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SOCIAL SECURITY REFORMS, RETIREMENT AND SECTORIAL DECISIONS

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JEL Codes: J26, H55, J45, J62

Keywords: Social security reform, public employment, public deficit, informality

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Social Security Reforms, Retirement and Sectoral Decisions*

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September 27, 2023

Abstract

In many countries, the regulations governing public and private pension systems, hiring procedures, and job contracts differ. Public sector employees tend to have longer tenures and higher wages compared to workers in the private sector. As such, social security reforms can affect both retirement decisions and sectoral choices. We study the effects of social security reforms on retirement and sectoral behavior in an economy with multiple pension systems. We develop a life-cycle model with three sectors - private formal, private informal and public - and endogenous retirement. In a model calibrated to Brazil, we quantitatively assess the long-run effects of reforms being discussed and implemented across countries. Among them, we study the unification of pension systems and increasing the minimum retirement age. We find that these reforms affect the decision to apply to a public job, the profile of savings over the life cycle, and informality. In the long run, these reforms lead to higher output and capital, reduced informality, and average welfare gains. They also drastically reduce the social security deficit.

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1 Introduction

This paper investigates the quantitative impacts of social security reforms on retirement and sectoral behavior, a topic that has gained significant attention in recent years due to the demographic and financial challenges that many countries' social security systems face. Specifically, we study the effects of reforming pension systems with differing regulations for public and private sectors at both the micro and macro levels. The reforms analyzed include the unification of public and private systems and the raising of the minimum retirement age, which are currently under discussion or being implemented in several countries.

Population aging is the result of increasing life expectancy and falling fertility rates. According to the Nations (2013), the world's life expectancy at birth increased from 65 years in 2000 to over 70 years nowadays. Consequently, the old age support ratio[1] is expected to decrease from 3.8 nowadays to 2.1 in 2050 and 1.6 in 2100 for OECD countries. Moreover, according to OECD (2021) average income has grown faster for older people than for the total population.

Pay-as-you-go (PAYG) social security systems exist in most countries in the world. Some of them have differentiated pension rules for public workers. Kings et al. (2007) document that over half of the OECD countries have different pension rules for workers in the public and private sectors. For instance, some jurisdictions in the US have segregated pension plans. Furthermore, Beshears et al. (2011) report that the majority of public sector pension schemes are still defined-benefit, unlike the private sector retirement plans.[2]

Holzmann and Hinz (2005) and Pilipiec et al. (2021) argue that most public pension systems are not financially sustainable. Of course, considering sustainability issues, government's budgetary deficit problems are even more problematic. An important example is the 1998 Brazilian crisis, in the aftermath of the East Asian and Russian financial meltdowns. It was documented that a fiscal deficit of more than 6% of GDP triggered this crisis and that two-thirds of this deficit was due to pension costs. Likewise, in Lebanon, public retirees' pensions are the third greatest expenditure item in the government's budget, even though they account for less than 3% of the population.

Given the upcoming demographic pressures and the financial situation of pension systems, it is important to evaluate the long run consequences of potential reforms.[3] We provide a tool for analyzing these reforms by developing a life-cycle model with heterogeneous agents and endogenous sectoral and retirement decisions. Our model economy has three working sectors: private formal, private informal and the government, each with its own retirement system. We calibrate the model to match key aspects of Brazil in 2013. With rapid aging population and over

[1] The ratio of individuals aged between 20 and 64 and those with more than 65.

[2] Some countries have deployed other pension reform schemes. See for example, the means-tested pension system analyzed by Kudrna et al. (2022).

[3] See OECD (2015) for a summary of pension reforms being implemented around the world.

generosity of its pension system – especially that of public workers – this country can be seen as a typical case of the social security problems countries are currently facing, or will face in the near future. The calibrated model closely reproduces the observed distribution of individuals across sectors, their decisions to apply for a public sector job and to retire from the labor force, as well as aggregate moments on the social security deficit and inequality.

We evaluate the potential long-term impacts on the overall economy of various social security reforms. Specifically, we examine two reforms in Brazil: the 2013 removal of the "integrality" provision for public retirees, which imposed a cap on their benefits and aligned the public and private pension schemes; and the 2019 reform that increased the minimum retirement age to 65. These changes are similar, for instance, to the proposal put forth by the Macron administration in France for a universal retirement system that eliminates special regimes for the public sector and the recent increase of the minimum retirement age to 64.

We find that the 2060 demographic changes alone will increase the social security deficit, without reforms, from 2.1% to 16.6% of GDP. The participation rate is projected to decline by 13 percentage points, while the proportion of retirees is predicted to rise from 13.9% in 2013 to 31.5% in 2060. The public sector portion of the deficit will increase from 1.3% to 5.2% of GDP, while the private sector portion will increase from 0.8% to 11.5% of GDP.

Implementing reforms can help reduce the social security deficit, with a potential decrease of almost 40% when both reforms - unifying the pension systems and raising the minimum retirement age to 65 - are jointly implemented. A key driver of this reduction is the decrease in the proportion of retired people from 31.5% to 18.7%. We also find that the share of applicants to a government job falls by 15%, as a public sector career becomes less attractive.

While both reforms contribute to reducing the social security deficit, only the imposition of a higher minimum age can effectively decrease the number of retired people, as this reform forces people to stay longer in the labor market. However, despite the potential reduction in the social security deficit, it will still remain high at 10.2% of GDP. To completely eliminate the deficit, the minimum retirement age would need to increase to 72. This may be challenging politically, as evidenced by the protests in France over a two-year increase in the minimum retirement age from 62 to 64. We find, however, that the positive impact on GDP would be sizable, as taxes decreases and people remain in the labor market for a longer. But not every age group would gain: while young cohorts would see their welfare increase with the reforms, older cohorts would lose.

Following the approach developed by Huggett (1996), Huggett and Ventura (1999), Conesa and Krueger (1999), among others, studied the macroeconomic consequences of pension reforms. This strand of literature, however, treats retirement as an exogenous behavior. This is not a plausible assumption to address early retirement provisions, a common feature in many pension systems.[4] Even though agents' earnings influence their retirement benefits, these papers do not consider the trade-off between working one extra year to raise future pension benefits versus retiring sooner to collect benefits for longer.

Imrohoroglu and Kitao (2012), Ferreira and Santos (2013), Jung and Tran (2012) and Gustman and Steinmeier (2005) deal with endogenous retirement. Imrohoroglu and Kitao (2012) study the impact of two social security reforms on the US economy. They introduce health heterogeneity, as well as medical expenditures, which could act as determinants of social security benefit claims decisions. Another branch of literature explores the interaction of heterogeneous mortality and heterogeneous demographic change on pensions reform (Laun et al., 2019; Sanchez-Romero et al., 2020; Jones and Li, 2023). However, none of these studies addresses retirement choices when agents face more than one working sector. This is important as social security reforms can induce workers to reallocate across different sectors, potentially having significant macroeconomic consequences on the fiscal deficit.

Our quantitative framework introduces a unique approach to the analysis of pension reforms by considering both the role of informality and the public sector. Previous literature has explored the impact of the informal sector on economic development, such as Franjo et al. (2022), who developed a life-cycle model to investigate the interplay between informality and financial development in Brazil. They also used cross-country data analysis to demonstrate that accounting for the informal sector is essential to comprehend the financial and economic development relationship. Additionally, McKiernan (2021) has emphasized that the informal sector should be a critical factor in assessing pension reform outcomes. Our contribution to this literature is the inclusion of the public sector, which allows us to examine the effects of unifying pension systems that have been under discussion in various countries worldwide.

The unification of pension systems may lead to fewer benefits for public workers, causing high-skill individuals to opt out of the public sector, which could ultimately affect the productivity of the economy. Several articles, including Glomm et al. (2009), Santos and Pereira (2010), Dos Reis and Zilberman (2014), and Santos and Cavalcanti (2021), have examined the issue of segregated social security systems and the case of Brazil. They highlight how an overpaid and secure (in terms of job stability) public sector attracts the best human capital in the economy, and the macroeconomic consequences of such a sector. However, none of these studies have developed a model, as we do, to analyze the sectoral and retirement choices of agents who face multiple working and retirement sectors, including an informal sector where workers can avoid taxation. Furthermore, we also contribute to the literature evaluating the optimal minimum age policy to eliminate the pension deficit.[5]

The remaining of the paper proceeds as follows. Section 2 establishes key facts about social

[4] See Vestad (2013) for a list of countries that have early retirement possibilities.

[5] Other papers have focused on different optimal policies. See for example, Hosseini and Shourideh (2019) for a Pareto optimal reform approach.

security systems around the world, as well as some peculiarities of the Brazilian system. Section 3 presents the key features of our model, including the problem of the agents and the stationary distribution. Section 4 presents the equilibrium. Section 5 describes the calibration procedure, and Section 6 the external validation, comparing non-targeted moments that our model generates to the data. In Section 7, we evaluate the steady state macroeconomic consequences of different social security reforms. Finally, Section 8 concludes.

2 Motivation and Data

This section empirically motivates why quantifying the macroeconomic consequences of pension reforms while considering the allocation of labor across sectors is of first order for economies around the world. We proceed in three steps. First, we show that the combination of population aging and the current state of public pension expenditure will constrain government budgets in the near future. Second, we note that countries tend to have segregated retirement systems for workers in the public and private sectors, and that understanding the consequences of reforms that unify them is important. Finally, we detail the Brazilian case, explain why we focus on this country, and highlight some aspects of the Brazilian economy that guide our theoretical and quantitative analysis.

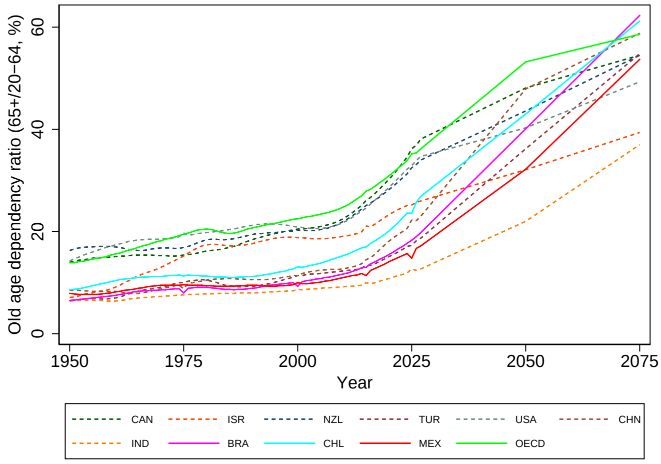
Population aging is a widespread phenomenon that governments throughout the world will have to deal with in the next decades. According to OECD data, the share of the elderly in the population is expected to increase substantially until 2100. For the OECD countries, the share of individuals over seventy is expected to increase by almost 70% until 2050, from 12.2% to 20.6%. Considering the world as a whole, this share is expected to move from 5% in 2015 to 11% in 2050.

The increase in the elderly population will be followed by a likely reduction in the proportion of working-age individuals in the future. Figure 1 shows that countries such as Brazil, Mexico and Chile expect large increases in the old-age dependency ratio, the ratio between the population over 65 to the population between 20 and 64. The old age dependency ratio is an important indicator for the financial sustainability of pension systems, as most countries adopt PAYG schemes, where intergenerational transfers between the working population and the retired population occur.

Government expenditures on retirement benefits constitute a large share of GDP. Figure 2 shows that the average share is projected to increase 17% by 2050, going from 8.1% to 9.5%. The average increase across the sample is 44% and, in the case of Brazil, this share will go from less than 9% to 16%. The demographic changes and higher expenditures with the elderly population will push toward worsening the social security deficit in the future.

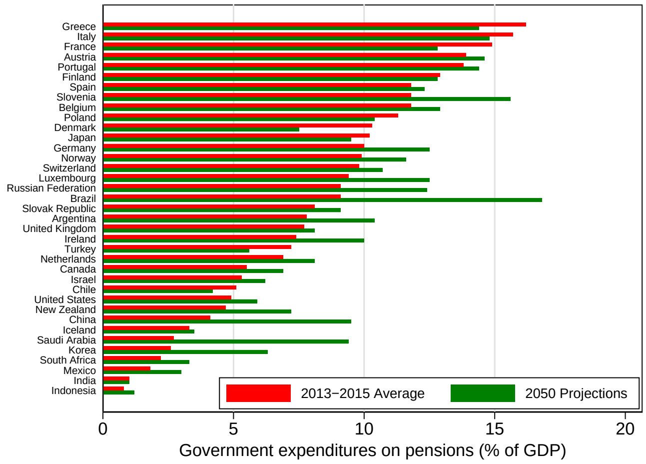
Several countries have retirement systems that vary according to the employment sector. In

Figure 1: The evolution of the old age denpendecy ratio



Notes: This figure plots the evolution of the fraction of individuals aged 65+ to those aged 20-64 years old across different countries in the world. Source: OECD (2017).

Figure 2: Public expenditures on retirement pensions



Notes: This figure plots the share of GDP governments spent on retirement pensions across 2013-2015, and those projected for 2050. Sources: OECD (2017).

