



# The competition between legal and illegal firms in the market: Theoretical models and empirical evidence

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## ABSTRACT

The main aim of this paper is to study the phenomenon of the coexistence of firms with illegal characteristics and firms showing legal behaviour in developed economies by using an evolutionary competition model known as the Lotka-Volterra equations. Enterprises in a 'legal system' obey the extant laws, particularly market rules, while enterprises in an 'illegal system' disregard them. Illegal enterprises have a competitive advantage over legal enterprises, yet they cannot survive if legal enterprises disappear completely. The numerical solution of the Lotka-Volterra equations are used to show how there can be a situation of coexistence between legal and illegal enterprises and how state intervention can help reduce illegality in an economic system. This paper outlines a new approach to address the problem of coexistence because it uses non-linear and evolutionary tools to define the competition between legal and illegal firms. The research gap presented in this paper is addressed using the predator-prey scheme to model the competition between legal and illegal firms, which is perceived as the competition between two populations with different fitness levels (survival probabilities). This competition gives rise to three different types of possible equilibrium outcomes: survival of only legal firms, survival of only illegal firms and coexistence of these two types of firms. An empirical analysis of an Italian case study confirms the results of this paper's theoretical model.

## 1. Introduction

The traditional 'Beckerian' approach of crime and punishment, which constituted the 'mainstream' of the criminal economy literature over a considerable period in history, moved from the original (Becker, 1968) to the more advanced (e.g. Schmidt-Witte 1984) and finally to the 'strategic-structural' approach (Schelling, 1984) that could engage more deeply with the problem of the criminal organisation, its internal structure, the strategic interaction within 'gangs' and the relationship between the state and the subjects of the criminal economy. Today, the contributions of these abovementioned studies remain important at a descriptive level as well as in terms of their possible implications for economic policy, as they have significantly impacted the development of the subsequent literature on criminal economy. In fact, the interest in models that attempt to explain the dynamics of organised crime with regard to competing behaviours between agents has not only established itself but also grown considerably in the recent years. This is evident, for

example, in the contributions of Blackburn et al. (2017), who develop a model to study the interaction between crime and corruption through which they explain the coexistence of legal and illegal firms in a given market, with illegal firms using corruption to compete. Moreover, Lewitt (2017) provides a comprehensive literature review spanning the last 20 years on the economics of crime, while Chalfin et al. (2017) offer an interesting survey (also covering a period of 20 years) of the effect of police, punishments and work on crime. Motivated by the predator-prey model, Abbas et al. (2017) construct their model to study the interaction between criminal and non-criminal populations. Sooknanan et al. (2016) base their model on a situation in which the police are the predators and gang members the prey; the authors examine whether it is possible for the police to effectively control gangs.

Further, in Mastrobuoni et al. (2019), the individual disutility resulting from a prison sentence is estimated using microdata. Jawadi et al. (2021) show that crime and unemployment are positively correlated. Rey (2018) points to the inefficiency that results from connections

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between legal and illegal enterprises.

However, this paper claims that the aforementioned literature has hitherto overlooked one area where further investigation is possible. It departs from the observation that legal and criminal economies together explain the level of development of an economic system as a whole. In this respect, Skaperdas (2001) points out that organised crime emerges from the absence of state enforcement and that the control of organised crime is necessary because it can easily corrupt existing government institutions. Delving further, Becket et al. (2013) analyse illegal markets on the basis of the three coordination problems – valuation, competition and cooperation – and conceptualise the structure of illegal markets on the basis of these problems, identifying the systematic differences in the functioning of illegally operating markets. Hudson (2020), starting from the consideration that the literature on urban and regional development lacks a reflection on the effects of illegal activities on the economies of successful cities and regions, focuses on two sets of questions: the significance of illegality to the economic practices of ‘successful’ cities and regions and the relationship between state policies and illegality. Moreover, Reuter (1983) couples his examination of illegal markets with an analysis of the economic consequences of product illegality and examines the effect of the illegal status of underworld markets on such organisations. Furthermore, Miranda et al. (2019, 2022) show that organised crime seeks out companies facing financial difficulties in order to acquire them and that the emergence of organised crime has a long-term negative effect on economic growth at the local level.

The legal operator behaves according to market rules, whereas the illegal operator uses non regular competitive means to purchase or procure inputs. Predictably, his costs would apparently be lower than those of a legal producer, including his perceived risk of being penalised for his behaviour (van Winden et al., 2012). Evidently, it is the inefficiency of the justice system in prosecuting such a crime, reflected in the entrepreneur’s greater propensity for risks or the fact that he estimates the expected cost of preventing and punishing his illegal behaviour to be low, that determines a possible ‘competitive advantage’ of an illegal economy over a legal one (D’Orsogna et al., 2015).

While moving in a ‘Beckerian’ line of research, this paper presents an innovative approach to addressing the problem of competition between legal and illegal firms; for this purpose, it introduces non-linear and evolutionary tools that revolutionises the way we understand and analyse such competition. By adopting this fresh perspective, the paper offers an original framework that is used to model and examine the dynamics of such competition. In fact, one of the key contributions of this research is the conceptualisation of the competition between legal and illegal firms as a clash between two distinct populations, each characterised by different levels of fitness or survival probabilities. This paradigm shift allows for a more nuanced understanding of the complex interplay between the aforementioned two types of firms, shedding light on their coexistence or dominance in the marketplace. In the new framework conceptualised by this paper, the interaction between legal and illegal firms gives rise to three potential equilibrium outcomes, each of which has interesting aspects and implications. First, the survival of only legal firms highlights the scenario where legal entities outperform their illegal counterparts, resulting in a market predominantly composed of lawful businesses. This equilibrium demonstrates the ability of legal firms to effectively compete and thrive within the existing legal and regulatory frameworks. Second, the survival of only illegal firms showcases a contrasting scenario wherein illicit entities surpass legal firms in terms of fitness and successfully establish dominance in the marketplace. This equilibrium underscores the potential vulnerabilities within the existing regulatory systems and the challenges they face in deterring illegal activities. In turn, it prompts policymakers and law enforcement agencies to reassess their strategies and devise more robust measures to combat illicit practices. Finally, the coexistence of both legal and illegal firms emerges as a dynamic equilibrium, highlighting the intricate balance between the two types of entities. This equilibrium signifies a scenario where legal and illegal firms coexist, each occupying

