The Effects of Education Subsidy in an R&D Based Economy

Hansi Wan

June 15, 2024

Abstract This paper aims to study the effects of education subsidies in an R&D-based economy. I develop a Schumpeterian growth model that incorporates both variety innovations and quality improvements, with endogenous population growth and the accumulation of human capital. The results show that the effects of education subsidy policy may be contrary to its intended objectives. Specifically, it hinders long-term economic growth when the population growth rate is constant or decreases with the education subsidy. This occurs because the subsidy expands variety innovation but impedes quality improvements, and also reduces the value of the quality-adjusted market size of intermediate goods, which determines the long-term economic growth rate. However, the effects become ambiguous if the population growth rate increases with the education subsidy, which tends to reduce or even reversing the subsidy's impact.

Keywords Education subsidy, endogenous fertility, variety innovation, quality improvements

JEL Classification J13, J24, O30, O31

1 Introduction

Many countries provide education subsidies with the aim of encouraging more people to pursue higher levels of education, thereby promoting human capital, advancing technology development, and driving economic growth. For example, China has implemented nine-year compulsory education since 1986, offering free tuition for students in primary and secondary schools. The government also maintains low tuition fees for high school and university students and offers low-interest loans to students with financial difficulties. All these policies can be regarded as education subsidies, reducing the opportunity cost of receiving an education. In fact, education expenditure accounts for a significant proportion of GDP in most developed countries. Figure 1 displays the proportion of GDP spent on education in OECD countries, averaging around 5%, with some of the most developed countries, such as the US, Canada, and Australia, reaching around 6%. Figure 2 illustrates the distribution of public and private expenditure on education in OECD countries. [1] On average, nearly 84% of the expenditure across OECD countries comes from public sources, although the share of public and private expenditure varies widely across countries.

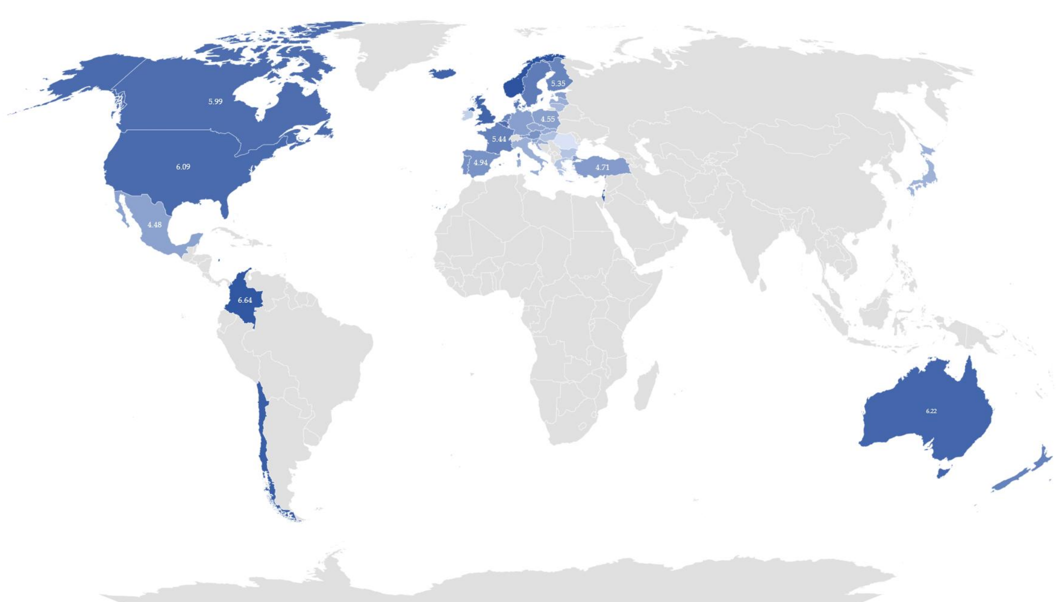

Figure 1: Expenditure on education %GDP in OECD Countries

The aforementioned description raises some intriguing questions. Does the education subsidy truly achieve its objectives, and what mechanisms underlie its effects? What is the optimal education subsidy rate to maximize long-term economic growth? One might initially think that education subsidies increase the average education level of the populace, thereby fostering

[1]Data of Figure 1 and 2 is showed in the OECD website: https://data.oecd.org/eduresource/public- spending-on-education.htm#indicator-chart

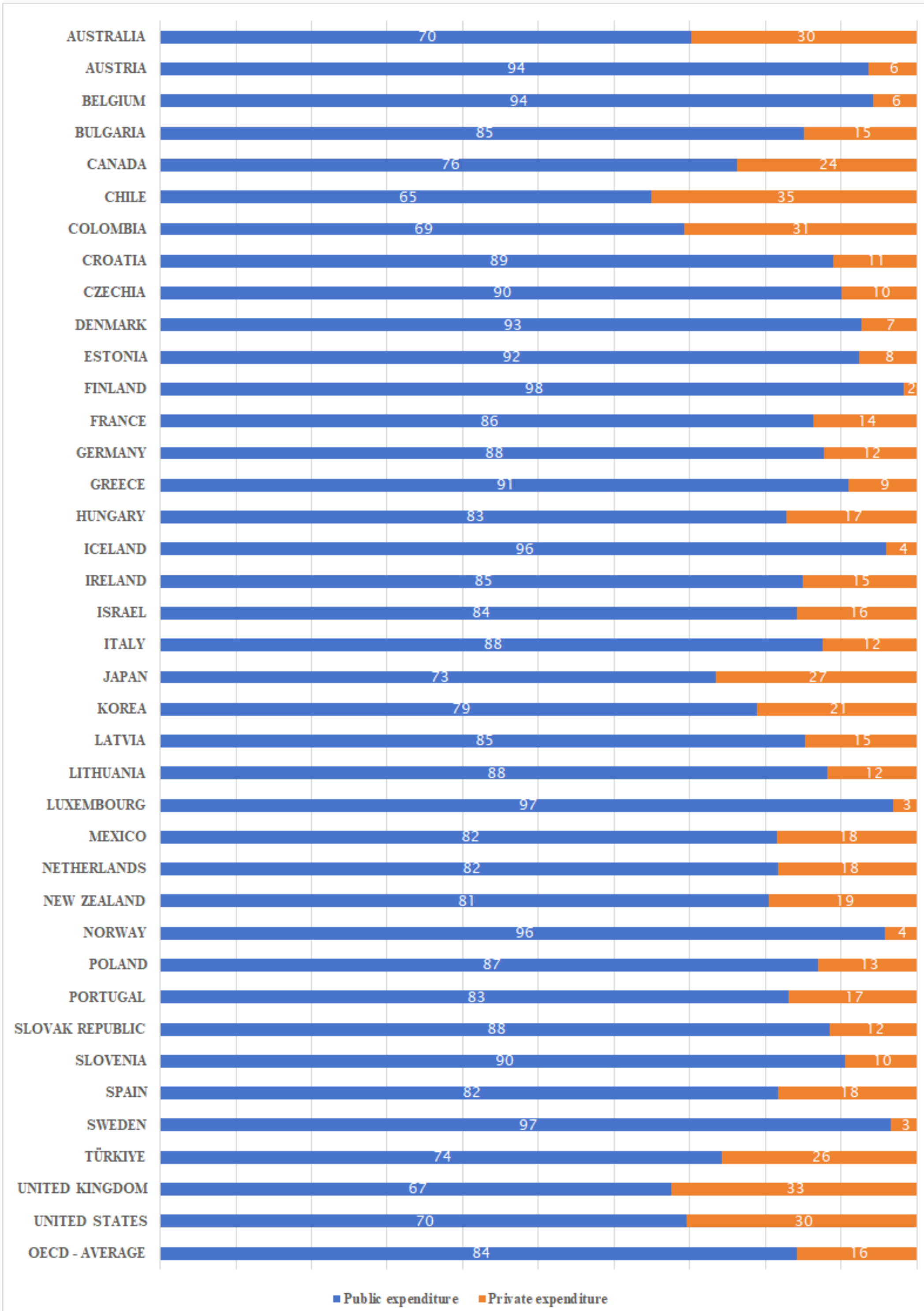

Figure 2: Proportion of public and private expenditure on education

economic growth through human capital accumulation. Lucas (1988) models the role of human capital in economic growth and proposes a law of motion for human capital, which accumulates through schooling. On the other hand, technological advancement has been recognized as the main driver of long-term economic growth [e.g., Solow (1956)]. Furthermore, Grossman and Helpman (1991) and Aghion and Howitt (1992) highlight the positive impact of vertical innovations, where entrepreneurs improve the quality of existing goods, on driving economic growth. Romer (1990) models horizontal innovations by entrepreneurs who create new designs of goods, and it also shows the interaction between human capital and technological progress in the economy. Therefore, it is crucial to explore the effects of education subsidies on economic growth through the mechanisms of human capital and both horizontal and vertical innovations. Additionally, education subsidies are likely to impact the population growth rate, which in turn affects human capital accumulation. Thus, the population growth rate should be endogenized to achieve a more general and comprehensive analysis. However, as far as I know, existing literature has only partially considered these mechanisms when exploring the effect of education subsidies on economic growth, which reduces the reliability and validity of the research.

