African Journal of Business Management Vol.5 (11), pp. 4033-4043, 4 June, 2011
Available online at http://www.academicjournals.org/AJBM
ISSN 1993-8233 ©2011 Academic Journals

Review

A new aggregation method for strategic decision making and its application in assignment theory

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Accepted 8 February, 2011

A new aggregation method for decision making is presented by using induced aggregation operators and the index of maximum and minimum level. Its main advantage is that it can assess complex reordering processes in the aggregation that represent complex attitudinal characters of the decision maker such as psychological or personal factors. A wide range of properties and particular cases of this new approach are studied. A further generalization by using hybrid averages and immediate weights is also presented. The key issue in this approach against the previous model is that we can use the weighted average and the ordered weighted average in the same formulation. Thus, we are able to consider the subjective attitude and the degree of optimism of the decision maker in the decision process. The paper ends with an application in a decision making problem based on the use of the assignment theory.

Key words: Index of maximum and minimum level, ordered weighted average, induced aggregation operators, weighted average, decision making, assignment theory.

INTRODUCTION

Aggregation operators (Beliakov et al., 2007) are very useful for decision making (Chen, 2009; Demir and Bostanci, 2010; Kacprzyk and Zadrozny, 2009; Liu, 2009, 2010; Sreekumar and Mahapatra, 2009; Wang et al., 2009; Xu and Cai, 2011; Xu and Hu, 2009, 2010; Yang et al., 2010). They are able to fuse the available information in order to obtain a representative result that permits us to make decisions. A very practical aggregation operator is the ordered weighted averaging (OWA) operator (Yager, 1988; Yager and Kacprzyk, 1997). It provides a parameterized family of aggregation operators between the minimum and the maximum. The OWA operator can be extended by using order inducing variables in the reordering step of the aggregation obtaining the induced OWA (IOWA) operator (Yager and Filev, 1999). Since its appearance, it has been studied by a lot of authors.

Yager developed further improvements by using other aggregation operators such as the Choquet integral (Yager, 2004). Tan and Chen (2010) also presented a generalization with Choquet integrals. Merigó and Gil-Lafuente (2009) generalized it by using generalized and quasi-arithmetic means. Merigó and Casanovas (2009) implemented this approach in Dempster-Shafer theory of evidence. They also developed further extensions by using uncertain information (Merigó and Casanovas, 2010a, b, 2011a, c) and distance measures (Merigó and Casanovas, 2010c, d, 2011b). Wei developed several applications by using intuitionistic fuzzy information (Wei, 2010; Wei et al., 2010). Wu et al. (2009) developed several extensions by using continuous aggregations.

Another useful technique for decision making is the index of maximum and minimum level (IMAM) (Gil-Lafuente, 2001, 2002). It uses similarity measures like the Hamming distance (Hamming, 1950) for making decisions. Recently, Merigó and Gil-Lafuente (2009) have suggested the use of the OWA operator in the IMAM operator. They called it the ordered weighted averaging index of maximum and minimum level (OWAIMAM) operator. It provides a parameterized family of similarity measures between the minimum and the maximum. Therefore, we can aggregate the information considering ideals in the information and the attitudinal character of the decision maker in the specific problem considered.

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Thus, it is able to include a wide range of particular cases including the OWA distance (OWAD) operator (Merigó and Gil-Lafuente, 2007, 2010; Xu and Chen, 2008) and the OWA adequacy coefficient (OWAAC) operator (Gil-Lafuente and Merigó, 2010; Merigó and Gil-Lafuente, 2008, 2010).

The aim of this paper is to present a new development based on the use of the IOWA operator in the IMAM operator. We call it the induced ordered weighted averaging index of maximum and minimum level (IOWAIMAM) operator. Its main advantage is that it can deal with complex reordering processes in the aggregation based on the use of order inducing variables. Thus, we can represent more complex environments that consider the degree of optimism of the decision maker and a wide range of other situations such as psychological and personal factors. Moreover, it also includes a wide range of particular cases such as the weighted IMAM (WIMAM), the normalized IMAM (NIMAM) and the OWAIMAM operator. We study some of its main properties.

We further extend this approach by using the hybrid average (Xu and Da, 2003). Thus, we are able to deal with the weighted average and the IOWA operator in the same formulation. We called it the induced hybrid averaging index of maximum and minimum level (IHAIMAM) operator. Moreover, we also present another approach by using immediate weights that also consider the use of the weighted average and the IOWA operator. We call it the immediate weighted induced OWAIMAM (IWIOWAIMAM) operator. It is similar to the IHAIMAM operator but it has some technical differences in its formulation. We study the applicability of the new approach in a decision making problem regarding the selection of strategies in an assignment process. Thus, we can assign several elements of one set (enterprises) to another set of elements (strategies). We see that each aggregation operator may lead to different results because depending on the assumptions made by the decision maker the decisions may be different.

