La Palma 2021 Eruption Dike Modeling

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Abstract: Magnetotelluric surveying has a huge potential for studying subsurface structures and processes due to its wide range of depths. It is particularly useful in igneous and volcanic events due to the considerable resistivity changes that rock undergoes at high temperatures. Despite that, few models and simulations solving the temperature-resistivity evolution with time have been proposed. The 2021 volcanic eruption in La Palma provided an excellent opportunity to develop a model of these characteristics given the constant monitoring and surveying data available of the periods during and after the eruption. The presence of electrical resistivity anomalies during the cooling process of the igneous intrusion made it even more interesting to develop a more complete and general model for these types of systems.

I. INTRODUCTION

One of the most important challenges in volcanology is understanding the magma reservoir and dikes system (known as volcanic plumbing) that supply volcanic eruptions. Dikes are local-scale structures that control the outflow of lava from vents and their closing and opening are critical in predicting subsequent lava flows as shown in Loncar & Huppert [1] and Bruce & Huppert [2]. Therefore, modeling the cooling of the dikes is important, since it is the process that controls the rate of dike closure and its possible reopening in case of new magma injections. The 2021 volcanic eruption of La Palma (Canary Islands, Spain) provides a unique opportunity to study the cooling of a dike, since the magnetotelluric (MT) monitoring experiment conducted during and after the eruption has reported significant changes in electrical resistivity over time [3]. The Arrhenius equation [4] correlates both temperature and electrical conductivity (or its inverse, the electrical resistivity). Therefore, the combination of MT results from La Palma and the modeling of the dike cooling and its effects on electrical resistivity could provide valuable information about the structure of the volcano and help to understand other similar volcanic systems.

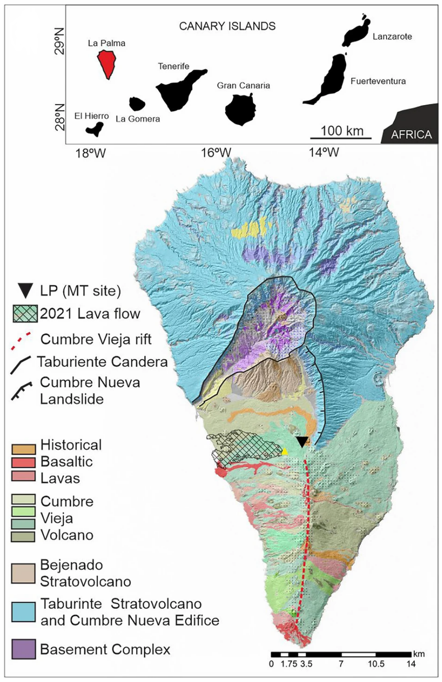

FIG. 1: La Palma Island map displaying its major geological features. The black triangle is the long-period MT site and the yellow triangle is the 2021 Cumbre Vieja volcano. The original map and the legend can be found at https://idecan2.grafcan.es. From [3]

A. La Palma 2021 volcanic eruption. Overview

The volcanic eruption of 2021 in La Palma started in the western flank of Cumbre Vieja, an approximately N-S oriented active volcanic ridge in the southern part of the island.

The volcanic eruption started in September 19, 2021 and lasted 85 days [3]. Magnetotelluric monitoring in the zone has been conducted since November 5, 2021 through a long-period MT site placed 2.4 km away from the volcanic cone providing with valuable post-eruptive data up until now [3]. The MT results initially suggested a low resistivity and thus high temperature complex system of differently oriented structures considered as several dikes connected between them and going up to the surface, consistent with the volcanic plumbing system of the island proposed by other surveying methods [5, 6]. The most superficial structure that gave rise to the recent eruption has been interpreted as a dike, an intrusive sheet of rock encased in a fracture of a larger rock body, with a marked N-S dominant orientation [3]. This structure and the associated geometry are what facilitated the ascent of magma that, driven by isostatic pressure, gave rise to the 2021 eruption, though the precise geometry is still under debate.