To study the effects of education subsidies on the economy, I employ the second-generation Schumpeterian growth model with both variety innovations and quality improvements, aligning with pioneering literature such as Peretto (1998), Segerstrom (1998), and Howitt (1999). In this model, households derive utility from both consumption and fertility, allocating time and human capital among child-rearing, working, and education. Human capital is accumulated through schooling, and the population growth rate negatively impacts the growth rate of human capital accumulation through two mechanisms. First, a higher fertility rate exerts a crowding-out effect on households' time endowment, reducing the time available for human capital accumulation. Second, a higher fertility rate dilutes human capital per capita, as newborns are completely uneducated and join the workforce or population with no human capital [Strulik (2005)]. The effects of education subsidies on the economy are ambiguous and closely depend on the endogenous population growth rate. If the population growth rate is constant with respect to education subsidies, the subsidies foster variety innovations but impede quality improvements, ultimately deterring long-run economic growth. If the population growth rate decreases with education subsidies, it enhances the positive effects of the subsidies on the long-run aggregate economy. Conversely, if the population growth rate increases with education subsidies, it reduces the positive effects of the subsidies on the long-run aggregate economy. In this case, the net effects of education subsidies depend on the parameters of the economy and the actual values of the population growth rate and subsidy rate. Thus, the effects of such a policy may be contrary to its intended objectives.

The intuition behind this result is as follows. If the population growth rate remains constant with respect to education subsidies, the subsidy lowers the cost of education. This increases the share of human capital allocated to education and decreases the share devoted to work. As a result, with a constant population growth rate and an increased share of human capital going to education, human capital accumulation accelerates, leading to a higher growth rate of human capital. Although the share of human capital devoted to work decreases, the overall increase in aggregate human capital eventually raises the amount of human capital allocated to the production of final goods, causing wages in terms of final goods to decline. Consequently, the cost for both in-house R&D and new entrants falls, known as the cost reduction effect, which promotes quality improvements and variety innovations in the short run. However, the expansion of variety results in the entry effect, causing the average market share for each intermediate goods firm to shrink. This, in turn, inhibits quality improvements. The negative entry effect outweighs the positive cost reduction effect, thereby lowering the growth rate of quality improvements in the long run. Also, the quality-adjusted market size of intermediate goods decreases with an increase in the education subsidy rate but increases with the population growth rate. Since the population growth rate is constant, the education subsidy negatively affects the quality-adjusted market size of intermediate goods, leading to a long-term decline in the economy's growth rate.If the population growth rate decreases with an education subsidy, it further accelerates human capital accumulation and reduces the quality-adjusted market size of intermediate goods, thus enhancing the effects of the education subsidy on the aggregate economy. However, the effects of the education subsidy on per capita GDP are ambiguous, determined by the relative strength of the decreasing population growth rate and the increasing total GDP. Conversely, if the population growth rate increases with the education subsidy, the rising population growth rate would impede or even reverse human capital accumulation and increase the quality-adjusted market size of intermediate goods, making the ultimate impact of the mechanism ambiguous.

This paper is connected to several branches of literature. Firstly, it relates to the literature on endogenous growth theories, which identify technological progress as the driver of long-run economic growth, as discussed in canonical papers such as Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Recent literature emphasizes the interaction between human capital and innovations, proposing two engines of economic growth: human capital accumulation and R&D-based innovation [e.g., Lloyd-Ellis and Roberts (2002), Mattali (2012), Chu et al.(2013), Bucci and Raurich (2017)]. In particular, this paper is closely related to previous works that models the endogenous market structure in the Schumpeterain economy[e.g., Peretto (2007, 2015), Chu et al. (2016, 2022)]. Peretto (2007, 2015) sets a Schupeterian growth model where the final goods are consumed by households or used as a factor input for entry, in-house R&D and the production of intermediate goods. The market structure, measured by the equilibrium number of firms, is endogenous and directly determined by variety innovation. Chu et al. (2016, 2022) apply the same model and derive the conditions under which a semi-endogenous or fully endogenous growth regime is likely to emerge. Drawing on their work, I endogenize the population growth rate and incorporate human capital accumulation into the model to study the interaction between human capital and R&D-based innovation, which enriches the model and makes it more general and applicable.

Secondly, the paper relates to pioneering literature that analyzes the effect of education subsidies on economic growth. Hanushek and Woessmann (2011) demonstrate that cognitive skills, as measured by international achievement tests, have a positive impact on economic growth. They also argue that these test score measures (e.g., PISA test scores) are measured at the primary and secondary levels of schooling, highlighting the significance of primary and secondary education for economic growth. Holmes (2013) argues that there is no statistically significant relationship between the level or expansion of higher education and growth rates, both within the OECD and globally, suggesting that widespread higher education does not necessarily lead to higher economic growth. Del Rey and Lopez-Garcia (2016) use an overlapping generations model incorporating endogenous growth to assess the welfare implications of intergenerational transfers and education subsidies, showing that education subsidies have mixed effects on economic growth as they redistribute resources across generations and alter the relative price of investing in human capital. Chen (2015) examines the effects of child allowances and educational subsidies on economic growth through the channels of occupational choice and capital accumulation, finding that these policies may trap countries in underdevelopment, with the tax rate being a key determinant of long-run economic growth. In contrast to these studies, I utilize a Schumpeterian growth model with an endogenous market structure, studying the effects of education subsidies through human capital accumulation and technological innovations. These differences enable me to propose an alternative mechanism explaining the effects of education subsidies on economic growth. Nonetheless, the outcome of the paper is consistent with empirical evidence, considering that primary and secondary education subsidies tend to increase the population growth rate, while tertiary education subsidies decrease it.

Thirdly, the paper is closely related to Morimoto and Tabata (2020) and Hashimoto and Tabata (2016), both of which study the effects of education subsidies on economic growth using an overlapping generations model in an R&D-based economy. Morimoto and Tabata (2020) shows that the effect of a tertiary education subsidy on economic growth is ambiguous when the market structure adjusts partially in the short run. However, the education subsidy expands the number of firms but reduces economic growth when the market structure adjusts fully in the long run. Hashimoto and Tabata (2016) endogenizes the birth rate and demonstrate that child education subsidies consistently foster economic growth. Although my paper shares some research interests with these studies, it differs in several aspects. First, Morimoto and Tabata (2020) does not endogenize the population growth rate and it distinguishes skilled and unskilled workers instead of adopting a law of motion for human capital accumulation, while Hashimoto and Tabata (2016) does not include quality improvements in their model. Second, I focus on a more general education subsidy rather than one specific type. These differences allows me to shed light on various dimensions of how education subsidies impact economic growth and obtain a more general and comprehensive outcome.

The rest of the paper is organized as follows. Section 2 outlines the model and characterizes the dynamics of the economy in the balanced growth path. Section 3 theoretically investigates the effects of education subsidies on the long-run economy. Section 4 employs numerical analysis to supplement the theoretical analysis. Section 5 concludes the paper.

2 The Model

I consider infinitely lived households that gain utility from both consumption and fertility, allocating their time and human capital among child-rearing, working, and education, with human capital accumulated through schooling. The economy comprises three sectors: the final goods sector, the intermediate goods sector, and the entrants. The final goods sector produces homogeneous goods in a perfectly competitive market, which are either consumed by households or used as factor inputs for entry, in-house R&D, and the production of intermediate goods. The intermediate goods sector consists of monopolistically competitive firms that produce differentiated intermediate goods used in the production of final goods. This sector focuses on vertical innovation, improving the quality of intermediate goods to maintain its monopolistic position in the industry. The entrants focus on horizontal innovation, creating new product designs for firms entering the intermediate goods sector with the goal of earning monopolistic profits.

