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Electoral competition with costly policy changes: A dynamic perspective ☆

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Abstract

We analyze two-party electoral competition with a one-dimensional policy space, costly policy changes, and random negative shocks to a party's viability over an infinite horizon. We show the existence and uniqueness of stationary Markov perfect equilibria in which parties use so-called simple strategies. Regardless of the initial policy, party choices converge in the long run to a stochastic alternation between two policies, with transitions occurring if and only if parties suffer a negative shock to their viability. Although costs of change have a moderating effect on policies, full convergence to the median voter position does not take place when parties are polarized.

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1. Introduction

Policy changes are often costly, and the costs increase with the extent of the policy shift. The reason is that changes in policy may *(a)* render original investments in human and physical capital by the private and public sectors incrementally obsolete, *(b)* entail new fixed costs and variable investments, or *(c)* require increasing effort from the government to design new policies and overcome resistance from groups intent on preserving the status quo and refusing to create new institutions or dismantle old ones. These costs of change are borne (close to) uniformly by all the citizens, including party members.

In this paper we develop an infinite-horizon election model with a representative voter and two policy-motivated political parties in which implementing a policy that differs from the one chosen in the previous period is costly for the parties and the voter (see below for more details). Policies are part of a one-dimensional space over which the voter and the two parties have standard quadratic utility functions that are defined by their peaks. The parties' peaks differ significantly from each other (and from the voter's peak), so we have party polarization. In each period, the two parties compete for office in an election decided by the voter. The winning party chooses a policy for that period. Neither the parties nor the voter can commit to particular policies before the election takes place, so political campaigns are irrelevant. Citizens are short-sighted, while parties are long-lived and forward-looking.[4]

During tenure, exogenous events may negatively affect the incumbent party in the form of an electoral shock. For instance, a corruption scandal may damage a party's reputation; a failed public project may indicate that the party is not fit to be in power; a recession may cast doubts on the incumbent administration's abilities, and so forth. We assume that in every period there is a constant probability that the incumbent party's perceived suitability for holding power declines sufficiently for the voter to elect the challenger party in the next election, regardless of any other consideration. Parties recover from negative shocks at the end of the next period. Although unsuitability for office is a *sufficient* condition for the incumbent party to be ousted, it is not clear a priori whether, and if so, under what circumstances, it is also *necessary*.

Policy choices made in one period become the status quo for the next period. We assume that the farther away the next policy choice is from the status quo, the higher the costs of implementing this new policy. These *costs of change* are incurred by the two parties and the voter, and they establish a dynamic link between policies across periods. We also assume that costs of change are linear in the extent of the policy shift, but this assumption is not critical. As we explain in Section 5, if parties are myopic and costs of change are modeled as a power function, our main insights hold as long as the exponent is less than two.

[4] Our model is best suited to describe elections for an executive office. Yet, one could also use our model for legislative bodies by assuming that elections determine who the median member of the legislature will be.

Our goal is to study the short- and long-term impact of costs of change on the policy choices and turnover of parties in power. For the game-theoretic analysis of our model, we introduce the concept of a *simple stationary Markov perfect equilibrium (SSMPE)*. It is a stationary Markov perfect equilibrium in which the forward-looking parties use *simple strategies*. These strategies are characterized by a closed interval $[m, M]$ of the policy space—identified by a minimum policy m and a maximum policy M, with $M \geq m$—containing all the status-quo policies the party will not change once in office. When a party is in office and encounters a status-quo policy below m, the former is replaced by policy m. Status-quo policies above M are replaced by policy M. Restricting attention to equilibria in which parties use simple strategies enables us, under the focus on Markov dynamics, to solve an otherwise complex dynamic problem.

We focus our attention on the case in which party polarization is large enough in comparison with the marginal cost of change, as this is where the most interesting dynamics occur. Our main result—Theorem 1—shows that the game described above has a unique SSMPE, which enables us to obtain comparative statics on the magnitude of costs of change, party polarization, the probability of electoral shocks, and the parties' discount factor. Theorem 1 provides four significant findings.

First, costs of change generate an advantage for incumbents since office-holders are re-elected by the voter on the equilibrium path if and only if they have not suffered an electoral shock. With the possible exception of the initial period, incumbents inherit in equilibrium a policy which was chosen by themselves and is close to, albeit potentially on either side of, their peak. This means that, at the margin, the (linear) costs of change matter more than the (quadratic) benefit of adjusting policy for the incumbent, whereas for the challenger it is the other way around. Thus, the voter anticipates that the incumbent party will not adjust the policy if it is re-elected, but the opposition party will if the voter decides to elect the challenger instead. Therefore, in equilibrium, the voter will maintain the party in office to avoid incurring costs of change, provided both parties are considered equally suitable for office.[5]

Second, in the long term, policies converge to a stochastic alternation between two states or policies, with the transition probability between the two states being equal to the probability of an electoral shock. These states are independent of the initial policy and are located on either side of the median voter's peak. In equilibrium, incumbent parties move the policy to a point where—given the change the other party would enact—the anticipated policy shift from switching parties would not be worth the cost for the voter unless the incumbent party suffers an electoral shock. This means that to maintain office in the long term, incumbent parties do not have to enact a policy corresponding to the median voter's peak, as a more extreme policy cannot be beaten by the challenger party. Therefore, no convergence to the median voter's peak takes place.

Third, the parties' long-term equilibrium policy choices are more moderate (i.e., closer to the median voter's peak) than their own peaks. In other words, policy polarization—i.e., the distance between the policies implemented by the two parties—is lower than party polarization—i.e., the distance between the parties' peaks. These two measures of polarization coincide when there are no costs of change, which means that from a long-term perspective costs of change can have

[5] The property that, in elections decided by the voter, incumbents have an advantage over challengers does not hold for all functional forms of costs of change. Suppose, for example, that parties are myopic, costs of change are quadratic, and the policy loss function is linear. Then, the incumbent party would generically move the policy toward its own peak (and, hence, away from the voter's peak), while the challenger party would make the same policy shift but toward *its* own peak (and, hence, toward the voter's peak). In such cases, the voter will prefer to elect the challenger party instead of the incumbent party that chose the previous policy (see our longer working paper, Gersbach et al., 2023).

positive consequences for welfare. The explanation is as follows: Parties themselves face costs when the policy changes, so moving the policy toward their peak has diminishing returns since parties not only incur the costs of adjusting policy today, they also have to incur the costs of policy change carried out by the other party in the future. The latter is a novel dynamic incentive identified by our analysis.

Fourth, policy paths sometimes display history dependence in the initial periods comprising the transition phase from the initial policy to the long-term sequence of moderate policies. If the initial policy is much more extreme than the initial incumbent's peak, the equilibrium policy path starts with a short sequence of extreme policies followed by an infinite sequence of moderate policies. In the other cases, the moderate policy stage is reached in the first period. In Section 5 we show that the moderate policy stage is reached immediately no matter the initial policy if the voter is sufficiently forward-looking (and the parties are myopic).

The paper is organized as follows: In Section 2 we review the strands of literature related to our paper. In Section 3 we present our model and introduce our equilibrium concept. In Section 4 we conduct the equilibrium analysis. Section 5 contains a brief discussion of some extensions to our baseline model. Section 6 concludes. The main part of the proof of Theorem 1 is in Appendix A. Appendix B (see online material) contains the proof of equilibrium uniqueness together with some other technical findings.

2. Related literature

Our paper contributes to the literature on dynamic elections with endogenous state variables (e.g. see Battaglini et al., 2012) by showing that costs of change give incumbents the opportunity to choose policies that create an electoral advantage.[6] The first papers to highlight strategic incentives for office-holders to manipulate variables such as the debt level for electoral gains are Persson and Svensson (1989) and Alesina and Tabellini (1990) (see also Bouton et al., 2016). These articles indicate that political competition results in higher debt accumulation, which can negatively affect welfare. This contrasts with our paper, in which costs of change—which entail burning utility—can have positive welfare effects, at least in the long term.

Two closely related papers are Forand (2014) and Nunnari and Zápal (2017). Both consider policy-makers committed to the same policy in all periods. Although costs of change reduce—but do not eliminate—the flexibility of *all* future office-holders in engineering policy changes, in equilibrium incumbent parties (unlike challenger parties) never choose a new policy if they are re-elected. This rationalizes the assumption of policy persistence by the incumbent party (but not by the challenger party) made in the aforementioned papers. As in our model, Forand (2014) shows that policy converges to an alternation between two limit points, but for different reasons.[7] And in contrast to our model, these two points generally depend on the initial policy. The model in Nunnari and Zápal (2017) also features policy alternation, but exhibits the property that policies converge to the median voter's position. Like our paper, both Forand (2014) and Nunnari and Zápal (2017) focus on Markovian equilibria, which is standard in the literature investigating elections and/or bargaining (see e.g. Baron (1996); Battaglini et al. (2012); Duggan and Kalandrakis (2012) and the recent survey by Eraslan et al. (2022)).

[6] There is a large literature on the existence and the causes of incumbency advantage and one has to distinguish between incumbency advantages in legislatures and in executive offices (see e.g. Gelman and King, 1990; Alford and Brady, 1989; Levitt and Wolfram, 1997; Cox and Katz, 1996; Erikson et al., 1993; Ansolabehere and Snyder, 2002).

[7] The initial policy matters in the long run if costs of change are very large, but then policy never shifts.

