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Experimental study of spatial and temporal coherence in a laser diode with optical feedback

MARÍA DUQUE GIJÓN,[1] CRISTINA MASOLLER,[1,*] AND JORDI TIANA-ALSINA[2]

[1] *Departament de Fisica, Universitat Politecnica de Catalunya, Rambla Sant Nebridi 22, 08222 Terrassa, Barcelona, Spain*
[2] *Department de Física Aplicada, Facultat de Fisica, Universitat de Barcelona, Marti i Franques 1, 08028 Barcelona, Spain*
[*] *cristina.masoller@upc.edu*

Abstract: Optical feedback can reduce the linewidth of a semiconductor laser by several orders of magnitude, but it can also cause line broadening. Although these effects on the temporal coherence of the laser are well known, a good understanding of the effects of feedback on the spatial coherence is still lacking. Here we present an experimental technique that allows discriminating the effects of feedback on temporal and spatial coherence of the laser beam. We analyze the output of a commercial edge-emitting laser diode, comparing the contrast of speckle images recorded using a multimode (MM) or single mode (SM) fiber and an optical diffuser, and also, comparing the optical spectra at the end of the MM or SM fiber. Optical spectra reveal feedback-induced line broadening, while speckle analyses reveal reduced spatial coherence due to feedback-excited spatial modes. These modes reduce the speckle contrast (SC) up to 50% when speckle images are recorded using the MM fiber, but do not affect the SC when the images are recorded using the SM fiber and diffuser, because the spatial modes that are excited by the feedback are filtered out by the SM fiber. This technique is generic and can be used to discriminate spatial and temporal coherence of other types of lasers and under other operating conditions that can induce a chaotic output.

1. Introduction

In semiconductor lasers, optical feedback either reduces or increases the linewidth, depending on the laser and feedback parameters [1,2]. Optical feedback can reduce the width of the laser line, but it can also induce emission in a new set of temporal modes (so-called external cavity modes [3,4]) and can also cause emission of a chaotic broadband output (so-called coherence collapse regime [5]).

These effects of optical feedback on the temporal coherence of the laser have been studied in different types of lasers [6–17] and have found a number of applications, such as self-mixing interferometry [18–21], random-number generation [22–24], photonic microwave generation [25,26] and neuromorphic information processing [27]. However, less attention has been paid to the effects of feedback on spatial coherence. In broad-area lasers, filamentation due to the excitation of spatial modes can be either enhanced or suppressed, depending on the feedback conditions [28–31].

The analysis of speckle patterns allows to quantify the degree of coherence of a laser beam. Speckles are intensity variations formed by coherent waves that interfere with each other, whose phase differences are random [32]. Optical feedback affects the coherence of the laser light, and thus, affects the speckle contrast [33]. Using the speckle technique, we recently studied how coherence emerges at threshold, during the laser turn-on, and found, under strong feedback, a sharp increase of the speckle contrast that indicates that the coherence increases abruptly. In contrast, in the solitary laser (or when the laser is under weak feedback) we found a gradual

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increase of the speckle contrast, which reveals a gradual increase of the coherence of the laser light [34]. In [35], we have also found that well above the threshold, strong optical feedback can reduce the amount of speckle by more than 50% over speckle generated by the solitary laser. However, we do not know the mechanisms by which feedback decreases the amount of speckle, either due to spectral broadening (i.e., feedback-induced excitation of temporal modes) and/or due to spatial instabilities (i.e., feedback-excited spatial modes). The different spatial modes excited in the laser cavity will also have different frequencies as it is unlikely that nearly degenerate modes are excited. On the other hand, in [34] the medium used to generate speckle was a multimode (MM) fiber, and we do not know how the medium used affects the speckle measurements.

Therefore, to clarify these points, here we compare the contrast of speckles generated by the solitary laser and by the laser with feedback, using three configurations (see Fig. 1): i) a MM fiber, ii) a MM fiber and a diffuser, and iii) a single mode (SM) fiber and a diffuser. In the first case, the speckles are generated by multiple reflections inside the fiber; in the second and third cases, also by the scattering produced by the diffuser. As spectral and speckle measurements do not provide independent, but complementary information about the spatial and temporal coherence of light, to complement the speckle information, we also perform measurements of the optical spectrum.