2.1 Households

Consider a representative household with the following utility function:

$$U = \int_0^\infty e^{-\rho t} u(c_t, n_t) dt \tag{1}$$

where $u(c_t, n_t) = \ln c_t + \lambda \ln n_t$, c_t, n_t denote the consumption per capita of final goods and population growth rate in time t respectively.[2] $\lambda > 0$ is a fertility-preference parameter and $\rho > 0$ is the time preference discount rate. The initial population size L_0 is normalized to unity. Each household is endowed with one unit of time and supplies h_t units of efficient labor in time t. The household allocates time and human capital between child-rearing, working (production of final goods) and education:

$$h_t(1 - n_t/\gamma) = h_t^Y + h_t^E \tag{2}$$

h_t^Y, h_t^E is the human capital allocated to work and education respectively. $(1 - n_t/\gamma)$ is the time spent on child-rearing, where γ inversely measures the time cost of child-rearing. Human capital follows the law of motion:

$$\dot{h}_t = \varepsilon h_t^E - (\delta + n_t) h_t \tag{3}$$

where ε measures the return of human capital on education, δ is the depreciation of human capital, $M_t h_t$ captures the diluting-effect of population growth on human capital as in Strulik(2005).

The household maximize utility subject to the following asset-accumulation equation:

$$\dot{a}_t = (r_t - n_t) a_t + (1 - \tau) \left[w_t h_t^Y + s w_t h_t^E \right] - c_t \tag{4}$$

where a_t is the real value of asset owned by the household, r_t is the real return on asset, and w_t is the wage per efficient unit of labor. τ is the exogenous

[2] If we only assume the households gain utility from bearing children, n_t should denote the birth rate, which equals to the sum of population growth rate and mortality rate. This does not change the results of the comparative statics and numerical analysis. However, if we assume the households gain utility from bearing children and loses utility from the death of loved ones, n_t should denote the population growth rate. To be consistent with the extant literature such as Chu et al (2013), n_t denotes population growth rate in the paper.

income tax rate and s is the exogenous education subsidy rate. By solving the optimization problem, I get the following equations:[3]

$$\frac{\dot{c}_t}{c_t} = r_t - n_t - \rho \tag{5}$$

$$\frac{\lambda}{n_t} = \frac{1}{c_t}\left[a_t + (\frac{1}{\gamma} + \frac{1-s}{\varepsilon})\,(1-\tau)\,w_t h_t\right] \tag{6}$$

$$r_t = \frac{\dot{w}_t}{w_t} - \delta + \frac{\varepsilon}{1-s}(1 - \frac{n_t}{\gamma}) \tag{7}$$

2.2 Final Goods

I follow Peretto (2007) to assume that final goods Y_t is produced by competitive firms using the following production function and the price of final goods is the numeraire (normalized to 1).

$$Y_t = \int_0^{M_t} X_t^{\theta}(i)\left[Z_t^{\alpha}(i) Z_t^{1-\alpha} H_t^Y / M_t\right]^{1-\theta} di, \quad 0 < \alpha, \theta < 1 \tag{8}$$

where H_t^Y is the aggregate human capital employed for producing final goods, with $H_t^Y = L_t h_t^Y$ and L_t is the population size in time t. M_t is the varieties of intermediate goods, which is denoted by $X_t(i)$ with $i \in [0, M_t]$ and are vertically differentiated in quality $Z_t(i)$. $Z_t \equiv \frac{1}{M_t}\int_0^{M_t} Z_t(i)di$ measures the average quality of all intermediate goods and it captures the R&D spillovers with degree of technology spillovers determined by $1-\alpha$. The quality term $Z_t^{\alpha}(i) Z_t^{1-\alpha}$ determines the productivity of labor using intermediate goods to produce final goods and works as the labor augmenting technology. From profit maximization of final goods firm, the equilibrium wage rate and conditional demand function are derived:

$$w_t = (1-\theta) Y_t / H_t^Y \tag{9}$$

$$X_t(i) = \left(\frac{\theta}{p_t(i)}\right)^{1/(1-\theta)} Z_t^{\alpha}(i) Z_t^{1-\alpha} H_t^Y / M_t \tag{10}$$

where $p_t(i)$ is the price of $X_t(i)$ and perfect competition implies that final goods producer makes zero profit and pay $\theta Y_t = \int_0^{M_t} p_t(i) X_t(i) di$, $(1-\theta)Y_t$ to intermediate goods firms and the labor for producing final goods respectively.

[3] See Appendix A.1 for proof

2.3 Intermediate Goods and In-House R&D

There is a continuum of industry leaders producing differentiated intermediate goods acting as a monopolist, with a technology where one unit of final goods is needed to produce one unit of intermediate goods, $X_t(i)$. The intermediate goods firms also spend $R_t(i)$ units of final goods in R&D improving the quality of the product. The innovation process is as follows:

$$\dot{Z}_t(i) = R_t(i) \tag{11}$$

The dividend flow $\Pi_t(i)$ at time t is

$$\Pi_t(i) = [p_t(i) - 1]\, X_t(i) - R_t(i) \tag{12}$$

The value of the monopolistic firm in industry i is:

$$V_t(i) = \int_t^{\infty} \exp\left(-\int_t^u r_v dv\right) \Pi_u(i) du \tag{13}$$

where the monopolistic firm maximizes (13) subject to (10) (11) and (12). By solve the optimization problem, I get the price of intermediate goods and the return on in-house R&D expenditure, r_t^q:[4]

$$p_t(i) = 1/\theta \equiv p_t \tag{14}$$

$$r_t^q = \frac{\alpha(1-\theta)}{\theta} \theta^{2/(1-\theta)} H_t^Y / M_t = \frac{\alpha(1-\theta)}{\theta} \frac{X_t}{Z_t} \tag{15}$$

where I follow the previous studies to consider a symmetric equilibrium in which $Z_t(i) = Z_t$ for $i \in [0; M_t]$ and the size of each intermediate goods firm is identical across all industries, $X_t(i) = X_t$. Substitute (14) into (10), and (16) into (8):

$$X_t = \theta^{2/(1-\theta)} Z_t H_t^Y / M_t \tag{16}$$

$$\theta^2 Y_t = M_t X_t \tag{17}$$

2.4 Entrants

Symmetric equilibrium also implies $\Pi_t(i) = \Pi_t$, and $V_t(i) = V_t$ for $i \in [0, M_t]$. A new firm invests βX_t units of final goods to develop a new variety of intermediate goods and establish its operation, where x_t captures the scale of

[4] See Appendix A.2 for proof

the initial operation, and $\beta > 0$ is a cost parameter. In equilibrium, the value of new varieties of intermediate goods should be equal to the research and development cost, so the free-entry condition is:

$$V_t = \beta X_t \tag{18}$$

The no-arbitrage condition equates the interest rate to the rate of return on V_t, which is the sum of monopolistic Π_t and capital gain $\dot{V}_t$:

$$r_t = \frac{\Pi_t + \dot{V}_t}{V_t} \tag{19}$$

Substitute (11), (12), (14) and (18) into (19), I get the rate of return on entry as:

$$r_t^e = \frac{1-\theta}{\beta\theta} - \beta\frac{\dot{Z}_t}{Z_t}\frac{Z_t}{X_t} + \frac{\dot{X}_t}{X_t} \tag{20}$$

2.5 Government

The government subsidizes education with an exogenous proportion $s \in (0,1)$ and collects income tax from the household with a fixed tax rate. The balanced-budget condition is:

$$\tau\{w_t[H_t(1-n_t/\gamma) - H_t^E] + sw_tH_t^E\} = (1-\tau)sw_tH_t^E + G_t$$

which can be simplified as follows:

$$\tau w_t[H_t(1-n_t/\gamma) - (1-2s)H_t^E] = sw_tH_t^E + G_t \tag{21}$$

where G_t is the unproductive government spending and change endogenously to balance the fiscal budget equation.

2.6 Decentralized Equilibrium

The equilibrium is a time path of allocations $\{A_t; C_t; Y_t; X_t; R_t; G_t; H_t^E; H_t^Y; H_t^M, n_t\}$ and a time path of prices $\{w_t; r_t; p_t; V_t\}$ that satisfy:

- households maximize utility taking $\{w_t; r_t\}$ as given;
- competitive final goods firms maximize profits taking $\{p_t, w_t\}$ as given;
- incumbents in the intermediate goods sector choose $\{p_t, R_t\}$ to maximize $\{V_t\}$ taking $\{r_t\}$ as given;

- entrants make entry decisions taking $\{V_t\}$ as given;
- the value of all existing monopolistic firms adds up to the value of the household's assets such that $A_t = M_t V_t$;
- the market-clearing condition for final goods holds: $Y_t = C_t + G_t + M_t(X_t + R_t) + \dot{M}_t \beta X_t$;
- the market-clearing condition for human capital holds: $H_t = H_t^E + H_t^Y$.