Literature on political competition explicitly considering the costs associated with policy changes has been relatively scant until recently (see Glazer et al., 1998; Gersbach and Tejada, 2018; Gersbach et al., 2020; Gersbach et al., 2019; Eraslan and Piazza, 2023; Gersbach et al., 2021; Dziuda and Loeper, 2022; Gersbach et al., 2022). We are the first to address the effect of such costs on elections and policy in an infinite-horizon framework. Our contribution to the literature on costs of change consists in proving *(a)* the existence of a Markov perfect equilibrium and *(b)* that there is only one such equilibrium if the parties' strategies are simple.[8] This enables us to describe the long-term behavior of political competition with costs of change and to characterize the transition phase towards the steady state. Our analysis also yields new comparative static results on elections and policy.[9]

In electoral competition models, it is questionable whether candidates can commit to policy positions before elections (as in the classical formulation by Hotelling, 1929; Downs, 1957) or not (e.g. as in Barro, 1973; Ferejohn, 1986; Austen-Smith and Banks, 1989; Persson et al., 1997; Ashworth, 2012). Our model can be interpreted as a model of imperfect (and asymmetric) accountability in which the costs of policy change enable incumbents (but not challengers) to commit to a particular policy that—if they stay in office—will generate small costs of change, or none at all. The asymmetric equilibrium-commitment power yields an electoral advantage for the incumbent and can moderate policies without reducing welfare.

Finally, party polarization and policy polarization are important parameters/variables in our model. A large body of literature has examined the causes of both phenomena (see e.g. Roberts and Smith, 2003; Theriault, 2006; Heberlig et al., 2006; Jackson et al., 2007) and their consequences (see e.g. Jones, 2001; Binder, 2003; Fiorina et al., 2005; Testa, 2012; Hetherington, 2001). Our paper adds to this literature by investigating how in the long term, policy polarization is determined by various levels of party polarization, costs of change, the parties' discount factor, and the probability of electoral shocks.[10]

3. Model

3.1. Setup

We examine a dynamic model of electoral competition ($t = 1, 2, \ldots$). In each period, an election takes place in which a society elects one of two political parties denoted by L and R. The society is represented by the median voter v. An arbitrary party is denoted by K, with $K \in \{L, R\}$. We assume that party R is in power in period $t = 1$. At the end of each period, the incumbent party suffers a negative shock with probability $\lambda \in [0, 1]$ causing it to lose the election (see below for more details). Electoral shocks are i.i.d. and are therefore sufficient for power turnover. Parties recover from the negative shock at the end of the next period.

[8] In a related model, Gersbach et al. (2020) cannot guarantee uniqueness of equilibrium outcomes because they consider a larger strategy space.

[9] The implications of costly policy changes in election models are part of a broader theme on dynamic links across periods in political agency models (Bowen et al., 2014, 2017; Chen and Eraslan, 2017; Buisseret and Bernhardt, 2017; Callander and Raiha, 2017; Glaeser and Shleifer, 2005; Zápal, 2016; Cho, 2014; Dziuda and Loeper, 2016, 2018; Austen-Smith et al., 2019; Strulovici, 2010).

[10] There is a growing literature on behavioral political economy (see e.g. Ortoleva and Snowberg, 2015; Attanasi et al., 2016). The paper most closely connected to ours is Alesina and Passarelli (2017), who investigate public-good provision when citizens are loss-averse with respect to changes in the status-quo policy. They find that this behavioral feature moderates policies, as do the costs of change in our model.

In a given period $t \in \{1, 2, \ldots\}$, the incumbent party K chooses a policy i_{Kt}, with $i_{Kt} \in [0, 1]$. We use i_t instead of i_{Kt} if the incumbent party's identity does not matter. We interpret $[0, 1]$ as the usual policy space ranging from liberal ($i_t = 0$) to conservative ($i_t = 1$). Before the election, parties cannot commit to carrying out a particular policy. Hence, political campaigns are irrelevant and we do not model them. Similarly, the voter cannot commit his/her vote before the election. We break ties in favor of the incumbent when the voter is indifferent between voting for either party.[11]

Voter v obtains the following (instantaneous) utility in any period $t \in \{1, 2, \ldots\}$:

$$U_v(i_{t-1}, i_t, s_{t-1}, K_{t-1}, K_t) = -\left(i_t - \frac{1}{2}\right)^2 - c \cdot |i_{t-1} - i_t| - A \cdot \mathbb{1}_{K_{t-1}}(K_t) \cdot s_{t-1}, \tag{1}$$

where $i_{t-1} \in [0, 1]$ is the status-quo policy, $i_t \in [0, 1]$ is the policy chosen in the current period, $K_{t-1}, K_t \in \{L, R\}$ are the parties in office in the previous period and in the current period, respectively, $\mathbb{1}_x(y) = 1$ if $x = y$ and $\mathbb{1}_x(y) = 0$ otherwise, and $s_{t-1} = 1$ with probability λ and $s_{t-1} = 0$ with probability $1 - \lambda$. That is, $s_{t-1} = 1$ indicates that the party in office at the end of period $t - 1$, party K_{t-1}, suffered an electoral shock in this period, while $s_{t-1} = 0$ indicates that it did not. For simplicity, we assume henceforth that $A \gg 0$, so that a party that receives an electoral shock while in office and is re-elected yields a very low utility for the voter (hence, the party is ousted in equilibrium if it receives such a shock). We also assume that $s_0 = 0$, which means that party R enters period $t = 1$ as an incumbent that has not suffered an electoral shock. Whenever possible, we simplify and slightly abuse notation and write $U_v(i_{t-1}, i_t)$ to denote the voter's utility excluding electoral shocks. In Equation (1), we have assumed for simplicity that the voter's peak is $1/2$, but this is not crucial for our results.

For $K \in \{L, R\}$, party K's (instantaneous) utility in any period $t \in \{1, 2, \ldots\}$ is that of its median member, a citizen like the median voter, and is denoted by

$$U_K(i_{t-1}, i_t, s_{t-1}, K_{t-1}, K_t) = -(i_t - \mu_K)^2 - c \cdot |i_{t-1} - i_t| - A \cdot \mathbb{1}_{K_{t-1}}(K_t) \cdot s_{t-1}.$$

Hence, μ_K denotes the peak of (the median member of) party K. We assume that

$$\frac{1}{2} \leq \mu_R < 1 \quad \text{and} \quad \mu_L = 1 - \mu_R.$$

These assumptions facilitate presentation but can be dispensed with. Assuming that both parties have peaks located on different sides of the political spectrum relative to the median voter's peak is standard. The symmetry assumption enables us to disregard exogenous differences between parties, but is not crucial for the dynamics of the model. If voter preferences are biased towards one party, the equilibrium policies will still fall on either side of the voter's peak, depending on which party is in office. Throughout the paper, we use $\Pi := \frac{1}{2} \cdot (\mu_R - \mu_L) = \mu_R - \frac{1}{2}$ to denote the level of *party polarization*. The larger Π is, the more marked is the opposition between the two parties' peaks and the farther away such peaks are from the median voter's peak.

Equation (1) implies a crucial feature that distinguishes our model from most existing dynamic models of electoral competition: Changing the policy from i_{t-1} in period $t - 1$ to i_t in period t is costly for the voter and the two parties, and such costs increase with the extent of policy change, $|i_{t-1} - i_t|$. Parameter $c \geq 0$ corresponds to the marginal cost of a policy change. In the first period, costs of change are equal to $c \cdot |i_0 - i_1|$, where $i_0 \in [0, 1]$ is the (exogenously given)

[11] Results about long-run policies on the equilibrium path do not change if we use a different tie-breaking rule.

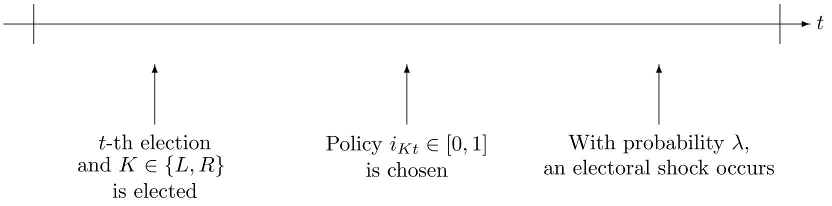


Fig. 1. Sequence of events in period t, for $t \in \{1, 2, \ldots\}$.

status-quo policy in $t = 1$. The larger the value of $|i_0 - 1/2|$ is, the more distant the initial policy is from the median voter's peak.

Both parties are forward-looking and discount future payoffs with a common discount factor, which we denote by $\psi \in [0, 1)$. Party K's lifetime utility in period t is therefore the expected present discounted value of the party's per-period utility, i.e.,

$$\mathbb{E}_t \left[\sum_{\tau \geq t} \psi^{\tau - t} \cdot U_K(i_{\tau-1}, i_\tau, s_{\tau-1}, K_{\tau-1}, K_\tau) \right]. \tag{2}$$

By contrast, the voter is short-sighted or myopic in the sense that s/he only foresees (and cares about) the policies that each party will implement in the current period if it is elected. Hence, the voter's utility in every period is simply his/her instantaneous utility. Having a myopic voter and forward-looking parties yields a tractable model that is arguably not an implausible case.

Given the status-quo policy i_0, we use $\mathcal{G}^{i_0}$ to denote the dynamic game described above. Since i_0 can be any policy in $[0, 1]$ and party R is in office in period $t = 1$, we incorporate the possibility of exogenous preference shocks happening at the beginning of the game. Fig. 1 summarizes the sequence of events of game $\mathcal{G}^{i_0}$ in period t for $t \in \{1, 2, \ldots\}$.

3.2. Equilibrium

As already mentioned, we follow the literature and focus on Markovian dynamics. Our dynamic political game, $\mathcal{G}^{i_0}$, has the following (Markovian) state vector in period t:

$$(i_t, K_t, s_t) \in [0, 1] \times \{L, R\} \times \{0, 1\}, \tag{3}$$

where $i_t \in [0, 1]$ is the policy chosen, $K_t \in \{L, R\}$ denotes the party that wins the election and is in power, and $s_t \in \{0, 1\}$ captures whether or not party K_t has suffered an electoral shock. We stress that the initial state is $(R, i_0, 0)$, with $i_0 \in [0, 1]$. In each period t, the median voter then elects a party that chooses a policy and may suffer an electoral shock according to the rules described above.

For our analysis of game $\mathcal{G}^{i_0}$, we further assume that the forward-looking parties use so-called simple strategies, which we now define.