Furthermore, the magnetotelluric study also showed an electrical resistivity anomaly not expected in a cooling process such as a cooling magma intrusion after the associated eruption. An initial decrease in resistivity followed by an increase, suggest initially the presence of meltback followed by the cooling of the intrusion[3]. Meltback is a process in which the heat conducted to the walls is high enough to start melting the surrounding country rock of the dike instead of cooling and solidification of the magma intrusion [1]. This anomalous behaviour is the motivation of this study, aiming to elaborate a temperature and resistivity model for the volcanic eruption in La Palma that helps to understand the structure and the processes suggested by the MT measurements. For this purpose I have written and proposed a code to compute a model for this system based on the physical mechanisms governing the cooling of dike systems.

II. METHODOLOGY

A. Physical description of the problem

A dike can be modelled as a three-dimensional system oriented vertically relative to the Earth's surface, composed of two plano-parallel walls with roughly the same separation or width (first dimension) along the second vertical dimension (which represents the depth in the Earth's crust). The third dimension spanning along the fracture (going in the N-S direction in our case) plays a less relevant role as it is supposed to be much larger than the others thus its boundaries have little effect on the intrusion tip where the eruption took place and the model is centered on. The margins or side walls, referred to as the surrounding country rock, are also a crucial factor and can heavily influence the entire eruptive and cooling processes of the intrusion depending on their initial conditions [2].

When studying cooling dikes, two regimes can be found, *blocking* where the crystallization of the magma flow keeps narrowing the dike and ends up completely blocking it, thus ending the eruption, or *meltback* where high amounts of heat conducted by the magma and the release of latent heat from crystallization starts melting the surrounding country rock, a process that continues until the flow decreases and meltback stops. The width of the dike and the initial temperature of the surrounding country rock are crucial for determining which regime will dominate [2]. Also, the successive reheating of the dike and the host rock via reinjection of magma is known as *baked margins* and is the driving factor for long lasting eruptions due to meltback. Thus, the problem to overcome to obtain a useful model for the volcanic eruption is to define and solve the governing equations for all these processes.

B. Governing equations

1. *The heat equation*

The heat equation can be used to describe the evolution of the temperature with time in dikes, although several adjustments must be made. The heat equation considers both transmission and generation of heat in the aforementioned volume and has the following form according to Fowler [7]:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\nabla^2 T + \frac{A}{\rho c_p} - u \cdot \nabla T, \tag{1}$$

where T is the rock temperature at the point of study and t represents time, k is the thermal conductivity, c_p the specific heat and A the heat generation coefficient. The expression is simplified by assuming no heat generation in the flow thus $A = 0$ and neglecting the advection term $u \cdot \nabla T$ as convective heat transfers represent a small fraction of total transfer due to the reduced dimensions in dikes [8]. These terms are further negligible considering the static nature of stagnant magma in the cooling process after the eruption has ceased. The heat equation then takes the form:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\nabla^2 T, \tag{2}$$

where the factor $\frac{k}{\rho c_p}$ is the thermal diffusivity coefficient κ. This is still not accurate enough to provide a useful model because it does not consider the latent heat released in solidification or absorbed during melting of the magma and of the rock wall if meltback is present.

2. *Meltback and latent heat of crystallization*

The variation in the width of the dike due to melting or solidification of the rock walls in the surrounding country rock has been mentioned in section II A and is known as *meltback* in the first case. To model this process, it is necessary to consider the release of latent heat in the heat equation on melting-solidification processes during the eruption. When introducing this, we both consider the energy release related to the change of phase of the magma flow and changes in the wall at the same time. The latent heat release gives rise to a jump condition in the melting-crystallization heat versus temperature curves [1] and, since both crystallization and melting occur not at a fixed temperature but throughout an entire range, it is necessary to define the extremes of this range as solidus (100% of rock fraction solidified) (T_s) and liquidus (100% of rock fraction melted) (T_l) temperatures. Both values depend on the rock type involved. On the same topic, Mobile Melt

