Journal of Macroeconomics 77 (2023) 103542

Contents lists available at ScienceDirect

Journal of Macroeconomics

journal homepage: www.elsevier.com/locate/jmacro

Misallocation of talent, teachers' human capital, and development in Brazil[☆]

Fernando Barros Jr[a,*], Bruno R. Delalibera[b], Luciano Nakabashi[a], Marcos J. Ribeiro[a]

[a] FEARP/USP, Brazil

[b] Universitat de Barcelona, BEAT and CREB, Spain

ARTICLE INFO

JEL classification:
J20
J24
O11
O40

Keywords:
Misallocation
Human capital
Labor
Externalities
Growth

ABSTRACT

In this study, we investigate the allocation of talent in an economy where teachers play a critical role in developing the human capital of the workforce. To this end, we formulate a Roy model with externality in the occupational choice, as the quantity and quality of teachers are key determinants of workers' human capital. Our analysis suggests that when individuals with greater abilities opt for teaching careers, the entire workforce benefits. However, frictions in the labor and educational goods markets may lead to a suboptimal allocation of talent and hinder economic growth and development. Our model is calibrated to the Brazilian economy, and our findings reveal a negative correlation between frictions in the teacher's occupation and per capita output in the Brazilian states. Our results indicate that eliminating friction in the labor market could result in a 16.94% increase in Brazilian income.

1. Introduction

Many studies analyze the hindrances to economic growth, and one relevant approach is that of resources misallocation. Misallocation of capital, credit, and talent has been pointed out as possible barriers to growth (Banerjee and Duflo, 2005; Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Hsieh et al., 2019). Misallocation of talent across occupations and sectors may be a consequence of race and gender discrimination, social norms and culture, and barriers to the human capital formation (Restuccia and Rogerson, 2017).[1] In the present paper, we study the allocation of talent in an economy where individuals choose their occupation facing different barriers among professions, and teachers play an explicit role in the human capital formation of all workers.

Based on Hsieh et al. (2019), we build a general equilibrium model where individuals choose consumption, time at school, investment in education, and the sector to work. We introduce two barriers that influence individuals' occupational choices, affecting talent allocation in the economy. First, we consider frictions in the labor market, which can be interpreted as the relative difficulty

☆ We thank the co-editor, Chong K. Yip, and an anonymous referee for their insightful and constructive comments that have significantly improved the paper. We also thank Luiz Brotherhood, Tiago Cavalcanti, Pedro Ferreira, Fábio Gomes, Vahagn Jerbashian, Lourenço Paz, Xavier Raurich, and seminar participants at FEARP/USP-RP and 44th Meeting of the Brazilian Econometric Society for their helpful comments. The remaining errors are our own. This research was carried out with the support of Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), Brazil - Financing Code 001. Fernando Barros Jr thanks CNPq-Brazil for partial financial support. Delalibera thanks the financial support from AGAUR-Generalitat de Catalunya through grant 2021 SGR 00862.

* Correspondence to: Faculdade de Economia, Administração e Contabilidade de Ribeirão Preto da Universidade de São Paulo, Avenida Bandeirantes, 3900 - Vila Monte Alegre, Ribeirão Preto SP, 14040-905, Brazil.

E-mail addresses: fabarrosjr@usp.br (F. Barros Jr), bruno.delalibera@ub.edu (B.R. Delalibera), luciano.nakabashi@gmail.com (L. Nakabashi), mjribeiro@usp.br (M.J. Ribeiro).

[1] In the context of developing economies, Hnatkovska et al. (2012) show that the misallocation of talent in India comes from the caste system. In Brazil, Café (2018) shows an overqualification of workers in the public sector in relation to the private sector, especially when the evaluation in the public sector is not related to the worker's performance.

https://doi.org/10.1016/j.jmacro.2023.103542
Received 10 February 2023; Received in revised form 10 May 2023; Accepted 31 May 2023
Available online 16 June 2023

of finding a job in a given occupation and region. This barrier can result from social status or discrimination. The second barrier appears in the educational market. It is related to the costs of human capital formation in a given region and occupation.

In our model, the number of workers choosing an occupation decreases with higher barriers. Moreover, frictions in the teacher's occupation would harm the whole economy since it is essential to the human capital formation of all workers. Furthermore, following Eckstein and Zilcha (1994), we consider the quality of teachers as an input to human capital formation. Based on Gilpin and Kaganovich (2012) and Hatsor (2012), we also consider the number of teachers as input.

Our model is calibrated to the Brazilian economy, where our baseline calibration demonstrates a positive correlation between barriers to the teacher's occupation and per capita output in the Brazilian states. Specifically, these barriers are lower in the less developed regions of Brazil, leading to the accumulation of more human capital among teachers in these areas. Conversely, the more developed states exhibit higher levels of productivity, owing to elevated Total Factor Productivity (TFP).

Brazil confronts several microeconomic challenges, such as a distortionary tax burden, high levels of labor market regulation with expensive firing costs, varying regulations across different sectors, different levels of union market power across sectors, limited competition in several industries (like the energy sector), regional disparities in infrastructure quality, and a significant informal sector. These obstacles make Brazil an important country for research and analysis.

Barros and Delalilbera (2018) have also identified an inverse relationship between the relative wage of teachers and the Brazilian states' economic development. They point out that the occupational choice of workers with multiple skills is driven by labor market incentives (net wage) and the costs of investing in education. Our study differs from Barros and Delalilbera (2018) in two ways. First, in addition to considering that teachers' human capital is a source of positive externalities, we explicitly model the importance of the number of teachers in the workforce's human capital formation. Second, we use our model to study differentials in the relative workers' wages to better understand the relationship between market frictions and the misallocation of talent in Brazil.

We show that the frictions related to the teacher's occupation have a more relevant influence on the economy's output than those of other occupations. We run a series of counterfactual exercises, and we find that the complete removal of frictions in the Brazilian economy would generate an increase of 16.94% in GDP. Furthermore, we calibrate our model with data from different periods to study the evolution of the allocation of talent in Brazil. We argue that the reduction of the barriers over time could be one of the drivers of absolute income convergence across the Brazilian states.[2]

Although we based our model on Hsieh et al. (2019), we are interested in understanding the impact of misallocation of talent in the teacher's occupations. In contrast, Hsieh et al. (2019) study the economic performance related to the reduction in gender and race discrimination over time in the United States. They find that between 20% and 40% of GDP per capita growth over the last five decades is due to declining occupational barriers, causing women and blacks to occupy highly qualified positions over time. Abdulla (2019) also investigates the misallocation of talent in Brazil and India. Their results show that removing all frictions of the labor market and human capital accumulation in Brazil and India would increase average output by 22%–52% and 38%–53%, respectively. We extend the analysis of the above studies by modeling the tradeoff between quality and quantity of teachers in human capital formation.

The misallocation literature has traditionally focused on the role of individual choices in explaining the economic outcome. However, recent studies have highlighted the importance of financial constraints in shaping the educational choices of families (Soares et al., 2012; Ponczek and Souza, 2012; Hanushek et al., 2014; Delalibera and Ferreira, 2019; Brotherhood and Delalibera, 2020). For example, Soares et al. (2012) provide micro-evidence that children from disadvantaged families are associated with more child labor and less schooling, while Ponczek and Souza (2012) shows that twins in the family have adverse consequences for children's education. Hanushek et al. (2014) provide an overlapping generations model that demonstrates how different college funding rules can affect aggregate outcomes and individual welfare. Brotherhood and Delalibera (2020) also build an overlapping generation model to study the optimal allocation of public expenditure across schools and universities.[3] Our findings complement this literature by studying how barriers in educational markets, which can also be viewed in part as financial constraints, can affect the occupational choice of multi-ability workers and generate a misallocation of talent.

Human capital is crucial for economic development by increasing labor productivity, besides facilitating innovation and diffusion of technology as in Romer (1990), Mankiw et al. (1992), Borensztein et al. (1998), and Benhabib and Spiegel (2005). We contribute to this literature by showing how regional disparities in the labor and educational markets can generate talent misallocation in the teacher's occupation and, in turn, affect aggregate human capital.

The recent literature has emphasized the relevance of education quality in economic growth. For example, Hanushek and Woessmann (2012) argue that Latin American countries lagged behind because of their students' poor performance in educational achievement. In addition, many studies point to the relevance of teachers in the students' learning process (Woessmann, 2016; Barros and Delalilbera, 2018; Hanushek et al., 2019). Indeed, Hanushek et al. (2019) find a robust and positive relationship between the teachers' cognitive skills and student performance measured by the Programme for International Student Assessment (PISA) scores. The cognitive skills of teachers are even more critical to students' performance than the cognitive skills of their parents (Hanushek et al., 2019).

Using the PISA's mathematics test score, Woessmann (2016) points to the relevance of teachers' quality measured by their relative wage and human capital on students' performance. Woessmann (2016) argues that higher teacher wages positively influence

[2] See Ferreira (2000) and Ribeiro and Almeida (2012) for evidence of income convergence in Brazil.

[3] In this context, Brotherhood et al. (2022) claim that when there is a high proportion of credit-constrained students, a reallocation of expenditure towards public schools positively affects GDP.

recruiting higher-ability individuals into teaching. For Brazil, Menezes-Filho and Pazello (2007) find that the relative wage of teachers positively affects the proficiency of public school students. Machado and Scorzafave (2016) point out that wages may affect the decision of the most talented individuals to become teachers. In addition, after an individual becomes a teacher, the wages affect their effort in the classroom and the turnover rate. Several other studies also indicate that the ability of teachers is related to their relative wage, as Stoddard (2003), Lakdawalla (2006), and Bacolod (2007).

Tamura (2001) examines the role of education and the quality and quantity of teachers in economic growth and income convergence. Following Card and Krueger (1992a) and Card and Krueger (1992b), Tamura (2001) formulates a function of human capital formation, where teachers' quality and class size interact with private investment to produce human capital. Then, the author shows that human capital convergence across regions occurs if teachers' quality is relatively more important than class size in human capital production. He argues that poor school districts have relatively better teachers than wealthier districts, driving the income convergence observed in the data. We also consider teachers' quality and quantity to study income convergence across the Brazilian states. We find that income convergence is due to human capital convergence because teachers of poorer Brazilian states have a higher quality.

Besides this introduction, the present paper is organized as follows. Section 2 presents our general equilibrium model. Section 3 explains how this model is calibrated using data from the Brazilian economy. The calibration results, some stylized facts, and the counterfactual exercises are presented in Section 4. Section 5 presents the robustness checks of our main exercises. Finally, Section 6 brings our final remarks.

2. Model

This section provides an overview of the model and its fundamental assumptions. We discuss the behavior of both firms and workers and highlight the main implications of the model. Additionally, we define the competitive equilibrium.

2.1. Firms

We begin by considering a country divided into $R \in \mathbb{N}$ independent regions (states). Each region comprises a continuum of workers who choose one of the $N \in \mathbb{N}$ available occupations in the economy. It is assumed that workers born in a particular region, r, can only work there.[4] Multiple homogeneous competitive firms hire workers from all regions and occupations to produce a single product. The production function for each firm is defined by

$$Y = \sum_{r=1}^{R} \sum_{i=1}^{N} A_r H_{ir}, \tag{1}$$

where Y is output, A_r is Total Factor Productivity (TFP) of region r, and H_{ir} is the aggregate human capital of people working in occupation i at region r. Output can be consumed or used as an educational good. The firm's problem is choosing labor in terms of efficient units (aggregate human capital) to maximize profit, taking wages (w_{ir}) of each occupation in each region as given.

$$\max_{H_{ir} \geq 0} \left[\sum_{r=1}^{R} \sum_{i=1}^{N} A_r H_{ir} - \sum_{r=1}^{R} \sum_{i=1}^{N} w_{ir} H_{ir} \right]. \tag{2}$$

The solution to the problem described above is simple. The demand for human capital is given by:

$$H_{ir}^{d} = \begin{cases} 0 & \text{if } A_r < w_{ir} \\ x \in \mathbb{R}_+ & \text{if } A_r = w_{ir} \\ \infty & \text{if } A_r > w_{ir} \end{cases}. \tag{3}$$

2.2. Workers

Each worker in our model has idiosyncratic abilities for each occupation. In a world with multiple occupations, some workers possess a high talent for many occupations, while others may lack the skills for any occupation. Individuals value both consumption and leisure, which we model as the time not spent at school. Each worker is endowed with one unit of time, which can be allocated to either studying or leisure. The following equation gives the utility of a worker:

$$U(c, s) = c^{\beta}(1 - s), \tag{4}$$

where c represents consumption, s is time spent at school, and β is a parameter giving the relative importance of consumption to leisure.

We adopt the approach of Hsieh et al. (2019) and introduce two frictions in our model. First, we assume that a person working in occupation i in region r is paid a net wage of $(1 - \tau_{ir}^{w})w_{ir}$, where τ_{ir}^{w} is a barrier specific to occupation i and location r. This can

[4] In Appendix D, we discuss migration and argue that the fraction of the Brazilian population that migrates is relatively small. Therefore, our assumption of no migration is consistent with the available data.

be interpreted as an unobserved cost (or benefit) of working in occupation i at region r, which can arise due to various factors such as social status or barriers to finding a job in a given occupation and region.

The educational market in our model also experiences friction in the form of τ^h_{ir}, which captures the barriers to acquiring human capital for different occupations and regions. These barriers may include difficulties in finding quality educational institutions or suitable training programs for a particular occupation. Additionally, τ^h_{ir} may represent the costs of developing the necessary skills for specific occupations.

Following Tamura (2001) and Barros and Delalilbera (2018), we assume that the quality of teachers is a crucial input to human capital formation. In addition, we extend the existing literature by incorporating the number of teachers as a determinant of workers' human capital formation. Therefore, the human capital of workers in each region can be represented by the following expression:

$$h_{ir}(e,s) = T_r^{\varphi} s_i^{\phi_i} e_{ir}^{\eta}, \tag{5}$$

where e represents the consumption of educational goods, s is the time spent at school, η is the elasticity of the human capital concerning the consumption of educational goods, and $\phi_i > 0$ is the elasticity of human capital concerning the time spent at school. This parameter varies among occupations and generates differences in schooling. Finally, T_r represents the role of teachers in the workers' human capital formation. We set $T_r = p_{tr}^{\alpha} H_{tr}^{1-\alpha}$ where $\alpha \in (0,1)$, p_{tr} is the fraction of people working as teachers, and H_{tr} is the teachers' aggregate human capital. We use this functional form to incorporate the quality and quantity of teachers into the workers' human capital formation.[5]

Following McFadden (1974), Eaton and Kortum (2002), and Hsieh et al. (2019), abilities dispersion is modeled as a multivariate Fréchet distribution. Let ϵ_i be the ability of an individual in occupation i, then the distribution of abilities across occupations is:

$$F(\epsilon_1, \dots, \epsilon_N) = \exp\left[-\sum_{i=1}^{N} \epsilon_i^{-\theta}\right], \tag{6}$$

where θ governs the skill dispersion.

