Study of the dispersion relation for an interacting magnon gas in a YIG film at room temperature.

Author: Joaquim Villena Galimany.
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.[*]

Advisor: Dr. Marius V. Costache
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Abstract: The focus of this *Treball Fi de Grau* (TFG) is the study of magnon's dispersion relation in a thin film of YIG at room temperature when applying an in-plane magnetic field. The magnon dispersion relation was first analyzed in order to better understand the conditions for the BEC: material thickness and applied magnetic field direction and intensity. Secondly, the ferromagnetic resonance was experimentally studied in a thin YIG film with the theoretically analyzed geometry, more specifically the uniform mode ($k = 0$) and spin-wave resonance.

I. INTRODUCTION

The ground state for a ferromagnetic material is a uniformly magnetized state, at zero temperature. As the temperature increases, there appear fluctuations on the individual magnetic moments - also called spins - that precess around their equilibrium directions. The lowest excited state allows for the appearance of magnons throughout the material. A magnon is the quasiparticle produced by the collective fluctuation of the spins in a crystal lattice. This quasiparticle behaves as a boson with spin-1 and can also be interpreted as a spin wave (SW), due to the quantum mechanical wave-particle duality. Every wave follows a dispersion relation which relates its frequency f_k to its wavelength λ or wave number k, and has different normal modes.

The accurate dispersion relation for a magnon takes into account the same interactions that have to be considered in a spin system: a dipolar interaction and an exchange interaction. The first one is linear to the momentum, whereas the second one scales quadratic to it. Therefore, depending on the domain, one interaction will prevail over the other: at low k the dipolar interaction overshadows the exchange interaction and at high k the opposite happens. There is a transitional regime where both have the same order of magnitude and their competition results in a minimum in the dispersion relation with degeneration two [4].

Magnons can be injected through many different ways, one of the most relevant being ferromagnetic resonance (FMR) pumping. Here one applies an alternating (AC) microwave pumping magnetic field into the material, perpendicular to an additional static field. The static field will magnetize the magnetic moments in a particular direction while the AC field will change their precession if the field's frequency is equal to the Larmor frequency. In that case magnons will be excited into the material.

In some particular conditions, the injected magnons rapidly decay into the minimum with two degeneration described by the dispersion relation with opposite wave vectors thus condensing into a Bose-Einstein condensate (BEC). This may come as a surprise as BECs are usually linked to very low temperatures, but that is not the case for these quasiparticles. However, magnons can also be lost due to lattice interactions, and will only reach the minimum if the magnon-phonon interaction timescale is much greater than the magnon-magnon interaction. Yttrium-iron garnet (YIG), $Y_3Fe_5O_{12}$, is an interesting ferrimagnet candidate because the timescales previously mentioned are on the order of microseconds and nanoseconds respectively, allowing for a consistent formation of the BEC [1].

II. MAGNON DISPERSION RELATION

A thin ferromagnetic film with thickness d is considered. A Cartesian coordinate system is used where the x,z coordinates lie in the film's plane and z is always facing the direction of the applied static magnetic field H. There is a spin wave (SW) that propagates through the x-z plane with an angular frequency $\omega_k = 2\pi f_k$ and a stationed spin wave (SSW) along the y direction. Damon and Eshbach [2] and Hurben and Patton [3] studied this problem by solving the Landau-Lifshitz-Gilbert equation of motion

$$\frac{d\vec{M}}{dt} = \gamma(\vec{M} \times \vec{H}) - \alpha\vec{M} \times \frac{d\vec{M}}{dt}, \tag{1}$$

where the first term considers a steady state precession and the second term is a dissipative damping. When considering low values of k, thus only taking into account the dipole-dipole interaction, a transcendental equation can be obtained [2-4] that relates the frequency with the wave vector components

$$2(1+\kappa)(-\delta)^{1/2}\cot(k_y d) + \delta(1+\kappa)^2 - \nu^2\sin^2\theta_k + 1 = 0, \tag{2}$$

where θ_k is the angle between the wave vector $\vec{k}$ in the plane and the z direction, k_y is the wave number that

