition occurs, maintaining a constant level of transfers with a higher population growth implies future higher savings that put more pressure on the real interest rate and higher government debt rates.

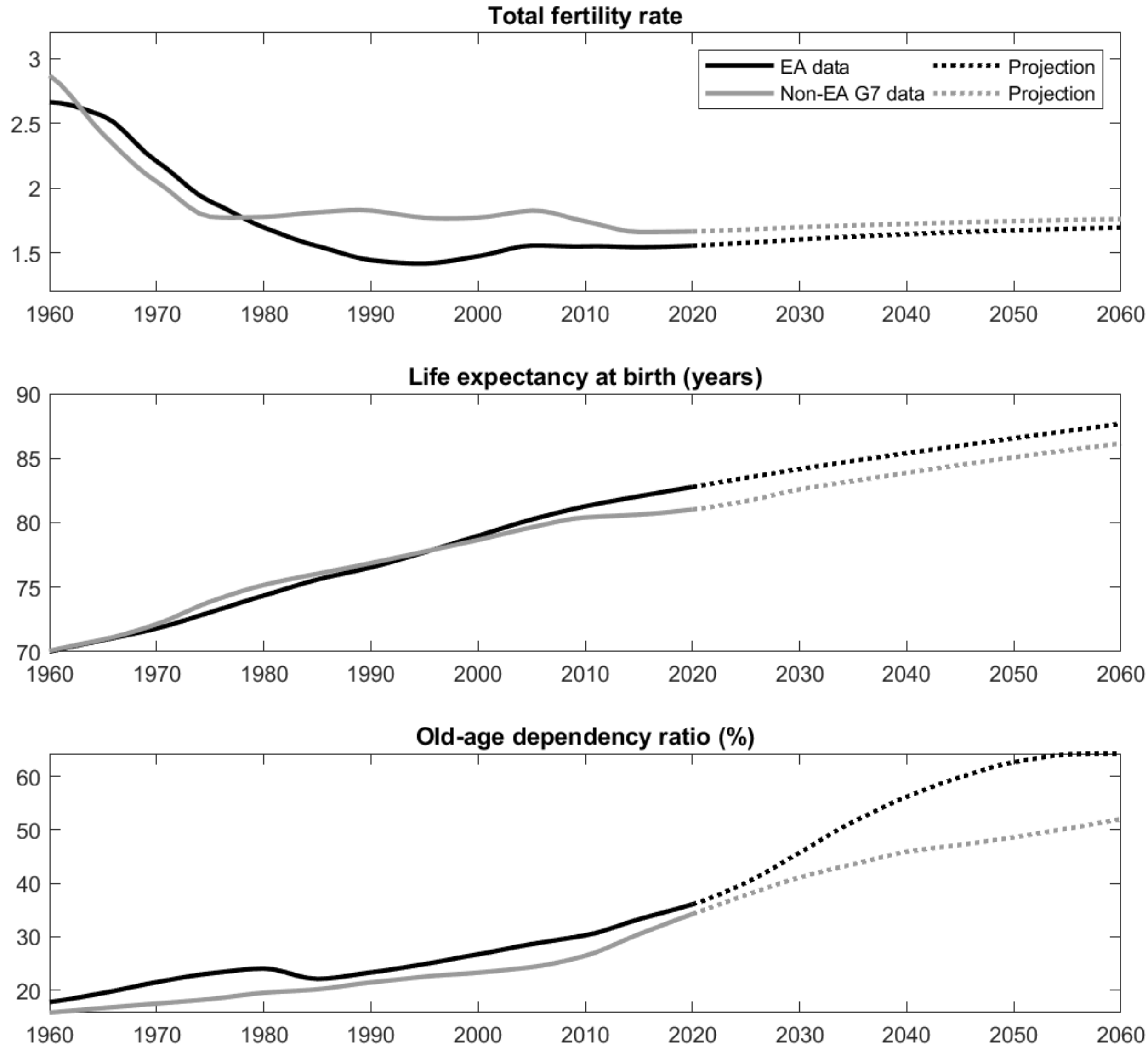

Figure 1: Evolution of main Demographic Rates. Source: United Nations World Population Prospects 2019. (Replicated Figure)

These papers highlight the importance of demographics as an explanatory factor for the decreasing trend of the natural rate of interest in advanced economies. The authors advise on the significant role that demographics will have in the ongoing years. Precisely because of this, understanding the dynamics and influence of such a correlation is crucial to address present and future economic challenges.

2.4 Empirical studies on the Euro area

In the Euro area, the concept of the natural rate of interest has received significant attention in recent years, with a growing number of studies examining its determinants and evolution. In this section, the main studies on the natural rate of interest in the Euro area will be reviewed, focusing on both, theoretical and empirical contributions.

One of the first papers that talk about the correlation between the interest rate effects of demographics is the Kara and von Thadden (2010)[11] one. In this paper, the authors construct a model within a closed economy New-Keynesian framework, using Gertler's [5] Overlapping Generation Model, which incorporates age-independent life-cycle behaviour, working age and retirement, and a PAYGO pension system. The main findings of the article are that both a decrease in population growth and an increase in life expectancy will contribute to a gradual increase in the old-age dependency ratio, accompanied by a gradual decline in the equilibrium interest rate over a period of about 20 years.

Another interesting scope is the one made by Holston et al. (2017)[9], but focusing just on the Euro area analysis. In this case, the model is built on the New Keynesian framework of a Philips curve relationship and an intertemporal IS equation to describe the dynamics governing the output gap and inflation as a function of the real rate gap. Also, the natural rate can be affected by low-frequency nonstationary processes. The implementation of the model ends up once the Kalman filter is used to estimate the natural rate of output, the growth rate trend and the natural rate of interest for each country. The results of this article are similar to the ones made before, in concrete those pointed out in the first section of this chapter.

A different scope is the one made by Krueger and Ludwig (2007)[14]. Here, the authors use demographic projections from the United Nations and an extension of the Auerbach and Kotlikoff[1] Overlapping Generations model through a quantitative model. In this model, households live up to 95 years and face uninsurable idiosyncratic uncertainty in the forms of labour productivity and mortality risk. The baseline model also considers the free flow of capital across regions. Finally, the authors incorporate a pure Pay-As-You-Go public pension system adjusted to the demographic changes in each country, but with a common retirement age of 65. The results show that, for the period between 2005 and 2080, the world interest rate is expected to fall by up to 1 percentage point.

Overall, the empirical studies reviewed in this section have shed light on the determinants and evolution of the natural rate of interest in the Euro area. Comparing the expected future trends in the Euro area to the rest of the advanced economies, it is difficult to see any difference. Though, because of the relatively elderly age structure of the Euro area concerning the rest of the economies, this fall in the natural rate of interest is expected to be before that in the rest. Thus, ”exporting” the effects of the demographic transition to other countries in the form of larger factor price changes.

3 Data and Methodology

This chapter provides a full overview of the data and methodology used in the analysis of the relationship between demographics and the NRI in the Euro area, as presented in the paper by Bielecki et al. (2020)[3]. The importance of using appropriate data and methodology in economic research cannot be overstated, since these elements have a big impact on the validity and reliability of the results obtained.

The rest of this chapter is divided into the following sections. The specification of the empirical model is fully described in Section 3.1, together with information on the variables that make up the model, and its underlying assumptions. The data sources utilized in the analysis are discussed in Section 3.2, along with the econometric methods used to estimate the model parameters and exogenous variables. Finally, Section 3.3 analyse the goodness of fit of the model selected.

3.1 Economic Model

Bielecke et al. (2020)[3] constructed a structural open economy model with life-cycle features. The model is populated by several agents, including households, producers, investment funds, and fiscal authority, and is linked to the rest of the world through international trade in goods and assets. Below, I present the most important features of each agent on the economy as well as some of their limitations.

It is important to notice that it has followed the rule of using upper case letters for aggregates and lower case letters for variables stated in per capita terms for designating real allocations.

3.1.1 Households

The model consists of a life-cycle framework in which each household is represented by a single agent who enters the model at the age of 20 with an age index of $j = 1$. The agents can live up to 99 years (where $j = J = 80$) and are subject to age- and time-dependent mortality risk $\omega_{j,t}$, resulting in the model economy being populated by 80 cohorts of overlapping generations at any given point in time, with the size of the cohort aged j denoted by $N_{j,t}$ (at time t).

$$U_{j,t} = \sum_{i=0}^{J-j} \beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \ln c_{j+i,t+i} \tag{1}$$

Each representative household, with age j, aims to maximize its expected remaining lifetime utility, which is dependent on its future consumption stream $c_{j,t}$ and on the β^i subjective discount factor, which represents the impatience of each agent concerning their consumption over time. The utility function is the summation of the logarithm of consumption at different ages i, weighted by the probability of surviving for at least i more years, represented by the ratio $N_{j+i,t+i}/N_{j,t}$.

As per the given information, households retire at the age of 63, denoted by $j = JR = 44$, and thereafter receive gross pension benefits that are not affected by their age. It should be noted that the pension system is compulsory, and all employed individuals must contribute to social security, which is charged along with other taxes on their labour income at a total rate of τ_t. To balance their expenses, households can trade different assets like domestic capital, government debt, and assets issued by other countries. In the deterministic environment of the model, these assets are considered perfect substitutes and pay the same real interest rate r_t, which will be examined in detail later. Borrowing across different cohorts is also allowed, as negative asset holdings $a_{j,t}$ are permitted at any age j. Additionally, households receive lump-sum and age-specific dividends $\pi_{j,t}$ from intermediate goods producers. As most individuals do not live up to their maximum age, they unintentionally leave bequests that are equally redistributed among all living individuals through lump-sum transfers beq_t.

$$c_{j,t} + a_{j+1,t+1} = (1-\tau_t)\left[\left(1-\mathbb{1}_{j\geq JR}\right) w_t z_j + \mathbb{1}_{j\geq JR}\text{pen}_t\right] + \pi_{j,t} + beq_t + (1+r_t)\, a_{j,t} \tag{2}$$

Analytically, the budget constraint is defined by Equation 2, where $\mathbb{1}_{j\geq JR}$ is an indicator of being retired. The intuition behind this is that their present consumption plus their future asset holdings equalize their net (from labour and pensions) income, dividends ($\pi = 0$ in perfect foresight), lump-sum bequests and interest obtained through assets holding.

3.1.2 Aggregation and Demographic Forces

In this model, the changes in the size of the youngest cohorts $n_{1,t+1} = \frac{N_{1,t+1}}{N_{t-1}}$ and the mortality risk $\omega_{j,t}$ are considered exogenous factors that govern demographic processes. The total number of living agents N_t and the population growth rate n_{t+1} are given by Equations 3:

$$N_t = \sum_{j=1}^{J} N_{j,t} \quad \text{and} \quad n_{t+1} = \frac{N_{t+1}}{N_t} - 1 \tag{3}$$

where $N_{j,t}$ represents the number of agents in each cohort, and the growth rate is determined by Equation 4:

$$N_{j+1,t+1} = (1-\omega_{j,t})N_{j,t} \tag{4}$$

The aggregate allocations over all living households can be expressed in per capita terms as follows:

Consumption: It is computed as the summation of all per capita consumption of a j-aged population at time t divided by the total population (N_t).

$$c_t = \frac{1}{N_t} \sum_{j=1}^{J} N_{j,t} c_{j,t} \tag{5}$$

Labour Productivity: It is computed as the summation of all active per capita productivity of a j-aged population at time t divided by the total population (N_t).

$$h_t = \frac{1}{N_t} \sum_{j=1}^{JR-1} N_{j,t} z_j \tag{6}$$

Asset Holdings: Is computed as the summation of all per capita asset holding of a j-aged population at time t divided by the total population (N_t).

$$a_{t+1} = \frac{1}{N_{t+1}} \sum_{j=1}^{J} N_{j,t} a_{j+1,t+1} \tag{7}$$

Bequests: They are computed as the summation of all the assets (including interests) of those that had died during one period of difference at time t divided by the total population (N_t).

$$beq_t = \frac{1}{N_t} \sum_{j=1}^{J} (N_{j,t-1} - N_{j,t})(1 + r_t) a_{j,t} \tag{8}$$

3.1.3 Firms

The model economy has two different sorts of firms: monopolistically competitive intermediate goods producers, which are indexed with ι, and perfectly competitive final goods producers. The mass of each type of enterprise is related to the size of the population, which is consistent with demographic processes in the household sector.

Final goods producers: According to the following CES aggregator, a representative final goods producer purchases intermediate goods for $y_t(\iota)$ and creates a homogenous final product for y_t:

$$y_t = \left[\int_0^{N_t} \left(\frac{y_t(\iota)}{N_t} \right)^{\frac{1}{\mu}} dl \right]^{\mu} \tag{9}$$

where the gross markup caused by insufficient substitution across intermediate types is denoted by $\mu \geq 1$. The amount of finished items produced is, therefore, the average of the purchases of intermediate goods, weighted according to the difference between the selling price and the purchased price. The answer to the profit maximization problem entails the following demand function for intermediate goods:

$$y_t(\iota) = p_t(\iota)^{\frac{\mu}{\mu-1}} y_t \tag{10}$$

where the producer of intermediate goods (ι) charges a real price that is expressed in terms of the level of the overall price ($p_t(\iota)$). An intuitive definition would be that the demand equalizes real prices of intermediate goods weighted by the gross markup of all the production of final goods.

Intermediate goods producers: Intermediate goods producers hire capital and labour, and produce differentiated output according to the following Cobb-Douglas production function:

$$y_t(\iota) = x_t k_t(\iota)^{\alpha} h_t(\iota)^{1-\alpha} \tag{11}$$

where x_t is exogenous total factor productivity, $k_t(\iota)$ denotes physical capital accumulated according to the standard law of motion:

$$k_{t+1}(\iota) = (1-\delta)k_t(\iota) + i_t(\iota) \tag{12}$$

and $h_t(\iota)$ denotes labor. The period profit flows are then:

$$\pi_t(\iota) = p_t(\iota)y_t(\iota) - w_t h_t(\iota) - i_t(\iota) \tag{13}$$

and each producer of intermediate items maximizes their present discounted value while accounting for the demand forecasts provided by Equation 10. It is assumed that households receive a percentage of profits each period in proportion to their labour income.