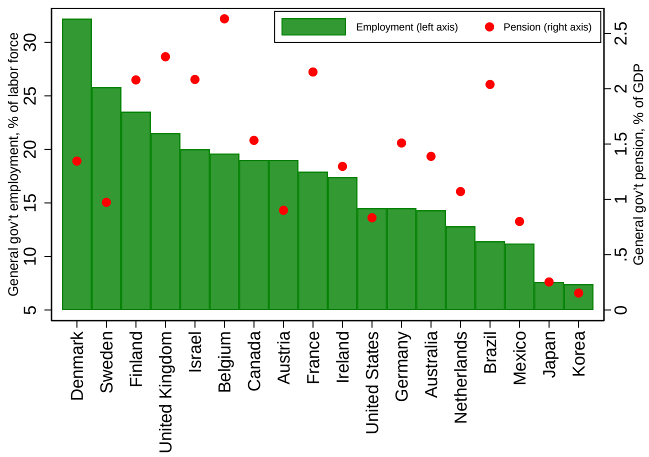
particular, public employees tend to have better retirement conditions relative to their private sector peers.[6] On average, public sector pensions account for 20% of overall pensions expenditures.[7] Furthermore, these pensions represent a large share of output. Figure 3 shows that the general government accounts for, on average, 17% of the labor force, and their retirement benefits sum up to 1.5% of GDP. Hence, it is not surprising that there are a large number of

[6]See OECD (2018) for a detailed analysis of the countries where the retirement conditions for public and private workers differ.

[7]This number was calculated by matching the data from Figures 1 and 2.

reforms - proposed or implemented - that emphasize the unification of the retirement schemes for private and public sector workers - e.g., in France and Brazil.

Figure 3: Demographics and pensions expenditure by country



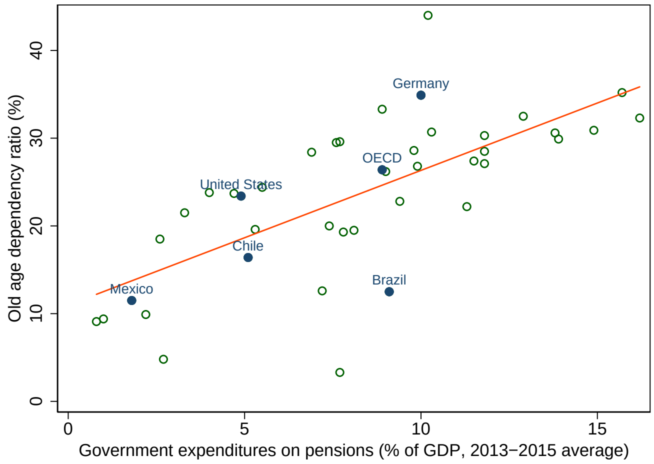
Notes: This figure plots the share of public expenditures on general government retirees to GDP (right) and the size of the general government in the labor force (left) in 2013. Sources: Social Security Expenditure Database (OECD), Brazilian National Accounts and Annual Report on Social Security (IBGE and Ministry of Social Security), and ILOSTAT Database (ILO).

The quantitative part of the paper specializes in the case of Brazil. We believe Brazil is representative of countries that already face or will face problems in their social security systems for three reasons. First, Brazilian demographics follow global trends. The country's life expectancy increased from 54.7 years in 1960 to 73.6 in 2012, and the Brazilian Institute of Geography and Statistics (henceforth IBGE) projects that the share of individuals aged 65 and over will grow from 4% in 1980 to 22% around 2050.

Second, given its current demographics, Brazil devotes a large share of its GDP to overall retirement pensions. Figure 4 plots the cross-country relationship between the 2013-2015 average share of GDP that retirement pensions represent and the old age dependency ratio in 2013. The figure shows that Brazil is as "young" as Chile and Mexico but spends a similar share of GDP with social security as Germany and the OECD average, countries and regions with considerably older populations.

Third, Figure 3 shows that Brazil has the widest discrepancy between the government size, which accounts for 11.4% of the labor force, and the size of government retirees' pensions, which represents 2% of GDP. Moreover, Table 1 shows that the social security deficit associated with both private and public workers' pensions has been systematically increasing over the last years, with similar contributions from each of the social security regimes. However, the number of retirees from the public sector, around one million people, is much smaller than those from the private sector, around 29 million people.

Figure 4: Public expenditures on retirement pensions



Notes: This figure shows the cross-country relationship between the old-age dependency ratio (ratio of 65+ to 20-64 years old individuals) and the government expenditures on pensions as a share of GDP. Sources: 2017 Pensions at a Glance (OECD).

Table 1: Social Security Deficit (Brazil, % GDP)

	Private	**Public**	**SS Deficit**
2013	0.94	1.18	2.12
2014	1.00	1.18	2.19
2015	1.44	1.22	2.66
2016	2.17	1.18	3.35

We now describe some key facts about the Brazilian economy used to guide our quantitative exercise. The lack of a minimum retirement age in the private sector until 2022 makes early retirement a prevalent feature of the Brazilian retirement system. Table 2 shows that the average age individuals claimed for retirement benefits under the contribution modality[8] is 56 years old for men and 52 for women. The expected duration of retirement is 23 years for men and 29 for women. The discrepancy between these numbers relative to the rest of the world is large. For example, the OECD's average retirement age is 64, with an expected duration of 16 years for men.

[8]This is a type of retirement Brazilians can opt to have, and it will be further detailed in the next sections.

Table 2: Brazilian Early Retirement

	Retirement Age		Expected Duration	
Location	**Men**	**Women**	**Men**	**Women**
OECD	64	63	16	21
Latin America	62	60	17	21
World	62	60	16	21
Brazil:				
Private: Contribution	56	52	23	29
Private: Age Modality (Rural)	60	55	19	26
Private: Age Modality (Urban)	65	60	16	22

Government sector jobs in Brazil tend to be relatively more stable and well-paid. Using data from the 2013 Pesquisa Nacional por Amostra de Domicílios (PNAD, a household survey), Figure 5 plots the job tenure of private and public workers. The average tenure of a private sector job is 5.3 years, whereas the average tenure for a public sector job is 13.6 years. On top of that, using data from the 2012-2018 Pesquisa Nacional por Amostra de Domicílios Contínua (PNADC),[9] we find evidence of a public sector wage premium. We regress log hourly earnings on a rich set of controls and individual-level fixed effects, and estimate a public sector wage premium of 9.4% (see Table 4).

Figure 5: Estimated tenure density of public and private jobs, 2013

Notes: This figure presents the density estimation of the on-the-job tenure for public (red) and private (green) workers. We restrict our sample to male employees between 16-75 years old. Bandwidth=1.5; Kernel=Epanechnikov; Sample weights are used. Source: 2013 PNAD (IBGE).

[9] See Section 5.1 for more details on the PNAD and the PNADC.

These facts, together with a more generous retirement scheme and a competitive hiring process,[10] make the public sector particularly attractive for more educated individuals. Even though the statutory public servants account for only 5% of the population, they represent nearly 20% of all individuals with more than 15 years of education. Moreover, the group of individuals that take the public exam (i.e., those trying to enter the public sector) represents 6% of the total population, but accounts for 23% of all individuals with more than 15 years of education.[11]

Finally, the Brazilian economy has a large share of informal workers that do not pay most taxes and are not, in principle, under the protection of the Social Security Administration.[12] According to the 2013 PNAD data, 18% of males with 16 to 75 years old are in the informal private sector.[13] Due to the lack of job security and the type of employers that typically hire in the informal sector,[14] it ends up absorbing individuals who have, on average, less schooling and, consequently, lower incomes.

Moreover, informal workers can access social security benefits, even if they have never contributed to it in their entire lives. Brazil has a social assistance program in which low-income individuals older than 65 can retire receiving as benefit the minimum wage. Hence, poor informal workers - the majority of them - can in principle retire receiving this benefit. And it does not require any contribution to the social security system in the past. The data shows that, even though contribution is voluntary in Brazil, the share of informal workers that contributed in 2013 was around 17% only. Clearly, the incentives are not there for them to contribute.

In conclusion, a model studying the macroeconomic and sectoral consequences of social security reforms should include the following features: (i) a well-paid and stable public sector with costly access, (ii) a public pension system that is relatively more attractive and expensive than the private pension system, and (iii) an informal sector with individuals that do not contribute the social security system, but may benefit from them. Although the model will be calibrated to Brazil, we think these features are common across a large number of poor-to-medium income countries.

[10]See Section 5.5 for details on the Brazilian retirement system, and Section 3 for a discussion on the public employment hiring procedure.

[11]These numbers were calculated using the 2013 PNAD data.

[12]Informality is not only a feature of the Brazilian economy. Bacchetta et al. (2009) study the informal sector in developing countries, and provide evidence that informality is negatively correlated with GDP and GDP growth. Moreover, the study shows that the share of informal employment in total employment in the 2000s was 52% for Latin America, 78% in Asia and 56% in Africa. Bacchetta et al. (2009) state that the informal sector (excluding agriculture) accounted for 26% of GDP in Latin America in 2006. This study also shows that the informal sector attracts less educated individuals: on average, around 65% of all informal workers in Latin America are "low skill" workers.

[13]An informal worker is defined as an employee that does not have a formal work contract. We do not consider self-employed informal workers, as we will not model entrepreneurial activity. The share of informality in the labor force goes up to 40% if you include these types of agents, but modeling them is out of the scope of this paper.

[14]See Ulyssea (2018) for firm-level analysis of the informal sector.

3 The Model

The economic environment in this paper consists of a life-cycle model of sectoral choice and retirement behavior. Individuals can be either in the private sector, working for the government, or retired from the labor force. All decisions are endogenous, so individuals will only retire from the labor force or work in a public sector job if it is worth it.

There are three working sectors in the economy: the public, the private formal and the private informal sectors. The government is responsible for paying non-competitive wages to its workers in exchange for the production of a public good, and for managing a PAYG retirement system for both public and private sector retirees. In order to balance its budget the government taxes consumption, capital and labor income. The formal private firms use a Cobb-Douglas technology with capital and labor, whereas firms in the informal sector produce with labor as their only input. All firms act competitively and produce goods that are perfect substitutes in consumption. Uncertainty in the economy comes from both idiosyncratic shocks private workers have in their labor efficiency and the life span of the agents. Agents can save in a risk-free asset to smooth consumption against these shocks.

3.1 Demography, Preferences and Choices

The economy is populated by a continuum of mass one agents who may live at most J periods. Each agent has a time endowment of $\bar{H}$ hours per period.

There is uncertainty regarding the time of death in every period, so that everyone faces a probability ψ_j of surviving to age $j+1$ conditional on being alive at age j. The age profile of the population, denoted by $\{\varphi_j\}_{j=1}^{J}$ follows the law of motion $\varphi_{j+1} = \frac{\psi_j}{1+g_n}\varphi_j$ and satisfies $\sum_{j=1}^{J} \varphi_j = 1$, where g_n denotes the population growth rate.[15] The lifespan uncertainty entails that a fraction of the population leaves accidental bequests, which are assumed to be equally distributed to all surviving individuals in a lump-sum basis (call it ζ). There are no private annuity markets in the economy.

Agents enjoy utility over effective consumption, $\tilde{c}_j$, and leisure, l_j. They maximize lifetime expected utility:

$$\mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^j \left(\prod_{k=1}^{j} \psi_k \right) u(\tilde{c}_j, l_j) \right]$$

Where β is the intertemporal discount factor and $\mathbb{E}_j$ is the expectation operator conditional

[15] The fraction of newborns is given by $\varphi_1 = \left[1 + \sum_{j=1}^{J-1} (1+g_n)^{-j} \prod_{i=1}^{j} \psi_i \right]^{-1}$.

on time j. The period utility takes the form:

$$u(\tilde{c}_j, l_j) = \frac{[\tilde{c}_j^{\gamma} \cdot l_j^{1-\gamma}]^{1-\sigma}}{1-\sigma}$$

The effective consumption is given by $\tilde{c} = c + \varepsilon Y_G$, with c being private consumption and Y_G the consumption of a public good. The parameter σ determines the risk aversion, γ denotes the share of consumption in the utility, and ε measures the relative importance of public consumption in overall consumption. All agents in the economy can save and lend their savings to a private competitive firm.

In this economy, agents can be private workers, public servants or retirees. Let $m \in \{P, G, R\}$ denote these individual states, respectively.[16] Agents choose how much to consume, $\tilde{c}_j \geq 0$, and how much to work h_j. We restrict the labor choice to be a discrete choice in $\{0, H < \bar{H}\}$. The remaining time is considered to be entirely leisure, so $l_j + h_j = \bar{H}$. We assume that it is mandatory for public servants to go to work. In contrast, private workers and retirees choose between working or staying at home.[17]

Agents can choose to take an open exam and try their luck into the public sector. Taking this exam is costly, where the time cost is a function of their current age, $c_{app}(j)$. We assume that retirees cannot enter the public sector, and that public workers cannot retake the exam.

As workers become older, and conditional on meeting the eligibility requirements, they can apply for social security benefits, becoming retirees within their respective sectors. We assume that informal private workers retire as private retirees, the same retirement sector as formal private workers (there is no informal retirement sector). Moreover, we assume that the public and the retirement sectors are absorbing states. For example, once individuals enter the public sector, there is no turning back until they retire from the labor force. At the same time, retirees cannot reapply retirement nor apply for public sector jobs.

3.2 Labor Income, Efficiency, and Budget Constraints

Conditional on their respective sector, individuals make decisions on whether to work or not, and on asset accumulation. Let w_f (w_i) denote the competitive wage paid by formal (informal) firms. An individual aged j who decides to work $h_j \in \{0, H\}$ hours produces a total of units of consumption before taxes given by:

[16] There is endogenous migration between sectors, to be detailed in the next sections.