a distinct niche within a market. Importantly, the coexistence equilibrium prompts further investigation into the factors that enable such a delicate balance and raises questions regarding the potential influence of external variables, such as socioeconomic factors or enforcement mechanisms, on this equilibrium (Marino & Trapasso, 2009, 2020). By elucidating these three potential equilibrium outcomes, this paper contributes significantly to our understanding of the competition dynamics between legal and illegal firms. It offers a comprehensive and multifaceted perspective that not only highlights the complexity of the topic at hand but also provides a foundation that can support further research on the same and can encourage the development of effective strategies to promote a legal and more sustainable marketplace.

This paper is structured in such a way that its theoretical framework is developed in Sections 2 and 3. In particular, Section 2 outlines the theoretical background and defines the legal and illegal behaviours of firms, while Section 3 provides an explanation of the biological competitive models and their implications for the economic field. Section 4 delineates the methods and samples used in this paper and presents the results of simulations based on the numerical solution of the Lotka-Volterra equations as well as the empirical tests of the competitive model. Section 5, titled ‘Conclusions’, identifies some indications for relevant policy and management, also highlighting the limitations of this paper and the future research prospects on the paper’s topic.

## 2. Background

First, it seems necessary and opportune to describe the theoretical framework within which the subsequent theoretical elaborations are developed. Thus, it is important to describe the particular market within which legal and illegal enterprises operate, their operating mechanisms and the role and action of the state in this context. Therefore, after describing the rationale of the considered model, this paper goes on to define the behaviour and choices of various enterprises in the given market, distinguishing legal behaviours from illegal ones. Thereafter, it shows how the oligopoly model (with exit) is the market form within which firms tend to develop their competition. Finally, the description of the nodes of state intervention in such a market (having the specific aforementioned characteristics) concludes the analysis of the preliminary theoretical framework.

### 2.1. The rationale of the model

The market behaviour and market decisions of illegal enterprises are quite complex and diverse. For this reason, it is important to define the maximising behaviour of illegal enterprises at the outset. Illegal enterprises do indeed maximise a certain objective function, but this function is not always profit. Table 1, in accordance with the available literature (Miranda et al., 2022; Becker, 1968; Beckert & Dewy, 2017; Beckert & Wehinger, 2012; International Monetary Fund, 2001; Santino, 1990),

**Table 1**  
Objective functions and indicators of illegal activity – our elaborations.

Objective functions	Indicators	Impact level
Profit	Net profit, Return on Equity (ROE), Return on Investment (ROI)	Low
Territorial control	No. of enterprises controlled by criminal gangs	High
Laundering	Volume of Financial Transaction	High
Market share acquisition	Sectoral mortality rate of enterprises	Medium
Coverage of criminal activities	No. of complaints for illegal business activities	Low
Job management	Black work, ‘Caporalato’	Medium
Merger of companies	Presence of nominees	Medium
Criminal Strategy	No. of economic crimes of an associative nature	High

contains a list of the target functions of illegal enterprises, a set of indicators capable of optimising their objective functions and a qualitative estimate of their levels of impact, with the aim of better highlighting the real behaviour of illegal enterprises:

Table 2, in accordance with the abovementioned literature, makes it possible to complete the qualitative description of the behaviour of illegal enterprises by describing a set of instruments that can be used to determine a certain outcome of such enterprises:

As per the data provided in these above tables, the behaviour of illegal enterprises is extremely complex. Their competitive strategies are varied. On one hand, a neoclassical enterprise has profit maximisation as its objective; the behaviour and instruments used by such an enterprise are relatively simple and functional with regard to this purpose. The illegal enterprise, on the other hand, is more like an organic system whose goal is to survive in a hostile environment by maximising its probability of survival. Its strategies can be classified as follows:

- (1) Expulsion of competitors from the market.
- (2) Incorporation of competitors within one’s own company.

Both these strategies can be implemented using market and non-market mechanisms. Moreover, in general, these two different mechanisms can be implemented jointly by the same illegal enterprise.

To elaborate further, market mechanisms are essentially based on the inherent cost differential of illegal enterprises. By exploiting the cost differential, illegal enterprises can drive legal enterprises out of the market at any time.

Extra-market mechanisms, on the other hand, are forms of persuasion or cost reduction that illegal enterprises may solely use due to their criminal nature. Usury, extortion, imposition of labour and threats are the instruments through which extra-market competition is generated.

### 2.2. Enterprises’ behaviour

In order to delineate the mechanism of market functioning in the presence of competition between legal and illegal enterprises, an economic modelling of the behaviour of enterprises is necessary.

In defining the formal structure of this model, it can be assumed that there are ‘legal’ (L) and ‘illegal’ (I) enterprise groups.

Notably, the objective of both types of enterprises (or groups of enterprises) is to maximise profits under the constraint of a production function, which is identical for both the abovementioned types of firms, considering a number (‘n’) of production inputs and at constant returns to scale.

