Mutual Information for Free Fields

Author: Pau Duran Garcia*
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Advisor: Dr. Pablo Bueno Gómez
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The entanglement entropy (EE) and mutual information (MI) of spatial regions in general conformal field theories contain universal information which can be used to characterize the corresponding theory. In this study, we perform lattice calculations of these quantities for free scalars and fermions in three spacetime dimensions, which allows us to probe some of these features. In particular, we verify the expected long- and short-distance behaviour of MI in each model, determining the relevant universal coefficients. Based on our results, we conjecture that the leading coefficient in the long-distance expansion of the MI is maximized when the regions are round disks. Additionally, we use the EE of regions with geometric singularities to compute the two-point function stress-tensor coefficient, C_T, for a free scalar, obtaining an agreement of $\sim 97\%$ with respect to the exact value.

I. INTRODUCTION

The building blocks of a quantum field theory (QFT) are: i) a Hilbert space of states which carries a unitary representation of the (covering of the) Poincaré group $\mathfrak{P}$ and, among those, a unique state $|\Omega\rangle$ (the vacuum) which is invariant under all unitary transformations $U(g)$, $g \in \mathfrak{P}$; ii) a set of operators obtained from the smearing of quantum fields $\Phi(x)$ over spacetime regions in Minkowski space. Wightman reconstruction theorem asserts that correlation functions, *i.e.* vacuum expectation values of products of quantum fields, uniquely determines a QFT [1]. Thereby, quantum fields and Hilbert spaces can be reconstructed from the statistics of vacuum fluctuations:

$$\{\phi(x), \mathcal{H}\} \leftrightarrow \langle\Omega|\phi(x_1)\dots,\phi(x_n)|\Omega\rangle \tag{1}$$

Alternatively, QFT can be interpreted in terms of ("type III") von Neumann algebras [2]. Roughly, a von Neumann algebra is a set of operators closed under linear combinations, products and taking adjoints, and such that it coincides with its double commutant. Within this description, the fundamental objects are algebras of local observables associated to spacetime regions rather than fields operators acting on points. The relations between such algebras characterize the QFT.

In order to make this precise, we would like to assign numerical values to the algebras. These would correspond to statistical measures of the algebras in the vacuum state, similarly to (1). An obvious candidate for doing this is the so-called entanglement entropy (EE) which, for quantum mechanical systems, assigns numbers to the density matrix induced by the vacuum state on a given algebra of operators associated to a subsystem —see below. In the QFT context, the EE associated to a subregion algebra suffers from ultraviolet (UV) divergences associated to short-range correlations accross the region boundary. In order to avoid this issue, we can either introduce a UV regulator and extract the "universal" pieces contained in the EE, which are well-defined in the continuum, or consider alternative well-defined quantities, such as the mutual information (MI). This is defined in terms of the EE of two regions A and B as:

$$I_2(A,B) \equiv S(A) + S(B) - S(A\cup B)\,. \tag{2}$$

The aforementioned UV divergences always cancel up in this expression. The MI allows for the study of the relations between algebras. Analogously to the Wightman reconstruction theorem, the algebraic description leads to the natural question of whether the MI contains enough information to uniquely characterize a given QFT:

$$\{\mathcal{A}(A), \mathcal{H}\} \overset{?}{\leftrightarrow} I_2(A,B) \tag{3}$$

In this work, we will provide numerical evidence supporting the idea that MI can serve as a tool for uniquely determining the structure of a QFT, focusing on conformal field theories (CFTs). We will also explore the universal content of EE and MI in various situations and present a new conjecture regarding the shape dependence of MI.

The remainder of the TFG is as follows. In Section II we briefly introduce CFTs and some of their defining building blocks and properties. In Section III, we define the EE and comment on its general structure for general CFTs, with special emphasis in its universal features. In Section IV, we review the long-distance expansion behaviour of MI and how this can be used to systematically obtain the CFT data of a given theory. In Section V we present our lattice results.

