

# Exploring a coupling between dark energy and photons: cosmic dynamics and the variation of $\alpha$

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**Abstract:** Dynamical dark energy (DE) is an alternative to a positive cosmological constant  $\Lambda$ . In this work we model the DE dynamics by means of a quintessence scalar field, considering a coupling between DE and photons that triggers the cosmological evolution of the fine structure constant  $\alpha$ . We derive the coupled system of differential equations that rules the universe expansion at the background level and solve it numerically. We also constrain the model with state-of-the-art cosmological data and obtain strong upper bounds on the parameters that control the DE dynamics and the coupling with the electromagnetic sector. Finally, we compare the fitting results with those obtained from  $\Lambda$ CDM and uncoupled quintessence. As original contribution of our work, we have developed a code that is publicly available in [[https://github.com/amunozna/alpha\\_phi\\_CDM\\_coupling.git](https://github.com/amunozna/alpha_phi_CDM_coupling.git)].

## I. INTRODUCTION

Cosmological observations reveal that the universe is undergoing accelerating expansion. In the standard cosmological model,  $\Lambda$ CDM, this acceleration is explained using a positive cosmological constant ( $\Lambda$ ), which behaves like a perfect fluid with negative pressure [1]. An alternative approach is to consider dynamical dark energy (DE) by introducing, e.g., a time-evolving scalar field  $\phi$  in the action. Couplings between  $\phi$  and other fields in the theory are possible in principle and could leave an imprint on cosmological observables. This coupling could induce time variations in some of nature's fundamental physical constants, the detection of which would be a smoking gun of the existence of dark energy. Consequently, this could have groundbreaking implications for fundamental physics.

This work focuses on the coupling between the DE scalar field and photons, which triggers the cosmic evolution of the fine-structure constant  $\alpha$  that controls the interaction strength between charged particles and light,

$$\alpha = \frac{e^2}{4\pi\epsilon\hbar c}, \quad (1)$$

where  $e$  is the electric charge of the electron,  $\hbar$  the reduced Planck constant and  $\mu$  and  $\epsilon = c^2/\mu$  are the permeability and permittivity of vacuum, respectively. As we will show in the following pages, in our model  $\mu$  and  $\epsilon$  pick a dependence on  $\phi$  and, thus, the dynamics of the scalar field translate into the variation of  $\alpha$ . In this work, we employ background cosmological data, including measurements of the variation of  $\alpha$  at different redshifts, to constrain our model.

The layout of this work is as follows. In section II, we present the action of our coupled DE model and the most relevant equations. In section III, we discuss the methodology used to solve the background equations numerically, as well as the data employed to constrain not only our model with and without coupling, but also the

$\Lambda$ CDM, which we treat as a benchmark model. Finally, in section IV, we discuss the results and present our conclusions.

## II. THEORETICAL FRAMEWORK

### A. Conventions

*Metric:* We use in this work the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which takes into account the symmetries imposed by the cosmological principle. The latter essentially states that space-time can be decomposed into completely homogeneous and isotropic hypersurfaces of constant cosmic time  $t$ , in agreement with the vast majority of cosmological observations. The line element reads,

$$ds^2 = a^2(\eta) \left[ -c^2 d\eta^2 + dx^i dx^j \delta_{ij} \right]. \quad (2)$$

where the scale factor  $a(\eta)$  encapsulates the dynamics of the universe,  $d\eta = dt/a(t)$  is the conformal time and  $x^i$  ( $i = 1, 2, 3$ ) are the comoving spatial coordinates.

As for the other geometrical quantities, the following sign conventions have been chosen:  $R^\lambda{}_{\mu\nu\sigma} = \partial_\nu \Gamma^\lambda{}_{\mu\sigma} + \Gamma^\rho{}_{\mu\sigma} \Gamma^\lambda{}_{\rho\nu} - (\nu \leftrightarrow \sigma)$  for the Riemann curvature tensor,  $R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$  for the Ricci tensor, and  $R = g^{\mu\nu} R_{\mu\nu}$  for the Ricci scalar.