For the legal enterprise, we have the following equation:

$$\begin{aligned} \text{Max}\Pi_L &= Py_L - w_1x_1 - w_2x_2 - \dots - w_nx_n \\ \text{s.t. } y_L &= f(x_1, x_2, \dots, x_n) \end{aligned} \tag{1}$$

On the other hand, the illegal enterprise pays different prices for the same inputs, while it manages to capture a market share that is by definition larger than that of the legal enterprise.

$$\begin{aligned} \text{Max}\Pi_I &= Py_I - w_1x_1 - w_2x_2 \dots w_nx_n \\ \text{s.t. } y_I &= f(x_1, x_2, \dots, x_n) \\ y_I - y_L &\geq 0 \end{aligned} \tag{2}$$

**Table 2**  
Tools and outcomes in the illegal competition – our elaborations.

TOOLS	OUTCOME
Competition below costs	Exit
Imposition of labour	Increased costs, internal control
Wear	Financial dependence
Threat	Distortion of competition
Extortion	Distortion of competition, additional costs
Laundering	Zero Cost of Capital
Incorporation of legal enterprises	Elimination of competitors
Disguised criminal activities	Protection for criminal activities

For the legal enterprise, the prices of the inputs are given by the market, whereas illegal enterprises themselves set the prices of their inputs and accomplish this task outside the market. Obviously, the inputs purchased by both types of enterprises are qualitatively equal. Thus, as per their definitions, we have the following equations:

$$\begin{aligned} w'_1 &< w_1 \\ w'_2 &< w_2 \\ w'_3 &< w_3 \end{aligned} \tag{3}$$

The market can, in the period under consideration, be set at a level  $y^*$  (the demand side), whereby the following equation emerges:

$$y^* = y_L - y_I \tag{4}$$

### 2.3. ‘Oligopoly with exit’ and ‘Oligopoly with asymmetric costs’ approaches

In this paper, coexistence is proposed on the basis of a rather traditional oligopolistic model in the literature, the so-called ‘oligopoly with exit’. This model is treated extensively in the available literature. Specifically, a fundamental concept on which our work in this paper is based is the so-called ‘War for Attraction’. In this regard, a further interesting approach can be attributed to the work of Fudenberg and Tirole (1983, 1986).

In the ‘oligopoly with asymmetric costs’ approach to modelling, starting from a situation where two firms have cost asymmetry between them, a dynamic selection process is generated in which a firm leaves the market when its profits as a member of an oligopoly are lower than its fixed costs. This process is slowed down by the lack of knowledge of the other firm’s fixed costs, being defined in the model as the costs of operating within the industry added to the opportunity costs of foregoing profits accrued from other activities.

To be precise, the firms do not know each other’s costs but possess only a probability distribution over them. As per this information asymmetry, the ‘oligopoly with asymmetric costs’ model implies that the only rational position is inaction.

Here, one must keep in mind that Fudenberg and Tirole (1983, 1986) assume the market is overall a potential natural monopoly but they also note that one cannot *a priori* impose ‘a unit probability on the cancellation of the present value of the duopolist’s profits’.

There is a certain level of similarity between the theoretical approach we propose in the next sections and that of the ‘oligopoly with exit’ approach. At the outset, we trace this similarity to the information asymmetry, although we remain aware that our model seems forced when we assume that the ‘illegal’ firms are not in a position to know the fixed costs of the ‘legal’ firms. This assumption seems like a stretch, as it were, that can be corrected by thinking of how the idea of fixed costs in Fudenberg and Tirole’s (1983, 1986) model incorporates both actual costs and the opportunity cost of foregoing other activities; this way, at the very least, the illegal firm can be assumed to have limited knowledge of this aspect of the legal firm’s cost structure.

On the other hand, the model is consistent in its results with our conclusions from the biological model. In fact, one can have both a coexistence result and a specialisation result between legal and illegal firms, similar to the predicament of the two firms in Fudenberg and Tirole’s (1983, 1986) model, while not having to resort strictly to the same assumptions, notably that of natural monopoly.

A later work by Maskin and Tirole (1988) makes a further contribution in this direction, which can be useful in justifying our own approach. As per these authors, under the assumptions of perfect demand and cost information, a dynamic oligopoly model is generated that can be traced back to the concept of ‘perfect Markovian equilibrium’ (1988).

Firms operate with different cost functions and compete on price in each period. Thus, the profit  $\Pi$  is a function of the prices of the two firms

and there are no capacity constraints in this situation; thus, the larger industry's profit, under certain conditions for demand assumptions, can be calculated as the follows:

$$\Pi(P) = (P - \sum_1^n C_i(S_i))D(P) \tag{5}$$

where  $C_i$  marks the cost of each firm,  $S_i$  is the market share of each firm,  $P$  the price and  $D$  the demand function.

For two firms, the joint maximum profit solution is possible when the price is equal to the monopoly price of the first firm, the market share is equal to 1 and there is a 'compensatory payment' to the second firm for its exit. Excluding the possibility of a 'compensatory payment', the coexistence of the two firms is possible as a collusive equilibrium with a price that lies between the two monopoly prices (Martini, 1991).

Although with essentially different motivations, the result of this study regarding pricing is similar to that provided by our model, given that (as we observe in our case) firms with lower (illegal) costs cannot adhere to the monopoly price as they must consider the expected value of their penalty.

#### 2.4. Public government behaviour

In this paper, the full description of the microeconomic behaviour of agents is completed with the description of the microeconomic behaviour of the state in the presence of legal and illegal enterprises. In our scheme, state intervention is perceived by firms as linked to the possible cost configurations of the legal and the illegal firms.

Here, the role of the state is to guarantee the legality of the given system through a sanction on illegal behaviour. One assumption is that state intervention is more incisive and stronger when the market and its illegality is more widespread. This assumption introduces a feedback mechanism that justifies the cycles in the fight against illegality to which we return at the end of this section.