3.1.4 Fiscal Authority

According to the model, the government controls the pension system and makes market purchases of products in the amount of G_t. To pay for its expenses, it levies taxes that are proportional to the labour income of households and issues public debt valued at B_t that is due the following period (B_{t+1}). The government's intertemporal budget restriction can be expressed as:

$$\tau_t w_t \sum_{j=1}^{JR-1} N_{j,t} Z_j + B_{t+1} = G_t + (1-\tau_t)\,\mathrm{pen}_t \sum_{j=JR}^{J} N_{j,t} + (1+r_t)\,B_t \tag{14}$$

where τ_t is the tax rate, w_t is the average wage rate, $z_{j,t}$ is the productivity of the j-th (active) household, $N_{j,t}$ is the size of the household j, B_t is the total amount of government debt owed at the time t, pen_t is the pension per retired household, G_t is government purchases at time t, and r_t is the real interest rate.

The pension per retired household is computed as the product of the replacement rate (ϱ_t), which is exogenously determined, and the average wage for the entire economy, $\widetilde{w}_t$. The second is described as:

$$\widetilde{w}_t = w_t \frac{\sum_{j=1}^{JR} N_{j,t} z_{j,t}}{\sum_{j=1}^{JR} N_{j,t}} \tag{15}$$

The pathways of public debt, government spending, and the replacement rate are also presumed to be exogenously determined, but the tax rate τ_t is expected to adapt to satisfy the government's budget constraint.

3.1.5 Rest of the World

Domestic agents can acquire overseas assets or borrow from foreign sources since they have access to international financial markets. To impose some realistic assumptions, international transactions are intermediated by specialized agents, which charge Γ_t per unit borrowed, sending the collected revenue to savers. This cost appears as a wedge in the following formula for the uncovered interest rate parity condition:

$$1 + r_{t+1} = (1 + \Gamma_t)(1 + r^*_{t+1}) \tag{16}$$

where r^*_t, or the global interest rate, is exogenous for the domestic economy. The intermediation wedge depends on the country's external debt, so that

$$\Gamma_t = \xi \left(\exp\left(-\frac{B^*_t}{Y_t} \right) - 1 \right) \tag{17}$$

where $\xi > 0$ and B^*_t denotes the aggregate net foreign assets position.

3.1.6 Market Clearing Conditions

A uniform set of market clearing conditions are used to close the model. Since all producers of intermediate goods are the same, their prices are established in equilibrium so that $p_t(\iota) = 1$ for all ι. As a result, it becomes straightforward to aggregate the production side of the economy, and in particular, this produces the common aggregate production function shown below.

$$Y_t = x_t K_t^{\alpha} H_t^{1-\alpha} \tag{18}$$

Therefore, asset market clearing become:

$$A_t = K_t + B_t + B^*_t \tag{19}$$

which, after combining all agents' constraints, yields the standard law of motion for net foreign assets, where the future net foreign assets are those that equalize the capitalization of the bonds and the remaining gross income.

$$B^*_{t+1} = (1 + r_t)B^*_t + Y_t - C_t - I_t - G_t \tag{20}$$

3.1.7 Exogenous Forces

It is critical to take into account both the internal dynamics that are influenced by actions made within the economy (endogenous variables) and the external forces that are outside of our control (exogenous factors) to provide a more realistic picture of how the economy is performing.

The growth rate of the young population ($n_{1,t}$) and the age-specific mortality risk ($\omega_{j,t}$ $(j = 1,\ldots,J)$) for all age groups are two key external factors that might have an impact on the economy.

The fiscal authority, on the other hand, is unaffected by internal dynamics when making decisions about the procurement of commodities (G_t), the replacement rate for the pension system, and the trajectory of the public debt (B_t). The total factor productivity (x_t) and the foreign real interest rate (r_t^*) are two more external factors that have an impact on the economy.

3.2 Data, Parameters and Exogenous Forces

An overview of the data, parameters, and exogenous variables employed in the model previously discussed is intended in this part. Understanding the source of the data, the parameter values utilized, and the exogenous factors selected is necessary before being able, in the next chapter, properly interpret the study's conclusions and evaluate its validity.

As the scope of the project is to study the NRI in the euro area, the model was developed only for this region. This implies the remaining four G7 economies (id est, Canada, Japan, the UK, and the US) will be referred to as non-EA G7 regions. There were two reasons that justifies making this decision. First of all, the essayist thought it was more pertinent to concentrate on the group of wealthy countries rather than on the more populous but financially poor countries because the study attempted to account for international capital flows. Second, they encountered difficulties with the scant supply of pertinent data for nations outside the EA and G7.

The model was first solved for the non-EA G7 as a closed economy, which generated the world interest rate. Then, it was solved again, this time with the EA treated as a small open economy that took the world interest rate as given.

Regarding the rest of this section, it will be first described the data incorporated in the life-cycle characteristics as well as the demographic dynamics of the model used by the authors. Then, the exogenous factors modelled will be described and, straightaway, explaining how those have been chosen and what might be the driving forces behind the negative trend on the NRI. Finally, it will be studied the exogenous forces and finally, the values for the parameters have been chosen, from a theoretical and empirical perspective.

Overall, this section aims to provide a comprehensive understanding of the data, parameters, and exogenous factors used in the study, and to highlight their importance for interpreting the results of the analysis.

3.2.1 Life-cycle characteristics

In their study, the authors calibrated the age profiles of the Euro Area (EA) based on calculations presented in Jabłonowski (2018)[10] using data from the Household Finance and Consumption Survey (HCFS) conducted between 2012 and 2014. The productivity measure used was hourly labour income, which includes wage employment and self-employment. It is important to notice that all the life-cycle characteristics are determined to match those of the household head. It is important to state that, since the model does not account for changes in the household composition, an equivalence scale is applied when working on household data.[2] The obtained profile was smoothed using the Hodrick-Prescott filter and extrapolated linearly for the age group 20-24, for which data was missing.

In the non-EA G7 countries case, there had been issues regarding obtaining comparable and consistent micro-level data. In the US case, the profiling of labour productivity has been done by using hourly wage profiles by age computed by Keane and Wasi (2016)[3][12]. Regarding the Japanese case, the data used is that of the Ministry of Health, Labour and Welfare.[4]. The resulting profile is essentially identical to the profile of EA workers, with active workers older than the average effective retirement age showing the biggest differences. This phenomenon could be happening because of the difference in social benefits across the studied countries/regions.

The below Figure 2, shows how the model has matched the Productivity, Consumption and Assets profiles. In particular, the first graph in the figure displays the matched age profile for labour productivity, which conforms to the typical pattern of increasing productivity up to the late middle age of a household head, followed by a decline until retirement. Even though assets and consumption are not directly the object of study of the project, both are computed as a matter of curiosity. Being the first the net wealth excluding pensions, and consumption the amount spend on food and utilities. Regarding the second graph, we can clearly state the expected increasing path regarding consumption during a lifetime until retirement[5]. Finally, the Assets graph also shows how, as agents age, they increase their asset accumulation until retirement, when they start to dissave. In the next chapter, I will discuss the reasons behind the negative values at the first stages of life.

[2]In the same line some similar papers, the age profiles are divided by the square root of the number of households members.

[3]Even though Keane and Wasi (2016)[12] work is specified and estimated under a life-cycle labour supply model, their estimates of the effect of human capital and wages on labour supply is used to calibrate the labour supply equation in the studied model.

[4]In particular, Year Book of Labour Statistics 2017, Table III.69. (Contractual cash earnings, scheduled earnings and annual special earnings by industry, size of the enterprise, sex, type of workers and age group)

[5]Some studies on developed countries find a "negative relation between the drop in consumption and changes in time devoted to activities that might be substitutes for market-purchased goods and services." (Schwerdt (2005)) [17].

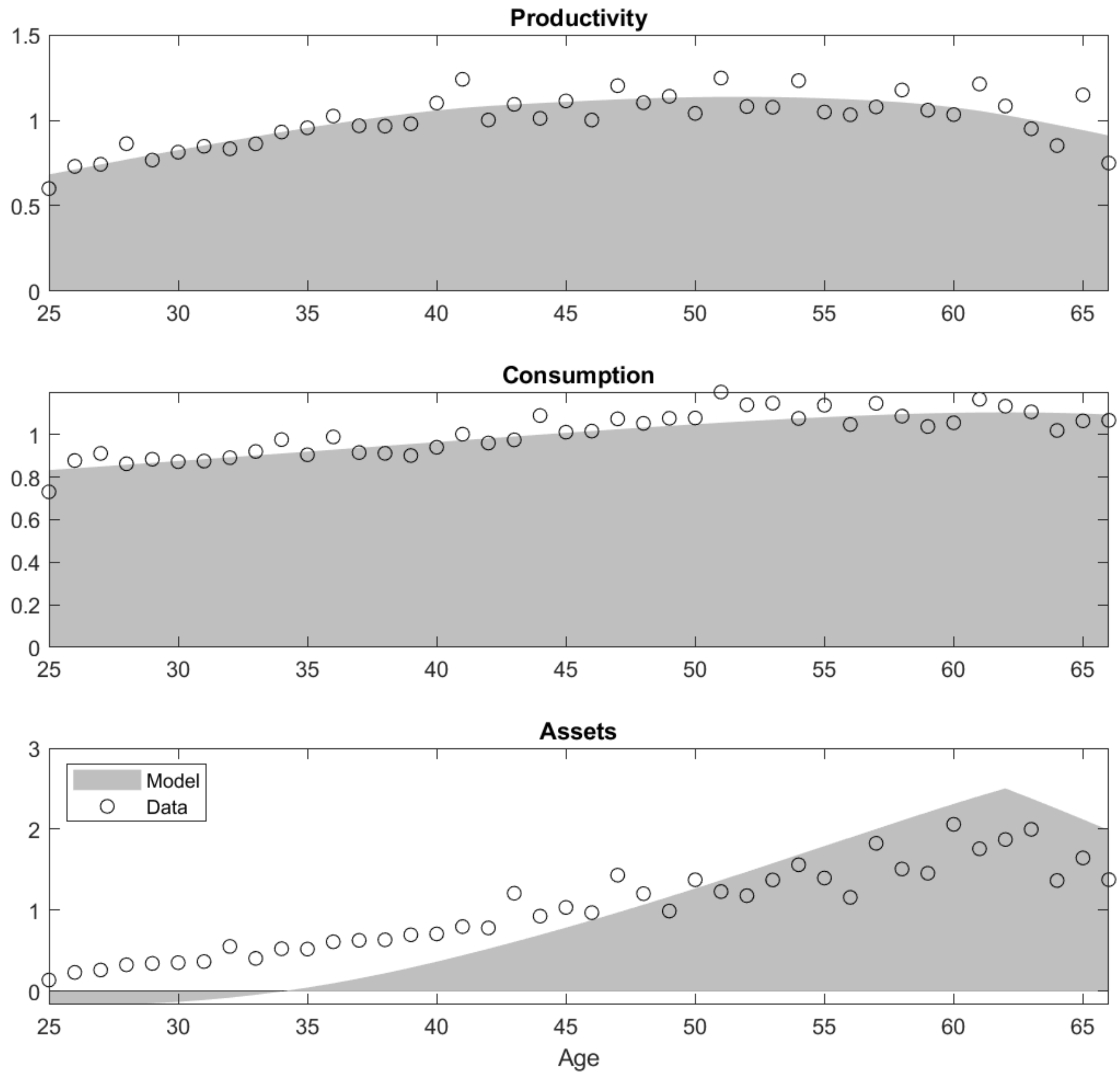

Figure 2: Decomposition of changes in EA NRI. *Notes:* All numbers are presented as a difference from the 1985 levels. (Replicated Figure)

3.2.2 Demographic forces

Therefore, the model demographic forces (mortality rates and cohort sizes) are based on the United Nations World Population Prospects (2019), which covers both, data from 1950 until 2020 and projections for the following decades. It is important to notice that cohort sizes have been normalized by the size of the 20-year-old cohort. Because the data is provided in 5-year intervals and for broad five-year age groups, it is first interpolated to an annual frequency through cubic splines. Secondly, population-weighted averages are then used to construct the data for both country groups. Finally, the resulting mortality rates and growth rates are smoothed using, as before, a Hodrick-Prescott filter.

The deterministic simulations begin in the year 1900 to guarantee that the model predictions for the years under consideration (id est, 1980 and after) are not tainted by frontloading effects and beginning circumstances. To properly match the current population structure, artificial population statistics for the years 1900-1950 are produced, and the sizes of historical 20-year-old cohorts are backcast while retaining the historical mortality rates at the earliest known date. The year 2010 is chosen as the base year and predicted population structures for subsequent years are built using data on mortality rates and growth rates of the initially young.

Figure 3 depicts the resultant age pyramids for the EA population in 1980, 2010, and 2040, confirming that the underlying demographic patterns are accurately preserved. As we can see, as decades advance, the movement of the latest Baby Boomers(1946-64) and those first of Generation X (1965-80) is well-represented by the biggest cohort sizes in the graphs. In concrete, the demographic issues that the 2040 plot is tackling, the fact that the EA will face the biggest amount ever seen of retirees with the higher ever life expectancy, is the main motivation behind this final degree project. Also, it is relevant to the gap between the 55 and 65-year-old cohorts in 1980, which would be explained because of the Second World War (1939 - 45).

3.2.3 Exogenous Variables

As stated in Chapter 2, between all the possible driving factors behind the negative trend between demographics and the natural rate of interest, two highlights overall: the productivity slowdown and shifts in saving-investment preferences due to fiscal measures. Because of that, the authors feed the model with several driving forces that capture both phenomena.

The first one, the total factor productivity (TFP), is used in the model to reflect worldwide technological development. It is assumed that the TFP growth rate for the EA and the other G7 nations is the same, therefore computed as a GDP-weighted index for the whole G7 group from 1960 to 2020. After a Hodrick-Prescott filter is used, the TFP growth rate is combined with Ageing Working Group forecasts for the TFP growth rate in the eurozone from 2016 to 2070.