PRELIMINARIES

Here we briefly revise the IMAM, the IOWA operator, the hybrid average and the immediate weights.

The index of maximum and minimum level

The NIMAM (Gil-Lafuente, 2001, 2002) is a similarity measure used for calculating the differences between two elements, two sets, etc. In decision making, we can use it for comparing alternatives in business decision making problems including strategic management, product management and financial management. In summary, we could define it as a measure that includes the Hamming distance (Hamming, 1950) and the adequacy coefficient (Gil-Aluja, 1998; Gil-Lafuente, 2005; Kaufmann and Gil-Aluja, 1986, 1987) in the same formulation. Sometimes, when normalizing the IMAM it is better to give different weights to each individual element. Thus, we get the WIMAM. It can be defined as follows. For two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$, it can be defined as follows:

Definition 1

A WIMAM of dimension n is a mapping K: $(0, 1)^n \times (0, 1)^n \rightarrow (0, 1)$ that has an associated weighting vector $W = V + U$ of dimension n with the following properties:

1) $\sum_{i=1}^{n} w_i = 1$,

2) $w_i \in (0, 1)$

and such that:

$$K(X,Y) = \sum_u Z_i(u) \times |x_i(u) - y_i(u)| + \sum_v Z_i(v) \times [0 \vee (x_i(v) - y_i(v))], \quad (1)$$

Where x_i and y_i are the ith arguments of the sets X and Y respectively, and $u + v = n$.

In the following, we present a simple numerical example of the WIMAM operator in order to see how this algorithm operates.

Example 1

Assume two sets of arguments X = (0.6, 0.4, 0.8, and 0.3) and Y = (0.9, 0.2, 0.7, and 0.5). Assume that the two first arguments have to be treated with the Hamming distance and the other two with the adequacy coefficient. We assume the following weighting vector: W = (0.3, 0.3, 0.2, and 0.2). Thus, the WIMAM is as follows:

$K(X, Y) = 0.3 \times |0.6 - 0.9| + 0.3 \times |0.4 - 0.2| + 0.2 \times (0 \vee (0.8 - 0.7)) + 0.2\ (0 \vee (0.3 - 0.5)) = 0.17$.

The induced OWA operator

The IOWA operator was introduced by Yager and Filev (1999). The main difference against the classical OWA operator (Yager, 1988) is that the reordering step of the IOWA is carried out with order-inducing variables, rather than depending on the values of the arguments a_i. It can be defined as follows.

Definition 2

An IOWA operator of dimension n is a mapping $IOWA$: $R^n \times R^n \rightarrow R$ defined by an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in (0, 1)$, and a set of order-inducing variables u_i, by a formula of the following form:

$$IOWA\ (\langle u_1,a_1\rangle, \langle u_2,a_2\rangle, \ldots, \langle u_n,a_n\rangle) = \sum_{j=1}^{n} w_j b_j , \quad (2)$$

Where ($b_1, \ldots, b_n$) is simply ($a_1, a_2, \ldots, a_n$) reordered in ascending order of the values of the u_i, u_i is the order-inducing variable and a_i is the argument variable.

Example 2

Assume a set of arguments A = (80, 40, 20, and 60) to be aggregated with the following weighting vector W = (0.2, 0.2, 0.3, and 0.3) and order inducing variables U = (20, 14, 29, and 17). Thus, we get:

$IOWA$ = 0.2 × 40 + 0.2 × 60 + 0.3 × 80 + 0.3 × 20 = 50.

Note that it is possible to distinguish between the ascending IOWA (AIOWA) and the descending IOWA (DIOWA) by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the AIOWA and w^*_{n-j+1} the jth weight of the DIOWA operator.

The hybrid average

The HA operator (Xu and Da, 2003) is an aggregation operator that uses the WA and the OWA operator in the same formulation. Thus, it is possible to consider in the same problem, the attitudinal character of the decision maker and the degree of importance of the variables. One of its main characteristics is that it provides a parameterized family of aggregation operators that includes the maximum, the minimum, the arithmetic mean (AM), the WA and the OWA operator. It can be defined as follows:

Definition 3

An HA operator of dimension n is a mapping HA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in$ (0, 1), such that:

$$HA\ (a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j , \quad (3)$$

Where b_j is the jth smallest of the $\hat{a}_i$ ($\hat{a}_i = n\omega_i a_i$, i = 1, 2, …, n), $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the a_i, with $\omega_i \in$ (0, 1) and the sum of the weights is 1.