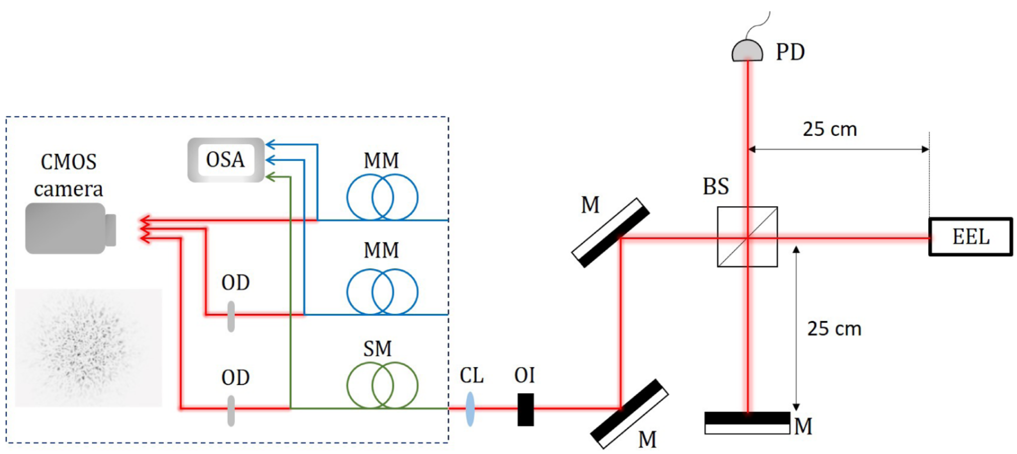

Fig. 1. Experimental setup. EEL: Edge-emitting laser, BS: Beam Splitter, M: Mirror, OI: Optical Isolator, CL: Collimation Lens, MM: Multi-Mode fiber, SM: Single-Mode fiber, OD: Optical Diffuser, OSA: Optical Spectrum Analyser, CMOS camera, PD: 2 GHz photodetector. The rectangle identifies the three configurations analyzed: MM fiber, MM and optical diffuser, and SM fiber and optical diffuser. The inset shows an example of a speckle image.

Well above the threshold and under strong feedback we find, as expected, that the optical spectrum measured after propagation in the MM or SM fiber is broad, confirming the feedback-induced decrease of temporal coherence of the laser. The analysis of the speckle contrast (SC) reveals, with the MM fiber, reduced coherence, but when the SC is measured after the light propagates in the SM fiber, the SC remains high. This difference indicates that optical feedback excites not only temporal but also spatial modes that can propagate in the MM fiber but which are filtered by the SM fiber. Therefore, feedback-excited spatial modes lower the speckle contrast in images recorded with the MM fiber, but do not affect the amount of speckle in images recorded with the SM fiber.

2. Experimental setup

The experimental setup is shown in Fig. 1. The output of an edge-emitting laser (AlGaInP MQW - Thorlabs HL6501MG), which emits at 653 nm when T = 18 °C with threshold 41.95 mA and whose pump current and temperature where controlled with 0.01 C and 0.01 mA accuracy (with Thorlabs ITC502 controller), is divided by a 90/10 beam splitter: 90% goes to a mirror that provides feedback (the external cavity length is 50 cm and the threshold reduction, ~15%), while the other 10% is used for analysis and is guided by two mirrors to an optical fiber that is either a MM fiber (Thorlabs M72L02, 0.39 NA, Φ_{core} = 200 µm, a step-index MM fiber) or a SM fiber (Thorlabs P1-630AR-2, 0.12 NA, Φ_{core} = 5 µm, AR coated for 500-800 nm). When we analyze the output of the solitary laser we block the path to the mirror with a dark, light-absorbing material.

The light is collimated to the fiber by adjustable collimators (for MM fiber, Thorlabs C340TMD-B with focal length f = 4 mm; for SM fiber, Thorlabs CFC-11X-B with variable focal length). Before the SM or MM fiber, an optical isolator (OI) is placed in order to avoid unwanted reflections.

At the end of the SM or MM fiber the optical spectrum or the speckle image is recorded, using an Optical Spectrum Analyzer (OSA, Anritsu MS9710B) or a 8-bit CMOS camera (IDS UI-1222LE-M, pixel size 5.3 µm with a resolution of 1280 × 1024 (h × v) pixels).