The individual decision is made in two steps. First, given the occupational choice i, for which the individual has an idiosyncratic ability ϵ_i, and taking wage w_{ir} as given, each worker chooses c, e, and s, to solve the following problem:

$$\max_{c,s,e} c^{\beta}(1-s) \tag{7}$$
$$\text{s.t. } c = (1-\tau^w_{ir}) w_{ir} h_{ir}(e_{ir}, s_i)\epsilon_i - (1+\tau^h_{ir}) e_{ir},$$

Solving the problem above yields the optimal time spent on school and the amount of educational goods purchased[6]:

$$s_i^* = \left(1 + \frac{1-\eta}{\beta\phi_i}\right)^{-1} \tag{8}$$
$$e_{ir}^*(\epsilon) = \left[\eta\left(\frac{1-\tau^w_{ir}}{1+\tau^h_{ir}} w_{ir}\right)(p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi}\left(1+\frac{1-\eta}{\beta\phi_i}\right)^{-\phi_i}\epsilon_i\right]^{\kappa} \tag{9}$$

where $\kappa = 1/(1-\eta)$.

A higher elasticity of human capital with respect to time for a given occupation (ϕ_i) leads to more time allocated to human capital accumulation. Individuals in occupations with a high ϕ_i acquire more schooling and have higher wages as compensation.

Using Eqs. (8), (9) and the budget constraint into the utility function, we have the following indirect utility function for occupation i:

$$D_{ir} = \left[\bar{\eta}\left(\frac{1-\tau^w_{ir}}{(1+\tau^h_{ir})^{\eta}} w_{ir}\right)(p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi_i}(1-s_i)^{\frac{1}{\beta\kappa}}\epsilon_i\right]^{\beta\kappa} \tag{10}$$

where $\bar{\eta} = \eta^{\eta}(1-\eta)^{1-\eta}$.

Therefore, the occupational choice problem reduces to picking the occupation that delivers the highest D_{ir}.[7] Since talent is drawn from an extreme value distribution, the highest utility can also be characterized by an extreme value distribution (McFadden, 1974). Proposition 1 states that the share of the workers in each occupation can be obtained by aggregating the individuals' optimal choices.

Proposition 1 (*Occupational Choice*)**.** *Let p_{ir} be the fraction of workers in occupation i in region r. Then, aggregating the solution of individual's occupational choice problem across workers, we have:*

$$p_{ir} = \frac{\tilde{w}_{ir}^{\theta}}{\sum_{j=1}^{N} \tilde{w}_{jr}^{\theta}} \tag{11}$$

[5] See Krueger (2003) and Lakdawalla (2006) for a discussion on teachers' quality and quantity.

[6] For a complete solution of the model, refer to the Online Appendix.

[7] Our model assumes a deterministic path for every feasible occupational choice without inherent risk. Additionally, we assume that there is no variation in the pre-existing wealth of workers. However, empirical evidence suggests that less wealthy individuals tend to select less risky income paths due to the higher marginal utility of consumption (Guo and Leung, 2021). For example, Cagetti and De Nardi (2006) find that restrictive borrowing constraints reduce the number of people engaging in (risky) entrepreneurial activities.

where

$$\tilde{w}_{ir} = \bar{\eta}\left(\frac{1-\tau^{w}_{ir}}{(1+\tau^{h}_{ir})^{\eta}}w_{ir}\right)(p^{\alpha}_{tr}H^{1-\alpha}_{tr})^{\varphi}s_i^{\phi_i}(1-s_i)^{\frac{1}{\beta\kappa}}$$

Proof. Let:

$$\tilde{w}_{ir} = \bar{\eta}\left(\frac{1-\tau^{w}_{ir}}{(1+\tau^{h}_{ir})^{\eta}}w_{ir}\right)(p^{\alpha}_{tr}H^{1-\alpha}_{tr})^{\varphi}s_i^{\phi_i}(1-s_i)^{\frac{1}{\beta\kappa}}$$

Then, we can rewrite Eq. (10) as:

$$D_{ir} = [\tilde{w}_{ir}\epsilon_i]^{\beta\kappa}$$

Therefore, the individual decision problem for worker i in region r consists of choosing the occupation that yields the highest value of $\tilde{w}_{ir}\epsilon_i$. Without loss of generality, let us consider the probability of an individual choosing occupation 1:

$$\begin{aligned} p_{ir} &= Pr(\tilde{w}_{1r}\epsilon_1 > \tilde{w}_{ir}\epsilon_i) \quad \forall i \neq 1 \\ &= Pr\left(\epsilon_i < \frac{\tilde{w}_{1r}}{\tilde{w}_{ir}}\epsilon_1\right) \quad \forall i \neq 1 \\ &= \int F_1(\alpha_1\epsilon, \alpha_2\epsilon, \dots, \alpha_N\epsilon)d\epsilon \end{aligned} \tag{12}$$

where F_1 represents the derivative of Eq. (6) with respect to its first argument, and $\alpha_i = \tilde{w}_{1r}/\tilde{w}_{ir}$ for $i \in \{1, 2, \dots, N\}$. Taking the derivative of Eq. (6) with respect to ϵ_1, and evaluating in ϵ:

$$\begin{aligned} F_1 &= \theta\epsilon_1^{-\theta-1}\exp\left(-\epsilon_1\hat{Z}\right) \\ F_1(\epsilon) &= \theta\epsilon^{-\theta-1}\exp\left(-\epsilon\hat{Z}\right) \end{aligned}$$

where $\hat{Z} = \sum_{i=1}^{n}\alpha_i^{-\theta}$. Then, Eq. (12) can be written as:

$$\begin{aligned} p_{1r} &= \int \frac{\hat{Z}}{\hat{Z}}\theta\epsilon^{-\theta-1}\exp\left(-\epsilon^{-\theta}\hat{Z}\right)d\epsilon \\ &= \frac{1}{\hat{Z}}\int \hat{Z}\theta\epsilon^{-\theta-1}\exp\left(-\epsilon^{-\theta}\hat{Z}\right)d\epsilon \end{aligned}$$

This expression is the derivative of Eq. (6) with respect to ϵ. Hence:

$$\begin{aligned} p_{1r} &= \frac{1}{\hat{Z}}\int dF(\epsilon) \\ &= \frac{1}{\hat{Z}} \\ &= \frac{\tilde{w}^{\theta}_{1r}}{\sum_{i=1}^{N}\tilde{w}^{\theta}_{ir}} \quad \square \end{aligned}$$

We can interpret $\tilde{w}_{ir}$ as a net reward of a person from region r and occupation i with average ability. Therefore, $\tilde{w}_{ir}$ is composed of wage per efficiency unit, schooling, teachers' human capital, and barriers. In this context, occupations with high w_i will attract more workers in all regions. On the other hand, differences in occupational choices are driven by frictions in the educational goods and labor markets. Therefore, the fraction of individuals choosing sector i is low when there are considerable barriers in human capital formation (τ^h is high) and in the labor market (τ^w is high). The following proposition defines the workers' human capital in each occupation in a given region.

Proposition 2 (*Average Quality of Workers*)**.** *For a given region, the human capital of workers in occupation i is:*

$$H_{ir} = p_{ir}\mathbb{E}[h(e_{ir}, s_i)\epsilon_i | \textit{person choices } i], \tag{13}$$

The average quality of workers is:

$$\mathbb{E}[h(e_{ir}, s_i)\epsilon_i | \textit{person choices } i] = \bar{\Gamma}\left[\left(\frac{1-\tau^{w}_{ir}}{1+\tau^{h}_{ir}}w_{ir}\right)^{\eta}\tilde{h}_{ir}p_{ir}^{-\frac{1}{\theta}}\right]^{\kappa} \tag{14}$$

where $\bar{\Gamma} = \Gamma(1-\kappa/\theta)$ *is related to the mean of the Fréchet distribution for abilities,* $\tilde{h}_{ir} = [(p^{\alpha}_{tr}H^{1-\alpha}_{tr})^{\varphi}s_i^{\phi_i}\eta^{\eta}]^{\kappa}$ *and* $\kappa = 1/(1-\eta)$.

Proof. We have:

$$h(e_{ir}, s_i)\epsilon_i = (p^{\alpha}_{tr}H^{1-\alpha}_{tr})^{\varphi}\left[\eta\left(\frac{1-\tau^{w}_{ir}}{1+\tau^{h}_{ir}}w_{ir}\right)(p^{\alpha}_{tr}H^{1-\alpha}_{tr})^{\varphi}s_i^{\phi}\epsilon_i\right]^{\eta\kappa}s_i^{\phi_i}\epsilon_i \tag{15}$$

H_{ir} is the total labor supply in efficiency units of occupation i in region r. Then,

$$\begin{aligned} H_{ir} &= p_{ir}\mathbb{E}\left\{ (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} \left[\eta \left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}} w_{ir} \right) (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi_i} \epsilon_i \right]^{\eta\kappa} s_i^{\phi_i} \epsilon_i \middle| \text{person choices } i \right\} \\ &= p_{ir} \left\{ (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} \left[\left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}} w_{ir} \right) \eta (p_{tr}^{\alpha} H_{tr}^{1-\alpha})^{\varphi} s_i^{\phi_i} \right]^{\eta\kappa} s_i^{\phi_i} \mathbb{E}\left[\epsilon_i^{\kappa} \middle| \text{person choices } i \right] \right\} \\ &= p_{ir} \tilde{h}_{ir} \left(\frac{1-\tau_{ir}^{w}}{1+\tau_{ir}^{h}} w_{ir} \right)^{\eta\kappa} \mathbb{E}\left[\epsilon_i^{\kappa} \middle| \text{person choices } i \right] \end{aligned} \tag{16}$$

To calculate this last conditional expectation, we use the Fréchet distribution. We suppress the region index r because this calculation is similar across all regions. Let $y_i = \tilde{w}_i \epsilon_i$. Since we are maximizing y_i, it also has the extreme value distribution:

$$\begin{aligned} \mathbf{Pr}\left(\text{Max}_i y_i < z\right) &= \mathbf{Pr}(\epsilon_i < z/\tilde{w}_i) \quad \forall i \\ &= F(z/\tilde{w}_1, \dots, z/\tilde{w}_N) \\ &= \exp\left[-\sum_{i=1}^{N} (z/\tilde{w}_i)^{-\theta} \right] \\ &= \exp\left[-kz^{-\theta} \right] \end{aligned}$$

where $k = \sum_i^N \tilde{w}_i^{\theta}$.

After some algebraic manipulation, we conclude that the distribution of ϵ^* (the workers' ability in their chosen occupation) has a Fréchet distribution:

$$G(x) = \mathbf{Pr}(\epsilon^* < x) = \exp\left[-k^* x^{-\theta} \right] \tag{17}$$

where $k^* = \sum_{i=1}^{N} (\tilde{w}_i/\tilde{w}^*)^{\theta} = 1/p^*$.

Finally, we calculate the expectation of Eq. (16). Let i be the occupation an individual chooses, and λ a positive exponent.

$$\begin{aligned} \mathbb{E}(\epsilon_i^{\lambda}) &= \int_0^{\infty} \epsilon_i^{\lambda} dG(\epsilon) \\ &= \int_0^{\infty} \theta \left(\frac{1}{p^*} \right) \epsilon^{(\lambda-\theta-1)} \exp\left[\left(\frac{1}{p^*} \right) \epsilon^{-\theta} \right] d\epsilon \end{aligned}$$

We set $x = \left(\frac{1}{p^*} \right) \epsilon^{-\theta}$ and rewrite the last expression as:

$$\begin{aligned} \mathbb{E}(\epsilon_i^{\lambda}) &= \left(\frac{1}{p^*} \right)^{\frac{\lambda}{\theta}} \int_0^{\infty} x^{-\frac{\lambda}{\theta}} \exp(-x) dx \\ &= \left(\frac{1}{p^*} \right)^{\frac{\lambda}{\theta}} \Gamma\left(1 - \frac{\lambda}{\theta} \right) \end{aligned}$$

Using this result in Eq. (16) completes the proof. □

This finding suggests that there is a selection effect at play in the economy. Eq. (14) reveals that the average quality of workers in occupation i and region r is inversely related to the proportion of workers in that occupation (p_{ir}). When there are significant frictions in occupation i and region r, only the most skilled workers are selected for that occupation. For example, if becoming a teacher is relatively easy in a particular region, the average human capital of teachers in that region will be low (intensive margin). On the other hand, if we keep the average human capital constant and increase the proportion of workers in an occupation, the aggregate human capital will be higher (extensive margin). The net effect depends on the values of the parameters. When $\theta(1-\eta) > 1$, the extensive margin dominates, while the intensive margin dominates otherwise. Having established this, we solve the model for the average wage in occupation i and region r.

Corollary 1 (*Gross Average Wages*). *Let W_{ir} be the gross average wage in occupation i in region r. Then:*

$$W_{ir} = w_{ir}\mathbb{E}[h(e_{ir}, s_i)\epsilon_i] = \bar{\Gamma}\eta \frac{(1-s_i)^{-1/\beta}}{(1-\tau_{ir}^{w})} \left(\sum_{i=1}^{N} \tilde{w}_{ir}^{\theta} \right)^{\frac{\kappa}{\theta}} \tag{18}$$

This result is a consequence of Proposition 2. As Eq. (18) indicates, the gross average wage varies across occupations in a region due to differences in schooling and labor market frictions. Occupations with higher levels of human capital offer more substantial gross average wages. Using Eq. (3), we deduce that in equilibrium, $A_r = w_{ir}$. As a result, $\tilde{w}_{ir}$ is a function of A_r, and consequently, W_{ir} depends on regional TFP. This implies that labor market frictions, average human capital, and TFP are all critical determinants of regional average wage disparities. Finally, we adopt a standard competitive equilibrium definition.

2.3. Equilibrium

Definition 1 (*Competitive Equilibrium*). A competitive equilibrium in this economy consists of:

(i) Given an occupational choice, w_{ir}, and the idiosyncratic ability ϵ, each worker chooses c, e, s to maximize utility in Eq. (7).
(ii) Given market friction, w_{ir}, H_{it}, and ϵ, a worker chooses the occupation that maximizes D_{ir}.
(iii) A representative firm hires H_{ir} to maximize profits.
(iv) The occupational wage, w_{ir}, clears the labor market in each occupation and region.
(v) Total output is given by the production function in Eq. (1).

3. Empirical investigation

This section describes how we calibrated the model to fit the Brazilian data. We used data from two distinct periods (2003 and 2015) to investigate the convergence of income and human capital across the Brazilian states.[8]

Our calibration strategy involved identifying appropriate values for frictions and TFP to ensure that the competitive equilibrium is consistent with the dataset of the Brazilian states in 2015. To achieve this goal, we used individual-level data from the Brazilian National Household Sample Survey (PNAD) for the following variables: years of schooling; work hours; gross earnings; and the share of workers in occupations.

After some adjustments,[9] our dataset consisted of 109,038 individuals belonging to eight occupational groups: (1) managers (excluding those in the public sector); (2) professionals in the fields of science and the arts; (3) middle-level technicians; (4) administrative service workers; (5) individuals in the service sector; (6) professionals in sales and service provision; (7) agricultural workers; and (8) workers in the goods and industrial production, services, and repairs/maintenance. To simplify our analysis, we combined groups 4, 5, and 6 into the service sector. Additionally, we separated individuals working as teachers in another category, resulting in the following list of occupational categories:

1. Managers (except public sector);
2. Professionals of sciences and arts (except teachers);
3. Middle-level technicians (except teachers);
4. Service sector;
5. Agriculture;
6. Goods and industrial production, services and repairs-maintenance;
7. Teachers.

The 26 Brazilian states and the Federal District (DF) are considered in the empirical analysis.[10] Therefore, the dataset contains a total of seven different occupations ($N = 7$) spread across twenty-seven regions ($R = 27$).

