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Essays on Monetary Economics

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Josep Miquel Navarro Ortiz | PhD in Economics

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Acknowledgements

Writing a PhD dissertation has been a long and exhausting journey, full of excitements and disappointments. Some days it seems like you are almost there and a week latter it looks like you have just started. I entered this journey thinking I knew a lot, but I am coming out thinking I know nothing. Every day I am closer to understand the motivations of Socrates when he said: "all I know is that I know nothing".

In all, with its ups and downs, it has been an enriching experience. Writing a thesis have made me grow not only as an scholar but also as a person. But, like anything worth mention, it was not a task to carry alone. I want to thank to the people whom, directly or indirectly, accompanied me through this long journey. You have made me a better person and have made this burden doable.

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Chapter 1

Introduction

Making the right decisions when monetary policy is designed is of vital importance for the well being of an economy. The right policy may solve significant current account imbalances and inflationary shocks but the wrong one may cause them. This makes monetary policy one of the most powerful tools to manage the economy but also one of its most feared enemies.

There are numerous examples of monetary policy decisions which created a worsening in the economic conditions of the country implementing them. One of the most famous cases was the British return to the Gold standard with pre-war convertibility in 1925. This decision caused an appreciation of British currency, causing problems for its exporting industry to compete on the international markets. To put it in Sir John Maynard Keynes words:

> "The policy of improving the foreign-exchange value of sterling up to its pre-war value in gold from being about 10 per cent. below it, means that, whenever we sell anything abroad, either the foreign buyer has to pay 10 per cent. *more in his money* or we have to accept 10 per cent *less in our money*. That is to say, we have to reduce our sterling prices, for coal or iron or shipping freights or whatever it may be, by 10 per cent. in order to be on competitive level, unless prices rise elsewhere. Thus the policy of improving the exchange by 10 per cent. involves a reduction of 10 per cent. in the sterling receipts of our export industry" Keynes (1925)

The monetary policy showed in this chapter of history its destructive powers. After years of deflation and stagnation, Britain finally decided to abandon the Gold standard in 1931. Being aware of the power to create and destroy of monetary policy, it is specially important to improve our knowledge about which policies are the right ones in each circumstances and the consequences this might have for the economy. The research on this topic is then of vital importance not just for the future of Economics as a science but for the future of the human societies.

In this Thesis I want to put my two cents studying three of the most important problems for nowadays monetary policy: the problem of external debt accumulation, the stabilization problems confronted by central banks in monetary unions and the effects of unemployment on unanticipated inflation.

In the second Chapter of this Thesis I am studying the problem of external debt accumulation. An excessive external debt may become a burden for the economy, specially if the interest rate rises. If the country under distress has its own currency, it will devaluate it to incentive exports and reduce imports, improving with this the current account balance and its capability to pay back the debt. An example of this was the Iberoamerican external debt crisis when, as a consequence of the 1970s oil crisis, the US started increasing their interest rate to fight the rising inflation. This rise on interest rates produced a huge problem for the Iberoamerican economies as more and more resources were needed to make the interest payments of the US loans. This rise of interest rates drove countries like Mexico, Argentina or Brazil to default and made necessary the intervention of the International Monetary Fund to stabilize the region. But, if the country does not have its own currency, a devaluation is not possible, making it necessary for the country to carry on an internal devaluation to improve its capability to pay back the debt. This situation puts pressure to the stability of the currency as the country under distress has incentives to abandon the currency. An example of this happened in Greece in the middle of 2010s when, as a consequence of the needed measures to stabilize the economy during the Euro Area debt crisis, some sectors of the society started advocating for leaving the Euro and going back to the Dracma.

The examples of external debt problems are not scarce during the last century. These problems have carried on extreme devaluations in some countries and bankruptcy in others. The abundance of these cases and the high cost of its consequences for the economy creates the necessity of improving our knowledge about them. In the second chapter I study the likelihood of one of these events to happen in a concrete economy. I develop a probabilistic approach to measure a country's external debt sustainability. Using data on international investment position and balance of payments from the International Monetary Fund, I estimate a vector autoregressive model for 38 countries (11 developed and 27 developing). Using the estimated parameters, I perform a Monte Carlo simulation to compute the distribution of the capacity to repay the debt for each country. A large portion of the projected distribution to the right of current debt is a warning indicator, signalling potential problems. In the case of countries with its own currency, these problems will mean a devaluation of this currency. While, in the case of countries sharing a currency or using a foreign currency as their own, this accounts for the extra cost of maintaining said currency instead of having their own.

I found that, countries like the US, with high levels of external debt and a currency of their own, are bound to need a devaluation in the near future, specially if the interest rates keep rising. While, countries like Spain are going to have problems to maintain themselves inside of a monetary union.

I devote the third chapter of this thesis to study the stabilization problems confronted by central banks inside of a monetary union. I use a two-country New Keynesian model to show that a monetary union without fiscal integration, like the Euro Area, may not fulfil the Blanchard-Kahn conditions causing problems of multiplicity of equilibria. I use this model to show that the terms of trade must be taken into consideration for setting the monetary policy in a monetary union because they are part of the welfare loss of the union. If the terms of trade are not taken into consideration when estimating the monetary policy rule, this estimation will be biased because of the omission of a relevant variable. Finally, I use Bayesian inference to estimate the Taylor rule with and without the terms of trade for the Euro Area and the US, I have found that while the US monetary policy is just slightly affected or even not affected by the terms of trade, the monetary policy in the Euro Area is strongly disturbed by it.

This incapability is aggravated by the monetary policy objectives of certain central banks like the ECB, which do not have in their mandate the objective of reacting to output/unemployment gaps. This decision of certain central banks to center all their attention on achieving constant inflation, even at the expense of high unemployment, adds another layer to the problems confronted by the ECB to fight against the Euro Area imbalances. The decision of leaving output/unemployment gap aside was not just a combination of political interests but, the result of the state of research of the Phillips Curve. When the Euro was designed, the relationship between unanticipated inflation and unemployment gap was unclear, with contradictory results depending on the studied country and time frame.

I devote the fourth chapter of this thesis to the study of this relationship in the Euro Area (EA). The main object of study in the Phillips Curve literature has always been the US, a country where the fiscal and monetary policy are set at the same level of government guaranteeing their objectives are aligned. However, this is not always the case, in currency areas like the EA monetary and fiscal decisions are taken at different levels. Monetary policy is set at the area level whilst fiscal policy is set at the country level, which destroys any guarantee for their objectives to be aligned.

In Chapter 4 I am going to include this trade-off between national and supranational entities in the estimation of the EA Phillips Curve. With this objective, I am first going to compare two extreme cases: the one where the EA is assumed to have its unique Phillips Curve with no country effects and the one where each country in the EA is assumed to have its own curve with no group effect. I then compare both estimations to see if they are statistically the same, showing the existence of a unique Phillips Curve for all the EA. Once this is established, the next step is to choose, based on goodness of fit criteria, which of both approaches is better. However, one problem still unsolved, each approach leaves aside one level of variability; the country by country estimation leaves aside the group-level variability while the pooled approach leaves aside the group-level one. In each case one level of variability is disregarded. To include simultaneously both levels of variability in the regression I introduce a multi-level or hierarchical approach. With this method it is possible to compute at the same time a common curve for all the EA and a specific curve for all EA countries as a function of the EA curve.

Finally, in the last chapter of this Thesis I will summarize the results and expose the conclusions of my research.

Chapter 2

Is external debt sustainable? A probabilistic approach[1]

2.1 Introduction

Countries' external debt sustainability is a major concern in current times. Over the last forty years, external debt has grown steadily in both developing and, especially, developed countries.

[1]This chapter is co-authored with Juan Sapena Bolufer. In 2020 it was published as an article on Economic Modelling Volum 93, Issue C, pages 142-153.

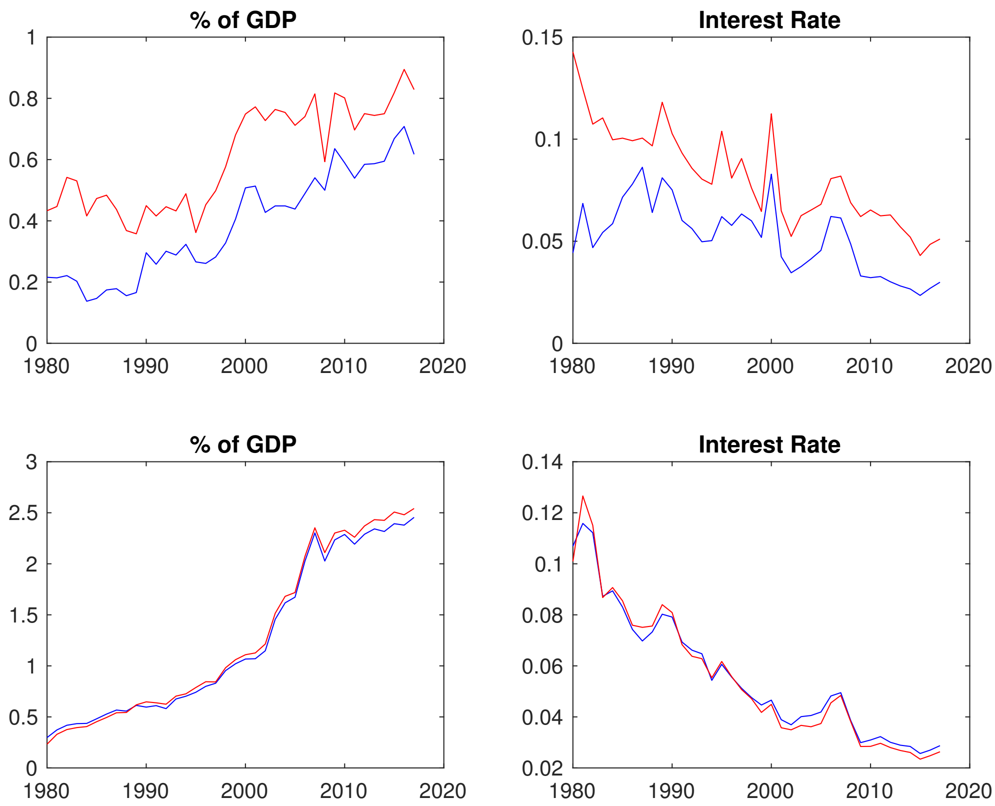

Figure 2.1: The red lines represent liabilities-to-GDP ratios and the interest rates on liabilities; the blue lines represent assets-to-GDP ratios and the interest rates on assets. The top plots refer to developing countries; the bottom plots refer to developed countries.

Figure 2.1 shows that both the external assets-to-GDP ratio and the external liabilities-to-GDP ratio for developing countries have doubled. Meanwhile, they have increased tenfold in developed countries. Although total external assets must equal total liabilities for the entire world economy (to the best of our knowledge, nobody owes money to Martians), the distribution of assets and liabilities across countries seems to be a concern.

There is also concern over the asymmetry of debt distribution. Some countries, such as Germany, are accumulating larger amounts of financial assets from foreign countries than assets issued by Germans to foreign investors. They are thereby taking a positive external position with respect to the rest of the world. On the contrary, others such as the United States or Spain are doing the opposite. The amount of foreign assets in national hands is notably lower than the amount of foreign assets belonging to nationals. Accordingly, these two countries are indebted to the rest of the world.

But the problem of external debt is not only debt size but also repayment. This debt burden results from the need to finance mounting current account deficits. The problem, then, lies in the difficulty in paying it back because the indebted country would require positive current account surpluses with creditor countries. In general, if an indebted country controls the issuing of its own currency, it could devalue its currency to recover price competitiveness and improve its current account status. Therefore, a negative external debt position may be an indicator of a need for currency devaluation.

However, as highlighted by Eichengreen & Hausmann (2005) and others in the related literature, the lack of monetary independence can become a problem. This problem is familiar to the member states of the European Economic and Monetary Union (EMU). These member states share a common currency, and monetary policy is managed at a supranational level by the European Central Bank (ECB). The situation becomes even more complex when, as in the case of the eurozone, the monetary union does not have a common fiscal policy such as the one existing in the United States at the Federal level. Accordingly, the external debt crisis has implications for the strength of the currency and the future of the currency union.

Under such circumstances, a highly indebted country cannot devalue the currency to restore equilibrium. Internal devaluation is the only measure that the government has at its disposal to reduce real costs. This measure means reducing nominal wages and suffering the public discontent that comes with it. The combination of public discontent and the difficulty in achieving a devaluation that redresses the balance in the country's national accounts poses a challenge for the survival of the common currency in the EMU; the public may start to wonder whether the cost of leaving the currency is lower than the price of staying. Therefore, high external indebtedness in a currency union signals wage adjustments and reductions of household consumption at the country level, together with legitimacy problems at the union level.

In this paper, we present a straightforward way to measure a country's external debt sustainability. Using a probabilistic approach, we compute the distribution of a country's capacity to repay its external debt and compare it with the current level of debt. From this comparison, we infer the potential repayment issues a country may have regarding its external debt. This framework may be useful for both policymakers and international investors. Using this approach, policymakers should be able to prevent excessive levels of external indebtedness by adopting the policies to return to an equilibrium before costs become unbearable. Meanwhile, international investors can use this approach to detect countries that may experience a future a devaluation or appreciation if debt is very low or if they have a positive position (negative debt).

For debt sustainability, one stability rule is that the interest rate is lower than the growth rate. However, there is a consensus in the literature that $r > g$ should generally prevail in the medium and long run and that the opposite could be associated with a dynamically inefficient allocation of resources. This fact is highlighted in the literature. We compare the projections of r – g and external debt for different economies. Our results suggest that a flight-to-safety effect is penalising some developing countries, whilst benefiting some industrialised economies.

The rest of the paper is organised as follows. In Section 2.2, we briefly review the literature on the topic. In Section 2.3, we derive the model and describe the variables of interest for the estimation. We also identify the difficulties associated with solving the problem of the Ponzi game condition and explain how to calculate the model without this condition. In Section 2.4, we explain the source and characteristics of the data employed to fit the model. In Section 2.5, we present the results of the estimation strategy that we use to solve the stationarity issues with the data. Finally, Section 2.6 concludes with a discussion of the main findings.

2.2 A brief review of the literature

Following the definition used by Roubini (2001), debt is sustainable if the borrower can service it now and in the future. In the case of a country's external debt, this definition is often stated as the fact that external debt is sustainable if net debt today is less than or equal to the present value of net exports. This definition offers the basis of the analysis presented in this paper.

The intertemporal approach to the current account was initially proposed by Sachs (1981) and Buiter (1981) and was extended by Obstfeld & Rogoff (1995). According to this approach, the notion of external solvency for a country is related to the long-term compliance of the intertemporal budget constraint. This constraint relies on a country's ability to generate sufficient trade surpluses in the future to repay existing debt, excluding potential discontinuities.

The concept of external solvency, which is derived from the intertemporal budget constraint, reflects the sustainability of current policies. As stated by Geithner (2002), when the expected value of the future resources devoted to debt service equals the current debt stock, the solvency condition is satisfied. Milesi-Ferretti & Razin (1996) distinguish between sustainability and solvency. For those authors, the notion of external solvency essentially refers to the ability to repay only. The willingness of debtors to do so is not considered, although it might not be politically feasible. Moreover, this definition of solvency also relies on foreign investors' willingness to lend to the country. They may not be willing to do so because of uncertainties about the country's ability to meet its debt obligations or because of a painful unexpected external shock. Indeed, changes in agents' perceptions of various factors may lead to variations or structural breaks that trigger dynamic adjustment to the current account equilibrium. Examples of such factors include risk, decisions about portfolio asset composition, economic policy variations, and changes in transaction costs in international financial markets.

Numerous empirical studies using time series analysis have addressed the issue of the sustainability of external imbalances. Unit root and cointegration techniques have provided insight into the solvency of government fiscal policy for a given time horizon. Such tests can be found in the literature on government solvency. The earliest contribution is that of Hamilton & Flavin (1986), which was subsequently developed by Trehan & Walsh (1991) and Wilcox (1989). This methodology has also been applied to evaluate the sustainability of external deficits and the problem of external solvency.

The traditional analysis of external sustainability, initiated by Trehan & Walsh (1991), focuses on the stationarity of the current account or external debt stock by applying unit root techniques (see Camarero et al. 2015, Holmes 2006 or Chen 2011). As Trehan & Walsh (1991) point out, the I(0) stationarity of the current account is a sufficient condition to ensure compliance with the intertemporal budget constraint (IBC). A sustainable current account is consistent with external debt sustainability and might suggest that a country has no incentive to default. Moreover, current account sustainability is also consistent with the intertemporal model of the current account. Methodologically, the issues considered would be analogous to the techniques used in the literature (See, for example, Barro 1979, Hamilton & Flavin 1986, Hakkio & Rush 1991, Trehan & Walsh 1991, and Wilcox 1989).

Some investigations have measured the dynamics of the adjustment process using flow data (e.g. Bussière et al. 2006 or Zanghieri 2004). However, a drawback of this approach is that it ignores changes in the valuation of foreign assets and liabilities. To solve this problem, scholars have started to employ stock variables instead of flows. Another advantage of stocks is that they are less volatile and can provide a better estimation of long-term relationships. Examples of this approach are provided by Lane & Milesi-Ferretti (2007) and Wickens & Uctum (1993). One notable approach is that of Gourinchas & Rey (2007), who used monthly data and an intertemporal budget constraint approach to assess U.S. external imbalances.

A second strand of the literature follows the approach pioneered by Bohn (1998) and adopted in subsequent studies. Bohn (2007) proposed the estimation of a linear reaction function for trade balance to measure external debt position. In this estimation, the reaction functions for external net foreign liabilities took the form of error correction models. Bajo-Rubio et al. (2014) or Durdu et al. (2013) applied this approach and obtained mixed results. Paniagua et al. (2017) estimated a time-varying fiscal reaction function, highlighting the possibility that non-linearities add complexities to the simple model.

If the past is completely ignored, then virtually any government could be deemed solvent, regardless of its debt level. All it takes is to consider as credible any public commitment to generate sufficiently high primary surpluses at some point in the future. Thus, giving an operational definition of debt sustainability instead of solvency means designing a procedure to adequately discipline and inform our judgment with "hard" data and "objective" criteria and indicators.

Recently, Borio & Disyatat (2016) questioned the appropriateness of popular analytical frameworks that use current accounts or net capital flows as a basis for assessing the pattern of cross-border capital flows, the degree of financial integration and the vulnerability of countries to financial crises. Borio & Disyatat (2011) criticised the view in Bernanke (2005). According to this view, large current account surpluses due to excess saving prompted the great financial crisis by loosening global financial conditions and fuelling credit booms in countries with current account deficits. In an interesting paper, Semmler & Tahri (2017) analysed the external debt for three euro area economies (Italy, Spain and Germany). They used debt over assets instead of the traditional debt-to-GDP ratio, finding an asymmetric evolution in these countries.

Recent research underscores the role of distrust and risk aversion, a lesson that has been learnt again after the global financial crisis. Lane & Milesi-Ferretti (2018) updated their data set on external assets and liabilities, showing how the share of domestic debt held by non-residents is negatively related to the size of the domestic debt market.

Monastiriotis & Tunali (2019) and Afonso et al. (2019) also focused on the issue of external sustainability for EMU countries.

Belke & Gros (2017) discussed the optimal external adjustment path for countries in a monetary union and the role of internal devaluation to rebalance external debt accumulation. From another perspective, Fatás et al. (2019) argued that political failures are a major cause of overborrowing, whilst showing how budgetary institutions and fiscal rules can help mitigate the tendency to borrow in excess.

Our paper draws upon the work of Blanchard & Das (2017), who offer a probabilistic definition of external debt sustainability as a high enough probability that net debt is less than or equal to the present value of net exports at the current exchange rate. Assibey-Yeboah et al. (2016) tested the current account sustainability hypothesis to show that positive inflation shocks, such as those due to the depreciation of the domestic currency, can lead to a decrease in the stock of real debt in domestic currency.

Applying this reasoning, we model the probability that external debt is repaid. To do so, we assess the sustainability of external imbalances in two samples, one of developed countries and another of developing countries. Nevertheless, we diverge from the solution offered by Blanchard & Das (2017) for situations characterised by "higher-than-one" discount rates. These rates occur for many of the developed countries because they are able to finance their debt at an interest rate that is lower than GDP growth. This issue of low interest rates is in fact the focus of an interesting discussion by Blanchard (2019).

2.3 The model

External debt is the sum of external liabilities owed by a country's residents to foreign creditors, after subtracting the value of external financial assets owned by domestic agents at a particular moment. This debt stock evolves over time as follows:

$$L_{t+1} - A_{t+1} = (1 + r_t)(L_t - A_t) - NX_t. \tag{2.1}$$

In Equation (2.1), A_t represents external assets in period t, whereas L_t is the value of external liabilities issued by residents of a country. The term r_t is the rate paid for assets and liabilities in period t. Finally, NX_t accounts for net exports. If external debt D_t is defined as $L_t - A_t$, (2.1) becomes:

$$D_{t+1} = (1 + r_t)D_t - NX_t. \tag{2.2}$$

The model in Equation (2.2) describes the behaviour of external debt, assuming that the rates of return on assets and liabilities are the same. In theory, therefore, we could just look at net debt. In practice, however, this is not necessarily the case. As noted by Gourinchas & Rey (2014), a sizeable difference between return of gross external assets over liabilities exists in most countries.

2.3.1 The model at different interest rates

The return on external liabilities essentially depends on borrowers' indebtedness and capacity to repay, as well as sovereign debt sustainability and currency exchange risks. If, as is normally the case, the rate of return on external assets differs from the rate of return on liabilities, the composition of net debt between assets and liabilities matters.

As some countries are perceived by international investors as safer than others, they have a higher lending interest rate than their borrowing rate. Conversely, countries that are perceived as riskier pay a higher rate on their liabilities than they receive on their assets. This difference was described by Gourinchas & Rey (2014), who showed that developing countries pay a higher interest rate for borrowing than they receive for lending. In contrast, developed countries receive a higher rate for their external assets than they pay for their liabilities.

Gourinchas & Rey (2014) found that the inherently higher risk of assets in developing countries is not the only explanation for this inequality. They observed that the underdevelopment of financial markets in most of the developing world, particularly the scarcity of "stores of value" in these countries, is also an important factor in explaining this difference. The combination of these two factors makes investors from developing countries look to the financial markets of developed countries for store-of-value assets. This capital movement pushes down interest rates in developed countries, especially those for which there is higher confidence in their ability to pay back, such as the United States.

But in fact, the consequences run deeper. These capital movements exacerbate inequalities between developed and developing countries, with rich countries paying part of their current account deficit using surpluses obtained from this favourable spread. This situation suggests that there is a reputation revenue for developed countries and a reputation cost for developing countries. According to Gourinchas & Rey (2014), this effect is helping finance half of U.S. current account deficit.

The findings reported by Gourinchas & Rey (2014) and the application of these findings by Blanchard & Das (2017) have been incorporated in our model. To rewrite the model to account for these differences, we define the interest rate for assets as r_{At} and the interest rate for liabilities as r_{Lt}. Using these terms, we can rewrite Equation (2.1) as:

$$L_{t+1} - A_{t+1} = (1 + r_{Lt})L_t - (1 + r_{At})A_t - NX_t. \tag{2.3}$$

Rearranging this equation and using the term D_t defined earlier gives:

$$D_{t+1} = (1 + r_{Lt})D_t - (r_{At} - r_{Lt})A_t - NX_t. \tag{2.4}$$

Dividing by GDP and iterating forward gives the following expression:

$$d_t = \sum_{j=0}^{n} \prod_{i=0}^{j} \frac{1 + g_{t+i}}{1 + r_{Lt+i}} \left[(r_{At+j} - r_{Lt+j})a_{t+j} + nx_{t+j}\right] + \prod_{i=0}^{n} \frac{1 + g_{t+i}}{1 + r_{Lt+i}} d_{t+n+1}. \tag{2.5}$$

This equation can be divided into two expressions. The first is the present discounted value of the current account plus what we term the "reputation revenue" (cost). This reputation revenue is the difference between the interest rate paid for the liabilities and the interest rate received on external assets. For countries where this difference is positive, part of their debt will be paid thanks to their reputation. In contrast, countries where this spread is negative will accumulate debt because of their bad reputation.

The second component in Equation (2.5) is the so-called Ponzi game condition. When time converges to infinity, this component may converge to zero or grow to infinity. Convergence to zero is the traditional case, when it is assumed that the interest rate is greater than the growth rate. This has been the most common situation throughout human history (Piketty 2014). Under this scenario, debt sustainability depends on the current level of debt and the expected capacity of payment (which, in this case, is the capacity to increase net exports). That is, economies with a powerful export sector may incur more debt because they have a higher capacity of payment. In contrast, economies with a small export capacity are in a worse position in terms of capacity of payment.

It is also possible to have a multiplier that does not converge to zero. Under this scenario, all levels of debt would be sustainable because the total size of debt would grow less than the GDP, making the debt-to-GDP ratio fall over time. This case is known as a Ponzi game because it is the expectation of future payment that makes the debt sustainable, regardless of the level of debt or net exports. This case has traditionally been omitted from the analysis under the premise that a situation where the growth rate is greater than the interest rate is unsustainable because investors would invest in other countries in search of more profitable investments. This is the case described by Blanchard & Das (2017), amongst others. However, the no-Ponzi game condition might not be realistic because, as noted earlier, some investors look to reduce risk rather than maximising profits. This incentive to look for risk-free assets opens the door to the appearance of bubbles in economies that are considered safe by international investors.

Financing debt at an interest rate that is lower than the growth rate implies that the discount factor is higher than 1. When this occurs, sustainability concerns are not an issue. This fact is highlighted by Blanchard & Das (2017), who argue that a discount factor above 1 (i.e. an interest rate that is lower than the growth rate) implies that debt is automatically sustainable because, for any arbitrary level of net exports, net debt eventually reaches a constant value. This constant value will be positive if net exports are positive, and negative otherwise. In the context of fiscal sustainability, Blanchard et al. (1990) report that for a negative value of $r - g$, the debtor government would no longer need to generate primary surpluses to achieve sustainability. With the primary balance in surplus, the debt-to-GDP ratio would steadily decline over time at the rate $g - r$. As these authors argue, the theory suggests that this case corresponds to dynamic inefficiency. In such a case, the debtor would issue more debt until the pressure on the interest rates forced these interest rates to a value at least equal to the growth rate.

More recently, Checherita-Westphal (2019) remarked that standard growth theory, initiated by Ramsey (1928) and developed in the subsequent literature, implies a positive difference of $r - g$ for economies that operate at their steady state (along a balanced growth path). This is true including for the safe rate because the Golden Rule level of capital is reached when the interest rate $r = g$. If the difference between the interest rate and the growth rate is strictly positive and the discount factor is below 1, then sustainability is still potentially an issue. A trade (primary) surplus is needed to stabilise or reduce the external debt-to-GDP ratio.