*Electromagnetic field:* We also present the electromagnetic tensor, which reads,

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (3)$$

with  $A^\mu = (\phi/c, A^i)$  ( $= g^{\mu\nu} A_\nu$ ) the 4-potential,  $\phi$  the electric potential and  $\vec{A}$  the vector potential. The relation between the elements of the electromagnetic tensor and the electric and magnetic fields depends on the observer's velocity. An observer with four-velocity  $u^\mu$

decomposes the electromagnetic tensor into electric and magnetic parts [2], as follows:

$$F_{\mu\nu} = \frac{1}{c^2} (u_\mu E_\nu - u_\nu E_\mu) - \frac{1}{c} \eta_{\mu\nu\kappa\theta} u^\kappa B^\theta, \quad (4)$$

with the 4-vectors  $E^\mu$  and  $B^\mu$  orthogonal to the observer's 4-velocity  $u^\mu$ , i.e.  $u^\mu E_\mu = u^\mu B_\mu = 0$ . This ensures that  $E$  and  $B$  are three-vectors in the observer's rest space. Thus, we have,

$$E_\mu = F_{\mu\nu} u^\nu, \quad B_\mu = \frac{1}{2c} \eta_{\mu\nu\theta\alpha} u^\nu F^{\theta\alpha}, \quad (5)$$

with  $\eta_{\mu\nu\theta\alpha}$  the covariant Levi-Civita tensor, which can be written in terms of the Levi-Civita symbols as follows,

$$\eta_{\mu\nu\theta\alpha} = -\sqrt{-g} \epsilon_{\mu\nu\theta\alpha}, \quad \eta^{\mu\nu\theta\alpha} = \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\theta\alpha}. \quad (6)$$

For a comoving observer in a FLRW universe, we can write,  $u^\mu = \frac{dx^\mu}{d\tau} = \left(\frac{c}{a}, \vec{0}\right)$  where  $\tau$  is the proper time of the observer. Hence, from Eq. (4) we obtain,

$$F_{0i} = -\frac{a}{c} E_i, \quad F_{ij} = a^3 \epsilon_{ijk} B^k. \quad (7)$$

Finally, we define the dual electromagnetic field tensor as  ${}^*F^{\mu\nu} = \frac{1}{2} \eta^{\mu\nu\alpha\beta} F_{\alpha\beta}$ .

## B. The action and the coupling

We consider the following action,

$$S = \int d^4x \sqrt{|g|} \left( \frac{R}{16\pi G c^{-3}} + \mathcal{L}_m + \mathcal{L}_\phi + \mathcal{L}_{\gamma\leftrightarrow\phi} \right) \quad (8)$$

The first term corresponds to the Einstein-Hilbert action, followed by  $\mathcal{L}_m$ , which represents the Lagrangian density of the non-relativistic matter components (dark matter and baryons). We will assume it can be described as a perfect fluid stress-energy tensor (SET),

$$T_{\mu\nu}^m = \left( \rho_m + p_m \right) \frac{u_\mu u_\nu}{c^2} + p_m g_{\mu\nu}, \quad (9)$$

where  $\rho_m$  and  $p_m$  are the matter energy density and pressure, respectively. Actually, non-relativistic matter does not exert pressure, so  $p_m = 0$ . On the other hand DE is modelled as quintessence,

$$\mathcal{L}_\phi = -\frac{1}{2c} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi), \quad (10)$$

This action yields the following SET,

$$T_{\mu\nu}^\phi = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left( \frac{1}{2} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi + V(\phi) \right) \quad (11)$$

with non-zero components

$$T_{00}^\phi = a^2 \left( \frac{\dot{\phi}^2}{2c^2} + V(\phi) \right) = a^2 \rho_\phi, \quad (12)$$

$$T_{ij}^\phi = a^2 \left( \frac{\dot{\phi}^2}{2c^2} - V(\phi) \right) \delta_{ij} = a^2 p_\phi. \quad (13)$$