Extent or MME is the threshold temperature between the aforementioned solidus and liquidus temperatures where 50% of the material fraction is melted and 50% is crystallized. Loncar and Huppert [1] proposed a heat profile expression including this latent heat related step with the form:

$$Q = \left\{ \begin{array}{ll} \rho c_p T & T < T_s \\ \rho c_p T + L\dfrac{T - T_s}{T_l - T_s} & T_s < T < T_l \\ \rho c_p T + L & T_l < T \end{array} \right\}, \tag{3}$$

where Q is the total heat, L the latent heat of crystallization of the rock and again T is the rock temperature at the point of study and c_p the specific heat. This ultimately leads to the adapted tri-dimensional heat equation shown in the appendix section V C, expressions (9) and (10). The heat equation obtained in (9) and (10) is the basis for the model. As the geometry of the dike leads to only two relevant dimensions, depth and width, the heat equation can be simplified to a single dimension of these two, width, and subsequently solved for different depths separately. This reduced form of the equation is:

$$(1+Z)\frac{\partial T}{\partial t} = \kappa\frac{\partial T}{\partial x}, \tag{4}$$

where:

$$Z = \left\{ \begin{array}{ll} 0 & T < T_s \\ \dfrac{L}{\rho c_p (T_l - T_s)} & T_s < T < T_l \\ 0 & T_l < T \end{array} \right\}. \tag{5}$$

Here x are the width coordinates. With all these considerations, we can proceed to solve numerically the adapted heat equation.

C. Numerical approach

To implement a numerical solution, an explicit finite difference method is applied, with progressive differences in time and centred in space. The equation expressed in differences is:

$$(1+Z)\frac{T_{i,k+1} - T_{i,k}}{\Delta t} = \kappa\frac{T_{i-1,k} - 2T_{i,k} + T_{i+1,k}}{(\Delta x)^2}, \tag{6}$$

where Δx and Δt are the sizes of the mesh discretization in (x,t), sub-index k is for time variation and i indicates spatial changes in x. Solving for $T_{i,k+1}$ leads to the equation

$$T_{i,k+1} = \lambda T_{i-1,k}(1-2\lambda)T_{i,k} + \lambda T_{i+1,k}, \tag{7}$$

where $\lambda = \frac{\kappa \cdot \Delta t}{(1+Z)(\Delta x)^2}$. This solution must satisfy a convergence condition defined by $0 < \lambda \leqslant 0.5$. It is the stability condition for a finite difference explicit scheme applied to an elliptical differential equation (with first derivative in time and second derivative in x) like the heat equation 4. It establishes a limit when choosing both time and width steps in the modelling.

D. Dike cooling model - Program flow

For the computing of the model, I have developed a code that solves the equations for the numerical approach using Matlab® (Mathworks). The parameters of the simulation must be set up in the preamble of the code, including time and position step for the mesh of the dike. Wall rock and magma physical parameters, boundary and initial conditions for wall temperatures and magma intrusion temperature are also defined in the preamble. The model then allows the selection of the intermediate times during the evolution of the simulation when it is desired to obtain a temperature profile plot (initial and steady/final state are always plotted). Taking into account the convergence condition $0 < \lambda \leqslant 0.5$, the solution is implemented by setting the initial temperature profile for both the intrusion and the surrounding country rock positions in the grid to their respective selected temperatures. This is extremely important as mentioned in section II A for the evolution and meltback of the rock walls. Material homogeneity must be considered as the surrounding country rock and magma flow are supposed to have the same physical properties in volcanic systems. Therefore, in my model we will choose basalt as our reference rock.

The code then proceeds to solve equation 7. The simulation starts by solving the aforementioned solution of the heat equation for every time step parting from the initial temperature profile. Reinjection of magma is also implemented. It works by stopping the evolution at a selected time (chosen in the preamble of the model as well) and restarting the simulation again. This time the initial temperature profile will be the last temperature profile from the previous injection cycle right when it stopped. The model proceeds to set only the grid positions from this profile that present a temperature higher than the MME to the intrusion temperature and then restarts the simulation. If no grid points reach the threshold temperature, reinjection will not be possible as the dike will be blocked and the model will keep running without the reinjection steps until the steady state is achieved (the steady state appears when two successive profiles calculated present a variation smaller than a minimum value set in the preamble). The full code is available at [9].