Definition 1. A stationary Markov strategy σ_K for party $K \in \{L, R\}$ is simple if it can be written as

$$\sigma_K(i_{t-1}) = \sigma_K(i_{t-1}, K_{t-1}, s_{t-1}) = \min\left\{\max\left\{\mu_K - \chi_K^-, i_{t-1}\right\}, \mu_K + \chi_K^+\right\} \tag{4}$$

for $\chi_K := (\chi_K^-, \chi_K^+) \in \mathbb{R}_+^2$.

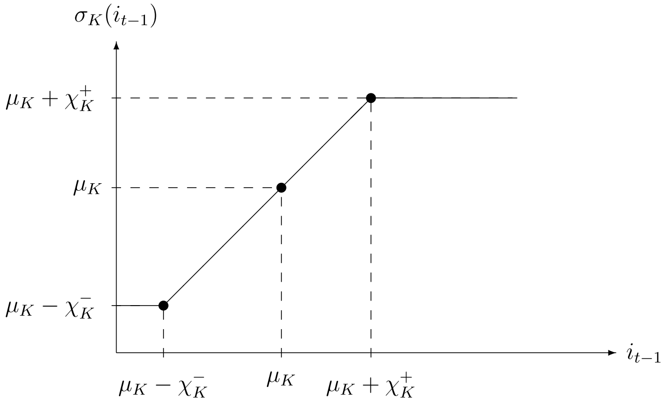


Fig. 2. A simple strategy for party K given the status-quo policy i_{t-1}.

Fig. 2 illustrates the shape of a party's simple strategy. A simple strategy for party K ignores which party was in power in the previous period (viz. K_{t-1}) and whether or not such a party received an electoral shock (viz. s_{t-1}). It is characterized by a closed interval $[\mu_K - \chi_K^-, \mu_K + \chi_K^+]$ containing all the status-quo policies i_{t-1} that the party will not change once in office. Accordingly, we often simplify and slightly abuse notation and write $\sigma_K = \chi_K$ for these strategies. An incumbent party encountering a status-quo policy below $\mu_K - \chi_K^-$ (above $\mu_K + \chi_K^+$) will replace the latter by the lower bound (upper bound) of the closed policy interval defined by the party's strategy. When parties use a simple strategy, they choose the range of persistence in policy-making that they apply whenever they are in power. In equilibrium, it must be optimal for parties not to change the policies in that range, so this persistence range can be credibly expected by the other party and the voter. It has been argued in the literature that incumbents always find it difficult to reverse the policies they have themselves chosen in the past (see e.g. Alesina, 1988; Miller and Schofield, 2003; Tavits, 2007). Simple strategies build on this rationale.

For voter v, a stationary Markov strategy can be written as a function

$$\sigma_v : [0,1] \times \{L, R\} \times \{0,1\} \to \{L, R\}$$

that maps current states $(i_{t-1}, K_{t-1}, s_{t-1})$ into electoral choices $K_t \in \{L, R\}$ for any period $t \in \{2, 3, \ldots\}$.

Finally, we introduce our equilibrium notion formally.

Definition 2. A simple stationary Markov perfect equilibrium (SSMPE) of $\mathcal{G}^{i_0}$ is a stationary Markov perfect equilibrium of $\mathcal{G}^{i_0}$, denoted by $(\sigma_v, \sigma_L, \sigma_R)$, in which the strategies of both parties are simple.[12]

4. Analysis

In this section we analyze the SSMPE of our political game and show the three main properties that characterize equilibrium behavior in the steady state, provided the marginal cost of change is not large relative to party polarization. First, policies eventually alternate between two

[12] Since the state defined in (3) does not include calendar time, Markov strategies in our setup are necessarily stationary.

points that are symmetrically located to the left and right of the median voter's peak. Second, policy changes occur if and only if the incumbent party suffers an electoral shock, in which case the challenger party is elected. While turnover in the case of an electoral shock follows directly from our assumption on the magnitude of such shocks, the interesting property is that the incumbent party cannot be challenged successfully without such a shock. This means that costs of change generate an incumbency advantage on the equilibrium path. Third, long-term policies are independent of the initial policy, but in the short term policies may display history-dependence.

To find the set of SSMPE of game $\mathcal{G}^{i_0}$, we need to characterize the optimal policy strategies for the two parties and the optimal behavior of the representative voter. This is done next.

Theorem 1. *Let* $c \in \left(0, \frac{2\Pi}{1+\psi}\right)$ *and* $i_0 \in [0, 1]$ *be the status-quo policy in* $t = 1$*. Then there is a unique SSMPE of* $\mathcal{G}^{i_0}$*, referred to as* $\sigma^* = (\sigma_v^*, \chi_L^*, \chi_R^*)$*, with*

$$\chi_L^* = \left(\underline{\chi}^*, \overline{\chi}^*\right) \quad \text{and} \quad \chi_R^* = \left(\overline{\chi}^*, \underline{\chi}^*\right),$$

and

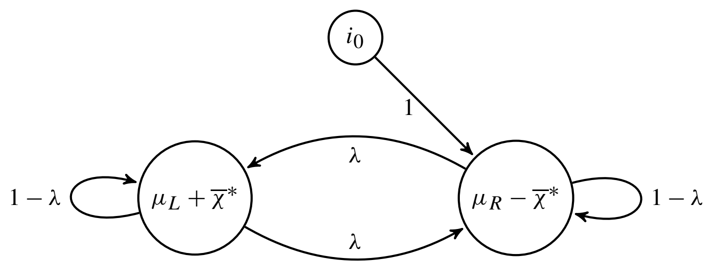
$$\underline{\chi}^* = \min\left\{\frac{1}{2} - \Pi, \frac{c}{2} \cdot (1 - \psi)\right\} \quad \text{and} \quad \overline{\chi}^* = \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)].$$

Moreover, in equilibrium the median voter v *using strategy* σ_v^* *will re-elect the incumbent party if and only if the latter did not suffer an electoral shock in the previous period.*[13]

The proof of Theorem 1 is standard. It first verifies that the simple strategies described in the statement of the theorem are part of an SSMPE (see Appendix A), and then it shows that no other SSMPE can exist (see Appendix B in the online material).

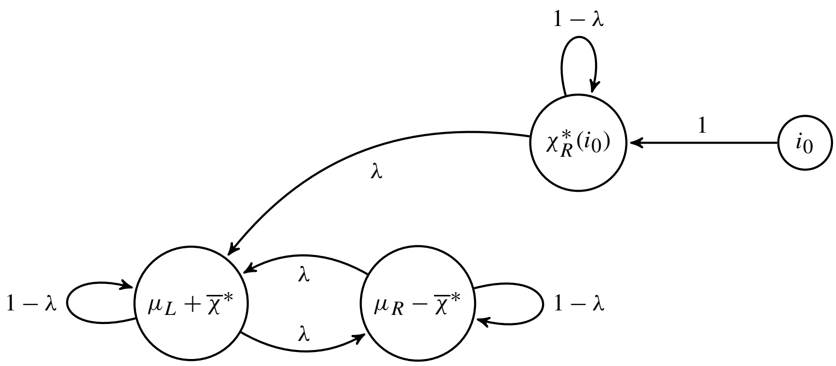
We start the discussion of Theorem 1 by noting that $c < \frac{2\Pi}{1+\psi}$ suffices for $\mu_L + \overline{\chi}^* < 1/2 < \mu_R - \overline{\chi}^*$. Hence, if marginal costs of change are sufficiently low or, equivalently, party polarization is sufficiently large, the parties' policy persistence regions defined by χ_L^* and χ_R^* do not overlap (and, moreover, the choices by the two parties fall on either side of the political spectrum relative to the median voter's peak).

To illustrate Theorem 1, it is useful to depict the equilibrium policy path of game $\mathcal{G}^{i_0}$. On the one hand, if $i_0 \leq \mu_R - \overline{\chi}^*$, we obtain the following Markov transition diagram:



On the other hand, if $i_0 > \mu_R - \overline{\chi}^*$, we obtain the following Markov transition diagram:

[13] We write $\underline{\chi}^* = \min\left\{\frac{1}{2} - \Pi, \frac{c}{2} \cdot (1 - \psi)\right\}$ to avoid $\mu_R + \overline{\chi}^* > 1$ and $\mu_L - \overline{\chi}^* < 0$. The same strategy is represented by $\underline{\chi}^* = \frac{c}{2} \cdot (1 - \psi)$.



In the two cases described above, the infinite sequence of policies converges to a set consisting of two policies: $\mu_L + \overline{\chi}^*$ (implemented by party L) and $\mu_R - \overline{\chi}^*$ (implemented by party R). Due to the positive costs of change, these policies are closer to the median voter's peak than the parties' peaks and are attained independently of the initial policy i_0. If the initial policy is sufficiently to the right, the infinite sequence of alternating policies is only reached after an initial phase of more extreme policies. Otherwise, the sequence where the above two policies alternate is immediately reached in period $t = 1$. Since an incumbent party maintains the status quo whenever it is re-elected, policy changes only occur when power shifts from one party to the other. Turnover occurs only when the incumbent party suffers an electoral shock, which in each period happens with probability λ. That is, in the absence of an electoral shock, costs of change enable the incumbent party to be re-elected by the median voter on the equilibrium path, regardless of the status quo.

If parties are forward-looking (i.e., if $\psi > 0$), their simple strategies are not symmetric with respect to the party's peak. Since $\overline{\chi}^* \geq \underline{\chi}^*$, a wider range of moderate status-quo policies (i.e., policies between the two parties' peaks) is preserved in equilibrium than extreme status-quo policies (i.e., policies beyond a party's peak). If the status quo is extreme, both parties agree on the direction (albeit not on the extent) in which the policy should be changed: toward the median voter's peak. This means that policy changes in such a direction will never credibly be reversed by either party, as such changes reduce the future costs of change for both parties. This enables the incumbent party to choose a policy that is closer, albeit not equal, to its own peak. It also implies that $\underline{\chi}^*$ is independent of λ. That is, the extent of policy adjustment implemented by the incumbent party when the status quo is extreme depends solely on its willingness to bear the costs of change now rather than later.