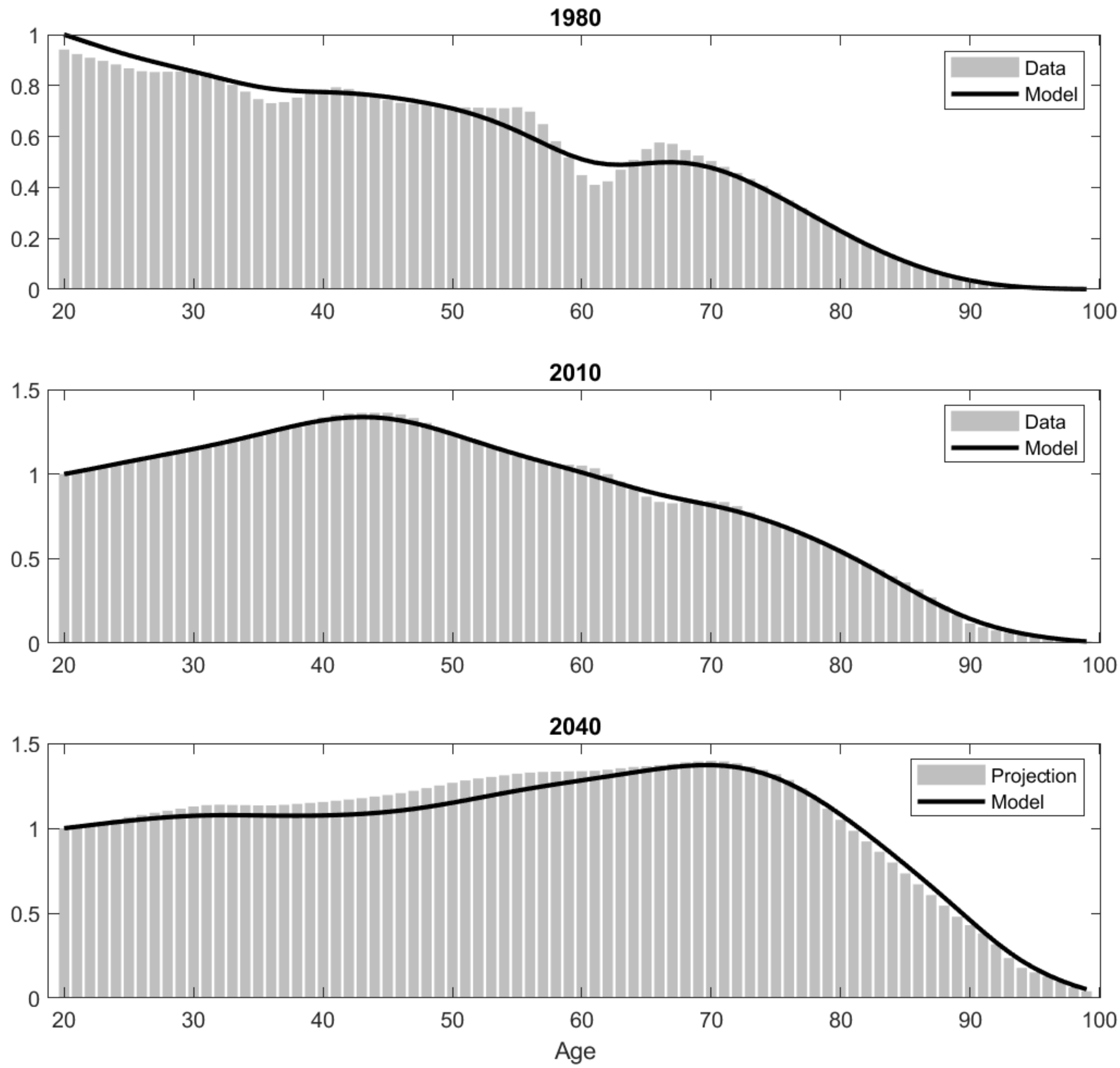

Figure 3: Evolution of demographic pyramids in the euro area. Source: United Nations World Population Prospects 2019 and model simulations. (Replicated Figure)

Second, the fiscal measures that could have an impact on the NRI, are mostly government gross and net debt. But it is important to state that, since the majority of households have no access to gross debt, is mostly held by central banks rather than private citizens. Therefore, net debt is only taken into account. While there is available information on net debt for the EA group, it is not the case for the non-EA group. In this line, it has been computed by first calculating GDP-weighted gross debt to GDP and then correcting this time series by the average observed ratio between net and gross debt across the entire group of advanced economies. Finally the path of public debt, for both regions, had been fixed to that of 1980 - 2024 levels for the remaining years.

Lastly, because of the lack of temporal volatility in the percentage of government purchases in EA output, is set it at 20%, which matches the data's long-term average. On the same line, the replacement rate is fixed at 44%, which corresponds to the average replacement rate found in EA nations during the previous decade.

3.2.4 Structural Parameters

Regarding the structural parameters calibration, their chosen values are presented in the following table.

Table 1: Calibrated structural parameters.

Parameter	Value	Description
β	1.0047	Discount factor in the EA
β^*	1.0093	Discount factor in the non-EA G7
δ, δ^*	0.1	Capital depreciation rate
α, α^*	0.25	Capital share in output
μ, μ^*	1.25	Product markup
ξ	0.038	International borrowing friction

Notes: Stars indicate the parameters used to calibrate the rest of the world, modelled as a closed economy version of the baseline model.

In summary, Table 1 describes that the model is calibrated with a standard set of parameters for labour income share, product markup, and capital depreciation, ensuring that they match observed values in the Eurozone. The discount factor is set to 1.0047 to match the average real interest rate of 1.2% observed in the Eurozone from 1999 to 2008, which is considered the period when interest rates were closest to their equilibrium values. The capital elasticity of output is calibrated at 0.25, resulting in an investment rate that is close to the Eurozone average. Together with a product markup of 1.25, this implies that the labour income share in output is 60%, which is consistent with national accounts. Also, the depreciation rate follows the common standard of 0.1 value.

The same parameter values are used for non-EA G7 nations as in the Eurozone, with the exception that the discount factor is adjusted to a slightly higher value to reflect the average real interest rate of 0.8% recorded in the non-EA G7 block from 1999 to 2008.

Finally, the risk premium parameter linked with foreign borrowing is set at 0.038, based on the Eurozone's average international investment position of around -10% of GDP from 1999 to 2008.

3.3 Model Analysis and Justification

An in-depth analysis of the model presented by Bielecki et al. (2020)[3] is provided in this section. The analysis is broken down into three phases, beginning with a ”Model Analysis” which gives a general overview of the model’s structure and underlying assumptions, assesses the model’s capacity to capture historical data and important economic characteristics, and explores its constraints and future possibilities. The second portion, titled ”Data Analysis,” handles the sources and standards of the data used in the model, as well as the techniques employed for data cleaning and processing.

Finally, the section ”Parameter Analysis”, describes how main parameters were selected and calibrated, performs sensitivity analyses on those parameters and how some of the main results are biased because of that, and finally contrasts the values of the parameters selected with those found in other relevant studies.

3.3.1 Model Analysis

The model used by Bielecki et al. is an enriched Overlapping Generations Model with two economies, one representing the Euro Area, and the other representing non-EA G7 countries. The economic model is settled up on an open economy style, which includes several key features of the economy, such as imperfect competition, nominal rigidities, and frictional labour markets since it is designed to capture key features of the euro area economy and its interactions with the global economy.

To evaluate the model’s ability to capture these features, a comparison between model outputs and historical data has been done. The simulations indicate that the model does a reasonable job of replicating the dynamics of key macroeconomic variables, such as GDP, inflation, and interest rates. However, there are some areas where the model falls short. For example, the model’s predictions of the unemployment rate are less accurate, particularly during times of economic crisis.

However, the authors also note that the model has some limitations. One limitation is that the model assumes that all households and firms are homogeneous, which may not be realistic in a real-world scenario. Another important is that all countries’ behaviours (by regions) are assumed to follow an equal path, which is not very realistic. Additionally, the model does not include some important factors that can affect economic behaviour, such as financial frictions (rather than the international ones), heterogeneous agents, and non-standard monetary policy measures, like the recent Quantitative Easing[6].

Another important limitation is the fact that, while being under the open economy assump-

[6] Quantitative easing (QE) is a monetary policy tool used by central banks to stimulate the economy by increasing the money supply and lowering interest rates. The European Union has implemented several rounds of QE since the global financial crisis of 2008 through the European Central Bank.

tion, authors impose strict exogeneity of the world interest rate from the EA perspective. This is shocking, especially since the EA region economy is seen as relatively small from an economic perspective. Also, even though the appearance of unintentional bequests is common in the literature, it does not stop being controversial since those are not homogeneous among all households, which could lead to divergences between capital accumulation among them.

Despite these limitations, the model represents an important tool for understanding the behaviour of the euro area economy. An example of that is the well-predicted pattern that Figure 2 shows. One potential extension to the model could involve incorporating more detailed representations of financial markets and their interactions with the real economy. Another area for potential improvement could be to incorporate a richer representation of the global economy, including some other regions apart from those that are already studied. This last point is especially important since, even though the EA is computed through a weighted mean, the policies, importance, and behaviour among those are so diverse. Overall, the progressive incorporation of heterogeneous agents would add some realistic assumptions to the model.

Overall, the model used in the paper provides a useful framework for analyzing the macroeconomic dynamics of the Euro Area and non-EA G7 countries. Ultimately, is able to capture key features of the studied economies and provides a solid foundation for future research.

3.3.2 Data Analysis

The quality of data used in a model is critical to the accuracy of its predictions.

Bielecki et al. (2020) used a vast array of data sources, including national accounts data from Eurostat, life-cycle profiles from the Household Finance and Consumption Survey database, and the mortality and cohort size from the United Nations World Population Prospects 2019 among others. The authors also employed a sophisticated data cleaning and processing procedure, which included checking for outliers and inconsistencies. In order to do that, whenever it has been necessary a Hodrick-Prescott (HP) filter has been applied, which is a statistical tool used to decompose a time series into a trend component and a cyclical component, mainly used to remove short-term fluctuations associated with business cycle fluctuations. They then created a harmonized dataset for all the countries included in the analysis for comparability reasons.

However, there are some important keys that are missing from the data. The first is that, because of using wide data sources, it could be the case that some of the results are slightly biased. The reason behind this is the scarcity of information for some countries, especially for those of the non-EA regions. Related to that, the standardization of a wide variety of countries into one single region could have a significant impact on the results, precisely because of the differences in economic structure among them. But the biggest threat to the data is coming from the presence of future migratory fluxes, which could diverge the results obtained, especially in the long run.

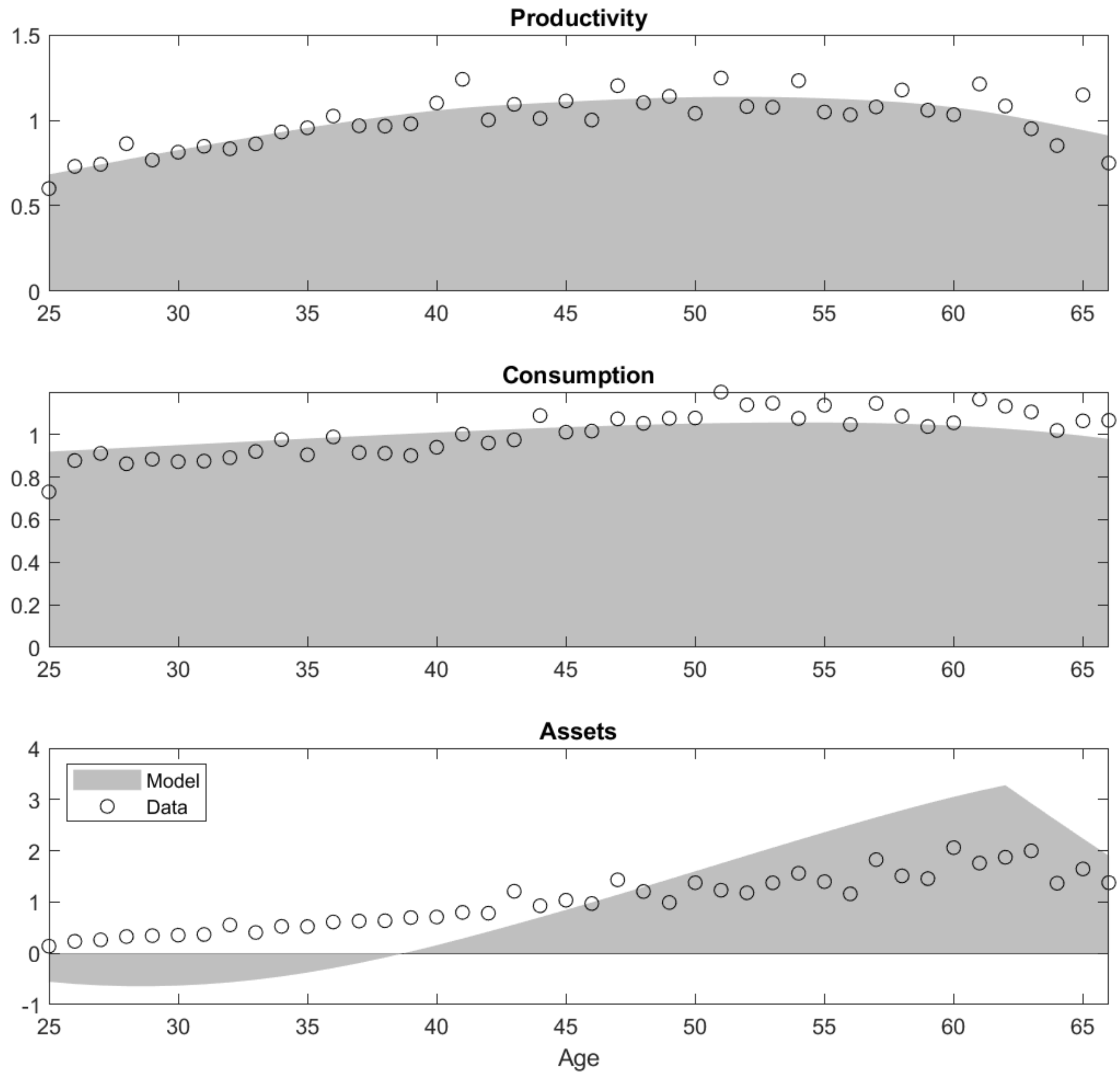

Figure 4: Decomposition of changes in EA NRI. *Notes:* All numbers are presented as a difference from the 1985 levels. Simulation for $\delta = 0.75$.