II. CONFORMAL FIELD THEORY

Conformal field theory (CFT) is a theoretical framework that describes physical systems with invariance under conformal transformations: coordinate transformations that preserve angles locally, but not necessarily distances. Conformal transformations form a group which

*Electronic address: pduranga29@alumnes.ub.edu

is an extension of Poincaré's. Any CFT is described by the spectrum of the local primary operators and the operator product expansion (OPE) coefficients. The set of operators and OPE coefficients is called "CFT data".

A primary field operator is an operator $\mathcal{O}$ with the lowest scaling dimension Δ in a given representation of the conformal algebra. Its dimension describes how it transforms under scale transformations. For Lorentz invariant theories, if the operator has intrinsic spin it will become in irreducible representation of the $SO(d)$ group. All other operators in a representation are called descendants, since they can be obtained from the primaries by acting with spatial derivatives. For the free scalar and the free Dirac fermion fields in d spacetime dimensions, the lowest conformal dimensions read:

$$\Delta^{\text{scalar}} = (d-2)/2\,, \quad \Delta^{\text{fermion}} = (d-1)/2\,. \tag{4}$$

Conformal symmetry strongly constraints the form of correlation functions. For instance, the two-point function of two primary operators vanishes if their conformal dimensions are different, and it is given by [3]:

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta}}\,. \tag{5}$$

if $\Delta_i = \Delta_j \equiv \Delta$. The OPE allows us to compute any n-point function by recursively reducing it to $(n-1)$, $(n-2)$, ... and finally to a 2pt-function. The OPE replaces a product of two local operators by a convergent series of operators inserted at the mid-point in the limit where the operators become close to each other:

$$\mathcal{O}_i(x)\mathcal{O}_j(y) \simeq \sum_k C_{ij}^k(x-y)\mathcal{O}_k(y)\,, \tag{6}$$

where $C_{ij}^k(x-y)$ are the "OPE coefficients". The sum is done over the primary operators, and it would be an exact equal if we added the small contribution of the descendants. In essence, the OPE allows the calculation of all the correlators of the theory.

The stress tensor $T_{\mu\nu}$ plays an important operator in a CFT. Its existence is a natural extra assumption which ensures that the theory is local. The stress tensor must be conserved, $\partial_\mu T^{\mu\nu} = 0$, and it has to be traceless, $T^\mu_\mu = 0$, which ensures that the theory does not possess a preferred scale. For any CFT, the stress tensor vacuum 2pt-function in flat space is given by [4]:

$$\langle T_{\mu\nu}(x)T_{\lambda\rho}(0)\rangle = \frac{C_T}{|x|^{2d}}\mathcal{I}_{\mu\nu,\lambda\rho}(x)\,, \tag{7}$$

where $\mathcal{I}_{\mu\nu,\lambda\rho}$ is a dimensionless tensor structure fixed by symmetry (and hence, theory-independent). The charge C_T is a theory-dependent and universal quantity. Computing C_T for all kinds of CFTs has been a major target of the community for decades. For the free scalar and the free fermion, one finds [4] $C_T|_{\text{scalar}} = 3/(32\pi^2)$, $C_T|_{\text{fermion}} = 3/(16\pi^2)$, respectively.

III. ENTANGLEMENT ENTROPY IN CFT

A global state $|\Psi\rangle$ describing a quantum system consisting of two subsystems A and B is called separable if it can be written as a tensor product of pure states $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$. Otherwise, it is called "entangled".

A natural measure of entanglement of two subsystems is given by the von Neumann entropy of the reduced density matrix $\rho_A = \text{tr}_B\rho$, obtained by tracing over B,

$$S_A \equiv -\text{tr}_A(\rho_A \log \rho_A)\,. \tag{8}$$

This entanglement entropy is always positive and vanishes for a global separable state. It remains finite in a finite-dimensional quantum system, but it suffers from UV divergences in QFT due to the short rang interaction near the boundary ∂A of the subsystem A. For a d-dimensional CFT, the leading term in EE is always [5]:

$$S_{\text{EE}}(A) = c_0\,\frac{\text{Area}(\partial A)}{\epsilon^{d-2}} + \dots\,, \tag{9}$$

where ϵ is a UV cutoff and c_0 is a non-universal quantity (ill-defined in the continuum). This is valid in any state and it is sometimes called the *area law of EE*. Obviously, the continuum limit $\epsilon \to 0$ is divergent.