We assume that the presence of the state imposes a sanction  $s$ , which is understood as a monetary measure of the cost of being discovered and which also depends on the speed and effectiveness of state action – on the illegal firm that is 'discovered' and on its probability of being discovered – or even the level of 'efficiency' of the illegal firm in defending itself against justice. The abovementioned probability keeps increasing in size. Hence, the expected value of the penalty is given as follows:

$$V(s) = \Pi(\vartheta)s \tag{6}$$

Furthermore, we have the following:

$$d\pi/d\vartheta > 0; \quad d^2\pi/d\vartheta^2 > 0 \tag{7}$$

In dynamic terms, this relationship is defined as follows:

$$V(s) = e^{-\rho t} \Pi(\vartheta)s \tag{8}$$

In the above equation,  $\rho$  is a discount factor that depends on the illegal company's consideration of the penalty. Moreover, the higher the discount rate, the lower the illegal firm's concern about the penalty. This is a measure of such a firm's risk aversion.

In a static version, one can distinguish between the production cost of the legal firm ( $C_L$ ) and that of the illegal firm ( $C_I$ ). The actual cost of the illegal enterprise ( $C_I^R$ ) is given as follows:

$$C_I^R = C_I + V(s) \tag{9}$$

In this context, the following may be the possible cases:

- a)  $C_I + V(s) = C_L$  which signifies a situation of pure competition;
- b)  $C_I + V(s) < C_L$  e  $C_I + V(s) > C_L$  in which case the possibility of coexistence or specialisation is configured.

Here, we admit that we are more interested in describing a situation

of 'coexistence' between enterprises with different cost structures where legal enterprises do not know about the additional costs of illegal enterprises.

In order to 'defend' itself against a sanction, the illegal firm may be willing to 'spend' some amount of capital to nullify the sanction's effect by incurring a cost  $A(s)$  that replaces  $V(s)$ , a cost that serves as a kind of insurance against the sanction. Thus, the following equations may emerge:

- (a) If  $A_s < V(s)$  then  $C_I^R = C_I + A_s$  e. the illegal enterprise pays the cost of nullifying the enforcement action;
- (b) If  $A_s > V(s)$  the illegal enterprise may have the expected cost of the sanction defined as  $V(s)$  and the actual cost as follows:  $C_I^R = C_I + V(s)$

In this respect, in a borderline situation, if

$$C_I + s < C_L \tag{10}$$

then there is a certainty of the illegal firm being 'caught'; however, this reduces the costs of the illegal firm and, in turn, all firms may then tend to behave illegally.

In other words, the costs curve of illegal firms in the absence of a sanction lies below the cost curve of legal firms; therefore, the latter have a cost advantage. State intervention can change this scenario by introducing a sanction that can, at some point, bring the cost curve of illegal firms above the legal ones, restoring the competitive conditions in the market.

Thus, from a policy point of view, it is necessary to identify an optimal level of public expenditure regarding the prevention and security of criminal activities that would make the additional cost of the illegal enterprise higher than that of the legal enterprise. However, we also want to remain within the framework of the problem of the coexistence of the two abovementioned types of enterprises and discuss our case using a very particular modelling approach, that of 'biological' models.

### 3. Biological models in economics and their explanation of the competition between legal and illegal firms

Biological models make it possible to study the relationships between two populations living on the same set of resources. These models are belong to the so-called evolutionary approach originally developed by Maynard Smith and Price (1973), Taylor and Jonker (1978) and Maynard Smith (1974, 1982). In the field of economics, the evolutionary game has proved very useful as a tool for studying various economic phenomena. To elaborate, in this game, agents choose between different actions whose payoffs depend on the choices of other agents; their behaviour evolves in relation to the prevailing strategies. In the context of evolutionary games, payoffs are also defined as fitness, i.e. the probability of survival.

A very rich and comprehensive review of the economic applications of evolutionary games can be found in the work of Friedman (1998). A relevant part of this literature concerns individual behaviour. Another important topic concerns selection phenomena in the market in the evolutionary context. Although our work is more closely related to the second problem, we also use some of the concepts implemented with regard to individual behaviour.

Previously, Cressmann (1995) studied an evolutionary game with two groups of individuals; in particular, he studied a local version of the Pareto optimal solution and found a dynamically stable solution for the replicator dynamics. Cannings and Whittaker (1995) focused their research on a particular form of conflict between individuals, termed 'war of attrition'. Moreover, Binmore and Samuelson (1997) examined a noisy equilibrium selection problem between individuals, modelling their selection strategy as a birth-and-death process in which mutations

would be allowed. They established the condition whereby the dominant equilibrium for risk or the dominant equilibrium for payoff would be selected. In this context, as soon as selection occurred, the modeller would have to introduce differences in the behaviours of groups of individuals.

In this regard, amongst the best-known biological models is the 'prey-predator' model (also known as the Lotka-Volterra model), to which Samuelson (1971) had repeatedly attributed interesting descriptive potentialities with respect to competitive phenomena. This model configures the existence of a predator that eats the prey, threatening its survival. Together, these aforementioned biological models make it possible to study the dynamic evolution of populations and the stability conditions of an economic system (Cellier, 1991).

Biological models also include the logistic model that robustly explains urban development in terms of a first phase of increasing returns with 'agglomeration economies' and a second phase in which diseconomies prevail instead, causing the system to tend towards a saturation asymptote (Maggioni, 1993).

In the work of Weisberg et al. (2011), there are two interesting surveys on biological models in economics. Moreover, Watanabe et al. (2005) try to integrate economic, biological and physical models in order to explore the most efficient combination of these models through which one can ensure sustainability. Robson (2001) considers the implications of biological evolution for economic preferences. Further, Wilkinson (2022) attempts to explain the concepts of poverty and development using biological paradigms.

On its part, this paper uses more sophisticated classifications of these models and develops an economic scheme that can be applied to study the relationship between legal and illegal enterprises. In this respect, the basic idea is to imagine two groups of enterprises (legal and illegal) competing in a market (also understood in a broad sense), each based on different characteristics. The 'legal' enterprises have their own growth dynamics but are also weakened by the erosion action that the 'illegal' enterprises execute by virtue of their 'competitive advantage'. 'Illegal' enterprises, on the other hand, have their own growth dynamic, which is reinforced by the fact that they express greater voracity than 'legal' enterprises (Chakra et al., 2015).