We divided the parameters into three distinct groups. The first group contains the preferences and technology parameters (η, θ, φ, β, α). The second group consists of the elasticity of human capital in relation to time spent at school (ϕ_i), as well as the frictions (τ^w_{ir} and τ^h_{ir}). The third group includes TPF (A_r).

3.1. Preferences and technology parameters

The model's parameters define the functional forms of various equations, such as those governing the distribution of abilities and the utility function. We set the first group of parameters (η, θ, φ, β, α) to evaluate income convergence by taking the mean of specific statistics from the period between 2003 and 2015. To estimate the skill dispersion parameter (θ) and the elasticity of human capital to educational goods (η), we follow the approach of Hsieh et al. (2019). We assume that wages within a specific occupation and region follow a Fréchet distribution shaped by θ and η in a multiplicative form: $\theta(1-\eta)$. Therefore, the dispersion of wages depends on $1/\theta$ and $1/(1-\eta)$, and the coefficient of variation (CV) of wages within a particular occupation and region is given by:

$$CV = \frac{\Gamma\left(1 - \frac{2}{\theta(1-\eta)}\right)}{\left(\Gamma\left(1 - \frac{1}{\theta(1-\eta)}\right)\right)^2} - 1, \tag{19}$$

where γ represents the Gamma function.

[8] We chose this period because the Brazilian National Household Sample Survey (PNAD) methodology underwent changes before 2003 and after 2015.

[9] We removed individuals with no occupation and those whose wages were less than 60% of the minimum wage to eliminate cases of underreported wages, leading us to drop individuals receiving considerably less than the minimum wage. We also limited our sample to individuals between the ages of 25 and 65, and excluded individuals in occupations that were not well-defined or in the military. In Appendix E, we present the results of an alternative calibration of our model, which demonstrates the robustness of our results to the data filtering process, including individuals that earn less than 60% of the minimum wage.

[10] Acre (AC), Alagoas (AL), Amapá (AP), Amazonas (AM), Bahia (BA), Ceará (CE), Espírito Santo (ES), Goiás (GO), Maranhão (MA), Mato Grosso (MT), Mato Grosso do Sul (MS), Minas Gerais (MG), Pará (PA), Paraíba (PB), Paraná (PR), Pernambuco (PE), Piauí(PI), Rio de Janeiro (RJ), Rio Grande do Norte (RN), Rio Grande do Sul (RS), Rondônia (RO), Roraima (RR), Santa Catarina (SC), São Paulo (SP), Sergipe (SE), Tocantins (TO).

Table 1
Baseline constant parameters.

Parameters	Value	Description	Source
η	0.129	Elasticity of educational goods in the human capital function	Estimated using data from PNAD 2015 and 2003
φ	0.48	Elasticity of teacher's human capital in the human capital function	Tamura (2001)
θ	2.52	Dispersion of skills	Estimated using data from PNAD 2015 and 2003
α	0.31	Weight of the share of teachers in T_r	Tamura (2001)
β	0.231	Consumption preference	Hsieh et al. (2019)

Table 2
Descriptive statistics of years of schooling among occupations and implied ϕ.
Source: Elaborated by the authors with data from PNAD 2015.

Occupation	Parameter	Schooling statistics				
	ϕ_i	Mean	1° Quartile	Median	3° Quartile	Variance
Managers	0.28	11.77	11	11	15	3.36
Sciences and arts	0.35	13.98	15	15	15	2.39
Middle-level technicians	0.28	11.67	11	11	14	2.58
Service-sector	0.22	8.91	6	11	11	3.79
Agriculture	0.12	5.07	2	4	8	3.92
Industrial production and services	0.18	7.75	5	8	11	3.69
Teachers	0.35	14.13	14	15	15	1.69

Each ϕ_i is computed using Eq. (8).

Afterward, we compute the mean and variance of the exponent of the regression residuals and use a root-finding algorithm to solve Eq. (19) for $\theta(1-\eta)$. The value for 2003 is 2.39, for 2015 it is 2.00, and the average of the two years is 2.19.

We adopt the approach of Hsieh et al. (2019) to estimate η as the ratio of educational expenditure to labor compensation. The total amount of public and private educational expenditures as a share of GDP was 0.064 in 2003, 0.079 in 2015, and its average was 0.072. The ratio of labor compensation to GDP was 0.53 in 2003, 0.58 in 2015, and averaged 0.56.[11] We set η to 0.129 based on these values. With $\theta(1-\eta)$ and η in hand, we can easily compute θ as 2.52.

Table 1 presents the remaining functional parameters of the model. To specify the parameters related to the teacher's role in human capital formation, we adopt the values suggested by Tamura (2001): $\alpha = 0.31$; and $\varphi = 0.48$. We also set $\beta = 0.231$, following Hsieh et al. (2019). In Section 5, we investigate the robustness of our results by varying the values of α, β, θ, η, and φ.

3.2. Estimation of ϕ_i's

To calculate the second group of parameters, which represents the elasticity of human capital to time spent at school for each occupation (ϕ_i's), we begin by computing the average years of schooling for each occupation and then the study hours. We assume that a typical individual studies six hours a day on weekdays, so the number of study hours in a year is $252 \times 6 = 1512$. Therefore, of the 8760 h available in a year, the time studying represents 17.26%.

We assume that the schooling period occurs in the first 25 years of an individual's life, which is the upper bound of years of education in our model. We then divide the average years of schooling in the dataset by 25 and multiply it by the share of studying time in a year (0.1726). Finally, we use Eq. (8) to calculate the ϕ_i's.[12] Table 2 brings the results.

3.3. Calibration of τ's and A's

We calibrate the remaining parameters, τ's and A's, using the Method of Moments, which involves minimizing the difference between the statistics of our model and those of the Brazilian data. In the calibration procedure, we use two statistics groups for each occupation and region: the workers' share; and the average gross wage.

We utilize the PNAD microdata to compute the average hourly wage for each occupation in each region.[13] In our model, those statistics are described by Eqs. (11) and (18). We use the First Order Conditions (FOC) of the firm's maximization problem, where $w_{i,r} = A_r \, \forall \, i, r$, to recover the equilibrium wage rate, which allows us to use Eqs. (11) and (18) to compute the model's statistics that represents the competitive equilibrium.

The sum of the occupations' share in each region equals one, $\sum_{i=1}^{N} p_{ir} = 1$, implying that each region has $(N-1)R$ independent statistics. Thus, we assume that $\tau_{1r}^{h} = 0, \forall \, r$. Also, we assume that $\tau_{1r}^{w} = \tau_{1}^{w}$ for all regions, implying that the frictions in occupation 1 are equal across regions. Also, we fix the TFP of the last region to a constant value, denoted as A_R.

[11] The data for labor compensation as a share of GDP was obtained from the PennWorldTable10.0.

[12] By rewriting Eq. (8) as $\phi_i = \frac{(1-\eta)s_i}{\beta(1-s_i)}$, we can substitute the time spent on education, as calculated previously, and the other parameters into this expression.

[13] Appendix A brings the average hourly wage and the share of workers by occupation and region.

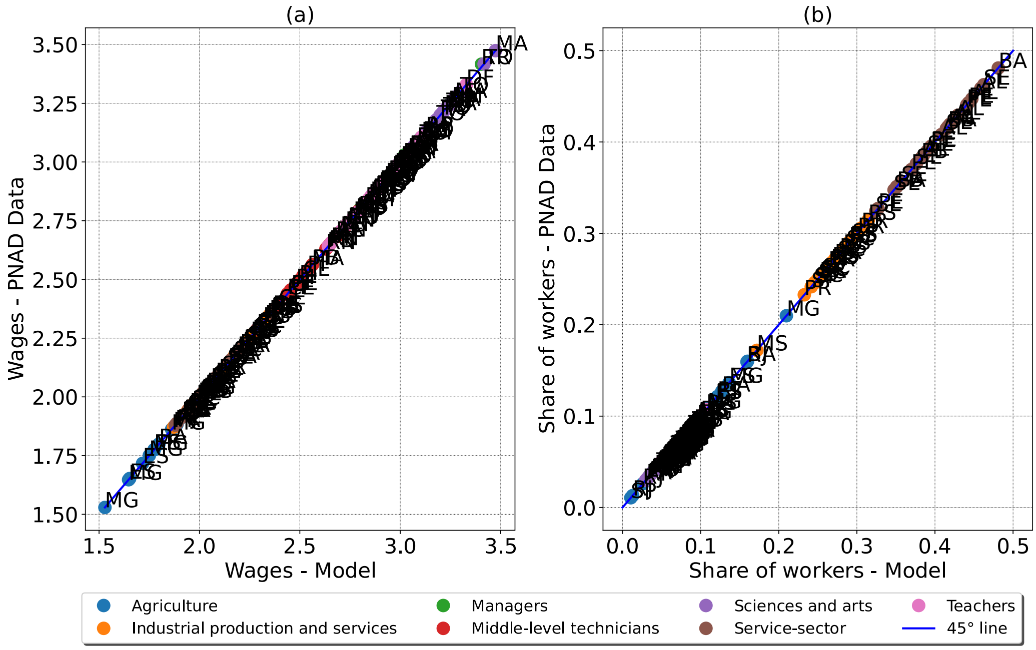


Fig. 1. Model adjustment to data - wages and share of workers.

We define the following objective function to our numerical routine:

$$\mathcal{M} = \sum_{i=1,r=1}^{N,R} \left(\frac{W_{ir}^M - W_{ir}^D}{W_{ir}^D} \right)^2 + \sum_{i=1,r=1}^{N-1,R} \left(\frac{p_{ir}^M - p_{ir}^D}{p_{ir}^D} \right)^2 \tag{20}$$

The superscripts M and D in Eq. (20) represent the model and target statistics, respectively.[14] We utilize the Nelder–Mead algorithm to minimize Eq. (20), resulting in $\mathcal{M} = 0.00092$, which is considered to be a small number, as we have a total of 378 different targets.

Fig. 1(a) displays the average hourly wage from the empirical data (vertical axis) and the estimated average hourly wage from the model (horizontal axis). Fig. 1(b) presents the empirical data (vertical axis) and model-estimated data (horizontal axis) for the share of workers in each occupation and region. The model fits the empirical data well, as indicated by the points being close to the 45° line. Appendix B brings the calibrated values of τ_{ir}^w, τ_{ir}^h, and A_r.

4. Results

This section presents and discusses the results of the numerical exercises. Firstly, we compare the results of our simulations with a set of stylized facts. Additionally, we perform a series of counterfactual exercises to assess the sensitivity of simulated GDP to changes in the labor market and educational frictions. Furthermore, we calibrate the model using 2003 data and compare it to the previous calibration to analyze the income convergence process across Brazilian states. Finally, we evaluate the robustness of our results.

4.1. Comparing model results with a set of stylized facts

The calibrated model produces a good fit for GDP per worker, as illustrated in Fig. 2(a). Moreover, as expected, Fig. 2(b) shows that the model's results indicate a positive correlation between GDP and TFP.

Fig. 3 presents the model's results and data on teachers' wages relative to other occupations.[15] The model's results also display a good fit for relative wages, and as shown in Fig. 3, on average, teachers have a higher relative wage in low and middle-income states[16] than in high-income states.[17]

---

[14] We apply the logarithm in Eq. (20) to improve the algorithm's numerical stability.

[15] In Brazil, the LawN11.738 of 2008 regulates the national minimum wage for public teaching professionals in basic education. However, Table A.1 in Appendix A shows that there is wage dispersion among teachers across regions.

[16] We rank the 27 Brazilian states using 2015 GDP *per capita* data. The first nine states are considered high-income, the middle nine are middle-income, and the last nine are low-income.

[17] To further investigate the relationship between teachers' relative wages and GDP per capita, we conduct a panel regression analysis with the results presented in Appendix F, which provides a more rigorous analysis than our earlier findings. The results of the econometric estimations support our earlier findings and provide additional evidence of the negative relationship between teachers' relative wages and GDP per capita.

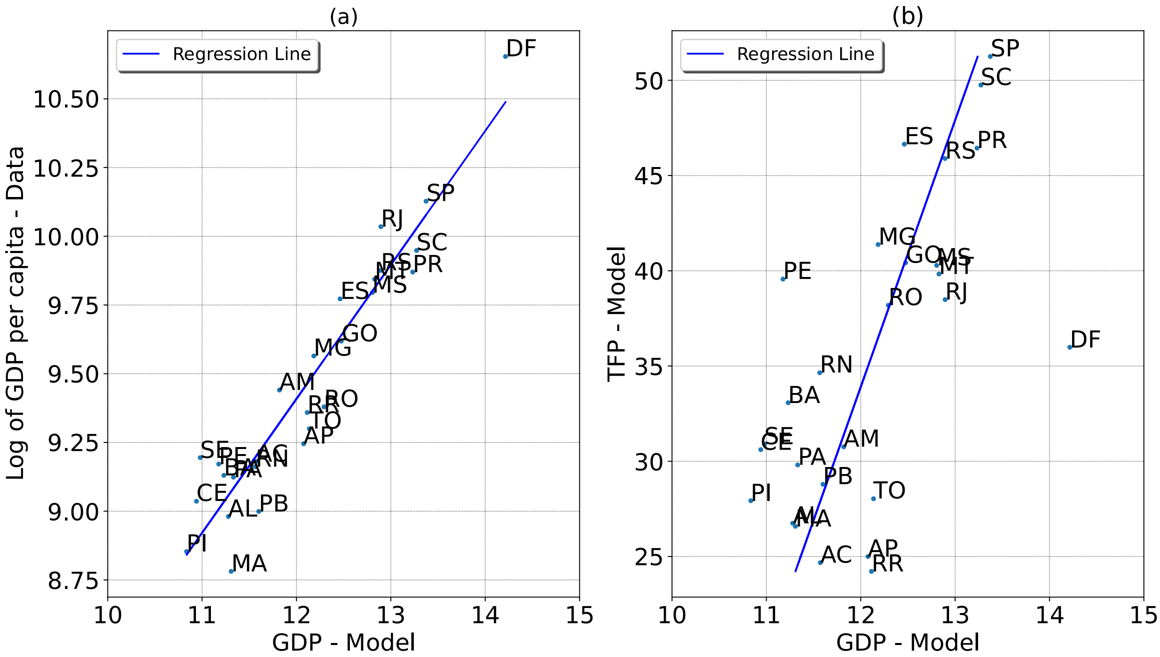


Fig. 2. GDP - Model.

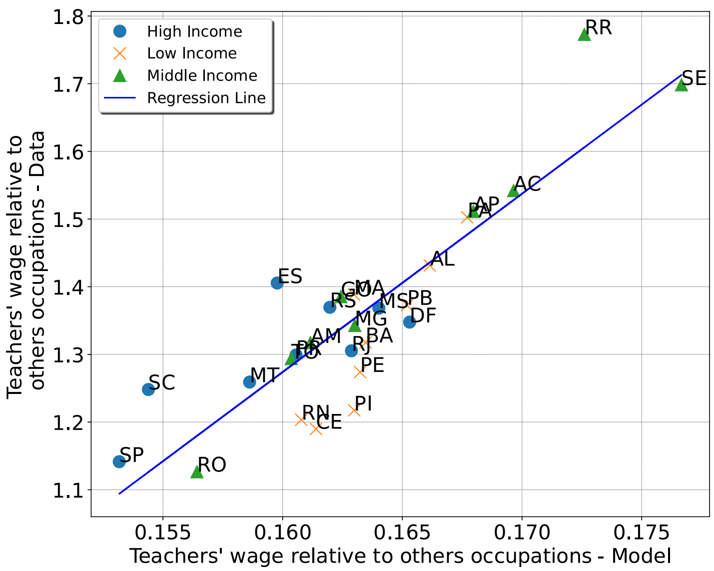


Fig. 3. Teachers' relative wages.