2.7 Balanced Growth Path

Proposition 1 *There exists a unique balanced growth path, along which each variable grows at a constant rate, and the share of human capital allocated to education and work is also constant, with the following equalities hold:*

$$g_Y = g_H + g_Z = g_M + g_X = g_C = g_A = g_G \tag{22}$$

$$g_H = g_M = (1 - \frac{n}{\gamma})\frac{\varepsilon}{1-s} - (\rho + \delta) \tag{23}$$

$$g_w = g_R = g_\Pi = g_V = g_X = g_Z = \frac{\alpha(1-\theta)}{\theta}\frac{X_t}{Z_t} - g_M - \rho \tag{24}$$

$$\frac{X_t}{Z_t} = \frac{\delta - (1 - n/\gamma)\varepsilon/(1-s)}{(1-\alpha)(1-\theta)/\theta + \beta[\delta - (1-n/\gamma)\varepsilon/(1-s)]} \tag{25}$$

$$s^E = \frac{1 - n/\gamma}{1-s} - \frac{\rho}{\varepsilon}; \quad s^Y = \frac{\rho}{\varepsilon} - \frac{s(1-n/\gamma)}{1-s} \tag{26}$$

$$\begin{aligned}\frac{s\beta\theta}{(1-s)}n^2 + \{[\frac{\rho}{\varepsilon} - \frac{s}{1-s} - \frac{\lambda\rho s}{(1-s)\theta}]\beta\theta^2 + (\frac{1}{\gamma} + \frac{1-s}{\varepsilon})(1-\tau)(1-\theta)\}n \\ -[(\frac{\lambda\rho^2}{\varepsilon} - \frac{\lambda\rho s}{1-s})\beta\theta^2 + \frac{\lambda(1-\tau)(1-\theta)(1-s)\rho}{\varepsilon}] = 0\end{aligned} \tag{27}$$

where g_B is the growth rate of variable B, and B denotes the variables such as $\{Y, M, X, H, Z, C, A, G, R, V, \Pi, \omega\}$.

Proof See Appendix A.3

Equation (22) shows that there are two drivers of economic growth: the accumulation of human capital and quality improvements. The growth rate of human capital equals the growth rate of variety innovations, and the growth rate of intermediate goods coincides with the growth rate of quality improvements [(23),(24)]. However, (24) indicates that variety innovations deter quality improvements. Intuitively, the expansion of varieties shrinks the market share of intermediate goods, which reduces the profits of intermediate goods firms [(12)] and thereby decreases the incentives for quality improvements. Equations (22), (23) and (24) together show that the long-run growth rate of the economy depends solely on the quality-adjusted market size of the intermediate goods $\frac{X_t}{Z_t}$.[5] This quality-adjusted market size decreases with the education subsidy rate s but increases with the population growth rate n.[6]

3 The Effects of Education Subsidy

In this section, I analyze the effects of education subsidies on the economy. To study these effects, I differentiate the growth rates of the economy with respect to the education subsidy rate s and conduct a comparative statics analysis. Proposition 2 summarizes the main results:

Proposition 2 *The effects of education subsidy on the economy is ambiguous and closely depends on the endogenous population growth rate. If population growth rate is constant,*[7] *the subsidy fosters the variety innovations, however, it impedes the quality improvements and deters the long-run economic growth; If population growth rate is decreasing in the education subsidy, it enhances the effects of education subsidy on the long-run aggregate economy; If population growth rate is increasing in the education subsidy, it reduces the effects of education subsidy on the long-run aggregate economy.*

[5] Peretto and Connolly (2007) justifies that the long-run economic growth depends only on the quality improvements. In the context of my paper, $\frac{X_t}{Z_t}$ represents the positive component of the growth rate of quality improvements, on which long-run economic growth solely depends.

[6] Proof is included in Appendix A.4.

[7] This is is analogous to treating the population growth rate as exogenous, I use the word "constant" instead of "exogenous" as the population growth rate is endogenized in this model. However, It is also feasible to set the population growth rate exogenous to achieve a more concise but less general result, which is just a special case of this model when it is assumed that population growth rate n is constant and the derivative with respect to n is zero.

Proof See Appendix A.4

The existing literature identifies two types of education subsidies: one for primary and secondary schooling, and the other for tertiary education [e.g., Hashimoto and Tabata (2016)]. Subsidies for primary and secondary schooling represent parental investments in children's education. Since parents typically invest considerable money and time in their children's basic education, subsidies in this area reduce the financial burden on parents, potentially increasing the population growth rate. Consequently, the net effects of these subsidies on the economy are more likely to be positive in the long run. Conversely, tertiary education is usually self-supported. Individuals pursuing higher education incur additional opportunity costs by investing more time and money, leaving them with less time to raise children. Therefore, subsidies for tertiary education, which encourage higher education, tend to lower the population growth rate and negatively impact the long-run economy. These predictions regarding the different types of education subsidies align with empirical studies. In this paper's setting, the costs attributed to primary, secondary, and tertiary education, as well as the corresponding subsidies, are not distinguished, making the effect on population growth rate ambiguous.

If the population growth rate remains constant despite education subsidies, increased subsidies reduce education costs and stimulate educational attainment. This leads to a higher share of human capital allocated to education and a corresponding decrease in the share devoted to work [(26)]. With a constant population growth rate and an increased share of human capital directed towards education, human capital accumulation accelerates, resulting in a higher growth rate of human capital [(3)]. Although the share of human capital devoted to work decreases, the overall increase in aggregate human capital eventually boosts the human capital allocated to the production of final goods [(A.3.1)], causing a decline in wages relative to final goods [(9), which could also be interpreted as the increase in the production of final goods, with the price of final goods declining, though it is normalized to 1 in the setting]. Consequently, the costs for both in-house R&D and new entrants decrease, known as the cost reduction effect, which promotes quality improvements and variety innovations in the short run. However, the expansion of variety results in the entry effect, which reduces the average market share for each intermediate goods firm, inhibiting quality improvements. The negative entry effect outweighs the positive cost reduction effect, thereby lowering the growth rate of quality improvements in the long run. As previously mentioned, the quality-adjusted market size of intermediate goods $\frac{X_t}{Z_t}$ However, the expansion of variety results in the entry effect, which reduces the average market share for each intermediate goods firm, inhibiting quality improvements. The negative entry effect outweighs the positive cost reduction effect, thereby lowering the growth rate of quality improvements in the long run. As previously mentioned, the quality-adjusted market size of intermediate goods s but increases with the population growth rate n [(25)]. Since the population growth rate is constant, the education subsidy negatively affects the quality-adjusted market size of intermediate goods, leading to a long-term decline in the economy's growth rate. [(22),(24),(25)]. If the population growth rate decreases with an education subsidy, it further accelerates human capital accumulation and reduces the quality-adjusted market size of intermediate goods $\frac{X_t}{Z_t}$, so the decreasing population growth rate enhances the effects of education subsidy on the aggregate economy. enhancing the effects of the education subsidy on the aggregate economy. However, the effects of the education subsidy on per capita GDP are ambiguous, depending on the relative strength of the decreasing population growth rate and the increasing total GDP. if the population growth rate increases with the education subsidy, the rising population growth rate would impede or even reverse human capital accumulation [(2) and (3)] and increases the quality-adjusted market size of the intermediate goods $\frac{X_t}{Z_t}$, making the ultimate impact of the mechanism ambiguous. The net effects of education subsidies depend on the economy's parameters and the actual values of the population growth rate and subsidy rate.

4 Numerical Analysis

In this section, to deepen our understanding of the impact of education subsidies on the economy, I conduct numerical simulations on the model. I calibrate the parameters of the model to match several target values of the US economy. Next, I evaluate the effects of education subsidy on the economy. Nonetheless, this paper does not simulate the real economy but only aims to complement the qualitative results of the theoretical model. Despite the careful choice of parameter values, caution is needed in interpreting the quantitative results derived in this paper.