[*]Electronic address: jvilgal@gmail.com

characterizes the normal mode pattern in the direction normal to the film and d is the thickness of the film. The other parameters are related to characteristic frequencies by

$$\kappa = \frac{\omega_H \omega_M}{\omega_H^2 - \omega_k^2}, \quad \nu = \frac{\omega_k \omega_M}{\omega_H^2 - \omega_k^2}, \quad \delta = \frac{1 + \kappa \sin^2 \theta_k}{1 + \kappa} \quad (3)$$

where $\omega_H = \gamma H$, $\omega_M = \gamma 4\pi M_S$, $\gamma = g\mu_B/\hbar = 2.8$ GHz/kOe is the gyromagnetic ratio for the YIG and $4\pi M_S = 1.76$ kOe is a term associated with the saturation magnetization M_S of the YIG [4]. The perpendicular wave number k_y follows the expression

$$k_y = (-\delta)^{-1/2} k, \quad (4)$$

where k is the wave vector of the spin-wave propagation in the plane. Its value can be either imaginary or real depending on the value of the parameter δ. For real values of k_y the SSW transverse to the film follows volume magnetostatic modes with a magnetization transverse component like a sine or cosine function, and for imaginary values it follows surface modes with an exponential dependence decaying from the surface.

When considering the exchange interaction, an additional effective field can be applied such that the parameter ω_H becomes

$$\omega_H = \gamma(H + Dk^2), \quad (5)$$

where $D = 2JSa^2/g\mu_B = 2 \times 10^{-4}$ kOe μm^2 is the exchange stiffness for the YIG, J being the nearest-neighbor exchange constant and a the lattice parameter of the film. After introducing this new effective field and considering $\theta_k = 0$, the obtained dispersion relation is

$$\omega_k^2 = \gamma^2 (H + Dk^2)(H + Dk^2 + 4\pi M_S). \quad (6)$$

In order to obtain an explicit expression that includes both the dipole-dipole and the exchange interactions and is valid throughout all values of k, a microscopic view must be used considering the Heisenberg hamiltonian that incorporates both the dipole-dipole interaction and the exchange interaction at all values of k. After the use of the second quantization of the spin excitations revolving in the magnon creation and annihilation operators, one obtains an equation for the dependence of the spin-wave frequency on the wave vector in the x-z plane [4-6]

$$\begin{aligned} \omega_k^2 = \gamma^2 [&H + Dk^2 + 4\pi M_S (1 - F_k) sin^2 \theta_k] \\ &\times [H + Dk^2 + 4\pi M_S F_k] \end{aligned} \quad (7)$$

where

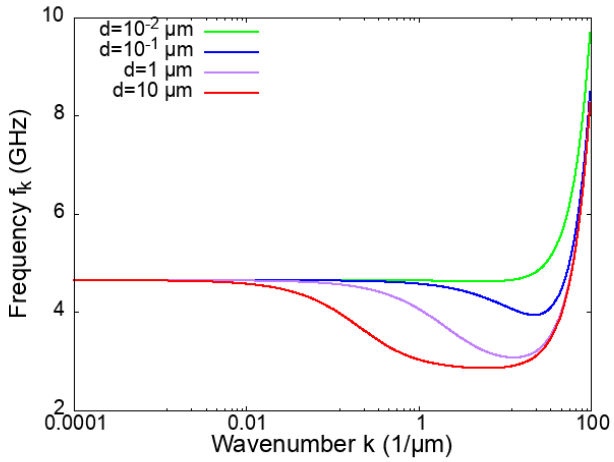
$$F_k = \frac{1 - e^{-kd}}{kd}. \quad (8)$$

When studying the YIG there are three main parameters that influence in the depicted relation: the angle θ_k, the applied magnetic field H and the thickness d of the film. The discussion, experimental data and analysis that follows has been treated with an angle $\theta_k = 0°$, which means that the transverse wave number k_y is imaginary and therefore the SSW is set in the bulk of the film. Then equation (7) becomes