There is a general consensus that the condition of a greater interest rate than growth rate probably holds, if not always, at least in the medium and long run. Nevertheless, Paul Krugman[2] and others have noted that, for some countries such as the United States, the UK and Japan, a high debt ratio does not necessarily lead to an increase in interest rates. However, even if a permanent negative difference can be rejected on theoretical grounds, this condition has not always held in the past, particularly for some economies.

2.3.2 The Ponzi game Condition

In Geerolf (2018) it is defined a dynamically efficient economy as the economy where interest rate is above growth rate. This is, the economy where investors try to maximize profits, no to store capital. On the other hand, the economies where this is not fulfilled and the investors are looking for storing his capital and not to maximize profits, are considered inefficient because they does not exploit all his potential.

In a dynamically efficient economy, where the aim of investment is to maximise returns, the possibility of an interest rate that is constantly below the growth rate is unimaginable.[3] This is the rationale behind the no-Ponzi game condition found in much of the literature, including the study by Blanchard & Das (2017). Under this assumption, bubbles are not allowed to form. Sustainability is therefore an outcome of the current account and the reputation revenue or cost. However, as suggested recently by scholars such as Geerolf (2018), it is difficult to defend the idea that savings are dynamically efficient in some advanced economies. In the words of Geerolf, *"sufficient conditions for dynamic efficiency are verified for none of the advanced economies and to the contrary, that Japan and South Korea verify the criterion of dynamic inefficiency"* (Geerolf 2018).

The efficient interest rate using the approach from the perspective of maximising consumption is the one that obeys the Golden Rule of capital. In other words, consumption will be maximised when the growth rate equals the interest rate ($r = g$). Dynamic efficiency is described as the case where the interest rate is greater than or

[2] See Krugman(2013).

[3] In classical economic theory, this situation can happen during an unexpected rise in inflation due to the rigidity of the nominal interest rate. However, it cannot be sustained over time. Agents would readjust to this situation, demanding higher interest rates to lend their funds and thereby forcing the economy back to equilibrium.

equal to the growth rate. This idea implies that with the increase of capital endowment through investment, the marginal productivity of capital decreases until it converges to the growth rate. Once the interest rate has converged to the growth rate, it follows this pattern in the steady state.

The literature explains cases where the interest rate is below the growth rate as the consequence of bubbles caused by the excess supply of savings. This excess supply of savings produces an excess demand of saving instruments, which may produce a bubble in the financial markets.

The dynamic inefficiency literature may help us explain the low interest rate in liabilities with respect to the growth rate for developed economies. It may also help explain the opposite effect for developing economies, where the interest rate on liabilities is above the growth rate. From an orthodox perspective, we would expect a flow from developed to developing economies in pursuit of higher return on assets. Then, in the long run, the interest rates on liabilities would converge to the growth rates, and the expected convergence would occur. As explained earlier, however, this phenomenon has not been observed empirically. Therefore, a heterodox perspective is needed to allow for dynamically inefficient behaviour in our model. This dynamically inefficient behaviour is supported by the results of Geerolf (2018) and Gourinchas & Rey (2014), who report that developing economies lack stores of value and must look for these assets in developed economies: "*The developing countries suffer from a shortage of 'stores of value'. This shortage tends to drive up the price of financial assets, that is, to drive down the equilibrium interest rate.*" (Gourinchas & Rey 2014). Translating this scenario into our theoretical model sometimes means a growth rate above the interest rate on liabilities. This situation creates some problems with estimation.

$$g_t \geq r_L, \tag{2.6}$$

$$1 - g_t \geq 1 - r_L, \tag{2.7}$$

$$\frac{1 - g_t}{1 - r_{Lt}} \geq 1, \tag{2.8}$$

$$\prod_{i=0}^{\infty} \frac{1 - g_{t+i}}{1 - r_{Lt+i}} \nrightarrow 0. \tag{2.9}$$

As a result, the discount factor will not converge to zero in the long run. Furthermore, discarding the case of $r_{Lt+i} = g_{t+i}$ for all i means that the discount factor will converge to infinity as t tends to infinity. These cases will represent the unlikely situation where developing economies are lending to developed economies because they lack store-of-value assets. To estimate the model whilst allowing for this possibility, we plot a distribution of possible futures, distinguishing those where the second part of Equation (2.5) converges to zero and those where it does not.

2.4 The data

Our probabilistic framework for the analysis of external debt sustainability requires estimation using empirical data. One unusual aspect of analysing external debt is the fact that debt issuing and repayment do not depend on a single entity, but a bundle of different types of debt involving different agents and maturities. Accordingly, a country's external debt is the difference between a conglomerate of national liabilities held by foreign residents and the foreign external assets held by the residents of an economy. Therefore, there may be national companies with huge external debt from borrowing abroad but low debt if national and foreign debt is considered as a whole. One national bank may be exporting home savings to other countries whilst another bank imports savings from abroad to finance borrowers in its own country. This situation makes it difficult to analyse external debt to determine its sustainability.

To compute the external debt and, more importantly, the interest rate associated with external assets and liabilities, we used International Monetary Fund (IMF) data on international investment position (IIP) and balance of payments (BoP) for the period 2001 to 2018/2019 depending on data availability. The data covered 38 countries, 11 developed and 27 developing. The developed countries were Australia, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Spain, the United Kingdom and the United States. The developing countries formed a heterogeneous group, including Argentina, Bolivia, Brazil, Chile, Colombia, Croatia, India, Indonesia, Mexico, Philippines, Poland, Romania, Saudi Arabia, South Africa, Thailand, Turkey and Venezuela.

One potential limitation of our empirical calculations was the short period. However, we prioritised the richness of our analysis afforded by a large number of countries, despite the limited length of the time series.

For convenience, we used the value of all assets and liabilities in U.S. dollars (USD) instead of using each country's national currency. USD is the most widely used currency in international transactions, and a large fraction of the world's financial assets are denominated in USD. Therefore, USD values of the assets and liabilities offers the best alternative to compare the values of assets from different countries.

We define the asset position with the rest of the world as the IIP financial accounts asset position at the end of the period. Likewise, the liability position was taken to be the IIP financial accounts liability position at the end of the period.

The interest rates were computed using the primary income position from the balance of payments (BoP). Therefore, the interest rates for assets were obtained from the primary income (credit) from each country's BoP, and the interest rates for liabilities were obtained from the primary income (debt) from each country's BoP divided by total assets and liabilities. Finally, the value of net exports was the difference between the current account credit and debit of the goods and services balance. These definitions are consistent with those used in the recent study by Blanchard & Das (2017).

Gourinchas & Rey (2014) and the early work of Bernanke (2005) document how the real interest rate has been declining over the last few decades. Analysing the G7 countries' real interest rate, Gourinchas & Rey (2014) observed a dramatic decline from 5% or 6% at the beginning of the 1980s to -2% by the end of 2011. This finding is consistent with the view of Geerolf (2018) that excess savings are causing a rise in debt in advanced economies driven by the need to find investment instruments.

Figure 2.1 shows how the interest rates on assets and liabilities have evolved over the last 40 years in developed and developing countries. This figure shows how interest rates have fallen in recent decades whilst external assets and liabilities have risen spectacularly.

However, although the tendency is the same in both sets of figures, the distribution of the interest rate is different for developing and developed countries. In developing countries, as already noted by Gourinchas & Rey (2014), the interest rate for assets is low. Specifically, the interest rate received from citizens of developing countries for investments in both developed and developing countries is lower than the interest rate paid to external lenders. Therefore, there is a major difference between how much developing countries are paying from their funds and how much they are receiving from their investments.

On the contrary, the lowermost graphs in Figure 2.1 show that the interest rate on assets is similar to the interest rate on liabilities. This is because developed countries are paying and receiving a similar interest rate on their investments. Notably, the inverse of the huge difference observed in the uppermost graphs in Figure 2.1 is not observed here because the value of the assets and liabilities in developing countries is smaller than the value of the assets and liabilities held by citizens of developed countries.

This difference across time and type of country motivated us to divide the data between developed and developing countries and between the periods before and after 2000. Table 2.2 shows the data from the beginning of the series to 2000 and from 2001 to 2018 for a selection of countries. Examining specific countries reveals some differences between countries within each group.

In the first sub-period shown in Table 2.2, some developed countries, such as Canada and the Netherlands, had an interest rate on liabilities that was above the interest rate in other countries such as Australia. This difference is mostly due to the downward trend of the interest rate. Notably, countries in the second group had a shorter period than the those in the first. Conversely, the distortion of time was less important in developing economies, where the data were more homogeneous. As expected, the interest rate on liabilities was higher than the interest rate on assets. In some cases, this difference was as much as 100%.

The second sub-period was identically defined for all countries, except Indonesia, Brazil and Philippines, for which there were no data for 2001. Similarly, we cut the sample at 2018 because few countries had data for 2019. The difference between the two groups was smaller. The interest rate on assets in developed countries was between 3% and 4%, whilst the interest rate on liabilities was between 2.4% and 4.5%. These figures imply that the difference between the interest rates on assets and liabilities between developed countries was negligible. For the developing countries, there was once again a difference between interest rates on assets and liabilities in the second sub-period. For instance, on average, Argentina and Brazil were paying twice as much for borrowed funds as they were receiving for loans. This is the consequence of Brazilian and Argentinian investors' search for safe investment abroad, where investment does not rely on an inflationary monetary policy by the central bank. An example of these risks is the infamous "corralito" that Argentina suffered at the beginning of this century. In an attempt to keep their investments safe from inflation and other exchange risks, investors look for safer environments to place their money, despite the possibility of not receiving any real reward for it.

2.5 Model estimation and simulation

In this section, we present the model employed to calculate the distribution of the capacity to repay for each country considered in our research. To perform similar calculations, Blanchard & Das (2017) used the IMF World Economic Outlook (WEO) five-year forecast to compute the average actual value of external debt. Then, using a vector autoregressive model, they obtained the error terms for the variables. Finally, using these estimated disturbances, they computed the distribution of future external debt around the previously calculated average value.

This method is very simple and accurate, but it can only be computed for five periods ahead because this is what is available from the WEO forecasts. The inability to extend this method to longer periods is a restriction. Additionally, this method does not consider the possibility of dynamically inefficient scenarios, such as those described in Section 2.3.2. These elements led us to develop a new approach that explicitly considers this possibility and that is able to account for more than five periods ahead.

2.5.1 VAR estimation

The first problem in applying VAR methodology is the requirement of stationarity of variables. When stationarity tests were performed, we could not reject a unit root for all variables, d_{t+1}, nx_{t+1}, r_{At+i}, r_{Lt+i}, a_{t+i} and g_{t+i}. Furthermore, a cointegrating relationship was not found. Therefore, we had to make some adjustments before estimating the model. Instead of forecasting each variable individually, we constructed two stationary clusters of variables. These are the first and second components of the first part of Equation (2.5).

$$\left.\begin{array}{l} c_{1,t+i} = \frac{1+g_{t+i}}{1+r_{Lt+i}} \\ c_{2,t+i} = (r_{At+j} - r_{Lt+j})a_{t+j} + nx_{t+j} \end{array}\right\}. \tag{2.10}$$

The first component is the discount factor, and the second is the addition of net exports to the repricing of net foreign assets for the period. This division has the desirable effect of leaving us with two variables that are stationary by construction. We can therefore specify our VAR model as follows:

$$\begin{bmatrix} c_{1,t} \\ c_{2,t} \end{bmatrix} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} c_{1,t-1} \\ c_{2,t-1} \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} c_{1,t-2} \\ c_{2,t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}. \tag{2.11}$$

Here, α_{11} and α_{12} are the constants of the model, β_{11}, β_{12}, β_{21} and β_{22} represent the slopes of the VAR model, and $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are white noise errors.

The estimated parameters for some countries are presented in the Appendix. The complete results are available upon request. According to the Akaike and Bayesian information criteria, the optimal number of lags differed across countries. Based on these criteria, the optimal number of lags was between one and three, depending on the specific characteristics of each country. To ensure comparable results, we set the number of lags to two for all countries. Modifying this assumption does not affect the validity of the analysis.

Some consideration must be given to endogeneity. As may have become apparent, it is impossible to discern the direction of the causal relation between $c_{1,t}$ and $c_{2,t}$. Economic theory suggests that the interest rate may affect the growth rate in the same way that it may affect the size of external assets. At the same time, however, the amount of external assets and liabilities held by a country may affect that country's lending and borrowing interest rate. Accordingly, causal identification of this model is difficult to determine because the two components of the VAR regression behave simultaneously. To tackle this problem, we tested the robustness of our model to changes in the order of the variables. The results were consistent with our findings. Thus, although endogeneity is always of concern in all external and sovereign debt analyses, our model seems to be robust to its effects.

After estimating the model, we used the resulting parameters to simulate the future values of the variables. Using Monte Carlo techniques, we drew 500,000 observations for each variable over 50 periods (years). Through this approach, we computed the future values of two out of the three components of Equation 2.5.

2.5.2 Simulation

We used the estimations from the above VAR model to run a Monte Carlo simulation for 50 periods ahead with 500,000 observations for each of the variables (c_t, c_2). Using this forecast, we computed the expected net present value of the external debt repayment performance of each of the 500,000 paths and the corresponding probability distributions of the capacity to repay. We used Equation (2.5) for this purpose. Using the simulated variables, we computed the first component of Equation (2.5), namely the net present value of the expected net exports for each of the simulated paths. However, we could not estimate the second part of Equation (2.5) because it depended on d_{t+n+1}, which in a 50-period forecast would be the expected value of the debt 50 periods ahead. To solve this problem, we divided the scenarios between those where the multiplier of d_{t+n+1} converged to zero and those where it did not. In the cases where it did converge to zero, the actual possibility of repayment was equal to the first part of Equation (2.5). In the cases where it did not converge to zero, the economy would be a dynamically inefficient economy where the savings bubble would make the debt perfectly sustainable.

To distinguish this two situations, we have plotted the areas in two different colors. For a simulated interest rate below the growth rate, the potentially dynamically inefficient areas, the areas are plotted in red. Under these scenarios the debt is perfectly sustainable because it is expected to be repaid using future revenues. On the other hand, the dynamically efficient scenarios, where the growth rate is below the interest rate, are painted in blue. These are the most interesting scenarios because it is under those scenarios where default is possible. Then, default risk is a potential treat for countries presenting an important size of their blue area at the right hand side of the current debt level.

In the first case, the economy would be dynamically efficient (there was no bubble). For this case, we plotted a histogram of the distribution of a country's capacity to repay its external debt in blue. A completely blue distribution would indicate that the economy would have dynamically efficient savings. Hence, the main determinant of investment would be the interest rate, not the store of value. This situation is common in unsafe economies where store-of-value assets are rare and investors prefer to look for these assets abroad.

On the contrary, when the economy was dynamically inefficient (there was a savings bubble), we plotted the distribution of the capacity of the country to repay its external debt in red. In such cases, the main determinant of investment would be the store of value, not the interest rate. This situation arises in major developed economies that enjoy high investor confidence, such as the United States or Australia. This effect occurs because investors from more uncertain economies look for safe assets to store savings, placing greater importance on their confidence in the assets than the interest rate they are paying. A good example of this kind of asset is U.S. sovereign bonds, which pay a negligible interest rate but provide almost guaranteed repayment.

For simplicity, the value of the first part of Equation (2.5) is plotted in the graphs for both cases. Hence, the graphs represent the actual net exports for both scenarios without considering the second part of Equation (2.5). If the second part of Equation (2.5) were considered, the red area would go to infinity when time converged to infinity. Plotting only the first part of Equation (2.5) had the advantage of revealing which proportion of scenarios were dynamically efficient, which were dynamically inefficient, and how they were distributed.

The following figures show the present value distribution of the capacity to repay external debt and the actual external debt position for a set of countries. We divided these countries into two groups: developed (Australia, Italy, Spain and the United States) and developing (Brazil, Chile, Mexico and Thailand).

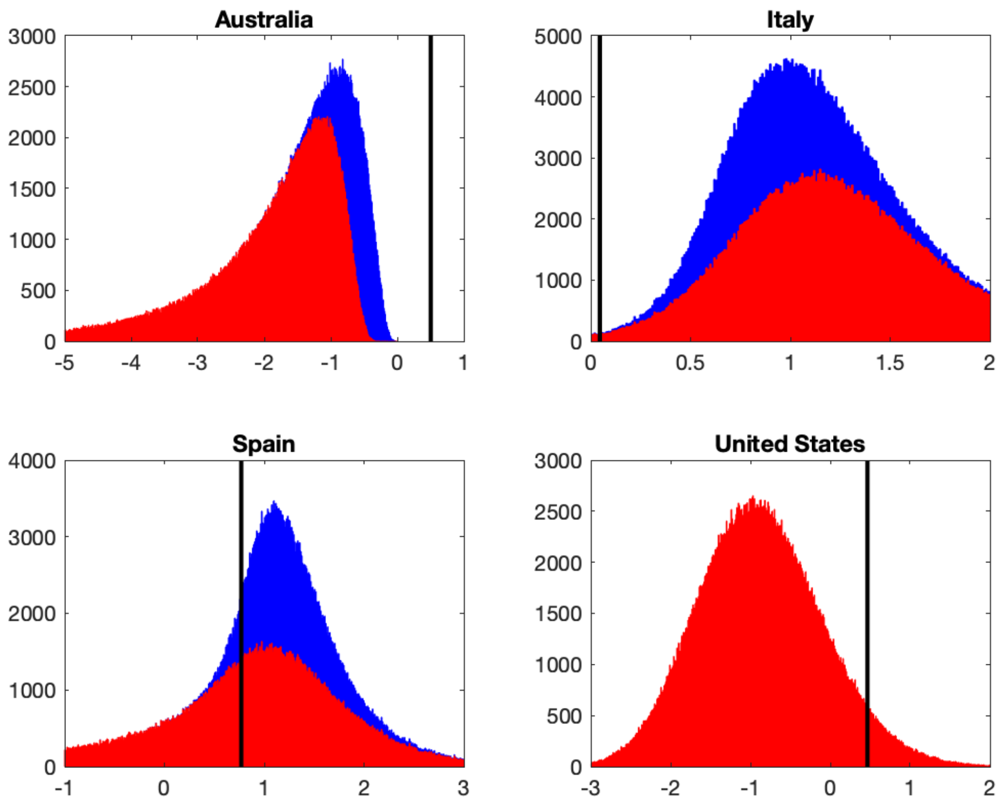

Figure 2.2: Actual external debt density. The red line represents the actual level of external debt.

Figure 2.2 shows the probability density distributions of the capacity to repay external debt for Australia, Italy, Spain and the United States. The first notable observation is that the whole area is plotted in red for the United States. This is because U.S. savings were dynamically inefficient in all studied scenarios. This situation occurs because of the constant flow of savings from the rest of the world to the U.S. financial markets in search of safe assets. This flow would imply that the United States is a safe place to store savings, as long as confidence in its payment capabilities remains. However, the United States would encounter problems repaying money abroad as soon as this confidence in payment capabilities disappeared.

The black line represents the actual external debt of the United States. Comparing this line with the distribution shows that a proportion of the scenarios lie to the right of the line. This area would be the probability of default if dynamic inefficiency were not considered. This finding is consistent with the results obtained by Blanchard & Das (2017), who argued that the United States should devalue the U.S. dollar to maintain perfect solvency.

A similar effect is observed in Australia, where the red area is much bigger than the blue area. This distribution implies that Australia is perceived as a safe destination for savings. In addition, unlike for the United States, the distribution for Australia lies to the left of the external debt threshold. Therefore, Australia would be a perfect destination for savings because its probability of default would be zero. Moreover, the financial markets could eventually push Australia through a revaluation of its currency.

Spain and Italy offer quite different cases. Despite being developed countries, both Spain and Italy have a blue area that is larger than the red area. This situation would imply a lack of trust in these countries as potential destinations for national and foreign savers who seek safe assets. This lack of confidence is a direct consequence of the euro area crisis and the tensions amongst its members.

However, although neither country is perceived as an ideal place to buy safe assets, the positions of both countries are different. Italy is in a decent situation. Despite its poor forecast, which would imply almost certain default, its actual debt is low, leaving room for any structural reforms that may be needed to improve its exports. Spain, on the other hand, has a huge external debt burden, which is close to 100% of its GDP. The mean of the distribution of its capacity to repay is to the left of current debt, but only slightly. Hence, the probability that Spain will have problems servicing its external debt should be a concern for national and international investors.

We now turn to the developing countries, observing the consequences of the findings of Gourinchas & Rey (2014). As a result of the capital flight from developing to developed economies, the net asset position in the developing world is positive in almost all cases. As always, however, one country bucks the trend. Only Thailand was found to have an expected discounted external debt distribution above its present external debt situation. All other developing countries had a present external position that was above their discounted external debt distribution.

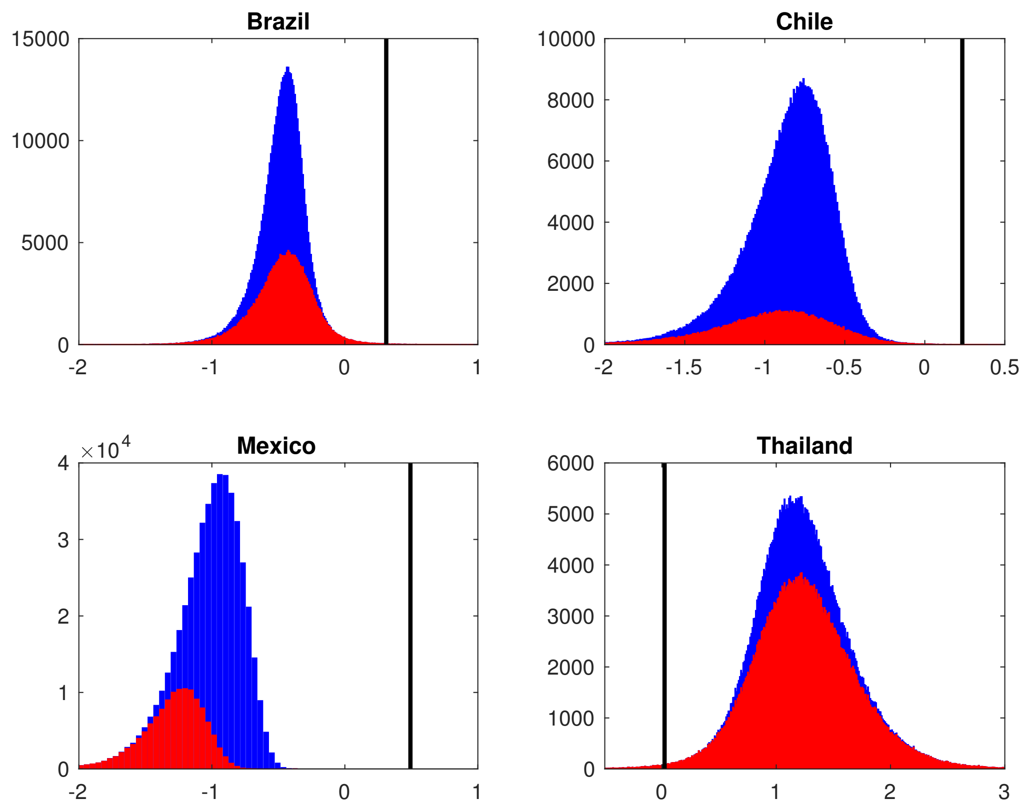

Figure 2.3: Actual external debt density. The red line represents the actual level of external debt.

As Figure 2.3 illustrates, Brazil follows a similar pattern to Australia. In recent decades, Brazil has received a large inflow of savings from the rest of the world, with investors looking for a profitable location for their money. This inflow has pushed down its interest rates and has enhanced its reputation. Brazil has become one of the safest places in Latin America, which has helped the region attract large amounts of savings. In addition, the current Brazilian external debt is in the tail of the distribution, making default highly unlikely. In contrast, Chile and Mexico both have a blue area that is bigger than the red area. Thus, scenarios where savings are dynamically inefficient are scarcer than dynamically efficient scenarios. This result implies that these countries are not considered safe places for investment because the majority of investments are made for high returns, not for storing the value of current wealth.

Despite reputational issues, the situation of both countries is reasonable. Their current possibilities of payment of external debt lie to the left of their current external debt. This situation is the result of the good performance of their external sector, which counterbalances the problem of capital outflow due to the search for safer places to store wealth. Therefore, the probability of default in these countries is mathematically zero.

Finally, Thailand offers a perplexing case amongst the developing economies. Of the developing economies examined in this study, it is the only one for which the distribution of the possibility of repayment of external debt lies to the right of current external debt. Unlike for other countries such as Spain, however, this situation should not be a concern given the large dynamically inefficient area and the small current external debt. Structural reforms are thus possible to help the exporting sector to switch the trend.

2.6 Conclusions

This paper uses a probabilistic approach to enhance our understanding of the sustainability of external debt. We apply this approach to a panel of 38 countries, including both developed and developing countries. Based on estimates from a vector autoregressive model, we use Monte Carlo techniques to compute the possible futures regarding a country's capacity to repay external debt. Using these simulated paths, we plot the distribution of the current value of external debt. Our calculation of this distribution and its application to a set of countries reflects our approach to the study of external debt sustainability.

The construction of a probability distribution for countries' external repayment performance enables analysis of the external debt situation of each individual country. A large part of the distribution to the right of the current external debt is interpreted as a sign of potential problems: Countries in this situation will experience potential problems to repay their debt at the current exchange rate. Countries controlling their own monetary policy will be forced to devalue their currency to improve their current account balance and service their external obligations. By contrast, for countries that are restricted in their use of monetary policy, such as the eurozone member states, this distribution signals future internal devaluations and structural reforms aimed at pushing down production costs.

Moreover, For the countries for which the entire distribution lies to the left of the current debt, a revaluation of their currency is likewise expected. They are expected to be lending to the rest of the world, and, at some point, they may consider improving their consumption instead of lending.

Our paper contributes to the literature by separating the country simulations into two groups of scenarios. In some scenarios, the discount factor is less than 1. According to the literature, this discount factor should generally prevail in the medium and long run. In other situations, external sustainability is achieved by a simulated discount factor above 1. This value indicates that interest paid on debt is lower than GDP growth. This situation is associated with dynamic inefficiency.

This analysis can be used by policymakers to understand when countries need devaluation to continue servicing their external debt. International investors can also use this analysis to anticipate devaluations or revaluations.

We have also shown how developing countries are financing developed countries through a desire to access safe financial assets. This demand for external financial assets is creating a perplexing situation whereby poorer countries are financing the consumption of richer ones. This puzzling situation arises because citizens of developing countries are uncertain about the future behaviour of their own country. Thus, they prefer to hold external assets to protect themselves against the risk of future instability.