4.1 The Model Parameterization

The model involves the following relevant parameters $\{s, \lambda, \rho, \gamma, \varepsilon, \delta, \theta, \alpha, \beta, \tau, \}$. Table 1 lists these preset parameters, which are based on available data or the results of previous numerical studies. In order to estimate the effects of education subsidy, I approximate the proportion of public education expenditure to total education expenditure as the education subsidies rate. According to the OECD, 92% of primary and secondary education is financed by publicly funded, while for tertiary education, the proportion drops to 36%. Overall, the proportion of public education expenditure to total education expenditure averages 69.6%, which I set as the initial education subsidy rate s.[8] For the time preference discount rate, I follow Acemoglu and Akcigit (2012) to set it to a standard value of 0.05. I then follow Chu et al.(2013) to set the depreciation rate of human capital $\delta = 0.055$ for the US economy. I also set the degree of technology spillovers $1 - \alpha = 0.833$ by refering to Iacopetta et al. (2019), thus $\alpha = 0.167$. According to the OECD, the average tax rate on wage income in the US from year 2001-2022 is 25.6%.[9] The last preset parameter is λ, which I set to 0.1 to ensur a smooth graphical representation in the subsequent subsections. It is possible to conduct a sensitive analysis by varying the value of λ, but it does not affect the trends of variables concerned and the corresponding analysis.

Table 1: Preset Parameters

Parameter	Description	Value
s	Education subsidy rate	69.6%
ρ	Time preference discount rate	0.05
δ	Depreciation rate of human capital	0.055
α	1-α measures the degree of technology spillover	0.167
τ	Average income tax rate	25.6%
λ	Fertility-preference parameter	0.1

Table 2: Targeted Values

Targeted variables	Description	Value
$w_t H_Y / Y_t$	Labor income share	60%
n	Population growth rate	1%
g_Y	Growth rate of the economy	3%
g_H	Growth rate of human capital	0.54%

Table 2 lists the five target variables. Based on the "Penn World Table", the labor income share denoted by $\frac{w_t H_t}{Y_t}$ is approximately 60%, the annual

[8]Data is showed in the website: https://data.oecd.org/eduresource/private-spending-on-education.htm#indicator-chart

[9]Data is showed in the website: https://data.oecd.org/tax/tax-revenue.htm

Table 3: Calibration of Baseline Parameters

Parameter	Description	Value
θ	Elasticity of intermediate goods to final goods	0.4
γ	Inversely measures the time-cost of child-rearing	0.785
ε	Productivity of human capital accumulation	0.034
β	Cost parameter of entrants	25.685

population growth rate in the US is $n = 1\%$, and the long-run annual growth rate of the US economy, stands around $g_Y = 3\%$. Using the human capital index from "PWT," the growth rate of human capital, g_H, is estimated at 0.54%.[10]

With these preset parameters and target variables, I calibrate the remaining four parameters of the baseline model as showed in Table 3. I first identify the elasticity of intermediate goods to final goods by setting the target value of labor income share $\frac{w_t H_t}{Y_t}$ in (9), resulting in $\theta = 60\%$. Then I determine the parameters $\{\gamma, \varepsilon, \beta\}$ by imposing the target value of n, g_Y and g_H on (22), (23), (24),(25),and (27). These calculations yields $\gamma = 0.785, \varepsilon = 0.034, \beta = 25.685$.

4.2 Simulation of the long-run Effects

The calibrated model above provides a framework to assess the impact of education subsidy on the economy in balanced growth path. In this subsection, I conduct the sensitive analysis of the education subsidy rate s, varying s from 0 to 1,to examine the responses of key variables. Figure 3 presents the results of this analysis, illustrating how various variables change in response to different education subsidy rates. These variables include the population growth rate n, share of human capital to education s^E, share of human capital to work s^Y, quality-adjusted market size of the intermediate goods $\frac{X_t}{Z_t}$, growth rate of human capital g_H, growth rate of quality improvements g_Z, growth rate of GDP g_Y and growth rate of GDP per capita g_y. It is important to emphasize that this simulation focuses exclusively on long-run effects, specifically how these dependent variables evolve in the balanced growth path across varying education subsidy rates.

As depicted in Figure 3,[11] the population growth rate exhibits varying

[10]PWT version 10.01 is a database provided by "Groningen Growth and Development Centre" at the University of Groningen, with information on relative levels of income, output, input and productivity, covering 183 countries between 1950 and 2019. It is published in the website: https://www.rug.nl/ggdc/productivity/pwt/?lang=en

[11]The vertical axis values in Figure 3 are omitted due to the simulation results not falling

trends in response to changes in the education subsidy rate. At low subsidy rates, the population growth rate decreases. As the subsidy rate surpasses a minimum threshold, the population growth rate begins to increase sharply. Once the subsidy rate reaches a sufficiently high level, the population growth rate stabilizes, indicating an exogenous or independent relationship with the education subsidy rate. Correspondingly, other variables adjust according to these trends in the population growth rate with respect to the education subsidy rate. In the subsequent paragraphs, I analyze the effects of education subsidies under three scenarios based on their impact on the population growth rate discussed above.

When the education subsidy rate is quite low, Figure 3-1 shows the population growth rate slumps as the education subsidy rate rises, accompanied by a dramatic increase in the share of human capital allocated to education, as shown in Figure 3-2. Extant literature illustrates that parents face a trade-off between children's quality and quantity and typically prioritize higher quality through enhanced education in contemporary society when the expenditure on children increases dramatically[e.g., Becker (1960), Hanushek (1992)]. In this scenario, a low education subsidy rate does not sufficiently mitigate the cost of education for children, thereby incentivizing higher educational attainment and suppressing fertility rates, resulting in a higher share of human capital allocated to education. Moreover, Figure 3-3 indicates a decrease in the share of human capital allocated to work, displaced by the increased share allocated to education. Figure 3-5 demonstrates a rapid increase in the growth rate of human capital, accompanied by a corresponding sharp decline in the growth rate of quality improvements, as shown in Figure 3-6. The value of quality adjusted market size of the intermediate goods also declines causing the growth rate of GDP slowing down, as presented in Figure 3-4 and Figure 3-7 respectively. Nonetheless, the slump of population growth rateleads to a rapid increase in per capita GDP growth, as depicted in Figure 3-8.

When the education subsidy rate is relatively high, Figure 3-1 illustrates a rapid increase in the population growth rate with rising education subsidy rates, and the share of human capital to education tumbles as presented in Figure 3-2. In this case, the relative high education subsidy rate reduces

within reasonable limits. This discrepancy may stem from inadequately set parameters or target values, or more likely because some of the parameters are directly or indirectly influenced by the education subsidy rate or the population growth rate, necessitating their endogenization for a more comprehensive and realistic simulation outcome. However, this does not detract from the analysis in this section, which aims to illustrate the direct and indirect impacts of education subsidies on the economy with only the endogenous population growth rate.