The dots denote derivatives with respect to cosmic time. We have neglected the spatial derivatives of  $\phi$  to respect the FLRW symmetries. The dynamics of the scalar field clearly depend on the shape of the potential. Here we opt to study the Peebles-Ratra (PR) potential,  $V(\phi) = \frac{1}{2} \kappa \phi^{-\lambda}$  [1]. In the last term of the action (8) we include the kinetic electromagnetic term and its coupling with  $\phi$ ,

$$\mathcal{L}_{\gamma\leftrightarrow\phi} = \frac{-1}{4\mu_0 c} \mathcal{G}(\phi) F_{\mu\nu} F^{\mu\nu}, \quad (14)$$

with  $\mathcal{G}(\phi) = e^{-\tau(\phi-\phi_0)}$ , being  $\tau$  the coupling parameter and  $\phi_0$  the current value of the scalar field. Its corresponding SET reads,

$$T_{\mu\nu}^{\gamma\leftrightarrow\phi} = \frac{\mathcal{G}(\phi)}{\mu_0} \left( F_{\mu\alpha} F_{\nu\beta} g^{\alpha\beta} - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g_{\mu\nu} \right). \quad (15)$$

For  $\tau = 0$  we retrieve the usual electromagnetic SET, as expected. If we define  $\mu(\phi) = \mu_0 / \mathcal{G}(\phi)$ , Eqs. (14) and (15) take the same form as in the uncoupled case, but  $\mu$  is promoted to a function of  $\phi$ . This dependence allows us to write the variation of  $\alpha$  as follows,

$$\frac{\Delta\alpha(\phi)}{\alpha_0} = \frac{\alpha(\phi) - \alpha_0}{\alpha_0} = e^{\tau(\phi-\phi_0)} - 1, \quad (16)$$

with  $\alpha_0$  the current value of  $\alpha$ . If  $\lambda \neq 0$  and  $\tau \neq 0$ ,  $\alpha$  becomes a function of the scale factor due to the dynamics of  $\phi$ . The components of Eq. (15) written in terms of the electric and magnetic fields read,

$$T_{00}^{\gamma\leftrightarrow\phi} = \frac{a^2}{2\mu(\phi)} \left( B^2 + \frac{E^2}{c^2} \right), \quad T_{0i}^{\gamma\leftrightarrow\phi} = \frac{\epsilon_{ijk} E_j B_k}{\mu(\phi)c}, \quad (17)$$

$$T_{ij}^{\gamma\leftrightarrow\phi} = \frac{-1}{\mu(\phi)} \left[ \frac{E_i E_j}{c^2} + B_i B_j - \frac{a^2}{2} \left( B^2 + \frac{E^2}{c^2} \right) \delta_{ij} \right]. \quad (18)$$

However, to be consistent with the cosmological principle, we need to consider spatial averages of the above quantities to obtain the background expressions. We use the following relations,

- $\langle E_i \rangle = \langle B_i \rangle = 0$ ,
- $\langle E_i E_j \rangle = \langle B_i B_j \rangle = 0$ , for  $i \neq j$ ,
- $\langle E^2 \rangle = 3 \langle E_i^2 \rangle \neq 0$  and  $\langle B^2 \rangle = 3 \langle B_i^2 \rangle \neq 0$ ,
- $\langle \frac{E^2}{c^2} - B^2 \rangle = 0$ . This implies  $\langle F_{\mu\nu} F^{\mu\nu} \rangle = 0$ .

The cancellation of the Lorentz invariant  $\langle F_{\mu\nu} F^{\mu\nu} \rangle$  in the presence of a non-zero coupling is ensured if the average amplitude of electric and magnetic fields are the same. We will show that this is actually the case in Sec. II D. In view of these arguments, the SET reduces to a diagonal tensor with the following non-zero components:

$$T_{00}^{\gamma\leftrightarrow\phi} = \frac{a^2 \langle E^2 \rangle}{\mu(\phi)c^2}, \quad T_{ij}^{\gamma\leftrightarrow\phi} = \frac{a^2 \delta_{ij}}{3\mu(\phi)} \frac{\langle E^2 \rangle}{c^2}. \quad (19)$$

Now, we can compare this result with the general form of the energy-momentum tensor of a perfect fluid, i.e. Eq. (9), finding that at the background level radiation behaves as a perfect fluid with energy density  $\rho_{\gamma\leftrightarrow\phi} = \frac{\langle E^2 \rangle}{\mu(\phi)c^2}$  and a constant equation of state (EoS) parameter  $w_{\gamma\leftrightarrow\phi} = \frac{p_{\gamma\leftrightarrow\phi}}{\rho_{\gamma\leftrightarrow\phi}} = \frac{1}{3}$ .