Here, one must bear in mind that to give substance to the problem, a further assumption must be introduced: 'illegal' enterprises need 'legal' enterprises to exist in order to ensure their own existence. From a descriptive point of view, this means that a long-run equilibrium cannot be marked by the exclusive existence of 'illegal' enterprises (market specialisation assumption).

Thus, we start from the consideration that enterprises are 'illegal' not because of the object of production but because of the way in which an enterprise is conducted in general (in terms of both the procurement of inputs and the conquest of markets). On the other hand, the mere existence of illegal enterprises can negate the very idea of the existence of markets. Thus, at the sector level only illegal enterprises may remain, but this cannot happen at the system level.

This condition of biological models is typical of the 'prey-predator' configuration, as per which predators become extinct if their prey becomes extinct. On the other hand, if we accept the description made about the action of the public operator, it must be considered that as far as the system as a whole is concerned, the state 'defends' the existence of the legal business sector by limiting the cost advantages of 'illegal' businesses. Evidently, the characteristic of this assumption is a kind of 'transversality condition'.

It should also be noted that in the explicit model, size considerations are disregarded. In terms of biological models, the model proposed by this paper is of the type that can be termed 'competitive with dependence'. This model is expressible in terms of a system of two non-linear differential equations and represents the dynamics of two populations competing for the same resource. The analytical form of the model is given as follows:

$$\begin{aligned} dx_I/dt &= ax_I - bx_Ix_L \\ dx_L/dt &= cx_L - dx_Ix_L \end{aligned} \tag{11}$$

where  $b = kd$ ,  $k > 1$ ,  $I$  stands for 'illegal',  $L$  stands for 'legal' and  $x$  is the number of firms in the respective populations.

In the above equation, the coefficients of the cross terms indicate the 'voracity' of the two populations. In our case study, the voracity of illegal firms is assumed to be higher than that of legal firms due to the 'competitive advantage' resulting from the use of illegality. One can understand  $k$  as the efficiency factor related to 'competitive advantage'.

The biological model considered in this paper makes it possible to study the spread of the abovementioned two types of enterprises in the presence of resources common to both of them. If one of them dies out, the other grows at an exponential rate (the parameter  $a$  in the case of illegal enterprises and the parameter  $c$  in the case of legal enterprises). Specifically, the solutions for the two populations can be given as follows:

$$\begin{aligned} x_I(t) &= x_{0I}e^{at} \text{ for } x_L = 0 \\ x_L(t) &= x_{0L}e^{at} \text{ for } x_I = 0 \end{aligned} \tag{12}$$

In the presence of quadratic terms, one cannot have exponential growth but can achieve growth based on a logistic function, i.e. growth limited by available resources. This term will be introduced in the following section.

As per the above discussion, the result of the economic system cannot exclude the possibility that only illegal enterprises exist. On the other hand, since it is our intention to exclude this case at the level of the entire system, the equations at the aggregate level must contain a variant that allows us to do the same ('transversality condition').

This is achieved with the following model:

$$\begin{aligned} dx_I/dt &= -ax_I - bx_Ix_L \\ dx_L/dt &= cx_L - dx_Ix_L \end{aligned} \tag{13}$$

In the above equations, for  $x_L = 0$ ,  $x_I(t) = x_0 e^{-at}$

Importantly, competitive ecological models can lead to three types of equilibrium outcomes:

- (a) stable equilibrium;
- (b) unstable equilibrium;
- (c) survival of a single species.

Thus, in relation to the model considered by this paper, the following equations emerge:

$$\begin{aligned} x^I &= a_1x_I - a_{12}x_Lx_I - bx_I^2 \\ x^L &= a_2x_L - a_{21}x_Lx_I - cx_L^2 \end{aligned} \tag{14}$$

For the sake of simplicity, we assume the following:

$$a_1 = a_2 = 1 \tag{15}$$

The following are the conditions required for a stable equilibrium to occur:

$$b > a_{21}; c > a_{12} \tag{16}$$

The conditions of unstable equilibrium are the following:

$$b > a_{21}; c < a_{12} \tag{17}$$

Regarding the conditions leading to the supremacy of one species, for the stable equilibrium we have the following:

$$c > a_{12}; a_{21} > b \tag{18}$$

For the unstable equilibrium, on the other hand, we have the following:

$$b > a_{21}; c < a_{12} \tag{19}$$

The evolution of our model can therefore lead either to situations of

coexistence of the two species or to the supremacy of a single species. To return to the economic dimensions of our project, it must be remembered that the coexistence situation results in the continuation of the duopoly, while the survival of only one species results in the establishment of a monopoly. Moreover, in both cases there is a social loss: in the first case this occurs due to the dissipation of the oligopoly income, while in the second case it occurs due to the net loss of the monopoly. Therefore, a system of illegal economy reveals, whatever the outcome, a situation of Pareto inefficiency.

#### 4. Methods and sample

##### 4.1. Results of simulations

In this section, we present the results of our simulation, obtained by numerically solving the Lotka-Volterra equations under various conditions and with different values assigned to the competition coefficients. To accomplish the same, we utilise a specialised code developed in MATLAB. As already evidenced by the available literature, the Lotka-Volterra equations capture the dynamics of predator-prey interactions in an ecosystem. By solving these equations numerically, we gain valuable insights into how different factors and coefficients affect the stability and behaviour of the given ecological system. By altering the competition coefficients, we are able to investigate the influence of interspecies competition on the overall dynamics of the system. The numerical solution, provided by our MATLAB code, enables us to observe and analyse the intricate relationships between predators and their prey. By examining the simulated outcomes, we can study the oscillations in population sizes, observe the existence of equilibrium points and determine the long-term behaviour of the ecosystem. These results are important for understanding the complex dynamics of predator-prey relationships in a system constituted by both legal and illegal enterprises. They provide valuable insights into the effects of competition coefficients on the stability and coexistence of various species of firms within an ecosystem. Furthermore, our findings can propel the development of policies and strategies to fight illegality in the economic marketplace.