This frame of analysis for the question of debt sustainability could be applied to any type of debt, be it public or private. In future research, this method can be applied to public debt sustainability in the eurozone. Further research could likewise explore the potential of this probabilistic approach in the analysis of corporate debt sustainability.

Appendix

Developed economies								
		r_A	r_L	g		r_A	r_L	g
Australia	1990-2000	4.5%	5.9%	2.6%	2001-2018	3.5%	4.5%	8.0%
Belgium	1996-2000	3.2%	3.1%	2.8%	2001-2018	3.5%	3.9%	5.4%
Canada	1981-2000	5.2%	6.6%	5.1%	2001-2018	3.0%	4.0%	5.2%
France	1981-2000	8.2%	9.8%	4.1%	2001-2018	3.3%	2.4%	4.4%
Germany	1981-2000	6.4%	6.6%	5.2%	2001-2018	3.5%	3.2%	4.4%
Italy	1981-2000	6.0%	8.0%	5.2%	2001-2018	3.7%	3.3%	3.8%
Japan	1997-2000	3.5%	2.8%	1.2%	2001-2018	3.5%	2.3%	2.0%
Netherlands	1981-2000	6.1%	7.6%	3.3%	2001-2018	3.9%	3.9%	4.8%
Spain	1982-2000	5.3%	6.3%	6.7%	2001-2018	3.9%	3.1%	5.7%
United Kingdom	1981-2000	7.8%	8.0%	5.6%	2001-2018	2.7%	2.7%	3.4%
United States	1981-2000	6.7%	5.8%	6.6%	2001-2018	4.0%	2.7%	3.9%
Developing economies								
Argentina	1992-2000	3.9%	5.0%	4.4%	2001-2018	2.6%	5.7%	4.9%
Bolivia	1998-2000	2.9%	5.4%	6.5%	2001-2018	2.5%	5.7%	5.6%
Brazil					2002-2018	2.4%	4.9%	5.2%
Chile	1997-2000	2.6%	5.0%	3.8%	2001-2018	2.9%	6.3%	5.0%
Colombia	1981-2000	4.6%	7.3%	2.8%	2001-2018	3.1%	8.0%	7.8%
Croatia	1999-2000	3.2%	7.9%	7.1%	2001-2018	3.5%	6.0%	7.1%
India	1997-2000	3.5%	5.9%	5.6%	2001-2018	3.4%	6.5%	9.7%
Indonesia					2002-2018	2.8%	7.0%	9.9%
Mexico	1997-2000	2.5%	7.6%	3.7%	2002-2018	2.8%	5.7%	6.2%
Philippines					2002-2018	5.9%	6.3%	8.9%
Poland	1995-2000	4.8%	6.2%	8.5%	2001-2018	5.6%	7.1%	7.9%
Romania	1991-2000	3.5%	6.4%	4.9%	2001-2018	4.7%	6.2%	9.2%
Saudi Arabia	1994-2000	2.7%	7%	1.8%	2002-2018	4.2%	5.8%	8.5%
South Africa	1981-2000	4.6%	7.9%	3.8%	2001-2018	2.5%	5.0%	6.9%
Thailand	1996-2000	3.9%	4.9%	1.2%	2001-2018	2.8%	6.0%	7.1%
Turkey	1984-2000	4.6%	6.0%	7.1%	2001-2018	3.2%	3.4%	6.7%
Venezuela	1984-2000	4.6%	7.3%	3.2%	2001-2018	2.2%	6.9%	5.0%

Table 2.1: IMF international financial statistics and balance of payment statistics.

Developed economies							
		r_A	r_L	g	$real\ g$	π	$\frac{1+g}{1+r_L}$
Australia	2001-2018	3.5%	4.5%	8.0%	5.1%	2.9%	1.03
Belgium	2003-2018	3.5%	3.6%	5.2%	3.4%	1.7%	1.02
Canada	2001-2018	3.0%	4.0%	5.2%	3.2%	2.0%	1.01
France	2001-2018	3.3%	2.4%	4.4%	3.1%	1.3%	1.02
Germany	2001-2018	3.5%	3.2%	4.4%	3.1%	1.3%	1.01
Italy	2001-2018	3.7%	3.3%	3.8%	3.6%	0.2%	1.00
Japan	2001-2018	3.4%	1.6%	0.5%	−0.3%	0.8%	0.99
Netherlands	2001-2018	3.9%	3.9%	4.8%	3.4%	1.4%	1.01
Spain	2001-2018	4.0%	3.1%	5.5%	3.8%	1.6%	1.02
United Kingdom	2001-2018	2.9%	2.7%	3.9%	2.1%	1.8%	1.01
United States	2001-2018	4.0%	2.7%	3.9%	2.0%	2.0%	1.01
Developing economies							
Argentina	2001-2018	2.0%	7.9%	5.9%	3.6%	2.4%	0.98
Bolivia	2001-2018	2.4%	7.4%	7.6%	3.6%	4.0%	1.00
Brazil	2002-2018	2.4%	5.7%	6.9%	4.7%	2.1%	1.01
Chile	2001-2018	2.9%	8.4%	8.4%	4.6%	3.8%	1.00
Colombia	2001-2018	3.1%	8.0%	7.8%	4.0%	3.9%	1.00
Croatia	2001-2018	3.9%	4.0%	6.4%	4.5%	1.9%	1.02
India	2001-2018	3.6%	5.1%	10.4%	3.1%	7.3%	1.05
Indonesia	2002-2018	2.5%	6.5%	10.9%	5.3%	5.6%	1.04
Mexico	2002-2018	2.4%	3.8%	3.2%	0.7%	2.5%	0.99
Philippines	2002-2018	8.9%	5.0%	8.9%	3.3%	5.6%	1.04
Poland	2001-2018	7.4%	6.2%	7.3%	3.5%	3.8%	1.01
Romania	2001-2018	5.6%	5.5%	11.9%	7.8%	4.2%	1.06
Saudi Arabia	2008-2018	4.4%	4.6%	5.7%	2.0%	3.7%	1.01
South Africa	2001-2018	3.2%	5.2%	6.8%	3.6%	3.2%	1.02
Thailand	2001-2018	3.3%	7.2%	8.1%	4.1%	4.0%	1.01
Turkey	2001-2018	3.4%	3.4%	6.7%	1.7%	5.0%	1.03
Venezuela	2001-2018	2.4%	7.4%	4.7%	3.1%	1.6%	0.97

Table 2.2: IMF international financial statistics and balance of payment statistics.

Australia				
Effective sample size: 16				
Number of estimated parameters: 10				
Log-likelihood: 67.79				
AIC: -115.58				
BIC: -107.85				
	Value	SD	t stat	p val
α_{11}	0.89	0.25	3.55	0.00
α_{21}	0.03	0.03	0.91	0.36
β_{11}	0.45	0.21	2.14	0.03
β_{21}	-0.02	0.03	-0.76	0.45
β_{12}	-4.84	2.08	-2.33	0.02
β_{22}	0.60	0.25	2.41	0.02
γ_{11}	-0.37	0.20	-1.82	0.07
γ_{21}	-0.02	0.02	-0.74	0.46
γ_{12}	2.00	1.97	1.02	0.31
γ_{22}	-0.16	0.24	-0.68	0.50

Italy				
Effective sample size: 16				
Number of estimated parameters: 10				
Log-likelihood: 70.01				
AIC: -120.03				
BIC: -112.30				
	Value	SD	t stat	p val
α_{11}	0.92	0.32	2.86	0.00
α_{21}	0.05	0.04	1.21	0.23
β_{11}	0.27	0.25	1.08	0.28
β_{21}	-0.01	0.03	-0.37	0.71
β_{12}	-0.98	2.01	-0.49	0.63
β_{22}	0.88	0.24	3.69	0.00
γ_{11}	-0.19	0.25	-0.74	0.46
γ_{21}	-0.03	0.03	-1.05	0.29
γ_{12}	1.23	2.19	0.56	0.57
γ_{22}	-0.08	0.26	-0.29	0.77

Spain				
Effective sample size: 16				
Number of estimated parameters: 10				
Log-likelihood: 67.09				
AIC: -114.18				
BIC: -106.46				
	Value	SD	t stat	p val
α_{11}	0.79	0.41	1.92	0.06
α_{21}	0.12	0.06	1.98	0.05
β_{11}	0.42	0.28	1.49	0.14
β_{21}	-0.08	0.04	-1.90	0.06
β_{12}	-0.99	1.96	-0.50	0.62
β_{22}	0.92	0.28	3.29	0.00
γ_{11}	-0.19	0.30	-0.63	0.53
γ_{21}	-0.03	0.04	-0.77	0.44
γ_{12}	0.62	1.77	0.35	0.73
γ_{22}	-0.13	0.25	-0.53	0.59

United States				
Effective sample size: 16				
Number of estimated parameters: 10				
Log-likelihood: 116.19				
AIC: -212.38				
BIC: -204.65				
	Value	SD	t stat	p val
α_{11}	0.31	0.30	1.02	0.31
α_{21}	0.38	0.11	3.34	0.00
β_{11}	0.91	0.33	2.79	0.01
β_{21}	-0.57	0.12	-4.65	0.00
β_{12}	1.36	0.63	2.14	0.03
β_{22}	-0.04	0.24	-0.15	0.88
γ_{11}	-0.22	0.22	-0.96	0.34
γ_{21}	0.20	0.08	2.39	0.02
γ_{12}	-1.08	0.73	-1.49	0.14
γ_{22}	1.11	0.27	4.07	0.00

Table 2.3: VAR Results of selected developed countries.

Brazil				
Effective sample size: 15				
Number of estimated parameters: 10				
Log-likelihood: 68.13				
AIC: -116.27				
BIC: -109.19				
	Value	SD	t stat	p val
α_{11}	1.57	0.39	4.01	0.00
α_{21}	0.06	0.02	2.29	0.02
β_{11}	-0.27	0.27	-1.00	0.32
β_{21}	-0.02	0.02	-1.00	0.32
β_{12}	1.50	3.04	0.49	0.62
β_{22}	0.95	0.19	4.97	0.00
γ_{11}	-0.24	0.24	-1.00	0.32
γ_{21}	-0.04	0.01	-2.62	0.01
γ_{12}	5.31	3.83	1.39	0.17
γ_{22}	0.02	0.24	0.09	0.93

Chile				
Effective sample size: 16				
Number of estimated parameters: 10				
Log-likelihood: 62.62				
AIC: -105.23				
BIC: -97.51				
	Value	SD	t stat	p val
α_{11}	1.00	0.26	3.80	0.00
α_{21}	0.09	0.07	1.17	0.24
β_{11}	0.30	0.22	1.34	0.18
β_{21}	-0.02	0.06	-0.35	0.73
β_{12}	2.22	0.85	2.60	0.01
β_{22}	0.44	0.24	1.85	0.06
γ_{11}	-0.28	0.19	-1.46	0.14
γ_{21}	-0.07	0.05	-1.30	0.19
γ_{12}	-0.75	0.94	-0.80	0.42
γ_{22}	0.33	0.26	1.29	0.20

Mexico				
Effective sample size: 15				
Number of estimated parameters: 10				
Log-likelihood: 82.67				
AIC: -145.34				
BIC: -138.26				
	Value	SD	t stat	p val
α_{11}	1.60	0.50	3.17	0.00
α_{21}	-0.01	0.03	-0.28	0.78
β_{11}	-0.29	0.24	-1.20	0.23
β_{21}	-0.01	0.01	-0.54	0.59
β_{12}	14.40	5.13	2.81	0.00
β_{22}	0.73	0.28	2.55	0.01
γ_{11}	-0.17	0.26	-0.68	0.49
γ_{21}	0.00	0.01	0.30	0.76
γ_{12}	-8.19	6.14	-1.33	0.18
γ_{22}	-0.20	0.34	-0.59	0.55

Thailand				
Effective sample size: 16				
Number of estimated parameters: 10				
Log-likelihood: 57.44				
AIC: -94.88				
BIC: -87.15				
	Value	SD	t stat	p val
α_{11}	0.59	0.32	1.87	0.06
α_{21}	0.41	0.17	2.50	0.01
β_{11}	0.36	0.26	1.40	0.16
β_{21}	-0.14	0.13	-1.06	0.29
β_{12}	0.65	0.44	1.48	0.14
β_{22}	0.28	0.23	1.22	0.22
γ_{11}	0.05	0.23	0.20	0.84
γ_{21}	-0.25	0.12	-2.05	0.04
γ_{12}	-0.28	0.46	-0.60	0.55
γ_{22}	-0.08	0.24	-0.35	0.72

Table 2.4: VAR Results for selected developing countries.

Chapter 3

Is the Euro Area diverging? An empirical analysis

The existence of a tradeoff between output and inflation gaps has been a central object in monetary policy analysis over the last 70 years. Following the publication of Phillips (1958) and Phelps (1968) this tradeoff and its effect on the real economy has been extensively studied. Most of these studies have been carried on looking to the aggregate data of one economy, usually the United States, without taking into consideration its effects on the internal entities conforming this economy. In the case of the US, these entities would be the individual states forming the US currency area. Even more, this currency area could be considered to embrace all additional countries and regions using the US dollar as its official currency, like Ecuador, El Salvador, Zimbabwe or Puerto Rico. The effects of monetary policy observed in each individual member of the union may or may not match the effect observed and studied in the union as an aggregate. The study of the reaction of each sub-entity to the common monetary policy has been left aside with the assumption that in the long run all sub-entities would converge to the equilibrium of the aggregate economy as predicted by Mundell (1961).

However, not all currency areas are optimal and some areas, which currently may be considered optimal, may become suboptimal at some point in time Alesina et al. (2002). This leaves open the question of how the common monetary policy affects economies which are not converging even when they share the same currency. It could be argued that El Salvador, Ecuador and Zimbabwe are part of the US currency area because they share the same currency. But, the Federal Reserve (Fed), does not take into consideration the best interest of these economies when it sets its monetary policy, leaving them with no guarantee about the optimality of the monetary policy. Leaving these particular cases aside, some questions remain unanswered: do all of the states of the US have the same necessities from a monetary policy point of view? are all them reacting to monetary policy in the same way? Even more, are all of them converging to its aggregate equilibrium?

This work studies the asymmetries between the countries/states inside a monetary union and whether those asymmetries are increasing, decreasing or they stay constant over time. With this objective, I compute the correlations of Real GDP, Real GDP per capita, GDP deflator and population between the different countries/states inside the US and the EA. These correlations give me essential information to understand how the economies inside the union interrelate between them. According to Mundell (1961), it is expected from monetary union for their subentities to behave in the same direction in the long run.

> *"... consider a simple model of two entities (regions or countries), initially in full employment and balance-of-payments equilibrium, and see what happens when this equilibrium is disturbed by a shift of demand from the goods of entity B to the goods of entity A. Assume that money wages and prices cannot be reduced in the short run without causing unemployment, and that monetary authorities act to prevent inflation.*
>
> *...*
>
> *Contrast this situation (the one where the countries do not share a common currency) with that where the entities are regions within a closed economy lubricated by a common currency; and suppose now that the national govermnent pursues a full-employment policy. The shift of demand from B to A causes unemployment in region B and inflationary pressure in region A, and a surplus in A's balance of payments. To correct the unemployment in B the monetary authorities increase the money supply. The monetary expansion, however, aggravates inflationary pressure in region A: indeed, the principal way in which the monetary policy is effective in correcting full employment in the deficit region is by raising prices in the surplus region, turning the terms of trade against B. Full employment thus imparts an inflationary bias to the multiregional economy or (more generally) to a currency area with common currency."* Mundell (1961)

The same should happen with Real GDP, if some country/state inside the union improves its production technology, this improvement should flow crossing the national/state borders to improve the productivity of the whole union. To test if these variables follow their expected behaviour, I am going to compute the correlation of Real GDP and GDP Deflator between countries/states. But, the most relevant matter is not the behaviour of each particular country/state inside of the monetary union by itself, but the distortion that these negative correlations may have on the effects of the monetary policy. In a monetary union, where the relevant variables of the countries/states are not highly correlated or are even uncorrelated or negatively correlated, the central bank will have problems to properly stablish an optimal monetary policy to stabilize the economy.

There exist an extensive literature studying the relationship between two or more countries using DSGE models. This literature could be divided in three main groups. In the first group, there are models considering an infinite number of small open economies without power to change the aggregate equilibrium. This is the case in the model of Gali & Monacelli (2005) where they study a monetary union with an infinite number of economies and shared monetary authority. They found that "despite this joint monetary-fiscal regime being designed in order to maximize the welfare of the union as a whole, optimal fiscal policy in each member country calls, in equilibrium, for a macroeconomic stabilization role that goes beyond the one consistent with the optimal provision of public good." This conclusion has obvious implications for the stabilization of a monetary union without a strong fiscal authority. A similar result was found by Faia & Monacelli (2008) where, in the same infinite economies framework, they allow for each economy to have its own monetary authority. They found that "home bias in consumption is a sufficient condition for inducing the monetary policy maker of an open economy to deviate from a strategy of strict markup stabilization". In essence, both of them show that a long list of conditions is necessary for fiscal and monetary authorities to conduct a policy which would be the optimal for the aggregate union but not necessarily for each individual entity inside it. In these cases, deviations from the aggregate equilibria yield a better outcome for the individual country.

The second main group is the most extensive one, it consists of models with two countries where each of the countries has its own currency and their central bank has to decide to commit to a common objective or not. This is the case in the model of Clarida et al. (2002) where two central banks have to decide if they should cooperate or not. They found that there are gains from cooperation but to achieve it the central banks have to react to foreign inflation as well as to the national one. While under the Nash equilibrium, the central banks only need to react to domestic inflation. Ida (2013) followed a similar approach and found that cooperation is not optimal when both economies hit the zero lower bound. Under similar conditions Pappa (2004) conduct a welfare analysis to compare the equilibrium in the three possible arrangements: a cooperative equilibrium, a non-coperative one and a monetary union. He finds that "the non-cooperative equilibrium may be suboptimal because of terms of trade spillover effects, while the monetary union may be suboptimal because of the sluggishness of relative prices."

In order to guarantee the existence of an equilibrium, the above authors need for each economy to have its own central bank. When both countries have its own central bank following an inflation targeting rule, but they do not have a common or individual fiscal authorities powerful enough to stabilize the economy, uniqueness of equilibria is not guaranteed. Bullard & Schaling (2009) studied this problem in the environment created by Clarida et al. (2002) and described under which conditions uniqueness of equilibrium can be achieved. They found that if a country inside of a union deviates from the equilibrium, all the other countries will be unable to attain the equilibrium by themselves. Batini et al. (2004) took a different approach to the same problem and studied the determinacy problem using inflation-forecast-based rules instead of the common Taylor Rules used in one country New Keynesian models. Under this methodology first employed by Batini & Pearlman (2002) they are able to obtain a unique stable equilibria with a two-world economy.

The third group consists of models where some conditions are added to the policy rule in order to guarantee the equilibrium. This is the case of Benigno (2004) where he proposes to give a higher weight in the policy rule to the inflation of the country with higher degree of nominal rigidity inside of the monetary union. With this modification he is able to guarantee determinacy and a nearly optimal equilibrium. Another possible approach to guarantee equilibrium determinacy is the one proposed by Ferrero (2009). In a monetary union with individual fiscal authorities, he advocates for those fiscal authorities to be in charge of smoothing the impact of idiosyncratic exogenous shocks while the monetary authority focuses on maintaining overall price stability.

The only proposed solution for equilibrium determinacy consistent with the European Central Bank (ECB) mandate is the one proposed by Ferrero (2009). To guarantee the equilibria, this approach needs for all the individual fiscal authorities to be powerful enough to stabilize their economy on their own, as a substitute for common fiscal authorities used in one-country models. This last case is the one observed in the US where the US Treasure plays the role of a common and powerful fiscal authority. However, the EA, where such an authority does not exist, needs to assume powerful enough individual authorities to stabilize each individual economy. Looking to the recent past, the EA sovereign debt crisis showed that this assumption may be too strong to hold. During this crisis, some countries like Greece or Portugal where not able to finance themselves in the international financial markets, making it necessary the intervention of international institutions to guarantee their solvency.

The present paper proceeds as follows. In Section 3.1 I present the correlation of some relevant variables between the US and the EA states/countries to show how they interrelate inside of its respective unions. In Section 3.2 I develop a two-country New-Keynesian model with a common central bank suffering multiplicity of equilibria. From this model I compute the welfare loss function to see which variables are driving it. I, then, use this function to build a policy rule which I estimate for the US and the EA in Section 3.4. Finally, Section 3.5 concludes.

3.1 Stylized Facts

To illustrate the problems faced by the ECB to achieve convergence in the EA I have plotted annual inflation represented by the GDP deflator, for a subset of the EA countries for the period 1998-2022. All the EA data comes from Eurostat, while the data for the US comes from the Bureau of Economic Analysis and the Federal Reserve. In Figure 3.1 it is observed how the inflation for the whole EA is close but below the 2% inflation rate, which is considered the equilibrium level by the ECB.[1] But, while the EA as a whole is in equilibrium, some countries inside the union are not.

[1]"Price stability is defined as a year-on-year increase in the Harmonised Index of Consumer Prices (HICP) for the euro area of below 2%. Price stability is to be maintained over the medium term." ECB (2003).

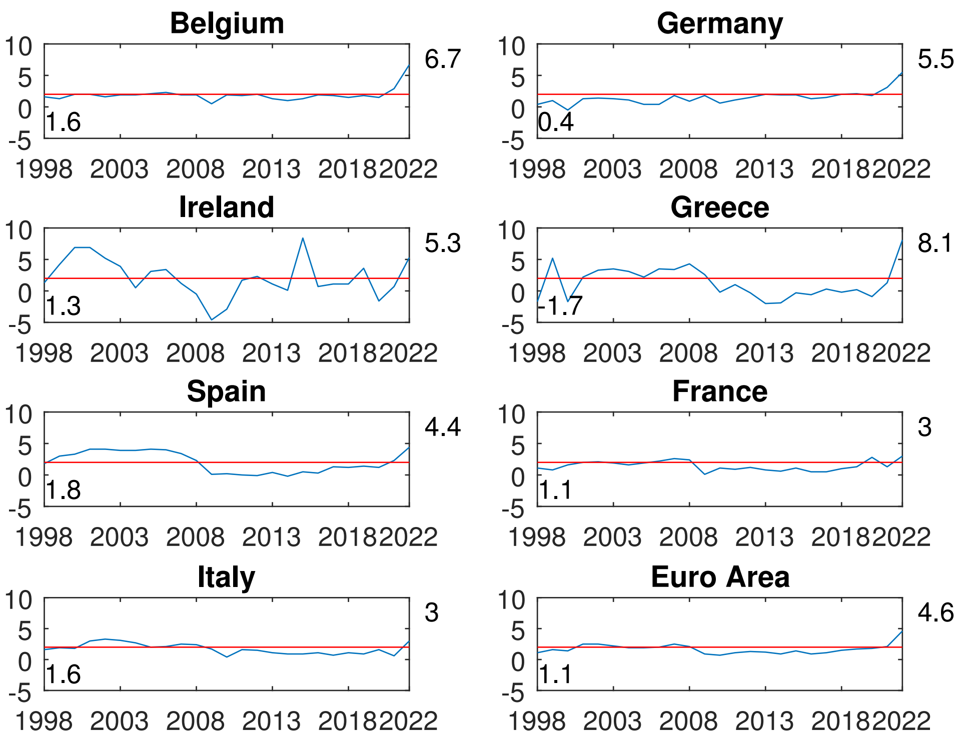

Figure 3.1: Yearly GDP deflator for a set of Euro Area countries.

For the first part of the series, it is observed how inflation in Spain and Greece is recurrently above the target, while inflation in Germany is below it. But, in the second period of the series, it is Germany the one around or even above the 2% mark while Spain and Greece are recurrently below the target. it is also interesting to note how inflation in Ireland is all over the place. It grew during the first years of the Euro to stabilize around the 2% mark in 2004-2005 and suffer a big fall during the financial crisis of 2008. Then, after 2012, it seemed to stabilize around the 2% mark with a big deviation in 2015. On the other hand, France and Italy, despite showing a behaviour similar to that of Spain and Greece, had a very small upwards deviations during the first period of the Euro to fall slightly below the objective during the second period, having a behaviour very similar to the one followed by the average of the EA. Finally, Belgium is the only one, among the subset of the selected countries, which in always in equilibrium for ECB standards. Having its only significant deviation form the 2% objective during the COVID and the subsequent supply crisis. The rest of the EA countries show a similar behaviour to the ones shown above.

In addition, looking to Table 3.7, it can be observed how the average inflation for the EA in the considered period is below but close to the 2% objective. This is also the case of Belgium which has an average inflation of 1.9% over the considered period. But, the inflation of other countries like Germany or Greece is below the 2% objective with 1.5 and 1.38 % respectively. On the other side of the coin, countries like Estonia, Lithuania or Latvia are above the 2% mark established by the ECB. Even when the average inflation for the EA is close to the objective, the average inflation for most of the countries in the EA is not in equilibrium for ECB standards.

Looking to the plots of Figure 3.1 some countries in the EA seem to be going in different directions. While the inflation is growing in Spain or Greece, it is falling in Germany and the other way around but, at the same time, it is constant in Belgium. This supposes a big challenge for the economic stabilization policies, as the ECB is only able to react to aggregate inflation, leaving all the weight to reconduct deviations of a particular country or group of countries on the hands of national fiscal authorities, as stated by Ferrero (2009).

A good way to observe how inflation and output gap behave inside of a monetary union is computing the correlation of these variables between countries. To show this relation, I am displaying the correlations of Real GDP per capita and the GDP deflator between the above subset of countries and the aggregate of the EA.

	Belgium	Germany	Ireland	Greece	Spain	France	Italy	EA
Belgium								
Germany	0.65							
Ireland	0.23	0.10						
Greece	0.41	0.21	−0.03					
Spain	0.42	0.11	0.35	0.51				
France	0.33	0.30	0.20	0.27	0.54			
Italy	−0.06	−0.10	0.08	0.30	0.51	0.49		
EA	0.72	0.72	0.35	0.47	0.69	0.74	0.41	

Table 3.1: GDP deflator correlations.

In Table 3.1, I put numbers to what it was observed in Figure 3.1. The inflation correlations between some countries is weak, even negative in some cases. Looking to the aggregate, the inflation correlations of the selected countries with the EA inflation is somewhat low. For Belgium, Germany, Spain and France this correlation is close to 0.7; but it goes even lower for other considered countries, like Ireland which is the lowest among the selected countries. The inflation in Ireland, Greece or Italy, among others, is weakly correlated with the one of the whole union. This effect creates a big problem for inflation stabilization, as the ECB is only able to react to the aggregate of the monetary union and does not take into consideration the particular deviations of an individual country. It is then possible for a subset of countries to experience an inflation different to the equilibrium one. A country or a group of countries may have an inflation recurrently above the one of equilibrium, while this is compensated for another country or subset of countries having an inflation below the one of equilibrium. This compensation between an inflationary and a non-inflationary group of countries would leave the EA inflation on its expected mark. This equilibrium in the EA would be treacherous though, because non of the internal subentites of the union would be in equilibrium by itself. This is a very dangerous situation. Bullard & Schaling (2009) showed how if one country inside of a union is not in equilibrium, the whole union cannot be in equilibrium, as there is some imbalance growing inside of it.