The EE contains universal terms which capture well-defined information about the theory. In even dimensions, the universal contributions consist of a logarithmic term which captures UV physics. In odd dimensions, the universal contribution is a constant term which, on the other hand, captures IR effects. This constant term depend on the theory and the shape of the entangling region. For $d = 3$ CFTs, one finds:

$$S_{EE}(A) = c_0\frac{\text{perimeter}(\partial A)}{\epsilon} - F(A) + \mathcal{O}(\epsilon)\,, \tag{10}$$

where $F(A)$ is a universal quantity that characterizes the theory. The value of $F(A)$ is minimized when A is a disk [6], $F(\partial A = \mathbb{S}^1) \equiv F_0$, and in that case it matches the Euclidean free energy on the sphere [7]. Consequently, F_0 provides a canonical normalization for $F(A)$. For the free scalar field and the free fermion, one finds $F_0^s \simeq 0.0638$ and $F_0^f \simeq 0.1095$ [8], respectively.

When regions have geometric singularities, like i corners with opening angles $\theta_i \in (0,\pi)$, a new logarithmic universal contribution appears in the EE [9]:

$$F(A) = -\sum_i a(\theta_i)\log\left(\frac{R}{\epsilon}\right)\,, \tag{11}$$

with R some characteristic length of A. The coefficient $a(\theta)$ is a cutoff-independent function. It is positive and satisfies $a(2\pi-\theta) = a(\theta)$. In the limits of a nearly smooth surface and a sharp corner, it behaves as $a(\theta \to \pi) \simeq \sigma(\pi-\theta)^2$ and $a(\theta \to 0) \simeq \kappa/\theta$. The nearly smooth coefficient is actually proportional to the central charge of the stress tensor through the general relation [10]:

$$\sigma/C_T = \pi^2/24\,. \tag{12}$$

IV. MUTUAL INFORMATION

Divergent terms of EE are always local and extensive along the boundary. Linear combinations of entropy for different regions can be taken to make each boundary contribution appear with a vanishing coefficient. The natural quantity with this property is MI and has the expression (2). The MI is a positive semidefinite quantity. For a given state, it is a measure of correlations (both classical and quantum mechanical) between the two algebras. For general theories, it satisfies the bound [11]:

$$I_2(A,B) \geq \frac{1}{2}\frac{(\langle \mathcal{O}_A \mathcal{O}_B\rangle - \langle \mathcal{O}_A\rangle\langle \mathcal{O}_B\rangle}{||\mathcal{O}_A||^2||\mathcal{O}_B||^2}\,, \tag{13}$$

where $\mathcal{O}_{A_i}$ is any operator with support in A_i. For CFTs, using the 2pt-function expression (5), it becomes: $I_2(A,B) \geq \alpha/|x_A - x_B|^{4\Delta}$, where α is a proportional constant and Δ is the lowest conformal dimension of the model. Cardy showed that this inequality is in fact saturated [12]. Using the OPE, the MI can be written as a long-distance expansion where the leading term reads

$$I_2(A,B) = c(d,\Delta)\frac{R_A^{2\Delta}R_B^{2\Delta}}{L^{4\Delta}} + \dots\,. \tag{14}$$

where R_A and R_B are characteristic sizes of the regions, $L \gg R_A, R_B$ is the separation between them and $c(d,\Delta)$ is a universal coefficient which depends on their shape. For arbitrary boosted spherical regions A and B, the coefficient $c(d,\Delta)$ can be identified exactly for general theories and dimensions. In the case in which the lowest-dimensional operator is a scalar, it reads

$$c(d,\Delta) = \frac{\sqrt{\pi}}{4}\frac{\Gamma(2\Delta+1)}{\Gamma(2\Delta+\frac{3}{2})}\,. \tag{15}$$

Generalizations for arbitrary spins are also known [13, 14]. Hence, if we are given the mutual information of a pair of distant spherical regions for an unknown theory, we can determine both the conformal dimension and the spin of the leading primary. From subleading orders in the expansion and with generalizations of the MI to more subregions one can reconstruct further information about the CFT, such as subleading scaling dimensions and OPE coefficients —presumably, the full CFT data [15–17].