To ensure the simplicity of analysis, we consider only two cases, the case of coexistence and the case of survival of only one species. We particularly highlight the case of stable equilibrium and the case of the survival of legal firms. The qualitative results in the other cases should not differ substantially.

##### Case 1. Coexistence Stable Equilibrium - Initial Conditions (1, 1).

The results our case study are shown in the following two graphs, where on the x-axis the time is measured and on the y-axis the number of enterprises is evaluated. The highest curve, which stabilises at the level of 6 enterprises, concerns legal enterprises; the lowest curve, which stabilises at a level between 1 and 2, concerns illegal enterprises. Under the parameter assumptions highlighted and for any value of the initial conditions, the model is in stable equilibrium. This can also be seen from initial conditions that are absolutely unbalanced towards illegal enterprises (40,000 vs. 1). Evidently, based on this model, an intervention capable of acting on the parameters in order to make them compatible with each other should allow a transformation of the system in favour of legal enterprises (Fig 1, Fig 2)

##### Case 2. Survival of Legal Firms Only - Initial Conditions (20, 20).

In this case, starting from initial conditions of parity between the two groups, a survival result of only legal firms is obtained based on the values of the chosen parameters (Fig. 3).

The abovementioned simulation confirms the previously made theoretical assumptions about the model considered by this study. This result is important because it shows that the competition between legal

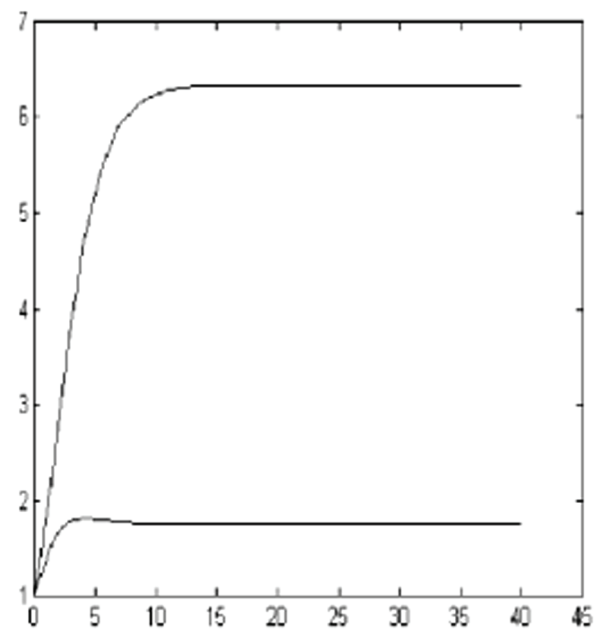


Fig. 1. Equilibrium stable initial conditions (1, 1).

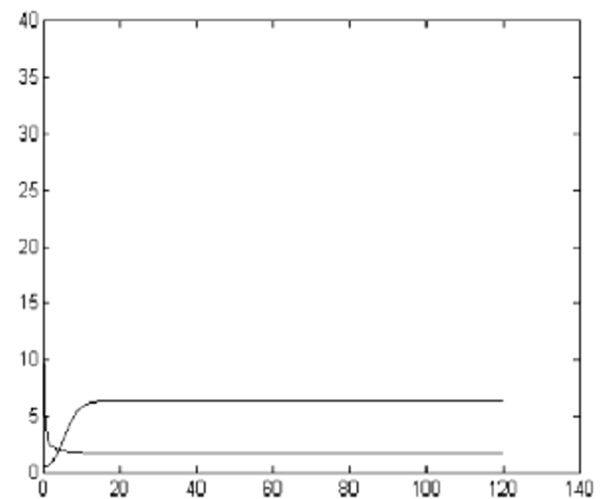


Fig. 2. Equilibrium stable initial conditions (1, 40,000).

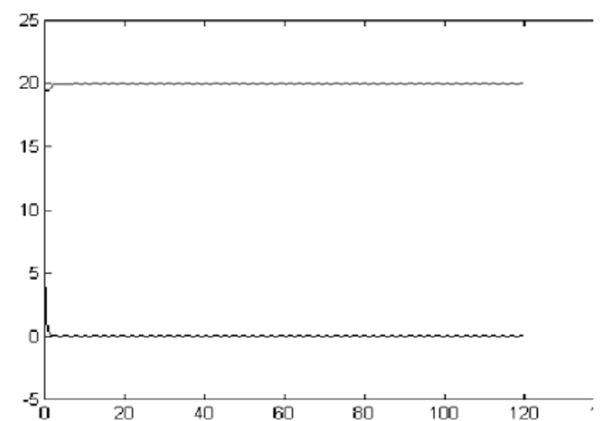


Fig. 3. Survival of legal firms only - initial conditions (20, 20).

and illegal firms in an industry can have multiple outcomes that do not depend on the initial configuration (i.e. how many legal or illegal firms exist at the start) but are simply based on cost advantages. Legal and illegal behaviour amongst enterprises can, therefore, spread by quickly infecting other enterprises. Finding coexistence amongst the possible outcomes is an important result because this fact shows that it is possible to find a balanced market in which legal and illegal enterprises can coexist, which is normally the case. Notably, the ratio between the number of legal enterprises and the number of illegal enterprises in a coexistence equilibrium identifies the degree of legality of an industrial sector. The state can, therefore, intervene in order to make an industrial sector or even the entire economy 'more legal' by increasing its ambit of control and thus making illegal behaviour more costly. Thus, by moving across different balances of coexistence, the state can steer an economic system towards states of 'zero' illegality.