When performing speckle measurements, between the end of the fiber and the camera, an optical diffuser (Thorlabs Ground Glass Diffuser DG10-120, Φ = 25.4 mm and width = 2 mm) is placed or removed, in order to record speckle images after propagation in the fiber, or after propagation in the fiber and scattering in the diffuser.

A LabVIEW control routine was used to vary the laser current, set the optimal exposure time and acquire the experimental data; the data (optical spectra and speckle images) were processed and analyzed with MATLAB. For each pump current value, the camera exposure time was automatically adjusted to the light intensity (controlled by the pump current) as explained in [34]. Then, 8 speckle images were recorded. For each image, the speckle contrast was calculated, in a circle of radius 200 pixels in the center of the image, as the ratio between the standard deviation of the values of the pixels in the center of the image, and the average value, $SC = \sigma/\langle I \rangle$. The SC values presented in the next section are the average of the 8 SC values. After the acquisition of the 8 images, the process was repeated: the pump current was increased in a 0.5 mA step, and after ~30 seconds to let transients die away, the camera exposure time was adjusted to the new illumination conditions, and another set of 8 speckle images was recorded.

3. Results

Figure 2 shows the optical spectra (in color code) and the speckle contrast (in white) vs. the laser current, for the three configurations analyzed: a) MM fiber, b) MM fiber and diffuser and c) SM fiber with diffuser. Figures 2(a)-(c) present measurements for the solitary laser, and Figs. 2(d)-(f), for the laser with optical feedback. In the experiments with the MM fiber, the threshold reduction due to feedback is 15.22%, in the experiments with the MM fiber and diffuser, 15.02%, and in the experiments with the SM fiber and the diffuser, 15.30%. The spectra shown in panels (a) and (b), and in (d) and (e), are the same because the OSA is at the end of the MM or SM fiber (see Fig. 1).

For the solitary laser, in Figs. 2(a)-(c) we see that the spectrum measured at the end of the MM fiber and at the end of the SM fiber are very similar, and also, we see that the variation of the speckle contrast with the laser current is similar in the three configurations (the gradual SC increase reveals a gradual increase of the coherence of the light).

Nevertheless, when the MM fiber is used, with or without diffuser (Figs. 2(a,b)), in the increase of the speckle contrast there are some fluctuations that correlate well with changes in the spectrum, where mode switching and a red-shift is observed (well known thermal-induced phenomena).

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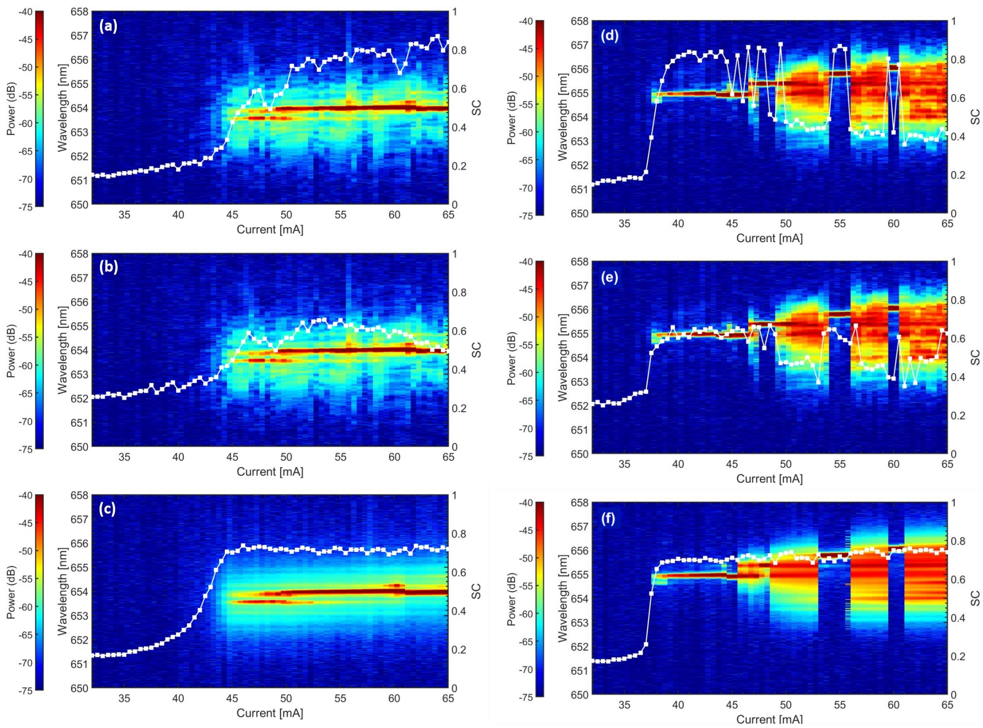

Fig. 2. Speckle contrast (white) and optical spectra (color code) recorded with the solitary laser by using (a) a MM fiber, (b) a MM fiber and a diffuser, (c) a SM fiber and a diffuser, and with the laser with optical feedback using (d) a MM fiber, (e) a MM fiber and a diffuser and (f) a SM fiber and a diffuser.