### C. Friedmann, pressure and conservation equations

In this section we find the equations that govern the dynamics of the universe and the energy densities of the various species. Firstly, we extremize Eq. (8) with respect to  $g^{\mu\nu}$  to obtain the Einstein field equations,

$$G_{\mu\nu} = \frac{8\pi G}{c^4} (T^m + T^\phi + T^{\gamma\leftrightarrow\phi})_{\mu\nu}. \quad (20)$$

Given the symmetries of a FLRW universe, we only are left with two independent equations, the Friedmann and pressure equations, respectively,

$$H^2 = \frac{8\pi G}{3c^2} (\rho_m + \rho_\phi + \rho_{\gamma\leftrightarrow\phi}), \quad (21)$$

$$-2\dot{H} - 3H^2 = \frac{8\pi G}{c^2} (p_m + p_\phi + p_{\gamma\leftrightarrow\phi}). \quad (22)$$

From the zeroth component of the Bianchi identity  $\nabla_\mu G^{\mu\nu} = 0$ , and the independence of the matter term from the rest of contributions of the right-hand side of Eq. (20), we obtain the matter conservation equation,

$$\dot{\rho}_m + 3H\rho_m = 0. \quad (23)$$

Hence,  $\rho_m \sim a^{-3}$ . If we extremize the action with respect to  $\phi$ , instead, we obtain the modified Klein-Gordon equation,

$$\square\phi - \frac{\partial V(\phi)}{\partial\phi} = \frac{1}{4\mu_0} \frac{\partial\mathcal{G}(\phi)}{\partial\phi} F^{\mu\nu} F_{\mu\nu}. \quad (24)$$

We can expand the d'Alembert operator from the last expression and rewrite it as

$$\ddot{\phi} + 3H\dot{\phi} + c^2 \frac{\partial V(\phi)}{\partial\phi} = -\frac{c^2}{4\mu_0} \frac{\partial\mathcal{G}(\phi)}{\partial\phi} F^{\mu\nu} F_{\mu\nu}. \quad (25)$$

Using Eq. (21), (22) and (25) we get,

$$\dot{\rho}_{\gamma\leftrightarrow\phi} + 4H\rho_{\gamma\leftrightarrow\phi} = \frac{c^2}{4\mu_0} \frac{\partial\mathcal{G}(\phi)}{\partial\phi} F^{\mu\nu} F_{\mu\nu}, \quad (26)$$

which corresponds to the conservation equation for photons. As pointed out in the previous section, the right-hand side of Eqs. (25) and (26) vanish, see also Sec. II D. Therefore the scalar field evolves as in the standard Peebles-Ratra model and radiation gets diluted following the usual law  $\rho_r \sim a^{-4}$ , since we are considering massless neutrinos. The contribution of the latter does not appear explicitly in Eq. (8).

### D. Modified Maxwell Equations

If we extremize our action with respect to the photon field  $A^\mu$  we find the modified inhomogeneous Maxwell equations,  $\nabla_\mu (\mathcal{G}(\phi)F^{\mu\nu}) = 0$ , which encapsulate the Gauss and Ampères laws,

$$\vec{\nabla} \cdot \vec{E} = 0, \quad (27)$$

$$\frac{-1}{\mathcal{G}(\phi)} \frac{\partial\mathcal{G}(\phi)}{\partial\eta} \frac{\vec{E}}{c^2} = \frac{3\mathcal{H}}{c^2} \vec{E} + \left( \frac{1}{c^2} \frac{\partial\vec{E}}{\partial\eta} - \vec{\nabla} \times \vec{B} \right), \quad (28)$$

where  $\mathcal{H} \equiv aH$ . Comparing Eq. (28) with the equation found in Minkowski we find two extra terms: the one in the left-hand side, which is proportional to the derivative of  $\mathcal{G}$  with respect to the conformal time; and the first term in the right-hand side. Both are actually proportional to  $\mathcal{H}$ , since  $\partial/\partial\eta = a\mathcal{H}\partial/\partial a$ . The last two terms of Eq. (28), instead, are proportional to the frequency of light, which is much larger than  $\mathcal{H}$ . This allows us to neglect the aforementioned extra terms and recover the Minkowskian equation. The homogeneous Maxwell equations can be obtained from the geometrical condition  $\nabla_{[\mu} F_{\nu\sigma]} = 0$ , where [...] stands for the antisymmetrisation of indices, which can be expressed in terms of the dual electromagnetic field tensor as  $\nabla_\mu {}^*F^{\mu\nu} = 0$ . Similarly, the modifications in the homogeneous equations vanish at the background level. They can be combined with Eqs. (27)-(28) to get the standard propagation equation of electromagnetic waves, with a constant  $c$ . Hence, we find that the amplitude of the electric and magnetic fields is the same and, therefore,  $\langle F_{\mu\nu} F^{\mu\nu} \rangle = 0$ .