#### 4.2. An empirical verification

The empirical verification of the theoretical model of competition can be carried out by checking whether there is evidence of a different degree of concentration of illegal enterprises within the different economic sectors. However, the first problem that arises when studying the criminal economy sector from an empirical point of view is certainly the identification of a dataset that provides reliability in the presence of a phenomenon that necessarily tends to conceal its true dimensions and characteristics.

To attempt an initial empirical verification of the theoretical model, we use the Open Data of the National Agency for the Administration and Destruction of Seized and Confiscated Assets (ANBSC), Italy. The primary source of the data presented here is the 'ReGIO' system, the information system that the Agency has been given as per law to provide operational support to the administration and destination of assets that are seized and confiscated from the domain of organised crime and to create a census of the confiscated enterprises. The confiscation is a measure adopted by a judge following the seizure of a company whereby, following the outcome of the adversarial process with the involved parties, the prerequisites for the applicability of the dispossession measure against the owner are verified. Confiscation is ordered in the event where the person against whom the proceedings are brought is found to be the owner of assets or to have assets at his disposal (for any reason) that are disproportionate to his income (declared for income tax purposes) or to his economic activity, as well as those assets that are found to be the proceeds of unlawful activities. The ANBSC database, being based on administrative sources, guarantees good-quality data and, in addition, ensures the possibility of classifying companies on a sectoral basis; thus, it can be a useful tool of analysis. Owing to the Company Registry, i.e. the economic registry and instrument of legal publicity provided by the Italian Civil Code art. 2188 et seq. (which contains the official data of all Italian companies), the information held by ANBSC is enriched with data on their nature and location. Notably, there are currently 2949 companies in the ANBSC database.

The large number of companies in the database and their direct relationship with illegal activities make the gathered data extremely useful and indicative with regard to the empirical verification of the model considered in this paper, whereby the greater or lesser concentration of confiscated companies based on sector and territory becomes an excellent proxy for the greater or lesser specialisation of the economy with respect to the legality/illegality dynamics.

Therefore, using territorial and sectoral concentration/specialisation indicators we can assess whether there is empirical evidence of the presence of three possible states mentioned before: coexistence of legal and illegal enterprises, specialisation of illegal enterprises and specialisation of legal enterprises. In Table 3, we tabulate the values obtained for the regional sectoral concentration indicator in Italy. The sectoral specialisation index is given as the number of confiscated enterprises of a sector divided by the number of total enterprises in the sector. The

**Table 3**

Sectoral concentration indicator - sector average = 100; source: our calculations based on ANBSC open data.

Agriculture, forestry, fishing	96
Mining and quarrying	13
Manufacturing	148
Electricity, gas, steam supply	25
Water supply; sewerage, network management activities	24
Construction	499
Wholesale and retail trade; car repair	457
Transport and storage	122
Accommodation and food service activities	213
Information and communication services	23
Financial and insurance activities	37
Real estate activities	174
Professional, scientific and technical activities	41
Rental, travel agencies, business support services	90
Public administration and defence; social insurance	0
Education	3
Health and social work	12
Arts, sports, entertainment and recreation activities	87
Other service activities	34
Activities of families and households as employers	0
Extraterritorial organisations and bodies	0
Unclassified enterprises	102
<b>TOTAL</b>	<b>100</b>

numerical result is normalised and standardised by setting the sectoral average of confiscations as equal to 100. Further, in Table 4 we tabulate the values of the regional concentration indicator. The regional specialisation index is calculated by dividing the number of enterprises in each region by the number of total enterprises in that region; again, the result is subsequently normalised and standardised by setting the value for Italy as equal to 100.

On the basis of the results described in Table 3 the following explanations emerge: we define cases with a score > 150 as Sectors with Specialisation in illegal enterprises; cases with a score > 50 and < 150 as Sectors showing Coexistence of legal and illegal enterprises; and cases with a score < 50 as Sectors with Specialisation in legal enterprises (Table 5).

Similarly, with reference to the results described in Table 4, we define cases with a score > 150 as Regions with Specialisation in illegal enterprises, cases with a score > 50 and < 150 as Regions with Coexistence of legal and illegal enterprises and cases with a score < 50 as Regions with Specialisation in legal enterprises (Table 6).

Together, the above data show that the illegal economy in Italy

**Table 4**

Regional concentration indicator Italian average = 100; source: our elaborations based on ANBSC open data.

Piemonte	23
Valle d'Aosta	17
Liguria	40
Lombardia	54
Trentino-Alto Adige	4
Veneto	10
Friuli-Venezia Giulia	8
Emilia-Romagna	39
Toscana	37
Umbria	9
Marche	4
Lazio	147
Abruzzo	32
Molise	12
Campania	174
Puglia	92
Basilicata	3
Calabria	390
Sicilia	379
Sardegna	20
<b>Italy</b>	<b>100</b>

**Table 5**

Classification of sectors in relation to the degree of illegality; source: our elaborations on ANBSC open data.

Specialisation in illegal enterprises	Coexistence of legal and illegal enterprises	Specialisation in legal enterprises
1 Construction	1 Agriculture, forestry, fishing	1 Extraction of minerals from quarries and mines
2 Wholesale and retail trade; car repairs	2 Transport and storage	2 Supply of electricity, gas, steam
3 Accommodation and food service activities	3 Manufacturing activities	3 Water supply; sewerage, network management activities
4 Real estate activities	4 Rental, travel agencies, business support services	4 Information and communication services
	5 Arts, sport, entertainment and recreation activities	5 Financial and insurance activities
	6 Unclassified enterprises	6 Professional, scientific and technical activities
		7 Public administration and defence; social insurance
		8 Education
		9 Health and social work
		10 Other service activities
		11 Other service activities
		12 Extraterritorial organisations and bodies

represents a minority of enterprises, though it is not a residual portion of economic activity in this nation. We can clearly observe the existence of the three situations previously highlighted by the theoretical model: coexistence of illegal and legal enterprises, specialisation in legal enterprises, specialisation in illegal enterprises. Evidently, some sectors and regions do have a high presence of illegal enterprises. In particular, the southern regions of Italy show the highest rates of illegality; notably, the sectors with the lowest knowledge impact have the highest values of illegality (Construction, Real Estate, Catering and Great Distribution). On the other hand, cases of coexistence are situations that can slowly evolve more or less towards a greater or lesser condition of illegality. In summary, this paper achieves an exploratory empirical verification (the first of its kind) of its theoretical model by conducting a case study concerning the enterprise economy of Italy, for which it uses the Open Data of the ANBSC, Italy.