E. Temperature and resistivity. Arrhenius formula

The temperature profiles obtained in the model must be transformed into resistivity profiles to correlate with MT measurements. The electrical resistivity is the inverse of electrical conductivity, and conductivity in rocks depends on various mechanisms. When reaching temperatures above $400°C$ the conductivity of the rock starts to become important due to the high melt fraction [4]. The expression that relates temperature and conductivity in this case is the Arrhenius formula, and it has the following form according to Pineda [4]:

$$\sigma = \sigma_0 \exp \frac{-E_a}{k_B T} \tag{8}$$

where σ_0 is the conductivity at infinite (or very high) temperature (T) and E_a is the activation energy, k_B being the Boltzmann constant. These parameters are determined in my case by using previously measured resistivity values of $1000\Omega m$ at $400°C$ and $10\Omega m$ at $800°C$ [10]. Solving an equation system for both pairs of values yields $\sigma_0 = 231.74 \quad S \cdot m^{-1}$ and $E_a = 1.1473 \cdot 10^{-19} J$. The Boltzmann constant is taken as $k_b = 1.38 \cdot 10^{-23} J \cdot K^{-1}$. The reference resistivity values have been obtained from Scarlato et al. [10] and are measured for basaltic rocks at Mount Etna in Sicily, Italy. Due to the absence of data from the La Palma volcano, these values are considered adequate given the similarity in the systems and hence used in my model.

Using the Arrhenius formula (taking resistivity as the inverse of the conductivity) and our temperature profiles obtained in section II D that vary with time, the resistivity profiles can be implemented in the code and will already be time dependant and useful for comparison with MT measurements.

III. RESULTS AND DISCUSSION

To analyse the model results, I present the simulation obtained with my code for the cooling process of the proposed dike structure from La Palma 2021 eruption[3]. For the simulation parameters, a dike width of $7m$ is chosen as proposed in Montesinos et al.[11] for the supposed width of the intrusion in La Palma. The surrounding country rock initial temperature has been set to $T_0 = 710°C$, which is the first temperature that provided a valid solution with the presence of meltback. This choice of temperature is also based on the model for minimum halfwidth of a dike for a given host rock temperature proposed by Loncar and Huppert[1]. Both values are within the same range. Parameters of the rock are taken as those of basalt, with a specific heat capacity of $c_p = 1480 J \cdot kg^{-1} \cdot K^{-1}$, a thermal diffusivity of $\kappa = 5.3 \cdot 10^{-}7 m^2 \cdot s^{-1}$ and latent heat of crystallization of $L = 4 \cdot 10^5 J \cdot kg^{-1}$. Solidus and liquidus temperatures are chosen as $T_s = 950°C$ and $T_l = 1250°C$, and $MME = 1100°C$ [1]. For the Arrhenius formula, the constants are those calculated in section II E. I set the grid parameter to $\Delta x = 0.001m$ which fixes time step to $\Delta t = 0.9s$, to obtain a detailed result for meltback as it represents a width variation of the order of centimetres usually. Intrusion temperature is set to $1200°C$[3]. The resulting temperature profile evolution for one year of the dike section can be seen in figure 2.

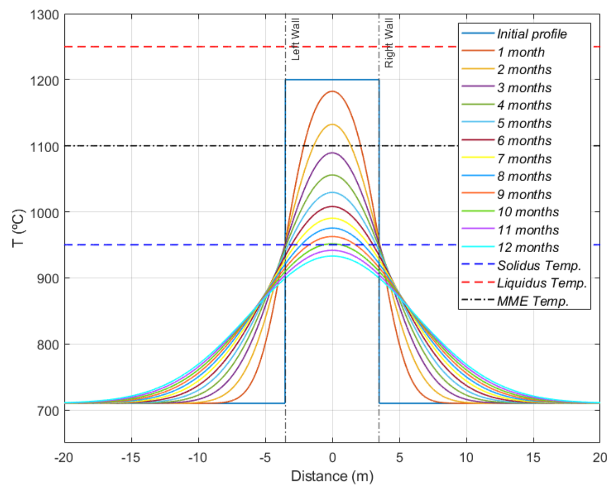

FIG. 2: Temperature profile evolution for the section of the modeled dike. Different colours represent a different month over the course of one year evolution. Solidus and liquidus temperatures as well as Mobile Melt Extent temperature are shown respectively as deep blue, red and black discontinuous lines.

Computing with a step of one month over the period of one year yields a temperature profile evolution consistent with the analytical solution proposed in [12].