[17] We allow for retirees to endogenously choose whether to work or not. This is the case in Brazil, even though in some countries retirees are not allowed to work. We can easily shut down this option if we are interested in studying some other country's social security reforms, or introduce a punishment in terms of retirement benefit reduction in the case the retiree works.

$$y_j(\omega, m) = \begin{cases} \omega e^{z+\eta_j} \cdot h_j, & \text{if} \quad m = \textit{Private, Retirement} \\ (1+\theta) w_f e^{z_G+\eta_j} \cdot H, & \text{if} \quad m = \textit{ Government} \end{cases}$$

For $\omega \in \{w_f, w_i\}$.

We assume that the idiosyncratic productivity $\{z\}$ follows a first order Markov process with transition matrix Π. There is no uncertainty regarding the public sector, and z_G is the productivity that the private worker had when she decided to take the admission test for the public sector and succeeded. The function η_j is a deterministic age-specific component of labor efficiency. The parameter θ corresponds to the wage premium or economic rent that public sector workers receive relative to their counterparts in the formal private sector.

All agents in the economy pay capital income tax τ_k and consumption tax τ_c. Workers face labor income tax rate of $\tau_y(m)$, and those who are not retired must additionally contribute a fraction $\tau_{ss}(m)$ of their labor income (up to a maximum taxable income, or social security ceiling, y_{max}) to the social security system. The revenue from $\tau_{ss}(m)$ is used to finance the social security benefits of the retirees, and the revenue from $\tau_y(m)$ finances overall government expenditures not related to the social security system. Retirees pay a tax rate of τ_b over their social security benefits. Informal workers do not pay labor income taxes, nor contribute to the SS system.

We assume that individuals save in a risk-free asset which pays a competitive interest rate r. They cannot have negative assets at any age, so that the amount of assets carried over from age j to $j+1$ is such that $a_{j+1} \geq 0$. Furthermore, given that there is no altruistic bequest motive and death is certain at age $J+1$, agents at age J consume all their assets, that is, $a_{J+1} = 0$. We normalize the continuation value after age J as zero.

The budget constraint for the non-retired individuals in the private sector is given by:

$$(1+\tau_c)c + a' = \begin{cases} [1+(1-\tau_k)r]a + (1-\tau_y(P))y_j(w_f, P) - \tau_{ss}(P)\min\{y_j(w_f, P), y_{\max}\} + \zeta, & \text{if formal} \\ [1+(1-\tau_k)r]a + y_j(w_i, P) + \zeta, & \text{if informal} \end{cases}$$

The budget constraint for the public sector workers is:

$$(1+\tau_c)c + a' = [1+(1-\tau_k)r]a + (1-\tau_y(G) - \tau_{ss}(G))y_j(w_f, G) + \zeta$$

Lastly, for the retirees receiving benefits b,, the budget constraint is:

$$(1+\tau_c)c + a' = \begin{cases} [1+(1-\tau_k)r]a + (1-\tau_y(P))y_j(w_f, R) + (1-\tau_b)b + \zeta, & \text{if formal} \\ [1+(1-\tau_k)r]a + y_j(w_i, R) + (1-\tau_b)b + \zeta & \text{, if informal} \end{cases}$$

where b denotes the retirement benefits the retiree is entitled to. These benefits are calculated upon retirement, with rules that depend on the worker's state vector, to be discussed further in Section 5.

3.3 Public Sector Recruitment

We now describe how the public sector recruitment works. We model this procedure as close as possible to the Brazilian case. According to constitutional rules, the hiring process of civil servants in Brazil is given by public competition. Candidates must take a nationwide open exam, and only those who scored the best fill up the job vacancies. In the model, agents who want to work in the public sector must perform well at costly "exams", and only those who obtain the best grades become eligible to fill a predetermined number of job positions. Once a private worker takes the test and succeeds, she becomes a public servant next period, and must work for the government until retirement.

The timing is the following. First, an agent chooses to apply, at age j, paying the time cost $c_{app}(j)$. Her score q_{jz} is then revealed in the next period, according to:

$$log(q_{jz}) = log(e^{z+\eta_j}) + \nu,$$

where $\nu \sim N(0,1)$. The above equation decomposes the test score into two parts: one that is linked to the individual's skill and age, and another that is random in nature.[18] If $q_{jz} \geq \bar{q}$, she will necessarily work for government from age $j+1$ onward. Otherwise, she will remain in the private sector. The threshold score, $\bar{q}$, is chosen by the government in equilibrium to balance the demand and supply of public servants. In what follows, we define the probability of a worker being approved in the public exam as: $P_{app} \equiv Pr(q_{jz} \geq \bar{q})$.

3.4 Social Security

Private and public sector workers have social security systems with different rules. Workers in the private sector have two retirement modalities, contribution and age. In the first scheme, the worker may apply for retirement after contributing for a number of years (35, in the case of Brazil) even if she has not reached the normal retirement age. In the age modality, similar to the rest of the world, a worker may apply for retirement only after reaching the normal retirement age (65 years old in the case of Brazil) and having contributed for a minimum number of years. In the model, agents are free to chose the retirement modality that suits them the best.

The value of the retirement benefits, b, is calculated as a replacement rate over average past earnings, x. This replacement rate in turn depends on the age of retirement, the modality, the number of years of contribution, among other institutional factors to be detailed in Section 5.

[18] An equivalent approach to modeling public exams in Brazil can be found in Brotherhood and Delalibera (2020) and in Brotherhood et al. (2023).

Public workers also have two retirement modalities. If she is older than a certain age (60 years old in the case of Brazil) and contributed for a minimum number of years she can retire under the contribution modality with a replacement rate of 100% over her current labor income. Civil servants older than 65 can retire under the age modality. In this case, however, individuals are entitled only to a proportion of their last wage, depending on the number of years of contribution.

3.5 Value Functions

We now detail the agents' problems. We first describe the state space and the policy functions, then we detail the value functions for each agent in the economy.

We divide an individual state depending on what sector of the economy she is located. The state of an agent in the private sector is $s_P = (j,a,z,x,t_C) \in S_P \equiv \{1,...,J\} \times \mathbb{R}_+ \times \mathscr{Z} \times \mathscr{X} \times \{0,...,J\}$, where j is her age, a her asset holdings, z is the agent's idiosyncratic productivity, x is her average past earnings, and t_C is the number of years contributed to the SS system. The state of a public worker is $s_G = (j,a,z,t_C,t_G) \in S_G \equiv \{1,...,J\} \times \mathbb{R}_+ \times \mathscr{Z} \times \{0,...,J\} \times \{0,...,J\}$, where t_G is the number of years worked in the government. As for the retirees, their relevant state is given by $s_R = (j,a,z,b) \in S_R \equiv \{1,...,J\} \times \mathbb{R}_+ \times \mathscr{Z} \times \mathscr{B}$, where b denotes the retirement benefits.

Solving the recursive problem yields the policy functions for working hours $d^h(s_m) \in \{0,H\}$, asset holdings $d^a(s_m) \in \mathbb{R}_+$ and consumption $d^c(s_m) \in \mathbb{R}_{++}$ for all $m \in \{P,G,R\}$; retirement $d^{ss}(s_m) \in \{0,1\}$ for $m \in \{P,G\}$; informality $d^{inf}(s_m) \in \{0,1\}$ for $m \in \{P,R\}$ and the public sector application $d^{app}(s_P) \in \{0,1\}$.

3.5.1 Retired Workers

A retiree chooses between working formally, informally, or staying at home. Her value function is given by:

$$V(s_R) = \max_{c,a',h,\text{inf}} u(\tilde{c}, \bar{H} - h) + \beta \psi_j \cdot \mathbb{E}\left[V(s'_R)\right]$$

s.t.

$$(1+\tau_c)c + a' = \begin{cases} [1+(1-\tau_k)r]a + (1-\tau_y(P))y_j(w_f,P) + (1-\tau_b(R))b + \zeta, & \text{inf} = 0 \\ [1+(1-\tau_k)r]a + y_j(w_i,P) + (1-\tau_b(R))b + \zeta, & \text{inf} = 1 \end{cases}$$

$$(c,a') \geq 0$$

$$h \in \{0,H\}$$

$$\text{inf} \in \{0,1\}$$

where the evolution of the state follows $s'_R = (j+1, a', z', b)$ and $\mathbb{E}[V(s'_R)] = \sum_{z'} \Pi(z,z')V(s'_R)$ is the standard expected value conditional on the current productivity, z.

3.5.2 Public Servants

At each age j, a public sector worker decides whether to retire from the labor force or not. Letting *ret* be an indicator function that assumes the value of 1 if the individual decides to retire and the zero otherwise, the value function of the public employee is given by:

$$V(s_G) = \max_{c,a',ret} u(\tilde{c}, \bar{H} - H) + \beta\psi_j \cdot \{(1-ret)\cdot V(s'_G) + ret \cdot V(s'_R)\}$$

s.t.

$$(1+\tau_c)c + a' = [1 + (1-\tau_k)r]a + (1 - \tau_{ss}(G) - \tau_y(G))y_j(w_f, G) + \zeta$$
$$(c, a') \geq 0$$
$$ret \in \{0,1\}$$

with next period state being $s'_G = (j+1, a', z, t_C + 1, t_G + 1)$ when she does not retire and $s'_R = (j+1, a', z, b' = b(s_G))$ in case of retirement. The variable $b(s_G)$ denotes the retirement benefits the agent will receive as a function of her state variables.[19]

3.5.3 Private Workers

At each age j, the formal private worker makes decisions in three dimensions, in addition to the consumption/saving choice. First, if eligible, she decides whether to retire from the labor force or not. Second, she decides whether to work (formally or informally) or to stay at home. Third, she decides if she takes the public exam. Letting *app* be an indicator function that assumes the value of 1 if the individual decides to take the public exam and the zero otherwise, the private worker value function can be written as:[20]

$$\begin{aligned} V(s_P) = \max_{c,a',h,ret,app,inf} u(\tilde{c}, \bar{H} - h) + \beta\psi_j \cdot \{ \quad & ret \cdot \mathbb{E}\left[V(s'_R)\right] \\ & +(1-ret)\cdot app \cdot P_{app} \cdot V(s'_G) \\ & +(1-ret)\cdot app \cdot (1-P_{app}) \cdot \mathbb{E}\left[V(s'_P)\right] \\ & +(1-ret)\cdot(1-app)\cdot \mathbb{E}\left[V(s'_P)\right] \} \end{aligned}$$

[19]The benefits function $b(\cdot)$ for both private and public workers are specified in Section 5.

[20]Because retirement is a deterministic choice and we do not allow retirees to work in the government, there is no continuation value when the agent retires from the labor force and takes the public exam.

s.t.

$$(1+\tau_c)c + a' = \begin{cases} [1+(1-\tau_k)r]a + (1-\tau_y(P))y_j(w_f,P) - \tau_{ss}(P)min\{y_j(w_f,P), y_{max}\} + \zeta, & \text{inf} = 0 \\ [1+(1-\tau_k)r]a + y_j(w_i,P) + \zeta & , \text{inf} = 1 \end{cases}$$

$$(x', t'_C) = \begin{cases} \left(\frac{(t_C-1)x + min\{y_j(w_f,P), y_{max}\}}{t_C}, t_C + 1\right), & inf = 0 \text{ and } h = H \\ (x, t_C) & , \text{otherwise} \end{cases}$$

$$l + h + app \cdot c_{app}(j) = \bar{H}$$
$$(c, a') \geq 0$$
$$h \in \{0, H\}$$
$$ret, app, inf \in \{0, 1\}$$

The first term of the continuation value inside the brackets corresponds to retirement, the second to not retiring, applying to a public job and passing the exam, the third to not passing the exam (and not retiring), and the fourth and final term corresponds to the decision of not retiring and not applying to a public job.

The evolution of the state variable is, for the case of retirement, $s'_R = (j+1, a', z', b' = b(s_P))$; for the case of entering the public career, $s'_G = (j+1, a', z, t'_c, 1)$; and, finally, $s'_P = (j+1, a', z', x', t'_c)$ for the case where she continues a private sector agent.

3.6 Agents' Stationary Distribution

The stationary distribution of agents is characterized by probability distribution functions $\mu_m : S_m \to [0,1]$, for all $m = \{P, G, R\}$, such that $\sum_{(m,s_m)|j} \mu_m(s_m) = \varphi_j$ for all $j \in \{1, ..., J\}$. That is, $\mu_m(s_m)$ is the measure of individuals in sector m and state s_m in the population.

In equilibrium, the stationary distribution of agents is constructed by forward induction using the policy functions derived in the previous section. However, we have to take a stance on the distribution of agents entering the economy (i.e., such that $j = 1$). We assume that: (i) every agent starts her life-cycle with zero initial assets, zero average past earnings and zero time of contribution; (ii) everybody starts as a worker in the formal private sector; and (iii) the initial distribution of the idiosyncratic productivity is the invariant distribution of the Markov process for $\{z\}$.

For each of the remaining ages, the distributions is derived using forward induction, considering the agents' policy functions, the transition matrix for the income process, the survival probabilities, and the probability of succeeding in the public exam and entering the public sector. A formal derivation of the equilibrium distribution can be found in Appendix A.

3.7 Technology

We assume that there are two representative firms producing perfect substitute goods. One operates in the formal sector and one in the informal sector. The first one produces using capital and labor, whereas the second one uses only labor. Both of them act competitively and maximize profits given input prices.

The production function of the formal sector is Cobb-Douglas: $Y_f = F(K, N_f) = A_f K^{\alpha} N_f^{1-\alpha}$, where K and N_f are the aggregate capital and private labor inputs, α is the capital's share in output, and A_f denotes the formal sector total factor productivity (TFP). Capital is assumed to depreciate at a rate δ each period. The problem of the formal firm is:

$$\max_{K,N_f} K^{\alpha} N_f^{1-\alpha} - (1+\tau_w) w_f N_f - (r+\delta)K \tag{1}$$

where τ_W is a tax rate charged by the government to finance the social security system.