As argued by Barros and Delalilbera (2018), one possible reason for the negative relationship between teachers' relative wages and GDP per capita is that the teaching profession is labor-intensive and less affected by technological and structural changes compared to other occupations. Therefore, in states with more advanced technologies, the relative teachers' wage is lower than in less developed states.

In low-income states, the teaching profession may be more accessible to individuals due to lower labor and educational market barriers compared to richer states. As a result, a higher share of talented people may choose the teaching profession in these states compared to high-income ones, as illustrated in Fig. 4. Therefore, providing more incentives for talented individuals to become teachers in high-income states may lead to even higher incomes in these regions.

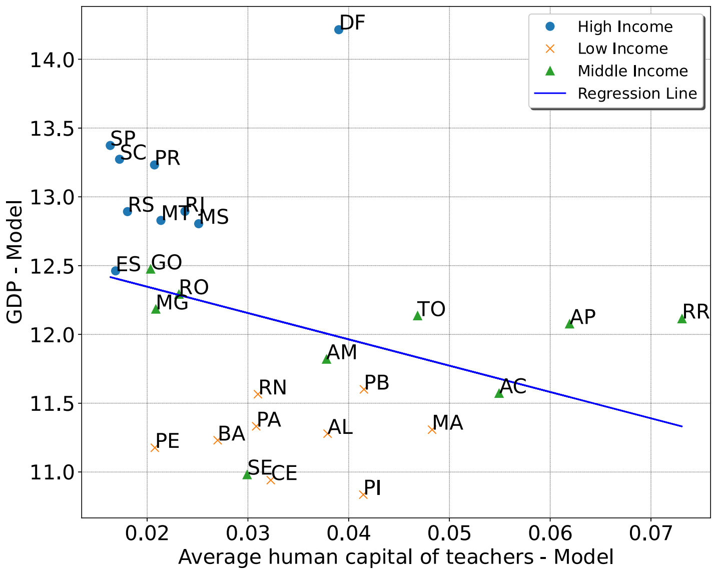


Fig. 4. GDP per worker and teachers' human capital.

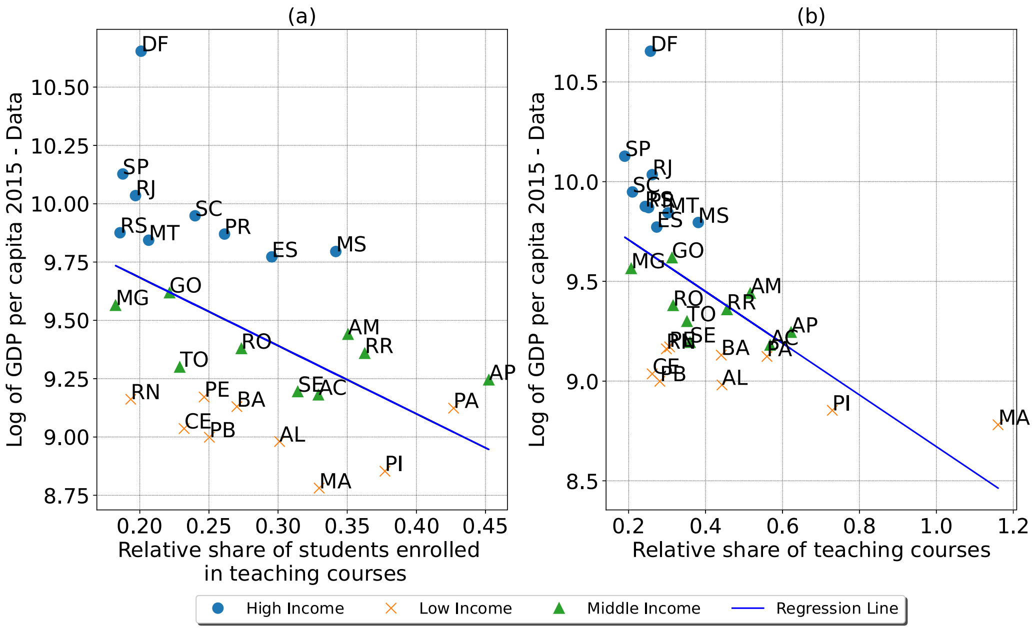


Fig. 5. Share of students in teaching courses, and share of teaching courses offered. Note: The data used to create this figure was obtained from the Higher Education Census of 2015, which was provided by the National Institute of Educational Studies and Research Anísio Teixeira (INEP).

The results obtained are consistent with the available data. Fig. 5(a) illustrates that, on average, more students are enrolled in teaching courses in the poorest states than in the wealthiest ones, which can be explained by the fact that the poorest states tend to have a greater number of institutions offering teaching courses, as depicted in Fig. 5(b).

Additional data findings support the calibration of our model. For instance, in the model, agriculture has the highest average frictions in the educational market, which is consistent with the fact that around 65% of workers in this occupation lived in rural areas in 2015 (PNAD), where access to education is often more challenging.[18] In addition, research has shown that the quality

[18] See Appendix B for more details.

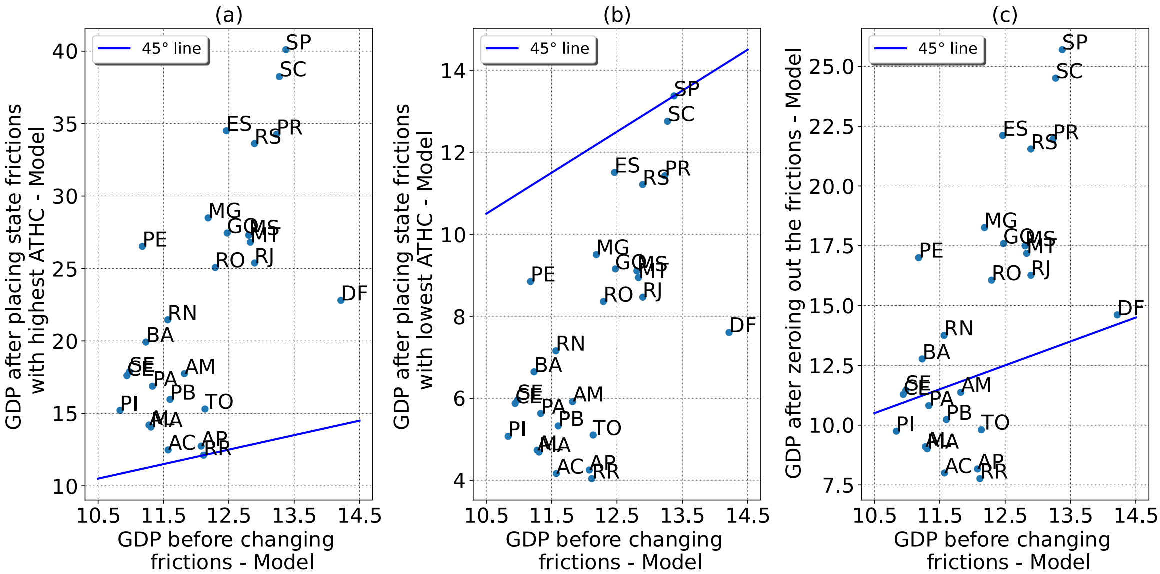


Fig. 6. GDP before and after placing state barriers with the highest and lowest ATHC in all states.

of education in rural schools is generally lower than that in urban areas.[19] On the other hand, at least 92% of workers in other occupations reside in urban areas.

4.2. Frictions and GDP

To investigate the economy's sensitivity to frictions, we conduct counterfactual exercises. In Fig. 6(a), we assume that all states have the same frictions (τ^w and τ^h) as the one with the highest Average Teachers Human Capital (ATHC), which is Roraima (RR). In this case, our results show that all states would have a significantly higher GDP, with the Brazilian GDP increasing by 87.85%. Moreover, the relative wage of teachers in all states would be equal to the level observed in Roraima.

Conversely, if all states had the same frictions as São Paulo (SP), the state with the lowest ATHC, the GDP of all states would decrease (Fig. 6(b)), with the Brazilian GDP declining by 59.62%. Finally, in 6(c), we explore a counterfactual scenario where all frictions are eliminated, resulting in a 16.94% growth in the Brazilian output.

The above counterfactual exercises illustrate the significant impact that frictions have on the economy. However, these exercises involve changing the entire structure of economic incentives, which is a complex matter of public policy. While the results make sense, they cannot be easily implemented.

Based on this exercise, we can infer that labor misallocation is a significant issue across Brazilian states. When the barriers to entry in the teaching profession are altered, the relative wage for teachers changes, which leads to a reallocation of talent across different occupations. Since becoming a teacher has an externality, meaning that it impacts society as a whole, a modification in frictions that encourages more talented individuals to choose this occupation significantly impact regional GDP.

In the next exercise, we examine how market frictions across all occupations affect GDP *per capita*:

1. We calculate the GDP *per capita* assuming no frictions in both labor and educational markets.
2. We set the educational market frictions to zero and vary the labor market friction from −0.9 to 0.9.
3. Similarly, we conduct an exercise where labor market frictions are set to zero, and we analyze the impact of educational goods market frictions.

Frictions in the educational market act as a "price", enabling consumers to adjust their demand for educational goods. On the other hand, due to the inelastic labor supply, frictions in the labor market only affect the net wages. Therefore, alterations in educational frictions tend to exert a more substantial influence on GDP. Fig. 7 shows the results.

4.3. Teacher's human capital

This section examines the impact of changing frictions in each occupation while keeping the frictions in the other occupations at zero. Fig. 8 reveals that changes in frictions in the teacher's occupation, particularly in the educational goods market, have a more significant effect on GDP than changes in other occupations' frictions. Hence, public policies should incentivize more qualified people to become teachers to promote GDP growth. It should be noted that reducing frictions in other occupations could adversely affect economic performance by reducing the incentives for individuals to become teachers.

[19] See, for example, Williams (2005) and Zhang (2006).

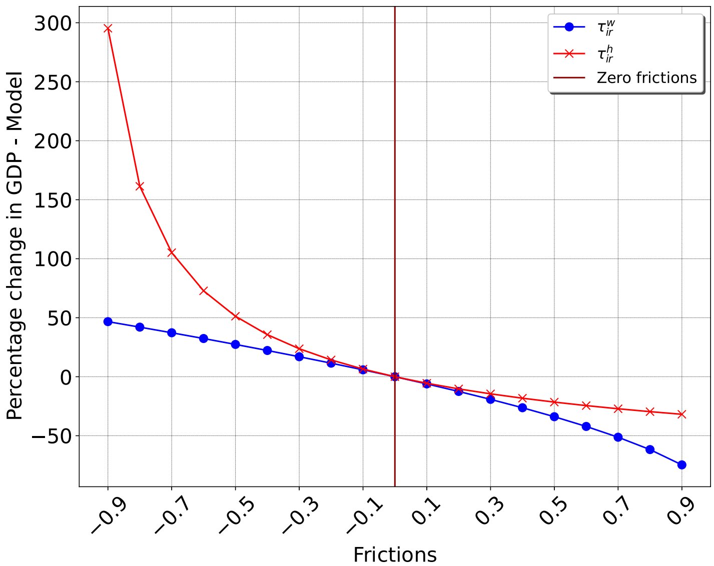


Fig. 7. Increases in the frictions of all occupations and the percentage effects on GDP - Model.

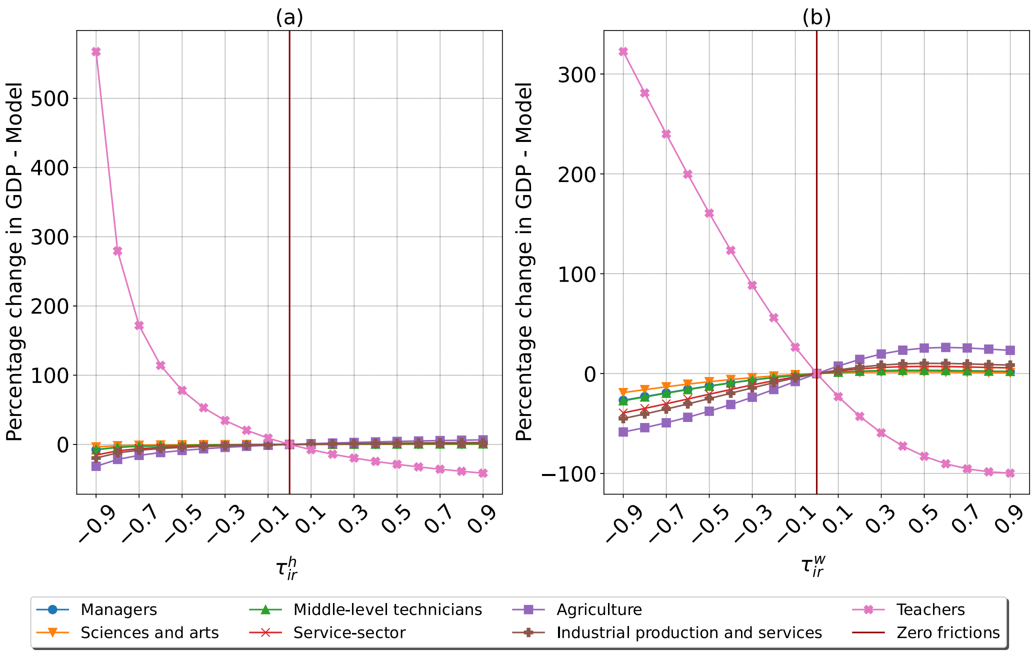


Fig. 8. Increases in the frictions of each occupation and the percentage effects on GDP.

In Fig. 9, we shift the focus from GDP sensitivity to the impact of frictions on teachers' human capital and the proportion of teachers in the workforce.

The teaching profession becomes more attractive with lower barriers, and as a result, more individuals choose this career path (extensive margin). However, the decrease in barriers may also lead to individuals with lower idiosyncratic skills choosing to become teachers, resulting in a lower average quality of teachers (intensive margin).