By contrast, if the status-quo policy is moderate, parties do not agree on the direction in which policy should be changed. Moving the status-quo policy closer to the incumbent party's peak reduces or maintains the costs of change that will occur in the next period if the incumbent party stays in power, but it also increases the costs of change that will occur whenever there is power turnover. Hence, compared to the case of an extreme status quo, the incumbent party retains a wider range of moderate status-quo policies, including policies that are farther away from its peak. The resulting moderate policy persistence range, $[\mu_R - \overline{\chi}^*, \mu_R]$ for party R, then depends on the probability of a given incumbent party suffering an electoral shock, λ. The larger λ is, the larger $\overline{\chi}^*$ will be and, hence, the wider the persistence range for moderate policies will also be.

Theorem 1 offers a variety of further comparative statics. First, consider the parties' discount factor, ψ, and without loss of generality focus on party R. Decreasing ψ increases the size of the interval $[\mu_R, \mu_R + \underline{\chi}^*]$, since the benefits of choosing a more moderate policy closer to the party's peak in the current period, and, hence, also in future periods, diminish as the future becomes less valuable for the parties. Thus, the costs of change that would need to be incurred today for choosing the more moderate policy matter more. By contrast, the impact of changing ψ on the size of the interval $[\mu_R - \overline{\chi}^*, \mu_R]$ is ambiguous. If $\lambda > 1/2$, and hence if in the next period it is more likely that the challenger party will be in power than the incumbent party, the size of the interval $[\mu_R - \overline{\chi}^*, \mu_R]$ decreases if ψ becomes lower. The reason is that if $\lambda > 1/2$, costs of change matter a lot more for the incumbent party's utility since power turnover is likely. But lowering ψ reduces this importance and induces the incumbent party to choose policies that are closer to its peak. If $\lambda < 1/2$, the logic is reversed, and lowering ψ increases the size of the interval $[\mu_R - \overline{\chi}^*, \mu_R]$.

Two further crucial parameters of our model are c, the marginal cost of a policy change, and Π, the level of party polarization. On the one hand, Theorem 1 implies that increasing c from 0 to $2\Pi/(1+\psi)$ widens the parties' persistence policy range, thereby reducing the distance between the policies chosen in the long run by the two parties, and thus the distance between these policies and the median voter's peak. The smaller this distance, the higher the utility for the median voter. It means that from a long-term perspective, levels of costs of change that are sufficiently close to the level of party polarization can be beneficial for the median voter.

On the other hand, increasing Π shifts the persistence policy range to the right (for party R) and to the left (for party L). This shift has negative consequences for the median voter. There are two reasons for this. First, the policies implemented by the two parties are farther away from the voter's peak, which reduces the utility the voter derives from these policies. Second, the costs of change generated every time there is turnover of parties in power will increase because the parties' policies move farther apart from each other. This increases the average costs of change incurred by the median voter in one period.

5. Extensions

The main insights of our analysis of the previous section remain valid for a number of extensions. They include office-motivated politicians, convex costs of change, and forward-looking voters (see Gersbach et al., 2023).

First, adding motivations for parties beyond policy issues can be easily accommodated in our model. The reason is that costs of change already generate an incumbency advantage and electoral shocks are large and exogenous, which means that parties cannot increase their re-election chances by choosing other policies than those in Theorem 1. This guarantees that the strategy profile of Theorem 1 will remain an SSMPE for any level of office rents that party members may derive from being in power.

Second, assume that for any period t and any status-quo policy $i_{t-1} \in [0, 1]$, the costs of change implied by policy choice $i_t \in [0, 1]$ in period t are

$$c \cdot |i_t - i_{t-1}|^\eta ,$$

with $1 < \eta < 2$. That is, costs of change are convex, albeit with a smaller exponent than the quadratic utility loss function for policies. For simplicity, we assume that $\psi = 0$, i.e., parties are myopic. Then the incumbent party is still ousted if and only if it has suffered an electoral shock, as in the case where costs of change are linear. This means that the transition probability from one party being in power to the other party being in power is again pinned down by λ. But now, parties during tenure will move policies closer to a limit point—their own peak—in a series of

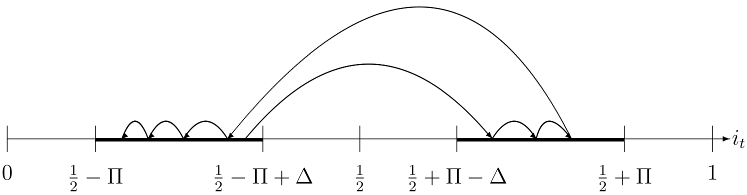


Fig. 3. A long-term policy path when Condition (5) holds and costs of change are convex.

moves that become lower in extent over time until the parties are ousted. Specifically, let Δ be the unique solution of the following equation:

$$\Delta = \frac{c\eta}{2}\,(2\Pi - \Delta)^{\eta-1}\,.$$

Fig. 3 depicts the long-term dynamics when costs of change have the form $c \cdot |i_t - i_{t-1}|^\eta$ and

$$c < \frac{2}{\eta} \cdot \Pi^{2-\eta}. \tag{5}$$

The latter condition guarantees that $\Delta < \Pi$, so the loci of the policies chosen by the two parties do not intersect.

Fig. 3 illustrates that in steady state, policies alternate between two regions of policies marked in black following a change of the party in power. These regions are the interval $[1/2-\Pi, 1/2-\Pi+\Delta]$ (for party L) and the interval $[1/2+\Pi-\Delta, 1/2+\Pi]$ (for party R). Since neither of these intervals contains the median voter's peak, there is no convergence to this position for any turnover probability λ. The turnover probability only determines how likely it is to observe particular policies of these regions along the equilibrium path. In the extreme case $\lambda = 1$, the sequence of policies chosen on the equilibrium path will converge in the long run to an alternation between policies $1/2 - \Pi + \Delta'$ and $1/2 + \Pi - \Delta'$, where Δ' is the unique solution of the following equation:

$$\Delta' = \frac{c\eta}{2}\left(2\Pi - 2\Delta'\right)^{\eta-1}\,.$$

One can easily verify that $\Delta \geq \Delta' \geq 0$. If $\lambda = 1$, parties are in office for only one period and they do not have enough time to move the policy farther toward their peak. In the extreme case $\lambda = 0$, there is no power alternation, and the policy converges to $1/2 + \Pi = \mu_R$, since party R is the incumbent in period $t = 1$.

Third and last, suppose that the median voter is forward-looking, and (for simplicity) that parties are myopic. Then, in the long term, policies alternate between two policies ($\mu_R - c/2$ chosen by party R and $\mu_L + c/2$ chosen by party L), and the voter re-elects the incumbent party if and only if it did not suffer an electoral shock in the previous period. The short-term dynamics of Theorem 1 also remain valid if either (1) $i_0 < \mu_R - c/2$ or (2) $\mu_R - c/2 \leq i_0 \leq \mu_R + c/2$ and the voter's discount factor is at most θ^{i_0}, with $\theta^{i_0} \in [0, 1]$ being a discount factor that depends on the initial policy. In such cases, party R is re-elected for as long as it does not suffer an electoral shock. Until then, party R chooses the same (extreme) policy in all periods, namely $\min\{\mu_R + c/2, i_0\}$. Otherwise, i.e., if either (1) $i_0 > \mu_R + c/2$ or (2) $\mu_R - c/2 \leq i_0 \leq \mu_R + c/2$ and the voter's discount factor is larger than θ^{i_0}, the median voter prefers to elect the challenger party L in period $t = 2$ regardless of electoral shocks, as this party will choose a policy that is closer to the median voter's peak. By doing so, the voter ensures a more moderate policy for a greater number of periods, which since the future is sufficiently valuable for the median voter compensates for incurring the costs of change associated with the large policy shift sooner rather than later. To sum up, with voters who are sufficiently forward-looking, no extreme policy is implemented along the equilibrium path from period $t = 2$ onward, and we already observe an alternation between the two long-term policies from this period, regardless of the initial status quo.

6. Conclusion

We developed a new infinite-horizon model of electoral competition to analyze the long-term consequences of costs of policy changes. We found that if costs of change are not too large and party polarization is large enough, the equilibrium policy path either is, or moves toward, an infinite sequence of (regions of) policies that are equidistant from the median voter's peak and more moderate than the office-holders' peaks. These properties are independent of the initial policy. The dynamics of policy and power turnover are determined by the incumbency advantage created by costs of change, on the one hand, and by the shocks that affect any other policy dimension, on the other.

Our analysis provides a series of testable hypotheses that serve as a basis for further inquiries and applications. Possible avenues for further research include the impact of costs of change in systems with more than two parties or the possibility of endogenizing party platforms (in particular, allowing entry at the median voter position). Our results also suggest that the major effects of costs of change on long-term policy outcomes already arise when voters (and to a lesser extent office-holders) concentrate on the current term. This may provide a rationale for behavioral approaches assuming that voters only concentrate on the immediate consequences of their voting decisions. Whether such a property can be further substantiated is a subject for future theoretical, empirical, or experimental work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix A

Proof of Theorem 1. The proof of the theorem involves three steps.
Step 1: We derive expression $\Delta_v(i_{t-1})$. It denotes the utility difference in some period t for voter v if s/he elects party R at the beginning of this period compared to electing party L, given the status-quo policy $i_{t-1} \in [0, 1]$, and assuming that the incumbent party did not suffer an electoral shock in period $t - 1$ and parties play simple strategies χ_L and χ_R throughout. That is,

$$\Delta_v(i_{t-1}) := U_v(i_{t-1}, \chi_R(i_{t-1})) - U_v(i_{t-1}, \chi_L(i_{t-1})). \tag{6}$$

Step 2: We show that the strategy profile described in Theorem 1 is an SSMPE of game $\mathcal{G}^{i_0}$.
Step 3: We show that game $\mathcal{G}^{i_0}$ has a unique SSMPE.