Informal firms have linear technology in labor: $Y_i = A_i N_i$ and maximize profits according to:

$$\Pi_i = \max_{N_i} A_i N_i - w_i N_i \tag{2}$$

As the focus of our analysis is on the household-level responses to social security reforms, we abstract from firm heterogeneity within the formal and informal sectors. By doing so, we deviate from the literature that looks at the impact of informality on development and firm-level behavior.[21] That literature typically considers an increasing, strictly convex labor cost for operating under informality, which represents the probability of being caught by government authorities. Our simplified assumption, together with our calibration exercise, implicitly embeds these costs on the informal sector productivity parameter, A_i.

3.8 The Government Sector

The government taxes consumption, capital, income, wages, and social security benefits to finance the social security coverage, the payroll of public servants and its own, non-productive consumption. We assume that the government consumes a constant fraction of the formal GDP: $C_g = \alpha_g Y_f$.

In the labor market, the government hires a share $\bar{N}_G \in [0,1]$ of the population as public servants, and uses them to produce a public good Y_G. We assume that the government production

[21] See Ulyssea (2018), Meghir et al. (2015), Almeida and Carneiro (2012) and de Paula and Scheinkman (2011).

function is linear in the effective labor supply: $Y_G = L_G$.

In equilibrium, the government is responsible to choose $\bar{q}$ in order to balance the demand and supply of public workers. This assumption hinges on the fact that, over the 2005-2013 period, the Brazilian government employed consistently around 5% of the population.[22]

4 Equilibrium

We now define the recursive competitive equilibrium in this economy. A recursive competitive equilibrium consists of allocations of households and firms, prices (wages and interest rate), government taxes and threshold score, a stationary distributions of agents, bequests and public goods such that: (i) households and firms optimize; (ii) individual and aggregate behaviors are consistent; (iii) the government sets threshold scores and consumption taxes to balance the size of the public sector in the population as well as its budget constraint; (iv) the stationary distributions evolve according to the policy functions of the agents; and (v) the amount of public good and the amount of bequests are consistent with individual behavior. A complete equilibrium definition can be found in Appendix B, and the algorithm used to compute the equilibrium is detailed in Appendix C.

5 Data and Calibration

We now describe the calibration procedure. Using publicly available micro and macro data from different sources, we calibrate the model to match features of the Brazilian economy in 2013. Tables 5 and 6, by the end of this section, summarize the parameters values of this exercise.

5.1 Data Sources

At the aggregate level, we use data from the 2013 Brazilian National Accounts to calculate the fraction of GDP consumed by the government, the 2013 Social Security Annual Report to calculate the social security deficit and statistics on public versus private retirement, and the 2013 IBGE mortality tables to obtain the survival probabilities.

At the micro level, we use two data sets, both provided by the IBGE. The first is the 2013 PNAD, a cross-section household survey that is representative at the national level. We use this data set to calculate the distribution of agents across the three main sectors in the model (private, public and retirees) as well as the distributions of economic participation, test takers, and statistics on retirees by age.

[22] This figure was calculated using PNAD data from 2005-2013.

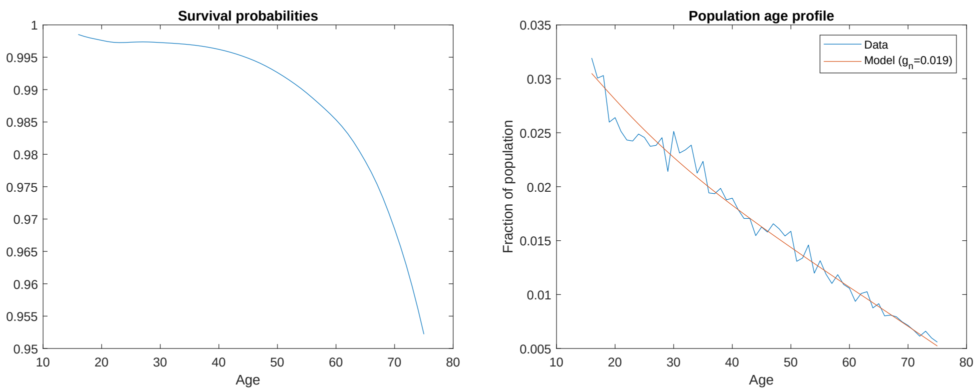
The second data set, used for the estimation of the income process, the age-efficiency profile and the public sector wage premium, is the 2012-2018 PNADC. This survey is a quarterly rotating panel with information on labor market outcomes for a nationally representative sample. Its main goal is to produce indicators that monitor quarterly fluctuations in the workforce. It follows a rotation scheme where each household is visited five times during five consecutive quarters, allowing us to construct a panel of individuals in an interval of one year.

Even though they are similar, the PNAD has better information on both demographics and individuals out of the labor force, as the main purpose of the PNADC is to generate labor force statistics. In both data sets we focus the analysis on men between 16 and 75 years old. We define an informal worker as a worker that does not have a legal working contract (carteira de trabalho), hence is not covered by the social security system and does not pay income taxes. A government agent is a statutory government employee, as these are subject to the segregated retirement scheme.[23]

5.2 Demography and Endowments

A model period corresponds to one year, and we assume agents live from 16 to 75 years old, so J = 60. The survival probabilities are taken directly from the IBGE's 2013 mortality tables. We set the population growth rate at $g_n = 0.019$ to match the population age profile obtained from the 2013 PNAD. Figure 6 plots the survival probabilities used in the model (left panel) and a comparison between the population age profile in the data and in the model (right panel).

Figure 6: Calibrated Survival Probabilities and Age Profile.



Notes: This figure shows the survival probabilities (left) and the population age profile (right), comparing the model implied profile (red) with the data (blue). Sources: 2013 IBGE Mortality Tables and 2013 PNAD.

We assume agents have $\bar{H}$= 8.760 hours per year, and that they can either work zero or 8 hours per day (the contractual full-time routine in Brazil in 2013), which implies H = 2.016

[23]Politicians, Central Bank workers and administrative/clerical jobs in the government are examples of statutory workers. Employees of publicly owned enterprises are not considered statutory public servants, and are subject to the private social security scheme.

hours.

5.3 Preferences and Technologies

We detail first the calibration of the preference parameters. The value of β is chosen so that the capital-to-output ratio is 2.5. This value is obtained from Morandi (2016), which applies the Perpetual Inventory Method to compute a historical series of the capital stock in Brazil. This number lies in the range of 2.5 and 3, values commonly used in the Macro literature for Brazil.[24] We set the risk aversion parameter at 2.5 based on the literature on consumption, surveyed by Attanasio (1999). This value is in line with estimates of the risk aversion parameter for the Brazilian economy.[25] The consumption share in the utility is chosen to match the participation rate of 72.3% in 2013, according to the PNAD. Lastly, we set ε to 0.5 following the work of Ferreira and do Nascimento (2005).

The technology parameters were calibrated as follows. For the formal sector, we set the capital share in output as $\alpha = 0.4$, and the depreciation rate at $\delta = 6\%$, as commonly used in the Macro literature.[26] We calibrate the formal sector TFP, A_f, to match the 2013 GDP per capita of R$26,520. As for the informal sector, we calibrate the informal firms' productivity of labor, A_i, to match a share of informal workers in the population of 18.9%.

5.4 Estimation of Labor Income

We now turn to the estimation of the the stochastic process for z, the age-efficiency profile, η_j, and the public sector wage premium, θ. We use micro data from the IBGE's Pesquisa Nacional por Amostra de Domicilios Continua (PNADC) from 2012q1 to 2018q4. We assume that the age-efficiency profile is quadratic: $\eta_j = \alpha_1^\eta j + \alpha_2^\eta j^2$, and use the model to write down the log hourly wages of individual i as:

$$\log\left(\frac{y}{h}\right)_{ij} = C + \alpha_1^\eta j + \alpha_2^\eta j^2 + z_{ij} \tag{3}$$

where C varies only with the working sector of the agent.

We estimate the state space, $\mathscr{Z}$, and the transition probabilities, $\Pi(z, z')$, of the idiosyncratic labor productivity non-parametrically,[27] following Hansen et al. (2014), De Nardi et al. (2016), and Ferreira and Gomes (2017). Because labor income fluctuation in the model only happens in

[24] See Santos and Cavalcanti (2021), Glomm et al. (2009) and Ferreira and do Nascimento (2005).

[25] See Gandelman and Hernandez-Murillo (2015) and Fajardo et al. (2012).

[26] See Parente and Prescott (2002) for evidence on the depreciation rate and Gomes et al. (2005) for evidence on the capital share of formal output.

[27] We believe a non-parametric estimation of the income process is important as it captures empirical deviations from the normality assumption highlighted, for the Brazilian case, by Gomes et al. (2020), Guvenen et al. (2021) and De Nardi et al. (2016) also find large deviations from normality in the U.S. labor market.

Table 3: Idiosyncratic productivity process

Transition matrix: $\Pi(z,z')$		z_1	z_2	z_3	z_4	z_5
	z_1	0.659	0.220	0.089	0.029	0.003
	z_2	0.220	0.473	0.233	0.070	0.004
	z_3	0.082	0.246	0.479	0.183	0.009
	z_4	0.038	0.090	0.248	0.573	0.051
	z_5	0.022	0.030	0.055	0.252	0.642
State space: $\mathscr{Z}$		-0.715	-0.249	0.074	0.548	1.542

Notes: This table shows the results for the estimation of the idiosyncratic labor income process. The first 6 rows display transition probabilities between every two periods. Rows represent productivities at t and columns represent productivities at $t+1$. The last row shows the elements in the discretized state space.

the private sector, we restrict the data to this sector. First, we obtain the empirical counterparts of z_{ij} as the residuals from a regression of log hourly wages on age and age-squared. We then split these residuals into five groups: the top 5%, the next 20%, 25%, 25%, and the bottom 25%, and set each element in the state space $\mathscr{Z}$ as the mean within each respective group. The construction of the transition probabilities explores the panel dimension of the data. We calculate the fraction of individuals that migrated between every two groups over a year.

The estimated grid and transition probabilities are shown in Table 3, and the regression results are displayed in the first column of Table 4. There is asymmetry in both the transition probabilities as well as the elements in the state space (around zero). This is fundamentally different than the symmetry imposed by traditional methods for estimating the income process, such as Tauchen (1986), and we believe is a better representation of the data. The estimation of the public sector wage premium and the age-efficiency profile also rely on taking Equation (3) to the data. The Markovian nature of the idiosyncratic productivity process allows us to write:

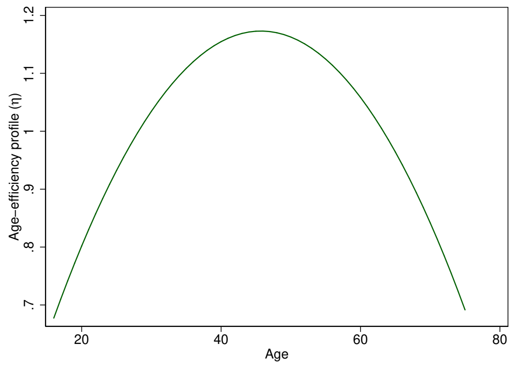
$$\log\left(\frac{y}{h}\right)_{ij} = C + \alpha_1^{\eta} j + \alpha_2^{\eta} j^2 + \underbrace{g(z_{ij-1}) + \varepsilon_{ij}}_{=z_t} \tag{4}$$

The $g(z_{ij-1})$ term above shows the potential omitted variable problem Regression (1) faces. To deal with this problem, we include a rich set of controls (such as education, occupation and sector), and individual fixed effects. A public sector wage premium of 9.8% ($= exp(0.094) - 1$) is estimated from the "Public servant" coefficient in Column (3) of Table 4. To avoid issues from selection of older people, we restrict our sample to 25-55 years old individuals and estimate the age-efficiency profile coefficients of $\alpha_1^{\eta} = 0.028617$ and $\alpha_2^{\eta} = -0.000313$, as shown in Column (4) of Table 4. Figure 7 shows the estimated age-efficiency profile.

Table 4 also highlights the importance of the panel dimension in the data. Column (2) calculates the public sector wage premium without controlling for individual fixed effects, and it finds a coefficient value of 17%, almost two times larger than the estimation of Column (3).

This clearly reflects the selection on fixed individual characteristics that drive high-type agents into the public sector, as discussed in Section 2.

Figure 7: Estimated age-efficiency profile



Notes: This figure plots the age-efficiency profile, $\hat{\alpha}_1^{\eta} j + \hat{\alpha}_2^{\eta} j^2$, implied by the age coefficients estimated in column (4), Table 4. Sources: 2012-2018 PNADC.

Table 4: Estimation of the income process variables

	(1)	(2)	(3)	(4)
Public servant		0.169*	0.094*	0.090*
		(0.004)	(0.005)	(0.006)
Informal worker		-0.188*	-0.056*	-0.053*
		(0.002)	(0.002)	(0.003)
Age	0.061*	0.032*	0.031*	0.029*
	(0.000)	(0.000)	(0.001)	(0.003)
Age^2	-0.00063*	-0.00028*	-0.00033*	-0.00031*
	(0.000)	(0.000)	(0.000)	(0.000)
Individual FE			✓	✓
Controls		✓	✓	✓
Time FE	✓	✓	✓	✓
Sample	Private	All	All	25-55
N	1,832,912	2,053,458	1,773,520	1,270,749
R^2	0.196	0.615	0.899	0.898

Notes: This table shows the main coefficients of Regression (3). We use the PNADC data from 2012q1-2018q4, focusing on men between 16 and 75 years old. Column (1) focus on individuals in the private sector, columns (2) and (3) consider the whole sample, and column (4) restricts the sample to 25-55 years old individuals. Control variables are: tenure, race (when there are no individual fixed effects), and occupation, sector, and education fixed effects. Robust standard errors in parentheses. $*p < 0.001$.