V. LATTICE CALCULATIONS

In this section, we perform lattice calculations of MI and EE for free scalars and fermions in 3 spacetime dimensions. Our lattice setup consists of a two-dimensional set of regularly spaced points with total size 200. We consider different regions (for various purposes) in the lattice: equilateral triangles, right triangles, squares, regular pentagons, regular hexagons, regular heptagons, regular octagons and circles.

For the free scalar, fields satisfy canonical commutation relations $[\phi_i, \pi_j] = i\delta_{ij}$ and $[\phi_i, \phi_j] = [\pi_i, \pi_j] = 0$,

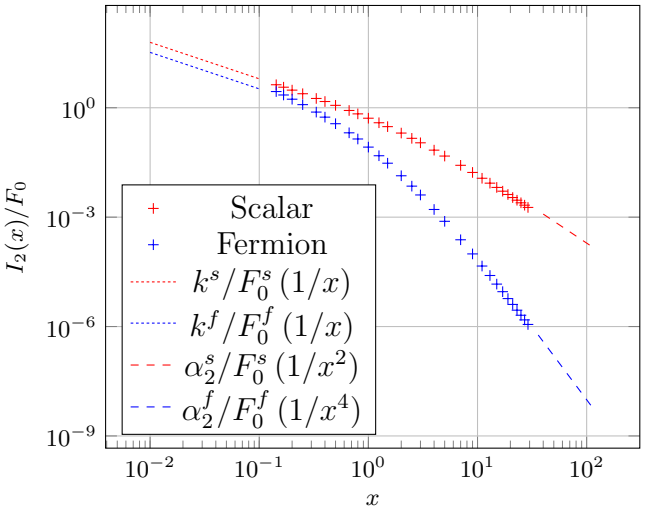

FIG. 1: For two squares in the lattice, the MI is computed as a function of the ratio x both for a free scalar (red) and a free fermion (blue). L is the length of the side of the squares, and L is the distance from one centre to the other. All MI values are normalized by its respective F_0. The theoretical behaviours for short and long distances are plotted as dashed lines and are found in good agreement with our results.

where i refers to a particular lattice point. Setting the lattice spacing to one, the Hamiltonian is given by:

$$H = \frac{1}{2}\sum_{i,j}[\pi_{i,j}^2 + (\phi_{i+1,j} - \phi_{i,j})^2 + (\phi_{i,j+1} - \phi_{i,j})^2]\,. \tag{16}$$

In an arbitrary region A, the vacuum EE can be computed using the correlators $X \equiv \langle \phi_i \phi_j\rangle$ and $P \equiv \langle \pi_i \pi_j\rangle$, with $C \equiv \sqrt{XP}$ as [9]:

$$S(A) = \mathrm{tr}[(C+1/2)\,\log(C+1/2) - \\ -(C-1/2)\,\log(C-1/2)]\,. \tag{17}$$

The vacuum correlators appear e.g., in [9].

The free fermion satisfies canonical anticommutation relations $\{\psi_i, \psi_j^\dagger\} = \delta_{ij}$, and their Hamiltonian is:

$$H = -\frac{i}{2}\sum_{n,m}\big[(\psi_{m,n}^\dagger \gamma^0\gamma^1(\psi_{m+1,n} - \psi_{m,n}) + \\ +\psi_{m,n}^\dagger\gamma^0\gamma^2(\psi_{m,n+1} - \psi_{m,n})\big) - \text{h.c.}\big]\,, \tag{18}$$

The EE in an arbitrary region A can be computed from the correlator $D \equiv \langle \psi_i \psi_j^\dagger\rangle$ as:

$$S(A) = -\mathrm{tr}\,[D\log D + (1-D)\log(1-D)]\,. \tag{19}$$

In each case, the MI can be evaluated as a function of the ratio $x \equiv L/R$, where L is the distance between regions and R is the characteristic size of the regions. Given a particular configuration keeping x fixed, MI is computed for increasingly greater values of R and L as MI asymptotically tends to a constant value as $R, L \to \infty$

in the continuum limit. We perform fits with the data calculated using functions $\{1, 1/x, 1/x^2, \dots\}$ and extract the continuum result in each case.