A magnified view of the temperature profiles around the right wall and close to the solidus isotherm is shown in figure 4 in the appendix section V A. The magnified figure indicates the possibility of meltback in the period between the first three months of evolution as the three corresponding curves are well above the solidus temperature for positions beyond the original wall, up to $4-5cm$ inside the host rock.

The final step is obtained after the model applies the Arrhenius formula and yields the resistivity profiles presented in figure 3. When analysing the zone between the walls corresponding to the dike, a clear increase in the resistivity of around one order of magnitude, from $1.4\ \Omega \cdot m$ right after the intrusion to $20\ \Omega \cdot m$ after one year of cooling can be seen. This is an obvious result since the cooling of the intrusive rock/magma leads to higher resistivity values according to the Arrhenius formula.

When studying closely the margins, two regimes arise. Far from the dike, the host rock heats up by conduction and keeps rising its temperature and thus reducing its resistivity over the course of the simulated

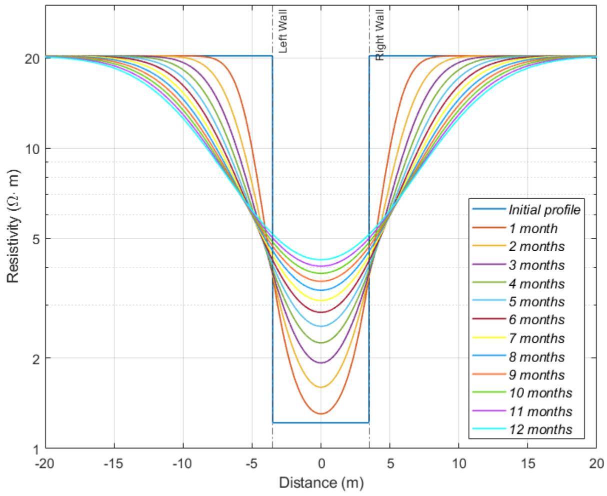

FIG. 3: Profiles for the evolution of resistivity with time obtained with the model. As in the temperature profiles from figure 2, each colour is assigned to the resistivity profile depending on the month in the cooling process it corresponds to.

year as shown in figures 6 and 7 in the appendix section V B. The second regime is found in the zone closest to the dike, from the walls up to around $1.5 - 2m$ from them. In this zone, temperature rises initially and at a certain time it starts cooling, which translates into an initial decrease in resistivity values followed by an increase that fits the anomaly from Piña-Varas et al.[3]. This is shown in figures 4 and 5 in the appendix section V A. Despite this behaviour being found only close to the walls, it does not guarantee the presence of meltback since the change from heating to cooling will be present in all positions in the cross-section if sufficient time for cooling is given.

IV. CONCLUSIONS

The model is capable of successfully simulating cooling processes on a transversal cross-section of a dike via temperature and resistivity profile evolution with time. The relevance of surrounding country rock initial temperature is confirmed to be crucial for the outcomes of volcanic eruptions as it regulates meltback regimes. On the same topic, another notable factor in the model is the importance of considering sufficiently large margins when computing, and not only the immediate surroundings of the walls, since the heat sink effect of a large rock body drastically changes the result of the simulation.

Furthermore, the model showed that it is possible to attribute the anomaly in resistivity detected in La Palma after the eruption to normal cooling processes. Meltback cannot be discarded as the model leaves a window of opportunity if the host rock is hot enough but cannot be confirmed either since the changes in resistivity can arise from the same cooling of the rock.

Acknowledgments

In loving memory of my father, without whom I would have never achieved this. I would like to thank my tutors Perla and Pilar for everything they have taught me and for giving me this opportunity and Marco Loncar for the advice provided. Finally many thanks to my family, especially my mother for her unconditional support.

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V. APPENDIX

A. Temperature and Resistivity profile evolution - Dike wall and nearby

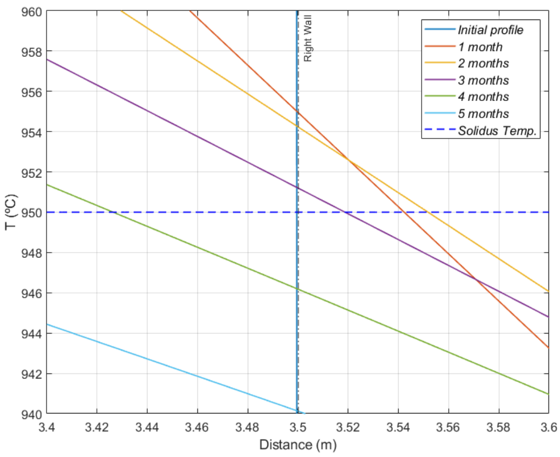

FIG. 4: Detail on the temperature profiles for the right wall of the dike and nearby showing the presence of slight meltback in the first three months. The initial increase and posterior decrease in temperature of the rock (right side of the wall) can be noticed.