5.5 Social Security System

The social security in Brazil is a pay-as-you-go system, which transfers income from workers to retirees. The system is financed with payroll taxes, and has two very different regimes - the private sector regime and the public sector regime. The benchmark year of our calibration is 2013, hence the retirement benefits structure is modelled in order to mimic the retirement rules that used to prevail in Brazil at that year, before the implementation of two major social security reforms.

5.5.1 Private Benefits

The private sector regime is organized under INSS, which stands for Instituto Nacional do Seguro Social (National Institute of Social Security). Under the INSS retirement sector, we have two modalities of retirement - the age modality and the contribution modality.

We first detail the eligibility requirements. If the worker is older than the normal retirement age, which is 65 years old, and has contributed to the retirement system for more than 15 years, she can apply for retirement under age modality. If the worker has not achieved the normal retirement age, but have contributed for more than 35 years to the social security system, she can retire under the contribution modality.

In both modalities, the value of the benefits is calculated as a fraction of average past earnings, x:

$$b(j^r, x, mod, t_C) = \Psi(j^r, mod, t_C) \cdot x$$

where $mod \in \{Contrib, Age\}$ stands respectively for the contribution and the age modalities of retirement, $\Psi(j^r, mod, t_C)$ denotes the retirement replacement rate as a function of the age in which the worker retires from the labor force, j^r, the retirement modality, mod, and the number of years that the worker contributed formally to the social security system, t_C.

The average lifetime earnings, x, is calculated by taking into account individual earnings up to the age of withdrawal from the labor force that are lower than the maximum taxable income, $y_{\max}$ and its law of motion can be written as:

$$x_{j+1} = \frac{x_j(t_C - 1) + min\{y_j(w_f, P), y_{\max}\}}{t_C}, \quad \text{for } j = 1, 2, ..., j^r \tag{5}$$

Only earnings from the formal sector are considered in the calculation of x, as we assume that individuals in the informal sector do not contribute to the social security system.[28]

For those who retire under the contribution modality, the replacement rate is given by:

$$\Psi(j^r, Contrib, t_C) = f(j^r, t_C)$$

[28] Even though informal workers can contribute so the SS system, only a small fraction do so. In 2013, nearly 84% of the informal workers have not contributed to the SS system. On average from 2002-2013, only 11% of the informal workers contributed.

where $f(j^r, t_C)$ is commonly known as the *fator previdenciário* (social security factor). Such discount was implemented by the Fernando Henrique Cardoso's presidency, in order to discourage the early retirement that occurred in Brazil. Its formula is given by:

$$f(j^r, t_C) = \frac{0.31 t_C}{E(j^r)} \left[1 + \frac{(j^r + 0.31 t_C)}{100} \right]$$

where $E(j^r)$ is the life expectancy of the individual at the retirement age j^r, and t_C is the number of years social security contributions. Depending on the number of years the worker has contributed to the social security system, and on the age of retirement, the social security factor can be greater than 1.[29]

Under the age modality, the worker can choose between the social security factor rule or an alternative replacement rate, which starts at[30] 85% of average earnings and can increase up to 100%. Hence, the replacement rate for the age modality reads:

$$\Psi(j^r, age, t_C) = \max\{f(j^r, t_C), \tilde{\Psi}(t_C)\},\ \tilde{\Psi}(t_C) = \min\left\{0.70 + \frac{t_C}{100}, 1\right\}$$

Individuals older than 65 earning less than one fourth of the minimum wage are also entitled to retirement with a pension equal to the minimum wage. This assumption replicates the LOAS (Lei Orgânica de Assistência Social), a social assistance program that provides financial assistance to elderly individuals who do not participate in the formal private retirement system and do not have the means to support themselves. In the model, we calibrate the minimum wage to its observed annual value in 2013 of R$ 8,136.

We calibrate the contributions to the private social security system following the 2013 social security rules. Private workers paid 8%, 9% or 11% of their labor income up to the social security ceiling y_{max} in the following manner:

$$\tau_{SS}(P) = \begin{cases} 8\%,\ if\, y \leq R\$14,972 \\ 9\%,\ if\, R\$14,972 < y \leq R\$24,954 \\ 11\%,\ if\, R\$24,954 < min\{y, y_{max}\} \end{cases} \tag{6}$$

Private retirees have no tax on their benefits. We set the private social security ceiling $y_{max} = R\$50,000$ per year following the 2013 social security rules.[31]

[29] For instance, a 58 years old worker who contributed for 35 years receives only 80% of her past earnings upon retirement. However, if the same worker contributed for 45 years, her replacement rate would be 106%.

[30] This number comes from the 70% in the formula for $\tilde{\Psi}(t_C)$ plus the 15% required for eligibility.

[31] The value was defined as R$ 4,157 per month, which sums up R$ 49,884 annually.

5.5.2 Public Benefits

According to the Brazilian Constitution public servants have rights to a different pension system. This pension system is also split into two modalities, a contribution and an age modality. The retirement modality determines the replacement rate applied to the wage base, determining the public retiree's benefits. Diferently from the private retirement sector, where the wage base is a bounded average of past earnings, the wage base in the public retirement sector, until 2013, did not have an upper limit and corresponded to the average of the 80% highest wages received during the public career. Since we assume that the productivity of a worker, after entering in the public sector, does not change this average equals the last wage.

Two other differences relative to the private retirement scheme are that retirement in the public sector is mandatory at age 70, and the individual must have at least 10 years working as a public servant to be eligible for retirement.

Civil servants older than 60 years old that contributed for at least 35 years can retire under the contribution modality. The benefits match the worker's current labor income, y:

$$b(y, Contribution) = y$$

Civil servants older than 65 can retire under the age modality. In this case, individuals are entitled to a proportion of their last wage:

$$b(y, t_C, Age) = min\{\frac{t_C}{35}, 1\} \cdot y$$

Finally, following the 2013 tax code, we assume that public workers compulsorily pay 11% of their income to the social security system, and that public retirees pay an 11% tax over the benefits they have in excess to the private social security ceiling.

5.6 Government Sector

The labor and capital tax rates are chosen based on the Brazilian macro literature.[32] We set them at $\tau_y(P) = 18\%$ and $\tau_k = 15.5\%$. Following Immervoll et al. (2006), we calibrate the labor income tax of the public servants to $\tau_y(G) = \frac{\tau_y(P)}{2} = 9\%$. The consumption tax rate is chosen to balance the government budget constraint in equilibrium. Lastly, we use the 2013 Brazilian National Accounts to calculate that government consumption accounts for $\alpha_g = 19\%$ of GDP.

We use the PNAD data to calibrate the variables associated with the government sector. We assume that the time cost for taking the public exam is quadratic: $c_{app}(j) = \alpha_2^{app} j^2 + \alpha_1^{app} j + \alpha_0^{app}$, and calibrate the parameters to minimize the distance between the fraction of test takers by each age in the model and in the data. This procedure implies a convex cost function with

[32] See, among many, Glomm et al. (2009), and Pereira and Ferreira (2010) .

Table 5: External calibration

Parameter	Description	Value	Source
J	Maximum age	60	Agents live from 16-75
$\{\psi_j\}$	Survival probabilities	Figure 6	IBGE
g_n	Population growth rate	1.9%	IBGE
$\bar{H}$	Time endowment	8,760 hours	24 hours/day x 365 days/year
H	Working hours	2,016 hours	8 hours/day x 252 weeks/year
σ	Risk aversion	2.5	Attanasio (1999)
ε	Public good coef.	0.5	Ferreira and do Nascimento (2005)
α	Capital share in output	0.4	Standard value
δ	Depreciation rate	0.06	Parente and Prescott (2002)
$\mathscr{Z}$	Productivity space	Table 3	PNADC
$\Pi(z,z')$	Transition probabilities	Table 3	PNADC
$\{\eta_j\}$	Age-efficiency profile	Figure 7	PNADC
θ	Public sector wage premium	9.8%	PNADC
$b(.)$	Benefits function	Section 5.5	2013 rules
$\tau_y(P)$	Private sector's income tax	18%	Literature
$\tau_y(G)$	Public sector's income tax	9%	Immervoll et al. (2006)
τ_k	Capital tax	15.5%	Literature
$\tau_{SS}(G)$	G worker SS contribution	11%	2013 tax rates
$\tau_{SS}(P)$	P worker SS contribution	Equation (6)	2013 tax rates
τ_b	G retiree benefit tax	11%	2013 tax rates
α_G	$\frac{\text{Govt. Consumpt.}}{\text{GDP}}$	18.9%	National Accounts
$\bar{N}_G$	Size of the government	5%	2005-2013 PNAD
y_{max}	Private SS ceiling	R$ 50,000	2013 rules

Notes: This table summarizes the externally calibrated parameters, discussed throughout Section 5.

coefficients $\alpha_2^{app} = 6$, $\alpha_1^{app} = -80$, and $\alpha_0^{app} = 266$. The size of the public sector is calculated directly from the PNAD data: $\bar{N}_G = 5\%$.

5.7 Calibration Results

Tables 5 and 6 summarize the external and internal calibration procedures, respectively.

6 Equilibrium Features and External Validation

This section details the quantitative features of the calibrated model. The model generates a distribution of agents across the three main sectors (private, public and out of labor force), a distribution of endogenous choices (public sector application and informality), and other macroeconomic aggregates (e.g., overall social security deficit and early retirement) that match the data closely.

Table 6: Internal calibration

Parameter	Target	Model	Data
$\beta = 0.96$	K/Y	2.4	2.5
$\gamma = 0.248$	Participation Rate	73%	72.3%
$A_f = 2.194$	GDP per capita	R$ 27,468	R$ 26,520
$A_i = 3.03$	Informal sector size	19.5%	18.9%
$\alpha_0^{app} = 266$			
$\alpha_1^{app} = -80$	Test takers age profile	(Figure 9)	
$\alpha_2^{app} = 6$			
$\tau_w = 0.085$	Private SS deficit/GDP	0.8%	0.9%

Notes: This table shows the internal calibration results. The numerical implementation of the model is discussed in Appendix C.

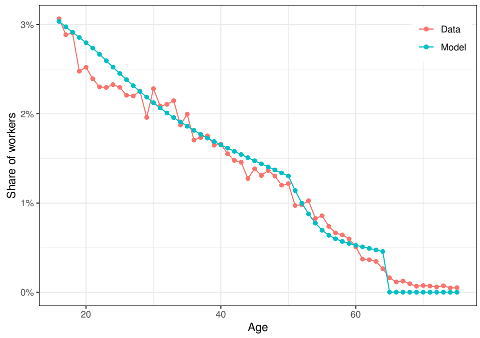
We first discuss the main equilibrium variables in the calibrated economy. The interest rate is 8.47%. This value is relatively high by international standards, but not for Brazil. The formal wage rate (4.03 Reais per hour) is 1.33 times larger than the informal wage. As discussed in the motivation section, this discrepancy in wage rates reproduces the fact that the informal sector absorbs low productivity individuals. Finally, the consumption tax rate is 34.86%.

How are agents distributed across the private, public and retirement sectors? Even though we target a government that absorbs 5% of the population, the model reproduces a private sector that accounts for 81.1%, and a retirement sector that accounts for the remaining 13.9% of the population. In the data, these numbers are 82% and 13%, respectively. Breaking down the agents' distributions by age, Figure 8 shows that our calibrated model matches the age profile of individuals across sectors well. It captures relevant features of the data such as the age profile of public servant, which is key in determining the public sector social security deficit. It also reproduces the early retirement in the private sector, which determines the extend to which minimum retirement age provisions are effective in reducing the private sector social security deficit.

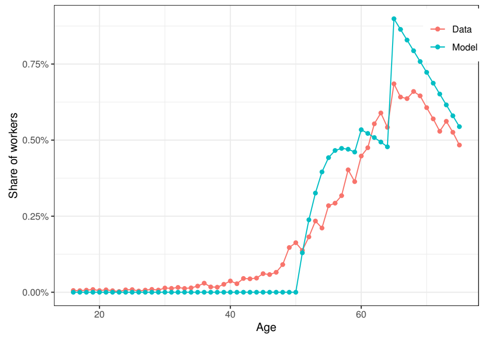
Even though the calibration procedure targets the overall participation rate, the model reproduces the labor force participation by age quite well, as shown in the top-left panel of Figure 9. However, it overestimates the number of young people in the labor market. Young agents in the model economy have low asset levels, and have to work early in life to compensate this lack of resources. Moreover, we do not consider human capital accumulation, nor schooling decisions. Most individuals between 16-18 years old are probably finishing their studies, preparing to enter the labor market, and still living with their parents, features that are not present in the model.