Example 3

Assume the same information than Example 2 and a weighting vector ω = (0.1, 0.2, 0.3, and 0.4). Thus, we get:

HA = 0.2 × (20 × 0.3 × 4) + 0.2 × (40 × 0.2 × 4) + 0.3 × (80 × 0.1 × 4) + 0.3 × (60 × 0.4 × 4) = 49.6.

Note that it is possible to distinguish between the ascending HA (AHA) operator and the descending HA (DHA) operator by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the AHA and w^*_{n-j+1} the jth weight of the DHA operator. For further information on the HA operator, for example (Merigó and Casanovas, 2010a; Merigó et al., 2010; Wei, 2009; Xu, 2010; Zhao et al., 2009, 2010).

Immediate weights

The immediate weight (IW) is an immediate probability (Engemann et al., 1995, Merigó, 2010; Yager et al., 1995) but focussed on the use of the weighted average instead of the probability. Thus, it is able to use the weighted average and the OWA operator in the same formulation. It can be defined as follows:

Definition 4

An IW operator of dimension n is a mapping IW: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $w_j \in$ (0, 1) and $\sum_{j=1}^{n} w_j = 1$, such that:

$$IW\ (a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j , \quad (4)$$

Where b_j is the jth smallest of the a_i, each a_i has associated a WA v_i, v_j is the associated WA of b_j, and $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$.

Example 4

Assume the same information than Example 2 and a weighting vector V = (0.3, 0.3, 0.3, and 0.1). Thus, we get:

$$IW = \frac{0.2\times0.3}{0.24}\times80 + \frac{0.3\times0.1}{0.24}\times60 + \frac{0.2\times0.3}{0.24}\times40 + \frac{0.3\times0.3}{0.24}\times20 = 45$$

.
As we can see, if $w_j = 1/n$ for all j, we get the weighted average and if $v_j = 1/n$ for all j, the OWA operator.

THE INDUCED ORDERED WEIGHTED AVERAGING INDEX OF MAXIMUM AND MINIMUM LEVEL

Here we introduce the IOWAIMAM operator and study some of its main properties and particular cases. The induced ordered weighted averaging index of maximum and minimum level (IOWAIMAM) operator is an aggregation operator that uses order-inducing variables with the IMAM and the OWA operator. Its main advantage is that it includes a wide range of aggregation operators and it is very useful when dealing with complex reordering processes where the highest result is not always the first (or the last) in the reordering process. By using the IMAM operator, we can deal with the Hamming distance and the adequacy coefficient in the same formulation depending on the interests of the decision maker in the aggregation.

For two sets $X = (x_1, x_2, \ldots, x_{n)}$ and $Y = (y_1, y_2, \ldots, y_{n)}$, it can be defined as follows:

Definition 7

An IOWAIMAM operator of dimension n is a mapping f: $(0, 10^n \times (0, 1)^n \rightarrow (0, 1)$ that has an associated weighting vector W of dimension n with $w_j \in (0, 1)$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$f(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \sum_{j=1}^{n} w_j K_j, \quad (5)$$

Where K_j represents all the $|x_i - y_i|$ and the $(0 \vee (x_i - y_i))$, reordered in ascending order of the values of u_i, u_i is the order-inducing variable and x_i and y_i are the argument variables.

Example 5

Assume the same information than Example 1 and the order inducing variables U = (16, 27, 12, and 36). Thus, we get:

IOWAIMAM = $0.3 \times (0 \vee (0.8 - 0.7)) + 0.3 \times |0.6 - 0.9| + 0.2 \times |0.4 - 0.2| + 0.2\ (0 \vee (0.3 - 0.5)) = 0.16$.

Note that it is possible to distinguish between ascending (AIOWAIMAM) and descending (DIOWAIMAM) orders by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DIOWAIMAM and w^*_{n-j+1} the jth weight of the AIOWAIMAM operator. Moreover, we can suggest an equivalent removal index that it is a dual of the IOWAIMAM because $Q(X, Y) = 1 - K(X, Y)$. We call it the induced ordered weighted averaging dual IMAM (IOWADIMAM). Note that if the weighting vector is not normalized, that is, $W = \sum_{j=1}^{n} w_j \neq 1$, then, the IOWAIMAM operator can be expressed as:

$$f(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \frac{1}{W} \sum_{j=1}^{n} w_j K_j, \quad (6)$$

Note also that the IOWAIMAM operator is commutative, monotonic, bounded and idempotent. A further interesting aspect is the reflexivity property, that is:

IOWAIMAM $((u_1, x_1, y_1), \ldots, (u_n, x_n, y_n)) = 0$ if and only if $x_i = y_i$ for all $i \in (1, n)$.