Although there is a trade-off between the quality and quantity of teachers in our model, a lower τ^h results in a lower "price" of educational goods (see Eq. (9)). As a result, all workers who choose to become teachers invest more in human capital, compensating for the potential decrease in quality due to the increase in quantity (extensive margin). Therefore, the net effect of reducing barriers

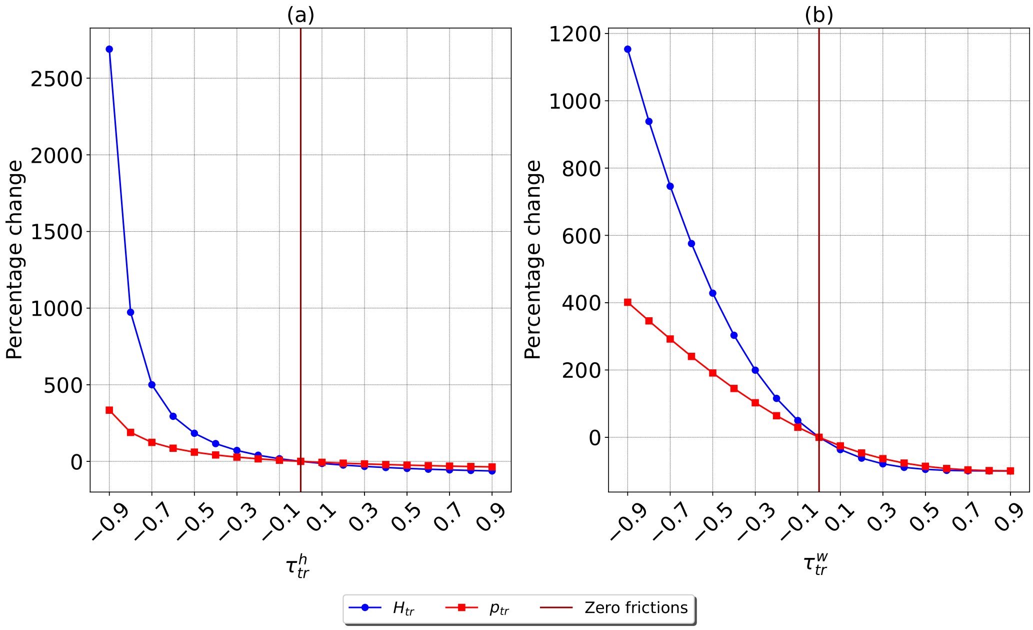


Fig. 9. Increases in frictions and percentual effects on H_{tr} and p_{tr}. Note: H_{tr} = Human capital of teachers, p_{tr} = proportion of workers in teacher occupation.

is an increase in the average quality of teachers (intensive margin). It is important to note that all these effects are amplified due to the positive externality of teachers on the entire workforce.

4.4. Income convergence

According to the research conducted by Barro and Sala-i-Martin (1992), absolute income convergence happens when economies with lower income grow faster than those with higher income per capita, leading to a decrease in the income gap between poor and wealthy regions over time. To test whether income convergence occurs across the Brazilian states, we utilize data from a model calibrated for 2015 and 2003. To achieve this, we estimate the following equation using Ordinary Least Squares (OLS):

$$\frac{1}{T}\log\left(\frac{Y_{r,2015}}{Y_{r,2003}}\right) = a + b\log(Y_{r,2003}) + \epsilon_r \tag{21}$$

Here, $Y_{r,2015}$ and $Y_{r,2003}$ represent the GDP of region r in the years 2015 and 2003, respectively. The model is estimated over T years, with a and b being constants and ϵ_r representing the error term. A negative value of b provides support for the convergence hypothesis.

The findings in Table 3 support the hypothesis of absolute income convergence among the Brazilian states since the estimated b is negative and statistically significant. In addition, we have calculated the speed of convergence of this economy, which is $\beta_s = 4.01\%$.[20] The half-life concept can be used to interpret this result, which represents the time required to reduce the income gap by half. The half-life is calculated as $HL = \log(2)/\beta_s$, and we find that it equals 17.3 years.

The occurrence of absolute income convergence among Brazilian states from 2003 to 2015 is further evidenced by Fig. 10. This plot clearly shows that low-income states, such as Paraíba (PB), Rio Grande do Norte (RN), Maranhão (MA), Alagoas (AL), Piauí(PI), and Ceará (CE), experienced relatively fast income growth in this period. Conversely, high-income states, including the Federal District (DF), São Paulo (SP), Santa Catarina (SC), Rio de Janeiro (RJ), and Rio Grande do Sul (RS), exhibited slower growth rates.

The occurrence of absolute income convergence among Brazilian states from 2003 to 2015 can be explained by multiple factors, including a reduction in educational market frictions and an increase in Total Factor Productivity (TFP).[21] Our analysis reveals that educational frictions have decreased more sharply in the poorest states and Rio de Janeiro. Additionally, there has been an average reduction in frictions for occupations in the teaching occupation. However, the most substantial reduction in friction occurred in the agriculture sector, likely due to the increase in the average years of education of Brazilian agricultural workers.

Our analysis also indicates a slight increase in labor market barriers from 2003 to 2015, on average. However, as depicted in Fig. 7, the impact of increases in labor market barriers on GDP is relatively small compared to reductions in educational market frictions.

[20] The formula for the speed of convergence is given by: $\beta_s = -\frac{\log(Tb+1)}{T}$.

[21] Appendix B provides the calibrated frictions and TFP for 2003.

Table 3
Absolute income convergence across Brazilian states from 2003 to 2015.
Source: Search results.

	$\frac{1}{T}\log\left(\frac{Y_{r,2015}}{Y_{r,2003}}\right)$
a	0.0880***
	(0.0114)
b	−0.0318***
	(0.0049)
R-squared	0.6284
R-squared Adj.	0.6136

Notes: Standard errors in parentheses. Single (*), double (**) and triple (***) asterisk denote statistical significance at 10%, 5% and 1%, respectively.

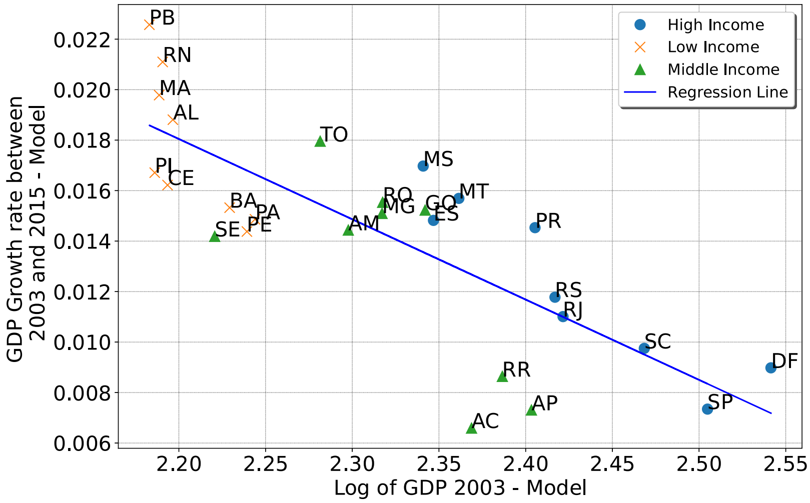


Fig. 10. Growth rate from 2003 to 2015 and Log of GDP 2003.

5. Robustness check

We present the results of our robustness analysis in Table 4. To conduct this analysis, we performed a counterfactual exercise similar to that presented in Section 4.2, where we set the market frictions (τ^w and τ^h) to match those of the states with the highest and lowest Average Teachers' Human Capital (ATHC).

In our counterfactual exercise, when we change the educational goods elasticity in the human capital function η using the frictions of the state with the highest ATHC (Roraima), GDP increases substantially from 2.71% ($\eta = 0.05$) to 87.85% with our baseline $\eta = 0.129$, and up to 162.36% when $\eta = 0.25$. The results are analogous when we use the frictions of the state with the lowest ATHC (São Paulo). Therefore, we can conclude that changes in η substantially impact GDP, regardless of the specific state used in the counterfactual exercise. Among all parameters tested in our robustness analysis, we find that GDP is most sensitive to changes in η and φ.

In the following two lines of Table 4, we analyze the sensitivity of skill dispersion, represented by the parameter θ. In the second column, we observe that when skill dispersion is given by $\theta = 2$, GDP is 57.95% higher and 90.41% greater when $\theta = 3$. The third column shows that changes in θ have a more significant impact on GDP in an economy with higher frictions. Moreover, the parameter β also positively affects GDP.

One critical parameter in the human capital function is the one measuring the trade-off between the quantity and quality of teachers. By increasing α, we place more weight on the number of teachers in relation to their quality (average human capital of teachers).[22] When we set $\alpha = 0.2$, GDP increases by 107.55%, and it is 57.87% higher when $\alpha = 0.6$.

Finally, we examine the sensitivity of our results to the teacher's contribution to human capital formation, denoted by φ. As shown in Table 4, an increase in this parameter has a significant positive influence on GDP. Therefore, teachers play a critical role in driving economic performance in our economy.

[22] Recall that $T_r = p_{tr}^{\alpha} H_{tr}^{(1-\alpha)}$, where H_{tr} is the average human capital of teachers and p_{tr} is the proportion of teachers in region r.

Table 4
Robustness check for constant parameters.
Source: Search results.

Parameter	GDP variation (Largest ATHC)	GDP variation (Lowest ATHC)	GDP variation (Zero frictions)
$\eta = 0.05$	2.71%	−14.07%	9.31%
$\eta = 0.25$	162.36%	−108.50%	20.22%
$\theta = 2.0$	57.95%	−48.50%	21.19%
$\theta = 3.0$	90.41%	−60.59%	16.68%
$\beta = 0.1$	80.46%	−52.38%	−0.29%
$\beta = 0.3$	88.52%	−61.37%	18.35%
$\alpha = 0.2$	107.55%	−73.20%	18.29%
$\alpha = 0.6$	57.87%	−38.79%	14.41%
$\varphi = 0.1$	8.44%	−16.06%	20.20%
$\varphi = 0.6$	162.13%	−99.39%	13.19%
Benchmark	87.85%	−59.62%	16.94%

Notes: ATHC is Average teacher human capital. The baseline values are $\eta = 0.129$, $\beta = 0.231$, $\theta = 2.52$, $\varphi = 0.48$ and $\alpha = 0.31$.

6. Final remarks

In this paper, we develop a Roy model to investigate the influence of market frictions on labor and educational markets in Brazil. Additionally, we incorporate a function where teachers play a crucial role in the human capital formation of the entire workforce. After calibrating the model to Brazilian data, we find a positive correlation between barriers related to the teacher's occupation and GDP across Brazilian states. We also show that increasing the attractiveness of the teaching occupation results in higher GDP. When more individuals with higher idiosyncratic abilities pursue teaching careers, they directly affect the workforce's productivity. These findings highlight the importance of addressing market frictions in the education and labor sectors and underscore the critical role of teachers in promoting economic growth.

Furthermore, our calibrated model for 2015 suggests that the main driver of absolute income convergence was the reduction of frictions related to the teaching profession. This reduction led to an increase in the average human capital and productivity of the entire economy. Therefore, policymakers should focus on increasing incentives for individuals to pursue teaching as a career, particularly for those with higher idiosyncratic abilities, to attract more talented people.

Due to the practical challenges involved in selecting high-quality teachers, it is important to interpret our findings with caution. Identifying individuals with high ability in this occupation is a challenging task. For instance, Rivkin et al. (2005) estimate teacher quality using a detailed micro dataset and find that factors such as teachers' experience and education explain very little of teacher quality. Further research is needed to determine the most effective strategies for enhancing the attractiveness of the teaching profession. For instance, policymakers could consider implementing strategies to enhance the attractiveness of the teaching profession, such as creating career paths for teachers based on their performance and offering salaries comparable to those of similarly qualified professionals in other fields. Additionally, providing a work environment that fosters collaboration among teachers, investing in their training, and offering a good retirement plan could also be effective incentives. Further research is necessary to better understand the factors that drive individuals to choose a career in teaching and how to improve the quality of the teacher workforce.

There is potential for further research that accounts for differences in risk levels across occupational options and incorporates heterogeneity in workers' wealth. Such an extension could shed light on how wealth, risk, and the marginal utility of consumption influence occupational choices, and it is a promising direction for future research. Furthermore, investigating the misallocation of teachers across different educational stages could offer further insight into the efficiency of the education system and provide opportunities for targeted policy interventions. For instance, analyzing the allocation of teachers between primary, secondary, and tertiary education, and evaluating how this allocation affects human capital accumulation could be a fruitful area of research. Therefore, future studies could explore these extensions to expand upon our findings and provide a deeper understanding of the dynamics of occupational choices and teacher allocation, which would help inform policy decisions to improve the allocation of human capital and promote economic growth.

CRediT authorship contribution statement

Fernando Barros Jr: Conceptualization, Methodology, Investigation, Writing – original draft, Writing - review & editing. **Bruno R. Delalibera:** Conceptualization, Methodology, Software, Investigation, Writing – original draft, Writing – review & editing. **Luciano Nakabashi:** Investigation, Writing – original draft, Writing – review & editing. **Marcos J. Ribeiro:** Methodology, Software, Data curation, Investigation, Writing – original draft, Writing – review & editing.

Data availability

Data will be made available on request.

Table A.1
Descriptive statistics of teachers' hourly wages by state.
Source: Elaborated by the authors with data from PNAD 2015.

State	Relative wage	Mean	1° Quartile	Median	3° Quartile	Variance	Income group
AC	1.57	17.53	9.72	14.29	23.81	10.46	Middle income
AL	1.45	16.04	9.38	13.91	20.37	9.65	Low income
AM	1.30	16.13	9.52	14.29	20.24	8.63	Middle income
AP	1.48	19.77	13.17	17.80	23.53	10.03	Middle income
BA	1.33	15.40	8.33	11.90	17.86	11.97	Low income
CE	1.21	14.04	8.33	11.90	15.87	11.17	Low income
DF	1.49	27.93	13.69	23.81	35.71	17.42	High income
ES	1.29	18.16	9.68	15.01	21.33	12.99	High income
GO	1.42	19.30	10.19	14.29	22.55	15.18	Middle income
MA	1.30	15.93	8.93	11.90	20.40	12.50	Low income
MG	1.37	18.42	9.52	14.29	21.65	13.78	Middle income
MS	1.45	21.84	11.11	17.86	26.19	16.22	High income
MT	1.31	18.93	11.90	17.06	21.43	10.58	High income
PA	1.50	17.81	9.38	14.29	21.71	13.82	Low income
PB	1.37	17.13	9.04	12.50	21.60	12.55	Low income
PE	1.29	14.99	7.28	11.43	19.05	11.57	Low income
PI	1.28	14.24	9.52	13.10	15.67	7.72	Low income
PR	1.36	21.04	11.90	17.27	23.81	14.86	High income
RJ	1.33	20.00	9.52	15.87	23.81	15.05	High income
RN	1.24	15.53	7.37	11.90	18.45	13.25	Low income
RO	1.25	16.21	10.39	14.07	17.86	10.30	Middle income
RR	1.62	22.34	9.72	20.22	29.17	14.21	Middle income
RS	1.38	20.40	10.84	15.16	23.81	15.12	High income
SC	1.21	18.15	11.90	14.88	20.83	11.36	High income
SE	1.70	19.61	9.40	16.67	26.19	13.52	Middle income
SP	1.13	18.61	9.52	14.88	23.15	14.07	High income
TO	1.30	17.32	9.38	14.58	19.05	13.09	Middle income

Notes: Relative wage is the average hourly wage of teachers divided by the average hourly wage of other six occupations. Acre (AC), Alagoas (AL), Amapá (AP), Amazonas (AM), Bahia (BA), Ceará (CE), Distrito Federal(DF), Espírito Santo (ES), Goiás (GO), Maranhão (MA), Mato Grosso (MT), Mato Grosso do Sul (MS), Minas Gerais (MG), Pará (PA), Paraíba (PB), Paraná (PR), Pernambuco (PE), Piauí(PI), Rio de Janeiro (RJ), Rio Grande do Norte (RN), Rio Grande do Sul (RS), Rondônia (RO), Roraima (RR), Santa Catarina (SC), São Paulo (SP), Sergipe (SE), Tocantins (TO).