The timing of the public sector application decision determines the duration of the careers in the sector. This, in turn, will govern the amount of social security taxation and spending related

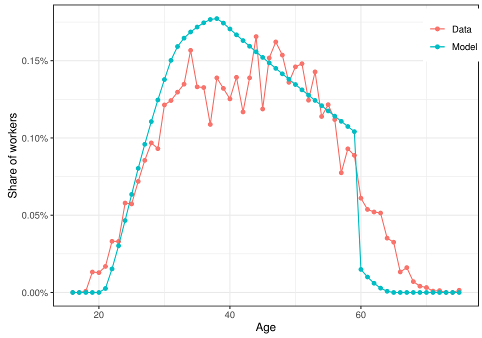
Figure 8: Agents' equilibrium distribution by age and sector



(a) Private Sector



(b) Retirees



(c) Public sector

Notes: This figure plots, for each age, the percentage of the population allocated in each sector.

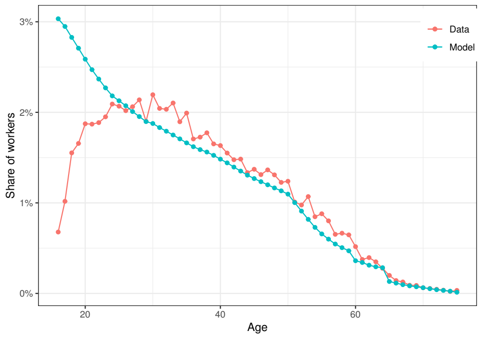
to public servants. Figure 9 shows that the calibrated economy generates a life cycle profile of applicants that follows closely to the data. The nonlinearities over the life cycle are well captured by a convex application cost function.

The bottom panel of Figure 9 plots the size of the informal sector by age. In the model, informality decreases significantly as individuals become older, a pattern that is also observed in the data, even though the intensity of the reduction is somewhat different. This is because, from the agent's point of view, it is too easy to alternate between formal and informal sector, whereas in reality this transition should be more sluggish. Moreover, as discussed above, the model overestimates the share of young agents in the labor force and given their relative low productivity and experience, they tend to work in this sector in greater proportion than what is observed in the data.

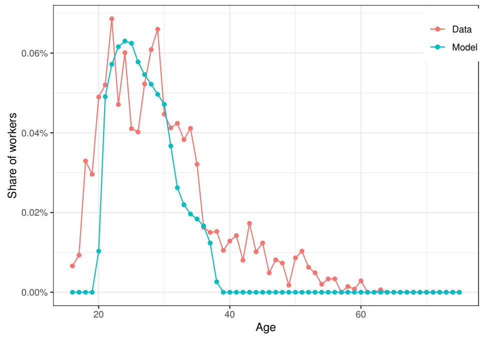
We now turn to non-targeted outcomes related to the social security system. Table 7 shows how the model performs in replicating the social security deficit breakdown, the retirement modality choices, the early retirement in the private sector observed in the data, and the income GINI coefficient.

The estimated overall social security deficit is 2.09% of GDP, matching the 2013 data - 2.12% of GDP - closely. Moreover, the model breakdown of social security deficit across public

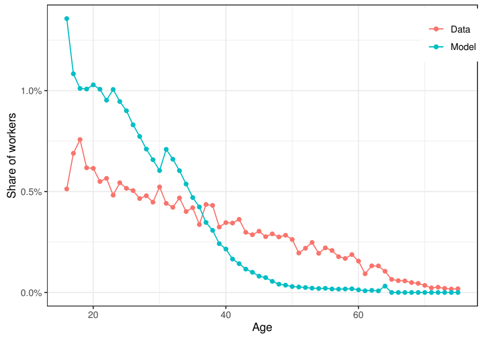
Figure 9: Participation rate, test taking behavior and informality



(a) Participation rate



(b) Test takers



(c) Informal sector

Notes: This figure shows, for each age, the fraction of the population participating the labor markets (top left), taking the public exam (top right), and working in the informal sector (bottom), in the calibrated model (blue) and in the data (red).

Table 7: Non targeted moments: model vs. data

	Model	Data
Public sector SS deficit (%GDP)	1.3%	1.2%
Overall SS deficit (% GDP)	2.1%	2.1%
Fraction of contrib. modality claims	59%	61%
Average age at retirement	52.2	54.8
Average years contributed at retirement	35.0	35.3
Mean effective earnings Formal/Informal	1.4	1.5
Income GINI coefficient	53.2	52.7

Notes: This table shows moments related to the social security system in the model and in the data. "Average age years contributed" are for the contribution modality only. Sources: model simulations, PNAD, World Bank, and Tafner et al. (2015).

and private retirement sectors is also very close to what was observed in the data in that year, with the public (private) share slightly larger (smaller) than the official figures. The model also matches closely the retirement age, years of contribution at retirement, and the fraction of retirement claims relative to the contribution modality. Lastly, the model replicates the average wage ratio between formal and informal workers and the GINI coefficient.

Overall, the model replicates well the data in all relevant dimensions for our analysis. We now turn to the evaluation the effects of social security reforms.

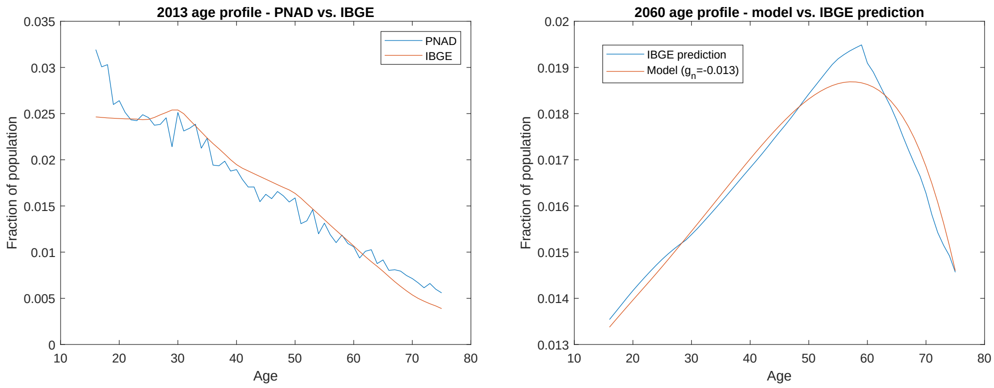
7 Social Security Reforms

What are the macroeconomic effects of reforming the social security system? In this section, we use the calibrated model to quantify the long-run effects of two policies: the unification of the public and private retirement systems, and the imposition of the minimum retirement age. We describe how we compute the counterfactual demographics and detail the policy reforms analyzed. We then discuss the quantitative results.

7.1 The Social Security Reform Experiments

Using data from the IBGE's Tables of Population Projections 2000-2060 and the model-implied age distribution, we find the population growth rate that matches Brazil's demographic profile in 2060. This procedure yields a counterfactual growth rate of $g_n = -0.013$. Because the age distribution used in the model was obtained from the PNAD, we compare the age distribution in 2013 from these two sources. Figure 10 shows that the demographic structures are not far from each other (left panel), which gives us confidence in using these predictions for our counterfactual analysis, and shows that the new age profile from the model fits the Brazilian age profile in 2060 calculated by the IBGE (right panel).

Figure 10: 2013 and 2060 population age profiles, model vs. data



Notes: The left panel compares the age profile in the 2013 PNAD with the one in the IBGE's Tables of Population Projections 2000-2060. The right panel plots the model-implied population under the counterfactual growth rate of $g_n = -0.013$ versus the IBGE's projection for the male Brazilian population in 2060. Sources: 2013 PNAD and IBGE's Tables of Population Projections 2000-2060.

For each demographic structure, we compare the general equilibrium behavior of the model economy with and without social security reforms. The first reform analyzed is the unification of the social security schemes. In 2013, Law 12.618 imposed a ceiling on the social security benefits received by new entrants in the public sector. This cap is the same that limits private sector benefits, y_{max}. The eligibility conditions did not change, neither did the benefits' formulas. The contribution to the social security system was limited to 11% of the minimum between the earnings of the public worker and y_{max}. This reform approximated the two social security regimes, and intends to alleviate, at least in an ex-ante manner, the fiscal pressure of public retirement on public accounts. The second reform is at the core of most discussions regarding social security: an increase in the minimum retirement age. We choose to impose a minimum retirement age of 65, as it mimics what was implemented by the Brazilian government with the Constitutional Amendment 103 of November 12th, in 2019.[33] In section 7.4, we assess the economic implications in a scenario where the retirement age is raised further, up to 72 years.

7.2 2013 Demography

In our first counterfactual exercise, we keep the 2013 demography constant and implement the two social security reforms, unifying the pension systems and increasing the minimum retirement age to 65. This will provide a benchmark for comparison with the main exercises ahead, where we simulate population aging on top of the pension reforms. The results are summarized in Table 8.

Table 8: The effects of social security reforms on equilibrium variables with 2013 demography

2013 Demography (Values in %)				
	Benchmark	**Unification**	**Minimum age**	**Both reforms**
SS deficit/GDP	2.1	1.6	-1.0	-1.1
Public SS deficit/GDP	1.3	1.0	-0.2	0.1
Private SS deficit/GDP	0.8	0.6	-0.7	-1.2
Consumption tax	34.9	34.0	30.8	30.4
Avg. Consumption (C)	100.0	102.3	99.1	100.7
Avg. Capital (K)	100.0	104.5	100.1	107.2
Avg. Output (Y)	100.0	108.9	98.9	102.3
K/Y	100.0	104.2	101.2	104.8
Participation rate	73.0	73.5	71.5	71.5
Fraction of retirees	13.9	15.1	8.0	8.0
Informality	19.5	14.5	20.5	16.1
Public sector aplicants	0.72	0.74	0.67	0.69

Notes: This table shows the effects of different social security reforms on a selected group of aggregate variables. Avg. Consumption (C), Avg. Capital (K), Avg. Output (Y), and the K/Y ratio, are normalized to the benchmark value.

Unifying the pension systems in the public and private sectors reduces the social security

[33] This is the age for men. For women, the minimum retirement age is 62 years old. We are also abstracting from a large number of transition rules.

deficit by 0.5 percent of GDP. The public pension deficit decreases by 0.3 percent of GDP and is the largest contributor to the overall deficit reduction, as public workers are now subject to a relatively low pension ceiling. Increasing the minimum retirement age to 65 generates a total pension surplus of 1 percent of GDP. In the benchmark calibration, nearly 60% of retirees opt for the contribution modality, with an average retirement age of 52.2 years. With the imposition of a 65 minimum retirement age, formal workers contribute to the pension system for longer and retire later, increasing revenues and reducing expenditures in the private pension system. However, this reform discourages labor force participation and increases informality, so in equilibrium output falls by 1.1%.

These findings suggest that reforms to the pension system can improve its financial situation. We find that early retirement seems to be more costly today than the generosity of the public retirement system. Moreover, reforming the pension scheme affects the decision of agents to stay in the workforce and the choice of which sector and formality status to work on.

7.3 Long-Run Demography

We now quantify the macroeconomic impacts of social security reforms with the 2060 demography. First, we evaluate the changes in aggregate variables such as the social security deficit over GDP, aggregate capital, the participation rate, and others. We then look at the changes in the life cycle behavior of agents. Table 9 and Figures 11 and 12 summarize the main findings of this section.

Table 9: The effects of social security reforms on equilibrium variables with 2060 demography

		2060 demography (%)			
	Benchmark (%)	**w/o reforms**	**Unification**	**Minimum age**	**Both reforms**
SS deficit/GDP	2.1	16.6	14.6	10.8	10.2
Public SS deficit/GDP	1.3	5.2	3.2	1.8	1.4
Private SS deficit/GDP	0.8	11.5	11.4	9.0	8.9
Consumption tax	34.9	54.6	51.7	46.1	45.4
Avg. Consumption (C)	100.0	106.7	110.9	107.0	109.6
Avg. Capital (K)	100.0	109.7	120.4	112.0	119.7
Avg. Output (Y)	100.0	99.3	104.7	99.4	103.0
K/Y	100.0	110.5	115.0	112.7	116.2
Participation rate	73.0	60.0	60.9	58.5	58.7
Fraction of retirees	13.9	31.5	33.7	18.7	18.7
Informality	19.5	7.1	3.8	8.6	6.3
Public sector aplicants	0.72	0.46	0.47	0.38	0.39

Notes: This table shows the effects of different social security reforms on a selected group of aggregate variables. In every column but "Benchmark (%)", the model is calibrated with the 2060 demography. Avg. Consumption (C), Avg. Capital (K), Avg. Output (Y), and the K/Y ratio, are normalized to the benchmark value.

Between 2013 and 2060, Brazil's demographic composition is set to undergo a significant transformation, with the share of people aged 65 or older increasing by 10.7 percentage points, from 8% to 18.7%. This demographic shift is expected to have substantial impacts on the economy. The participation rate is projected to decline by 13 percentage points, while the proportion of retirees is predicted to rise from 13.9% in 2013 to 31.5% in 2060. Additionally, individuals who stay at home are expected to account for 40% of the population. These demographic changes will increase the overall social security deficit from 2.1% to 16.6% of GDP, with the public portion of the deficit rising from 1.3% to 5.2% of GDP and the private sector portion increasing from 0.8% to 11.5% of GDP. These changes will place significant pressure on the government's budget, forcing an increase in the consumption tax rate from 34.9% to 54.6%.

The scale of these changes underscores the importance of addressing the challenges an aging population poses. Many countries around the world are experiencing a similar process of population aging. Several have overly generous dual social security systems that typically favor public sector workers. The results above indicate that these economies, and not only Brazil, will experience a severe deterioration of their fiscal conditions if they keep their retirement rules the same.

Despite the large increase in the consumption tax rate, there is only a slight fall in aggregate GDP, while consumption and capital are projected to increase between 2013 and 2060. This is so because the 2060 demography has relatively more (fewer) people aged 35-55 (16-30) than the 2013 demography (see Figure 10). Therefore, there are more people at the peak of their labor productivity, increasing the aggregate productivity of the workforce and partially offsetting the negative effect of tax increases.