Note also that *IOWAIMAM* $((u_1, x_1, y_1), \ldots, \langle u_n, x_n, y_n\rangle) =$ *IOWAIMAM* $((u_1, y_1, x_1), \ldots, (u_n, y_n, x_n))$.

Other interesting generalizations can be developed following Spirkova (2009) and Torra and Narukawa (2010). For example, we can develop the function induced OWAIMAM operator, which uses a generating function r for the order inducing variables such that r: $I \rightarrow R$, being that $I \subset R$ is a closed interval $I = (a$, and $b)$ and a generating function for the arguments such that s: $R^m \rightarrow R$. This generating function expresses the formation of the arguments when a previous analysis exists, such as the use of a multi-person process where each argument is constituted by the opinion of m persons. Moreover, we use a weighting function f for the weighting vector (Spirkova, 2008).

In this case, we directly extend the approach by obtaining the function induced mixture IMAM (IMIMAM) operator as follows. In this definition, we refer to the arguments as two sets $X = (x_1, x_2, \ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$.

Definition 8

An IMIMAM operator of dimension n is a mapping IMIMAM: $(0, 1)^n \times (0, 1)^n \times (0, 1)^n \rightarrow (0, 1)$ that has an associated vector of weighting functions f, r: $I \rightarrow)0, \infty($, is a some positive continuous function, s: $R^m \rightarrow R$, such that:

IMIMAM $((r_o(u_1), s_p(x_1), s_q(y_1)), \ldots, (r_o(u_n), s_p(x_n), s_q(y_n))) =$

$$\frac{\sum_{j=1}^{n} f_j(s_y(b_j)) s_y(b_j)}{\sum_{j=1}^{n} f_j(s_y(b_j))}, \quad (7)$$

Where $s_y(b_j)$ is the $|s_p(x_i) - s_q(y_i)|$ and the $(0 \vee (s_p(x_i) - s_q(y_i)))$ value of the IMD triplet $(r_o(u_i), s_p(x_i), s_q(y_i)))$ having the jth smallest $r_o(u_i)$; u_i is the order-inducing variable; $|s_p(x_i) - s_q(y_i)|$ is the argument variable represented in the form of individual distances; and o, p and q indicate that each order-inducing variable and each argument is formed by using a different function. A further interesting issue is the use of infinitary aggregation operators (Mesiar and Pap, 2008). In this case, we can represent an aggregation process where there are an unlimited number of arguments that appear in the aggregation process.

Note that $\sum_{j=1}^{\infty} w_j = 1$. By using, the IOWAIMAM operator we get the infinitary IOWAIMAM (∞-IOWAIMAM) operator as follows:

$$\infty\text{-}IOWAIMAM\,(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \sum_{j=1}^{\infty} w_j b_j \,, \quad (8)$$

However, note that the reordering process is much more complex, that is, we never know which argument is the largest argument because we have an unlimited number of arguments. This problem can be partially solved by using the order inducing variables. For further reading about the usual OWA by Mesiar and Pap (2008). A further interesting issue to analyze is the different measures used for characterizing the weighting vector of the IOWAIMAM operator, based on the measures developed for the OWA operator by Yager (1988). Thus, we can analyze the degree of orness of the IOWAIMAM, the entropy of dispersion and the balance of the weighting vector.

Families of IOWAIMAM operators

A wide range of families of IOWAIMAM operators can be used by using a different manifestation in the weighting vector (Emrouznejad and Amin, 2010; Merigó et al., 2010; Yager, 1993, 2009, 2010; Zhou and Chen, 2010). For example, it is possible to obtain the maximum, the minimum, the IMAM, the WIMAM operator and the OWAIMAM operator.