When the MM fiber and the optical diffuser are used (Fig. 2(b)), the SC values are, below the threshold, slightly higher, and above the threshold, slightly lower with respect to those obtained when using only the MM fiber to generate speckle (Fig. 2(a)). On the other hand, when the SM fiber and the diffuser are used to generate speckle (Fig. 2(c)), the variation of the SC with the pump current is less noisy, and after the laser turns on the speckle contrast remains nearly constant, a fact that can be understood as due to the spatial filtering effect of the SM fiber.

When the laser is subject to optical feedback (Figs. 2(d)-(f)), instead of a gradual increase of the SC at threshold, we observe a sharp increase (as we have recently reported [34]). The optical spectra measured at the end of the MM or SM fiber are again very similar. We observe just above threshold mono-mode behavior (which is consistent with high SC values), but as the pump current increases the spectrum broadens (except in some narrow intervals of the pump current, to be discussed later). The broadening of the spectrum reveals the well-known onset of feedback-induced temporal instabilities. It is interesting to note that the laser emits a single mode up to the solitary threshold. A similar behaviour was seen in [34]. To the best of our knowledge, this has not been previously reported and merits further theoretical and experimental studies.

The variation of the speckle contrast with the pump current, after showing an abrupt increase at threshold, differs in the three configurations. When the MM fiber is used to generate speckle, Fig. 2(d), the SC tends to decrease with the pump current (except for some values of the pump current). Comparing Figs. 2(a) and 2(d) we see that for large pump current the SC decreases from 0.8 (solitary laser) to 0.4 (laser with feedback). This is consistent with the feedback-induced SC reduction reported in [35].

When the MM fiber and the diffuser are used to generate speckle, we see in Fig. 2(e) that the diffuser either decreases or increases the SC, with respect to the SC value without diffuser (shown in Fig. 2(d)): when, without diffuser the SC is high, the diffuser decreases the SC; however, when without diffuser the SC is low, the diffuser does not produce a further reduction of the speckle, but it leaves the SC unaffected, or it even increases slightly the SC (a fact that can be understood as due to additional interference produced by the diffuser).

As expected, in Figs. 2(d),(e), low SC values are in general correlated with broad spectra, and vice versa, high SC values are correlated with narrow spectra. However, for some pump currents (i.e. 60 mA in Fig. 2(a)), this correlation does not hold, and high (low) SC values correspond to broad (narrow) spectra.

To gain insight into these observations, we performed a second set of experiments where we modified the MM fiber setup, placing a 2 GHz photodetector (Thorlabs Det10A/M Silica based photodetector) and a 1 GHz digital oscilloscope (Agilent Technologies Infiniium DSO9104A) in the available port of the first beam splitter. The setup allowed us to inspected the time traces of the laser intensity, Fig. 3(a), and the corresponding speckle images, Fig. 3(b),(c), and optical spectra, Fig. 3(d); however, it do not allow simultaneous spectral and speckle measurements.