Overall, despite the limitations and challenges in the data, the model benefits from a good quality and reliable source of information. The data cleaning and processing procedures were well-performed, ensuring that the final dataset used in the model is accurate and representative of the underlying economic reality, at least for the time horizon required for this research.

3.3.3 Parameter Analysis

The key parameters used in the model were carefully chosen and calibrated to ensure that the model produces realistic and meaningful outputs, as seen in Table 1. Parameter estimation procedures involved the use of both historical data and a priori assumptions based on economic theory.

Sensitivity analysis was conducted to test the robustness of the model to changes in key parameters, and the results indicate that the model is generally stable and produces reasonable outcomes even with some degree of parameter variation. But when some ”extreme” values were tested, for example, a null -zero- capital depreciation rate, the model fails to match the reality and the results are illogical. Rather than that, after slowly increasing δ, Figure 4 shows how, in comparison to Figure 2, agents prefer to consume more in the early stages of life instead of buying assets, which it seems rational under the assumption that δ now is set to 0.75. In other simulations, in this case regarding the assumption that there is no international borrowing (id est, some ”autarchy” economy), the results obtained show that agents do not have any foreign assets, which seems logical, but the interest rates remained unchanged, which clearly is not.

All in all, the parameters chosen appear to be sensible, consistent with economic theory, and backed by historical facts. It is crucial to note, however, that some extreme values might provide implausible findings. Furthermore, it is critical to evaluate the model's limitations, such as assumptions and simplifications that may not adequately depict the economy's complexity. Future research may focus on improving the model's accuracy and utility for policy analysis by refining it and including new elements, like a parameter to limit the amount of bequest or some other to tax differently cohorts depending on their age[7].

[7] Those kinds of parameters are already present on some other papers, see Eggertsson et al. (2019)[6]

4 Results

Throughout the entire paper, it has been demonstrated that population ageing can significantly impact economics, specifically the Natural Interest Rate. As shown in Figure 1, this project attempts to address the issue of the inexorable decline in population pyramids. The consistently declining fertility rates, coupled with the highest life expectancy ever experienced by humans, lead to a substantial increase in the old-age dependency ratio. This chapter aims to examine the political, social, and economic consequences arising from the current situation and anticipated in the near future.

Thus, this chapter describes the empirical findings of the study on the connection between demography and the natural interest rate in the eurozone, building on the theoretical framework presented in the preceding chapter while advancing the knowledge frontier through several model extensions.

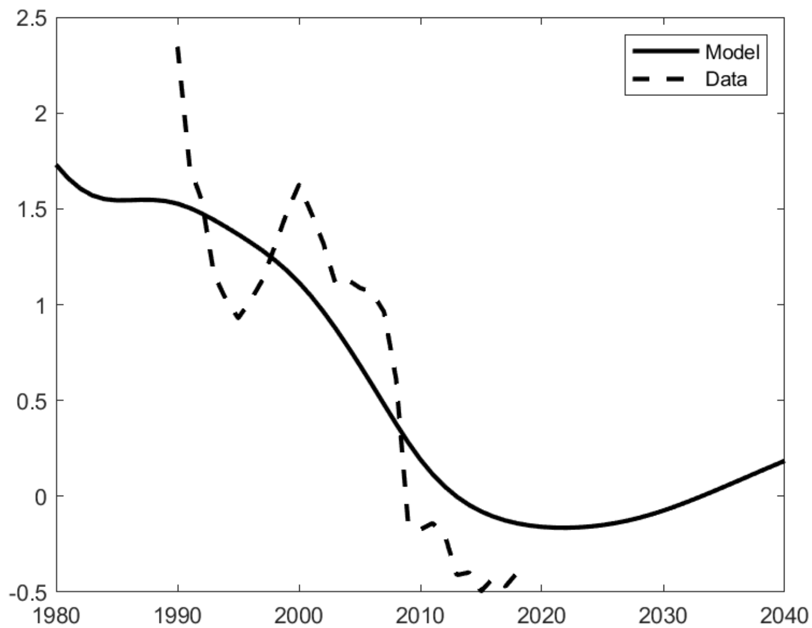

Figure 5: Natural rate of interest in non-EA G7 countries. Source: Holston et al. (2017) [9] and Han (2019)[8].*Notes:* Computation through a GDP-weighted average of the countries. (Replicated Figure)

In order to accomplish this, the chapter is organized into three subsections. The first subsection presents the baseline results of the study paper. The second subsection presents increasingly structural extensions of the model, beginning with several simulations of the effects of agents' risk aversion (σ) and concluding with the modelling of substitution elasticity (ε) parameters on the model economy.

4.1 Baseline Model

This section is dedicated to simulating and analyzing the results obtained through the main paper model. First, the main results will be described, and several possible driving factors behind the fall in the NRI will be explored, including openness scenarios, pension system reforms, and the role of migration in the economy. It is important to note that all simulations are produced by running variations of the model in stochastic scenarios, which assumes that all driving factors are known in advance to agents.

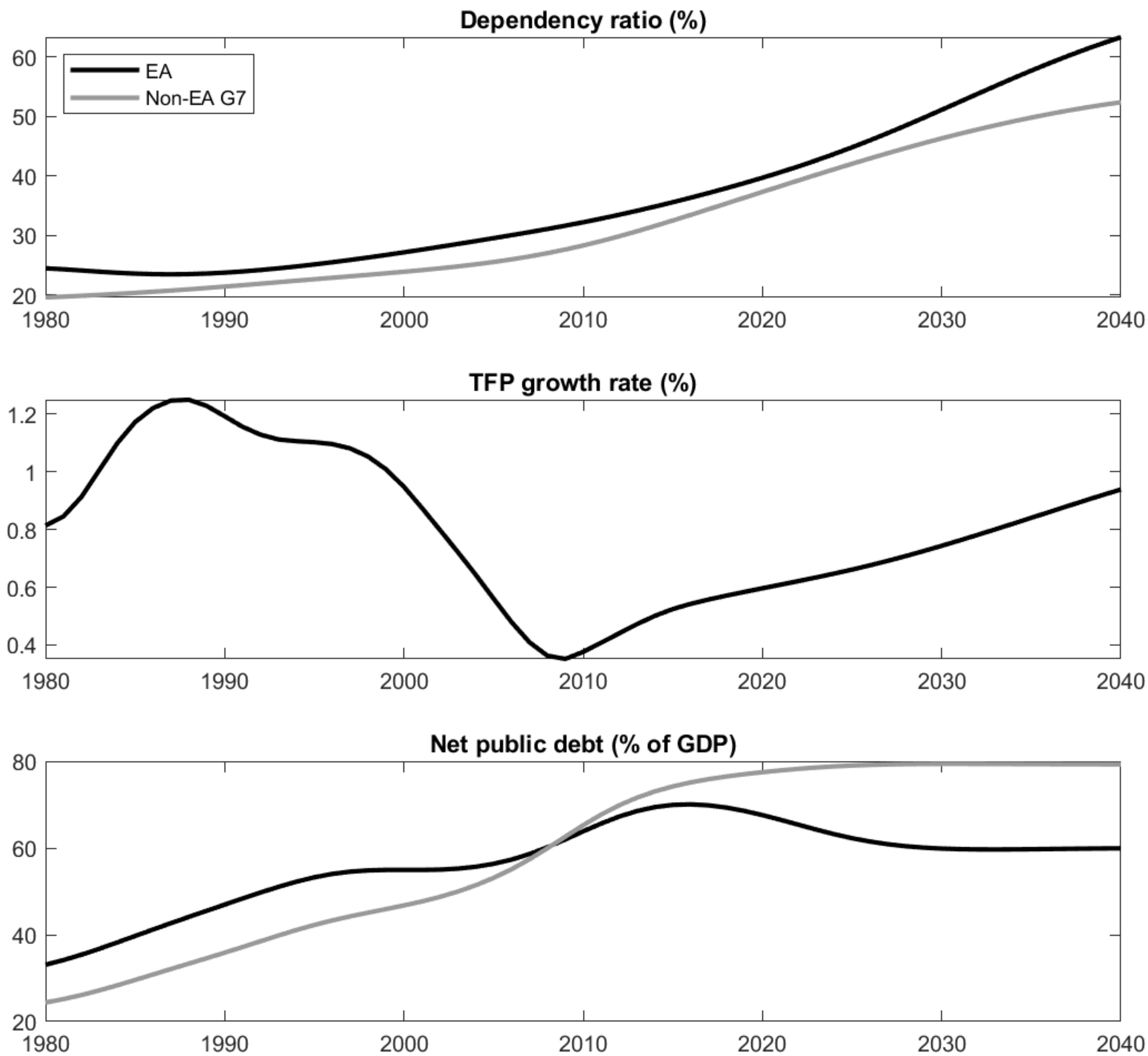

Figure 6: Exogenous driving forces from EA and non-EA G7 perspective (Replicated Figure)

4.1.1 Baseline Scenario

This section presents the first scenario, which focuses on the main purpose of the project: the NRI. As shown in Figure 5, the model smoothly matches the declining trend. The declining trend starts as early as 1980 and was predicted to hit rock bottom between 2020 and 2021, as seen in the figure. The Euro Area region takes the world NRI as given for its agents, given its small economy. The obtained results suggest that a behind-driving factor exists for such a decrease in a relatively short time period. Following the main hypothesis, the declining trend in the first 40 years coincides with demographic changes and declining rates on the TFP, as shown in Figure 6. The increasing trend in the TFP also matches the expected trend, but the growth rate of the mid-1980s will never be reached again. Thus, it can be seen as the driving factor from the NRI perspective.

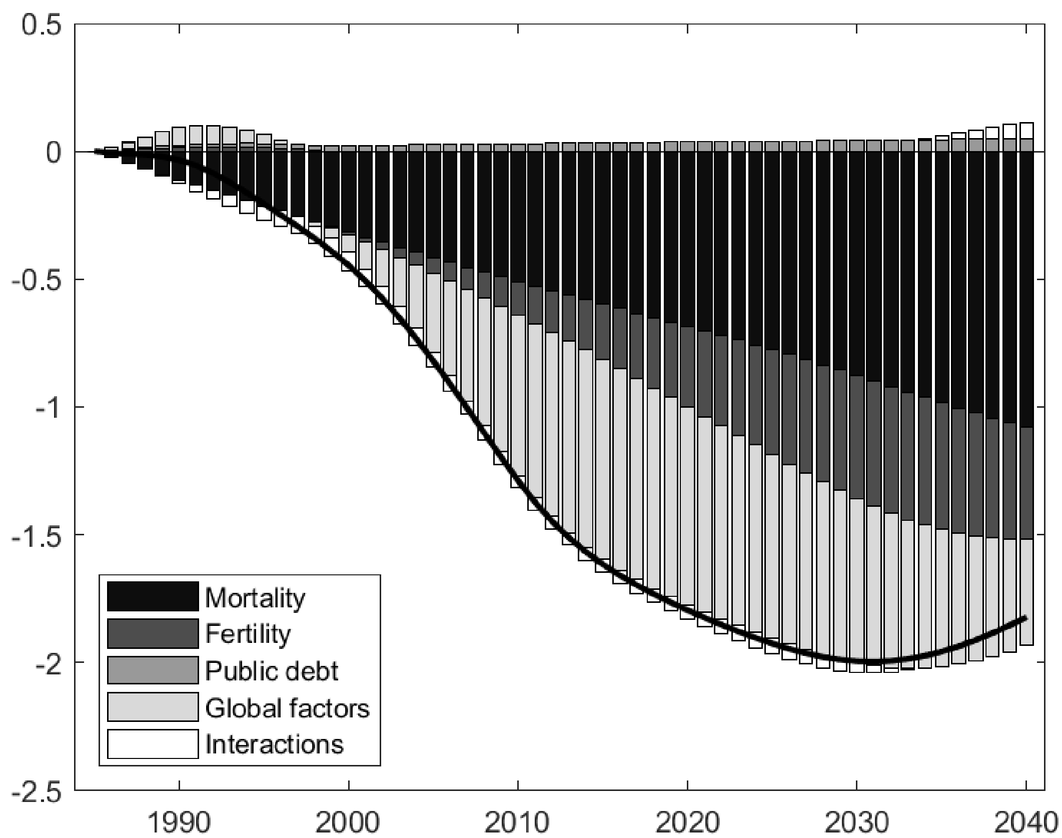

Figure 7: Decomposition of changes in EA NRI. *Notes:* All numbers are presented as a difference from the 1985 levels. Global factors combine the impact of TFP growth and foreign interest rate. (Replicated Figure)

The previous figure illustrates the same observations that were made in the Literature Review chapter, namely that lower fertility rates and higher life expectancy in the Euro Area (EA) compared to the non-EA G7 countries result in higher dependency ratios. It is worth noting that this trend began in the late 1980s with the ageing of the Baby Boomer generation. Additionally, the expected stability of net public debt with respect to GDP appears to contradict the current economic situation, but as mentioned earlier, the COVID-19 pandemic caused a sudden change in economic behaviour.[8]