Remark 1

For example, it is possible to obtain the maximum, the minimum, the IMAM, the WIMAM operator and the OWAIMAM operator.

i) The maximum is found if $w_p = 1$ and $w_j = 0$, for all $j \neq p$, $u_p = \text{Max}((x_i, y_i))$.
ii) The minimum, if $w_q = 1$ and $w_j = 0$, for all $j \neq q$, $u_q = \text{Min}((x_i, y_i))$.
iii) If $w_k = 1$ and $w_j = 0$, for all $j \neq k$, we get the step-IOWAIMAM operator.
iv) The IMAM is found when $w_j = 1/n$, for all $\langle x_i, y_i\rangle$.
v) The WIMAM when the ordered position of I is the same as the ordered position of u_i.
vi) The OWAIMAM is found when the ordered position of j is the same as the ordered position of u_i.
vii) If we use the Hamming distance in all the arguments, then the IOWAIMAM operator becomes the IOWAD operator (Merigó and Casanovas, 2010f).
viii) If we use the adequacy coefficient (Kaufmann and Gil-Aluja, 1986) in all the arguments, then, we get the IOWA adequacy coefficient (IOWAAC) operator.
ix) The median-IOWAIMAM, if n is odd we assign $w_{(n+1)/2} = 1$ and $w_{j^*} = 0$ for all others. If n is even we assign, for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j^*} = 0$ for all others.
x) When $w_{j^*} = 1/m$ for $k \leq j^* \leq k + m - 1$ and $w_{j^*} = 0$ for $j^* > k + m$ and $j^* < k$, we are using the window-IOWAIMAM operator. Note that k and m must be positive integers such that $k + m - 1 \leq n$.
xi) If $w_1 = w_n = 0$, and for all others $w_{j^*} = 1/(n-2)$, we are using the Olympic-IOWAIMAM. Note that it is possible to present a general formulation of the Olympic-IOWAIMAM considering that $w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $w_{j^*} = 1/(n-2k)$, where $k < n/2$. Note that if $k = 1$, then, this general form becomes the usual Olympic-IOWAIMAM.
xii) The centered-IOWAIMAM operator is found if the aggregation is symmetric ($w_j = w_{j+n-1}$), strongly decaying (when $i < j \leq (n+1)/2$ then $w_i < w_j$ and when $i > j \geq (n+1)/2$ then $w_i < w_j$) and inclusive ($w_j > 0$).
xiii) The generalized S-IOWAIMAM operator is obtained when $w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for $j = 2$ to $n - 1$ where α, $\beta \in (0, 1)$ and $\alpha + \beta \leq 1$.

Note that if $\alpha = 0$, the generalized S-IOWAIMAM operator becomes the "andlike" S-IOWAIMAM operator and if $\beta = 0$, it becomes the "orlike" S-IOWAIMAM operator.

THE INDUCED HYBRID AVERAGING INDEX OF MAXIMUM AND MINIMUM LEVEL

The IOWAIMAM operator can be further generalized by using the HA operator (Xu and Da, 2003). Thus, we can use the weighted average and the IOWA operator in the IMAM, considering both the attitudinal character of the decision maker and its subjective probability (or degree of importance). We call this new aggregation operator the induced hybrid averaging index of maximum and minimum level (IHAIMAM) operator.

Definition 9

An IHAIMAM operator of dimension n is a mapping *IHAIMAM*:

$(0, 1)^n \times (0, 1)^n \rightarrow (0, 1)$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in (0, 1)$, such that:

$$IHAIMAM\,(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \sum_{j=1}^{n} w_j K_j \,, \quad (9)$$

Where K_j represents all the $|x_i - y_i|^* = n\omega_i\,|x_i - y_i|$ and the $(0 \vee (x_i - y_i))^* = n\omega_i\,(0 \vee (x_i - y_i))$, reordered in decreasing order of the values of the u_i, u_i is the order-inducing variable, with $i = 1, 2, \ldots, n$, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the $(0 \vee (x_i - y_i))$, with $\omega_i \in (0, 1)$ and the sum of the weights is 1.

Example 6

Assume the following fuzzy sets in an aggregation process:

X = (0.8, 0.4, 0.8, and 0.5), and Y = (0.6, 0.5, 0.7, and 0.9).

The following weighting vectors:

W = (0.3, 0.3, 0.3, and 0.1) and ω = (0.4, 0.3, 0.2, and 0.1), and the order inducing variables U = (14, 17, 22, and 19).

If we calculate the similarity between X and Y using the IHAIMAM operator and assuming that the first two arguments of the sets X and Y has to be treated with the Hamming distance and the other ones with the adequacy coefficient, we get the following:

First, we weight the arguments with the weighted average:

$(0 \vee (0.8 - 0.7)) \times 0.4 \times 4 = 0.16$,
$(0 \vee (0.5 - 0.9)) \times 0.3 \times 4 = 0$,
$|0.8 - 0.6| \times 0.2 \times 4 = 0.16$,
$|0.4 - 0.5| \times 0.1 \times 4 = 0.04$.