We prove Steps 1 and 2, while the proof of Step 3 can be found in Appendix B (see online material). Step 3 builds on arguments similar to those used in Steps 1 and 2.

Step 1: Voter utility difference

Consider party strategy profiles $\chi_K = (\chi_K^-, \chi_K^+)$, with $K \in \{L, R\}$, and some period $t \geq 2$. We assume that

$$\mu_L - \chi_L^- \leq \mu_R - \chi_R^- \tag{7}$$

and

$$\mu_L + \chi_L^+ \leq \mu_R + \chi_R^+, \tag{8}$$

which is satisfied by χ_L^* and χ_R^*. We distinguish different cases depending on the status-quo policy i_{t-1}.

Case 1: $i_{t-1} < \mu_L - \chi_L^-$

By Condition (7), we also have $i_{t-1} < \mu_R - \chi_R^-$. Hence, if party R is in power in period t, it will choose policy $\mu_R - \chi_R^-$. Similarly, if party L is in power in period t, it will choose policy $\mu_L - \chi_L^-$. Using the fact that $\mu_R + \mu_L = 1$,

$$\begin{aligned}\Delta_v(i_{t-1}) &= c \cdot [(\mu_L - \chi_L^-) - i_{t-1}] + \left[(\mu_L - \chi_L^-) - \frac{1}{2}\right]^2 \\ &\quad - c \cdot [(\mu_R - \chi_R^-) - i_{t-1}] - \left[(\mu_R - \chi_R^-) - \frac{1}{2}\right]^2 \\ &= [(\mu_L - \chi_L^-) - (\mu_R - \chi_R^-)] \cdot [c - \chi_L^- - \chi_R^-].\end{aligned} \tag{9}$$

Note that the above expression is independent of i_{t-1}.

Case 2: $\mu_L - \chi_L^- \leq i_{t-1} \leq \mu_L + \chi_L^+$ and $i_{t-1} < \mu_R - \chi_R^-$

In this case, if party R is in power in period t, it will choose policy $\mu_R - \chi_R^-$. By contrast, if party L is in power in period t, it will choose policy i_{t-1} also in period t. Then,

$$\begin{aligned}\Delta_v(i_{t-1}) &= \left(i_{t-1} - \frac{1}{2}\right)^2 - c \cdot [(\mu_R - \chi_R^-) - i_{t-1}] - \left[(\mu_R - \chi_R^-) - \frac{1}{2}\right]^2 \\ &= [i_{t-1} - (\mu_R - \chi_R^-)] \cdot [c + i_{t-1} + (\mu_R - \chi_R^-) - 1] \\ &\begin{cases} > 0 & \text{if } i_{t-1} < \mu_L + \chi_R^- - c, \\ = 0 & \text{if } i_{t-1} = \mu_L + \chi_R^- - c, \\ < 0 & \text{if } i_{t-1} > \mu_L + \chi_R^- - c. \end{cases}\end{aligned} \tag{10}$$

Case 3: $\mu_L - \chi_L^- \leq i_{t-1} \leq \mu_L + \chi_L^+$ and $\mu_R - \chi_R^- \leq i_{t-1} \leq \mu_R + \chi_R^+$

In this case, both parties will also choose policy i_{t-1} in period t. Then,

$$\Delta_v(i_{t-1}) = \left(i_{t-1} - \frac{1}{2}\right)^2 - \left(i_{t-1} - \frac{1}{2}\right)^2 = 0. \tag{11}$$

By the tie-breaking rule, the incumbent is re-elected in this case.

Case 4: $\mu_L + \chi_L^+ \leq i_{t-1} \leq \mu_R - \chi_R^-$

In this case, if party R is in power in period t, it will choose policy $\mu_R - \chi_R^-$. By contrast, if party L is in power in period t, it will choose policy $\mu_L + \chi_L^+$. Then,

$$\begin{aligned}\Delta_v(i_{t-1}) &= c\cdot[i_{t-1}-(\mu_L+\chi_L^+)]+\left[(\mu_L+\chi_L^+)-\frac{1}{2}\right]^2\\ &\quad - c\cdot[(\mu_R-\chi_R^-)-i_{t-1}]-\left[(\mu_R-\chi_R^-)-\frac{1}{2}\right]^2\\ &= c\cdot[2i_{t-1}-1]+[\chi_R^- - \chi_L^+]\cdot[c+(\mu_R-\chi_R^-)-(\mu_L+\chi_L^+)]. \end{aligned} \tag{12}$$

Note that if $\chi_R^- = \chi_L^+$, then the sign of $\Delta_v(i_{t-1})$ is the same as the sign of $2i_{t-1}-1$.
Case 5: $\mu_R - \chi_R^- \le i_{t-1} \le \mu_R + \chi_R^+$ and $\mu_L + \chi_L^+ < i_{t-1}$

In this case, if party R is in power in period t, it will choose policy i_{t-1} as in the previous period. By contrast, if party L is in power in period t, it will choose policy $\mu_L + \chi_L^+$. Then,

$$\begin{aligned}\Delta_v(i_{t-1}) &= c\cdot[i_{t-1}-(\mu_L+\chi_L^+)]+\left[(\mu_L+\chi_L^+)-\frac{1}{2}\right]^2-\left(i_{t-1}-\frac{1}{2}\right)^2\\ &= [i_{t-1}-(\mu_L+\chi_L^+)]\cdot[c+1-i_{t-1}-(\mu_L+\chi_L^+)]\\ &\begin{cases} >0 & \text{if } i_{t-1} < \mu_R-\chi_L^+ + c,\\ =0 & \text{if } i_{t-1} = \mu_R-\chi_L^+ + c,\\ <0 & \text{if } i_{t-1} > \mu_R-\chi_L^+ + c.\end{cases}\end{aligned} \tag{13}$$

Case 6: $\mu_R + \chi_R^+ < i_{t-1}$

By Condition (8), we also have $\mu_L + \chi_L^+ < i_{t-1}$. Hence, if party R is in power in period t, it will choose policy $\mu_R + \chi_R^+$. Similarly, if party L is in power in period t, it will choose policy $\mu_L + \chi_L^+$. Using the fact that $\mu_R + \mu_L = 1$,

$$\begin{aligned}\Delta_v(i_{t-1}) &= c\cdot[i_{t-1}-(\mu_L+\chi_L^+)]+\left[(\mu_L+\chi_L^+)-\frac{1}{2}\right]^2\\ &\quad - c\cdot[i_{t-1}-(\mu_R+\chi_R^+)]-\left[(\mu_R+\chi_R^+)-\frac{1}{2}\right]^2\\ &= [(\mu_R+\chi_R^+)-(\mu_L+\chi_L^+)]\cdot[c-\chi_L^+-\chi_R^+]. \end{aligned} \tag{14}$$

Note that the above expression is independent of i_{t-1}.

Step 2: Existence of SSMPE

Let $t \ge 1$ be any period and $i_{t-1} \in [0, 1]$ be the status-quo policy in this period. We show that if voters behave according to Equations (9)–(14) (see Step 1) and the parties' choices from period $t+1$ onward are in accord with χ_L^* and χ_R^* on and off the equilibrium path, it is optimal for the party $K \in \{R, L\}$ that is in office in period t to choose its policy according to $\chi_K^*(i_{t-1})$ on and off the equilibrium path. Moreover, no other policy choice is optimal for this party (see Step 2.1). After this, in Step 2.2 we show that if parties behave according to χ_L^* and χ_R^*, then voter v elects the incumbent party on the equilibrium path if and only if it did not suffer an electoral shock in the previous period.[14] As stated in the theorem, we assume

$$c < \frac{2\Pi}{1+\psi}. \tag{15}$$

[14] For our arguments, we therefore use the one-stage deviation principle (see Theorem 4.2 in Fudenberg and Tirole, 1991).

Step 2.1: Parties

It suffices to consider that party R is in office in period t, since the behavior of party L follows by symmetry. Given a policy choice $x \in [0, 1]$ for party R in period t, we let $U_R(x)$ be the lifetime utility that the party derives from choosing policy x, from the perspective of period t (see Expression (2)). We abuse notation and henceforth omit in $U_R(x)$ the term that captures the electoral shocks. Since electoral shocks are very large, such a term is independent of x, and thus it does not affect the choice of x that maximizes $U_R(x)$. Similar comments apply throughout the proof. We distinguish the following cases depending on the status-quo policy.

Case 1: $i_{t-1} \le \mu_L - \frac{c}{2} \cdot (1-\psi)$,
Case 2: $i_{t-1} \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right)$,
Case 3: $i_{t-1} \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right]$,
Case 4: $i_{t-1} \in \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R + \frac{c}{2} \cdot (1-\psi)\right)$,
Case 5: $i_{t-1} \ge \mu_R + \frac{c}{2} \cdot (1-\psi)$.

We start with Cases 1 and 5 and then discuss the remaining cases based on these two cases.

Case 1: $i_{t-1} \le \mu_L - \frac{c}{2} \cdot (1-\psi) = \mu_L - \underline{\chi}^*$

We show that $U_R(x)$ is maximized for $x^* = \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] = \mu_R - \overline{\chi}^*$. We can focus on the case where $x \ge i_{t-1}$, since it is never optimal for party R to choose a policy lower than $\mu_L - \frac{c}{2} \cdot (1-\psi)$. This follows from Equation (9), which shows that choosing a policy in period t lower than $\mu_L - \frac{c}{2} \cdot (1-\psi)$ *(a)* does not affect party R's re-election probability in period $t+1$, while it *(b)* increases the costs of change in period t and period $t+1$ without affecting further policy changes and *(c)* moves the policy in period t further away from party R's peak. We distinguish several subcases.