## 5. Conclusions

Concerning the topic of this paper, the Beckerian literature recognises the existence of a competitive advantage for firms that is driven by their engagement in illegal behaviour, stemming from the possibility of utilising the means and/or capital of illegal origin to conduct lawful activities; this advantage can be constrained by state actions that punish such illegal behaviour (Becker, 1968). Three distinct cases can be identified in light of the competitive dynamics between legal and illegal enterprises, at both territorial and sectoral levels. The first case involves the specialisation of a sector or territory, wherein only legal enterprises operate, indicating a clear demarcation between legal and illegal activities and effective state enforcement. The second case illustrates the specialisation of a sector or territory with exclusively illegal enterprises, implying the prevalence of illicit practices and the challenges faced by the state in curbing them (Reuter, 1983). The third case, the most intricate of the three, depicts the coexistence of legal and illegal enterprises within a sector or territory, reflecting the complex interplay between regulatory efforts and the resilience of illegal actors (Mirenda et al., 2019, 2022). In fact, the third case highlights the challenges to fully eradicating illegal activities and also the blurred boundaries between legal and illegal conduct. An understanding of these three cases provides valuable insights into the competitive dynamics between legal and illegal enterprises; this understanding can help policymakers, law enforcement agencies and researchers analyse the relationship between legality, illegality and competition across economic sectors and territories.

In this paper, coexistence is proposed on the basis of various references in the literature that refer to the models of ‘oligopoly with exit’ (Fudenberg & Tirole, 1983, 1986) and ‘oligopoly with asymmetric costs’ (Maskin & Tirole, 1988), wherein firms with different costs can either coexist or ‘exit’ depending on their structure, their preferences, the resources available in the system and the degree of state intervention.

The results of this study’s simulation show all three possible

outcomes of the dynamic evolution of the model that is finally utilised by this study, in terms of both the coexistence of the two types (legal and illegal) of enterprises and the specialisation in one type of enterprise. The numerical result shows that the corrective intervention of the state, aimed at eliminating illegal firms, can alter the structure of competition between legal and illegal firms, reducing or eliminating the cost asymmetry between such types of firms and, therefore, favouring the specialisation of the given economic sector in legal firms only. In turn, the available data reveal that the illegal economy in Italy constitutes a minor portion of the nation’s overall economic activity, albeit not a negligible one. Moreover, the presence of the three situations highlighted by the theoretical model – coexistence of illegal and legal enterprises, specialisation in legal enterprises and specialisation in illegal enterprises – is clearly observable. However, certain sectors and regions exhibit a notable prevalence of illegal enterprises. In particular, the southern regions of Italy demonstrate the highest rates of illegality, with sectors such as Construction, Real Estate, Catering and Great Distribution showing the highest levels of illegal activity. It is worth noting that these sectors, marked by lower levels of knowledge impact, seem to be correlate with higher levels of illegality. To sum up, the cases of coexistence of legal and illegal enterprises represent dynamic situations that can gradually evolve towards increased or decreased levels of illegality. An exploratory empirical verification of the study’s theoretical model is achieved through the study of the enterprise economy of Italy, by utilising the Open Data provided by the ANBSC, Italy. These findings shed light on the dynamics of the illegal economy in Italy and contribute to an improved understanding of the theoretical framework underpinning the coexistence of and specialisations in legal and illegal enterprises. Hence, this study lends itself to interesting future developments, from both theoretical and empirical standpoints. From a theoretical perspective, a

**Table 6**

Classification of regions in relation to the degree of illegality; source: our elaborations on open data ANBSC.

Specialisation in illegal enterprises	Coexistence of legal and illegal enterprises	Specialisation in legal enterprises
1 Calabria	1 Lazio	1 Piemonte
2 Sicilia	2 Lombardia	2 Valle d’Aosta
3 Campania	3 Puglia	3 Liguria
		4 Molise
		5 Abruzzo
		6 Basilicata
		7 Sardegna
		8 Umbria
		9 Toscana
		10 Veneto
		11 Marche
		12 Trentino Alto Adige
		13 Emilia Romagna
		14 Friuli Venezia Giulia



significant avenue of research advancement lies in the enrichment of the existing theoretical framework via the incorporation of game theory. By employing game-theoretical models, future researchers on this topic can delve deeper into the dynamics of competition, cooperation and strategic behaviour between legal and illegal enterprises. On the empirical front, direct surveys offer a promising avenue through which one can obtain more reliable and even qualitative data on the larger issue. By directly collecting data from relevant stakeholders, such as businesses, consumers or law enforcement agencies, researchers can gather first-hand information regarding the prevalence, nature and dynamics of illegal activities in the enterprise economy. Such surveys can also provide insights into the motivations, strategies and operational characteristics of both legal and illegal actors, thereby enhancing our understanding of the intricacies that inform the coexistence of and the competition between legal and illegal enterprises.

In conclusion, the prospects of this study are twofold. Theoretical advancements on the given research topic can be achieved by incorporating game theory, which can enable a more nuanced analysis of strategic interactions. On the other hand, empirical progress can be made through the use of direct surveys, enabling researchers to gather more robust and qualitative data, thereby deepening the understanding of the phenomenon under investigation. Finally, by combining theoretical and empirical approaches, we can further advance our concomitant knowledge and contribute to a broader understanding of the dynamics between legal and illegal enterprises.

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