Next, we develop the aggregation with the IOWA operator:

$IHAIMAM$ (X, and Y) = 0.3 × 0.16 + 0.3 × 0 + 0.3 × 0.16 + 0.1 × 0.04 = 0.1.

Note that if $w_j = 1/n$, for all j, we obtain the WIMAM operator and if $\omega_i = 1/n$, for all i, we obtain the IOWAIMAM operator. If $w_j = 1/n$ and $\omega_i = 1/n$, for all i and j, we get the NIMAM operator. The IHAIMAM operator accomplishes similar properties than the IOWAIMAM operator.

However, it is not idempotent nor commutative. Moreover, we can also study a wide range of families of IHAIMAM operators following the methodology explained in induced ordered weighted averaging index of maximum and minimum level.

IMMEDIATE WEIGHTS IN THE IMAM OPERATOR

A similar extension than the IHAIMAM operator can be suggested by using immediate weights in the aggregation. Thus, we get the immediate weighted IOWAIMAM (IWIOWAIMAM) operator. Its main advantage is that it can deal with the weighted average and the IOWA operator in the same formulation. It can be defined as follows:

Definition 10

An IWIOWAIMAM operator of dimension n is a mapping *IWIOWAIMAM*: $(0, 1)^n \times (0, 1)^n \rightarrow (0, 1)$ that has an associated weighting vector W of dimension n with $w_j \in (0, 1)$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$IWIOWAIMAM(\langle u_1, x_1, y_1\rangle, \ldots, \langle u_n, x_n, y_n\rangle) = \sum_{j=1}^{n} \hat{v}_j b_j, \quad (10)$$

Where b_j is the $|x_i - y_i|$ and the $(0 \vee (x_i - y_i))$ value having the jth smallest u_i, u_i is the order-inducing variable, x_i, $y_i \in (0, 1)$, each $|x_i - y_i|$ and $(0 \vee (x_i - y_i))$ has associated a WA v_i, v_j is the associated WA of b_j, and $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$.

Note that it is also possible to consider descending and ascending orders and the dual by using $Q(X, Y) = 1 - K(X, Y)$. It also accomplishes similar properties than the IOWAIMAM operator excepting that it is not commutative. Furthermore, it is possible to study a wide range of families of IWIOWAIMAM operators by using a different expression in the weighting vector as explained in induced ordered weighted averaging index of maximum and minimum level.

Example 7

Assume the same information of Example 1. With the IWIOWAIMAM operator we get the following:

First, we calculate the weights. Thus, we have to adapt the ordering of W to V (or vice versa). W* = (0.3, 0.1, 0.3, and 0.3). Now we mix the weighting vectors by using $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$. Therefore, we obtain:

$$\hat{v}_1 = \frac{0.3 \times 0.4}{0.24} = 0.5,$$

$$\hat{v}_2 = \frac{0.1 \times 0.3}{0.24} = 0.125,$$

$$\hat{v}_3 = \frac{0.3 \times 0.2}{0.24} = 0.25,$$

$$\hat{v}_4 = \frac{0.3 \times 0.1}{0.24} = 0.125.$$

Once we have the new weighting vector, we can aggregate the information according to the reordering established in the order inducing variables. That is,

$IWIOWAIMAM$ (X, Y) = 0.5 ×((0 ∨ (0.8 – 0.7)) + 0.125 × (0 ∨ (0.5 – 0.9)) + 0.25 × |0.8 – 0.6| + 0.125 × |0.4 – 0.5| = 0.1.

Note that in Examples 1 and 2 we get the same result

Table 1. Characteristics of the markets.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.6	0.6	0.8	0.6	0.8	0.5
A_2	0.5	0.7	0.6	0.7	0.6	0.7
A_3	0.4	0.4	0.9	0.5	0.7	0.8
A_4	0.7	0.6	0.7	0.7	0.4	0.6
A_5	0.5	0.6	0.7	0.6	0.5	0.7
A_6	0.4	0.6	0.8	0.3	0.6	0.6

with both methods, but sometimes these methods may lead to different results depending on the assumptions made in the analysis.

APPLICATION IN ASSIGNMENT THEORY

In the following, we are going to develop a brief illustrative example of the new approach in a decision making problem concerning strategy selection. We focus on the use of the assignment theory where we want to assign several elements of one set to other elements of a second set. Note that in the literature we may find a wide range of methods for doing so (Gil-Aluja, 1999), but in this paper we will focus on a very simple algorithm that assigns the results by elimination of rows and columns (Gil-Aluja, 1999). Note that other applications could also be considered (Gil-Aluja et al., 2009).