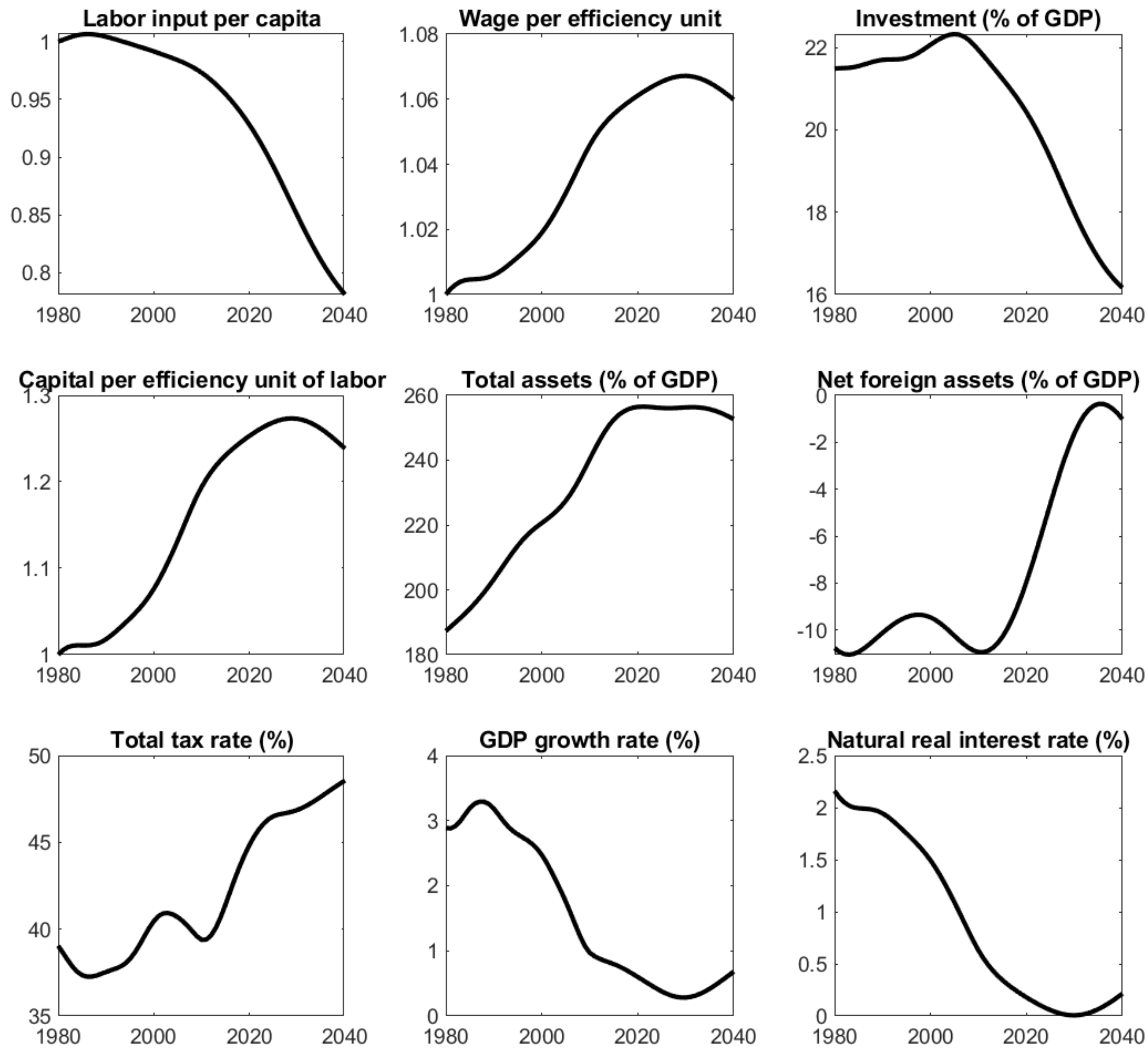

Figure 8: Baseline model simulation results. (Replicated Figure)

It has cleared the path of the NRI and its main driving forces, but we need to investigate which ones are the most significant. Figure 7 precisely graphs the decomposition of the NRI that the EA considers as exogenous.

The real interest rate in the EA follows a clear decreasing trend over the simulation period, with

[8]The authors assumed that, since IMF projections only extended to 2024, the path of public debt would stabilize at around 60% thereafter.

a significant drop of about 2 percentage points between 1985 and 2030. On the one hand, among all the demographic profile variables studied, Mortality and Fertility alone constantly explain more than two-thirds of the overall negative trend, with Mortality accounting for more than half of it. Note that Fertility, while mostly negative throughout the entire time period, plays a positive role in the '90s due to the post-baby boom effect. Finally, public debt has a permanent positive impact, but their weight is too low to make a significant difference.

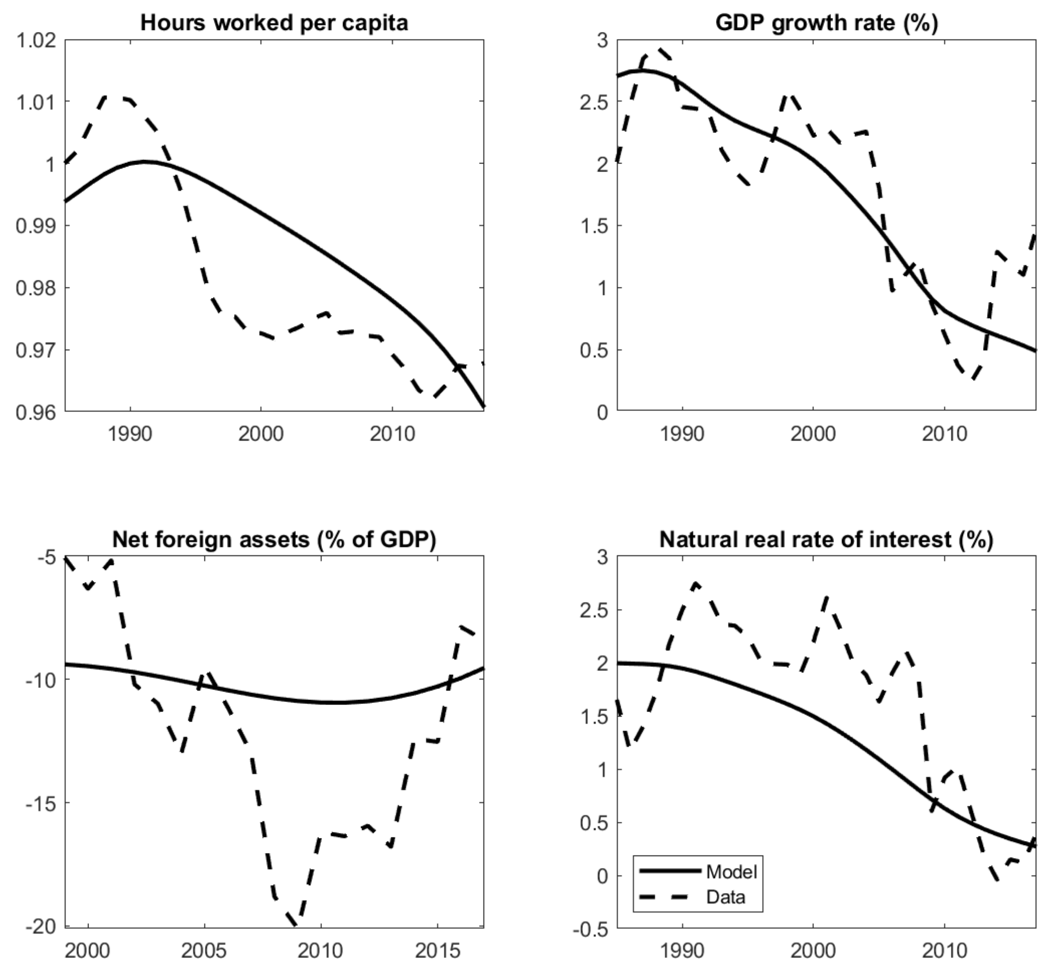

Figure 9: Simulated paths versus actual developments in the EA. *Notes:* Hours worked per capita are normalized and computed for France, Germany and Italy. EA Data is available only since 1999. Source: Holston et al. (2017) [9]. (Replicated Figure)

On the other hand, Global factors, which combine the effect of TFP and the foreign interest rate, have an initial positive impact due to the highest rates of TFP growth, even as they began to decline.

In order to illustrate the impact of demographic factors such as mortality and fertility, Figure 8 is presented. The figure shows that as life expectancy increases and retirement age remains fixed, the dependency ratio will also increase, making labour a scarce resource. As a result, wages will rise in real terms, and firms will consequently invest in capital to achieve higher capital-labour ratios.

Around 2010, the increasing number of retirees will cause the investment ratio to decline, leading to a decrease in productivity and GDP. Simultaneously, households will increase their savings due to the higher life expectancy shock and the lower labour income. Since demographic changes occur earlier in the EA, these savings are primarily invested in foreign assets, resulting in a quasi-positive net foreign asset balance for 2030. Finally, due to the fixed replacement rate and retirement age assumptions in the baseline scenario, the total tax rate will rapidly increase from around 38% in 2008 to almost 50% in 2040 due to the higher dependency ratio and the social pension system in place.

However, is the data following the same paths as those shown by the model? Well, as presented in Figure 9, there are some discrepancies, but the results generally match those of the data. The predicted decline in hours worked per capita, as well as that of the GDP growth rate, is almost perfect from the perspective of the data. Both capture the declining path from 1990 until 2010 very well. However, although the model predicts a continued decline, the data shows a slow increase. Comparatively, the model is less successful when it comes to the NRI, which is reasonable since the data fluctuates rapidly in the short term and is not well captured by the repeated Hodrick-Prescott filter. Finally, a similar effect seems to be happening with the net foreign asset position, which remains flat instead of following the trend.

Despite these shortcomings, the model fits the reality in the medium and long term, and it only appears to fail when there are short-term fluctuations.

4.1.2 The Role of Openness

Since most economies, especially those in the EA, have experienced the most globalized decades in Earth's history, it is reasonable to move from a closed to an open economy. What implications will this have? Well, the liberalization of capital flows will link domestic and foreign interest rates, specifically with the world interest rate. However, even under perfect assumptions regarding the free movement of capital, the real economy is not as simple, and there will always be some degree of friction. Even in the unrealistic case where all capital returns are equalized, investors will have risk aversion and preferences regarding which country to invest in.

To capture this, the parameter ξ, which represents international borrowing friction, was included as part of the intermediation wedge charged by specialized agents on international transactions[9]. Therefore, ξ can be viewed as all the friction that capital has when moving internationally,

[9] See Equation 17

whether borrowing or not.

As shown in Table 1, the parameter was first calibrated to match the Eurozone's average international investment position at 0.038, which corresponds to the baseline simulation. To observe the effects of ξ on the model, several calibrations were made and simulated in Figure 10.

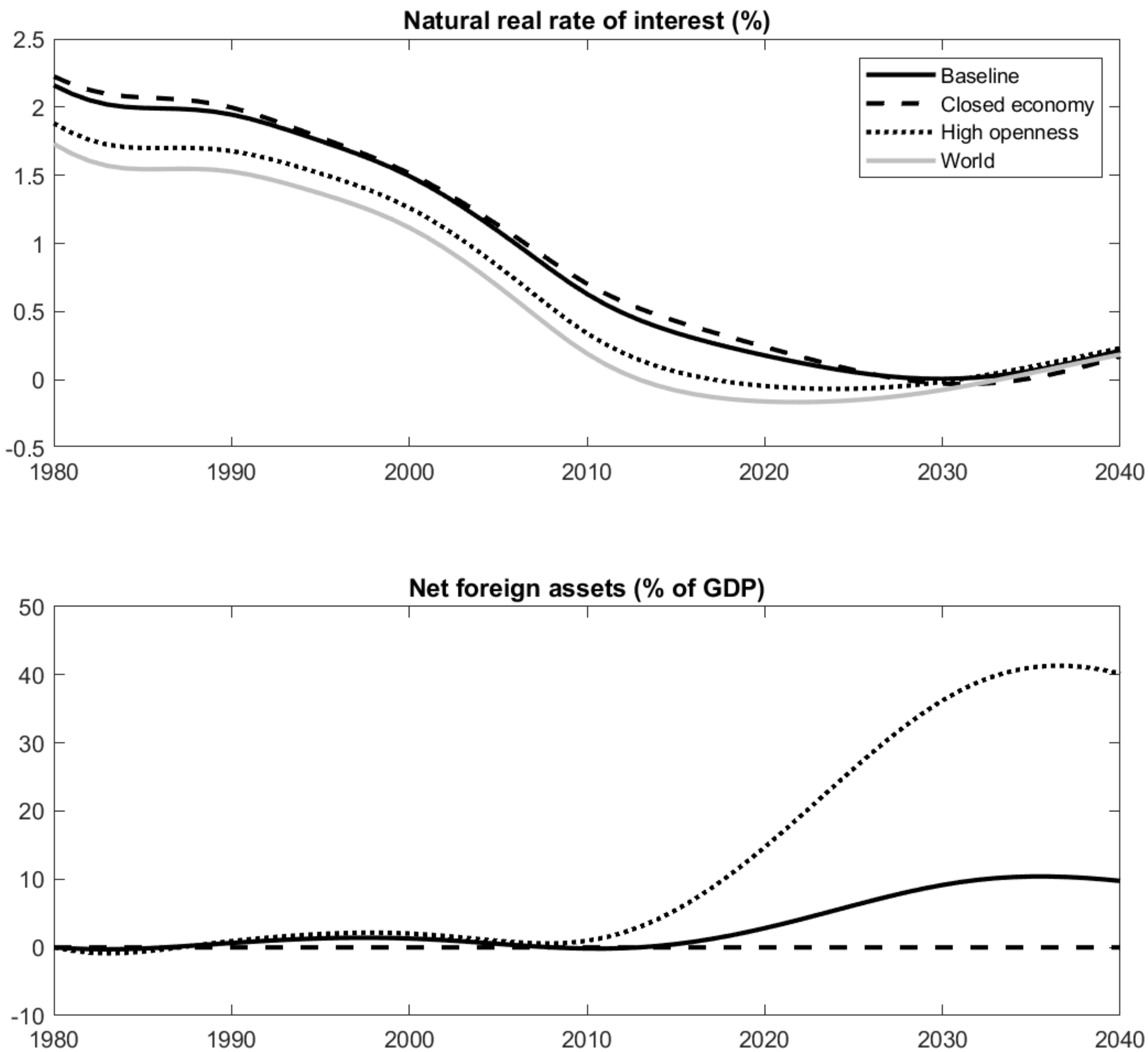

Figure 10: Financial Openness Scenarios on the EA. (Replicated Figure)

The first simulation studied how our economy would be affected if the degree of openness were increased, rather than the baseline. In this case, ξ was set four times lower than the baseline, at 0.0016. It is reasonable to assume that the higher the degree of openness, the closer the NRI is to that of the world. The results show that the expected recovery is faster than in the baseline model, occurring around 10 years earlier. This occurs because, given the dynamics of the model explained earlier, EA households tend to increase their foreign assets position. This is precisely what happens, and in this case, the net foreign assets held by households differ by 30pp from the baseline model. This process continues until the world NRI is higher than that of the EA economy, at which point domestic agents' preferences for foreign assets begin to disappear.