A. MI versus ratio for 2 squares

The MI for two regions with parallel sides at arbitrary separations [17] behaves as follows. For short distances for both free scalar and fermion fields we have:

$$I_2(x)/F_0 = \kappa/F_0\,(1/x) + \dots\,, \tag{20}$$

where κ is the universal sharp corner coefficient of $a(\theta)$. For long distances, we have in turn:

$$I_2(x)/F_0^s = \alpha_2^s/F_0^s\,(1/x^2) + \dots\,, \tag{21}$$

$$I_2(x)/F_0^f = \alpha_2^f/F_0^f\,(1/x^4) + \dots\,, \tag{22}$$

where we used the general result in (14). All results are normalized by the respective F_0 in order to compare both fields. In a doubly logarithmic scale, we should have linear behaviours with -1 slopes for short distances for both fields, and -2 and -4 slopes for long distances for the free scalar and fermion respectively. The lines should intercept the Y axis at a value $\log(\kappa/F_0)$ for short distances and $\log(\alpha_2/F_0)$ for long distances. The constants κ and α_2 have been previously obtained numerically [9, 16]:

$$\kappa^s \simeq 0.0397\,, \quad \kappa^f \simeq 0.0722\,, \tag{23}$$

$$\alpha_2^s \simeq 0.124\,, \quad \alpha_2^f \simeq 0.108\,. \tag{24}$$

Carrying out the aforementioned procedure, we display our lattice results in Fig. 1 for two square regions and arbitrary separations.

As shown in the figure, our numerics fit the theoretical behaviours nicely. We observe that the MI for the scalar is above the fermion one for general separations. This supports the conjecture proposed in [18], which states that for any three-dimensional CFT, the universal coefficient in the entanglement entropy of an arbitrary region A normalized by the disk result is bounded above by the free scalar result:

$$\frac{F(A)}{F_0} \leq \left.\frac{F(A)}{F_0}\right|_{\text{free scalar}} \quad \forall \quad \text{CFT}\,. \tag{25}$$

B. MI as a function of shape

In this subsection, focusing on the free scalar, we present results for different region shapes. We define now the long-distance coefficient as:

$$I_2(x) = \widetilde{\alpha_2}\,\frac{\text{perimeter}\cdot\text{perimeter}}{L^2} + \dots \tag{26}$$

namely, we use the perimeter of each figure as our characteristic scale. Figure 2 shows coefficient $\widetilde{\alpha}_2$ for identical pairs of regions at long distances. The values we find are: $\tilde{\alpha}_2 = 0.00490, 0.00656, 0.00701, 0.00747, 0.00768, 0.00782$ for regular polygons of $n = 3, 4, 5, 6, 7, 8$ sides, respectively. The value of $\widetilde{\alpha}_2$ grows as the number of sides increases, and it tends to the theoretical value corresponding to two identical disks. It is natural to speculate that this may be a general feature extending beyond free fields, similar to the global maximization of the EE for round disks occurring for general CFTs [6].

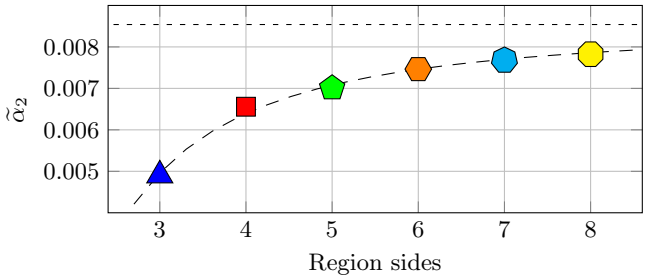

FIG. 2: For pairs of regular polygons in the lattice, the coefficient $\widetilde{\alpha}_2$ is plotted as a function of the number of sides. We consider equilateral triangles, squares, pentagons, hexagons, heptagons, octagons, and disks. The horizontal dashed line shows the limiting case of round disks, for which $\widetilde{\alpha}_2(\text{disks}) \simeq 0.00851$.