Assume that a group of companies are planning an investment strategy for the next year and they consider the possibility of expanding to several markets but they do not want to compete between them. The key idea is that each company should focus in one market. After careful analysis of the information, the companies consider that the main markets are the following:

i) Expand to the Nigerian market: A_1.
ii) Expand to the South African market: A_2.
iii) Expand to the Egyptian market: A_3.
iv) Expand to the Argelian market: A_4.
v) Expand to the Kenian market: A_5.
vi) Expand to the Tanzanian market: A_6.

After careful review of the information, the group of experts of the company establishes the following general information regarding the strategies. They have summarized the information of the strategies in six general characteristics $C = (C_1, C_2, C_3, C_4$, and $C_5)$.

i) C_1: Benefits in the short term.
ii) C_2: Benefits in the mid term.
iii) C_3: Benefits in the long term.
iv) C_4: Risk of the strategy.
v) C_5: Size of the market.
vi) C_6: Other factors.

The companies involved in the following decision process that can invest in these markets are the following:

i) E_1: Enterprise A.
ii) E_2: Enterprise B.
iii) E_3: Enterprise C.
iv) E_4: Enterprise D.

With this information, the group of experts of the company describes each market and each company according to the characteristics established. That is, with the markets we get the expected results when investing in these markets and with the enterprises, the results they would like to obtain in this strategy. We assume that each company has similar characteristics so they are more or less equally qualified for carrying out the strategic investment process. The results are shown in Table 1 for the markets and in Table 2 for the enterprises. The results are valuations (numbers) between 0 and 1 being 1 the best result and 0 the worst result.

With this information, it is possible to develop different methods for calculating the similarity between the enterprises and the markets. In this example, we consider the WIMAM, the IOWAD operator and the IOWAIMAM operator. We consider that the first three characteristics have to be treated with the adequacy coefficient and the other three with the Hamming distance. We assume that W = (0.1, 0.1, 0.1, 0.2, 0.2, and 0.3) and V = (0.2, 0.2, 0.2, 0.2, 0.1, and 0.1). With the WIMAM operator we get the following similarities between the enterprises and the markets shown in Table 3. With this information, now we can develop an assignment process by using one of the algorithms available in the literature. For example, we will use the algorithm presented by Gil-Aluja (1999) regarding elimination of rows and columns. By using this algorithm, we always select the lowest similarity and then eliminate its row and its column until we have assigned all the elements of one of the sets.

Note that in Table 3 we already show the first assignment process where we write in parenthesis the element selected and in which position in order to see how we have eliminated rows and columns. Once we have found the first assignment, we eliminate its row and its column (E_3 and A_1) and we repeat the process with the

Table 2. Characteristics of the enterprises.

	C_1	C_2	C_3	C_4	C_5	C_6
E_1	0.6	0.7	0.8	0.7	0.6	0.6
E_2	0.3	0.6	0.9	0.6	0.8	0.7
E_3	0.5	0.6	0.7	0.7	0.7	0.5
E_4	0.4	0.7	0.7	0.6	0.6	0.6

Table 3. Aggregated results and assignment process (step 1).

	A_1	A_2	A_3	A_4	A_5	A_6
E_1	0.07	0.07	0.17	0.06	0.1	0.17
E_2	0.04	0.1	0.05	0.13	0.07	0.14
E_3	0.03 (1º)	0.05	0.13	0.04	0.06	0.15
E_4	0.05	0.05	0.11	0.06	0.04	0.11

Table 4. Aggregated results and assignment process (step 2).

	A_1	A_2	A_3	A_4	A_5	A_6
E_1	0.07	0.07	0.17	0.06	0.1	0.17
E_2	0.04	0.1	0.05	0.13	0.07	0.14
E_3	0.03 (1º)	0.05	0.13	0.04	0.06	0.15
E_4	0.05	0.05	0.11	0.06	0.04 (2º)	0.11

Table 5. Aggregated results and assignment process (step 3).

	A_1	A_2	A_3	A_4	A_5	A_6
E_1	0.07	0.07	0.17	0.06 (4º)	0.1	0.17
E_2	0.04	0.1	0.05 (3º)	0.13	0.07	0.14
E_3	0.03 (1º)	0.05	0.13	0.04	0.06	0.15
E_4	0.05	0.05	0.11	0.06	0.04 (2º)	0.11

rest of the Table.

The second assignment process is presented in Table 4. Next, we eliminate the row and the column of the second assignment and repeat the process until we have assigned all the possible elements. The third and fourth assignments are presented in Table 5.