Alternatively, if we consider an autarchic economy, that is, one that is completely closed, the net foreign assets are zero[10]. The economic rationale behind this is that specialized agents charge the full amount of the country's external debt, which provides no incentives for households to purchase them. The implications for the NRI are slightly different from those of the baseline scenario. This occurs because demographic changes in the EA and G7 countries are almost identical, but they occur earlier in the EA. As a result, the NRI will be lower when it reaches its lowest point around 2030.

In conclusion, financial openness does not have a significant impact on the NRI, except in extreme cases. However, the higher the degree of openness, the closer the NRI will be to the world average. This will lead to an earlier expected reversal of the downward trend due to increased foreign asset accumulation.

4.1.3 Pension Systems Reforms

One of the consequences of increasing old-dependency ratios, where pension systems are managed by the government, is a constant increase in taxes to maintain pension income. This would imply severe financial problems for the public coffers, which, under the model assumptions, would have to be supplied through debt. In the baseline model, a Pay-As-You-Go public pension system is set up, where the retirement age is fixed at 63 and the replacement rate is set at 44%.

As described in Section 3.1, the government constraint, once pensions are computed, is as follows:

$$(1+\gamma_{t+1})\,b^y_{t+1} - (1+r_t)\,b^y_t = g^y_t + \frac{1-\alpha}{\mu}\left[(1-\tau_t)\,\rho_t d_t - \tau_t\right] \tag{21}$$

where the government constraint, once pensions are computed, is determined by the government revenues, which are represented by γ, the growth rate of the aggregate GDP in real terms, and b^y_t, the share of public debt in output. The expenditures are determined by g^y_t, the share of government purchases in output, and d_t, the old age dependency ratio, which is computed in Eq. (22).

$$d_t = \frac{\sum_{j=1}^{JR} N_{j,t}}{\sum_{j=1}^{JR-1} N_{j,t}} \tag{22}$$

[10] It's important to note that the extreme value causes the model to fail when computing the rest of the results.

The intuition behind this is that there are two ways to tackle the increase in the dependency ratio (d_t): as the baseline model suggests, through increases in taxes (τ_t), or alternatively through increases in the retirement age (JR) or adjusting the replacement rate (ρ_t).

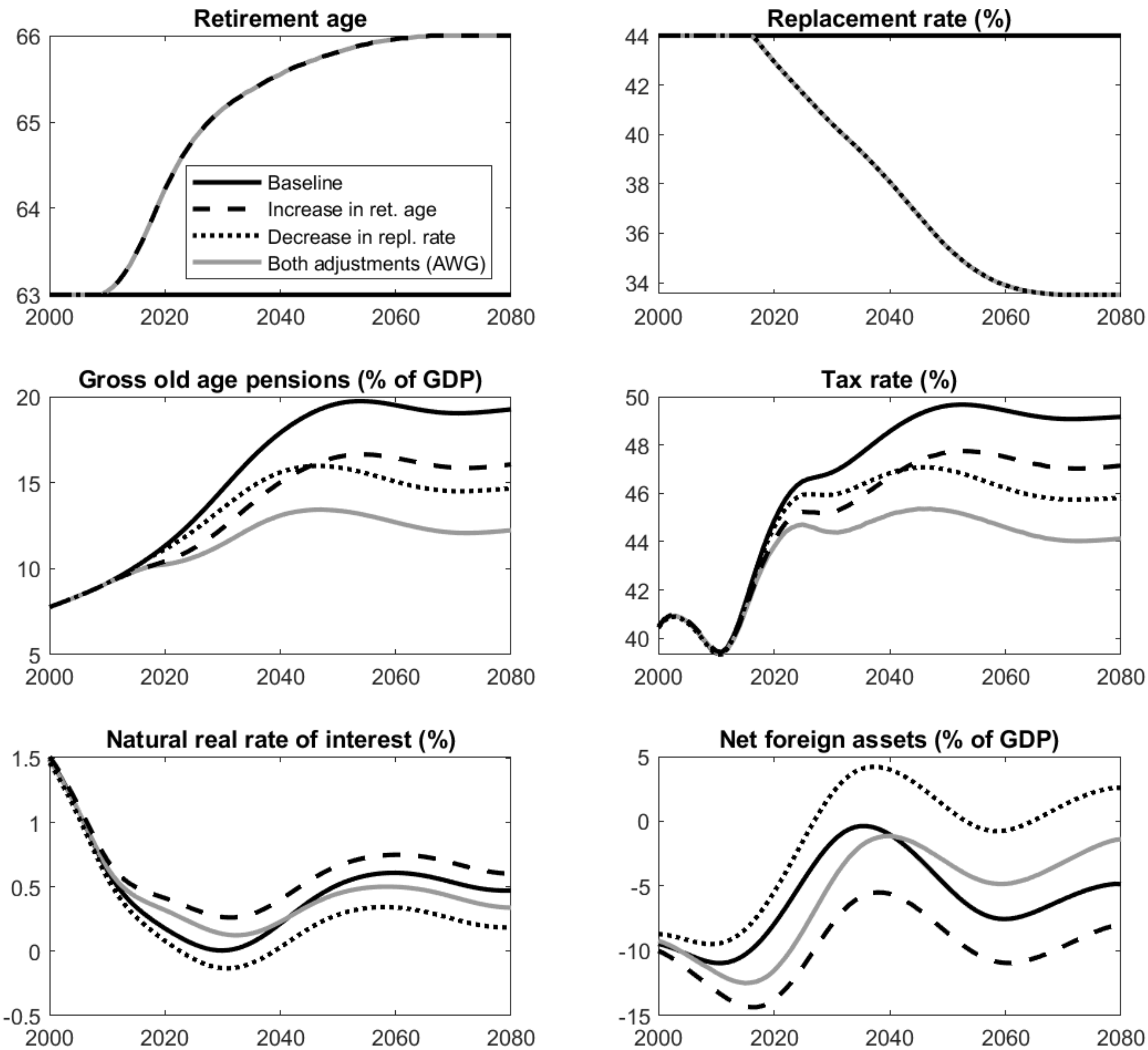

Figure 11: Pension System Reforms Projections. Source: Ageing Working Group (2018). (Replicated Figure)

In both cases, the authors propose implementing a fractional retirement age ($\rho_{j,t}$) rather than an abrupt retirement. Thus, the resulting renewed government budget constraint is Eq. (23). It is important to note that agents know the government policy in advance, allowing them to adjust their resource allocation accordingly. Additionally, all policies start in 2010 and are implemented gradually until 2060, when both policies are fully implemented.

$$c_{j,t} + a_{j+1,t+1} = (1-\tau_t)\left[\left(1-\rho_{j,t}\right) w_t z_j + \rho_{j,t} pen_t\right] + \pi_{j,t} + beq_t + (1+r_t)\, a_{j,t} \tag{23}$$

The economic rationale behind this is that as a representative household age, the proportion of time spent in retirement progressively increases until it is no longer participating in the labour market.

The results plotted in Figure 11 show how the main studied variables are being modified because of the application of one of them or both.

In the first policy, the replacement rate is reduced by 10 percentage points, from 44% to 34%. This results in agents having greater incentives to accumulate foreign assets in order to finance their remaining life expectancy. Moreover, the government can alleviate some of the baseline tax rates since the gross old age pensions are lower. As a result of increased savings, the NRI is expected to fall even more than in the baseline model, which is consistent with the model's paths and dynamics.

In the second policy, the retirement age is increased from 63 to 66. As with the first policy, this leads to lower tax rates than in the baseline model due to the reduction in gross old age pensions. However, the notable difference is that households no longer have the same incentives to accumulate assets since they do not need the same amount of savings to support their non-working lifespan. Consequently, the external balance position becomes more sensitive than in the baseline, but the NRI is higher than before.

Lastly, it is considered the effects of combining both policies. In this case, the gross old age pensions and tax rates are the sum of each policy. The asset accumulation closely matches that of the baseline scenario, owing to the cancellation of counteracting effects. Similarly, the NRI follows a similar pattern as in the baseline scenario. However, in both cases, the series experiences a time-lapse delay regarding the secular movements depicted in the baseline model. This delay is explained by the progressive implementation of the policies.

Overall, the analysis highlights the pension system's increasing problem, which the EA will inevitably face. Policymakers face a make-or-break decision between two options: a reduction in the replacement rate, which places more downward pressure on the NRI and constrains the monetary authority by directing more funds to investment, or an increase in the retirement age, which reduces the need to accumulate private savings, smoothes consumption, and ultimately leads to a higher NRI. However, it remains unclear whether any politician would accept the electoral cost associated with implementing such policies.

4.1.4 The migratory paper

In this final section, the focus is on studying the impact of migration on the economy and the extent to which the findings can be modified by alternative scenarios of future migratory fluxes. The declining labour force in the euro area could potentially be offset by a large pool of potential migrants from non-advanced economies. It is important to note that the model assumes that all migrants arrive before they turn 20 years old.

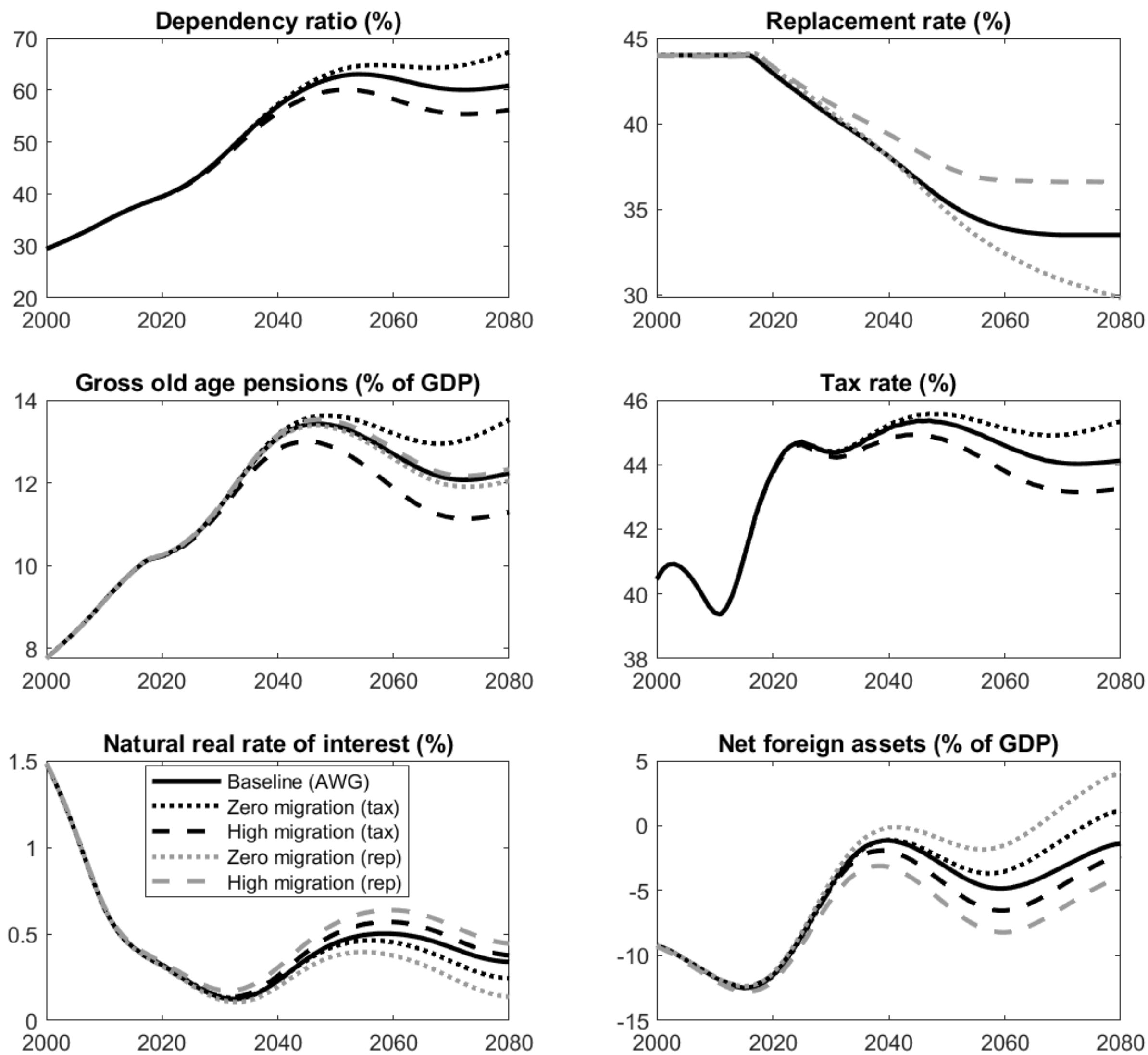

Figure 12: Migration effects on several pension reforms. Source: Zero migration and high migration data comes from Eurostat's EUROPOP 2018 projections. The baseline scenario is that projected by the Ageing Working Group (2018). (Replicated Figure)

To study the effects of migration, the authors used EUROPOP 2018 projections to construct two scenarios: ”zero migration” and ”high migration”. In the ”zero migration” scenario, migration of minors was ignored, while in the ”high migration” scenario, it was assumed that all migrants arrived at the age of 20 and had the same productivity and asset profile as workers born in the Euro area. Additionally, the two pension system policies examined earlier were studied simultaneously. The results of all four scenarios are presented in Figure 12.

On the one hand, if the ”High migration” scenario occurs and the replacement rate is maintained (i.e. ”tax” scenario), the dependency ratio is expected to decrease while maintaining the same cyclical fluctuations as the baseline. This is because the demographic process still occurs even when it is alleviated. Thanks to this decrease, the gross old age pensions are expected to decrease, which allows the tax rate to achieve lower pressure rates. If, on the contrary, the replacement rate is adjusted (i.e. ”rep” scenario), the gross old age pensions are expected to maintain slightly higher values than in the baseline.