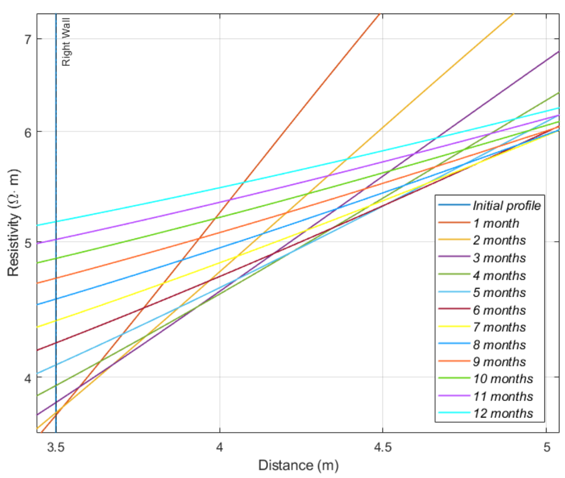

FIG. 5: Detail on the resistivity profiles for the right wall of the dike and nearby focusing on the decrease and increase of resistivity values as suggested by the anomaly in Cumbre Vieja volcano.

B. Temperature-Resitivity profile evolution - Far from the dike

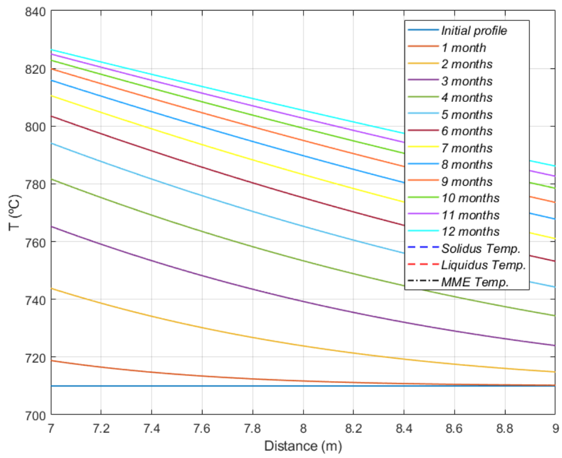

FIG. 6: Detail on the temperature profiles far from the dike wall showing the constant increase in temperature over the year.

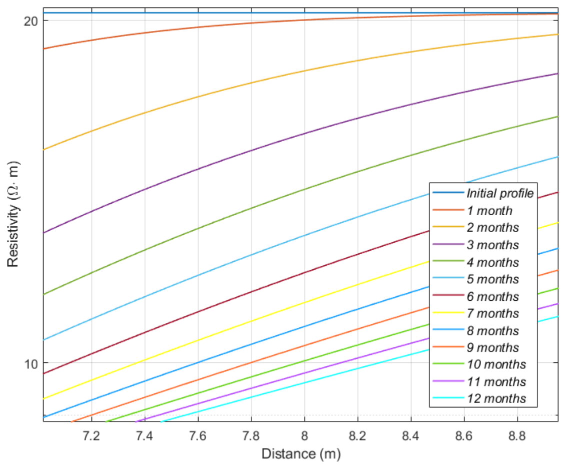

FIG. 7: Detail on the resistivity profiles far from the dike wall showing the slight decrease in resistivity over the year due to temperature increase

C. Heat equation with latent heat step applied

$$(1+Z)\frac{\partial T}{\partial t} = \kappa \nabla^2 T \tag{9}$$

$$Z = \left\{ \begin{array}{ll} 0 & T < T_s \\ \dfrac{L}{\rho c_p (T_l - T_s)} & T_s < T < T_l \\ 0 & T_l < T \end{array} \right\} \tag{10}$$