If we look in more depth to Table 3.1, it is observed how the inflation correlation between some countries is negative. This is the case of the correlation between Italy and Belgium, Italy and Germany, and Greece and Ireland. This implies that when inflation is growing in one of these countries, it is falling in the other one. This reinforces the problem of stabilization as the inflation in two countries inside of the union are going in different directions. When one of these countries needs an expansionary monetary policy, the other country is going to need a contractionary one. This dichotomy will be impossible for the monetary authority to achieve and, considering that the monetary authority reacts to the weighted average of the union, it will choose a policy less expansionary than necessary for the first country and less contractionary than necessary for the second one. Making monetary policy incapable of guaranteeing, by itself, convergence to the equilibrium.

It can also be noted some interesting facts about Table 3.1. Similar countries usually have a higher inflation correlation than different ones. For example, the inflation correlation between Spain and Greece is the highest between countries for Greece, and the second highest for Spain. The highest for Spain is the one between Spain and France which, additionally, is the highest for France. The same can be observed between Germany and Belgium or Italy with France and Spain. This was somehow predictable because similar countries, with close borders, tend to have stronger trade ties which favour the homogeneity of prices across borders. Despite this stronger links between some countries, it is not possible to divide the EA countries in two main groups because while inflation in country A might be correlated with inflation in country B and this at its time with inflation in country C, the inflation between country A and C might not be correlated. This is the case of Greece, Spain and France where the inflation correlation between Greece and Spain and Spain and France is relatively high but the one between France and Greece is not. It is clear then that the ECB must be having problem for stabilizing the inflation in the EA. Even when it seems that the aggregate inflation for the EA is converging, the inflation inside of the union is not. The main goal of the ECB is to maintain price stability but, as a secondary goal, it should try to reduce output gap as well. In the Table 3.2 have depicted the Real GDP per capita correlation between the same set of countries than in Table 3.1.

	Belgium	Germany	Ireland	Greece	Spain	France	Italy	EA
Belgium								
Germany	0.95							
Ireland	0.80	0.80						
Greece	−0.13	−0.40	−0.29					
Spain	0.79	0.64	0.59	0.34				
France	0.97	0.92	0.74	−0.09	0.85			
Italy	−0.07	−0.28	−0.12	0.77	0.47	0.08		
EA	0.98	0.94	0.84	−0.10	0.85	0.98	0.05	

Table 3.2: Real GDP per capita correlations.

The results observed in Table 3.2 are somehow more extreme than the ones observed in Table 3.1. The correlation between the Real GDP per capita in Belgium, Germany and France and the EA is almost perfect. We must keep in mind that Germany and France are the largest countries in the EA and thus the ones contributing the most to the EA aggregate, the ECB weights the contribution of each country to the aggregate depending on its size with respect to the union. Spain and Ireland Real GDP per capita are strongly correlated with the one of the EA as the correlation is above 0.8 in both cases. But this is not the case for Italy or Greece. The correlation between the Real GDP per capita in Italy and the aggregate for the EA is almost zero. This means that any monetary policy devoted to close the output gap in Germany, France or Spain will not be effective in Italy, as its Real GDP per capita is not behaving in the same direction than the aggregate of the EA or its other commercial partners. What is more disturbing, the correlation between the Real GDP per capita in Greece and the aggregate for the EA is weak but negative. This is, when Real GDP per capita is growing in the aggregate of the union, it is falling in Greece and the other way around. This implies that any monetary policy designed to stabilize the EA as a whole will be counterproductive for Greece, as it most likely needs the opposite policy to converge to its individual equilibrium.

If we look in the inside of the union, the results are even more disturbing. The Real GDP per capita of Greece is negatively correlated with Belgium, Germany, Ireland and France. These correlations are not weak any longer, the negative correlation between Greece and Germany is up to -0.4. But this is not all, the Real GDP per capita in Italy, the third biggest economy of the EA, is also negatively correlated with the ones of Belgium, Germany and Ireland with values up to -0.28. This is a big problem from an stabilization point of view as the Real GDP per capita of some of the biggest economies in the EA are not all behaving in the same direction, but in opposite directions.

It must be recalled that the ECB is an inflation targeting central bank and it does not pay special attention to output gap deviations. Some part of the behaviour of these negative correlations may be then explained by the lack of reaction from the ECB but, by itself, this does not paint the complete picture. Going back again to Ferrero (2009), for a monetary union like the EA to converge to its equilibrium, it needs strong fiscal authorities capable of closing the output gaps in order to guide all countries in the same direction. Then, if we look carefully to the data, it will not be surprising to realize that the countries with worst results mimiking the behaviour of the EA aggregate, with negative or almost zero correlations, are those with less fiscal muscle because of its high public indebtedness: Italy and Greece with 150% and 194% of its GDP respectively.

Tables 3.1 and 3.2 paint a partial picture of what is happening in the EA. But, to have a more general view, it is not necessary to compute the correlation for these and other variables for the whole union. To summarize all this information in a comprehensible and tractable way, I am going to plot an histogram with the correlations between countries/states for each variable of interest. Using histograms to summarize the information makes easier to see what is happening in each union and draw comparisons between them. To study the behaviour of the relationship between the countries/states inside of a monetary union I have plotted the correlations between countries/states of four variables of interest: Real GDP, Real GDP per capita, GDP deflator and population.

The first variable of interest is Real GDP, this shows if the GDP of the considered countries are growing/shrinking at the same time. If two countries have a highly correlated Real GDP, even when their levels are different, both of them will be in a similar position of their economic cycle. This high correlation is what might be expected in a fully functional monetary union with a common market. If this correlation is weak, both countries will be in the same side of the economic cycle, both growing or shrinking, but not exactly in the same position. But, what happens when two countries inside of the union are uncorrelated or, even worse, negatively correlated? If the Real GDP in two countries is uncorrelated their business cycle will not be related; they might be both growing at some point in time and one of them may enter in a recession while the other still growing. Finally, there remains the case where the Real GDP between two countries is negatively correlated. In this case, when a country is growing the other one is very likely to be in a recession and the other way around. Essentially, they are in opposite sides of their business cycle, stressing the stabilization capabilities of the central bank as both countries have conflicting interests and necessities.

The second variable of interest is the Real GDP per capita. This variable, like the above one, tells us which is the relationship between the business cycles of two different countries. High correlation implies a similar business cycle while a negative correlation implies that each country is in a complete different part of its business cycle. This measure has a problem though, if the union has a high migration rate, which is not driven by economic purposes, the statistic might be unclear. The changes in population in a specific state/country might increase/decrease the GDP per capita without affecting the GDP. If a migration is constituted by workers looking to improve their living conditions, the fall in GDP per capita would be representative of what is happening in the economy, as the increment of labour would have reduced its marginal productivity. But the same is not true if the movement is constituted by non-workers, like retires, looking for better weather drawing rents from other states/countries and not actively participate in the labour market. To address this issue, I have included the correlations of population size between countries/states. The population histogram shows how the population of the different country/states studied behave. If the weight of the distribution is on the extremes, the migrations will be unidirectional, the citizens from a subset of countries will be migrating to another subset but not the other way around. On the other hand, if the weight of the distribution is in the middle, the migrations will be bidirectional between countries/states and much more homogeneous.

Finally, the last variable of interest is the correlation between the GDP deflator between countries/states inside the monetary union. This histogram tells us how prices evolve in the union. In a fully functional monetary union with a common market it is expected for prices to move in the same direction in all countries/states of the union. Even more in the cases where the main goal of the central bank is to maintain price stability like the EA. A high correlation of GDP deflator between two countries means that both countries suffer inflation/deflation at the same time. This would be the expected outcome in a monetary union as both countries are subject to the same monetary policy and external sources of imported inflation. But, as shown in Tables 3.1 and 3.2 this is not the only possible outcome; in some cases the GDP deflator in two countries can be uncorrelated or even negatively correlated.

In a common market, by definition, there is perfect movements of goods and services. As a consequence, prices are expected to converge between countries, if prices are not converging any businessman can buy goods from the country where they are cheaper and sell them in the country where they are expensive, making a profit in the process. Arbitrage would make difficult for prices inside of a common market with a unique currency not to be correlated. If inflation is not correlated, it would mean that prices are growing/falling in one country while they stay constant in another one inside of the union. What is even worse, if inflation between countries is negatively correlated, prices would be growing/falling in one country while they fall/grow in other country inside of the union. A situation like this would put pressure on the stability of a common market and create an insolvable problem for the central bank, as both countries will have conflicting interests and need different monetary policies.

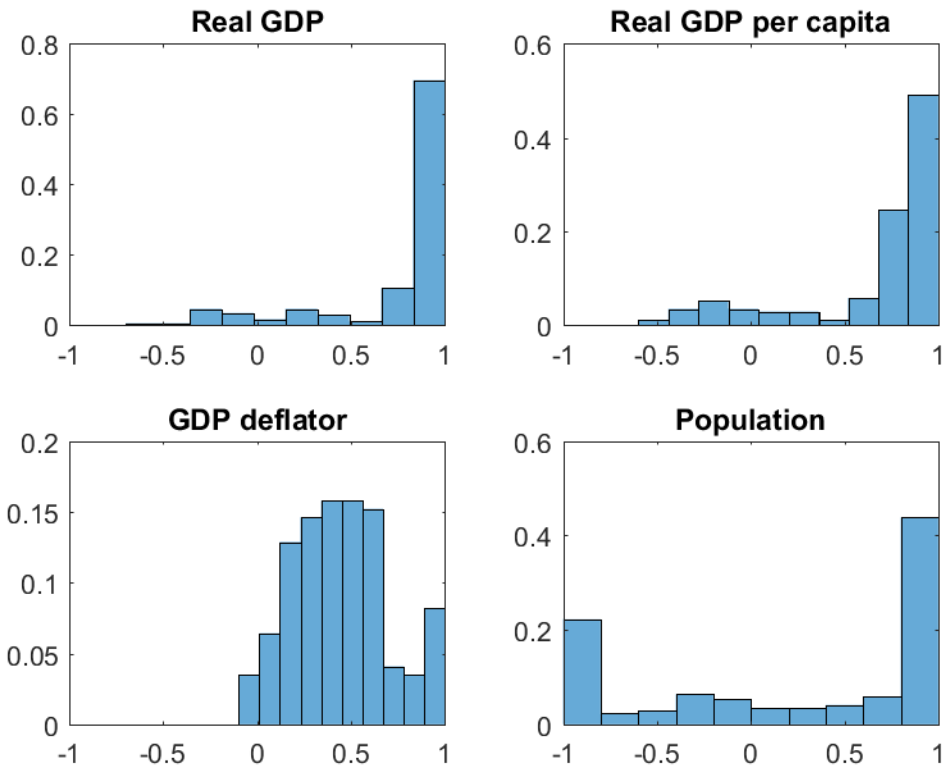

Figure 3.2: Histogram of the EA deflator, GDP per capita, GDP and population correlation across countries.

In the above Figure, I have plotted the histograms for the EA-19 correlations. In the first plot it can be observed the correlation of Real GDP between countries. The 58% of the Real GDP correlations in the EA are above 0.9 but the 9.3% are below 0. This shows a big disparity between countries, most of the EA countries behave in the same direction but an important fraction of them is behaving in the opposite direction. Leaving also a relevant fraction in the middle which is not going anywhere. These results are similar to the ones of the Real GDP per capita where roughly the same is observed. In the third plot of Figure 3.2, it can be observed how the inflation correlation between countries is above 0.9 in just the 8,2% of the cases and above 0.8 in the 10.5% of the cases. The ECB might be doing an excellent job reducing inflation in the EA as a whole but this plot shows that it is not doing a great job achieving price stability inside of the EA. Finally, the population correlation show how the population in some countries grow at the same time while the population in other countries shrinks. This paints the picture of the movements of workers across EA borders and how some countries are net exporters of workers while others are net importers with the 30% of the cases above 0.9 and 15% of the cases below -0.9.

Turning our view to the other side of the Atlantic, I have plotted the correlation for the same variables for the states of the US and the district of Columbia.

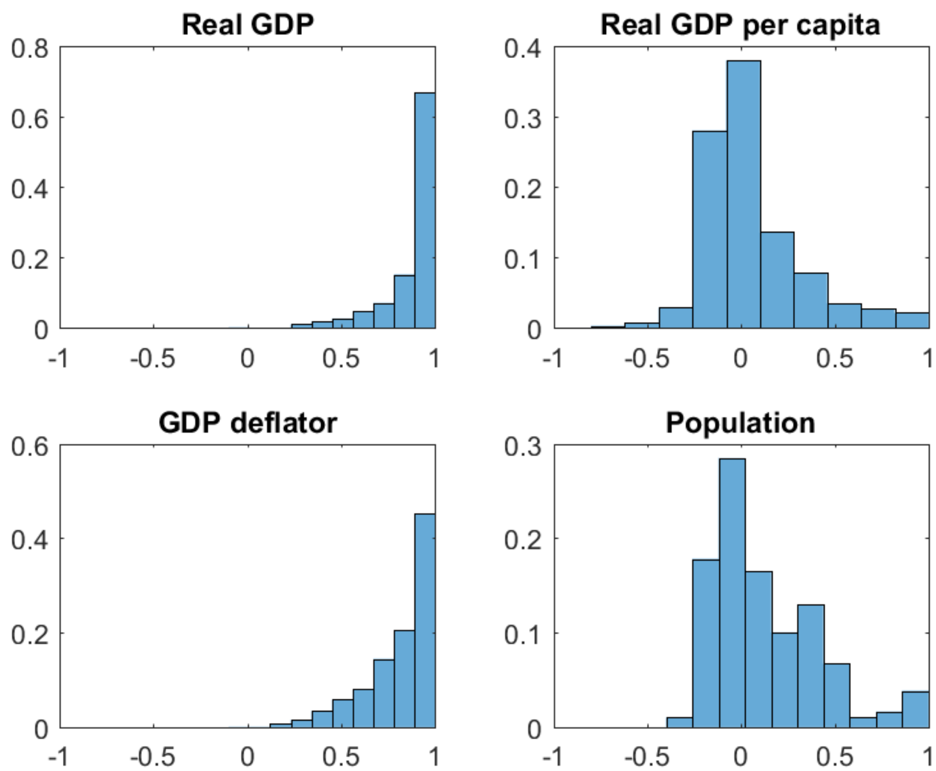

Figure 3.3: Histogram of the US deflator, GDP per capita, GDP and population correlation across states.

The first thing to note about the correlations between the states in the US is the one of the Real GDP. It is observed that non of the states have negative correlation between them and just 4.4% are below 0.5, while for the EA this happened in the 18.7% of the cases. In the same way, 64% of the correlation between states in the US are above 0.9 while for the EA this proportion is of 58%. This shows that in the upper side of the correlation distributions both unions are similar but, in the lower side, the EA has a higher mass than the US. This is, the EA has a mass of countries which are not converging with the rest of the union while the US does not have this problem.

If we observe the histogram for the Real GDP per capita in the US after examining the one for Real GDP the results might seem perplexing. In this case, it is observed how some states have negative correlations on its GDP per capita and the mass of correlations is around zero while the one for the EA was very similar to the histogram of its Real GDP. This is the reason why I have added the population correlation between states to Figures 3.5 and 3.3. In the US the movements of citizens are higher and more homogeneous than in the EA, affecting with it the Real GDP per capita correlations. The issue of the different labour market integration and the migrations in the US and Europe is broadly discussed in Dorn & Zweimüller (2021). This disparity in movements of persons between the two monetary unions makes the Real GDP correlations a better measure of comparison than per capita terms.

Turning, finally, our view to the inflation, it can be observed how the histogram is similar to the one of Real GDP. The inflation correlation grows in an exponential way from the positive neighbourhood of zero to 1, leaving most of its weight in the neighbourhood of 1. This distribution is not at all similar to the one observed for the inflation correlations in the EA, which looked more like a normal distribution. Looking more in depth to the data, non of the inflation correlations in the US are below zero, while 3% of the cases are below this threshold in the EA. The 57.3% of the cases in the EA are between 0 and 0.5 while just the 7.8% of the cases in the US are in this interval. If I take the 0.75 threshold, the 70% of the inflation correlations between states is above this mark while just the 12% of the EA countries are above it.

The comparison of both monetary unions are not flattering for the EA. While in the US non of their states have negatively correlated Real GDP or GDP deflator, the EA has some countries where these variables are negatively correlated. This negative correlation supposes a problem for national fiscal authorities in charge of stabilizing their economies as well as for the ECB which is incapable to stabilize countries with conflicting interests and necessities. The solution given by the European authorities are similar to the ones proposed by Ferrero (2009) where the ECB has to achieve price stability on average while the national authorities have to stabilize their economies by themselves. Looking to the data plotted in Figure 3.1, the ECB is doing an excellent job maintaining inflation under control. But, looking to Figures 3.2 and 3.3, it is clear that the EA national authorities are having problems achieving the convergence of their economies to the union aggregate.

These negative correlations by themselves are not an unsolvable problem. It is possible for two countries inside of the union to temporarily move in different directions as a consequence of particular shocks or different necessities. A good example is the internal devaluation carried on by counties like Greece or Spain during the first part of the last decade. The objective of this internal devaluation was to regain international competitiveness if compared to the other countries of the EA in order to improve their current account balance. This is an example of the intervention of national authorities to stabilize the economy. But this intervention had some negative effects also, the effort done by the countries which confronted the internal devaluation left deep scars in the society and high amounts of sovereign debt which reduced the capability of the national authorities to conduct any ambitious stabilization policy any time soon. The COVID shock and the supply crisis which followed have stressed the stabilization capabilities of some countries which were already highly indebted while other economies inside the EA had enough muscle to carry a broad reaction. This asymmetry in the capacity to react to shocks might cause additional imbalances inside the EA as the economies with more muscle can overreact to external shocks in order to obtain the better outcome for them, even at the expense of the rest of the union. This problem is largely discussed by Gali & Monacelli (2005).

The problem of weak or even negative correlations of inflation between countries is the incapability of the central bank to properly react to inflationary shocks. Even when, on average, the union is suffering an inflation above the desired one, the central bank might have problems to react to this inflation, as some countries inside of the union might be experiencing low inflation and needing a low interest rate. It is then necessary to study the source of these negative correlations and the distortion that they might have over the monetary policy. To study this problem I develop a New Keynesian model with a monetary policy determination consistent with the ECB mandate. I use this model to compute the policy function and compute the distortion caused by the asymmetric behaviour of the EA countries.

3.2 The model

New Keynesian models are the common approach to analyse the effect of choosing between different monetary policy rules. Using a New Keynesian model, I would be able to test different policy rules to find which yields a lower welfare social loss. However this approach has a problem: a two-country model with an inflation targeting central bank like the ECB does not fulfil the Blanckard-Khan conditions, resulting in multiplicity of equilibria. More than one equilibria is possible and the one reached depends on initial conditions. As I have discussed earlier, the only solution proposed in the literature consistent with the EA treaties is the one in Ferrero (2009). But for this to work, the EA needs individual authorities strong enough to stabilize their economy. If these authorities are not strong enough, the central bank will not be able to react with its full strength to output and/or inflation deviations in the union.

The incapability of the individual fiscal authorities to stabilize their economy will be patent when the heterogeneity between countries starts affecting the monetary policy determination for the whole union. This heterogeneity between countries can be captured in the terms of trade of the following two-country New-Keynesian model.

I am developing the simplest case of a Monetary Union, the case where two countries decide to share a unique currency without a political union. As a consequence of this union, the Central Bank has to decide which monetary policy to implement taking into account the interest of both countries. The two regions, home (H) and foreign (F), may differ in size but are otherwise symmetric. In the Home country, it is assumed to live the γ proportion of the total population, whereas in the Foreign country lives the remaining $1 - \gamma$ of the population. I assume this population to be a continuum in the interval $[0, 1]$. Therefore, the population on the segment $[0, \gamma]$ lives in the Home country and the one in the segment $[\gamma, 1]$ lives in the foreign one. Households in both countries also have access to Arrow-Debreu securities which can be traded both domestically and internationally.

3.2.1 Home country Households

I follow the lines of the model developed by Clarida et al. (2002) for a two-block world. The big difference between both is in how the monetary policy is set. While they assume two central banks setting individually their monetary policy, I assume a common central bank setting the monetary policy for the whole union. According to ECB Governing Council,[2] the central bank reacts to the weighted average of inflation. Households choose between home and foreign goods according to:

$$C_t \equiv \frac{C_{Ht}^{\gamma} C_{Ft}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}}, \tag{3.1}$$

where C_{Ht} and C_{Ft} are indexes of consumption across the continuum of differentiated goods produced in country H and F respectively.

$$C_{Ht} \equiv \left[\gamma^{-\frac{1}{\varepsilon}} \int_0^{\gamma} C_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3.2}$$

$$C_{Ft} \equiv \left[(1-\gamma)^{-\frac{1}{\varepsilon}} \int_{\gamma}^{1} C_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3.3}$$

with $C_t(h)$ denoting the quantity of good $h \in [0, \gamma]$, produced in the home country, consumed by each household in period t. In the same way, $C_t(f)$ stands for the quantity of foreign produced goods $f \in [\gamma, 1]$ consumed by the national household in period t. For convenience, I assume the number of firms to be equal to the number of households in each country.

The elasticity of substitution across goods produced within a country is ε. I assume there is not home bias in consumption, the elasticity of substitution between home and foreign consumption will then be the proportion of population in the national economy (γ).

Individual firms will minimize costs yielding the following relations:

$$C_t(h) = \frac{1}{\gamma}\left(\frac{P_t(h)}{P_{Ht}}\right)^{-\varepsilon} C_{Ht}, \tag{3.4}$$

$$C_t(f) = \frac{1}{1-\gamma}\left(\frac{P_t(f)}{P_{Ft}}\right)^{-\varepsilon} C_{Ft}. \tag{3.5}$$

From the above assumption derives the aggregate price index for the goods produced in each country:

[2] The ECB is tasked to achieve price stability as defined by ECB (2003). The inflation statistics followed by the ECB is the Harmonized Consumer Price Index (HCPI) which is a weighted average of the prices in the EA Eurostat (2018).

$$P_{Ht} \equiv \left[\frac{1}{\gamma}\int_0^{\gamma} P_t(h)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}},$$

$$P_{Ft} \equiv \left[\frac{1}{1-\gamma}\int_{\gamma}^{1} P_t(f)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}},$$

where $P_t(h)$ is the price for home country produced goods and $P_t(f)$ is the price for foreign country produced goods. In the same way, P_{Ht} is the aggregate price of the home country produced consumption goods while P_{Ft} is the aggregate price of the consumption goods produced in the foreign economy.

Conditional on optimal behaviour:

$$\int_0^{\gamma} P_{Ht}(i) C_{Ht}(i) di = P_{Ht} C_{Ht},$$

$$\int_{\gamma}^{1} P_{Ft}(i) C_{Ft}(i) di = P_{Ft} C_{Ft}.$$

Total consumption expenditure in each type of goods can be written as a product of the price index times the quantity index.

The maximization of Equation (3.1) s.t. $P_{Ht}C_{Ht} + P_{Ft}C_{Ft} \equiv X_t$ yields:

$$C_{Ht}P_{Ht} = \gamma P_t C_t, \tag{3.6}$$

$$C_{Ft}P_{Ft} = (1-\gamma) P_t C_t, \tag{3.7}$$

$$P_t \equiv P_{Ht}^{\gamma} P_{Ft}^{1-\gamma}. \tag{3.8}$$

Utility Maximization

The preferences of a representative household in the home country are given by the following function which is maximized with respect to the budget constraint.

$$\begin{array}{ll} \text{Max} & \sum_{k=0}^{\infty} \beta^k \left[\frac{C_{t+k}^{1-\sigma}-1}{1-\sigma} - \frac{N_{t+k}^{1+\varphi}}{1+\varphi}\right] Z_{t+k}, \\ \text{s.t.} & P_t C_t + \frac{B_t}{1+R_t} \leq B_{t-1} + W_t N_t, \end{array}$$

where Z_t is an exogenous preference shifter. $\beta \in (0, 1)$ is the discount factor. W_t is the nominal wage. B_t are the nominal riskless disccount bonds purchased in period t. R_t is the interest rate for the period bonds.

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$$\frac{W_t}{P_t} = N_t^{\varphi} C_t^{\sigma}, \tag{3.9}$$

$$C_t^{-\sigma} = \beta \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} (1 + R_t) C_{t+1}^{-\sigma}. \tag{3.10}$$

A first order approximation to Equation (3.9) yields:

$$\hat{w}_t - \hat{p}_t = \varphi \hat{n}_t + \sigma \hat{c}_t, \tag{3.11}$$

where hat lowercase letters represent deviations from the non-inflationary steady state. Equation (3.11) can be seen as a competitive labour supply schedule where the amount of labour supplied is a function of the real wage and the utility of consumption. Where the parameter φ is the inverse Fisher labour supply elasticity.

A first-order approximation around the zero inflation steady state to Equation (3.10) yields the Euler equation for consumption:

$$\hat{c}_t = E_t \{\hat{c}_{t+1}\} - \frac{1}{\sigma} \left(\hat{r}_t - E_t \{\pi_{t+1}\} - (1 - \rho_z) \hat{z}_t\right), \tag{3.12}$$

where $\pi_{t+1} = p_{t+1} - p_t$ is the rate of inflation between t and t+1 in the home economy. The preference shock is assumed to follow an autorregressive process according to $\hat{z}_{t+1} = \rho_z \hat{z}_t + \varepsilon_t^z$. In the same way, $\hat{c}_t$ is the home consumption deviation from the steady state and $\hat{r}_t$ is the interest rate deviation from the steady state.

3.2.2 National Firms

The national production of one specific variety of a good is the sum of its consumption on both countries:

$$Y_t(h) = \gamma C_t(h) + (1-\gamma) C_t(h)^* = \left(\frac{P_t(h)}{P_H t}\right)^{-\varepsilon} \left(\frac{P_{Ht}}{P_{Ft}}\right)^{\gamma-1} C_t^W = \left(\frac{P_t(h)}{P_H t}\right)^{-\varepsilon} S_t^{1-\gamma} C_t^W, \tag{3.13}$$

where $C_t(h)^*$ stands for the consumption in the foreign economy specific varieties of goods produced in the home country. C_t^W represents the aggregate world consumption and $S_t \equiv \frac{P_{Ft}}{P_{Ht}}$ represents the terms of trade.