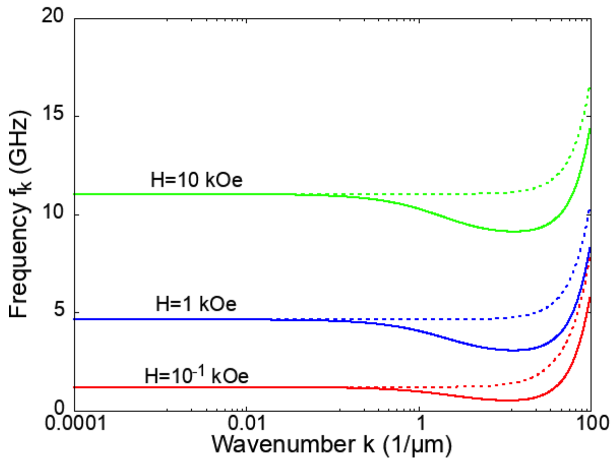
$$\omega_k^2 = \gamma^2 (H + Dk^2)(H + Dk^2 + 4\pi M_S F_k). \quad (9)$$

This last dispersion relation can be approximated as Eq.(6) for very small k, which is to be expected as the exchange interaction becomes negligible compared to the dipole-dipole interaction.

For different thickness d and different applied magnetic field H, the dispersion relation can be represented as shown in Fig.(1a) and Fig.(1b). Other plots of interest may address the deepness and width of the dips depicted for the study of the BEC of magnons, but that is not the focus of the current study.



(a)



(b)

FIG. 1: Representation of magnon dispersion relation Eq.(9) $f_k = \omega_k/2\pi$ for *a)* different thickness d and fixed field $H = 1$ kOe and *b)* different values of H and fixed $d = 1$ μm. The angle θ_k is fixed to 0 in both cases.

III. FMR THEORY AND SETUP

Ferromagnetic resonance (FMR) is a well-established experimental technique for magnetic sample characterization [7]. When a magnetic field is applied to a ferromagnet, its magnetic moments precess around the direction of that field. If an additional oscillating magnetic field is applied perpendicularly to the former, the moments' precession can change if the frequency of the oscillating magnetic field is equal to the Larmor frequency of the material. When all the previous criteria are met the material will absorb some energy to induce magnons.

Vector network analyzer-FMR (VNA-FMR) can sweep frequency and measure reflected/transmitted power while the external magnet can sweep the magnetic field providing 2D scans over both quantities allowing for a much bigger set of data. The sample is excited by passing the radio frequency (RF) signals through a coplanar waveguide (CPW) over which the sample is positioned (see Fig.(2)). On top of that AC field, a static DC in-plane field H generated by a Helmholtz coil and measured by a Hall probe is applied to magnetize the sample. When the VNA-FMR frequency $f = \omega/2\pi$ matches the Larmor frequency, magnon pumping occurs in the YIG film.

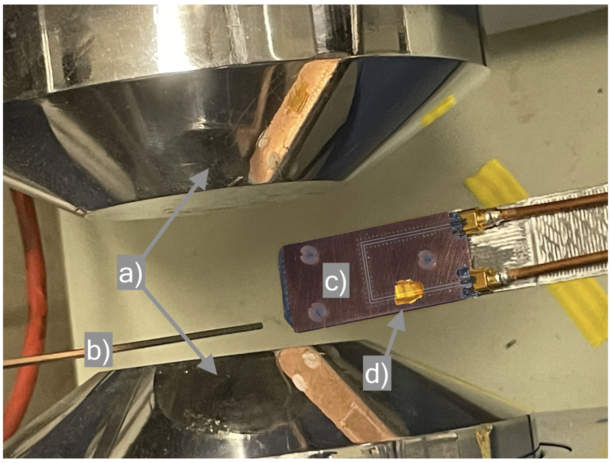


FIG. 2: Picture of the experimental setup with *a)* Helmholtz coils, *b)* Hall probe, *c)* CPW and *d)* YIG film in the described position.

For FMR to happen it is important that the static field is perpendicular to the alternating field in the film. The alternating field is closed around the CPW so there are two main components that influence the YIG's magnetization: one is in-plane and the other is transverse to the film (see Fig. (3)). Therefore, FMR pumping takes place through the perpendicular direction to the film.