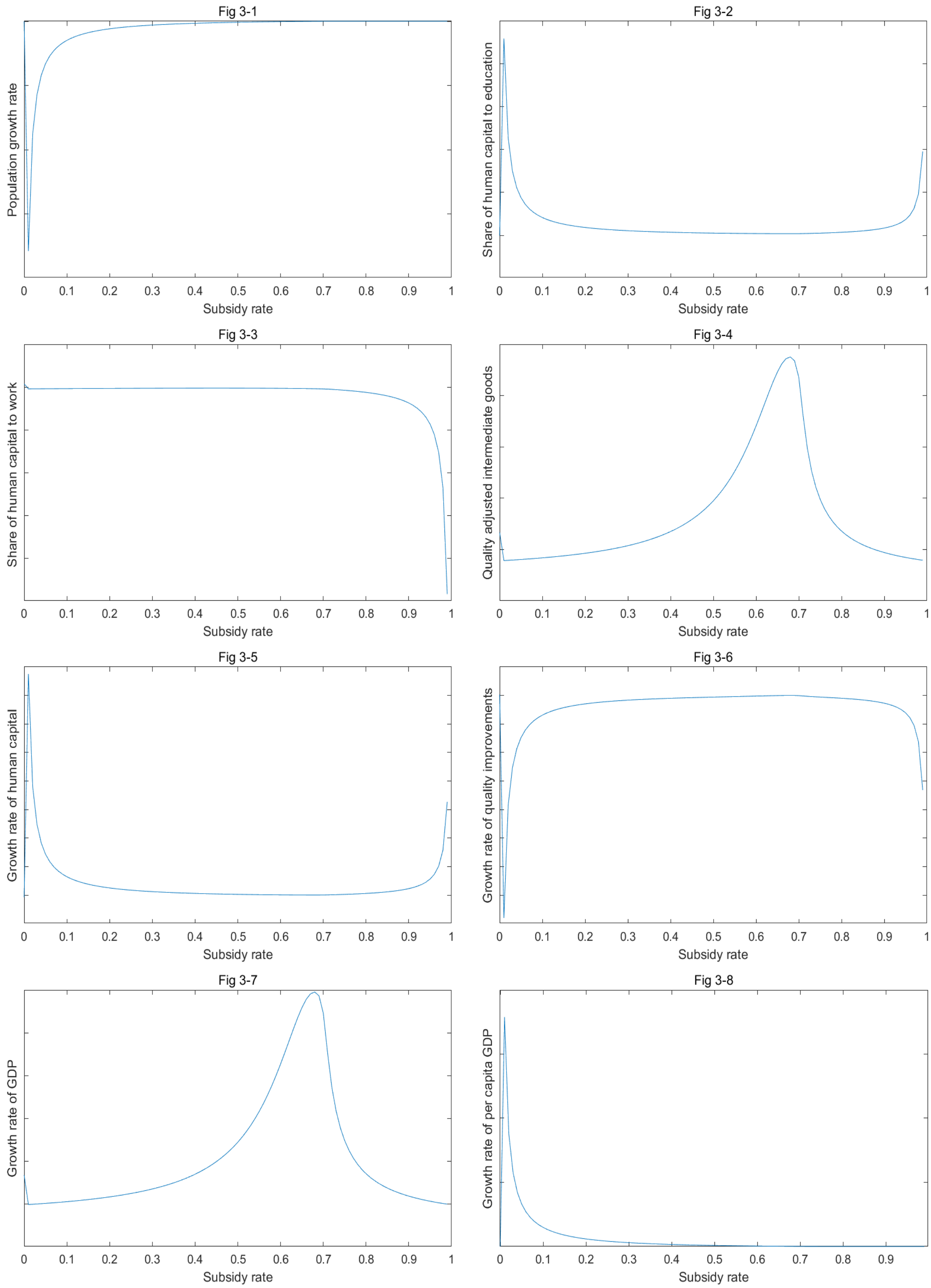

Figure 3: The Effects of Education Subsidy

the children's education cost significantly so that it stimulates an increase in fertility rates and potentially reduces investment in children's education, thereby lowering the share of human capital allocated to education. Figure 3-3 indicates that the share of human capital allocated to work remains nearly constant or decreases slightly. Figure 3-5 depicts a rapid decline in the growth rate of human capital, followed by a dramatic increase in the growth rate of quality improvements, as shown in Figure 3-6. The quality-adjusted market size of intermediate goods consistently increases, contributing significantly to the growth rate of GDP, as depicted in Figures 3-4 and 3-7, respectively. However, the rapid increase in the population growth rate leads to a significant decrease in the growth rate of per capita GDP, as illustrated in Figure 3-8.

When the education subsidy rate is very high, Figure 3-1 illustrates that the population growth rate increases slowly or remains constant as the education subsidy rate rises. Meanwhile, Figure 3-2 shows a consistent increase in the share of human capital allocated to education. Intuitively, when the education subsidy rate reaches a high level, both the cost of education and the share of human capital allocated to education decline to very low levels, and the fertility rate stabilizes near its upper limit. Consequently, further increases in the education subsidy do not significantly promote population growth. Figure 3-3 demonstrates a dramatic drop in the share of human capital allocated to work, a result of the significantly increased allocation to education. Figure 3-5 shows that the growth rate of human capital reverses its decline and increases rapidly, while Figure 3-6 indicates a corresponding decrease in the growth rate of quality improvements. The quality-adjusted market size of intermediate goods also experiences a significant decline, resulting in a sharp slowdown in the GDP growth rate, as illustrated in Figure 3-4 and Figure 3-7 respectively. Moreover, with a constant population growth rate, the slowdown of the overall economy is accompanied with a decrease in the growth rate of per capita GDP, as depicted in Figure 3-8.

In summary, when the population growth rate remains constant across varying education subsidy rates ($s > 0.74$ in Figure 3-1), the subsidy positively impacts human capital accumulation but negatively affects quality improvements, the quality-adjusted market size of intermediate goods, and GDP.Similarly, when the population growth rate decreases with the education subsidy rate ($0 \leq s \leq 0.1$ in Figure 3-1), these effects persist, albeit with steeper curves indicating amplified impacts as showed in Figure 3. Thus, decreasing population growth enhances the subsidy's effects. Conversely, when the population growth rate increases with the education subsidy rate, the effects reverse when $0.1 < s \leq 0.68$ as showed in Figure 3-4 to Figure 3-7. For $0.68 < s \leq 0.74$, although the population growth rate continues to rise, the increase is insufficient to reverse the subsidy's effects; instead, it merely mitigates them. These simulation results align with Proposition 2, thereby bolstering the reliability of the paper's conclusions.

The findings of this analysis have significant policy implications. Firstly, they suggest that the effects of education subsidies may contradict the typical policy objective aimed at promoting long-term economic growth through innovation and human capital accumulation. For example, policies advocating free education could inadvertently hinder long-term economic growth while not significantly boosting population growth compared to policies offering moderate educational subsidies. Secondly, governments should carefully calibrate education subsidy rates in line with specific policy goals. For instance, if the objective is to enhance long-term per capita GDP growth, a very low or no subsidy combined with higher education costs may be preferable. Conversely, if the goal is to stimulate overall GDP growth or address demographic challenges like population aging, a moderately higher subsidy rate could effectively increase population growth and counteract the negative economic impacts of subsidies. Nevertheless, decisions regarding education subsidies should be made within the broader socio-economic context to ensure comprehensive policy effectiveness.

5 Conclusion

This study employs a Schumpeterian growth model incorporating both variety innovations and quality improvements to investigate the impact of education subsidies on the economy, considering endogenous population growth and human capital accumulation. The model identifies two primary drivers of long-run economic growth: human capital accumulation and quality improvements. I demonstrate that the effects of education subsidies on the economy are actually ambiguous and heavily contingent on the endogenous population growth rate. When the population growth rate remains constant (akin to being exogenous) relative to the education subsidy, the subsidy paradoxically hinders long-term economic growth. It promotes variety innovations and increases the number of firms but diminishes each firm's market size, thereby impeding the growth rate of quality improvements and reducing the quality-adjusted market share of intermediate goods crucial for long-run economic growth. If the population growth rate decreases with the education subsidy rate, it enhances the effects of the education subsidy on the economy, whereas if the population growth rate increases in the education subsidy, it tends to reduce or even reverse the negative effects of education subsidy. Moreover, through model calibration and numerical analysis, I substantiate the findings from comparative statics analysis, reinforcing the reliability of the conclusions. This research introduces a novel approach by incorporating the human capital accumulation and endogenizing the population growth rate in the Schumpetarian growth framework with both variety innovations and quality improvements, which enriches existing literature lacking such integration. Furthermore, the study contributes to understanding the implications of education subsidies, suggesting potential policy reconsiderations given the observed divergence from policy objectives. However, it is acknowledged that the study has limitations; other parameters directly or indirectly influenced by education subsidies or population growth rates may warrant endogenous treatment, providing avenues for future research.

6 References

Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. Econometrica, 60, 323–351.

Becker, G. S. (1960). An economic analysis of fertility. In Demographic and economic change in developed countries (pp. 209-240). Columbia University Press.

Bucci, A., & Raurich, X. (2017). Population and economic growth under different growth engines. German Economic Review, 18(2), 182-211.

Chen, H. J. (2015). Child allowances, educational subsidies and occupational choice. Journal of Macroeconomics, 44, 327-342.

Chu, A. C., & Wang, X. (2022). Effects of R&D subsidies in a hybrid model of endogenous growth and semi-endogenous growth. Macroeconomic Dynamics, 26(3), 813-832.

Chu, A. C., Cozzi, G., & Liao, C. H. (2013). Endogenous fertility and human capital in a Schumpeterian growth model. Journal of Population Economics, 26, 181-202.

Chu, A. C., Furukawa, Y., & Ji, L. (2016). Patents, R&D subsidies, and endogenous market structure in a Schumpeterian economy. Southern Economic Journal, 82(3), 809-825.

Del Rey, E., & Lopez-Garcia, M. A. (2016). Endogenous growth and welfare effects of education subsidies and intergenerational transfers. Economic Modelling, 52, 531-539.

Grossman, G. M., & Helpman, E. (1991). Quality ladders in the theory of growth. The review of economic studies, 58(1), 43-61.

Hanushek, E. A. (1992). The trade-off between child quantity and quality. Journal of political economy, 100(1), 84-117.

Hanushek, E. A., & Woessmann, L. (2011). How much do educational outcomes matter in OECD countries?. Economic policy, 26(67), 427-491.

Hashimoto, K. I., & Tabata, K. (2016). Demographic change, human capital accumulation and R&D-based growth. Canadian Journal of Economics/Revue canadienne d'économique, 49(2), 707-737.