The last three columns of Table 9 present the aggregate implications of the social security reforms. In all of them, we keep the 2060 demography and all other parameters unchanged. The third column presents the results for the unification of the two social security systems, the fourth refers to the implementation of a minimum retirement age of 65, and the last column to the right presents the case in which both reforms are implemented.

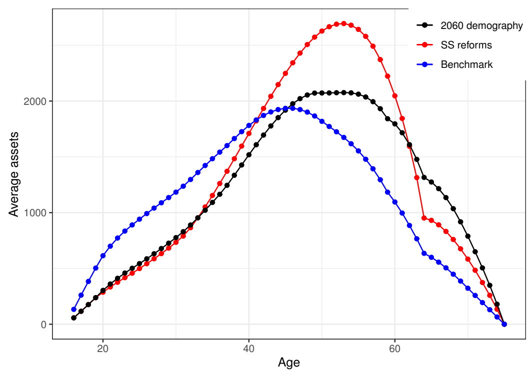
A key result of the reforms is the reduction of the social security deficit. When both reforms are jointly implemented, the social security deficit falls by almost 40% compared to a scenario without reforms. One of the main reasons for this result is the sharp decrease in the share of retired people, from 31.5% to 18.7%. Both reforms reduce the social security deficit, but only the imposition of a higher minimum age reduces the number of retired people, as this reform forces people to stay longer in the labor market. Nevertheless, the aggregate social security deficit will remain high as a proportion of GDP, around 10.2%.

We find that increasing the minimum retirement age reduces the public pension deficit by more than unifying the pension systems. Because the social security ceiling is relatively high for low-productivity public employees, imposing it in the public pension system changes little their decision to apply for a public job. However, increasing the minimum retirement age across the board reduces incentives to take the costly public exam, as public workers now have to work five more years before reaching the retirement age for the contribution modality. This causes a sectoral reallocation of low-productivity individuals away from the public sector into the informal sector, whose size increases from 7.1% to 8.6%, or out of the labor force. This, in turn, leads to a lower deficit in the public pension system.

We now discuss the response of savings, both in aggregate and over the life cycle. The 2060 economy has a larger share of workers in the most productive part of their careers, where they save the most out of labor income. Therefore, the reforms will lead to further savings expansion, as shown in Table 9. In the simulation with both reforms being implemented, aggregate savings increase by 9.1% with respect to the no-reform scenario. In addition to the incentives from lower taxation, agents increase their savings to smooth consumption as the pension system becomes less generous.

The above rationale is clear in Figure 11. The figure displays average savings over the life cycle. An older population induces more savings for people older than 45 years old, as the 2060 line is above that of 2013. However, for our purposes, the most interesting feature is the jump of the savings profile in a 2060 world with social security reforms. There is a significant increase in total average assets from around 33 years old to 62 years old. After this age, individuals' assets fall below the levels of the no-reform world. This is because people are saving more to compensate for the fall in retirement income caused by the changes in social security.

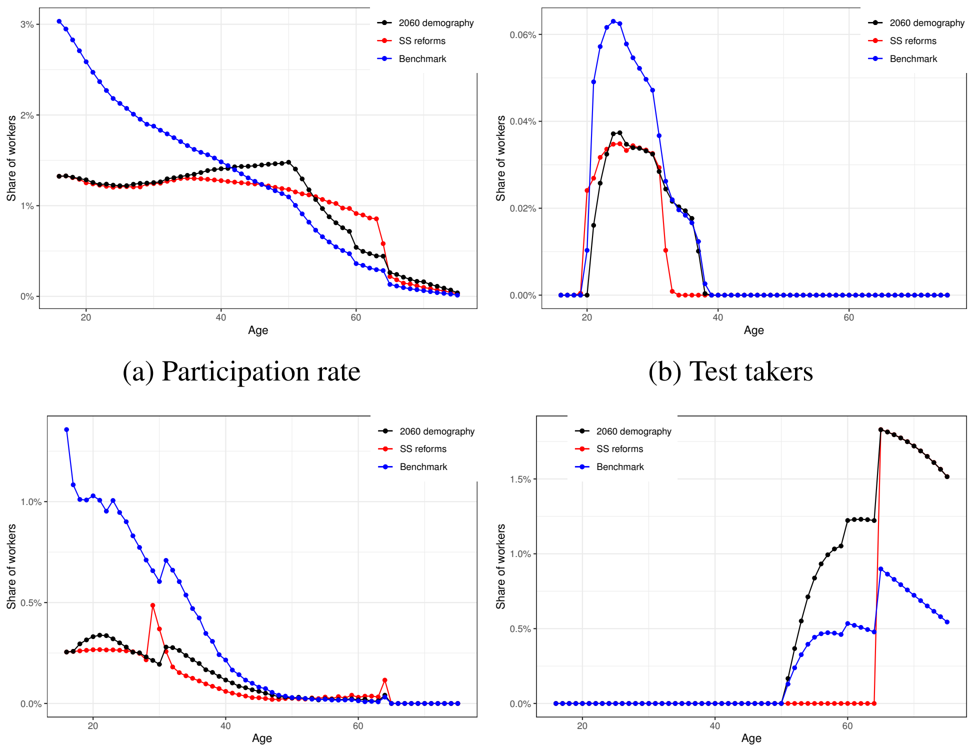
Figure 11: The response of average savings over the life cycle



Notes: This figure plots asset accumulation over the life cycle in 2013 Brazilian Reais.

Figure 12 depicts the economy's responses with respect to labor market participation, public sector application decisions, informality, and retirement. As also displayed in Table 9, informality falls between 2013 and 2060. This is mostly due to population aging: in the 2013 benchmark economy, a large part of the informal workforce was composed of younger people. Comparing the 2060 economy with and without reforms, the latter will not modify significantly informality over the life cycle. The main difference is around 30 years of age, when agents try their chances in the public sector. To compensate for the fall in utility, they migrate to the informal sector, avoiding taxes and increasing their net income.

Figure 12: The responses of participation rate, test taking behavior and informality



(a) Participation rate

(b) Test takers

(c) Informal sector

(d) Retirees

Notes: This figure shows, for each age, the fraction of the population participating the labor markets (top left), taking the public exam (top right), working in the informal sector (bottom left), and retiring (bottom right) in the calibrated model (blue), in the economy with 2060 demography (black) and in the 2060 demography with both reforms (red).

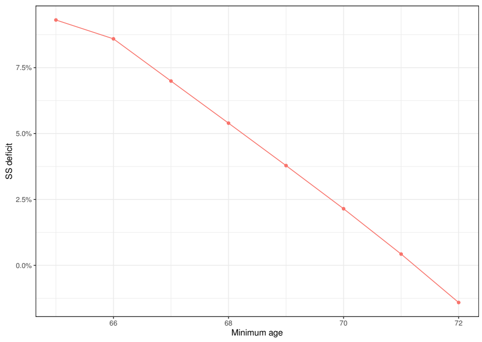
Public sector application decisions change significantly in each scenario. From 2013 to 2060, there is a substantial reduction in the proportion of applicants, but the shape of the age profile remained mostly the same. The social security reform also reduces the average age of applicants. Although there is now a concentration of younger applicants, which implies longer worker tenures in the public sector, the mass of applicants is 14% lower.

The most relevant change caused by the reforms is a significant increase in the labor market participation rate of people between 60 and 65 due to the new minimum retirement age, as Figure 12a shows. This can also be seen in Figure 12d, which displays a stark reduction in the number of retirees after the imposition of the minimum retirement age. Indeed, 12.8% of the population between 51 and 64 years old would be retired in the 2060 economy without the reforms. After them, by the force of the new law, this figure falls to zero.

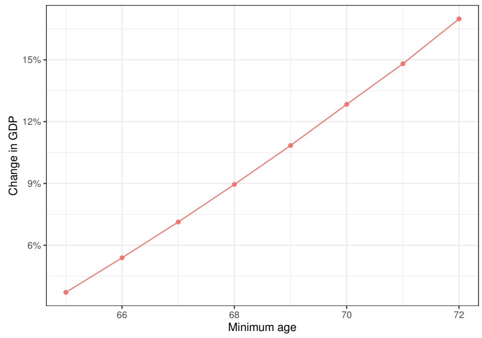
7.4 The zero deficit policy

Demographic changes will considerably affect the social security deficit, and the 2013 and 2019 reforms, according to our simulations, will not be able to fully offset these demographic pressures. In this section, we simulate the 2060 economy for different minimum retirement ages to find the one that eliminates the deficit in the long run and assess its general equilibrium consequences.

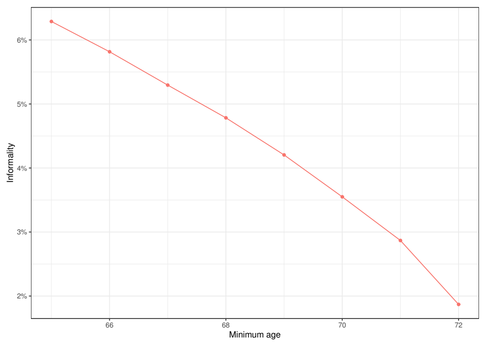
Figure 13: Minimum retirement age: The responses of deficit, GDP and informality



(a) SS deficit



(b) GDP



(c) Informal sector

Notes: This figure shows, for each minimum retirement age on the horizontal axis, the Social Security deficit (top left), the percentage change in GDP comparative to 2060 demography without reform (top right), and the informality (bottom).

Figure 13 presents the results for the joint reform discussed in the last section, but changing the minimum retirement age from 65 to 72. Figure 13a shows that a minimum retirement age higher than 71 would be necessary to eliminate the social security deficit in Brazil by 2060. This finding is important for many countries striving to balance their own social security deficits. For instance, in 2020, men who entered the labor market at age 22 had an average retirement age of 64.2 years across OECD countries. Countries such as Iceland, Norway, and Israel had a higher normal retirement age of 67 (Publishing, 2021). In France, it was still 62 by 2022, but changed to (only) 64 in 2023. According to our model, a minimum age requirement above these figures would probably be necessary to balance the social security system's revenues and expenditures.

Figures 13b and 13c display the effects of different retirement ages on GDP and informality, respectively. GDP increases from 3.7% to 17% as the minimum retirement age varies from 65 to 72 years of age. As people work longer, they contribute more to the economy through taxes and production, increasing government revenues, reducing retirement expenses and the equilibrium consumption tax. Additionally, as the workforce ages, a larger pool of experienced and more productive workers stays longer in the labor market, increasing output. Moreover, they save for retirement over a longer time period, increasing aggregate savings and capital. The increase in capital and reduction of interest rates, in turn, boost the demand of formal firms for workers, raising the formal wage rate, and reducing the size of the informal sector from 6.3% to 1.9%.

7.5 Welfare

We now analyze the long-run welfare consequences of the social security reforms. We measure welfare as the proportional change in consumption for all economic agents, Δ_w, that equalizes the average utility between the steady-state equilibria. To ease notation, let the prime superscripts denote equilibrium variables in the steady states where reforms are implemented, and variables without the prime superscripts denote the steady state without reforms. The change in welfare is given by:

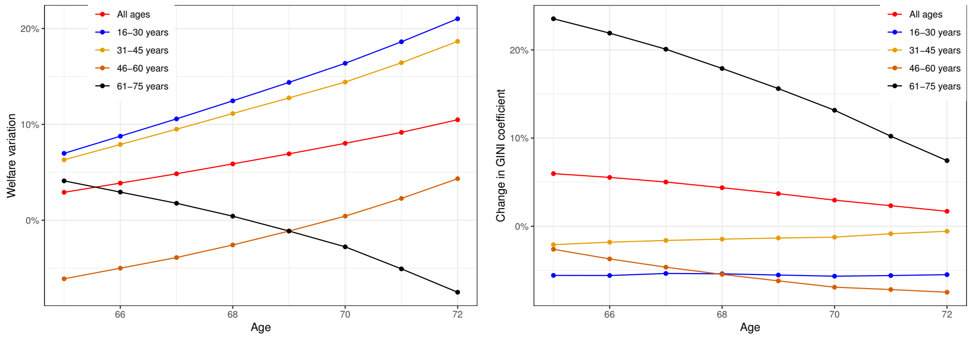
$$\Delta_w \in \mathbb{R}: \quad \sum_{\tilde{s}} \mu(\tilde{s}) u(d_{c,t}(s), d_{l,t}(s), Y_G) = \sum_{\tilde{s}} \mu'(\tilde{s}) u((1-\Delta_w) d'_{c,t}(s), d'_{l,t}(s), Y_G)$$

Figure 14a shows the welfare changes between equilibria with different social security reforms relative to the equilibrium where the Brazilian population has grown older, but the government did not adjust the social security rules. The long-run welfare could increase by 2.9% with the current reform and by 10.5% if the retirement age were 72.

However, the welfare consequences of the reforms differ by age group. Figure 14a, shows that younger cohorts will benefit from these reforms independent of the minimum retirement age. In contrast, in some cases the population groups aged 45-60 and 61-75 can be worse off. The 61-75 age group is the one directly affected by the retirement reform. Thus, most of the negative effects of higher minimum retirement ages are concentrated in the elderly population. At the retirement age of 69, the welfare of the elderly starts to be lower than the case without reforms. For the 45-60 age group, the slope with respect to the minimum age is positive because the negative effect of increasing the retirement age will not affect this group anymore, meaning that only the reform's positive side counts to this group. However, implementing the policy of a minimum retirement age of 65 (from the case without policy) can have a significant negative impact on this group as 7.9% of the population was retiring between 52 and 60 years old.