Assignment process 1

1º- Enterprise 3 assigned with market A_1 (Nigeria).
2º- Enterprise 4 assigned with market A_5 (Kenia).
3º- Enterprise 2 assigned with market A_3 (Egypt).
4º- Enterprise 1 assigned with market A_4 (Argelia).

Next, we develop a similar assignment process but with the IOWAD operator. We assume that U = (13, 16, 17, 27, 22, and 19). The results are shown in Table 6. In this case, by using the assignment algorithm by elimination of rows and columns, we obtain the following results.

Assignment process 2

1º- Enterprise 2 assigned with market A_1 (Nigeria).
2º- Enterprise 3 assigned with market A_5 (Kenia).
3º- Enterprise 4 assigned with market A_2 (South Africa).
4º- Enterprise 1 assigned with market A_4 (Argelia).

By using the IOWAIMAM operator, we can also develop a similar analysis in order to obtain an optimal assignment process. Note that with the IOWAIMAM we are using the Hamming distance and the adequacy coefficient in the same formulation and aggregated with the OWA operator. The results are presented in Table 7. By using the assignment algorithm commented before, we get the

Table 6. Aggregated results with the IOWAD operator.

	A_1	A_2	A_3	A_4	A_5	A_6
E_1	0.06	0.08	0.2	0.09 (4º)	0.1	0.15
E_2	0.04 (1º)	0.18	0.13	0.24	0.13	0.14
E_3	0.07	0.17	0.17	0.1	0.05 (2º)	0.14
E_4	0.13	0.07 (3º)	0.15	0.11	0.07	0.1

Table 7. Aggregated results with the IOWAIMAM operator.

	A_1	A_2	A_3	A_4	A_5	A_6
E_1	0.06	0.08	0.18	0.06(3ab)	0.1	0.15
E_2	0.04	0.09	0.08 (4ºb)	0.1	0.07 (4ºa)	0.09
E_3	0.02 (1º)	0.05	0.13	0.04	0.05	0.12
E_4	0.05	0.04 (2ºa)	0.11	0.05	0.04 (2ºb)	0.08

following assignment results.

Assignment process 3

1º- Enterprise 3 assigned with market A_1 (Nigeria).
2º- Two optimal assignments:
2ºa) Enterprise 4 assigned with market A_2 (South Africa).
3ºa) Enterprise 1 assigned with market A_4 (Argelia).
4ºa) Enterprise 2 assigned with market A_5 (Kenia).

or

2ºb) Enterprise 4 assigned with market A_5 (Kenia).
3ºb) Enterprise 1 assigned with market A_4 (Argelia).
4ºb) Enterprise 2 assigned with market A_3 (Egypt).

As we can see, we obtain different results with each aggregation method. Thus, depending on the method used, our decisions may be different. Note that with the IOWAIMAM we find that there are two optimal assignment processes. However, if we analyze both assignments in detail, we see that the degree of optimality of the first assignment process is 0.02 + 0.04 + 0.06 + 0.07 = 0.19, and for the second assignment process is 0.02 + 0.04 + 0.06 + 0.08 = 0.20. Thus, although both assignment processes are optimal, it seems that the first one is better.

CONCLUSIONS

We have presented a new aggregation method very useful for decision making problems. We have introduced the IOWAIMAM operator. It is an aggregation operator that uses the IOWA and the IMAM operator in the same formulation. Thus, it uses the Hamming distance and the adequacy coefficient in the same formulation. Moreover, it also uses a complex reordering process in the aggregation of the information by using order inducing variables. We have also seen that it includes a wide range of particular cases such as the OWAIMAM, the WIMAM, the IOWAD and the weighted Hamming distance. We have further extended this approach by using the hybrid average obtaining the IHAIMAM operator. Thus, we have been able to use weighted averages and IOWA operators in the IMAM operator. We have also presented a similar model by using immediate weights that we have called the IWIOWAIMAM operator. We have studied the applicability of this new approach in a decision making problem regarding the selection of strategies in an assignment process. We have seen that depending on the particular type of aggregation operator used the results may lead to different decisions. Therefore, we have seen that it is very important that the decision maker appropriately selects the method that is in closest accordance with his interests.

In future research, we expect to develop further developments by using other types of aggregation operators such as probabilistic ones or the use of unified aggregation operators. We will also consider other decision making applications in business problems and by using other assignment and grouping algorithms.

ACKNOWLEDGEMENTS

We would like to thank the anonymous referees for valuable comments that have improved the quality of the paper. Support from the projects JC2009-00189 from the Spanish Ministry of Education and A/023879/09 from the Spanish Ministry of Foreign Affairs is gratefully acknowledged.

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