Holmes, C. (2013). Has the expansion of higher education led to greater economic growth?. National Institute Economic Review, 224, R29-R47.

Howitt, P. (1999). Steady endogenous growth with population and R. & D. inputs growing. Journal of Political Economy, 107(4), 715-730.

Lloyd-Ellis, H., & Roberts, J. (2002). Twin engines of growth: Skills and technology as equal partners in balanced growth. Journal of Economic Growth, 7, 87-115.

Mattalia, C. (2012). Human capital accumulation in R&D-based growth models. Economic Modelling, 29(3), 601-609.

Morimoto, T., & Tabata, K. (2020). Higher education subsidy policy and R&D-based growth. Macroeconomic Dynamics, 24(8), 2129-2168.

Peretto, P. F. (1998). Technological change and population growth. Journal of Economic Growth, 3(4), 283-311.

Peretto, P. F. (2007). Corporate taxes, growth and welfare in a Schumpeterian economy. Journal of Economic Theory, 137(1), 353-382.

Peretto, P. F. (2015). From Smith to Schumpeter: A theory of take-off and convergence to sustained growth. European Economic Review, 78, 1-26.

Peretto, P. F.,& Connolly, M. (2007). The manhattan metaphor. Journal of Economic Growth, 12, 329-350.

Romer, P. M. (1990). Endogenous technological change. Journal of political Economy, 98(5, Part 2), S71-S102.

Segerstrom, P. S. (1998). Endogenous growth without scale effects. American economic review, 1290-1310.

Solow, R. M. (1956). A contribution to the theory of economic growth. The quarterly journal of economics, 70(1), 65-94.

Strulik, H. (2005). The role of human capital and population growth in R&D-based models of economic growth. Review of international Economics, 13(1), 129-145.

A APPENDIX

A.1 Households Maximization

The current-value Hamiltonian for the representative households is:

$$H_t^c = \ln c_t + \alpha \ln n_t + \eta_t\{(r_t - n_t)a_t + (1-\tau)w_t[h_t(1 - \frac{n_t}{\gamma}) - (1-s)h_t^E] - c_t\} + \lambda_t[\varepsilon h_t^E - (n_t + \delta)h_t] \tag{A.1.1}$$

where η_t, λ_t are the multipliers on the constrains. The first-order conditions include:

$$\frac{\partial H_t^c}{\partial c_t} = 0 \Leftrightarrow \frac{1}{c_t} = \eta_t \tag{A.1.2}$$

$$\frac{\partial H_t^c}{\partial a_t} = -(\dot{\eta}_t - \rho\eta_t) \Leftrightarrow \dot{\eta}_t + (r_t - n_t - \rho)\eta_t = 0 \tag{A.1.3}$$

$$\frac{\partial H_t^c}{\partial n_t} = 0 \Leftrightarrow \frac{\alpha}{n_t} = \frac{1}{c_t}[a_t + \frac{(1-\tau)w_t h_t}{\gamma}] + \lambda_t h_t \tag{A.1.4}$$

$$\frac{\partial H_t^c}{\partial e_t} = 0 \Leftrightarrow \lambda_t = \frac{(1-\tau)(1-s)w_t}{\varepsilon c_t} \tag{A.1.5}$$

$$\frac{\partial H_t^c}{\partial h_t} = 0 \Leftrightarrow \dot{\lambda}_t - (\rho + n_t + \delta)\lambda_t = \frac{(1-\tau)w_t}{c_t}(1 - \frac{n_t}{\gamma}) \tag{A.1.6}$$

Combine (A.1.2) and (A.1.3), it yields (5); Combine (A.1.4) and (A.1.5), it yields (6); Combine (A.1.5), (A.1.6) and (5), it yields (7).

A.2 Intermediate Goods Firm Maximization

The current-value Hamiltonian for monopolistic intermediate goods firms i is:

$$H_t^c = \Pi_t(i) + \lambda_t(i)\dot{Z}_t(i) = [p_t(i) - 1]\, X_t(i) - R_t(i) + \mu_t(i)R_t(i) \tag{A.2.1}$$

where $\mu_t(i)$ is the multiplier on $\dot{Z}_t(i) = R_t(i)$. The first-order conditions include:

$$\frac{\partial H_t}{\partial p_t(i)} = 0 \Leftrightarrow p_t(i) = \frac{1}{\theta} \tag{A.2.2}$$

$$\frac{\partial H_t}{\partial R_t(i)} = 0 \Leftrightarrow \mu_t = 1 \tag{A.2.3}$$

$$\frac{\partial H_t}{\partial Z_t(i)} = -(\dot{\mu}_t - r_t^q \mu_t) \Leftrightarrow r_t^q = \frac{(1-\theta)\theta^{2/(1-\theta)}\alpha}{\theta} Z_t(i)^{\alpha-1} Z_t^{1-\alpha} H_t^Y / M_t \tag{A.2.4}$$

Imposing symmetry on (A.2.4), $Z_t(i) = Z_t$, it yields (15).

A.3 Proposition 1-BGP

To demonstrate the existence of balanced growth path(BGP), I first assume there is BGP (i.e. all of the variables grow at a constant rate), then I derive the growth rate of the variables and prove they are constant(no contradiction).

Proof: assume there is BGP, then all of the variables grow at a constant growth rate. In the following proof, I use g_B to denote the growth rate of the variable B. According to (2), I get:

$$g_h = g_{h^Y} = g_{h^E} = g_h \tag{A.3.1}$$

$$g_n = 0; n_t = n \tag{A.3.2}$$

hence, the share of human capital allocated to work($s_t^Y = h_t^Y/h_t = s^Y$) and education ($s^E = h_t^E/h_t = s^E$) are also constant. According to (6), it can be derived:

$$g_c = g_a = g_w + g_h \tag{A.3.3}$$

Substitute (5),(A.3.2) and (A.3.3) into (7), I get:

$$g_h = (1 - \frac{n}{\gamma})\frac{\varepsilon}{1-s} - (n + \rho + \delta) \tag{A.3.4}$$

as we assume the growth rate of consumption g_c is constant, according to (5) and (A.3.2), I get $r_t = r$ is also constant. According to (15):

$$g_H = g_M; g_X = g_Z \tag{A.3.5}$$

According to (18),(19), and (11), I get:

$$g_X = g_V = g_\Pi = g_R = g_Z \tag{A.3.6}$$

According to (17) and (A.3.5), it can be derived that:

$$g_Y = g_M + g_X = g_H + g_Z \tag{A.3.7}$$

According to (A.3.3) and the market-clearing condition for final goods:

$$g_Y = g_A = g_C = g_G \tag{A.3.8}$$

According to (A.3.3) (A.3.7) and (A.3.8), I get:

$$g_w = g_Z \tag{A.3.9}$$

Substitute (5) into (20), I get:

$$g_M = \frac{1-\theta}{\theta\beta} - \frac{g_Z}{\beta}\frac{Z_t}{X_t} - \rho \tag{A.3.10}$$

Substitute (5) into (15), I get:

$$g_Z = \frac{\alpha(1-\theta)}{\theta}\frac{X_t}{Z_t} - g_M - \rho \tag{A.3.11}$$

Substitute (A.3.11) into (A.3.10), I get:

$$g_M = \frac{[(1-\alpha)(1-\theta)/\theta - \rho\beta]\frac{X_t}{Z_t} + \rho}{\beta\frac{X_t}{Z_t} - 1} \tag{A.3.12}$$

Substitute (A.3.4) (A.3.5) and $g_H = g_h + n$ into (A.3.12), I get:

$$\frac{X_t}{Z_t} = \frac{\delta - (1-n/\gamma)\varepsilon/(1-s)}{(1-\alpha)(1-\theta)/\theta + \beta[\delta - (1-n/\gamma)\varepsilon/(1-s)]} \tag{A.3.13}$$

As long as n is constant, the ratio $\frac{X_t}{Z_t}$ is constant, thus g_Z and g_M are constant, then according to (A.3.5) to (A.3.8), the growth rate of all of the variables are constant and hence there is no contradiction, so the BGP exists. Now I derive the expression of n and prove it is constant.
According to the market clearing condition $A_t = M_t V_t$, (17), (18) and $y_t = \frac{Y_t}{L_t}$, $a_t = \frac{A_t}{L_t}$, I get:

$$a_t = \beta\theta^2 y_t \tag{A.3.14}$$

According to (9) and $y_t = \frac{Y_t}{L_t}$, $H_t^Y = \frac{s^Y H_t}{L_t}$, I get:

$$w_t h_t = \frac{(1-\theta)y_t}{s^Y} \tag{A.3.15}$$

Substitute (A.3.14) and (A.3.15) into (6), I get:

$$\frac{\lambda}{n} = [\beta\theta^2 + (\frac{1}{\gamma} + \frac{1-s}{\varepsilon})\frac{(1-\tau)(1-\theta)}{s^Y}]\frac{y_t}{c_t} \tag{A.3.16}$$