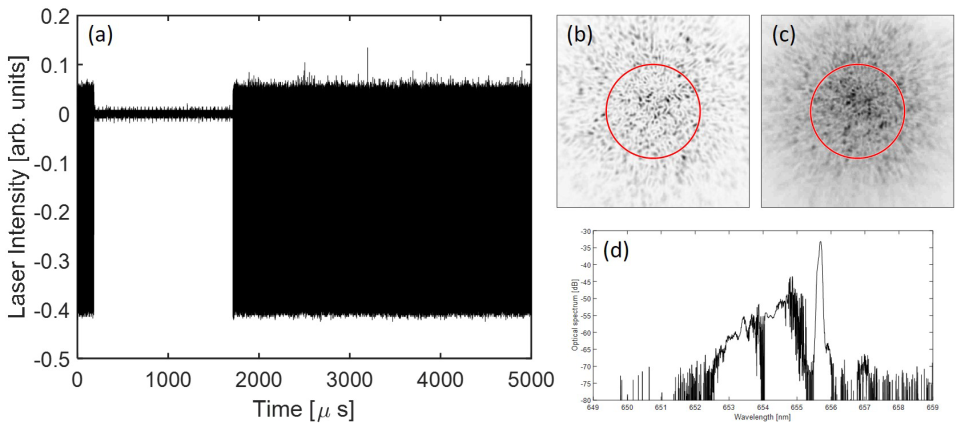

Fig. 3. (a) Time series of the laser intensity when stable emission alternates with low-frequency fluctuations. (b), (c) Speckle images recorded during stable emission and low-frequency fluctuations, respectively. (d) Optical spectrum.

We found, for similar experimental conditions as in Fig. 2(d) (15% threshold reduction and 55 mA), that the laser switches between two regimes: one in which the intensity is stable and displays only small fluctuations (this regime corresponds with high SC values, Fig. 3(b)), and another regime in which the intensity shows large spikes (known as low-frequency fluctuations, LFFs [2]), that presents low SC values, Fig. 3(c). The long time-scale needed for recording the optical spectrum (several seconds) makes both regimes visible: in the spectrum, Fig. 3(d), we see a broad spectrum and a narrow peak. Complementary to these measurements, we recorded a video of speckle images that reveals the alternation between two regimes (the video is available in [36]). Over time, the residence times in each regime show large variability: long intervals in the LFF regime may alternate with short or long intervals of stable emission, and vice versa. When the laser emits stable emission the images recorded at the output of the MM fiber show high SC, while when the laser displays LFFs emission, low SC is observed. Therefore, the laser may operate in one regime when the spectrum is recorded, and in the other regime when the speckle images are recorded. This switching may be the reason why, in Figs. 2(d), (e), we see that for some pump currents the SC is low (high) but the spectrum is narrow (broad).

When the SM fiber and the diffuser are used to generate speckle, the SC remains high after the laser turns on (Fig. 2(f)). This is interpreted as due to the filtering of the spatial modes done by the SM fiber: as explained before, optical feedback destabilizes spatial modes, and these modes lowered the SC value when the MM fiber was used (Figs. 2(d),(e)); however, they do not affect the SC when the SM fiber is used, because they are filtered by the fiber. Optical feedback also excites temporal modes, as revealed by broad spectra. While the excitation of temporal modes can also reduce the amount of speckle, this depends on the exposure time used to record the speckle images [37], and with the long exposure time of our CMOS camera the reduction of the speckle contrast produced by feedback-excited temporal modes can not be detected. As an example, for a pump current of 63 mA, the spectral width is about 0.2 nm (80 GHz), and the camera exposure time, about 2 μs.

4. Conclusions

We have studied the effects of optical feedback on the spatial and temporal coherence of the light emitted by a commercial edge-emitting laser diode. The optical spectra recorded after propagation in the MM fiber or SM fiber reveal the well-known feedback-induced regimes of operation (single-mode emission, mode-hopping, and broad-band emission). By comparing the contrast of speckle images recorded with or without feedback, using the MM fiber or the SM fiber and a diffuser, we have found that well above the threshold, in the images recorded with the MM fiber the speckle contrast decreases by about 50%, while it remains unchanged in the images recorded with the SM fiber. We interpret these observations as due to the fact that feedback excites temporal and spatial modes; several spatial modes can propagate in the MM fiber, and thus, decrease the amount of speckle in images recorded at the end of the MM fiber. In contrast, these modes are filtered by the SM fiber, and therefore, do not affect the speckle contrast. For this reason, this configuration (optical feedback and a SM fiber and a diffuser) can be attractive for applications that require laser light with high spatial coherence but low temporal coherence, and therefore, depending on the level of speckle suppression needed, optical feedback may be a cost-effective solution.

The technique proposed here can be used to discriminate spatial and temporal coherence of other types of lasers and under other operating conditions that can induce a chaotic output. Furthermore, since speckle is widely used for sensing applications, for future work it will be interesting to analyze the effect of optical feedback on the speckle contrast, when the laser beam propagates in free space instead of on a fiber.

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Data availability. The data is available from the corresponding author under reasonable request.

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