Case 1.a: $x \in \left[i_{t-1}, \mu_L - \frac{c}{2} \cdot (1-\psi)\right]$

In this case, we have

$$\begin{aligned} U_R(x) = & -c \cdot (x - i_{t-1}) - (\mu_R - x)^2 \\ & + \psi \cdot p_R \cdot \left[-c \cdot \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] - x\right) + C_R\right] \\ & + \psi \cdot p_L \cdot \left[-c \cdot \left(\mu_L - \frac{c}{2} \cdot (1-\psi) - x\right) + C_L\right], \end{aligned}$$

where C_R and C_L are constants independent of x. Due to Equation (9), also p_R and p_L are constants independent of x, with $p_R + p_L = 1$. Parameter p_R (p_L) denotes the probability that party R (L) will be in power in period $t+1$ if party R chooses policy x in period t satisfying the conditions of Case 1.a. The same notation is used throughout the proof. It immediately follows that

$$\frac{dU_R(x)}{dx} = -c + 2(\mu_R - x) + c \cdot \psi > 0,$$

where the inequality can be obtained from the following chain of inequalities:

$$x \le \mu_L - \frac{c}{2} \cdot (1-\psi) < \mu_R - \frac{c}{2} \cdot (1-\psi).$$

Hence,

$$x \in \left[i_{t-1}, \mu_L - \frac{c}{2} \cdot (1-\psi)\right] \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{16}$$

Case 1.b: $x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right)$

In this case,

$$x - \left(\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)] - c\right) > c - \frac{c}{2} \cdot (1-\psi) - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]$$
$$= c \cdot \psi(1-\lambda) \geq 0.$$

Equation (10) implies that the challenger party L is elected in period $t+1$ regardless of the electoral shocks in period t, since $\Delta_v(x) < 0$. We therefore obtain

$$U_R(x) = -c \cdot (x - i_{t-1}) - (\mu_R - x)^2 + \psi \cdot U_R^L(x), \tag{17}$$

where $U_R^L(x)$ denotes party R's expected utility from the perspective of period $t+1$ when party L is in office in period $t+1$ and the (status-quo) policy chosen in period t is x. By the assumptions of Case 1.b, party L will retain policy x in period $t+1$. Equation (10) then implies that party L is re-elected at the beginning of period $t+2$ (and is in office in period $t+2$) if it does not receive an electoral shock in period $t+1$, since $\Delta_v(x) < 0$. This means that

$$U_R^L(x) = -(\mu_R - x)^2 + \psi\lambda \cdot \left[-c \cdot \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] - x\right) + C_R\right]$$
$$+\psi(1-\lambda) \cdot U_R^L(x), \tag{18}$$

where C_R is independent of x. In Equation (18), we have used the fact that if party L receives an electoral shock in period $t+1$ and party R is elected in period $t+2$, then party R will move the policy in period $t+2$ to $\mu_R - \frac{c}{2} \cdot (1+\psi[2\lambda-1)]$, which is a policy that is independent of x. Note that Equation (18) can be explicitly solved to yield

$$U_R^L(x) = -\frac{(\mu_R - x)^2}{1-\psi(1-\lambda)} + x \cdot \frac{c\lambda\psi}{1-\psi(1-\lambda)} + C, \tag{19}$$

where C is independent of x. Plugging Equation (19) into Equation (17) and then differentiating, one obtains

$$\frac{dU_R(x)}{dx} = -c + 2(\mu_R - x) \cdot \frac{1+\psi\lambda}{1-\psi(1-\lambda)} + c \cdot \frac{\lambda\psi^2}{1-\psi(1-\lambda)} > 0. \tag{20}$$

By the assumptions of Case 1.b and using $2\Pi = \mu_R - \mu_L$, the above inequality is implied by

$$2\Pi \geq \frac{c}{2} \cdot [1+\psi(2\lambda-1)] + \frac{c}{2} \cdot \frac{1-\psi(1-\lambda)-\lambda\psi^2}{1+\lambda\psi}. \tag{21}$$

By Condition (15), it is sufficient, in turn, to show that

$$\frac{c}{2} \cdot [1+\psi(2\lambda-1)] \geq \frac{c}{2} \cdot \frac{1-\psi(1-\lambda)-\lambda\psi^2}{1+\lambda\psi}.$$

Using the fact that $c > 0$ and undertaking some algebraic manipulations, the latter inequality is equivalent to $2\lambda\psi(1+\lambda\psi) \geq 0$, so Inequalities (20) and (21) hold. To sum up,

$$x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{22}$$

Case 1.c: $x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right]$

Note that this case can occur since Condition (15) holds. Then, we have

$$\begin{aligned}U_R(x) =& -c\cdot(x-i_{t-1})-(\mu_R-x)^2\\ &+p_R\cdot\psi\cdot\left[-c\cdot\left(\mu_R-\frac{c}{2}\cdot[1+\psi(2\lambda-1)]-x\right)+C_R\right]\\ &+p_L\cdot\psi\cdot\left[-c\cdot\left(x-\mu_L-\frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right)+C_L\right],\end{aligned} \tag{23}$$

where C_R and C_L are two constants independent of x, $p_R = 1 - p_L$ and, by Equation (12) applied to $\Delta_v(x)$ and from our tie-breaking rule,

$$p_L = \begin{cases}1 & \text{if } x < \frac{1}{2},\\ \lambda & \text{if } x \geq \frac{1}{2}.\end{cases}$$

It immediately follows that for $x \neq 1/2$

$$\frac{dU_R(x)}{dx} = -c + 2(\mu_R - x) + c\cdot p_R\cdot\psi - c\cdot p_L\cdot\psi,$$

which is strictly positive if and only if

$$x < \mu_R - \frac{c}{2}\cdot[1+\psi(p_L-p_R)]. \tag{24}$$

Now, if $x > \frac{1}{2}$, Condition (24) is equivalent to $x < \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]$. If $x < \frac{1}{2}$, by contrast, Condition (24) reduces to

$$x < \mu_R - \frac{c}{2}\cdot[1+\psi], \tag{25}$$

which holds by Condition (15).

Finally, from the above analysis we know that

$$x \in \left[\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) \setminus \left\{\frac{1}{2}\right\} \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{26}$$

Moreover,

$$\left(\frac{dU_R}{dx}\right)_-\left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) = 0 \tag{27}$$

and

$$\begin{aligned}&\lim_{x\to\left(\frac{1}{2}\right)^+} U_R(x) - \lim_{x\to\left(\frac{1}{2}\right)^-} U_R(x)\\ &= (1-\lambda)\cdot\psi\cdot\left[-c\cdot\left(\mu_R-\frac{c}{2}\cdot[1+\psi(2\lambda-1)]-\frac{1}{2}\right)+C_R\right]\\ &\quad -(1-\lambda)\cdot\psi\cdot\left[-c\cdot\left(\frac{1}{2}-\mu_L-\frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right)+C_L\right]\\ &= (1-\lambda)\cdot\psi\cdot[-c\cdot(\mu_R+\mu_L)+c+C_R-C_L]\\ &= (1-\lambda)\cdot\psi\cdot[C_R-C_L] \geq 0,\end{aligned} \tag{28}$$

where $C_R > C_L$ since, due to the symmetry of the SSMPE (χ_L^*, χ_R^*), the expected utility for party R if such a strategy profile is played, is higher if it is in office and chooses policy $\mu_R - \frac{c}{2}\cdot$

$[1+\psi(2\lambda-1)]$ rather than if party L is in office and chooses $\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)]$. Indeed, one can easily verify that C_R and C_L are uniquely determined by the following equations:

$$C_R = -\left(\frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right)^2 + (1-\lambda)\psi\cdot C_R + \lambda\psi\cdot C_L$$

and

$$C_L = -\left(2\Pi - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right)^2 + (1-\lambda)\psi\cdot C_L + \lambda\psi\cdot C_R.$$

It is then a matter of simple algebra to show that

$$C_R - C_L = 2\Pi\cdot\frac{2\Pi - c\cdot[1+\psi\cdot(2\lambda-1)]}{1+\psi\cdot(2\lambda-1)} > 0,$$

where the inequality follows from Condition (15).

Case 1.d: $x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R + \frac{c}{2}\cdot(1-\psi)\right)$

In this case,

$$\begin{aligned} x - \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)] + c\right) &< \frac{c}{2}\cdot(1-\psi) + \frac{c}{2}\cdot[1+\psi(2\lambda-1)] - c \\ &\leq -c\psi(1-\lambda) \leq 0. \end{aligned}$$

By Equation (13) and Condition (15), it follows that since $\Delta_v(x) > 0$, the incumbent party R will be re-elected in period $t+1$ unless it suffered an electoral shock in period t. This means that

$$\begin{aligned} U_R(x) = &-c\cdot(x-i_{t-1}) - (\mu_R - x)^2 + (1-\lambda)\cdot\psi\cdot\left[U_R(x) + c\cdot(x-i_{t-1})\right] \\ &+ \lambda\cdot\psi\cdot\left[-c\cdot\left(x-\mu_L-\frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) + C_L\right], \end{aligned}$$

where C_L is a constant independent of x. Hence,

$$\begin{aligned} U_R(x) = &\frac{1}{1-(1-\lambda)\psi}\cdot\Bigg\{-c\cdot(x-i_{t-1}) - (\mu_R-x)^2 + (1-\lambda)\cdot\psi\cdot c\cdot(x-i_{t-1}) \\ &+ \lambda\cdot\psi\cdot\left[-c\cdot\left(x-\mu_L-\frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) + C_L\right]\Bigg\}. \end{aligned}$$

This implies that

$$\frac{dU_R(x)}{dx} = \frac{-c + 2(\mu_R - x) + \psi\cdot c\cdot(1-2\lambda)}{1-(1-\lambda)\psi} < 0,$$

where the inequality holds by the assumptions of Case 1.d. Therefore,

$$x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R + \frac{c}{2}\cdot(1-\psi)\right) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{29}$$

Moreover,

$$\left(\frac{dU_R}{dx}\right)_+ \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) = 0. \tag{30}$$