According to (2), (3) and (A.3.4), it can be derived that:

$$s^E = \frac{1-n/\gamma}{1-s} - \frac{\rho}{\varepsilon} \tag{A.3.17}$$

$$s^Y = \frac{\rho}{\varepsilon} - \frac{s(1-n/\gamma)}{1-s} \tag{A.3.18}$$

According to (4) and (A.3.3), it can be derived that:

$$\frac{c_t}{y_t} = \beta\theta^2\rho + \frac{(1-\tau)(1-\theta)[s^Y + s s^E]}{s^Y} \tag{A.3.19}$$

Substitute (A.3.17), (A.3.18) and (A.3.19) into (A.3.16), I get:

$$\begin{aligned} &\frac{s\beta\theta}{(1-s)}n^2 + \{[\frac{\rho}{\varepsilon} - \frac{s}{1-s} - \frac{\lambda\rho s}{(1-s)\theta}]\beta\theta^2 + (\frac{1}{\gamma} + \frac{1-s}{\varepsilon})(1-\tau)(1-\theta)\}n \\ &\quad -[(\frac{\lambda\rho^2}{\varepsilon} - \frac{\lambda\rho s}{1-s})\beta\theta^2 + \frac{\lambda(1-\tau)(1-\theta)(1-s)\rho}{\varepsilon}] = 0 \end{aligned} \tag{A.3.20}$$

which is a quadratic equation of n, so n is constant and there exists the BGP, Q.E.D.

A.4 Effects of Education Subsidy

According to (22)-(27), all of the growth rates of variables are functions of the population growth rate n and the education subsidy rate s, and n is also a function of s, so it is necessary to attain the derivative of n on s. According to (27), it can be derived that:

$$\frac{\mathrm{d}n}{\mathrm{d}s} = \frac{\{[\frac{1}{(1-s)^2} + \frac{\lambda\rho}{(1-s)^2\theta}]\beta\theta^2 + \frac{(1-\tau)(1-\theta)}{\varepsilon}\}n - [\frac{\lambda\rho\beta\theta^2}{(1-s)^2} + \frac{\lambda(1-\tau)(1-\theta)\rho}{\varepsilon}] - \frac{\beta\theta n^2}{(1-s)^2}}{\frac{2s\beta\theta}{1-s}n + [\frac{\rho}{\varepsilon} - \frac{s}{1-s} - \frac{\lambda\rho s}{(1-s)\theta}]\beta\theta^2 + (\frac{1}{\gamma} + \frac{1-s}{\varepsilon})(1-\tau)(1-\theta)} \tag{A.4.1}$$

which could be positive or negative. It depends on the parameters and the value of population growth rate n and the education subsidy rate s. According to (23), it can be derived that:

$$\frac{\mathrm{d}g_M}{\mathrm{d}s} = \frac{\partial g_M}{\partial s} + \frac{\partial g_M}{\partial n}\frac{\mathrm{d}n}{\mathrm{d}s} \tag{A.4.2}$$

$$\frac{\partial g_M}{\partial s} = \frac{(1-n/\gamma)\varepsilon}{(1-s)^2} > 0 \tag{A.4.3}$$

$$\frac{\partial g_M}{\partial n} = -\frac{\varepsilon}{\theta(1-s)} < 0 \tag{A.4.4}$$

however, since $\frac{\mathrm{d}n}{\mathrm{d}s}$ is ambiguous, $\frac{\mathrm{d}g_M}{\mathrm{d}s}$ is also ambiguous. According to (25), it can be derived that:

$$\frac{\mathrm{d}\frac{X_t}{Z_t}}{\mathrm{d}s} = \frac{\partial\frac{X_t}{Z_t}}{\partial s} + \frac{\partial\frac{X_t}{Z_t}}{\partial n}\frac{\mathrm{d}n}{\mathrm{d}s} \tag{A.4.5}$$

$$\frac{\partial\frac{X_t}{Z_t}}{\partial s} = -\frac{[(1-n/\gamma)\varepsilon/(1-s)^2](1-\alpha)(1-\theta)/\theta}{\{(1-\alpha)(1-\theta)/\theta + \beta[\delta - (1-n/\gamma)\varepsilon/(1-s)]\}^2} < 0 \tag{A.4.6}$$

$$\frac{\partial\frac{X_t}{Z_t}}{\partial n} = \frac{[\varepsilon/(\theta(1-s))][(1-\alpha)(1-\theta)/\theta]}{\{(1-\alpha)(1-\theta)/\theta + \beta[\delta - (1-n/\gamma)\varepsilon/(1-s)]\}^2} > 0 \tag{A.4.7}$$

however, as $\frac{\mathrm{d}n}{\mathrm{d}s}$ is ambiguous, $\frac{\mathrm{d}\frac{X_t}{Z_t}}{\mathrm{d}s}$ is also ambiguous. According to (24), it can be derived that:

$$\frac{\mathrm{d}g_Z}{\mathrm{d}s} = \frac{\partial g_Z}{\partial s} + \frac{\partial g_Z}{\partial n}\frac{\mathrm{d}n}{\mathrm{d}s} \tag{A.4.8}$$

$$\frac{\partial g_Z}{\partial s} = \frac{\partial g_Z}{\partial \frac{X_t}{Z_t}} \frac{\partial \frac{X_t}{Z_t}}{\partial s} + \frac{\partial g_Z}{\partial g_M} \frac{\partial g_M}{\partial s} \tag{A.4.9}$$

$$\frac{\partial g_Z}{\partial n} = \frac{\partial g_Z}{\partial \frac{X_t}{Z_t}} \frac{\partial \frac{X_t}{Z_t}}{\partial n} + \frac{\partial g_Z}{\partial g_M} \frac{\partial g_M}{\partial n} \tag{A.4.10}$$

$$\frac{\partial g_Z}{\partial \frac{X_t}{Z_t}} = \frac{\alpha(1-\theta)}{\theta} > 0 \tag{A.4.11}$$

$$\frac{\partial g_Z}{\partial g_M} = -1 < 0 \tag{A.4.12}$$

since $\frac{\partial \frac{X_t}{Z_t}}{\partial s} < 0$ and $\frac{\partial g_M}{\partial s} > 0$, so $\frac{\partial g_Z}{\partial s} < 0$; $\frac{\partial \frac{X_t}{Z_t}}{\partial n} > 0$ and $\frac{\partial g_M}{\partial n} < 0$, so $\frac{\partial g_Z}{\partial n} > 0$; however, since $\frac{\mathrm{d}n}{\mathrm{d}s}$ is ambiguous, $\frac{\mathrm{d}g_Z}{\mathrm{d}s}$ is also ambiguous. According to (22),(24), it can be derived that:

$$\frac{\mathrm{d}g_Y}{\mathrm{d}s} = \frac{\partial g_Y}{\partial s} + \frac{\partial g_Y}{\partial n} \frac{\mathrm{d}n}{\mathrm{d}s} \tag{A.4.13}$$

$$\frac{\partial g_Y}{\partial s} = \frac{\alpha(1-\theta)}{\theta} \frac{\partial \frac{X_t}{Z_t}}{\partial s} < 0 \tag{A.4.14}$$

$$\frac{\partial g_Y}{\partial n} = \frac{\alpha(1-\theta)}{\theta} \frac{\partial \frac{X_t}{Z_t}}{\partial n} > 0 \tag{A.4.15}$$

however, since $\frac{\partial \frac{X_t}{Z_t}}{\partial n}$ is ambiguous, $\frac{\partial g_Y}{\partial n}$ is still ambiguous.

If assuming n is constant, then $\frac{\mathrm{d}n}{\mathrm{d}s} = 0$, thus $\frac{\mathrm{d}g_M}{\mathrm{d}s} = \frac{\partial g_M}{\partial s} > 0$; $\frac{\mathrm{d}\frac{X_t}{Z_t}}{\mathrm{d}s} = \frac{\partial \frac{X_t}{Z_t}}{\partial s} < 0$; $\frac{\mathrm{d}g_Z}{\mathrm{d}s} = \frac{\partial g_Z}{\partial s} < 0$; $\frac{\mathrm{d}g_Y}{\mathrm{d}s} = \frac{\partial g_Y}{\partial s} < 0$.