Table A.2
Logarithm of average hourly wages by occupation and state.
Source: Elaborated by the authors with data from PNAD 2015.

	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers
AC	2.87	2.86	2.19	2.00	2.04	2.06	2.86
AL	2.68	2.99	2.29	1.95	2.06	1.95	2.77
AM	2.97	3.05	2.46	2.02	1.91	2.07	2.78
AP	3.11	3.13	2.50	2.08	1.86	2.10	2.98
BA	2.81	3.10	2.37	1.92	1.78	2.02	2.73
CE	2.83	3.13	2.44	1.92	1.53	1.87	2.64
DF	3.42	3.47	2.97	2.36	2.24	2.36	3.33
ES	2.92	3.21	2.74	2.06	2.03	2.27	2.90
GO	2.97	3.02	2.64	2.14	2.24	2.26	2.96
MA	3.10	2.99	2.44	1.99	1.75	1.95	2.77
MG	2.92	3.20	2.63	2.04	2.00	2.17	2.91
MS	3.05	3.27	2.70	2.14	2.29	2.26	3.08
MT	2.93	3.22	2.56	2.16	2.37	2.36	2.94
PA	2.94	2.92	2.48	1.96	2.00	1.99	2.88
PB	2.80	3.24	2.45	1.95	1.91	2.01	2.84
PE	2.91	3.07	2.36	1.89	1.72	1.94	2.71
PI	2.92	3.00	2.29	1.88	1.65	1.92	2.66
PR	3.10	3.23	2.79	2.23	2.26	2.32	3.05
RJ	3.03	3.42	2.65	2.15	1.88	2.27	3.00
RN	2.97	3.13	2.51	2.00	1.77	1.92	2.74
RO	2.89	3.00	2.56	2.08	2.20	2.28	2.79
RR	3.04	3.26	2.66	2.03	1.80	2.11	3.11
RS	3.05	3.24	2.68	2.19	2.22	2.22	3.02
SC	3.01	3.18	2.73	2.30	2.33	2.34	2.90
SE	2.92	3.02	2.39	1.91	1.65	1.97	2.98
SP	3.25	3.30	2.83	2.22	2.22	2.35	2.92
TO	2.90	3.20	2.54	2.10	2.01	2.19	2.85

Table A.3
Average years of schooling by occupation and state.
Source: Elaborated by the authors with data from PNAD 2015.

	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers
AC	9.82	10.82	10.69	8.71	4.26	6.79	14.40
AL	10.33	13.24	11.91	7.83	3.70	6.20	14.13
AM	11.48	13.48	11.24	9.09	4.39	8.40	14.20
AP	10.82	13.50	11.84	8.88	4.92	7.38	13.95
BA	11.09	13.51	11.27	8.80	3.52	7.19	13.69
CE	10.57	13.45	11.46	8.65	3.69	7.44	14.19
DF	12.76	14.18	11.88	9.55	5.61	7.90	14.47
ES	11.07	13.85	11.83	8.82	5.82	7.91	14.59
GO	11.69	13.43	11.47	8.77	5.85	7.73	14.45
MA	11.35	13.62	11.27	8.60	4.32	7.07	13.36
MG	11.57	13.97	11.54	8.62	5.02	7.41	14.11
MS	11.71	13.98	11.48	8.58	5.39	7.42	14.11
MT	11.08	13.79	11.37	9.14	5.71	7.54	14.39
PA	10.82	12.58	10.68	8.63	4.05	7.01	14.09
PB	11.88	13.82	10.81	8.51	3.41	6.29	14.32
PE	11.09	14.06	11.50	8.58	4.38	7.02	14.13
PI	11.03	13.35	11.37	8.21	4.05	6.13	14.27
PR	11.93	13.81	11.79	9.01	6.32	8.02	14.29
RJ	11.99	14.17	11.76	9.02	5.29	8.16	14.02
RN	10.47	13.57	11.13	8.85	3.67	7.09	14.03
RO	10.08	14.03	10.69	8.78	5.45	7.03	14.28
RR	10.83	12.95	12.05	9.34	4.83	7.23	14.16
RS	11.66	13.92	11.73	8.98	6.10	7.75	14.42
SC	11.60	13.69	11.57	9.20	6.57	8.18	14.37
SE	11.07	13.68	11.11	8.53	3.27	6.30	14.20
SP	12.48	14.27	12.02	9.18	6.46	8.38	14.09
TO	10.40	13.34	11.34	8.98	5.13	8.05	14.20

Table A.4
Share of workers in each occupation by state.
Source: Elaborated by the authors with data from PNAD 2015.

	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers
AC	0.04	0.03	0.05	0.42	0.13	0.25	0.09
AL	0.04	0.04	0.07	0.41	0.12	0.25	0.07
AM	0.05	0.05	0.08	0.38	0.07	0.29	0.08
AP	0.04	0.03	0.07	0.44	0.07	0.25	0.10
BA	0.05	0.04	0.06	0.45	0.08	0.26	0.06
CE	0.05	0.03	0.06	0.44	0.05	0.31	0.07
DF	0.06	0.10	0.10	0.48	0.01	0.17	0.08
ES	0.07	0.06	0.07	0.35	0.12	0.29	0.05
GO	0.05	0.05	0.06	0.42	0.08	0.29	0.05
MA	0.04	0.04	0.06	0.37	0.14	0.27	0.09
MG	0.06	0.06	0.06	0.39	0.10	0.28	0.06
MS	0.06	0.06	0.05	0.38	0.12	0.27	0.06
MT	0.05	0.05	0.06	0.35	0.16	0.28	0.05
PA	0.03	0.04	0.05	0.44	0.11	0.27	0.06
PB	0.05	0.05	0.07	0.42	0.08	0.25	0.08
PE	0.05	0.06	0.07	0.46	0.05	0.25	0.06
PI	0.04	0.03	0.05	0.40	0.10	0.30	0.08
PR	0.08	0.07	0.07	0.36	0.07	0.29	0.06
RJ	0.05	0.08	0.08	0.46	0.01	0.25	0.06
RN	0.06	0.05	0.07	0.42	0.06	0.26	0.07
RO	0.06	0.04	0.05	0.35	0.16	0.28	0.06
RR	0.05	0.03	0.07	0.39	0.11	0.24	0.11
RS	0.06	0.08	0.08	0.39	0.06	0.28	0.05
SC	0.08	0.06	0.07	0.32	0.09	0.31	0.06
SE	0.04	0.04	0.05	0.43	0.14	0.25	0.06
SP	0.07	0.09	0.08	0.41	0.03	0.27	0.05
TO	0.05	0.04	0.05	0.33	0.21	0.23	0.09

Table B.1
Labor market frictions τ_{ir}^{w} - 2015.
Source: Search results.

State	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers	Mean friction by state	Income level
AC	0.56	0.53	0.43	0.43	0.50	0.47	0.53	0.49	Middle income
AL	0.56	0.58	0.49	0.45	0.54	0.47	0.55	0.52	Low income
AM	0.56	0.54	0.48	0.42	0.45	0.45	0.50	0.49	Middle income
AP	0.56	0.53	0.46	0.41	0.41	0.43	0.51	0.47	Middle income
BA	0.56	0.57	0.48	0.42	0.44	0.47	0.52	0.50	Low income
CE	0.56	0.58	0.50	0.41	0.35	0.42	0.50	0.47	Low income
DF	0.56	0.54	0.50	0.43	0.47	0.45	0.52	0.50	High income
ES	0.56	0.57	0.54	0.44	0.49	0.51	0.53	0.52	High income
GO	0.56	0.54	0.51	0.45	0.53	0.50	0.53	0.52	Middle income
MA	0.56	0.51	0.45	0.38	0.38	0.39	0.47	0.45	Low income
MG	0.56	0.57	0.52	0.43	0.49	0.49	0.53	0.51	Middle income
MS	0.56	0.56	0.51	0.43	0.53	0.48	0.53	0.52	High income
MT	0.56	0.57	0.50	0.46	0.56	0.53	0.53	0.53	High income
PA	0.56	0.53	0.49	0.41	0.48	0.44	0.52	0.49	Low income
PB	0.56	0.60	0.51	0.43	0.49	0.47	0.54	0.51	Low income
PE	0.56	0.56	0.47	0.39	0.40	0.43	0.50	0.47	Low income
PI	0.56	0.54	0.45	0.38	0.38	0.42	0.48	0.46	Low income
PR	0.56	0.55	0.52	0.45	0.52	0.49	0.52	0.52	High income
RJ	0.56	0.59	0.50	0.44	0.44	0.49	0.53	0.51	High income
RN	0.56	0.56	0.49	0.41	0.41	0.41	0.49	0.48	Low income
RO	0.56	0.55	0.51	0.45	0.54	0.52	0.51	0.52	Middle income
RR	0.56	0.56	0.50	0.41	0.41	0.45	0.54	0.49	Middle income
RS	0.56	0.56	0.51	0.45	0.52	0.48	0.52	0.51	High income
SC	0.56	0.56	0.52	0.48	0.54	0.51	0.51	0.53	High income
SE	0.56	0.55	0.47	0.39	0.38	0.43	0.54	0.48	Middle income
SP	0.56	0.54	0.50	0.42	0.48	0.47	0.48	0.49	High income
TO	0.56	0.58	0.50	0.45	0.49	0.49	0.52	0.51	Middle income
Mean by occupation	0.56	0.56	0.49	0.43	0.47	0.46	0.52		

Acre (AC), Alagoas (AL), Amapá (AP), Amazonas (AM), Bahia (BA), Ceará (CE), Distrito Federal(DF), Espírito Santo (ES), Goiás (GO), Maranhão (MA), Mato Grosso (MT), Mato Grosso do Sul (MS), Minas Gerais (MG), Pará (PA), Paraíba (PB), Paraná (PR), Pernambuco (PE), Piauí(PI), Rio de Janeiro (RJ), Rio Grande do Norte (RN), Rio Grande do Sul (RS), Rondônia (RO), Roraima (RR), Santa Catarina (SC), São Paulo (SP), Sergipe (SE), Tocantins (TO).

Appendix A. Descriptive statistics

See Tables A.1–A.4.

Appendix B. Calibrated $\tau's$ and $A's$ to 2015 and 2003

See Tables B.1–B.6.

Appendix C. Public and private spending on education

We estimated private education expenditures in Brazil for 2003, 2009, and 2018 using data from Table 49 of the Family Budget Survey (POF).[23] Our estimates indicate that private education expenditures were approximately R$ 32.4 billion in 2003, R$ 40.5 billion in 2009, and R$ 145.4 billion in 2018. The private education expenditures as a percentage of GDP were 1.8%, 1.2%, and 2.0% for the respective years, with an average of 1.7%.

The National Institute of Educational Studies and Research Anísio Teixeira (INEP) provides data on public spending on education as a percentage of GDP. In 2003, public spending on education accounted for 4.6% of GDP, while in 2015, it increased to 6.2%. Therefore, Brazil's total public and private spending on education, as a share of GDP, was 6.4% in 2003 and 7.9% in 2015.

Appendix D. Migration between states

We analyzed the PNAD microdata from 2015 to assess the extent of worker migration, finding that, on average, 20.36% of workers moved to another state or country. Table D.1 presents the proportion of workers who relocated to another state or country by occupation. As illustrated, only a small proportion of employees relocated from their home state to another. Thus, assuming that workers do not migrate in the theoretical model is reasonable.

[23] Details about the POF can be found on the Brazilian Institute of Geography and Statistics (IBGE) website: https://www.ibge.gov.br/.

Table B.2
Education market frictions τ_{ir}^{h} - 2015.
Source: Search results.

State	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers	Mean friction by state	Income level
AC	0	0.09	1.83	−0.70	1.72	−0.35	−0.72	0.27	Middle income
AL	0	−0.52	−0.12	−0.77	0.92	−0.48	−0.69	−0.24	Low income
AM	0	−0.22	0.21	−0.55	9.96	−0.30	−0.47	1.23	Middle income
AP	0	0.14	0.43	−0.67	11.87	−0.18	−0.70	1.56	Middle income
BA	0	−0.46	0.29	−0.69	7.44	−0.34	−0.45	0.83	Low income
CE	0	−0.25	0.32	−0.68	33.70	−0.29	−0.46	4.62	Low income
DF	0	−0.62	−0.10	−0.60	170.40	0.95	−0.43	24.23	High income
ES	0	−0.42	0.34	−0.35	4.88	−0.31	0.12	0.61	High income
GO	0	−0.16	0.36	−0.68	3.88	−0.48	−0.26	0.38	Middle income
MA	0	−0.03	0.59	−0.57	4.48	−0.15	−0.60	0.53	Low income
MG	0	−0.51	0.44	−0.49	6.16	−0.24	−0.16	0.74	Middle income
MS	0	−0.32	1.11	−0.45	2.91	−0.15	−0.26	0.41	High income
MT	0	−0.38	0.70	−0.59	0.62	−0.53	−0.27	−0.06	High income
PA	0	−0.27	0.12	−0.77	1.93	−0.47	−0.62	−0.01	Low income
PB	0	−0.57	0.19	−0.62	6.90	−0.19	−0.60	0.73	Low income
PE	0	−0.43	0.56	−0.54	26.38	0.13	−0.09	3.72	Low income
PI	0	0.17	1.17	−0.56	9.07	−0.28	−0.54	1.29	Low income
PR	0	−0.25	0.58	−0.31	9.78	−0.05	0.11	1.41	High income
RJ	0	−0.76	0.03	−0.70	129.70	−0.34	−0.33	18.23	High income
RN	0	−0.24	0.51	−0.52	20.34	0.30	−0.27	2.88	Low income
RO	0	0.10	0.79	−0.54	1.11	−0.49	−0.22	0.11	Middle income
RR	0	0.17	0.22	−0.52	8.05	−0.04	−0.75	1.02	Middle income
RS	0	−0.55	0.20	−0.53	11.59	−0.17	0.01	1.51	High income
SC	0	−0.13	0.79	−0.32	6.14	−0.21	0.43	0.96	High income
SE	0	−0.36	0.36	−0.71	3.96	−0.32	−0.63	0.33	Middle income
SP	0	−0.49	0.46	−0.39	51.65	0.08	0.61	7.42	High income
TO	0	−0.29	0.79	−0.54	0.93	−0.27	−0.60	0.00	Middle income
Mean by occupation	0	−0.28	0.49	−0.57	20.24	−0.19	−0.33		

Table B.3
Total productivity factors - 2015.
Source: Search results.

State	A_r	State	A_r	State	A_r
AC	24.68	MA	26.59	RJ	38.49
AL	26.75	MG	41.38	RN	34.65
AM	30.76	MS	40.28	RO	38.19
AP	25	MT	39.83	RR	24.22
BA	33.07	PA	29.81	RS	45.89
CE	30.61	PB	28.79	SC	49.75
DF	35.99	PE	39.56	SE	30.90
ES	46.65	PI	27.93	SP	51.25
GO	40.42	PR	46.44	TO	28.03

Notes: Recall that in our model TFP is equal across occupations. The average of TFP is 35.4.

Appendix E. Alternative calibration

There are many microeconomic issues in Brazil, and informal work is an important one. Scholars have conducted extensive research to understand the relationship between economic development and the informal sector. For instance, Franjo et al. (2022) built a life-cycle model to explore the interplay between informality and financial development in Brazil. They also conducted cross-country data analysis, highlighting the importance of considering the informal sector when studying the relationship between financial and economic development.