Defining Y_t as:

$$Y_t \equiv \left[\frac{1}{\gamma}\int_0^\gamma Y_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3.14}$$

Using Equation (3.13) in the above equation, I obtain an equation linking output and consumption:

$$Y_t = \left(\frac{P_{Ht}}{P_{Ft}}\right)^{\gamma-1} C_t^W = S_t^{1-\gamma} C_t^W, \tag{3.15}$$

where $C^W \equiv \gamma C_t + (1-\gamma)C_t^*$ and

$$Y_t(h) = \left(\frac{P_{Ht}(h)}{P_{Ht}}\right)^{-\varepsilon} Y_t. \tag{3.16}$$

3.2.3 Aggregate employment

The number of firms is assumed to be equal to the number of households. The home country has a continuum of firms indexed by $i \in [0, \gamma]$. Each firm produces a differentiated good, but all of them use the same technology according to:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}, \tag{3.17}$$

where A_t represents the level of technology. Technology is assumed to be common to all firms operating in the home country. Aggregate employment in the home country is given by the sum of employment across firms:

$$N_t \equiv \int_0^\gamma N_t(i) di. \tag{3.18}$$

Using the production function from Equation (3.17) and Equation (3.16), I can write:

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^\gamma \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\frac{\varepsilon}{1-\alpha}} di, \tag{3.19}$$

which linearised with respect to the non-inflationary SS:

$$(1-\alpha)\hat{n}_t = \hat{y}_t - \hat{a}_t + \hat{d}_t, \tag{3.20}$$

where $\hat{a}_t$ is the technology deviation from the steady state. It evolves according to and autorregressive process $\hat{a}_{t+1} = \rho_a \hat{a}_t + \varepsilon_t^a$ with $\rho_a \in [0,1]$. Up to a first-order approximation, $d_t \equiv (1-\alpha) log \int_0^\gamma (P_t(i)/P_{Ht})^{-\frac{\varepsilon}{1-\alpha}} di$ is zero in the neighbourhood of the zero inflation SS. Then,

$$\hat{n}_t = \frac{1}{1-\alpha}\left(\hat{y}_t - \hat{a}_t\right). \tag{3.21}$$

Profit maximization

Using the formalism proposed by Calvo (1983), in any period, with probability $1-\theta$, each firm may reset its price. This probability is independent of the time elapsed since the last adjustment. Each period the proportion $1-\theta$ of the producers reset their prices while the other fraction θ does not. Considering this, the average duration of each price will then be $\frac{1}{1-\theta}$

Whenever firms reset their prices, they have to choose a price P_{Ht}^* which maximizes the current value of the profit that are going to be generated in the time this price remains effective.

$$\begin{array}{ll} \text{Max} & \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \frac{1}{P_{Ht+k}} \left(P_{Ht}^* Y_{t+k|t} - \mathcal{C}_{t+k}\left(Y_{t+k|t}\right)\right)\right\}, \\ \text{s.t.} & Y_{t+k|t} = \left(\frac{P_{Ht}^*}{P_{Ht+k}}\right)^{-\varepsilon} Y_{t+k}. \end{array}$$

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$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \frac{Y_{t+k|t}}{P_{Ht+k}} \left(P_{Ht}^* - \frac{\varepsilon}{\varepsilon-1} \Psi_{t+k|t}\right)\right\} = 0, \tag{3.22}$$

where $\Lambda_{t,t+k} \equiv \beta \frac{U_{c,t+k}}{U_{c,t}}$ is the stochastic discount factor, $U_{c,t}$ is the marginal utility of consumption in period t. $\Psi_{t+k|t} \equiv \mathcal{C}'_{t+k}\left(Y_{t+k|}\right)$ is the marginal cost function where $\mathcal{C}_t()$ is the nominal cost function and $Y_{t,t+k|t}$ is the output in period $t+k$ for a firm that last reset its prices in period t.

From the firm's cost minimization problem I obtain the individual firm marginal cost. The first-order approximation to the marginal cost function yields:

$$\hat{\psi}_{t+k|t} = \hat{w}_{t+k} - \hat{a}_{t+k} + \alpha \hat{n}_{t+k|t}, \tag{3.23}$$

where $\hat{\psi}_{t+k} \equiv \frac{\Psi_{t+k}-\Psi}{\Psi}$. Letting $\hat{\psi}_t \equiv \int_0^\gamma \hat{\psi}_t(i)di$.

$$\hat{\psi}_{t+k} = \hat{w}_{t+k} - \hat{a}_{t+k} + \alpha\hat{n}_{t+k}. \tag{3.24}$$

Substituting Equations (3.21) and (3.24) in Equation (3.23), I obtain:

$$\psi_{t+k|t} = \psi_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(p^*_{Ht} - p_{Ht+k}\right). \tag{3.25}$$

The Taylor expansion of Equation (3.22) around a zero inflation SS yields, after some manipulation:

$$p_{Ht} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\hat{\psi}_{t+k|t}\right\}.$$

Using the above equation with (3.25) and $p_{Ht} = \theta p_{Ht-1} + (1-\theta)p^*_{Ht}$.[3] I obtain:

$$\pi_{Ht} = \beta E_t\left\{\pi_{Ht+1}\right\} - \lambda\hat{\mu}_t, \tag{3.26}$$

with $\lambda \equiv \frac{1-\theta}{\theta}(1-\beta\theta)\frac{1-\alpha}{1-\alpha+\varepsilon\alpha}$, $\hat{\mu}_t \equiv p_{Ht} - \hat{\psi}_t$.

$$\hat{\mu}_t = -\frac{\psi+\alpha}{1-\alpha}\hat{y}_t + \frac{1+\varphi}{1-\alpha}\hat{a}_t - \sigma\hat{c}_t - (1-\gamma)\hat{s}_t. \tag{3.27}$$

3.2.4 Foreign Economy

The foreign consumers decide between national and foreign consumption goods according to:

$$C^*_t \equiv \frac{C^{\gamma}_{Ht}C^{1-\gamma}_{Ft}}{\gamma^\gamma(1-\gamma)^{(1-\gamma)}}. \tag{3.28}$$

As a consequence, as I already showed for the national economy:

$$C^*_{Ht}P_{Ht} = \gamma P_t C^*_t, \tag{3.29}$$

$$C^*_{Ft}P_{Ft} = (1-\gamma)P_t C^*_t, \tag{3.30}$$

where C^*_{Ht} is the quantity of goods produced in the home economy and consumed in

[3] proof in Galí (2015).

the foreign. In the same way, C^*_{Ft} is the quantity of goods produced and consumed in the foreign economy. C^*_t is the aggregate quantity of goods consumed in the foreign economy. From the law of one price derives that the prices of each variety is equal in both countries. As a consequence, the general prices must be also equal.

Considering foreign households have a utility function symmetrical to the national ones, their first-order necessary conditions from the maximization will be:

$$\frac{W^*_t}{P_t} = N_t^{*\varphi} C_t^{*\sigma}, \tag{3.31}$$

$$C_t^{*-\sigma} = \beta \frac{Z^*_{t+1}}{Z^*_t} \frac{P_t}{P_{t+1}} (1 + R_t) C_{t+1}^{*-\sigma}. \tag{3.32}$$

The first-order expansion around a non-inflationary steady state yields:

$$\hat{w}^*_t - p^*_t = \varphi \hat{n}^*_t + \sigma \hat{c}^*_t, \tag{3.33}$$

$$\hat{c}^*_t = E_t \left\{ \hat{c}^*_{t+1} \right\} - \frac{1}{\sigma} \left(\hat{r}_t - E_t \left\{ \pi^*_{t+1} \right\} - (1 - \rho_z) \hat{z}^*_t \right), \tag{3.34}$$

where W^*_t is the nominal wage for the workers in the foreign economy. N^*_t is the employment in the foreign economy and Z^*_t is the exogenous preference shifter. Like in the home economy, this last parameter follows an autorregressive process according to $\hat{z}^*_{t+1} = \rho^*_z \hat{z}^*_t + \varepsilon^{z*}_t$. Lowercase hat letters represent deviations from the steady state.

The firms in the foreign economy solve a symmetrical problem where the inflation of foreignly produced goods will behave according to:

$$\pi_{Ft} = \beta E_t \left\{ \pi_{Ft+1} \right\} - \lambda \hat{\mu}_t, \tag{3.35}$$

with:

$$\hat{\mu}_t = -\frac{\psi + \alpha}{1 - \alpha} \hat{y}_t + \frac{1 + \varphi}{1 - \alpha} \hat{a}_t - \sigma \hat{c}_t + \gamma \hat{s}_t. \tag{3.36}$$

3.2.5 Risk sharing

The interest rate of the monetary union must be equal in both countries. Then, using Equation (3.10) and Equation (3.32), I can write:

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{Z_{t+1}}{Z_t} = \frac{1}{1+R_t} = \beta \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{Z^*_{t+1}}{Z^*_t}.$$

Combining the above with the law of one price and considering the model around a symmetric equilibrium:

$$C_t = C^*_t \left(\frac{Z_t}{Z^*_t} \right)^{\frac{1}{\sigma}}, \tag{3.37}$$

which linearised around the symmetric non-inflationary steady state:

$$\hat{c}_t - \hat{c}^*_t = \frac{1}{\sigma}(z_t - z^*_t). \tag{3.38}$$

3.2.6 Equilibrium

The first-order approximation to Equation (3.15) and its corresponding foreign counterpart yield:

$$\hat{y}_t = (1-\gamma)\hat{s}_t + \gamma\hat{c}_t + (1-\gamma)\hat{c}^*_t, \tag{3.39}$$

$$\hat{y}^*_t = -\gamma\hat{s}_t + \gamma\hat{c}_t + (1-\gamma)\hat{c}^*_t. \tag{3.40}$$

I can use the linearized risk sharing condition from Equation (3.38) to write Equation (3.39), the output from the home economy, as a function of home consumption:

$$\hat{y}_t = (1-\gamma)\hat{s}_t + \hat{c}_t - \frac{1-\gamma}{\sigma}\left(\hat{z}_t - \hat{z}^*_t\right). \tag{3.41}$$

For the foreign economy, using Equation (3.38), I can write Equation (3.40), the output for the foreign economy, as a function of foreign consumption:

$$\hat{y}^*_t = -\gamma\hat{s}_t + \hat{c}^*_t + \frac{\gamma}{\sigma}\left(\hat{z}_t - \hat{z}^*_t\right). \tag{3.42}$$

Combining Equations (3.38), (3.39) and (3.40), I can write the terms of trade with respect to the difference between home and foreign output.

$$\hat{s}_t = \hat{y}_t - \hat{y}^*_t. \tag{3.43}$$

3.2.7 Welfare Analysis

The last piece to close the above model is to introduce a policy rule consistent with the ECB mandate. The seminal work of Rotemberg & Woodford (1999) established a second order approximation to utility losses as the baseline criteria to find the optimal policy rule. When I apply this methodology to this model, a new variable appears in the welfare loss function, the terms of trade. This variable is the one accounting for the trade between both economies produced by the price difference between home and foreign goods. For the model considered in this chapter, the second order approximation to the consumer welfare losses can be written as a function of steady state consumption as:

$$\mathbb{W} = -E_0\gamma \sum_{t=0}^{\infty}\left(\frac{U_t - U}{U_C C}\right) - E_0(1-\gamma)\sum_{t=0}^{\infty}\left(\frac{U_t^* - U^*}{U_C^* C^*}\right). \tag{3.44}$$

The second order Taylor expansion for the utility function in the home country around the steady state can be written with respect to output using Equation (3.20). After some manipulation I obtain:

$$\begin{aligned} U_t - U = {} & C^{1-\sigma} Z\left[\hat{c}_t(1-\hat{z}_t) + \frac{1-\sigma}{2}\hat{c}_t^2\right] \\ & - ZN^{1+\varphi}\left[\hat{y}_t(1-\hat{z}_t) + \hat{d}_t + \frac{1+\varphi}{2}\left(\hat{y}_t - \hat{a}_t\right)^2\right] + t.i.p. \end{aligned} \tag{3.45}$$

where $t.i.p.$ stands for terms independent of policy.

Efficiency in the SS implies $(1-\alpha)\left(Y/N\right) = \left(N^{\varphi}/C^{-\sigma}\right)$. From the risk sharing condition in Equation (3.37), $C = C^*$ in the SS, using this in Equation (3.15), I find $Y = C$. Using these conditions on Equation (3.45), I can write:

$$\frac{U_t - U}{U_C C} = -\frac{1}{2}\left[-(1-\sigma)(1-\gamma)^2\hat{s}_t^2 + \hat{d}_t + \left(\sigma\frac{\varphi+\alpha}{1-\alpha}\right)\hat{y}_t^2\right] + t.i.p. \tag{3.46}$$

A second order approximation around $\hat{d}_t$ yields $\hat{d}_t = \frac{\varepsilon}{\Theta} var\left\{p(h)_t\right\}$.[4]

[4]The proof can be found in Galí (2015).

$$\frac{U_t - U}{U_C C} = -\frac{1}{2}\left[-(1-\sigma)(1-\gamma)^2\hat{s}_t^2 + \frac{\varepsilon}{\Theta} var\left\{p(h)_t\right\} \right.$$
$$\left. + \left(\sigma\frac{\varphi+\alpha}{1-\alpha}\right)\hat{y}_t^2\right] + t.i.p. \quad (3.47)$$

As shown in Woodford (2003), $\sum_{t=0}^{\infty}\beta^t var\left\{p(h)_t\right\} = \frac{\theta}{(1-\beta\theta)(1-\theta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2$. Using this result in the above equation and the linearization of Equation (3.8), I can write Equation (3.47) as:

$$\frac{U_t - U}{U_C C} = -\frac{1}{2}\left[-(1-\sigma)(1-\gamma)^2\hat{s}_t^2 + \frac{\varepsilon}{\lambda}\left(\gamma^2\pi_{Ht}^2 + (1-\gamma)^2\pi_{Ft}^2\right) \right.$$
$$\left. + \left(\sigma\frac{\varphi+\alpha}{1-\alpha}\right)\hat{y}_t^2\right] + t.i.p. \quad (3.48)$$

In the same way, the equation for the foreign country will be:

$$\frac{U_t^* - U^*}{U_C^* C^*} = -\frac{1}{2}\left[-(1-\sigma)\gamma^2\hat{s}_t^2 + \frac{\varepsilon}{\lambda}\left(\gamma^2\pi_{Ht}^2 + (1-\gamma)^2\pi_{Ft}^2\right) \right.$$
$$\left. + \left(\sigma\frac{\varphi+\alpha}{1-\alpha}\right)\hat{y}_t^{*2}\right] + t.i.p. \quad (3.49)$$

Substituting Equations (3.48) and (3.49) into Equation (3.44), I obtain:

$$\mathbb{W} = \frac{1}{2}E_0\sum_{t=0}^{\infty}\left[-(1-\sigma)(1-\gamma)\gamma\hat{s}_t^2 + \frac{\varepsilon}{\lambda}\left(\gamma^2\pi_{Ht}^2 + (1-\gamma)^2\pi_{Ft}^2\right) \right.$$
$$\left. + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\left(\gamma\hat{y}_t^2 + (1-\gamma)\hat{y}_t^*\right)\right] + t.i.p. \quad (3.50)$$

The average welfare loss per period is given by a linear combination of the variances of output gap, inflation and terms of trade.

$$\mathbb{L} = \frac{1}{2}\Big[-(1-\sigma)(1-\gamma)\gamma\hat{s}_t^2 + \frac{\varepsilon}{\lambda}\left(\gamma^2\pi_{Ht}^2 + (1-\gamma)^2\pi_{Ft}^2\right) \\ + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\left(\gamma\hat{y}_t^2 + (1-\gamma)\hat{y}_t^*\right)\Big] + t.i.p. \quad (3.51)$$

In the case of a monetary union like the one considered above, I have shown that the welfare loss function is not only a function of output and inflation gap, but also of the terms of trade. If the central bank of a union like this one wants to find the best policy rule, it should take into account the terms of trade, in addition to the output and inflation gaps of the union.

From a mathematical point of view, it would be possible to study the optimal policy function for the monetary union from a benevolent central planner point of view. But, like in any other central planner equilibria, it would be necessary to assume the monetary authority to be able of choosing the desired levels of inflation and output gap at each point in time. In practice, without the existence of the central planner, the central banks cannot directly set either variable. The most common approach in the literature to this problem is to adopt an interest rate rule that guarantees the desired outcome is attained. The most common interest rules in monetary policy analysis are the ones derived from Taylor (1999) which relate interest rate to output and inflation gaps.

A regular Taylor rule weighted by the size of each economy in the monetary union, like the one established by the ECB Governing Council, would look like:

$$\hat{r}_t = \phi_\pi\left(\gamma\pi_{Ht} + (1-\gamma)\pi_{Ft}\right) + \phi_y\left(\gamma\hat{y}_t + (1-\gamma)\hat{y}_t^*\right). \quad (3.52)$$

With this rule, with the same structure than the one currently used by the ECB, the monetary authority reacts to the union output and inflation gaps. But this rule leaves aside a variable which appeared in the welfare loss function of the union, the terms of trade (s_t). The central bank could obtain a better outcome if it took into account all variables affecting the welfare of the union. If I introduce the terms of trade directly inside of Equation (3.52), I would have an equation like:

$$\hat{r}_t = \phi_\pi\left(\gamma\pi_{Ht} + (1-\gamma)\pi_{Ft}\right) + \phi_y\left(\gamma\hat{y}_t + (1-\gamma)\hat{y}_t^*\right) + \phi_s\hat{s}_t. \quad (3.53)$$

However, this policy function has a problem: based on their definition, $\hat{s}_t \equiv p_{Ft} -$

p_{Ht}, the terms of trade can be positive or negative depending on which country is suffering the shock. So, with this rule, if the home country is suffering the shock, the central bank would react increasing/decreasing the interest rate, but if it is the foreign country the one suffering the shock, the central bank reaction in terms of trade would be just the opposite because the sign of $\hat{s}_t$ would have switched. This issue can be easily addressed assuming the interest rate to be a function of the absolute value of the terms of trade instead of its actual value. Since the central bank cares about instability regardless of the direction of the fluctuation. With this modification the policy rule will become:

$$\hat{r}_t = \phi_\pi \left(\gamma \pi_{Ht} + (1-\gamma)\pi_{Ft}\right) + \phi_y \left(\gamma \hat{y}_t + (1-\gamma)\hat{y}_t^*\right) + \phi_s |\hat{s}_t|, \tag{3.54}$$

where the union interest rate is affected by the absolute value of the terms of trade. With this modification, it is no longer relevant which economy suffers the shock because the central bank reacts in the same way to any deviation, independently of its source. Once I have defined a new policy rule, in the next section, I estimate the current value of ϕ_s for the US and the Euro Area.

3.3 How to estimate the terms of trade

The next step in this analysis is to find a proxy variable for the terms of trade. In the model I have defined the terms of trade to be $\hat{s}_t \equiv p_{Ft} - p_{Ht}$. This is, the terms of trade is the difference between the prices in one country and the other in a two-country monetary union. In the real world, we do not have two-country unions but multilateral unions. The EA is constituted by 19 countries and the US has 50 states plus the District of Columbia. As a consequence, I have two options: to create two countries with an aggregate of countries/states or to find a way of computing the terms of trade for the whole union. The first option has a big problem, the countries/states are not easy to divide into two groups as I shown in Section 3.1. In addition, some effects could be masked by the aggregation as happens when I aggregate for the whole union. This makes the second alternative the most representative of what is happening in the union.

In order to estimate the terms of trade from the individual states/countries I need to use a statistic compiling all this information. From the definition of the terms of trade, the first option would be to use the average between the difference of each country prices and the union prices:

$$s = \sum_{i=1}^{n} \frac{p_i - p_{EA}}{n}.$$

This statistic would estimate the average price deviation of each country from the union. However, all countries/states have the same weight in the statistic. This is unreliable because some countries/states are much bigger than others in both GDP and population. The weight of Malta/Utah should not be the same than the weight of Germany/California. This calls for a weighting consistent with the different participation of each country/state in the union. It could be used both the participation in population or the participation in GDP. I am going to use the latter because it is the one used to aggregate the GDP and with it, the GDP deflator. The above equation will then become:

$$s = \sum_{i=1}^{n} (p_i - p_{MU}) \frac{RGDP_i}{RGDP_{MU}}.$$

This is an option to estimate the value of the terms of trade. But, what appears in the monetary policy rule, is not the terms of trade by itself, but its absolute value. I then propose to use the square root of:

$$s_p^2 = \sum_{i=1}^{n} \left\{ \frac{(p_i - p_{MU})^2}{n} \right\}, \tag{3.55}$$

$$s_{wp}^2 = \sum_{i=1}^{n} \left\{ (p_i - p_{MU})^2 \frac{RGDP_i}{RGDP_{MU}} \right\}. \tag{3.56}$$

In the first case I am giving the same weight to all states/countries while in the second case I am weighting them by their Real GDP (RGDP) participation in the union.

A new problem arises when looking for the data for prices. The obvious pick is the value in levels of the GDP deflator (Nominal GDP/ Real GDP), but, this statistic needs a base year. A year in which all the prices are equal to 100 for all states/countries. As I am interested in the evolution of the difference between prices and not in prices itself, I can homogenize the data for both unions using as a base year 1997, the year before the beginning of the studied series. With this homogenization, I am able to show how price dispersion has evolved in both unions after 1997. As a consequence of this choice, the dispersion in 1997 will be zero and it will necessarily grow afterwards.

For robustness, it would be good to have another way of estimating the terms of trade to see if the behaviour is similar to the one shown above. Given Equation (3.43), in a model with symmetric economies, the terms of trade could be thinked of as the dispersion on the GDP per capita in the union.[5] This statistic does not come from the definition of the variable, but from the particular setting of this model. It is then impossible to use it in the regression because, in the real world, there are not just two economies and they are not symmetric. But this statistic will help me to prove the rising heterogeneity between countries inside of the monetary union once it does not need a base year. If I take the Real GDP per capita, the statistic will be the square root of:

$$s^2_{WRGDP} = \sum_{i=1}^{n} \left\{ \left(\frac{RGDP_i}{Pop_i} - \frac{RGDP_{MU}}{Pop_{MU}} \right)^2 \frac{RGDP_i}{RGDP_{MU}} \right\}. \tag{3.57}$$

This statistic captures the dispersion in the Real GDP per capita. When this dispersion increases it means that the gap in Real GDP per capita increases between states/countries. While, when it falls, it means that the gap in Real GDP per capita between states/countries is closing. Finally, I add another statistic combining GDP and prices, the dispersion of Nominal GDP(NGDP) per capita.

$$s^2_{WNGDP} = \sum_{i=1}^{n} \left\{ \left(\frac{NGDP_i}{Pop_i} - \frac{NGDP_{MU}}{Pop_{MU}} \right)^2 \frac{RGDP_i}{RGDP_{MU}} \right\}. \tag{3.58}$$

This statistic captures the convergence/divergence of countries and states in both Real GDP and prices. It also has the advantage of not requiring a base year, making all periods comparable. Figure 3.3 shows the plots of the square root of these four statistics representing the terms of trade for the US and the EA.

[5]The effects of the terms of trade on GDP is a relationship established in the literature in works like Mendoza (1995).

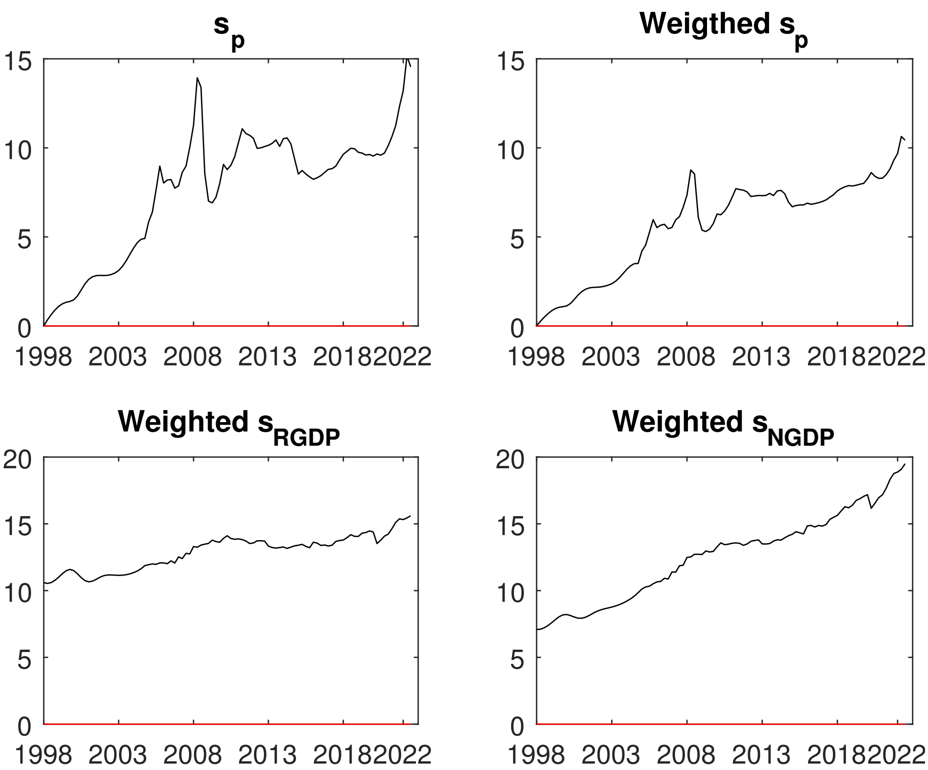

Figure 3.4: Estimation of the terms of trade for the US.

Comparing the first two plots in Figure 3.3 is observed how, thanks to weighting by the size of each state/country with respect to the union, the statistic is lower and less volatile. In both cases, the dispersion grows quickly during the first years as a consequence of the existence of a base year. If I could use 1970 or 1980 as the base year, this effect would be eliminated. Once the series arrives at 2005-2006 this effect dissipates and the series is relatively constant and below 10. There are some relevant deviations, like the one before the 2008 crack when the dispersion was growing, and just after the crack when the dispersion fell. Once it recovers in 2012, it is relatively constant until the COVID and the subsequent shortage supplies. In all, the weighted terms of trade computed using the price dispersion is relatively constant and bounded below 10.

Something similar can be observed if we turn our view to the second couple of plots in the above figure. Looking at the dispersion in the Real GDP, it grows very slowly during the considered period. These series are less volatile than the ones of prices and the effect of the 2008 crisis is not as clear as it is above. But it it is observed how the growth in dispersion is faster before 2008 than after it. It can also be observed how the dispersion falls after 2009-2010 like in the first and second plots. The effect of COVID is more vividly observed in the Real GDP dispersion plot than it was in the prices one. As above, it is observed both, the fall in the first quarters of 2020 and the rise shortly after. Finally, in the last plot of the figure it is observed roughly the same behaviour than in the Real GDP one but now, it is fluctuating around a trend. This trend is the consequence of including inflation in the analysis. Figure 3.5 reproduces the same plots for the EA.