Case 1.e: $x \geq \mu_R + \frac{c}{2}\cdot(1-\psi)$

This case is only feasible if $\mu_R + \frac{c}{2}\cdot(1-\psi) \leq 1$. By Equation (14),

$$\begin{aligned} \Delta_v(x) = &\left[\mu_R + \frac{c}{2}\cdot[1-\psi] - \mu_L - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right] \\ &\cdot\left[c - \frac{c}{2}\cdot[1-\psi] - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right]. \end{aligned}$$

Moreover,

$$\mu_R + \frac{c}{2} \cdot (1-\psi) - \mu_L - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] = 2\Pi - c\psi\lambda > 0$$

and

$$c - \frac{c}{2} \cdot (1-\psi) - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] = c\psi(1-\lambda) \geq 0,$$

where the first inequality holds by Condition (15). Accordingly, $\Delta_v(x) \geq 0$, so

$$\begin{aligned} U_R(x) = & -c \cdot (x - i_{t-1}) - (\mu_R - x)^2 \\ & + (1-\lambda) \cdot \psi \cdot \left[-c \cdot \left(x - \mu_R - \frac{c}{2} \cdot (1-\psi) \right) + C_R \right] \\ & + \lambda \cdot \psi \cdot \left[-c \cdot \left(x - \mu_L - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \right) + C_L \right], \end{aligned}$$

where C_R and C_L are constants independent of x. It then follows that

$$\frac{dU_R(x)}{dx} = -c + 2(\mu_R - x) - c \cdot \psi,$$

which is strictly negative because

$$x \geq \mu_R + \frac{c}{2} \cdot (1-\psi) > \mu_R - \frac{c}{2} \cdot (1+\psi).$$

To sum up,

$$x \geq \mu_R + \frac{c}{2} \cdot (1-\psi) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{31}$$

The combination of Conditions (16), (22), (26), (27), (28), (29), (30), and (31) implies that if the status-quo policy i_{t-1} satisfies $i_{t-1} \leq \mu_L - \frac{c}{2} \cdot (1-\psi)$, then

$$x^* = \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] = \chi_R^*(i_{t-1})$$

is the unique optimal policy choice for party R in period t. This completes the proof of Case 1.

Case 5: $i_{t-1} \geq \mu_R + \frac{c}{2} \cdot (1-\psi)$

Note that this case implies $\mu_R + \frac{c}{2} \cdot (1-\psi) \leq 1$. We show that $U_R(x)$ is maximized for $x^* = \mu_R + \frac{c}{2} \cdot (1-\psi) = \mu_R + \underline{\chi}^*$. By the same logic as in Case 1, we can focus on the case where $x \leq i_{t-1}$. Then we distinguish several subcases.

Case 5.a: $x \leq \mu_L - \frac{c}{2} \cdot (1-\psi)$

In this case, we have

$$\begin{aligned} U_R(x) = & -c \cdot (i_{t-1} - x) - (\mu_R - x)^2 \\ & + \psi \cdot p_R \cdot \left[-c \cdot \left(\mu_R - \frac{c}{2}(1+\psi(2\lambda-1)) - x \right) + C_R \right] \\ & + \psi \cdot p_L \cdot \left[-c \cdot \left(\mu_L - \frac{c}{2} \cdot (1-\psi) - x \right) + C_L \right], \end{aligned}$$

where C_R and C_L are constants independent of x. Due to Equation (9), also p_R and p_L are constants independent of x, with $p_R + p_L = 1$. It immediately follows that

$$\frac{dU_R(x)}{dx} = c + 2(\mu_R - x) + c \cdot \psi > 0.$$

The inequality follows from the following chain of inequalities:

$$x \leq \mu_L - \frac{c}{2} \cdot (1-\psi) < \mu_R + \frac{c}{2} \cdot (1+\psi).$$

To sum up,

$$x \leq \mu_L - \frac{c}{2} \cdot (1-\psi) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{32}$$

Case 5.b: $x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right)$

As in Case 1.b, Equation (10) implies that the challenger party L will be elected at the beginning of period $t+1$ regardless of electoral shocks, since $\Delta_v(x) < 0$. We therefore have

$$U_R(x) = -c \cdot (i_{t-1} - x) - (\mu_R - x)^2 + \psi \cdot U_R^L(x), \tag{33}$$

where $U_R^L(x)$ corresponds to Equation (18) (see Case 1.b). Plugging this latter expression into Equation (33) and then differentiating, one obtains

$$\frac{dU_R(x)}{dx} = c + 2(\mu_R - x) \cdot \frac{1+\lambda\psi}{1-\psi(1-\lambda)} + c \cdot \frac{\lambda\psi^2}{1-\psi(1-\lambda)} > 0,$$

where the inequality holds since

$$x < \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)] < \frac{1}{2} \leq \mu_R.$$

Note that the second inequality in the above chain of inequalities is implied by Condition (15) together with $\mu_R = 1 - \mu_L$ and $2\Pi = \mu_R - \mu_L$. To sum up,

$$x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{34}$$

Case 5.c: $x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right]$

Note that this case can only occur if

$$\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \leq \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)],$$

so it is implied by Condition (15). Then we have

$$\begin{aligned} U_R(x) = &-c \cdot (i_{t-1} - x) - (\mu_R - x)^2 \\ &+ p_R \cdot \psi \cdot \left[-c \cdot \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] - x\right) + C_R\right] \\ &+ p_L \cdot \psi \cdot \left[-c \cdot \left(x - \mu_L - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) + C_L\right], \end{aligned}$$

where C_R and C_L are two constants independent of x, $p_R = 1 - p_L$, and by Equation (12) applied to $\Delta_v(x)$ and from our tie-breaking rule,

$$p_L = \begin{cases} 1 & \text{if } x < \frac{1}{2}, \\ \lambda & \text{if } x \geq \frac{1}{2}. \end{cases}$$

It immediately follows that for $x \neq 1/2$

$$\frac{dU_R(x)}{dx} = c + 2(\mu_R - x) + c \cdot p_R \cdot \psi - c \cdot p_L \cdot \psi,$$

which is strictly positive if and only if

$$x < \mu_R + \frac{c}{2} \cdot [1 - \psi(p_L - p_R)]. \tag{35}$$

Now, if $x < \frac{1}{2}$, Condition (35) reduces to $x < \mu_R + \frac{c}{2} \cdot [1 - \psi]$, which is satisfied in Case 5.c. If $x > \frac{1}{2}$, by contrast, Condition (35) is equivalent to $x < \mu_R + \frac{c}{2} \cdot [1 - \psi(2\lambda - 1)]$, which is also satisfied in Case 5.c. Hence,

$$x \in \left(\mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)], \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right) \setminus \left\{\frac{1}{2}\right\} \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{36}$$

Finally, note that Equation (28) (see Case 1.c) implies that

$$\lim_{x \to \left(\frac{1}{2}\right)^+} U_R(x) - \lim_{x \to \left(\frac{1}{2}\right)^-} U_R(x) \geq 0. \tag{37}$$

Case 5.d: $x \in \left(\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)], \mu_R + \frac{c}{2} \cdot (1 - \psi)\right)$

In this case,

$$\begin{aligned} x - \left(\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] + c\right) &< \frac{c}{2} \cdot (1 - \psi) + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] - c \\ &\leq -c(1 - \lambda)\psi \leq 0. \end{aligned}$$

By Equation (13) and Condition (15), it follows that the incumbent party R will be re-elected in period $t + 1$ unless it suffered an electoral shock in period t, since $\Delta_v(x) > 0$. This means that

$$\begin{aligned} U_R(x) = &- c \cdot (i_{t-1} - x) - (\mu_R - x)^2 + (1 - \lambda) \cdot \psi \cdot \left[U_R(x) + c \cdot (i_{t-1} - x)\right] \\ &+ \lambda \cdot \psi \cdot \left[-c \cdot \left(x - \mu_L - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right) + C_L\right], \end{aligned}$$

where C_L is a constant independent of x. Hence,

$$\begin{aligned} U_R(x) = &\frac{1}{1 - (1 - \lambda)\psi} \cdot \Bigg\{ - c \cdot (i_{t-1} - x) - (\mu_R - x)^2 + (1 - \lambda) \cdot \psi \cdot c \cdot (i_{t-1} - x) \\ &+ \lambda \cdot \psi \cdot \left[-c \cdot \left(x - \mu_L - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right) + C_L\right] \Bigg\}. \end{aligned}$$

This implies that

$$\frac{dU_R(x)}{dx} = \frac{c + 2(\mu_R - x) - \psi \cdot c}{1 - (1 - \lambda)\psi} > 0,$$

where the inequality holds by the assumptions of Case 5.d. Hence,

$$x \in \left(\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)], \mu_R + \frac{c}{2} \cdot (1 - \psi)\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{38}$$

Case 5.e: $x \geq \mu_R + \frac{c}{2} \cdot (1 - \psi)$

In this case note that by Equations (13) and (14),

$$\begin{aligned} \Delta_v(x) = &\left[\mu_R + \frac{c}{2} \cdot [1 - \psi] - \mu_L - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right] \\ &\cdot \left[c - \frac{c}{2} \cdot [1 - \psi] - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]\right]. \end{aligned}$$

Moreover,

$$\mu_R + \frac{c}{2} \cdot [1-\psi] - \mu_L - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] = 2\Pi - c\psi\lambda > 0,$$

due to Condition (15), and

$$c - \frac{c}{2} \cdot [1-\psi] - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] = c\psi(1-\lambda) \geq 0.$$

Accordingly, $\Delta_v(x) \geq 0$, so

$$\begin{aligned} U_R(x) = &- c \cdot (i_{t-1} - x) - (\mu_R - x)^2 \\ &+ (1-\lambda) \cdot \psi \cdot \left[-c \cdot \left(x - \mu_R - \frac{c}{2} \cdot (1-\psi) \right) + C_R \right] \\ &+ \lambda \cdot \psi \cdot \left[-c \cdot \left(x - \mu_L - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \right) + C_L \right], \end{aligned}$$

where C_R and C_L are constants independent of x. Then it follows that

$$\frac{dU_R(x)}{dx} = c + 2(\mu_R - x) - c \cdot \psi,$$

which is strictly negative if and only if $x > \mu_R + \frac{c}{2} \cdot (1-\psi)$. That is,

$$x > \mu_R + \frac{c}{2} \cdot (1-\psi) \Longrightarrow \frac{dU_R(x)}{dx} < 0 \tag{39}$$

and, moreover,

$$\left(\frac{dU_R}{dx} \right)_+ \left(\mu_R + \frac{c}{2} \cdot (1-\psi) \right) = 0. \tag{40}$$

The combination of Conditions (32), (34), (36), (37), (38), (39), and (40) implies that $x^* = \mu_R + \frac{c}{2} \cdot (1-\psi)$ is the unique optimal policy choice of party R in period t when the status quo i_{t-1} satisfies $i_{t-1} \geq \mu_R + \frac{c}{2} \cdot (1-\psi)$. This completes the proof of Case 5.