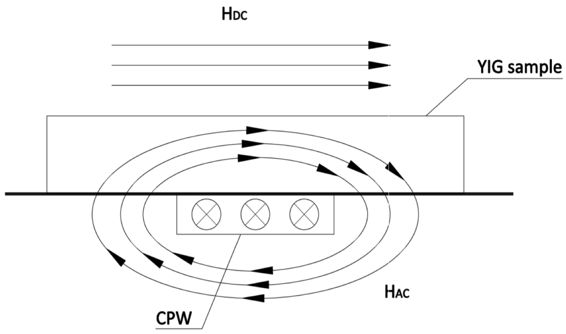


FIG. 3: Cross section of the CPW with the YIG sample and applied DC and AC magnetic fields.

IV. FMR ANALYSIS

The main measurements that were obtained by a computer connected to both the VNA-FMR and the Hall probe were the frequency of the AC current, the intensity of the DC field and the absorbed intensity by the YIG film to create the spin-waves. That intensity absorption results in an array of narrow peaks for different FMR pumping frequencies as shown in Fig.(4). As it can be inferred from Fig.(4), the peaks are symmetric for positive and negative values of the applied static field because when $\theta = 0$ or π, $\sin^2\theta = 0$.

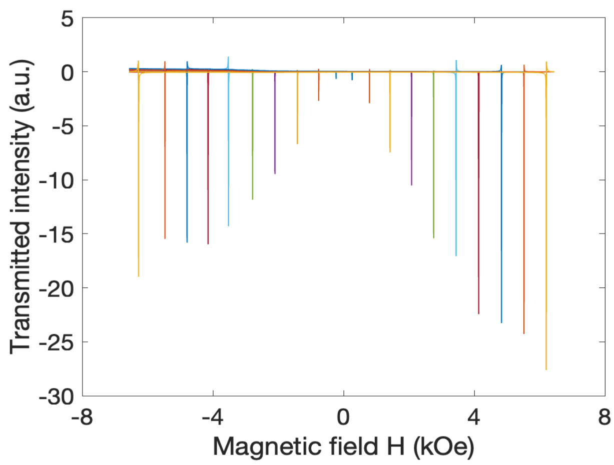
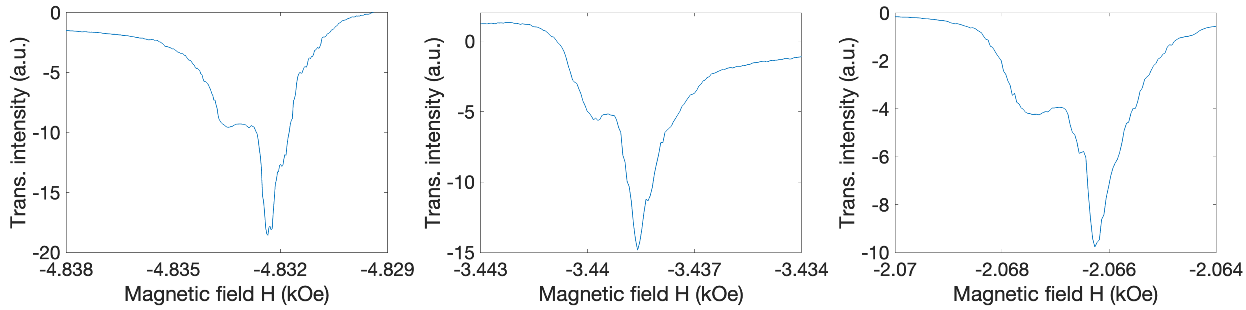


FIG. 4: Energy absorption by the magnon induction in the YIG film for different values of the frequency f_{FMR} as a function of the applied DC field H. From left to right, each minimum corresponds to frequencies 20 GHz, 18 GHz, ..., 2 GHz, 2GHz, ..., 18 GHz, 20 GHz.