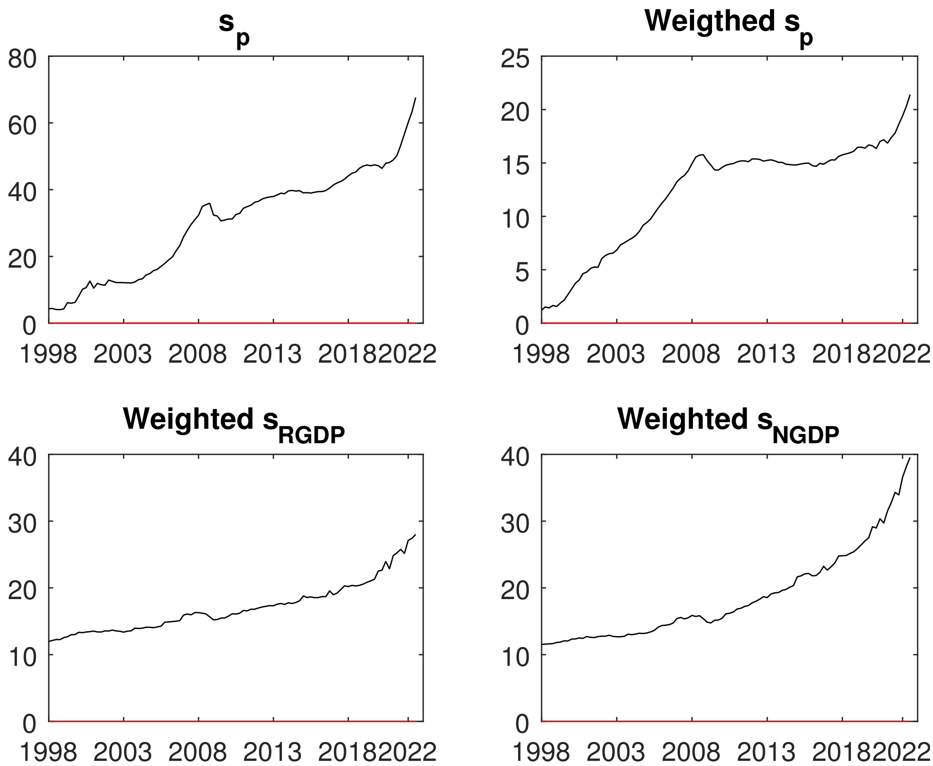

Figure 3.5: Estimation of the terms of trade for the EA.

The first thing to note is the huge difference between the first and second plots of Figure 3.5. The dispersion in the plot without weighting is huge if compared to the one where the dispersion is weighted by the size of each country, it should be noted the difference in the y-axis labels. Like in the US plot, it is observed how during the first years of the series the dispersion grows quickly. This is a bias caused by the existence of a base year for prices in 1997. Like in the US, this rise stops in 2008 as a consequence of the financial crisis but, unlike the US, the rise in the EA is faster, constant and without any relevant fluctuations. In addition, the levels of dispersion in the EA are higher than the ones in the US. The maximum in the US is a little above 10 in 2022 while in the EA it is already above 15 in 2008. Like in the US, the dispersion in the EA falls after the financial crisis of 2008, but the fall is smaller than in the US and it lasts less periods. After this, it stabilizes around 2010 and stays more or less constant until the COVID and subsequent shortages supplies shock.

Contrarily to the US, where the terms of trade seemed to be fluctuating around the mean, in the EA it seems like it is fluctuating around a trend. But, like in the US, this tendency may be caused by the existence of a base year. The best way to test the robustness of this findings is to compare it with a different statistic which does not need such a thing. Like before, I plot the weighted dispersion in the Real and Nominal GDP for the EA countries in the low panels of Figure 3.5.

Looking to the GDP dispersion plots, I observe how the dispersion grows for both variables during the studied period. While in the US the statistic based on the Real GDP grew from 10 to 15, which could just be a fluctuation around the mean, in the EA the same statistic grew from 10 to 30, this is, it tripled in roughly 20 years. Something similar is observed in the statistic based on the Nominal GDP, while the plot for the US seemed to be fluctuating around a trend, in the EA it looks like the rate of growth of this statistic is accelerating.

Based on this info, it can be concluded that the EA is a much more heterogeneous union than the US. In addition, I have shown that the states in the US may be experiencing some degree of divergence, but nothing comparable to the one experienced by the countries inside of the EA. The divergence observed in the EA is much more faster than the one observed in the US. This rising divergence, in a monetary union without a common fiscal authority, may put in danger the capabilities of the individual fiscal authorities to stabilize their economies. To show the strong link existent between the monetary policy and the terms of trade in Table 3.3 I have computed the correlations between them and the interest rate.

	s_p	Weighted s_p	Weighted s_{RGDP}	Weighted s_{NGDP}
r_{US}	-0.58	-0.62	-0.64	-0.63
r_{EA}	-0.74	-0.66	-0.72	-0.73

Table 3.3: Correlations between interest rate and terms of trade.

It can be seen how the correlation between the terms of trade and the interest rate is negative and strong independently of how I measure the former variable. This works as a robustness check to show that the influence of the terms of trade in monetary policy determination should be taken into consideration as I have argued using the union welfare loss function in Equation (3.50). This is consistent with the existent literature. See for instance De Fiore & Liu (2002) or Obstfeld (2009) where this relationship is broadly discussed. I have then a good reason to estimate the monetary policy rule using the terms of trade as an explanatory variable.

3.4 Estimation of the monetary policy rule for the US and EA

The litmus test to know which is the influence of the terms of trade in the monetary policy determination is to estimate the rule for the US and the EA and see if this variable is significant. The first step in this process is to transform the equations (3.52) and (3.54) into the regression functions:

Model 1:

$$r_t = \phi_\pi \pi_t + \phi_y y_t + \epsilon_t, \tag{3.59}$$

Model 2:

$$r_t = \phi_\pi \pi_t + \phi_y y_t + \phi_s s_t + \epsilon_t. \tag{3.60}$$

where π_t and y_t are the weighted average of output and inflation for the union and s_t is the absolute value of the terms of trade. It should also be noted that no intercept has been added as the model is defined in deviations with respect to the steady state.

For the US I use quarterly seasonally and calendar adjusted data from the Bureau of Economic analysis (BEA) and the Federal Reserve between 1998 and 2022. For inflation, I use the annual growth of quarterly GDP deflator; for interest rate, I use the effective federal funds rate. For the output gap, like in Taylor (1999), I have used the HP-filter cycle component of Real GDP per capita. The quarterly GDP deflator is not available for states in the US until 2005. I use cubic interpolation to estimate the quarterly data from the annual one for the period 1998-2004. In both cases the data is available at BEA. The US data is plotted in Figure 3.6:

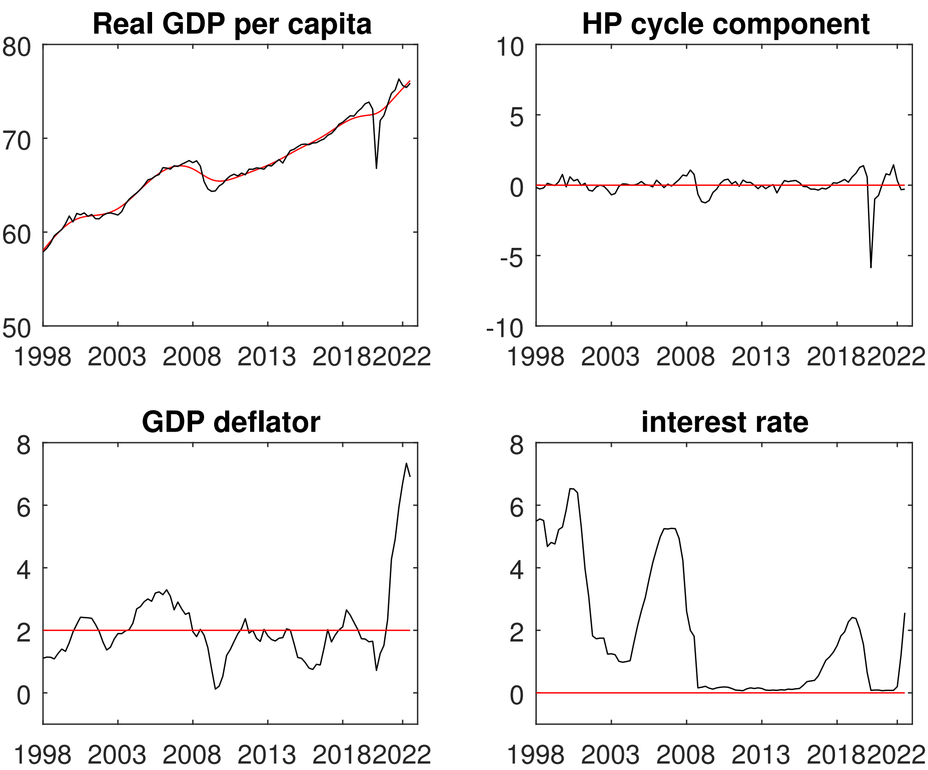

Figure 3.6: The first plot contains the Real GDP per capita of the US in black and the HP-filtter trend component in red.

In Figure 3.6 it is observed how the GDP per capita fell shortly after the crisis of 2008 and shortly after the COVID shock. I have computed the potential GDP per capita as the trend component of the HP-filter which is plotted in red in the first plot of the Figure. The cycle component of the HP-filter is available in the second plot. In the third plot, it can be observed how inflation, measured by the GDP deflator, was fluctuating around the 2% most of the current century. This fluctuation had two remarkable exceptions, the fall following the 2008 crisis and the current rise given by the shortages in input supplies. Finally, in the fourth plot, it is observed how the interest rate has evolved during the last two decades and how it was in the neighbourhood of zero for the lustrum following the 2008 financial crisis. It is observed how the interest rate fell due to the COVID crisis and the fast rise shortly after because of the supply crisis and the Ukraine War.

For the Euro Area, I use quarterly Real GDP per capita and GDP deflator from eurostat. The interest rate is the European Central Bank euribor. I am using the EA-19 as the representation of the EA.

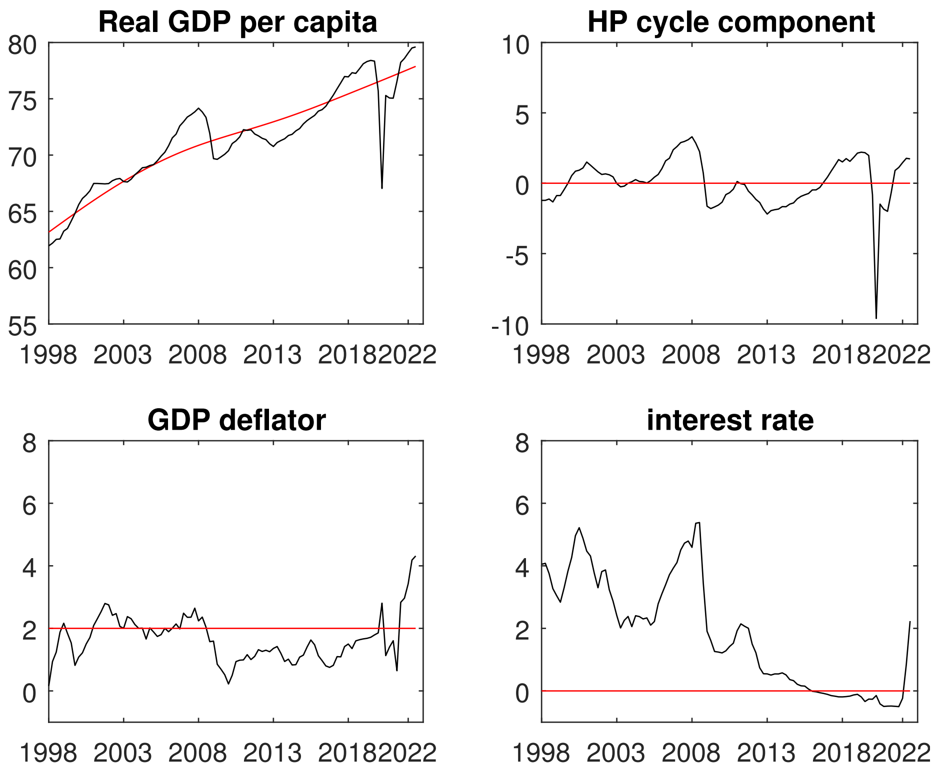

Figure 3.7: The first plot contains the Real GDP per capita of the EA and the HP-filter trend component in red. The y-axis are the same for both currency areas for consistency and comparability.

Figure 3.7 plots the same variables as in Figure 3.6 but for the Euro Area. In the first plot, it is observed how the deviations from the potential GDP per capita are larger in the EA than in the US. The fall in GDP per capita following the 2008 and the one following the COVID crisis observed in the US are not the only relevant deviations from its potential output. It is also observed a relevant fall on GDP per capita following the Euro Area sovereign debt crisis, an important set back in the EA recovery from the financial crisis. In the second plot of Figure 3.7, the cycle component of the HP-filter is provided. If our attention is turned to the GDP deflator, the plot is very similar to the one in the US. This is, inflation is around 2% in the studied period, with a rise in the last periods because of the supply shock affecting the world.

Lastly, but not least, in the fourth plot I have depicted the behaviour of interest rate. It is observed another important difference between the US and the EA, the EA interest rate fell below the 0% threshold after 2015, when the interest rate was already recovering in the US. This was one of the most important symptoms of the stabilization problems the EA confronted when fighting the liquidity trap which followed its sovereign debt crisis.

Up until now, I have described the data but I have not presented empirical conclusive evidence in favour of the importance of having into account the terms of trade when deciding the interest rate of the union. To show the importance of this variable in monetary policy determination, I estimate the Taylor rule (3.59) and the modified Taylor rule (3.60) through Bayesian methods. A Frequentist approach would allow for the parameter for inflation to be negative which would be contrary to ECB and FED mandates. I then exploit the capabilities of Bayesian statistics to include a positive prior distribution for this parameter. Following the results obtained by Taylor (1999) and monetary policy consensus, I have assumed the prior distribution of ϕ_π to be a $LogNormal(0, 1)$. This is, before putting the model through data, I expect the influence of inflation in interest rate to always be positive and to be, with the 95% of probability, between 0.14 and 7.1 in both monetary areas. I have chosen a log-normal prior distribution because both, the ECB and the Fed, are bound to increase/decrease the interest rate whenever their economy suffers a positive/negative inflationary shock. This mandate makes impossible, from an economic theory point of view, for the coefficient of inflation to be negative, making the log-normal distribution the most appropriate to describe this relationship. In the same way, I have assumed the prior distribution for the influence of output gap to be between -3 and 5 with the 95% of probability. This is, ϕ_y is assumed to have a prior distribution of a $N(1, 2)$. The effect of the terms of trade are less known, this is why I center the prior distribution around 0 and assume it to be between -4 and 4 with a 95% probability or, what is the same, I assume a prior distribution for ϕ_s of a $N(0, 2)$. These values are consistent with the Bullard & Mitra (2002) study. Finally, for the interest rate, I have chosen a normal distribution for the likelihood, instead of using a positively bounded distribution, like the log-normal, because the ECB has allowed for the interest rate to be negative.

Table 3.4 displays the results for the Bayesian estimation of the US Policy rules. In the above part of Table3.4, I display the results of estimating Equation (3.59), this is, Model 1. While in the below part of the Table I display the results of the estimation of Equation (3.60) Model 2. This model is divided in 2 submodels, depending on which of the estimators of the terms of trade is used in the regression.

		mean	sd	5%	95%
Model 1	ϕ_y	0.55	0.18	0.25	0.84
	ϕ_π	0.64	0.09	0.49	0.79
	σ	2.10	0.15	1.87	2.37
	WAIC		431.4		
Model 2.1 s_p	ϕ_y	0.40	0.20	0.08	0.73
	ϕ_π	0.91	0.19	0.61	1.22
	ϕ_s	-0.09	0.05	-0.17	0.00
	σ	2.09	0.15	1.86	2.36
	WAIC		431.6		
Model 2.2 Weighted s_p	ϕ_y	0.39	0.20	0.06	0.72
	ϕ_π	0.91	0.19	0.61	1.23
	ϕ_s	-0.12	0.07	-0.23	0.00
	σ	2.09	0.15	1.85	2.36
	WAIC		431.4		

Table 3.4: Bayesian estimation of the Taylor rule (3.59) for the US.

Table 3.4 shows how the coefficients for inflation and output gap are positive and significant for the first model. If the terms of trade are not considered, by each point output deviates from its steady state equilibrium, the Federal Reserve increases interest rate in 0.55 points. In the same way, by each point inflation deviates from its objective, the Fed reacts increasing interest rates in 0.64 points. These values are low if compared with the reaction to inflation and output gaps necessary to ensure uniqueness of equilibrium computed by Bullard & Mitra (2002). If the US was modelled as a country instead of as a group of states sharing a common currency, in most of the cases, the US would not fulfil the Blanchard-Kahn conditions.

This problem disappears when the terms of trade are taken into consideration. In Model 2.1 I have included the terms of trade using the non-weighted statistic developed in Section 3.3. While in Model 2.2 I have used the one weighted by the size of each country in the union. In both cases, when this variable is taken into consideration, the reaction of the Fed to inflation and output gap grows enough for the conditions established in Bullard & Mitra (2002) to be fulfilled. The inclusion of the terms of trade in the regression allows for the separation of the perturbation caused by the terms of trade on inflation and/or output stabilization policies.

The only relevant difference between Models 2.1 and 2.2 is on the size of the effect of the terms of trade on the interest rate. This difference is a direct consequence of the higher volatility of the statistic which does not take into consideration the size of each country in the union. When I look to the comparative statistics between models, I find that the three of them have almost the same values for WAIC. So, non of the three is better than the others from an statistical point of view. Nevertheless, the omission of the terms of trade may cause a problem of biaseness in Model 1 because of the omission of a relevant variable.

For comparing the reaction of the ECB to the terms of trade, I run the same regressions with the same prior distributions and time frame for the Euro Area. Table 3.5 depicts the results of the regression with and without the terms of trade.

		mean	sd	5%	95%
Model 1	ϕ_y	1.23	0.73	0.03	2.41
	ϕ_π	1.01	0.10	0.85	1.17
	σ	1.76	0.13	1.56	1.99
	WAIC		394.5		
Model 2.1 s_p	ϕ_y	0.74	0.85	-0.65	2.11
	ϕ_π	1.66	0.15	1.41	1.92
	ϕ_s	-0.04	0.01	-0.06	-0.03
	σ	1.56	0.11	1.39	1.76
	WAIC		371.8		
Model 2.2 Weighted s_p	ϕ_y	0.96	0.90	-0.52	2.43
	ϕ_π	1.51	0.18	1.22	1.80
	ϕ_s	-0.08	0.02	-0.12	-0.04
	σ	1.68	0.12	1.49	1.89
	WAIC		386.1		

Table 3.5: Bayesian estimation of the Taylor rule for the EA.

In the above table, it is observed how the coefficient for output gap is only significant for the 90% confidence level in the first model. This is not an unexpected outcome because the ECB is an inflation targeting central bank where output targeting is a secondary objective. Considering the coefficient for output deviations not to be significant, the EA would be on average just in the threshold of uniqueness of equilibria following the criteria established by Bullard & Mitra (2002). This is an strange outcome for an inflation targeting central bank as it would be incapable of achieving equilibria in the 50% of the cases: all the cases in the interval where ϕ_π is smaller than 1. But, like before, this changes when I introduce the terms of trade in the regression.

Once the terms of trade are taken into consideration, the reaction of the central bank to inflation grows from 1 to 1.5/1.6 with the 99.5% of the observations above 1. This is, uniqueness of equilibria would be guaranteed in the 99.5% of the cases in the EA if ϕ_y is equal to zero, in a higher proportion if it is bigger than zero. For this to be true, something else must be true: the coefficient for the terms of trade should not be significant because there exist a strong fiscal authority. However, unlike in the US, this is not the case for the EA. The inexistence of a common fiscal authority or coordinated and strong individual ones, leaves the monetary policy exposed to the effects of the terms of trade.

The mandate of the ECB establishes that it has to react to overall inflation in the EA and maintain it below but close to the 2%. This mandate has a problem though, it is possible for all inflation to come from a country or a subset of countries, being the rest of them in equilibria. In a complete monetary union like the US, or in one with powerful enough individual fiscal authorities like the one described by Ferrero (2009), the stability of the economy would be a problem for fiscal authorities and the monetary authority would react as strongly as needed to maintain overall inflation in equilibrium. This reaction would push the economies which are in equilibrium in the union out of it to help the ones out of the equilibrium to their convergence path. It would be then the job of the fiscal authority to ensure the convergence of all economies to their path.

However, this is not what is happening in the EA. When the ECB has to decide how to react to inflation deviations, they know that if the reaction is too strong it may cause problems to individual fiscal authorities and put their economy out of their equilibrium path for good. Then, the higher the number of economies inside of the union diverging, the less stronger the ECB can react to inflation deviations. This is a big problem for economy stabilization as the monetary policy chosen by the ECB is always going to be too weak to fulfil its objective in the optimal amount of time. However, if they react too strongly, some economies may be put out of their convergence path and the integrity of the union might be put in doubt as established by Bullard & Schaling (2009).

This incapability of the ECB to properly react to inflation deviations will produce longer convergence paths to the equilibrium. This effect was already observed in the aftermath of the 2008 financial crisis when the EA took much more time to recover than the US. It is also observed in the weaker reaction of the ECB, if compared to the Fed or the Bank of England, to inflation as a consequence of shortages of supplies following the COVID and Ukraine war. The ECB has tried to solve this problem during the last decade with non-conventional monetary policies like the LTRO. At the same time, the EA policymakers tried to address it for the long run, reformulating the Stability and Growth Pact and passing new legislation to create the European Stability Mechanism to strengthen the power of fiscal authorities. But the evidence seem to conclude this has not been enough to revert the divergent path the EA countries are at.

As a consequence of the supply shock crisis and the Russo-Ukraine war, the divergence between countries is growing at an even higher rate, as the evolution of the terms of trade show. This divergence has been increased by government response to this inflationary shocks. Some countries like Germany, with enough fiscal muscle, have announced measures to fight the effects of inflation on their economies. While others, with lower fiscal muscle like Italy or Greece, cannot follow this course of events. This divergence in the reaction only increases the actual rate of divergence and might put in question the existence of the Euro itself.

The EA countries have to accept that some sacrifices will be needed to be made for the preservation of the union. In the medium run it will be necessary to increase fiscal coordination even with the loss of some national sovereignty. Most importantly, in the short run, it is necessary for the EA countries to stop fiscally competing between them too, as this competition only increases the already existing gap between the countries and paves the way for further divergence. It is also necessary a mechanism for the highly indebted fiscal authorities to be able to access enough funds to stabilize their economy in the same conditions than the rest of the economies.

3.5 Conclusion

In this Chapter I have shown how traditional monetary policy without a fiscal counterpart may be ineffective in a monetary union like the EA. This ineffectiveness arises from the fact that the central bank only reacts to aggregate variables from the monetary union and does not take into consideration the specific deviations of each country from its objective which is left on the hands of the individual fiscal authorities. To put it in a real world example if, in the Euro Area, Spain has an inflation of 4% and Germany is having 1%, considering that Germany is twice the size of Spain, the Euro Area is perfectly in equilibrium around its objective of 2%. However, the Spanish economy is two points above its objective and Germany is one point below hers. The absence of a reaction produces an imbalance where prices from goods produced in Spain will rise faster than the ones produced in Germany, causing a loss of competitiveness in the former with respect to the latter.

Traditionally, this internal effect has been left aside from the monetary policy because it was considered a fiscal policy matter. However, the omission of the terms of trade from the monetary policy rule may cause a bias in their estimation. To show this, I computed the welfare loss function of a two-country New Keynesian model with monetary policy consistent with the ECB mandate. I have shown the welfare loss function to be a function of the terms of trade. Thus, the omission of a relevant variable such as the terms of trade causes an omission of relevant variable bias in any estimation of the policy rule in a monetary union. I have estimated the policy rule for the EA and the US with and without the terms of trade. I have found that in the US the terms of trade are not significant because they have a fiscal authority in charge of stabilizing the economy, guaranteeing homogeneous prices in the union. On the other hand, in the EA, where such an authority does not exist, the monetary policy is disturbed by this heterogeneity, difficulting the job of the central bank to manage price stability. The distortion caused by the terms of trade in the EA reduces the interest rate below its optimal rate, leaving longer convergence paths than in the US.

To solve the problem of multiplicity of equilibria stronger financial integration is required. A simple way of doing this, without the loss of national sovereignty, would be to redefine the concept of price stability. Right now price stability is defined as a constant growth of prices but it could be changed to an homogeneous constant growth of prices. With this modification, the ECB could argue the need to intervene the bonds market of certain economies as part of its mandate to maintain price homogeneity. With this modification of the definition of price stability, the ECB could buy/sell assets from a particular country in order to push inflation up or down as needed, guaranteeing the homogeneity of prices in the common market. In addition, if the ECB mandate comprises to buy sovereign debt of certain countries in distress, the individual fiscal authorities would be powerful enough to stabilize their economy without the need of the EA countries to loss sovereignty creating a common fiscal authority. The down side of this proposal is that some countries will be constantly indebted with the ECB and the rest of the union. But this seems like a low price to pay to maintain the integrity of the Euro. If no new convergence measures are established in the EA, the divergence will continue and the own existence of the Euro as a currency will be at stake.

Appendix

β	Discount factor.
φ	Curvature of labour disutility.
σ	Curvature of consumption utility.
α	Decreasing return of labour.
ε	Elasticity of substitution.
θ	Calvo index of price rigidity.
γ	Proportion of population living in the home economy.
ϕ_π	Inflation coefficient.
ϕ_y	Output coefficient.
ϕ_s	Terms of trade coefficient.
ρ_z	Demand shock home economy: Autoregressive coefficient.
ρ_z^*	Demand shock foreign economy: Autoregressive coefficient.
ρ_a	Technology shock home economy: Autoregressive coefficient.
ρ_a^*	Technology shock foreign economy: Autoregressive coefficient.