Case 2: $i_{t-1} \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \right]$

We show that $U_R(x)$ is uniquely maximized for $x^* = \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]$. We distinguish several subcases.

Case 2.a: $x \leq \mu_L - \frac{c}{2} \cdot (1-\psi)$

This case is equivalent to Case 5.a. We therefore obtain

$$x \leq \mu_L - \frac{c}{2} \cdot (1-\psi) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{41}$$

Case 2.b: $x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \right)$

Applying Cases 1.b (if $i_{t-1} \leq x$) and 5.b (if $i_{t-1} \geq x$), we obtain

$$x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{42}$$

Case 2.c: $x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \right]$

This case is equivalent to Case 1.c. We therefore obtain

$$x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \right) \setminus \left\{ \frac{1}{2} \right\} \Longrightarrow \frac{dU_R(x)}{dx} > 0 \tag{43}$$

and

$$\left(\frac{dU_R}{dx} \right)_- \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \right) = 0. \tag{44}$$

Moreover, it also holds that

$$\lim_{x\to\left(\frac{1}{2}\right)^+} U_R(x) - \lim_{x\to\left(\frac{1}{2}\right)^-} U_R(x) \geq 0. \tag{45}$$

Case 2.d: $x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R + \frac{c}{2}\cdot(1-\psi)\right)$

This case is equivalent to Case 1.d. We therefore obtain

$$x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R + \frac{c}{2}\cdot(1-\psi)\right] \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{46}$$

Case 2.e: $x \geq \mu_R + \frac{c}{2}\cdot(1-\psi)$

This case is equivalent to Case 1.e. We therefore obtain

$$x \geq \mu_R + \frac{c}{2}\cdot(1-\psi) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{47}$$

The combination of Conditions (41), (42), (43), (44), (45), (46), and (47) implies that $x^* = \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]$ is the unique optimal policy choice of party R in period t when the status-quo policy i_{t-1} satisfies $\mu_L - \frac{c}{2}\cdot(1-\psi) < i_{t-1} < \mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)]$. This completes the proof of *Case 2*.

Case 3: $i_{t-1} \in \left[\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right]$

We show that $U_R(x)$ is uniquely maximized for $x^* = \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]$. We distinguish several subcases.

Case 3.a: $x \leq \mu_L - \frac{c}{2}\cdot(1-\psi)$

This case is equivalent to Case 5.a. We therefore obtain

$$x \leq \mu_L - \frac{c}{2}\cdot(1-\psi) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{48}$$

Case 3.b: $x \in \left(\mu_L - \frac{c}{2}\cdot(1-\psi), \mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right)$

This case is equivalent to Case 5.b. We therefore obtain

$$x \in \left(\mu_L - \frac{c}{2}\cdot(1-\psi), \mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{49}$$

Case 3.c: $x \in \left[\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right]$

In this case, applying Cases 1.c (if $i_{t-1} \leq x$) and 5.c (if $i_{t-1} \geq x$), we obtain

$$x \in \left[\mu_L + \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) \setminus \left\{\frac{1}{2}\right\} \Longrightarrow \frac{dU_R(x)}{dx} > 0 \tag{50}$$

and

$$\frac{dU_R}{dx}\left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)]\right) = 0. \tag{51}$$

Moreover, it also holds that

$$\lim_{x\to\left(\frac{1}{2}\right)^+} U_R(x) - \lim_{x\to\left(\frac{1}{2}\right)^-} U_R(x) \geq 0. \tag{52}$$

Case 3.d: $x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R + \frac{c}{2}\cdot(1-\psi)\right)$

This case is equivalent to Case 1.d. We therefore obtain

$$x \in \left(\mu_R - \frac{c}{2}\cdot[1+\psi(2\lambda-1)], \mu_R + \frac{c}{2}\cdot(1-\psi)\right) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{53}$$

Case 3.e: $x \geq \mu_R + \frac{c}{2} \cdot (1-\psi)$

This case is equivalent to Case 1.e. We therefore obtain

$$x \geq \mu_R + \frac{c}{2} \cdot (1-\psi) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{54}$$

The combination of Conditions (48), (49), (50), (51), (52), (53), and (54) implies that $x^* = \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]$ is the unique optimal policy choice of party R in period t when the status quo satisfies $\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)] \leq i_{t-1} \leq \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]$. This completes the proof of *Case 3*.

Case 4: $i_{t-1} \in \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R + \frac{c}{2} \cdot (1-\psi)\right)$

We show that $U_R(x)$ is uniquely maximized for $x^* = i_{t-1}$. We distinguish several subcases.

Case 4.a: $x \leq \mu_L - \frac{c}{2} \cdot (1-\psi)$

This case is equivalent to Case 5.a. We therefore obtain

$$x \leq \mu_L - \frac{c}{2} \cdot (1-\psi) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{55}$$

Case 4.b: $x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right)$

This case is equivalent to Case 5.b. We therefore obtain

$$x \in \left(\mu_L - \frac{c}{2} \cdot (1-\psi), \mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{56}$$

Case 4.c: $x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right]$

This case is equivalent to Case 5.c, and thus we obtain

$$x \in \left[\mu_L + \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)]\right) \setminus \left\{\frac{1}{2}\right\} \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{57}$$

Moreover, it holds that

$$\lim_{x \to \left(\frac{1}{2}\right)^+} U_R(x) - \lim_{x \to \left(\frac{1}{2}\right)^-} U_R(x) \geq 0. \tag{58}$$

Case 4.d: $x \in \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)], \mu_R + \frac{c}{2} \cdot (1-\psi)\right)$

First, assume that $x < i_{t-1}$. Applying Case 5.d, we obtain

$$x \in \left(\mu_R - \frac{c}{2} \cdot [1+\psi(2\lambda-1)], i_{t-1}\right) \Longrightarrow \frac{dU_R(x)}{dx} > 0. \tag{59}$$

Second, assume that $x > i_{t-1}$. Applying Case 1.d, we obtain

$$x \in \left(i_{t-1}, \mu_R + \frac{c}{2} \cdot (1-\psi)\right) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{60}$$

Case 4.e: $x \in \left[\mu_R + \frac{c}{2} \cdot (1-\psi), 1\right]$

This case is equivalent to Case 1.e. We therefore obtain

$$x \geq \mu_R + \frac{c}{2} \cdot (1-\psi) \Longrightarrow \frac{dU_R(x)}{dx} < 0. \tag{61}$$

The combination of Conditions (55), (56), (57), (58), (59), (60), and (61) implies that $x^* = i_{t-1}$ is the unique optimal policy choice of party R in period t when the status-quo policy i_{t-1}

satisfies $\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] < i_{t-1} < \mu_R + \frac{c}{2} \cdot (1 - \psi)$. Together with the continuity of $U_R(x)$ at $x = x^*$, this completes the proof of *Case 4*.

Step 2.2: The voter

Consider that parties use strategies χ_L^* and χ_R^* and recall that we are assuming Condition (15). We can distinguish two cases. First, if $i_0 \geq \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$, then for as long as party R is in office, such a party will choose $\min\{i_0, \mu_R + \frac{c}{2} \cdot (1 - \psi)\}$. From the moment that power shifts to party L, the policy will be either $\mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$ (chosen by party L) or $\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$ (chosen by party R). Second, if $i_0 < \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$, then the policy chosen in any period is either $\mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$ (chosen by party L) or $\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$ (chosen by party R).

To verify that the voter will re-elect the incumbent party if and only if such party did not suffer an electoral shock in the previous period, we build on the analysis of Step 1 and distinguish a number of cases. We also recall that in period $t = 1$ party R is elected and hence there are no elections in this period. Hence, we focus on the case $t \geq 2$.

Case A: $i_{t-1} < \mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$

This case cannot occur on the equilibrium path if $t \geq 2$.

Case B: $i_{t-1} = \mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$

In this case, Equation (12) implies that the incumbent party L is re-elected.

Case C: $\mu_L + \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] < i_{t-1} < \mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)]$

This case cannot occur on the equilibrium path if $t \geq 2$.

Case D: $\mu_R - \frac{c}{2} \cdot [1 + \psi(2\lambda - 1)] \leq i_{t-1} \leq \mu_R + \frac{c}{2} \cdot (1 - \psi)$

In this case,

$$\begin{aligned} &i_{t-1} - \left(\mu_R - \frac{c}{2} \cdot [1 + \psi \cdot (2\lambda - 1)] + c\right) \\ &\quad \leq \mu_R + \frac{c}{2} \cdot (1 - \psi) - \left(\mu_R - \frac{c}{2} \cdot [1 + \psi \cdot (2\lambda - 1)] + c\right) = -c\psi(1 - \lambda) \leq 0, \end{aligned}$$

so $\Delta_v(i_{t-1}) \geq 0$. Then Equation (13) implies that the incumbent party R is re-elected.

Case E: $i_{t-1} > \mu_R + \frac{c}{2} \cdot (1 - \psi)$

This case cannot occur on the equilibrium path if $t \geq 2$. This completes the proof of existence of an SSMPE in Theorem 1. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2023.105716.

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