In order to compare the experimental results with the YIG parameters from the literature [4] one can fit Eq.(9) with $k \approx 0$ to obtain experimental values for the gyromagnetic ratio γ and the demagnetizing field $4\pi M_S$.



(a) $f_{FMR} = 16$ GHz (b) $f_{FMR} = 12$ GHz (c) $f_{FMR} = 8$ GHz

FIG. 5: Transmitted intensity as a function of the applied DC field H. FMR peaks appear for various values of the FMR pumping frequency of magnons.

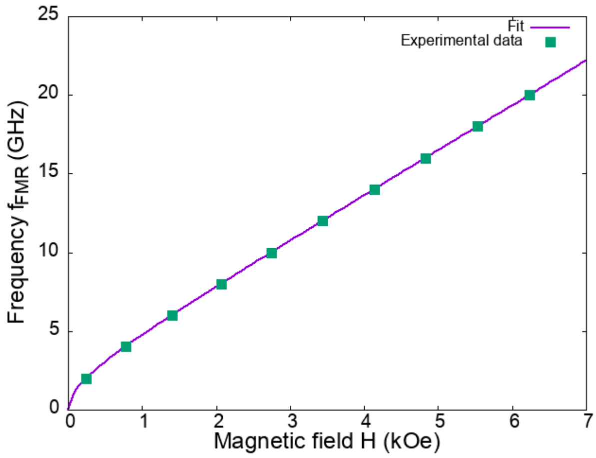


FIG. 6: Comparison of the pumping frequency f_{FMR} obtained experimentally as a function of the applied DC magnetic field H and its fit to Eq.(10). There is an error associated to both the frequency and the magnetic field, but it cannot be appreciated because they are of orders Hz and 10 Oe respectively.

Then Eq. (9) becomes

$$\omega_k^2 = \gamma^2 H(H + 4\pi M_S) \tag{10}$$

In Fig.(6) one can see that the results match up very well with the expected dispersion relation. From the fit the obtained parameters are $\gamma = 2.8209 \pm 0.0004$ GHz/kOe and $4\pi M_S = 1.825 \pm 0.002$ kOe.

After finding the FMR frequencies f_{FMR} for each applied DC magnetic field H, they were zoomed in to analyze the structure of the peaks with higher resolution. In Fig. (5) it can be seen there are additional peaks at around the same distances from the main minimum.

The local peaks that can be appreciated around the global minimum are related to the combined effect of both the dipole-dipole and exchange interactions. With that mixed interaction it is possible to induce dipole-exchange spin waves (DESW) into the YIG film near the FMR.

V. CONCLUSIONS

In this TFG, the magnon dispersion relation Eq.(9) for a thin film of YIG has been simulated, considering both the dipole-dipole and exchange interactions. Various thickness d and applied magnetic field H values have been considered, always maintaining the angle θ_k between wave propagation in the x-z plane and the field equal to zero.

In the experimental part, FMR pumping into a YIG film of thickness $d = 1$ μm was achieved with an AC current passing through a CPW while a static DC in-plane magnetic field generated by a Helmholtz coil magnetized the sample in the desired direction.

After the measurements were complete, the data was fitted to Eq.(10) and from the results both the gyromagnetic ratio and the saturation magnetization term where $\gamma = 2.8209 \pm 0.0004$ GHz/kOe and $4\pi M_S = 1.825 \pm 0.002$ kOe. From the literature [4] one finds that the expected values are $\gamma = 2.8$ GHz/kOe and $4\pi M_S = 1.76$ kOe. The relative discrepancy for γ is 0.7% and for $4\pi M_S$ is 3.70%. From these results it can be inferred that the assumptions taken with the choice of final dispersion relation to compare with and placement of the YIG film onto the CPW were correct.

Moreover, after zooming in the transmitted intensity peak, a rich structure can be appreciated for all pumping frequencies, showing a main minimum and various local minimums. The main peak is related to a pumping of magnons with wave number $k = 0$, therefore directly related with dipole-dipole interaction. From Eq.(4) we know that there is a transverse stationed wave with its own normal modes. Those local peaks may imply there is magnon pumping that depends on the exchange interaction, which is relevant for greater wave number.

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