Figure 14b presents the changes in the GINI coefficient between scenarios. Even though

Figure 14: Minimum retirement age: The responses of welfare and GINI



(a) Welfare changes (b) Changes in the GINI coefficient

Notes: This figure shows, for each minimum retirement age on the horizontal axis, the changes in welfare (left) and in the GINI coefficient (right) relative to the 2060 demography model.

total inequality increases for all minimum retirement ages, zooming into specific age groups uncover important heterogeneity. The older group is the one pushing inequality up, while the younger ones are pushing income inequality down. The explanation for the young cohort is that the ratio of consumption taxes to total income is high for the poorest household. As the consumption tax is equal to all consumers, a tax reduction can mitigate income inequality due to this proportional effect. Furthermore, the transfers are higher, which also helps the poorer. With respect to older people, the increase in retirement age forces the most productive workers to return to the labor market. Indeed, the participation rate increased for them, and those with higher productivity will now earn a higher income, increasing inequality.

8 Conclusion

Population aging poses a severe strain on the solvency of social security systems worldwide. The problem is expected to worsen in the future as the ratio of retired individuals to active workers continues to increase. This issue is compounded by overly generous retirement conditions, particularly within the public sector, resulting in financially unsustainable pension systems that are likely to consume an ever-increasing portion of tax revenues.

We modelled Brazil and its social security system in detail, a country similar to many others. We simulated the impact of the 2060 demography on the economy. We found that, if nothing is changed, population aging alone will increase the social security deficit from 2.1% of GDP in 2013 to 16.6% of GDP. Given the necessary increase in taxation to pay for the pension deficit, the estimated impact on the economy will be sizeable, as measured for instance by the reduction in household consumption. Hence, the first lesson is that inertia and inaction will have very high costs in the future.

We examined pension reforms similar to those implemented or discussed in many countries.

Specifically, we studied the impact of unifying social security systems and imposing a minimum retirement age of 65. These reforms substantially reduce the pension deficit over GDP. The largest impact comes from increasing the minimum retirement age, as a large proportion of workers in Brazil retire before 60. Eliminating early retirement increases the contributions to the system, reduces the average number of years a retiree receives benefits, and encourages productive agents to stay in the labor market for longer. Moreover, we find that the minimum retirement age that would balance the Brazilian pension system is substantially higher than the one implemented in the 2013 reforms. This is an issue that will also have an impact on different economies in the near future.

Social security reforms also affect sectoral choices. We find that these reforms anticipate the decision of households to apply for a public job, and reduce the size of the informal sector. At the same time, our simulations find that savings, capital and output increase in the long run, as households work longer and increase their savings to compensate for reduced pension benefits. Average welfare also increases, although some age groups (particularly the older population) are worse off when reforms are implemented.

The implementation of pension reforms will be challenging, if not impossible in the short term, due to political resistance from various groups of workers. This is because many people will experience losses during the transition to the new regime. Nevertheless, the figures presented in this paper demonstrate that the benefits of these reforms are significant, making them worth pursuing despite the opposition from interest groups.

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Appendix A Agents' Stationary Distribution

In this section, we formally derive the stationary distribution for the agents in the economy. Given the assumptions on the entrants detailed in the main text, we have:

$$\mu_P(s_P) = \begin{cases} \varphi_1 \bar{\Gamma}(z), \textit{ if } s_p = (1,0,z,0,0) \\ 0, \textit{ for all other } s_p = (1,a,z,x,t_C)\} \end{cases}$$

and $\mu_m(s_m) = 0$ for all other sectors $m = \{R, G\}$ at $j = 1$, where $\bar{\Gamma}$ represents the stationary distribution for the Markov process $\{z\}$. We now detail the construction of the measures of agents in all sectors when $j > 1$.

The measure of each private agent in the economy, (j, a', z', x', t'_C), can be written as the sum of two terms. The first one considers the mass of all agents that were in the private sector and voluntarily remained there. The second term takes into account the private workers that applied to the public sector and failed to get in. Both the terms take into account the measure $\mu_P(j-1, a, z, x, t_C)$. Within each term, we consider the transition probability of the idiosyncratic productivity, $\Pi(z, z')$, the optimal amount saved by the agents, $d^a(s)$, the updated average past earnings and updated time of contribution, conditional on optimal labor and informality choices, $\tilde{x}'$ and $\tilde{t}'_c$, retirement decisions $d^{ss}(s)$, the public sector application decision, $d^{app}(s)$ and the probability of succeeding in the public sector exam.

The distribution for the formal private sector workers across ages $j = \{2, ..., J\}$ is recursively given by:

$$\mu_P(j, a', z', x', t'_C) = \frac{\psi_{j-1}}{1+g_n} \cdot \Bigg\{ \sum_{s_P|j-1} \Pi(z, z') \mathbb{I}_{\{d^a(s_P) = a'\}} \mathbb{I}_{\{\tilde{x}' = x'\}} \mathbb{I}_{\{\tilde{t}'_C = t'_C\}} (1 - d^{app}(s_P))(1 - d^{ss}(s_P)) \cdot \mu_P(s_P) + \sum_{s_P|j-1} \Pi(z, z') \mathbb{I}_{\{d^a(s_P) = a'\}} \mathbb{I}_{\{\tilde{x}' = x'\}} \mathbb{I}_{\{\tilde{t}'_C = t'_C\}} d^{app}(s_P)(1 - P_{app}) \cdot \mu_P(s_P) \Bigg\}$$

where the updated average past earnings and length of contribution to the social security system can be written as a function of the optimal labor decision $d^h(s_P)$, informality decision $d^{inf}(s_P)$ and application to the public sector decision, $d^{app}(s_P)$:

$$(\tilde{x}', \tilde{t}'_C) = \begin{cases} \left(\frac{x(t_C - 2) + min\{y_{j-1}(w_f, P), y_{\max}\}}{t_C - 1}, t_C + 1 \right) & \text{if works formally} \\ (x, t_C) & \text{otherwise} \end{cases}$$

The distribution of the public servants has two components. The first one takes into account the private workers who took the test and succeeded. The second considers the decision of public workers who did not ask for retirement:

$$\mu_G(j,a',z',t'_C,t'_G) = \frac{\psi_{j-1}}{1+g_n} \cdot \Bigg\{ \sum_{s_P|j-1} \mathbb{I}_{\{z=z'\}} \mathbb{I}_{\{d^a(s_P)=a'\}} \mathbb{I}_{\{\tilde{x}'=x'\}} \mathbb{I}_{\{\tilde{t}'_C=t'_C\}} d^{app}(s_P) P_{app} \cdot \mu_P(s_P) + \sum_{s_G|j-1} \mathbb{I}_{\{z=z'\}} \mathbb{I}_{\{d^a(s_G)=a'\}} \mathbb{I}_{\{t_C+1=t'_C\}} \mathbb{I}_{\{t_G+1=t'_G\}} (1-d^{ss}(s_G)) \cdot \mu_G(s_G) \Bigg\}$$

where $\tilde{x}'$ and $\tilde{t}'_C$ have the same definition as before.

The distribution of the retirees is also composed by two parts. First, we have agents who already were retired from the labor force. Second, we account for the ones who recently asked for retirement:[34]

$$\mu_{RP}(j,a',z',b') = \frac{\psi_{j-1}}{1+g_n} \cdot \Bigg\{ \sum_{s_R|j-1} \Pi(z,z') \mathbb{I}_{\{d^a(s_R)=a'\}} \mathbb{I}_{\{b'=b\}} \cdot \mu_{RP}(s_R) + \sum_{s_P|j-1} \alpha_b \cdot \Pi(z,z') \mathbb{I}_{\{d^a(s_P)=a'\}} d^{ss}(s_P) \cdot \mu_P(s_P) \Bigg\}$$

where α_b is the interpolation coefficient of the function $b(\cdot)$ on the grid $\mathscr{B}$. A similar equation applies to compute the distribution of the retirees in the public sector, therefore we will omit it, for brevity.

Appendix B Defining the Stationary Competitive Equilibrium

In this Appendix, we define the recursive, stationary equilibrium of the model.

A *recursive competitive equilibrium* consists of value functions $V : S_m \to \mathbb{R}$ for all $m \in \{P,G,R\}$, policy functions: (i) $d^h : S_m \to \{0,H\}$, for the optimal labor decision for all $m \in \{P,G,R\}$; (ii) asset holdings $d^a : S_m \to \mathbb{R}_+$ for all $m \in \{P,G,R\}$; (iii) consumption $d^c : S_m \to \mathbb{R}_{++}$ for all $m \in \{P,G,R\}$; (iv) $d^{ss} : S_m \to \{0,1\}$ retirement decisions for $m \in \{P,G\}$; (v) $d^{inf} : S_m \to \{0,1\}$, $m \in \{P,R\}$ for the optimal decision of working for the informal sector; (vi) $d^{app} : S_P \to \{0,1\}$ for the optimal decision of application to the public sector; competitive prices $\{r, w_f, w_i\}$, age dependent but time invariant measures of agents $\mu_m(s_m)$, government transfers ζ, taxes, an amount of public good, Y_G, and a threshold score $\bar{q}$ such that:

(1) The value and policy functions solve the problem of the agents;

(2) Formal private firms maximize profits given $\{r, w_f\}$ and informal wages are given by $w_i = A_i$;

[34] When solving the model, we divide the retirees between private (*RP*) and public (*RG*) retirees. The stationary distributions of the retirees are similar, therefore we will only derive for *RP*.

(3) The individual and aggregate behavior are consistent:

$$K' = \sum_{m \in \{P,G,R\}} \sum_{s_m} d^a(s_m)\mu_m(s_m)$$

$$K = \frac{K'}{1+g_n}$$

$$N_f = \sum_{m \in \{P,R\}} \sum_{s_m} e^{z+\eta_j}(1-d^{inf}(s_m))d^h(s_m)\mu_m(s_m)$$

$$N_i = \sum_{m \in \{P,R\}} \sum_{s_m} e^{z+\eta_j} d^{inf}(s_m)d^h(s_m)\mu_m(s_m)$$

(4) The government chooses $\bar{q}$ in order to balance people coming in and out:

$$\bar{N}_G = \sum_{s_G} \mu_G(s_G)$$

(5) Public goods' consistency:

$$Y_G = \sum_{s_G} \mu_G(s_G)e^{z_G+\eta_j} \cdot H$$

(6) Final good market clears:

$$\sum_{m \in \{P,G,R\}} \sum_{s_m} d^c(s_m)\mu_m(s_m) + K' + C_g = Y_f + Y_i + (1-\delta)K$$

(7) τ_C balances the government budget constraint:

$$\sum_{m,s_m} \tau_C \cdot d^c(s_m)\mu_s(s_m) + \sum_{m,s_m} \tau_K \cdot d^a(s_m)\mu_s(s_m) + (\tau_y(P)+\tau_w)w_f N_f +$$
$$\tau_{ss}(P)\sum_{s_P}(1-d^{inf}(s_P))\min\{w_f e^{z+\eta_j} \cdot d^h(s_P), y_{max}\}\mu_P(s_P) =$$
$$\sum_{s_G}(1-\tau_y(G)-\tau_{ss}(G))(1+\theta)w_f e^{z_G+\eta_j} \cdot H\mu_G(s_G) + \sum_{s_R}(1-\tau_b)b \cdot \mu_R(s_R) + C_g$$

(8) Bequests are rebated to the living ones:

$$\zeta = \frac{1+r}{1+g_n}\sum_{m,s_m} d^a(s_m)(1-\psi_j)\mu_m(s_m)$$

Appendix C Computing the Stationary Competitive Equilibrium

In this appendix we detail the computational methods used to quantitatively assess the macroeconomic consequences of social security reforms.

We numerically solved the model in Fortran 90. In order to do so, we discretized the asset space, the average past earnings space, the income process space and the social security benefits space. We did so in 52, 10, 5 and 42 points, respectively.

The grid on assets goes from $R\$0$ to $R\$3,790,202$, with its points concentrated over the lower bound.[35] The grid for x is equally spaced between 0 and y_{max}. The grid for b linearly spaced between[36] $R\$0$ and $R\$100,000$. The algorithm to find the general equilibrium was an adaptation from the algorithm that is commonly used in the literature,[37] including a fixed point over $\bar{q}$ to match $\bar{N_G}$. The steps used to compute the stationary equilibrium are:

1. Guess initial values for $\Theta \equiv (r, Y_G, \zeta, \tau_c, \bar{q})$;
2. Use the formal firm first order conditions to obtain w_f ;
3. Solve the agents' problems backwards and find the respective policy functions;
4. Use the policy functions to compute the associated stationary distribution of households by forward induction;
5. Aggregate the individual decisions and find $\tilde{\bar{q}}$ such that $\bar{N}_G$ of the population is working as public servants;
6. Use individual decisions to calculate the implicit remaining variables $\tilde{\Theta} \equiv (\tilde{r}, \tilde{Y}_G, \tilde{\zeta}, \tilde{\tau}_c)$;
7. Check whether $||\tilde{\Theta} - \Theta|| < \varepsilon$. If not, update Θ, return to item 2 and iterate until convergence.

[35] We chose the upper bound as the smallest value such that no agent in any state chooses optimally this upper bound. The concentration in the grid follows a cubic polynomial.

[36] We chose a value for the upper bound large enough such that more than 99% of the benefits distributed in equilibrium are lower than that value.

[37] See Chen (2010) for an example.