In either case, the net foreign assets are expected to decrease because households do not have incentives to over-accumulate. This is because, in the ”tax” scenario, the dependency ratio is expected to decrease, pushing the economy forward. In the ”rep” scenario, the same effect occurs, but with less impact since the replacement rate is sustained above the baseline levels.

On the other hand, if the ”Zero migration” scenario occurs, the previously described cycles still occur but in the opposite direction, since the dynamics of the model are the same but fluctuate oppositely.

In summary, the effects on the NRI are positive only when there is a high migration scenario, and among those scenarios, the greater the policy regarding the adjustment of the retirement age, the more positive the effect. However, this mitigation is almost residual, and its effects are only obvious decades later, when migrants have accumulated enough assets to significantly impact the NRI.

4.2 Case of Study 1: Sigma

In this section, my focus centres on the adjustment of the sigma parameter in the CRRA utility function and its profound implications for the analysis. I acknowledge the significance of comprehending risk aversion and its impact on economic decision-making, while simultaneously providing a more profound understanding of the role that risk attitudes play within the model.

4.2.1 Theoretical Basis

As commonly employed in academic literature, the authors of the studied paper have proposed a CRRA utility function[11], which assumes a constant risk aversion parameter ($\sigma = 1$) for agents in the economy. This implies a constant relative risk aversion, where the degree of risk aversion remains consistent irrespective of the level of wealth or consumption.

$$U_{j,t} = \sum_{i=0}^{J-j} \beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \frac{\ln(c_{j+i,t+i})^{1-\sigma}}{(1-\sigma)} \tag{24}$$

Leaving sigma as a parameter instead of fixing it in the CRRA utility function offers several advantages. One of the primary benefits is the enhanced flexibility it provides in capturing a wide range of risk preferences among individuals. Different individuals exhibit varying degrees of risk aversion, and by allowing sigma to be a parameter, the model can accommodate this heterogeneity more effectively.

Another advantage is the model's adaptability to changing circumstances or contexts. Risk aversion can be influenced by various factors, such as economic conditions, financial markets, or personal experiences. By allowing sigma to be a parameter, researchers and policymakers can adjust its value to accurately reflect the prevailing conditions or specific situations under analysis.

In conclusion, by allowing changes in an individual's attitude towards risk, the model's results are expected to be more robust, as it increases the model's sensitivity. Additionally, it enables better predictions of consumption and savings patterns, thereby enhancing the accuracy of the overall analysis.

4.2.2 Empirical Results

In order to provide comprehensive insights into the effects of risk aversion on agents and its consequences in the model, various scenarios have been studied, which can be summarized as follows.

Increasing the value of sigma in the model, which represents higher risk aversion, leads to several effects. Firstly, individuals exhibit a stronger aversion to uncertain outcomes, resulting in more conservative decision-making and a preference for safer options. Secondly, there is a greater desire for consumption smoothing as individuals become more averse to fluctuations in consumption caused by income volatility or uncertainty. Additionally, higher sigma values tend to encourage higher levels of savings as risk-averse individuals prioritize building a buffer against uncertain future outcomes. Lastly, individuals prefer safer, low-risk assets in their portfolio choices, such as bonds or savings accounts, over riskier investments.

[11] See Equation 1.

As stated above, the results show that although consumption does not experience significant changes, there is an increase in asset accumulation. This leads to a smoothing effect on the Net Foreign Asset Position and in consequence on the NRI, with a 0.5 percentage point increase at the beginning of the '80s and a smaller but still higher effect in the rest of the time period. It is important to note that even though the NRI experiences some negative values, the subsequent rebound is greater than before. This trend is attributed to the fact that risk-averse agents allocate more of their portfolio to safer, low-risk assets like government bonds, which are used in the model to compute the NRI. Additionally, the increase in asset accumulation leads to a higher allocation of foreign assets, contributing to the smoothing of both the NRI and net foreign assets.

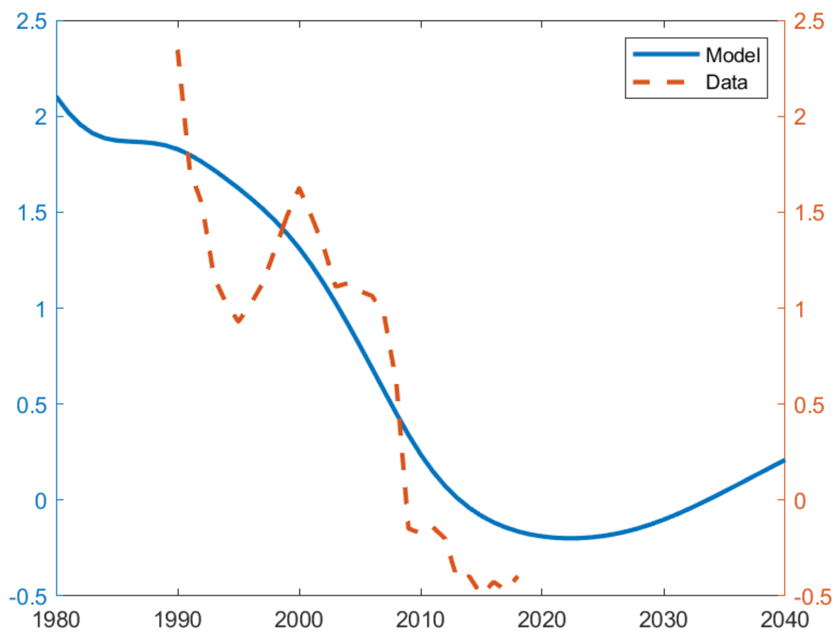

Figure 13: Natural rate of interest in non-EA G7 countries. Source: Holston et al. (2017) [9] and Han (2019)[8].*Notes:* Computation through a GDP-weighted average of the countries. Simulation for $\sigma = 1.25$

Consequently, the higher initial values compared to previous values contribute to the greater difference observed, as depicted in Figure 14. It is worth noting that these changes are expected to amplify the significance of Global Factors, namely TFP growth and foreign interest rates, due to the increased accumulation of foreign assets.

Overall, the model demonstrates fitting better the data compared to previous iterations, aligning with the prevailing academic consensus. Consequently, it is expected to have a positive impact on the natural rate of interest at any given point in time.

Conversely, decreasing the value of sigma, which signifies lower risk aversion, would yield contrasting effects. Individuals would display a greater tolerance for risk and a higher propensity for engaging in uncertain outcomes, leading to more risk-taking behaviour and a preference for

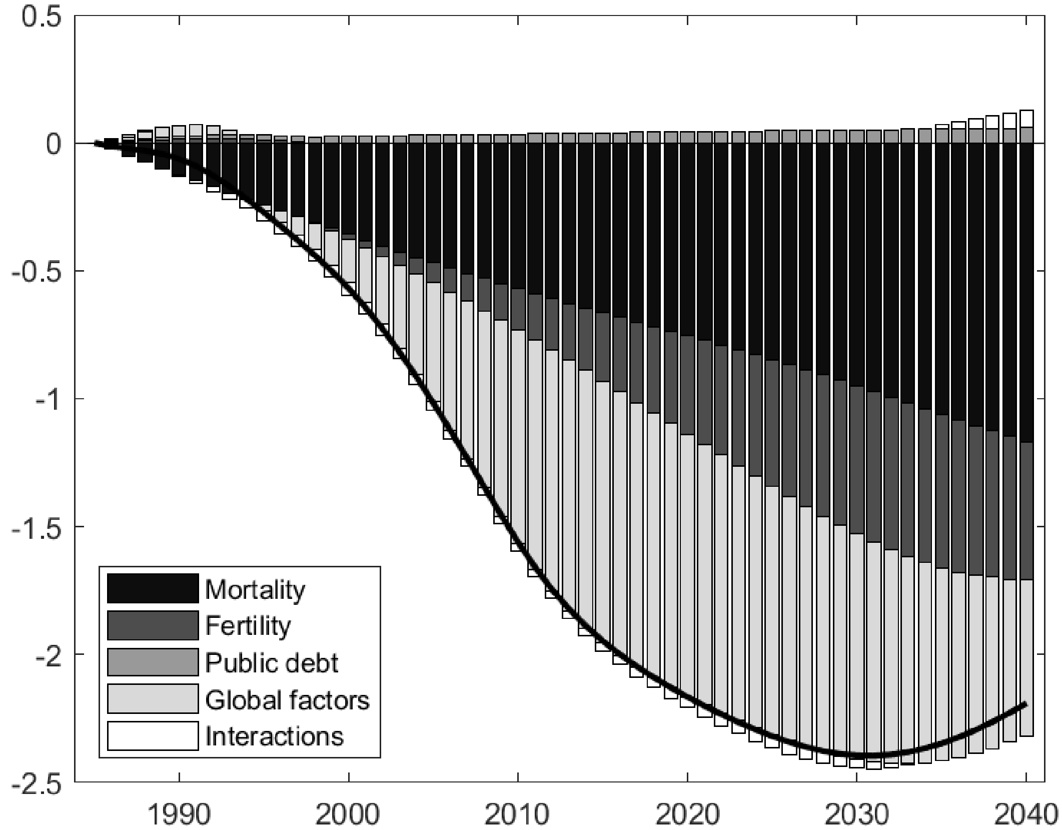

Figure 14: Decomposition of changes in EA NRI. *Notes:* All numbers are presented as a difference from the 1985 levels. Global factors combine the impact of TFP growth and foreign interest rate. Simulation for $\sigma = 1.25$

riskier options. With reduced risk aversion, the necessity for consumption smoothing would diminish as individuals become less averse to fluctuations in income and more willing to adjust their consumption accordingly. Lower sigma values could also result in decreased levels of savings as individuals with lower risk aversion are inclined to spend or invest their income rather than save it. Additionally, individuals would demonstrate a greater inclination to invest in riskier assets with the potential for higher returns but also higher volatility in their portfolio choices.

As stated above, the results show that although consumption does not experience significant changes, there is an increase in the early stages of asset accumulation, but an overall smaller growth. This leads to a worsening effect on the Net Foreign Asset Position and as a consequence on the NRI, with a 0.3 percentage point decrease at the beginning of the '80s and a smaller effect in the rest of the time period. It is important to note that even though the NRI experiences some negative values, the overall trend follows a flatter evolution. This trend is attributed to the fact that lower-risk-averse agents allocate more of their portfolio to unsaved, high-risk assets.

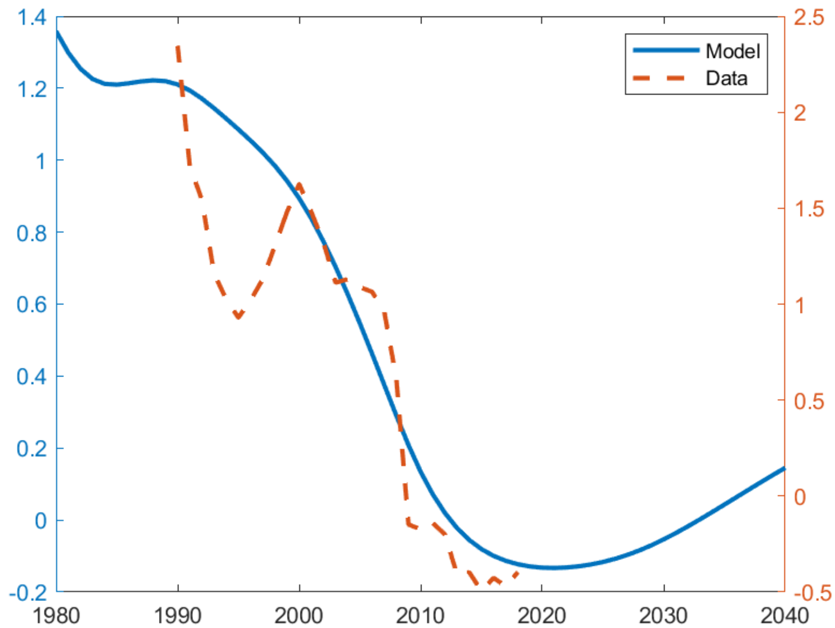

Figure 15: Natural rate of interest in non-EA G7 countries. Source: Holston et al. (2017) [9] and Han (2019)[8].*Notes:* Computation through a GDP-weighted average of the countries. Simulation for $\sigma = 0.75$

Interestingly, the model seems to struggle when it comes to fitting the data compared to previous iterations, which explains the non-ordinary results presented in Figure 16. This indicated that the agents in our economy are more likely to be highly averse to risk, which is on the same line that all literature has demonstrated when comparing several risk studies along the euro area and the rest of the world[12].

In conclusion, the examination of the sigma parameter in the empirical analysis has proven to be highly valuable, particularly when higher values of sigma were calibrated that seem to better fit the data. By allowing the sigma parameter to be an adjustable factor in the model, a deeper understanding of its impact on various economic outcomes has been gained. However, it should be noted that deviating from the range of 0.70 to 1.3 in the calibration of sigma leads to model compilation failures and renders the obtained results unreasonable. Despite this limitation, the empirical results have shed light on the effects of different levels of risk aversion on key variables, including savings behaviour, investment choices, and the NRI. These insights have proved to be more than useful in enhancing the model's understanding and analysis.