The prevalence of informal employment is not unique to Brazil; it is a common feature in many developing countries. Research conducted by Bacchetta et al. (2009) reveals that informality negatively correlates with GDP and GDP growth in developing countries. During the 2000s, the proportion of informal employment in total employment was 52% for Latin America, 78% for Asia, and 56% for Africa. The informal sector (excluding agriculture) accounted for 26% of Latin America's GDP in 2006. Moreover, the research suggests that informal employment tends to attract low-educated workers, with approximately 65% of all informal workers in Latin America classified as such.

The decision to exclude individuals earning less than 60% of the minimum wage could raise concerns as it may disproportionately impact workers in the informal sector. To address this concern and ensure the robustness of our findings, we provide an alternative calibration of our model that incorporates data from all workers with positive wages. Our primary analysis focused on a sample

Table B.4
Labor market frictions τ_{ir}^{w} - 2003.
Source: Search results.

State	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers	Mean friction by state	Income level
AC	−0.05	−0.06	−0.28	−0.72	−0.98	−0.68	−0.36	−0.45	Middle income
AL	−0.05	0.10	−0.34	−0.76	−0.94	−0.50	−0.38	−0.41	Low income
AM	−0.05	−0.12	−0.65	−0.82	−0.92	−0.75	−0.47	−0.54	Middle income
AP	−0.05	−0.15	−0.36	−0.68	−0.74	−0.45	−0.31	−0.39	Middle income
BA	−0.05	−0.08	−0.22	−0.69	−0.77	−0.54	−0.42	−0.40	Low income
CE	−0.05	−0.06	−0.22	−0.67	−1.00	−0.68	−0.38	−0.44	Low income
DF	−0.05	−0.02	−0.16	−0.50	−0.22	−0.45	−0.16	−0.22	High income
ES	−0.05	−0.15	−0.19	−0.58	−0.53	−0.50	−0.28	−0.33	High income
GO	−0.05	−0.05	−0.15	−0.57	−0.38	−0.48	−0.30	−0.28	Middle income
MA	−0.05	−0.01	−0.39	−0.63	−0.61	−0.54	−0.28	−0.36	Low income
MG	−0.05	−0.06	−0.20	−0.61	−0.58	−0.45	−0.20	−0.31	Middle income
MS	−0.05	0.02	−0.18	−0.50	−0.14	−0.52	−0.28	−0.24	High income
MT	−0.05	−0.06	−0.30	−0.64	−0.41	−0.47	−0.36	−0.33	High income
PA	−0.05	−0.02	−0.25	−0.64	−0.38	−0.54	−0.25	−0.30	Low income
PB	−0.05	−0.06	−0.25	−0.74	−0.87	−0.76	−0.34	−0.44	Low income
PE	−0.05	−0.07	−0.29	−0.67	−0.93	−0.64	−0.41	−0.44	Low income
PI	−0.05	0.02	−0.39	−0.74	−1.00	−0.93	−0.50	−0.51	Low income
PR	−0.05	−0.10	−0.21	−0.54	−0.29	−0.41	−0.22	−0.26	High income
RJ	−0.05	−0.04	−0.19	−0.53	−0.92	−0.38	−0.13	−0.32	High income
RN	−0.05	0.01	−0.22	−0.61	−1.00	−0.53	−0.23	−0.38	Low income
RO	−0.05	0.01	−0.15	−0.58	−0.18	−0.40	−0.16	−0.22	Middle income
RR	−0.05	−0.18	−0.27	−0.60	−0.61	−0.61	−0.29	−0.37	Middle income
RS	−0.05	−0.04	−0.18	−0.50	−0.34	−0.44	−0.19	−0.25	High income
SC	−0.05	−0.05	−0.10	−0.36	−0.14	−0.31	−0.19	−0.17	High income
SE	−0.05	0.04	−0.27	−0.61	−0.78	−0.57	−0.41	−0.38	Middle income
SP	−0.05	−0.11	−0.19	−0.49	−0.45	−0.39	−0.25	−0.28	High income
TO	−0.05	0.01	−0.18	−0.72	−0.56	−0.46	−0.38	−0.34	Middle income
Mean by occupation	−0.05	−0.05	−0.25	−0.62	−0.62	−0.53	−0.30		

Table B.5
Education market frictions τ_{ir}^{h} - 2003.
Source: Search results.

State	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers	Mean friction by state	Income level
AC	0	0.49	2.93	0.23	155.73	3.97	0.03	23.34	Middle Income
AL	0	−0.19	1.75	0.73	14.16	1.50	−0.15	2.54	Low Income
AM	0	0.71	3.83	0.24	205.14	1.32	0.80	30.29	Middle Income
AP	0	−0.00	0.49	−0.15	77.86	−0.16	−0.68	11.05	Middle Income
BA	0	0.33	1.24	0.34	16.33	1.66	0.86	2.97	Low Income
CE	0	−0.11	0.82	−0.17	51.68	0.99	0.13	7.62	Low Income
DF	0	−0.58	0.32	−0.02	253.03	3.08	−0.20	36.52	High Income
ES	0	0.37	1.60	0.46	12.16	1.39	0.92	2.41	High Income
GO	0	0.27	1.01	0.19	10.03	1.01	1.14	1.95	Middle Income
MA	0	−0.06	2.00	−0.05	5.79	0.68	−0.40	1.14	Low Income
MG	0	0.05	1.46	0.39	23.15	0.93	0.26	3.75	Middle Income
MS	0	0.78	3.69	0.35	7.17	2.07	1.26	2.19	High Income
MT	0	0.31	3.40	1.29	5.16	1.41	1.18	1.82	High Income
PA	0	−0.34	0.82	−0.26	28.28	0.52	0.03	4.15	Low Income
PB	0	0.38	1.39	0.56	28.44	3.18	−0.16	4.83	Low Income
PE	0	−0.19	0.93	0.00	54.79	2.18	0.45	8.31	Low Income
PI	0	−0.13	1.17	0.29	27.12	5.69	−0.22	4.84	Low Income
PR	0	0.21	1.20	0.55	14.87	1.27	0.83	2.70	High Income
RJ	0	−0.63	0.28	−0.33	360.08	0.31	−0.18	51.36	High Income
RN	0	−0.45	1.66	−0.32	28.81	0.43	−0.55	4.23	Low Income
RO	0	1.20	1.11	0.00	7.43	0.29	−0.28	1.39	Middle Income
RR	0	0.13	4.32	0.38	50.15	3.63	−0.41	8.32	Middle Income
RS	0	−0.08	0.86	0.39	21.53	1.06	0.78	3.51	High Income
SC	0	0.78	1.27	0.56	8.81	0.76	1.24	1.92	High Income
SE	0	−0.19	1.01	−0.13	16.92	0.81	−0.21	2.60	Middle Income
SP	0	−0.24	1.11	0.20	81.56	1.02	1.25	12.13	High Income
TO	0	0.30	1.87	1.62	13.17	2.16	0.37	2.79	Middle Income
Mean by occupation	0	0.12	1.61	0.27	58.49	1.60	0.30		

Table B.6
Total productivity factors - 2003.
Source: Search results.

State	A_r	State	A_r	State	A_r
AC	25	MA	19.65	RJ	28.93
AL	20.95	MG	31.16	RN	18.05
AM	31.24	MS	36.08	RO	24.47
AP	15.60	MT	34.85	RR	19.02
BA	30.05	PA	27.16	RS	37.54
CE	24.95	PB	21.09	SC	39.91
DF	26.92	PE	26.77	SE	19.57
ES	35.74	PI	17.88	SP	40.71
GO	36.73	PR	36.03	TO	25.16

Notes: Recall that in our model TFP is equal across occupations. The average of TFP is 27.82.

Table D.1
Share of workers who migrated and did not migrate to another state or country.
Source: Elaborated by the authors with data from PNAD 2015.

	Managers	Sciences and arts	Middle-level technicians	Service sector	Agriculture	Industrial production and services	Teachers
Migrated	0.21	0.18	0.19	0.22	0.20	0.22	0.15
Not migrated	0.79	0.82	0.81	0.78	0.80	0.78	0.85

Table D.2
Main counterfactual exercises – Sample with wages filter (Original calibration) vs. sample without wages filter.

	Main counterfactuals (%)	
	Main calibration	Alternative calibration
Highest ATHC	87.85	87.51
Lowest ATHC	−59.62	−53.74
Without frictions	16.94	18.30

Notes: This table compares the main counterfactuals of the main calibration of our model to the counterfactuals computed using the extended sample.

of 109,038 individuals after applying specific filters described in the main text. Here, we constructed a broader dataset comprising 115,994 individuals and compared the filtered (With filter) and unfiltered (Without filter) samples in terms of wages, educational attainment, and worker proportions, as shown in Fig. D1, where the red points represent teachers' statistics. We observed minor differences between the two samples for teaching professions, with the most significant disparities occurring in low-wage occupations, such as those in the agricultural sector.

We recalibrated our model using the extended sample. Fig. D2 demonstrates that the alternative calibration effectively fits the data. We also compared the calibrated parameters from the main text to those computed with the extended sample, as shown in Fig. D3. The parameters linked to teachers' occupations (marked in red) showed no substantial alterations. Moreover, the TPF (A_i) and educational market barriers (τ_h) are almost indistinguishable in both calibrations. While there are some differences in labor market barriers (τ_w), a high correlation is observed between the alternative calibration of these parameters.

We have verified that many important results from our analysis remain robust when using the alternative calibration. Table D.2 presents the outcomes of counterfactual experiments using both the primary and alternative calibrations. In the first scenario, we assumed that all states have the same frictions (τ_w's and τ_h's) as Roraima (RR), the state with the highest Average Teachers' Human Capital (ATHC). Using the alternative calibration, we find that the Brazilian GDP would increase by 87.51% (compared to 87.85% in our primary exercise).

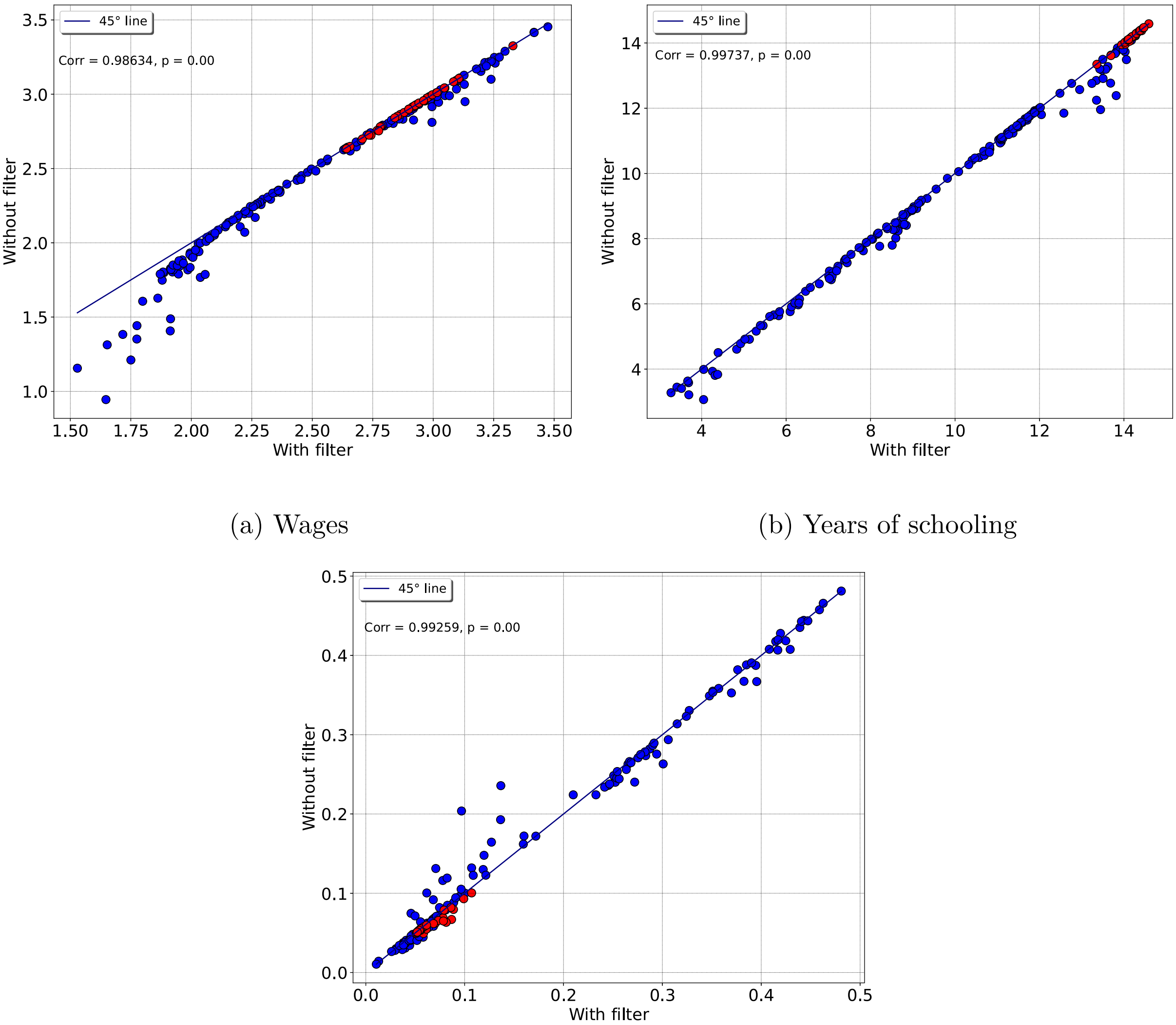
Suppose all states had the same frictions as São Paulo (SP), the state with the lowest ATHC. In that case, we find that the GDP of all states would decrease, and the Brazilian GDP would decline by 53.74% according to the alternative calibration (compared to 59.62% in our primary calibration).

Finally, in the exercise where we eliminate all frictions in the economy, the alternative calibration suggests that the GDP could increase by 18.3% (compared to 16.94% predicted by the same exercise using our primary calibration).

Therefore, the results in Table D.2 show that the main findings are not significantly affected by the different calibration choices, indicating that our model's results are not sensitive to excluding individuals earning less than 60% of the minimum wage.

Appendix F. Teachers' relative wages and economic development

In this section, to further investigate the link between economic development and the relative earnings of teachers, we conducted a panel regression analysis using all available data from the household survey (PNAD) covering the period from 2002 to 2015



(a) Wages

(b) Years of schooling

(c) Proportion of workers

Fig. D1. Data – Sample with wages filter vs. sample without wages filter. Notes: This figure compares our main calibration data (With filter) and the data without wage filters (Without filter). In the figure, the red dots represent observations of teachers, while the blue dots denote other occupations. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

(excluding 2010 due to lack of data). Our analysis includes information on the average wage of teachers relative to other occupations, the proportion of teachers in public institutions, and the ratio of teachers with labor municipality contracts to teachers with state contracts for all years and states.

The structure of the Brazilian education system differs from that of other economies. The Organization for Economic Cooperation and Development (OECD) notes that funding for different levels of education in Brazil is divided among the municipality, state, and federal governments. Typically, the municipal government provides funding for the lowest level of education, the state government funds the intermediate level, and the federal government funds higher education, such as colleges and universities. Moreover, the proportion of teachers financed by each administrative level also varies. In 2015, across the Brazilian states, the average percentages of teachers in public institutions were 55%, 39%, and 6% for the municipal, state, and federal levels, respectively. The standard deviations for these percentages were 16%, 15%, and 3%, respectively.