Table 3.6: Description of the parameters of the model.

	mean	sd	1998-2002	2003-2008	2008-2012	2013-2018	2018-2022
Euro Area	$1,72$	$0,81$	$1,82$	$2,1$	$1,22$	$1,1$	$2,34$
Belgium	$1,9$	$0,95$	$1,7$	$2,02$	$1,62$	$1,46$	$2,88$
Germany	$1,5$	$1,11$	$0,72$	1	$1,18$	$1,72$	$2,9$
Estonia	$5,07$	$3,7$	$5,86$	$7,18$	$3,56$	$2,74$	$6,02$
Ireland	$2,12$	$3,05$	$4,9$	$2,42$	$-0,8$	$2,28$	$1,82$
Greece	$1,38$	$2,54$	$1,46$	$3,14$	$1,48$	$-0,9$	$1,7$
Spain	$2,03$	$1,64$	$3,26$	$3,86$	$0,5$	$0,46$	$2,1$
France	$1,46$	$0,77$	$1,52$	$2,04$	$1,14$	$0,7$	$1,88$
Italy	$1,74$	$0,85$	$2,32$	$2,48$	$1,52$	$0,94$	$1,44$
Cyprus	$1,83$	$1,93$	$2,44$	$3,2$	$1,96$	$-0,52$	$2,1$
Latvia	$4,54$	$6,34$	$6,22$	$7,26$	$2,46$	$1,34$	$5,62$
Lithuania	$4,49$	$4,68$	$6,38$	$4,86$	$3,38$	$1,6$	$6,28$
Luxembourg	$3,06$	$2,3$	$2,04$	$3,7$	$3,76$	$1,66$	$4,16$
Malta	$2,24$	$1,45$	$1,23$	$1,12$	$2,32$	$2,58$	$2,64$
Netherlands	$1,98$	$1,28$	$3,02$	$2,02$	1	$0,84$	$3,04$
Austria	$1,82$	$0,87$	$1,1$	$1,92$	$1,74$	$1,78$	$2,56$
Portugal	$2,2$	$1,31$	$3,5$	$3,06$	$0,54$	$1,62$	$2,3$
Slovenia	$2,12$	$1,79$	$2,54$	$2,18$	$1,68$	$1,1$	$3,1$
Slovakia	$3,87$	$4,36$	$3,88$	$8,5$	$3,44$	$0,16$	$3,36$
Finland	$1,72$	$1,05$	$1,74$	$1,08$	$2,14$	$1,34$	$2,3$

Table 3.7: Average inflation measured through GDP deflator between 1998 and 2022 in the Euro Area. The last five columns have the average inflation during a 5 year interval.

Chapter 4

On the Phillips Curve in the Euro Area

The study of the relation between unanticipated inflation and unemployment gap commonly known as the Phillips Curve after Phillips (1958), has regained interest in the last years. First the supply chain problems caused by the COVID outbreak and latter the break of the Russo-Ukraine war has pushed inflation up in all the world. With the come back of inflation, it also came back the discussion about which is the optimal monetary policy in the current environment.

The existence of a distortionary effect of inflation over the real economy is widely agreed among economist. To fight these distortions, price stability is embedded in the foundation of all monetary authorities around the glove. In the advanced economies, these monetary authorities are created to be independent of political interference to guarantee the pursue of this goal even when it may have unpopular effects on the short run. With the objective of achieving price stability, monetary authorities increase/reduce the monetary supply to reduce/increase the interest rate on their economies. These increments/reductions of the interest rates affect the real economy under its supervision and their labour market with it. These policies then indirectly affect inflation as the employees increase/manage their salary claims.

The monetary authorities are the main source of policies dedicated to stabilize the economy but they are not the only ones. The fiscal authorities, when setting their budget, can include policies which may affect economic activity. Looking at a country as a whole, where it has its own monetary and fiscal policies, the scope of action for both authorities coincide, aligning their objectives with the ones of the country. Currently, in most of the advanced economies, monetary and fiscal policies are not designed by the same institutions. On the one hand, monetary policy is usually designed by a central bank, or a similar monetary authority, independently of the political government. Whereas, fiscal policy is designed by the government when setting their annual budget. This dichotomy on the design of monetary and fiscal policies is not relevant as long as the scope of action is the same for both institutions or, what is the same, if both policies are set at the same level of government. Whichever monetary or fiscal policy is implemented, this policy affects all the area under their supervision. Therefore, the adequate level of analysis to study the effects of this policies is the one at which these policies are implemented. In the case of the US, where the monetary policy is independently implemented by the FED while the fiscal policy is set by the US Treasury, these policies are set for all the states of the union at the union level. Some states may implement a different fiscal policy than the one implemented by the US Treasury, but the fiscal power of the US Treasury is superior to the one of any of its states.[1]

However, the EA is a different story. In the EA, the monetary policy is implemented by the European Central Bank having the best interest of the EA as a whole in mind. The objective of the ECB is to maintain average inflation in the EA close but below the 2%. Despite the existence of different objectives for the FED and the ECB, their action range is the same, the whole monetary area under their jurisdiction. However, the EA does not have a common fiscal authority like the US Treasury. Because of the inexistence of such an institution in charge of implementing a coordinated policy for all the members of the union, the fiscal policy is totally left in the hands of the government of each country. Each country then sets the policy better matching its individual objectives, even when these enter in conflict with the EA objectives. The absence of a unique fiscal authority or a highly coordinated network of national authorities on its defect, creates a new level of variation in the Phillips Curve which could potentially affect the relation between inflation and unemployment in the whole monetary union.

Researchers have approached the study of the EA Phillips Curve from two different perspectives. Some researchers have studied the Phillips Curve relation from an aggregate perspective, where a pooled approach is chosen, assuming this relation to be constant across all EA countries. This approach is very similar to the one where the Phillips Curve for the EA as a whole is estimated using the weighted average of each variable to create the EA variables. In both cases, the EA is treated as a unique entity with a common curve for all their subentities. This approach has been followed by

[1]According to the National Association of State Budget Officers, the total expenditure in fiscal year 2022 in the state with the largest budget, California, was 510 billion while the expenditure of the US treasury was above the 6 trillion dollars.

multiple researchers, a non-exhaustive list of them include Ball & Mazumder (2021), Moretti et al. (2019) or Passamani et al. (2022).

This is not the unique approach though, other researchers have taken the opposite path, where instead of assuming the existence of a common curve, they assume the existence of an individual curve in each EA country. The existence of a particular curve in each country is not incompatible with the existence of a common curve as long as these individual curves are a function of the common curve. However, this is not the path taken in the literature. Authors like Amberger & Fendel (2017*b*), Amberger & Fendel (2017*a*) or Hindrayanto et al. (2019) estimate the Phillips Curve for each country independently, assuming with it that what is happening with Spanish or French inflation and labour markets is irrelevant to explain what happens with German ones.

The coexistence of both approaches is not compatible. It is not possible for each country inside the EA to have its own independent Phillips Curve at the same time than a common Phillips Curve exist for all of them. In the former case, when the curves are looked at individually, the researcher is assuming that each country could influence and manage inflation on its own through fiscal policies with disregard to the common monetary policy. Whereas, in the latter case, the researcher would be assuming the actions of the fiscal authorities of each country to be irrelevant for the inflation in that same country. Which of these scenarios are we inhabiting? Is it perhaps the first, and the ECB is not at fault for any inflation deviation neither has the power to correct it? Is it the latter, and the national fiscal authorities can do whatever they want without worrying about the effect of its policies on their counties inflation? Or perhaps we are in an in-between scenario where even when the monetary policy sets the path for inflation, the actions of the national fiscal authorities are able to have a relatively small but significant effect on inflation?

To answer these questions, in this Chapter I am going to analyse these different approaches to find which of them better fits reality. The pooled or aggregated approach, where a unique Phillips Curve is estimated for all EA countries, the non-pooled or disaggregated approach, where the Phillips Curve for each country is independently estimated, and the mix approach, where the coefficients are allowed to be related across countries without the need of them being exactly equal.

The rest of the Chapter is going to proceed as follows: In Section 4.1 I develop the Phillips Curve specification that I will latter on estimate. In Section 4.2 I estimate the Phillips Curve using a pooled and a non-pooled approach. In Section 4.3 I estimate the multi-level approach and compare its statistics. Section 4.4 concludes.

4.1 The model

To estimate the Phillips curve I depart from the specification established by Friedman (1968) and Phelps (1968). In their work, they described a linear relationship between unanticipated inflation and unemployment gap. As a consequence, the economy was expected to be in equilibrium whenever inflation matched expected inflation and unemployment matched natural unemployment. If one of these two variables was put out of its equilibrium, the other one was expected to follow, as individuals reacted to this deviation. This relationship can be written in mathematical form as:

$$\pi_t = \pi_t^e + \gamma_1(u_t - u_t^*) + \varepsilon_t. \tag{4.1}$$

According to Friedman and Phelps, current inflation is a function of just expected inflation and unemployment gap. However, economies have become highly interdependent with globalization, making necessary to account for the inflation of imported goods when studying national inflation. This interdependence makes necessary to take into consideration the effect of imported inflation on national inflation when studying the evolution of prices in an economy. The inclusion of imported inflation in the Phillips Curve is common practice in the literature. Following Blanchard et al. (2015), I have added a variable accounting for imported inflation in my model, π_t^I. With the inclusion of this variable, I am capable of controlling the disturbances in national prices produced by changes in the prices of goods and services imported from abroad. With the inclusion of this variable, the above relationship then becomes:

$$\pi_t = \pi_t^e + \gamma_1(u_t - u_t^*) + \gamma_2\pi_t^I + \varepsilon_t. \tag{4.2}$$

It remains the question of how inflation expectations are formulated. The most extended solution is to assume inflation expectations to be a function of past inflation according to an autoregressive process. This is the approach followed by authors like Blanchard et al. (2015) and Ball & Mazumder (2011) among others. Following this approach, I assume inflation expectations to be a function of past inflation according to:

$$\pi_t^e = \delta_1 + \delta_2\pi_{t-1} + \xi_t. \tag{4.3}$$

Substituting this equation into Equation (4.2) I obtain the following equation:

$$\pi_t = \alpha_1 + \alpha_2\pi_{t-1} + \alpha_3(u_t - u_t^*) + \alpha_4\pi_t^I + \nu_t, \tag{4.4}$$

where ν_t is the combination of both error terms $\nu_t = \varepsilon_t + \xi_t$.

With the inclusion of the equation for expected inflation, inflation becomes a linear function of past inflation, imported inflation and unemployment gap. This Phillips Curve is very similar to the ones proposed by other authors like Blanchard et al. (2015), Ball & Mazumder (2011) or Coibion & Gorodnichenko (2015).

4.2 Estimation

To estimate the Phillips Curve in the EA a set of homogeneous data from the studied countries is needed. First of all, the EA is defined as the EA12, which includes the 11 founding fathers and Greece, which joined the Euro shortly after its creation in 2001, between 1998 and 2022. I use Eurostat CPI as proxy for inflation and the OECDs PMGS[2] growth as imported inflation proxy. Moreover, I use the OECD unemployment rate and the non-accelerating inflation rate of unemployment (NAIRU) as structural unemployment.

I have established the relation between unanticipated inflation and unemployment gap to behave according to Equation (4.4). But this equation does only account for a unique entity or level of variation, if I was to estimate the Phillips Curve for the EA as a whole, ignoring the country-level effects, Equation (4.4) would describe the relation between unemployment gap and unanticipated inflation for the EA as a whole. In this case, all data points are pooled together without differencing their country of origin. Whereas, if I was to estimate a Phillips Curve for each individual country in the EA, Equation (4.4) would describe the relation between unemployment gap and unanticipated inflation for each country, ignoring the effects common to all countries produced by a common monetary policy.

Under the pooled assumption, the Maximum Likelihood estimation of Equation (4.6):

[2] Imports of goods and services deflator on national accounts basis.

	mean	sd	5%	95%	p-val
β_1	0.96	0.17	0.68	1.24	0
β_2	0.54	0.08	0.41	0.66	0
β_3	-0.12	0.04	-0.19	-0.06	0
β_4	0.18	0.03	0.13	0.22	0

Table 4.1: Pooled estimation.

Under this assumption, both the intercept and the slopes of the Phillips Curve are assumed to be constant across countries. The coefficient measuring the persistence of inflation is 0.54. By each point of inflation in the current period, roughly speaking, half of it persists to the next period. In the same way, an extra point of unemployment gap reduces inflation in 0.12 points. Finally, the coefficient for imported inflation is estimated to be 0.18, national prices increase in 0.18 points by each point the prices of foreign goods increase. In addition, it is interesting to note that all coefficients are significant.

From an economic theory point of view, there are numerous reasons to defend the pooled approach if compared to the non-pooled. The principal reason for the pooled approach to prevail is the scope of action of the ECB monetary policy, where the EA is treated as a unique entity when objectives are set and policies are implemented. According with its mandate, the ECB has to set its monetary policy with the objective of stabilizing the weighted average of inflation for the whole EA ECB (2003). The greatest strength to manage inflation of monetary policy, if compared to fiscal policy, would make it prevail against fiscal policy if both were to be confronted in a particular country. This makes the pooled approach the most appropriate from an economic theory point of view.

Austria	mean	sd	5%	95%	p-val	Belgium	mean	sd	5%	95%	p-val
β_1	0.46	0.90	-1.02	1.94	0.30	β_1	0.90	0.76	-0.35	2.15	0.12
β_2	0.78	0.43	0.07	1.48	0.04	β_2	0.53	0.34	-0.02	1.09	0.06
β_3	-0.70	0.65	-1.77	0.37	0.14	β_3	-0.36	0.38	-0.99	0.25	0.17
β_4	0.28	0.15	0.03	0.53	0.04	β_4	0.25	0.09	0.10	0.41	0.00
Finland	mean	sd	5%	95%	p-val	France	mean	sd	5%	95%	p-val
β_1	0.35	0.65	-0.71	1.42	0.29	β_1	0.88	0.79	-0.42	2.19	0.13
β_2	0.73	0.35	0.16	1.29	0.02	β_2	0.50	0.44	-0.23	1.22	0.13
β_3	-0.89	0.53	-1.76	-0.03	0.04	β_3	-0.20	0.50	-1.02	0.63	0.34
β_4	0.15	0.10	-0.01	0.31	0.06	β_4	0.22	0.12	0.02	0.41	0.03
Germany	mean	sd	5%	95%	p-val	Greece	mean	sd	5%	95%	p-val
β_1	0.03	0.84	-1.35	1.42	0.49	β_1	2.09	0.58	1.15	3.05	0.00
β_2	1.17	0.46	0.41	1.92	0.01	β_2	0.22	0.20	-0.11	0.54	0.13
β_3	-0.30	0.54	-1.19	0.59	0.29	β_3	-0.22	0.08	-0.35	-0.09	0.00
β_4	0.27	0.11	0.08	0.45	0.01	β_4	0.11	0.08	-0.03	0.24	0.10
Ireland	mean	sd	5%	95%	p-val	Italy	mean	sd	5%	95%	p-val
β_1	0.94	0.52	0.09	1.80	0.04	β_1	0.61	0.86	-0.80	2.03	0.24
β_2	0.54	0.23	0.17	0.92	0.01	β_2	0.64	0.38	0.01	1.27	0.05
β_3	-0.14	0.11	-0.33	0.04	0.10	β_3	0.04	0.21	-0.31	0.39	0.43
β_4	0.08	0.13	-0.14	0.30	0.28	β_4	0.19	0.09	0.04	0.34	0.02
Luxembourg	mean	sd	5%	95%	p-val	Netherlands	mean	sd	5%	95%	p-val
β_1	0.91	0.74	-0.30	2.12	0.11	β_1	0.42	0.69	-0.71	1.55	0.27
β_2	0.50	0.26	0.07	0.93	0.03	β_2	0.89	0.31	0.38	1.41	0.00
β_3	-0.19	0.75	-1.44	1.04	0.40	β_3	-0.28	0.35	-0.86	0.29	0.21
β_4	0.16	0.09	0.01	0.31	0.04	β_4	0.15	0.11	-0.02	0.33	0.08
Portugal	mean	sd	5%	95%	p-val	Spain	mean	sd	5%	95%	p-val
β_1	1.11	0.55	0.21	2.03	0.02	β_1	1.02	0.69	-0.12	2.16	0.07
β_2	0.43	0.25	0.03	0.84	0.04	β_2	0.53	0.26	0.11	0.96	0.02
β_3	-0.09	0.15	-0.33	0.17	0.29	β_3	-0.09	0.08	-0.22	0.03	0.12
β_4	0.20	0.09	0.06	0.34	0.01	β_4	0.22	0.09	0.07	0.37	0.01

Table 4.2: Non-pooled estimation.

In Table 4.2 I have tabulated the results of the non-pooled or country by country estimation of the Phillips Curve. In this case, the common effects produced by the existence of a common monetary policy are overlooked in favour of each country individual effects. This assumption has the advantage of making it possible to compute a Phillips Curve for each country in the EA but it does so at the cost of overlooking the common effects produced by the ECB monetary policy.

From the estimation of the non-pooled regression in Table 4.2, one result stands out from the rest, the coefficient for lagged inflation in Germany is higher than one. This coefficient would imply that the German economy is in the middle of an hyperinflation. This implies that each inflation point in the current period creates 1.17 points in the following period. In addition, for the 90% confidence interval, some other countries like Austria, Finland or the Netherlands would have a sizeable probability of running the same luck. The existence of this parameter contradicting economic theory arises suspicions about the non-pooled estimates.

To test if both, the pooled and the non-pooled estimates, are statistically the same a likelihood ratio test is used, comparing both models. According to Wilks (1938), this test follows a χ^2 distribution with 44 degrees of freedom.[3] The Likelihood Ratio test statistics is computed to be $\lambda_{LR} = 2.15$ which gives a p-value of $1 - 1.59x10^{-21}$. Where the null hypothesis is for all country coefficients to be equal to the EA coefficients and the alternative for at least one of them to be different. I then cannot reject the null hypothesis of both models having the same coefficients, consequently the coefficients for all EA countries are not different from those of the EA showing that there exist a common latent curve for all EA countries. Having statistically the same coefficients it is then a matter of goodness of fit to know which method of estimation fits the data better.

4.2.1 Model Selection

To test which of these possible specifications fits better the data an information criteria is needed. When the different models have the same number of parameters, one might simply compare their log pointwise predictive densities (lppd). But, when the models have a different number of parameters, it is necessary to include some adjustment accounting for this difference to prevent overfitting.

In linear models, the most common information criteria to compare maximum-likelihood estimated models is the Akaike Information Criteria (AIC). This information criteria is penalized by the number of estimated parameters to correct for how much the fitting of k parameters increases the predictive accuracy.

$$AIC = -2 \log p(y|\hat{\theta}) + 2k. \tag{4.5}$$

In Table 4.3 I have tabulated the log predictive densities of all possible specifications, their number of parameters and the AIC statistic.

[3]The degrees of freedom are the result of subtracting the 4 estimated parameters in the pooled to the 48 estimated parameters in the non-pooled.

	lppd	k	AIC
pooled	$-551,04$	4	$1110,07$
non-pooled	$-549,98$	48	$1195,95$

Table 4.3: Model comparison statistics.

Looking to the lppd, the model which best fits the data is the non-pooled. But, the log pointwise predictive density has a problem as it does not control for overfitting. In the first case, the pooled approach only estimates 4 parameters while in the non-pooled 48 parameters are estimated. The disparity in the number of parameters makes necessary to include a penalization to prevent overfitting. Under the Akaike criteria, this overfitting is accounted for with the inclusion of the number of estimated parameters (k) which is available at the second column of Table 4.3. The application of Equation 4.5 to the first two columns of Table 4.3 results in the AIC statistic tabulated in the third column of said table. Once the penalization is included and the AIC criteria is estimated, the pooled specification is the one showing a better fit to the data.

Another possible way of analysing this problem is looking at significativity. When comparing the pooled and the non-pooled estimations displayed in Tables 4.1 and 4.2, the disparity on standard deviations stand out. The pooled estimation has a relatively low standard deviation where all coefficients are significant, whereas the non-pooled estimations have higher standard deviations with some of the parameters being not significant. To account for this, in Table 4.4 I have tabulated the Mean Square Errors (MSE) for all possible specifications in the first column and the MSE computed just with the coefficients which are significant at a 90% confidence interval (MSE90) in the second column.

	MSE	MSE90
pooled	$2,34$	$2,34$
non-pooled	$2,17$	$3,12$

Table 4.4: Model comparison statistics with MSE.

Like with the lppd, when the MSE criteria is used the model with a best fit is the non-pooled one, but it does so at the expense of overfitting. If I compute the same Mean Square Errors excluding the coefficients which are not in the 90% confidence intervals, the non-pooled model is not the best any longer. Like before, once overfitting is accounted for, the specification with a best fit to the data is the pooled. Both test push in the direction of the aggregate approach being the best to estimate the Phillips Curve in the EA. This is, according to these results, and considering that both specifications are not statistically different, the pooled approach gives a better fit to the data than the country by country or non-pooled approach.

4.2.2 Weighting the likelihood

Up until now, the weight of each country in the likelihood has been assumed to be the same. However, the ECB does not react to the average inflation in the EA but to the weighted inflation as specified in ECB (2003). This is why, in this section I am going to weight the likelihood according to the size of each country in the EA GDP. These weights affect the parameters of the pooled estimates but they do not affect the ones of the non-pooled as this estimator only uses the data from each country to estimate its individual curve. The likelihood will then become $L^{\omega}(\theta)$ where ω is the proportion of the GDP of each country with respect to the EA GDP. For this weight I could use both, the average proportion of each country GDP over the EA GDP or the specific proportion in each time period. However, in the last case, the weights for each country would not be constant over time as the participation of each country in the EA GDP may slightly change over time, creating a problem for the non-pooled estimation as the observations from some time periods would be slightly more influential on the likelihood than others. As a consequence, the coefficients of the non-pooled estimates would not be exactly the same than in the non-weighted estimation. This is why, without any loss of generality and for consistency, I have decided to use the average proportion across all studied periods instead of using the specific proportion for each year. The weight of the observations from each country in the likelihood will then be:

	ω
Austria	$3,21\%$
Belgium	$3,88\%$
Finland	$2,03\%$
France	$21,17\%$
Germany	$29,04\%$
Greece	$2,03\%$
Ireland	$2,22\%$
Italy	$16,86\%$
Luxembourg	$0,45\%$
Netherlands	$6,76\%$
Portugal	$1,84\%$
Spain	$10,52\%$

Table 4.5: Weight for each country.

In Table 4.5 it is observed how the countries dominating the Phillips Curve relation in the EA are the four biggest economies: Germany, France, Italy and Spain. While the weight of the smallest countries like Luxembourg is close to nil. Applying the weights to the pooled specification yields the following results:

	mean	sd	5%	95%	p-val
β_1	0,67	0,17	0,39	0,95	0
β_2	0,67	0,08	0,54	0,81	0
β_3	-0,08	0,04	-0,15	-0,01	0,02
β_4	0,21	0,02	0,17	0,25	0

Table 4.6: Pooled estimation with GDP weights.

If Table 4.6 is compared to Table 4.1 it stands out how the country intercept falls from 0.96 in the previous estimation to 0.69 in the current one. It is also interesting to note how the estimated influence of unemployment gap on inflation is smaller in the current case than in the previous one. These differences are the consequence of the weighting as now most of the information from the regression, the 77.6% to be exact, comes just from four countries: Germany, France, Italy and Spain. Whereas in the previous analysis, where the likelihood was not weighted by the GDP participation in the EA, these countries together just represented the 4/12 (33.33%) of the information. The opposite happens with the small countries like Luxembourg which falls from 1/12 (8.33%) in the non-weighted case to 0.4% in the weighted one.

Like in the previous section, I am going to use a Likelihood Ratio test to test if the coefficient of the pooled and the non-pooled still being statistically the same. This is, if the Phillips Curve of any of the EA countries estimated independently is different from the GDP weighted EA Phillips Curve. In this case, the Likelihood Ratio test statistics is $\lambda_{LR}^{\omega} = 1.73$ with a p-value of $1 - 1.61x10^{-23}$. Yet again, I then cannot reject the null hypothesis of both models having the same coefficients, concluding that non of the EA countries has a Phillips Curve different from the GDP weighted EA curve.

4.3 The Multilevel Estimation

The existence of more than one level of variability is not new in statistics, medical science or some social sciences, where these different levels of variation are included in the model through a multilevel/hierarchical approach. A non-exhaustive list of articles using a multilevel/hierarchical approach to control for two or more different levels of variation in multiple sciences includes: Bourne et al. (2017) which uses a hierarchical approach to study the global vision impairment and blindness. Flaxman et al. (2020) uses Hierarchical Bayesian models to estimate the tax of reproduction of COVID during the pandemic. Tauriello et al. (2018) use multilevel models to study colon cancer metastasis on mouses. Garibaldi et al. (2013) used a multilevel approach to study the bee pollination of fruits. Bandiera et al. (2021) uses a Bayesian Hierarchical model to study the influence of performance pay in men and women on gender earnings gap. Price et al. (1996) and Price & Gelman (2006) use a hierarchical approach to study the Radon concentration across the counties in the US state of Minnesota. The contribution of this Chapter to the literature is to follow their lead and use this method of estimation to include in the estimation the two levels of variation existent in the implementation of stabilization policies in the EA. The multi-level approach makes possible to include both: the group-level variability, caused by the existence of a common monetary policy set by the ECB, and the country level variability, caused by the existence of independent fiscal policy in each individual state.

A multilevel approach is a middle ground between the pooled and the non-pooled approaches, where both country and union level variability are jointly considered. Gelman & Hill (2006) shows how in the limiting case where the parameters across countries are not related, the multilevel model converges to the non-pooled estimate. In the same way, if the relation between the parameters across groups is perfect, the multilevel estimates converge to the pooled ones. With this methodology is then possible to estimate simultaneously the EA curve and the curve for each of its countries without the need of any assumption about the group-variance, as this variance is estimated from the data.

With the inclusion of methodology to the Phillips Curve literature, it is possible to estimate individual curves for each EA country without the need of assuming the Phillips Curve for each country to be independent from one to another.

In order to account for this double source of variability, Equation (4.4) must be modified to include the two levels of variation existent in the design and implementation of the stabilization policies in the EA. Following Gelman & Hill (2006), I am going to unify both approaches through a multilevel model from which both, the pooled and the non-pooled approaches become particular cases. Equation (4.4) can be written including two levels of variation as:

$$\pi_{i,t} \sim N\left(\beta_{1,i} + \beta_{2,i}\pi_{i,t-1} + \beta_{3,i}(u_{i,t} - u^*_{i,t}) + \beta_{4,i}\pi^I_{i,t}, \sigma^2\right), \quad for \;\; t = 1, ..., T$$
$$\begin{pmatrix} \beta_{1,i} \\ \beta_{2,i} \\ \beta_{3,i} \\ \beta_{4,i} \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_{\beta_1} \\ \mu_{\beta_2} \\ \mu_{\beta_3} \\ \mu_{\beta_4} \end{pmatrix}, \begin{pmatrix} \sigma^2_{\beta_1} & \sigma_{\beta_1\beta_2} & \sigma_{\beta_1\beta_3} & \sigma_{\beta_1\beta_4} \\ \sigma_{\beta_2\beta_1} & \sigma^2_{\beta_2} & \sigma_{\beta_2\beta_3} & \sigma_{\beta_2\beta_4} \\ \sigma_{\beta_3\beta_1} & \sigma_{\beta_3\beta_2} & \sigma^2_{\beta_3} & \sigma_{\beta_3\beta_4} \\ \sigma_{\beta_4\beta_1} & \sigma_{\beta_4\beta_2} & \sigma_{\beta_4\beta_3} & \sigma^2_{\beta_4} \end{pmatrix}\right) \quad for \;\; i = 1, ..., N. \tag{4.6}$$

where N is the number of countries inside of the union, 12 in our case, and T is the total time periods in the sample, 25 in the present case.