4.3 Case of Study 2: Epsilon

In this subsection, we shift our attention to the production side of the model and explore the implications of using a CES (Constant Elasticity of Substitution) production function with multiple

[12] See Beckhtiar et al. (2019)[2]

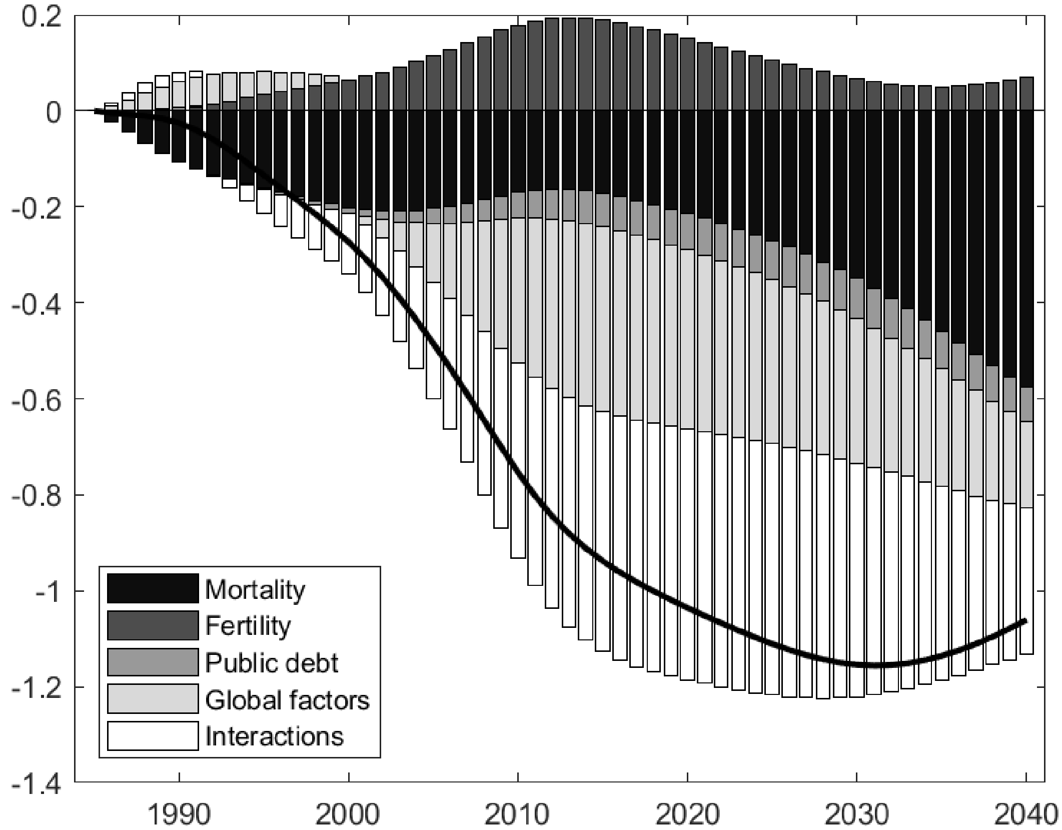

Figure 16: Decomposition of changes in EA NRI. *Notes:* All numbers are presented as a difference from the 1985 levels. Global factors combine the impact of TFP growth and foreign interest rate. Simulation for $\sigma = 0.75$

substitution (ε) calibrations. I, first, take into account the significance of the CES function, the theoretical expected results and the basis, and finally I explore several of the results obtained.

4.3.1 Theoretical Basis

As commonly employed in academic literature, the authors of the studied paper have proposed a Cobb-Douglas utility function[13] for intermediate goods producers, which assumes that there is no substitutability ($\varepsilon = 0$) between different inputs for producers in the economy. This implies that inputs are perfect complements, they must be used in fixed proportions to produce output, and that, because of the idiosyncrasy of the exponents, have constant returns to scale.

$$Y_t = A(\alpha \cdot K_{t-1}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha) \cdot H_t^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}} \tag{25}$$

[13] See Equation 11.

Leaving epsilon as a parameter instead of fixing the whole production function through a Cobb-Douglas form offers several advantages. The biggest benefit is that the CES production function offers a flexible framework for capturing different degrees of substitutability between inputs in the production process. By introducing multiple epsilon values, I can examine how varying degrees of substitutability impact key economic variables and shed light on the interplay between production factors.

Another advantage is that the elasticity of substitution can vary across different regions of the production function. This allows for more nuanced modelling, as it recognizes that substitution behaviour can differ depending on the level of inputs used. By allowing epsilon to be a parameter, researchers and policymakers can adjust its value to accurately reflect the prevailing conditions or specific situations under analysis.

In conclusion, by allowing changes in the substitutability of Labour and Capital, the model's results are expected to be more robust, as it increases the model's sensitivity. Additionally, it enables better predictions of the impact of changes in factor shares and substitution possibilities on output levels, factor allocations, and productivity.

4.3.2 Empirical Results

To provide comprehensive insights into the effects of the substitutability (ε) in the CES production function and its consequences in the model, several calibrations have been studied, which can be summarized as follows.

Decreasing the value of epsilon, indicating lower substitutability between inputs yields contrasting effects. Firstly, it is well known that the closer epsilon tends to zero, the closer the CES function tends to a Leontieff shape. Consequently, with lower substitutability, the production process becomes more specialized and interdependent, limiting the ability to substitute one input for another. This can lead to a less dynamic and flexible production process, potentially hindering productivity and economic growth. Lower productivity growth, in turn, may contribute to a lower natural rate of interest in the OLG model. The reduced substitutability between inputs also results in a more rigid capital-labour ratio, as firms have limited options for adjusting input combinations. This rigidity may impact the equilibrium interest rate, potentially leading to a lower natural rate of interest.

Contrary to the expected theory, the model does not capture this changing pattern when it comes to productivity since, because of the features of the paper, it is computed in household terms[14]. Though, since firms now cannot choose more optimal resource allocation because of the non-substitutability of Leontieff, we can clearly see the slowdown that the NRI suffers. In fact, this

[14] Productivity is measured in hourly labour income determined by the age of the household head.

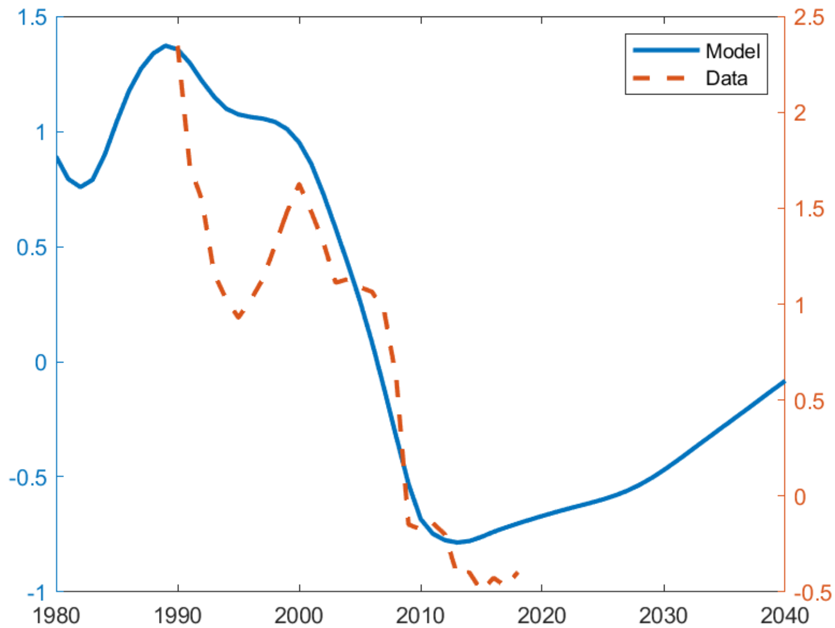

Figure 17: Natural rate of interest in non-EA G7 countries. Source: Holston et al. (2017) [9] and Han (2019)[8].*Notes:* Computation through a GDP-weighted average of the countries. Simulation for $\varepsilon = 0.01$

shift accounts for more than 1 p.p. at the beginning of the trend. However, it surprisingly goes up until 1990, contrary to what the data is telling. This shift already occurs in the non-modified graph, Figure 5, but now it has been magnified because of the features of the Leontieff production function. Excluding those decades, the model matches the slowdown of the data and the overall trend, but hits the bottom rock a couple of years earlier and with substantially lower values than before.

Overall, the model demonstrates not fitting better the data compared to previous iterations, even when it shifts according to economic theory. However, we can observe that the Leontieff production function is expected to have a negative impact on the natural rate of interest at any given point in time.

Conversely, increasing the value of epsilon, indicating higher substitutability between inputs has several implications for the model. Firstly, it is well known that the closer epsilon tends to one, the closer the CES function tends to a Cobb-Douglas shape. Consequently, it promotes a more dynamic and flexible production process, allowing firms to adjust their input combinations based on relative prices and other factors. This increased substitutability enables firms to increase their production through optimal resource allocation and adopt new technologies more readily, leading to higher productivity growth. Consequently, the higher productivity growth contributes to a higher natural rate of interest in the model. Moreover, the greater substitutability between inputs allows for a more responsive adjustment of the capital-labour ratio, influencing the equilibrium interest rate and resulting in a higher natural rate of interest.

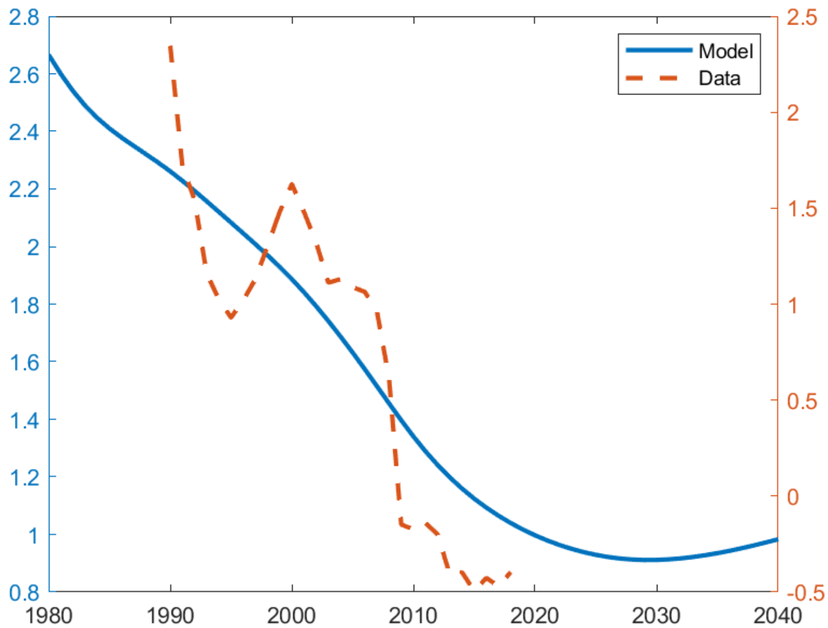

Figure 18: Natural rate of interest in non-EA G7 countries. Source: Holston et al. (2017) [9] and Han (2019)[8].*Notes:* Computation through a GDP-weighted average of the countries. Simulation for $\varepsilon = 2$

As before, the productivity shift predicted by the theoretical basis is not fulfilled. However, is worth noticing that now the NRI is higher than before, thanks to the better resource allocation that higher perfect substitution inputs have. In this case, the model also does not fit the data, furthermore, it does not even follow the same shape.

In summary, the level of substitutability, as determined by the epsilon parameter in the CES production function, has significant implications for the model. Higher substitutability promotes a more dynamic and flexible production process, contributing to higher productivity growth and a higher natural rate of interest. Conversely, lower substitutability limits flexibility and adaptability, potentially leading to lower productivity growth and a lower natural rate of interest. These effects' specific magnitude and direction depend on other model parameters and the broader economic context.

Through the calibration of several scenarios, we can clearly see that the more the production function (because of ε) differs from its original Cobb-Douglas form, the more the model mismatches the data and its pattern. In conclusion, even when exploring new lines of investigation is also worth it since it allows for possible better calibrations (e. g. the sigmma case), it is not always the case that the results obtained help to increase the robustness and quality of the model.

5 Conclusions

This final degree project aims to investigate the potential negative impact of current demographic changes on the natural interest rate in the euro area and its effects on pension systems, agent allocation, and policymakers' effectiveness. Throughout this study, I have presented a comprehensive analysis of the research topic, examined relevant literature, described, replicated and extended an OLG model, analysed the data and methodology employed, and presented the key findings.

It has been demonstrated and estimated a 2 percentage point decline in the EA equilibrium interest rate between 1985 and 2030. Of this loss, about two-thirds came from demographic trends, the main explanatory factor from a theoretical point of view. Furthermore, because of the relatively elderly age structure of the Euro area concerning the rest of the economies, this fall in the natural rate of interest is expected to happen before that in the rest. Thus, "exporting" the effects of the demographic transition to other countries through larger factor price changes through increasing agents' net foreign assets and improving its balance of payments by 10 percentage points.

However, it has been found that since those demographic secular movements have been happening among all advanced economies, the degree of openness to international capital flows does not alleviate the negative pressure on the EA NRI. On the same line, the NRI would not be alleviated because of the predicted decline in the GDP growth rate, even with a positive rebound coming from productivity increases.

Among the current literature, two other factors could reverse the so-called negative trend: fiscal measures and migration.

The first is studied through modifications in public system pensions because of its increasing weight on the importance and cost of public finances. It is found that the unique way to affect the NRI positively is through increasing the retirement age since it reduces the need to accumulate private savings and smoothes consumption, rather than reducing the replacement rate, which places more downward pressure on the NRI and constrains the monetary authority.

The second is studied by projecting several migratory scenarios. Among those, the only ones that had some positive impact were when there was expected high immigration. However, as most of the current literature is pointing out, this mitigation is almost residual, and its effects are only apparent decades later when migrants have accumulated enough assets to impact the NRIsignificantly.

Ultimately, it is essential to note that this main article has limitations. Among those, the most important would be that all countries within a group follow an equal path based on the assumption that all agents are homogeneous. Alternatively, while under an open economy premise, authors impose strict homogeneity of the world interest rate from the EA perspective since it is presupposed to be a relatively small region.

Working on this line, when studying the effects of risk aversion on the model, it is found that the model predicts better the path followed by the data when higher rates were studied. This indicates that the data tends to be biased regarding higher adversity levels. Contrary, the model struggles when the production function differs from its original Cobb-Douglas form, which indicates that it was already well-calibrated.

Despite these limitations, this project represents a significant step forward in studying interest rates and demographics. Even within the current high-interest environment produced by a series of unexpected shocks, the studied demographic trends will push downward the NRI at some point. Furthermore, if no actions are considered sooner rather than later, the consequences of hitting the zero lower bound, frequent financial bubbles due to the capital seeking riskier investment for higher profits, or secular stagnation scenarios will become progressively irremediable.

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