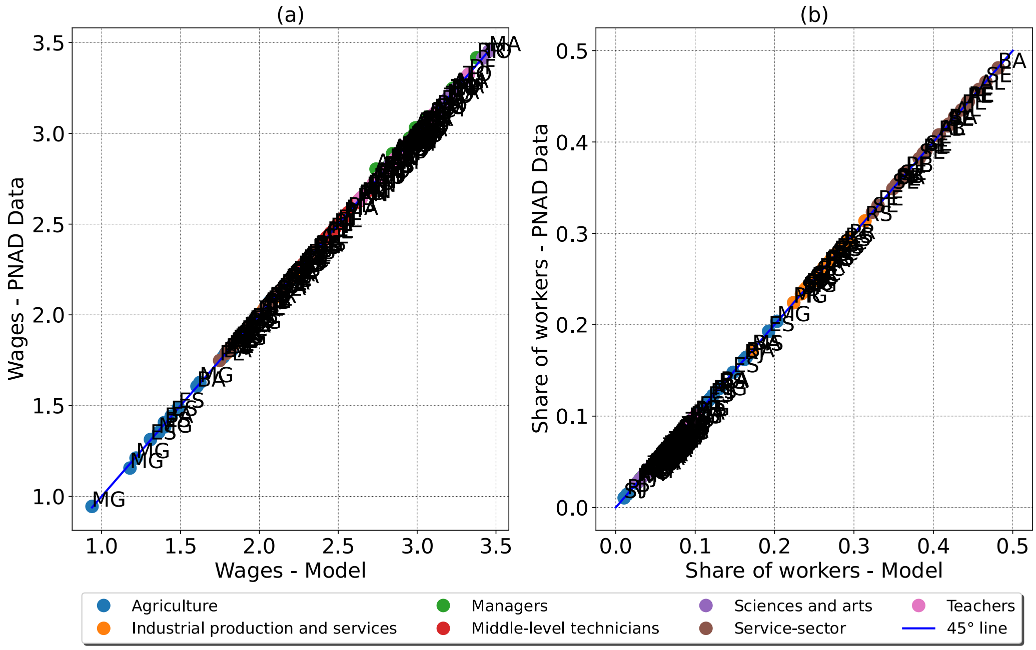


Fig. D2. Model adjustment to data with full sample- wages and share of workers.

Table D.3
The effect of teachers from public institutions on relative wages of teachers.

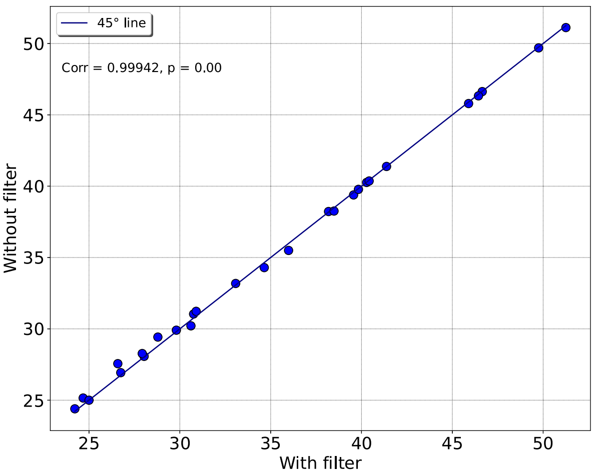
	(1)	(2)	(3)
Share of pub. teachers	−0.21 (0.27)	−0.21 (0.27)	−0.14 (0.26)
Municipality to state pub. teachers		0.001 (0.02)	−0.004 (0.021)
GDP per capita			−0.04*** (0.01)
Constant	✓	✓	✓
Time fixed effect	✓	✓	✓
State fixed effect	✓	✓	✓
Observations	351	351	351

Note: We report robust t-statistics in parentheses. Statistical significance is indicated at the $*** \ p < 0.01$, $** \ p < 0.05$, and $* \ p < 0.1$ levels. Our dataset comprises all household surveys conducted after 2000 with a consistent methodology (2002–2009, 2011–2015). We compute the weighted average of the relative wage of teachers to other occupations and the proportion of teachers in public institutions for all years and states. Our sample includes individuals aged 25 to 65 with positive wages, yielding 351 observations (13 years multiplied by 27 states). Our panel regression analysis consists of three models. In Model 1, we examine whether the proportion of public teachers explain differences in relative wages, controlling for time and state-fixed effects. Model 2 introduces the ratio of municipal to state-funded teachers across states. Finally, in Model 3, we extend the analysis by including the log of GDP per capita in the second model.

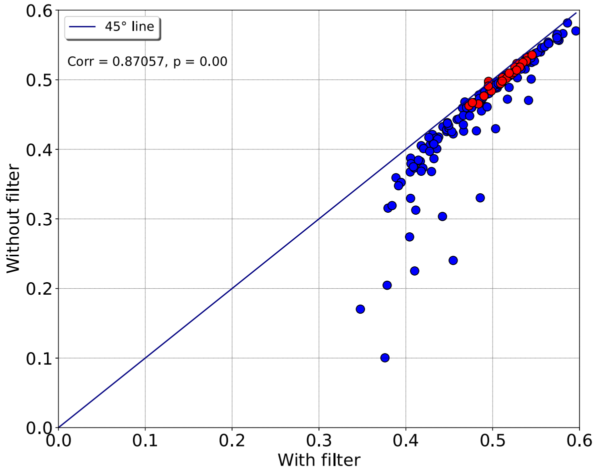
These numbers suggest significant variations in the distribution of teachers across administrative levels and regions. As teachers in different states or municipalities within the same state may receive different salaries, we consider the proportion of teachers in public institutions and the ratio of teachers with labor municipality contracts to teachers with state contracts in our analysis.

The dependent variable in our panel regression is the relative wage of teachers to other occupations. We explore whether the proportion of public teachers can account for the observed differences in relative wages, controlling for time and state-fixed effects. The first model (Model 1) examines this relationship. The second model (Model 2) expands on the first by adding the ratio of municipality teachers to state teachers. Finally, the third model (Model 3) further extends the analysis by including the log of GDP per capita as an additional regressor.

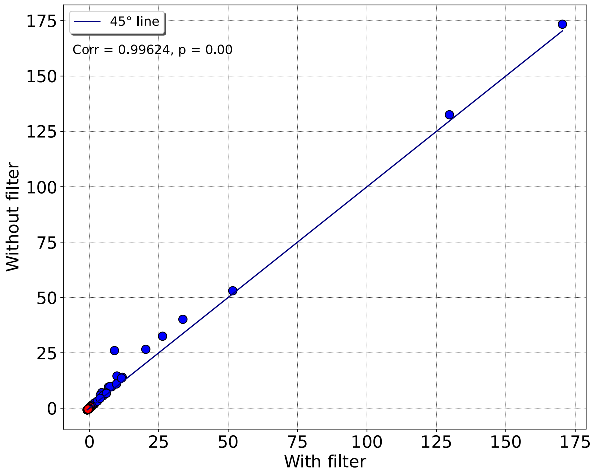
The regression results presented in Table D.3 indicate that neither the proportion of teachers in public institutions nor the ratio of municipality teachers to state teachers significantly impact the relative wages of teachers. However, we do find a negative correlation between GDP per capita and relative wages, which is consistent with our earlier findings.



(a) TFP (A_i)



(b) τ_w



(c) τ_h

Fig. D3. Calibrated parameters – Sample with wages filter vs. sample without wages filter. Notes: This figure compares the calibrated parameters obtained from the original calibration (with filter) to those obtained from the calibration without wage filters (without filter).

References

Abdulla, Kanat, 2019. Productivity gains from reallocation of talent in Brazil and India. J. Macroecon. 62, 103160.
Bacchetta, Marc, Ernst, Ekkehard, Bustamante, Juana P., et al., 2009. Globalization and Informal Jobs in Developing Countries. International Labour Organization, Geneva.
Bacolod, Marigee P., 2007. Do alternative opportunities matter? The role of female labor markets in the decline of teacher quality. Rev. Econ. Stat. 89 (4), 737–751.
Banerjee, Abhijit V., Duflo, Esther, 2005. Growth theory through the lens of development economics. In: Handbook of Economic Growth, Vol. 1. Elsevier, pp. 473–552.
Barro, Robert J., Sala-i-Martin, Xavier, 1992. Convergence. J. Polit. Econ. 100 (2), 223–251.
Barros, Fernando Jr., Delalilbera, Bruno R., 2018. Market frictions, misallocation of talent and development. Econ. Bull. 38 (4), 2410–2430.
Benhabib, Jess, Spiegel, Mark M., 2005. Human capital and technology diffusion. In: Handbook of Economic Growth, Vol. 1. Elsevier, pp. 935–966.
Borensztein, Eduardo, De Gregorio, Jose, Lee, Jong-Wha, 1998. How does foreign direct investment affect economic growth? J. Int. Econ. 45 (1), 115–135.
Brotherhood, Luiz, Delalibera, Bruno R., 2020. Minding the gap between schools and universities. J. Econom. Dynam. Control 120, 104010.
Brotherhood, Luiz, Delalibera, Bruno R., Torres de Mello Pereira, Luciene, 2022. Public overspending in higher education. Bull. Econ. Res..
Café, Renata Motta, 2018. A Alocação de Talentos No Setor Público Brasileiro (PhD dissertation).
Cagetti, Marco, De Nardi, Mariacristina, 2006. Entrepreneurship, frictions, and wealth. J. Polit. Econ. 114 (5), 835–870.
Card, David, Krueger, Alan B., 1992a. Does school quality matter? Returns to education and the characteristics of public schools in the United States. J. Political Econ. 100 (1), 1–40.

Card, David, Krueger, Alan B., 1992b. School quality and black-white relative earnings: A direct assessment. Q. J. Econ. 107 (1), 151–200.
Delalibera, Bruno Ricardo, Ferreira, Pedro Cavalcanti, 2019. Early childhood education and economic growth. J. Econom. Dynam. Control 98, 82–104.
Eaton, Jonathan, Kortum, Samuel, 2002. Technology, geography, and trade. Econometrica 70 (5), 1741–1779.
Eckstein, Zvi, Zilcha, Itzhak, 1994. The effects of compulsory schooling on growth, income distribution and welfare. J. Public Econ. 54 (3), 339–359.
Ferreira, Afonso, 2000. Convergence in Brazil: recent trends and long-run prospects. Appl. Econ. 32 (4), 479–489.
Franjo, Luis, Pouokam, Nathalie, Turino, Francesco, 2022. Financial frictions and firm informality: a general equilibrium perspective. Econ. J. 132 (645), 1790–1823.
Gilpin, Gregory, Kaganovich, Michael, 2012. The quantity and quality of teachers: Dynamics of the trade-off. J. Public Econ. 96 (3–4), 417–429.
Guo, Naijia, Leung, Charles Ka Yui, 2021. Do elite colleges matter? The impact on entrepreneurship decisions and career dynamics. Quant. Econ. 12 (4), 1347–1397.
Hanushek, Eric A., Leung, Charles Ka Yui, Yilmaz, Kuzey, 2014. Borrowing constraints, college aid, and intergenerational mobility. J. Hum. Cap. 8 (1), 1–41.
Hanushek, Eric A., Piopiunik, Marc, Wiederhold, Simon, 2019. The value of smarter teachers international evidence on teacher cognitive skills and student performance. J. Hum. Resour. 54 (4), 857–899.
Hanushek, Eric A., Woessmann, Ludger, 2012. Schooling, educational achievement, and the Latin American growth puzzle. J. Dev. Econ. 99 (2), 497–512.
Hatsor, Limor, 2012. Occupational choice: Teacher quality versus teacher quantity. Labour Econ. 19 (4), 608–623.
Hnatkovska, Viktoria, Lahiri, Amartya, Paul, Sourabh, 2012. Castes and labor mobility. Am. Econ. J.: Appl. Econ. 4 (2), 274–307.
Hsieh, Chang-Tai, Hurst, Erik, Jones, Charles I., Klenow, Peter J., 2019. The allocation of talent and US economic growth. Econometrica 87 (5), 1439–1474.
Hsieh, Chang-Tai, Klenow, Peter J., 2009. Misallocation and manufacturing TFP in China and India. Q. J. Econ. 124 (4), 1403–1448.
Krueger, Alan B., 2003. Economic considerations and class size. Econ. J. 113 (485), F34–F63.
Lakdawalla, Darius, 2006. The economics of teacher quality. J. Law Econ. 49 (1), 285–329.
Machado, Laura Muller, Scorzafave, Luiz Guilherme Dácar da Silva, 2016. Distribuição de salários de professores e outras ocupações: uma análise para graduados em carreiras tipicamente ligadas à docência. Rev. Bras. Econ. 70 (2), 203–220.
Mankiw, N. Gregory, Romer, David, Weil, David N., 1992. A contribution to the empirics of economic growth. Q. J. Econ. 107 (2), 407–437.
McFadden, Daniel, 1974. Conditional logit analysis of qualitative choice behavior. In: Zarembka, P. (Ed.), Frontiers of Econometrics. Academic Press, New York, NY, pp. 105–142.
Menezes-Filho, Naércio, Pazello, Elaine, 2007. Do teachers' wages matter for proficiency? Evidence from a funding reform in Brazil. Econ. Educ. Rev. 26 (6), 660–672.
Ponczek, Vladimir, Souza, Andre Portela, 2012. New evidence of the causal effect of family size on child quality in a developing country. J. Hum. Resour. 47 (1), 64–106.
Restuccia, Diego, Rogerson, Richard, 2008. Policy distortions and aggregate productivity with heterogeneous establishments. Rev. Econ. Dyn. 11 (4), 707–720.
Restuccia, Diego, Rogerson, Richard, 2017. The causes and costs of misallocation. J. Econ. Perspect. 31 (3), 151–174.
Ribeiro, Erika Cristina Barbosa de Almeida, Almeida, Eduardo Simões de, 2012. Convergência local de renda no Brasil. Econ. Apl. 16, 399–420.
Rivkin, Steven G., Hanushek, Eric A., Kain, John F., 2005. Teachers, schools, and academic achievement. Econometrica 73 (2), 417–458.
Romer, Paul M., 1990. Endogenous technological change. J. Polit. Econ. 98 (5, Part 2), S71–S102.
Soares, Rodrigo R., Kruger, Diana, Berthelon, Matias, 2012. Household choices of child labor and schooling: A simple model with application to Brazil. J. Hum. Resour. 47 (1), 1–31.
Stoddard, Christiana, 2003. Why has the number of teachers per student risen while teacher quality has declined?: The role of changes in the labor market for women. J. Urban Econ. 53 (3), 458–481.
Tamura, Robert, 2001. Teachers, growth, and convergence. J. Polit. Econ. 109 (5), 1021–1059.
Williams, J.H., 2005. Cross-national variations in rural mathematics achievement. J. Res. Rural Educ. 20 (5), 20–25.
Woessmann, Ludger, 2016. The importance of school systems: Evidence from international differences in student achievement. J. Econ. Perspect. 30 (3), 3–32.
Zhang, Yanhong, 2006. Urban-rural literacy gaps in Sub-Saharan Africa: The roles of socioeconomic status and school quality. Comp. Educ. Rev. 50 (4), 581–602.