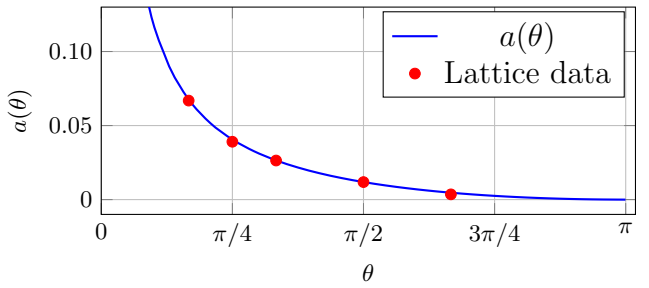

FIG. 3: Lattice values of $a(\theta)$ for different angles of θ (red dots) and exact theoretical function $a(\theta)$ (blue line) for a free scalar. Each value represented is found by computing the EE of a particular region for different sizes and making fits using functions $\{1, x, \log(x)\}$. The constant accompanying the log term is precisely $\sum_i a(\theta_i)$. $a(\pi/2)$ is found from 1/4 the EE of a square. $a(\pi/3)$ and $a(2\pi/3)$ are found analogously for the equilateral triangle and the regular hexagon dividing by 3 and 6 respectively. $a(\pi/4)$ is found from the EE of a right triangle with two $\pi/4$ angles by subtracting $a(\pi/2)$ off the total contribution and diving by 2 the result. $a(\pi/6)$ is found by computing a right triangle of angles $\pi/3$ and $\pi/6$ and subtracting $a(\pi/2)$ and $a(\pi/3)$ off the total contribution.

C. EE singularities

In this subsection, we perform lattice calculations of EE for singular regions, again for the free scalar. We compute several values of $a(\theta)$. These are extracted from the EE of various geometric regions, as explained in the

caption of Fig. 3. Our lattice data fits excellently the theoretical function [9]. Now, as shown in [19], it is possible to find accurate approximations of $a(\theta)$ by considering a linear combination of the form

$$a(\theta) = \lambda_1 \frac{(\theta-\pi)^2}{\theta(2\pi-\theta)} + \lambda_2[1 + (\pi-\theta)\cot\theta]\,, \qquad (27)$$

which has the correct functional behaviour in the sharp and almost-smooth limits of the corner. Using the theoretical value of κ shown in (23) and doing a fit of our data using the above trial function, we fit the values of $\lambda_{1,2}$ for the free scalar, and obtain a numerical value of the almost-smooth limit coefficient: $\sigma = 0.00377$. Finally, using the relation between this and the stress-tensor two-point function, we are able to extract the value of C_T for the free scalar from our lattice calculations. We find:

$$C_T^{\text{lattice}} = 0.00917\,, \quad C_T^{\text{theoretical}} \simeq 0.00950\,, \qquad (28)$$

which is in excellent agreement with the exact result.

VI. CONCLUSIONS

In this TFG we have performed various lattice calculations for free fields, with which we have probed several general features of EE and MI. We have verified the expected general behaviour of MI as a function of the regions separation and observed a consistent hierarchy of the scalar MI over the fermion one. We have also tested the dependence of MI on the region shape, obtaining a monotonically increasing result for the leading long-distance coefficient as a function of the number of sides of the region. As this tends to infinity, the value is maximized and agrees with the disks one. Finally, we have computed EE for singular regions, obtaining various data points for the corner function $a(\theta)$ for a free scalar. We found excellent agreement with the known results, and we were able to obtain the value of C_T with an accuracy of $\sim 97\%$. Overall, the lattice has proven to be an excellent tool for computing MI and EE. This experience opens the door to many additional studies, which we hope to explore in the future.

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