In Equation (4.6) I unify both, the pooled and the non-pooled approaches, under a common specification. This model converges to complete pooling in the limiting case where all countries in the EA are assumed to have the same coefficients, where the group-level variances ($\sigma^2_{\beta_1}$, $\sigma^2_{\beta_2}$, $\sigma^2_{\beta_3}$ and $\sigma^2_{\beta_4}$) are assumed to be zero. Under this assumption, the Maximum Likelihood estimator of Equation (4.6) converges to the efficient Least Squares pooled estimator. In the same way, if the group-level variances ($\sigma^2_{\beta_1}$, $\sigma^2_{\beta_2}$, $\sigma^2_{\beta_3}$ and $\sigma^2_{\beta_4}$) are assumed to be infinite, the Maximum Likelihood estimator converges to the corresponding efficient Least Square Fixed Effects estimator with varying intercepts and slopes.

The greater the relation of parameters across countries — the smaller the group-level variances ($\sigma^2_{\beta_1}$, $\sigma^2_{\beta_2}$, $\sigma^2_{\beta_3}$ and $\sigma^2_{\beta_4}$) is — the less efficient the non-pooled estimator is. This fall in efficiency, if compared with the pooled estimator, is a consequence of the existing relation of the parameters across countries. The true parameter is closer to the assumption of the group-level variances converging to zero, than to the assumption of them converging to infinity. Is in this case where the multilevel, which includes the group-level variances in the optimization process, produces a curve for each country as a function of the pooled estimate. In contrast, if the non-pooled approach was to show better statistics as a consequence of a high group-level variance, the multilevel estimates would become close to these estimates. Multilevel estimation is then a compromise between pooling and non-pooling estimates where, instead of providing a theoretical explanation for the necessity to use a pooled/non-pooled approach, the group-level variances are empirically estimated from the data.

According to the AIC and MSE90 information criteria, the pooled estimation is the one with a better fit to the data. From a goodness of fit criteria, I would then be tied to the pooled estimation, meaning that the common monetary policy is paramount to explain inflation in the EA. In this case I would be disregarding the effect that fiscal policy may have on unemployment gap and inflation. In addition, if the pooled hypothesis is accepted, it would mean to disregard the existence of an individual Phillips Curve for any country in the EA. However, according to economic theory, the national fiscal authorities have some power over their national inflation. A proof of this power is the limits to deficit and debt introduced in the European treaties during the foundation of the European Union and reinforced in 2012 with the modification of the Treaty on the Functioning of the European Union Comission (2012).

It is then convenient to use a less rigid estimator which does not need an assumption about the relation of the curve across countries but estimates this relation from the data. When the group-level variances are included in the weighted Maximum Likelihood optimization process, the following results are obtained:

Austria	mean	sd	5%	95%	p-val
β_1	0,67	0,23	0,3	1,03	0
β_2	0,68	0,12	0,49	0,88	0
β_3	-0,11	0,12	-0,32	0,05	0,11
β_4	0,21	0,05	0,14	0,29	0

Belgium	mean	sd	5%	95%	p-val
β_1	0,67	0,22	0,32	1,03	0
β_2	0,68	0,12	0,49	0,87	0
β_3	-0,11	0,11	-0,31	0,04	0,11
β_4	0,22	0,04	0,15	0,29	0

Finland	mean	sd	5%	95%	p-val
β_1	0,66	0,23	0,3	1,03	0
β_2	0,68	0,13	0,48	0,89	0
β_3	-0,11	0,12	-0,31	0,05	0,11
β_4	0,21	0,05	0,13	0,28	0

France	mean	sd	5%	95%	p-val
β_1	0,66	0,2	0,34	0,98	0
β_2	0,65	0,11	0,47	0,82	0
β_3	-0,1	0,1	-0,28	0,04	0,1
β_4	0,21	0,04	0,16	0,27	0

Germany	mean	sd	5%	95%	p-val
β_1	0,64	0,2	0,3	0,96	0
β_2	0,74	0,12	0,56	0,95	0
β_3	-0,14	0,11	-0,36	0	0,05
β_4	0,22	0,03	0,16	0,28	0

Greece	mean	sd	5%	95%	p-val
β_1	0,69	0,23	0,32	1,07	0
β_2	0,65	0,12	0,45	0,83	0
β_3	-0,09	0,07	-0,21	0,03	0,09
β_4	0,2	0,05	0,12	0,27	0

Ireland	mean	sd	5%	95%	p-val
β_1	0,67	0,22	0,32	1,04	0
β_2	0,68	0,12	0,49	0,88	0
β_3	-0,09	0,09	-0,24	0,04	0,11
β_4	0,21	0,05	0,13	0,28	0

Italy	mean	sd	5%	95%	p-val
β_1	0,66	0,21	0,32	1	0
β_2	0,65	0,1	0,48	0,82	0
β_3	-0,05	0,07	-0,16	0,08	0,2
β_4	0,2	0,03	0,14	0,25	0

Luxembourg	mean	sd	5%	95%	p-val
β_1	0,67	0,24	0,29	1,05	0,01
β_2	0,67	0,13	0,47	0,88	0
β_3	-0,1	0,12	-0,3	0,07	0,13
β_4	0,21	0,05	0,13	0,28	0

Netherlands	mean	sd	5%	95%	p-val
β_1	0,68	0,22	0,33	1,04	0
β_2	0,71	0,12	0,54	0,91	0
β_3	-0,11	0,11	-0,3	0,03	0,1
β_4	0,2	0,04	0,13	0,27	0

Portugal	mean	sd	5%	95%	p-val
β_1	0,67	0,23	0,31	1,04	0
β_2	0,67	0,12	0,47	0,86	0
β_3	-0,09	0,1	-0,26	0,06	0,13
β_4	0,21	0,05	0,13	0,28	0

Spain	mean	sd	5%	95%	p-val
β_1	0,68	0,22	0,34	1,04	0
β_2	0,66	0,1	0,49	0,82	0
β_3	-0,08	0,05	-0,16	-0,01	0,04
β_4	0,21	0,04	0,16	0,28	0

Table 4.7: Multi-level regression with GDP weights.

In addition to the weighted estimation, in the Table 4.15 in the Appendix I have included the non-weighted one for completeness. In this case, it is observed how the coefficients are more disperse than they were in the non-weighted estimation shown in Table 4.15. The multi-level estimator computes the Phillips Curve for each country as a function of the EA Phillips Curve. In this case each curve is not estimated independently any longer but dependent on what is happening in the rest of the EA.

With the introduction of the weights in the likelihood, the deviations of each country from the EA parameters does not have the same penalization. Like in the pooled estimation, the countries with a higher participation in the EA GDP have a higher penalization than the ones with smaller participation. As a consequence, the algorithm penalizes more the deviations in the big countries like Germany or France than the ones in small countries like Greece or Luxembourg. This is consistent with the ECB monetary policy which does not react the same to a deviation on inflation in Luxembourg than to the same deviation in Germany as each of them have a different effect on EA weighted inflation.

As a consequence of the joint estimation, in contraposition to the independent estimation shown in 4.2, the coefficient for lagged inflation in Germany is not any longer above 1. The multi-level estimation does not predict Germany to be in the middle of an hyperinflation as the non-pooled did. In addition, looking to the 90% confidence interval for this parameter in Germany, Austria, Finland and the Netherlands, which in the non-pooled estimation included both negative values and values above 1; in the multi-level estimation this 90% confidence interval does not include such values. These parameters are then more consistent with what could be expected from a theoretical point of view for a set of countries whose average inflation over the last twenty years is close to 2% than the ones shown in Table 4.2, giving an extra layer of legitimacy to the joint estimation if compared to the disjoint one.

In addition to the parameter estimates, the multi-level estimation also estimates the group-level variances. In this particular case, the group-level standard deviations are estimated to be:

	mean	sd	5%	95%
σ_{β_1}	0,11	0,1	0,01	0,3
σ_{β_2}	0,07	0,06	0,01	0,18
σ_{β_3}	0,08	0,07	0,01	0,21
σ_{β_4}	0,03	0,03	0	0,08

Table 4.8: Estimated group-level standard deviations.

In Table 4.8 I am showing the estimated group-level standard deviations from the weighted estimation. Looking to their values and confidence intervals, it is clear that the group-level standard deviations are closer to zero then they are to infinity. What is even more, in some cases zero is inside the 90% confidence interval shown in the table. Looking more carefully to Table 4.8, it is observed that the higher group-level standard deviation is the one for the intercepts, followed by the one for unemployment gap. This is consistent with what could be expected from an economic theory point of view, as the correlation of unemployment gap across some of the EA countries is negative as shown in Table 4.14. In addition, for completeness, in the Table 4.12 of the Appendix is available the group-level standard deviation estimates for the non-weighted multi-level estimation.

4.3.1 Model Selection

With the estimation of the group-level variances I have shown that the Phillips Curve coefficients are homogeneous across EA countries. This result reinforces the results obtained with the LR test, showing the EA parameters to be homogeneous across countries. It is then shown that there does not exist an independent Phillips Curve in each EA country but a perhaps a small deviation from the EA curve in each individual country. However, given the small group-level variances, it is possible for the pooled estimates to over-perform the multi-level ones.

From the estimation of the multi-level model, it is possible to compute the Mean Square Errors but it is not possible to compute the regular Akaike Information Criteria. This problem arises from the penalization factor in Equation 4.5. In the regular AIC, overfitting is controlled for with the introduction of the number of parameters estimated by the model. However, the multi-level model does not have a clear number of parameters. In the present estimation, the model could be considered to have both 4 and 48 parameters as the parameters for each country are not independently estimated. To put it in Gelman's words: "a batch of J parameters corresponds to one parameter if there is complete pooling, J independent parameters if there is no pooling, and something in between with partial pooling." Gelman & Hill (2006). It is then necessary to find a different penalization to prevent overfitting.

Recently a new approach has been developed to prevent overfitting in an AIC type information criteria, this is the case of the Widely Applicable Information Criteria (WAIC) Watanabe (2013). The difference between the WAIC and the AIC is that the former penalizes overfitting using the estimated variance of individual terms in the log pointwise predictive density summed over all n data points instead of the number of parameters like the AIC did. In some cases, this variance is referred to as the number of estimated parameters in the bibliography. The WAIC penalization takes the form of:

$$p_{WAIC} = \sum_{i=1}^{n} var_{post}(\log p(y_i|\theta)). \tag{4.7}$$

Using this penalization the WAIC can be written as:

$$WAIC = -2 \log p(y|\hat{\theta}) + 2p_{WAIC}. \tag{4.8}$$

The results for both, the pensalizations and the WAIC estimations are available at Table 4.9.

	lppd	penalty	WAIC
non-pooled	$-514,69$	$48,9$	$1127,19$
pooled	$-513,74$	$18,24$	$1063,98$
multi-level	$-509,7$	$26,54$	$1072,48$

Table 4.9: Widely Applicable Information Criteria and its components for the weighted models.

In Table 4.9 I have tabulated the WAIC for the three weighted regressions. It can be observed how the multi-level estimation is the one showing a higher log pointwise predictive density (lppd). This is, the multi-level estimation is the one with a better fit to the data if overfitting is not considered. However, when the penalization for overfitting is included, the pooled estimation is the one with a better fit followed closely by the multi-level and far from the non-pooled. The same can be observed in the MSE:

	MSE	MSE90
non-pooled	$2,17$	$5,01$
pooled	$2,37$	$2,37$
multi-level	$2,34$	$2,39$

Table 4.10: Mean Square Error and Mean Square Error for the 90% confidence interval for the weighted models.

In this case, like in the previous one, the non-pooled estimates are the ones showing a lower MSE when all coefficients are considered followed by the multi-level and the pooled. This hierarchy inverts when just the coefficients which are inside the 90% confidence interval are considered. In this case, the MSE of the non-pooled estimates goes to the roof while the ones of the multi-level slightly climbs up the ones of the pooled which stays constant. When the variance of the estimates is considered through significativity, the pooled and multi-level estimates behaves much better than the non-pooled or country by country estimates.

As a robustness check, the overfitting penalization or the effective number of parameters can also be computed from a completely different approach using Leave-one-out cross-validation (Loo) Vehtari et al. (2017) and Vehtari et al. (2015). Using this approach, the lppd, the effective number of parameters and the leave-one-out information criteria (looic) are:

	lppd	p_loo	looic
non-pooled	-565.16	50.31	1130.32
pooled	-531.87	17.89	1063.75
multi-level	-536.45	26.59	1072.9

Table 4.11: Leave-one-out cross-validation estimation of the effective number of parameters.

The results shown by the WAIC are consistent to the estimation of the information criteria using leave-one-out cross-validation. The estimated effective number of parameters for the pooled model using WAIC is 18.24 while using LOO is 17.89. The estimated effective number of parameters for the non-pooled estimation is 48.9 using WAIC and 50.31 using LOO. In the same way, for the multi-level estimation, the effective number of parameters is estimated to be 26.54 according to WAIC and 26.59 according to LOO. For completeness, the LOO estimates for the non-weighted estimations are available at Table 4.13 at the Appendix.

4.4 Conclusions and Discussion

In this chapter I have revisited the linear relationship between unanticipated inflation and unemployment in the EA. The estimation of this relation in a monetary union without a common fiscal policy has some challenges. The existence of two different levels of government, each with a different stabilization police at hand, creates a double source of variability, one at each level of government. The Phillips Curve can be then estimated using a pooled approach, where the data from all countries is pooled together to estimate a unique curve, or with a non-pooled approach, where the data from each country is independently used to compute an individual Phillips Curve for each country.

In this Chapter I have first shown that there exist a unique Phillips Curve for all the EA countries. I have done so by testing if jointly all coefficients of the country by country estimation were statistically the same than those of the EA estimates. I have found this null hypothesis could not be rejected for either, the weighted or the non-weighted estimates. As a result, statistically there only exist a unique Phillips Curve for all EA countries. The chosen specification, pooled or non-pooled, will then be based on goodness of fit criteria.

Using information criteria I have tested which approach showed a better fit to the data. I have found the country by country or non-pooled approach to be the one with the worst fit in all considered cases. I have also shown pooled estimates to overperform multi-level ones thanks to their lower variances. Lastly, but not least, I have shown that multi-level estimates give a better fit to the data than non-pooled estimates whenever the curves for an individual country are estimated.

As a future research I plan on using a DSGE model to simulate the data from a monetary union to show to which extent the multilevel approach is beneficial if compared with the pooled and the non-pooled ones. With Monte Carlo simulated data it will be possible to compute how a common monetary policy with different fiscal policies affect the Phillips Curve coefficients. The analysis of this chapter could be enhanced with the inclusion of non-informative prior in the Hierarchical model making it completely Bayesian. In this particular case, the results with and without informative prior were the same, which stopped me from proceeding with this approach. This could change if countries which joined the EA on a latter date were included or other relations are studied in the context of the EA.

Appendix

	mean	sd	5%	95%
σ_{β_1}	0.11	0.08	0.01	0.25
σ_{β_2}	0.04	0.03	0.00	0.1
σ_{β_3}	0.06	0.04	0.01	0.13
σ_{β_4}	0.06	0.02	0.03	0.09

Table 4.12: Estimated group-level standard deviations for the non-weighted estimation.

	lppd	p_loo	looic
pooled	-321.9	29.6	643.9
non-pooled	-335.2	70.7	670.4
multi-level	-316.7	46.9	633.4

Table 4.13: Leave-one-out cross-validation estimation of the effective number of parameters for the non-weighted estimation.

	Austria	Belgium	Finland	France	Germany	Greece	Ireland	Italy	Luxembourg	Netherlands	Portugal	Spain
Austria												
Belgium	0,71											
Finland	0,65	0,61										
France	0,46	0,67	0,68									
Germany	0,25	0,37	0,00	0,17								
Greece	0,21	0,32	0,61	0,71	-0,30							
Ireland	0,09	0,35	0,48	0,57	-0,37	0,56						
Italy	0,12	0,13	0,56	0,71	-0,28	0,88	0,46					
Luxembourg	0,50	0,46	0,60	0,68	-0,04	0,46	0,59	0,51				
Netherlands	0,73	0,78	0,62	0,66	0,00	0,52	0,42	0,34	0,73			
Portugal	0,34	0,44	0,48	0,54	-0,44	0,67	0,82	0,47	0,69	0,72		
Spain	0,16	0,41	0,64	0,71	-0,29	0,83	0,90	0,72	0,61	0,50	0,80	

Table 4.14: Unemployment gap correlations across countries between 1998 and 2022.

Austria	mean	sd	5%	95%	p-val
β_1	0,95	0,21	0,61	1,29	0
β_2	0,54	0,1	0,39	0,7	0
β_3	-0,15	0,1	-0,31	-0,02	0,03
β_4	0,18	0,04	0,12	0,26	0

Belgium	mean	sd	5%	95%	p-val
β_1	0,95	0,21	0,61	1,29	0
β_2	0,54	0,09	0,38	0,69	0
β_3	-0,15	0,09	-0,31	-0,03	0,03
β_4	0,19	0,04	0,13	0,26	0

Finland	mean	sd	5%	95%	p-val
β_1	0,93	0,21	0,59	1,27	0
β_2	0,54	0,1	0,39	0,7	0
β_3	-0,15	0,1	-0,32	-0,03	0,03
β_4	0,17	0,04	0,11	0,24	0

France	mean	sd	5%	95%	p-val
β_1	0,94	0,21	0,6	1,27	0
β_2	0,53	0,1	0,37	0,69	0
β_3	-0,14	0,09	-0,29	-0,01	0,04
β_4	0,18	0,04	0,12	0,25	0

Germany	mean	sd	5%	95%	p-val
β_1	0,94	0,21	0,61	1,28	0
β_2	0,55	0,1	0,39	0,72	0
β_3	-0,14	0,09	-0,3	-0,02	0,03
β_4	0,18	0,04	0,12	0,25	0

Greece	mean	sd	5%	95%	p-val
β_1	0,99	0,22	0,66	1,36	0
β_2	0,52	0,09	0,37	0,67	0
β_3	-0,13	0,05	-0,21	-0,04	0,01
β_4	0,16	0,04	0,09	0,22	0

Ireland	mean	sd	5%	95%	p-val
β_1	0,94	0,2	0,61	1,27	0
β_2	0,54	0,09	0,39	0,69	0
β_3	-0,14	0,06	-0,24	-0,05	0,01
β_4	0,17	0,04	0,09	0,24	0

Italy	mean	sd	5%	95%	p-val
β_1	0,95	0,21	0,61	1,28	0
β_2	0,54	0,09	0,38	0,69	0
β_3	-0,12	0,07	-0,23	0,01	0,06
β_4	0,18	0,04	0,11	0,24	0

Luxembourg	mean	sd	5%	95%	p-val
β_1	0,94	0,21	0,59	1,28	0
β_2	0,53	0,09	0,38	0,68	0
β_3	-0,14	0,09	-0,29	-0,01	0,04
β_4	0,17	0,04	0,11	0,23	0

Netherlands	mean	sd	5%	95%	p-val
β_1	0,96	0,21	0,62	1,31	0
β_2	0,56	0,1	0,41	0,73	0
β_3	-0,14	0,09	-0,3	-0,02	0,03
β_4	0,17	0,04	0,1	0,23	0

Portugal	mean	sd	5%	95%	p-val
β_1	0,95	0,21	0,62	1,29	0
β_2	0,53	0,09	0,38	0,68	0
β_3	-0,12	0,07	-0,23	-0,01	0,04
β_4	0,18	0,04	0,12	0,25	0

Spain	mean	sd	5%	95%	p-val
β_1	0,96	0,21	0,63	1,31	0
β_2	0,54	0,09	0,39	0,69	0
β_3	-0,12	0,05	-0,2	-0,03	0,01
β_4	0,18	0,04	0,12	0,25	0

Table 4.15: Multilevel estimation of the Phillips Curve.

Chapter 5

Concluding remarks

The design of monetary policy is not an easy task. Policymakers have to decide if they want to have a fixed exchange rate or if they prefer to have some control over the country's interest rate. In both cases, one of the main tasks of monetary policy is to manage the current account deficits and superavits. A devaluation of the national currency helps the national exporter sector to sell their goods abroad. In the same way, a revaluation of the national currency makes imports cheaper for national consumers. The proper management of the monetary policy is very important because, if current account imbalances are let unanswered, they accumulate in external debt. A sizeable external debt can become an important burden for a country because its currency may loose its value on international markets, making impossible for them to buy foreign goods which cannot be nationally produced. In cases like this, the accumulation of external debt becomes a burden for all the economy, making necessary unpopular policies to close the current account deficit and reduce the external debt.

In the second chapter, *Is external debt sustainable? A probabilistic approach*, I study the behaviour of external debt and its potential sustainability. To study this problem I have computed the current value of the future current account deficits/surpluses. I have then compared this future capability to repay the debt with the current level of external debt. Comparing these variables, I am able to compute the probability of default/devaluation as the proportion of scenarios at which the country might have problems meeting its obligations. In the case of a monetary union, where a devaluation is not possible, this could be seen as the stress supported by the currency. If a group of countries inside the union are highly indebted with the other countries inside of the union, but the union is close to the equilibrium as a whole, the survival of the currency will be put into question as there is not any monetary policy which can solve this situation.

The management of monetary policy becomes more difficult when a group of countries decides to merge under the same currency. In this case, a common central bank is in charge of the monetary policy for the whole union. In addition, this monetary policy has to be common as it is impossible to have more than one monetary policy with the same currency. This supposes a challenge for policymakers whenever the countries in the union have different necessities.

In chapter 3, titled *Is the Euro Area diverging? An empirical analysis*, I have studied the problems confronted by a central bank in a monetary union when the countries inside the union have different necessities. To analyse these problems I have built a two-country New Keynesian model without a fiscal authority. Using this model, I have shown how the central bank would be incapable of guaranteeing uniqueness of equilibria by itself because the model would not fulfil the Blanchard-Kahn conditions. This problem is solved in both the literature and the real world, like in the EA, assuming individual fiscal authorities strong enough to guarantee the equilibria. If these individual authorities are capable of stabilizing their economy by themselves, the central bank will be only responsible of managing inflation and uniqueness of equilibria is guaranteed. This is the case of the EA where the national fiscal authorities are the ones stabilizing the economies while the ECB's only objective is to guarantee low inflation. But this approach has a problem, if any of the fiscal authorities is incapable to stabilize its economy, the uniqueness of equilibria is not guaranteed.

This approach might seem robust from a theoretical point of view but it does not survive its touch with reality. In the aftermath of the 2008 financial crisis the Euro Area sovereign debt crisis was conceived. When the fiscal authorities of the individual countries had to rescue their financial institutions, some of these authorities did not have enough fiscal muscle to do so by themselves. In order to stabilize their economies, these fiscal authorities had to appeal for help to international institutions like the ECB and the IMF. These events made clear that the individual fiscal authorities where incapable to stabilize the economies by themselves, putting into question the stability of the EA. This could have been an isolated event but most of the EA fiscal authorities are still highly indebted, making it impossible for them to implement strong enough stabilization policies in the near future. They would need to make strong spending reductions to be able to reduce their debt to a manageable level in the long run, even more in a growing interest rate environment.

In Chapter 3, I have documented this problem computing the correlations of inflation and Real GDP between the EA countries and between the US states. In the EA I have found these variables to be in some cases negatively correlated. When the variables in two countries are negatively correlated, any policy addressed to stabilize one economy necessarily destabilizes the other one. But this is not all, the variables of some countries are not correlated with the EA aggregates. This is a challenge for the ECB stabilization policies because the ECB does react to the EA aggregate variables. The inexistence of a correlation between the behaviour of the EA aggregate variables and the variables of certain countries makes the monetary policy ineffective or even counterproductive for some of these countries. This effect is not observed in the US, where all the sates have positive correlations between them and with the US aggregate.

To study the effect of these asymmetries, I have computed the welfare loss function for a two-country New Keynesian model which I have found to be a function of the terms of trade. I have then argued that the monetary policy rule should be a function of the terms of trade as well because they appear in the welfare loss function of the union. I have estimated the policy rule for the EA and the US with and without the terms of trade. I have found that, without the terms of trade, the estimated reaction to output and inflation gaps was not strong enough to guarantee the uniqueness of equilibria if compared with the minimum needed reaction established by the literature. But if the terms of trade were included in the regression, the estimated parameters for output and inflation gaps were consistent with the literature. I then concluded that the terms of trade were affecting the monetary policy and should be taken into consideration when analysing the interest rate. Looking to the significance of the terms of trade in the two studied unions, this variable was significant for the EA but it was not significant for the US. This implies that the monetary policy of the ECB is disturbed by the terms of trade between their countries while the one of the Fed is not. The reaction of the ECB to a inflationary shock will be weaker the higher the terms of trade are because, if it reacts as strong as necessary, it could put some country out of its convergence path. This makes shocks in the EA intrinsically more persistent than in other countries where fiscal authorities are strong enough to stabilize the economy like the US.

Finally, in the fourth chapter of the Thesis, *The Phillips Curve in the EA* I have revisited the linear relationship between unexpected inflation and unemployment gap. The existence of this relationship is very important in macroeconomics as policymakers use it as an instrument to manage inflation. The estimation of this relation supposes an additional challenge whenever monetary and fiscal policies are not set at the same level of government. In the EA, monetary policy is set by the ECB for all countries while fiscal policy is set independently by each country. This creates a scenario where expansionary/contractionary monetary policies with contractionary/expansionary fiscal policies can coexist in some countries.

The existence of two levels of variability creates a disjunctive between estimating the Phillips Curve jointly for all EA countries or independently one country at a time. To address this issue, in Chapter 4 I estimated both specifications and tested if the coefficients for each country in the EA estimated independently were statistically the same than those of the EA. Based on the existing evidence, I could not reject this hypothesis, concluding that there exist a unique curve for all EA countries. Consequently, in order to choose which specification fits better the data I used goodness of fit criteria, finding the pooled specification or joint estimation to fit better the data than the independent or country by country estimation. Finally, I have proposed to use a multi-level approach to include both levels of variability in the regression function. With this, it is possible to estimate the relation between the Phillips Curve parameters across countries. This approach estimates the Phillips Curve for each country as a function of the EA curve, outperforming country by country estimation. However, given its higher variance if compared with pooled estimates, the latter shows a better fit to the data among all the studied.

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