## III. METHODOLOGY AND DATA

In the following lines we describe how to solve the system of coupled background cosmological equations in order to obtain  $\phi(a)$ ,  $H(a)$  and  $\alpha(a)$ . In general, Eq. (25) requires a numerical solution. Thus, it is convenient to rewrite it in terms of dimensionless variables. Using the convention  $' \equiv \partial/\partial a$ ,

$$\bar{\phi}'' + \left( \frac{4}{a} + \frac{\bar{H}'}{\bar{H}} \right) \bar{\phi}' - \frac{\lambda\bar{\kappa}}{2(a\bar{H})^2} \bar{\phi}^{-(\lambda+1)} = 0, \quad (29)$$

with  $\bar{\kappa} = \kappa\varsigma^{-2}\hbar^{3+\frac{1}{2}\lambda}M_{pl}^{-4-\lambda}c^{-5-\frac{3}{2}\lambda}$ ,  $M_{pl}$  the Planck mass and  $\bar{H} = H/\varsigma$ , expressed in terms of  $\varsigma = 1 \text{ Km/s/Mpc}$ . The latter also enables us to write the current value of the Hubble parameter as  $H_0 = 100h\varsigma$ , where  $h$  is the reduced Hubble parameter. To solve Eq. (29) with a finite step method we need an initial condition for the scalar field and its time derivative.

For  $\lambda > 0$  the PR potential fulfils the ‘‘tracker condition’’  $\Gamma = VV_{,\phi\phi}/(V_{,\phi})^2 > 1$  and the solution of Eq. (29) possesses the property of having an attractor-like behaviour, which means that a substantial family of functions overlap in the same trajectory. This mechanism

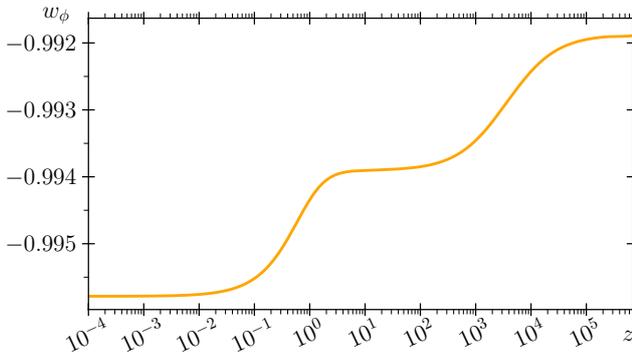


FIG. 1: Equation of state parameter of  $\phi$ ,  $w_\phi$ , as a function of  $a$ , obtained using the best-fit values from Table I. The plateaus in the radiation- and matter-dominated epochs are clearly visible (see Sec. III).

channels numerous initial conditions toward a shared final state, and we will make use of this fact. At  $a \approx 10^{-6}$  the Universe is deep in the radiation-dominated epoch. Power-law solutions of the form  $\phi(a) = A t^p$  are promising candidates during this period, where Eq. (21) simplifies considerably as we can take into account only the relativistic species. Considering this scenario in Eq. (29) we obtain the initial condition,

$$\bar{\phi}(a) = \left[ \frac{\lambda(\lambda+2)^2 \bar{\kappa}}{8 \cdot 10^4 (\lambda+6) \omega_r} \right]^{\frac{1}{\lambda+2}} a^{\frac{4}{\lambda+2}}, \quad (30)$$

with  $\Omega_r = \frac{8\pi G}{3H^2} \rho_r^0$  and  $\omega_r = \Omega_r h^2$  the density and reduced density parameters, respectively, and  $\rho_r^0$  the present value of the radiation energy density.  $w_r$  is fixed by the temperature of the cosmic microwave background (CMB) and the number of relativistic neutrino species.

Now that we have the initial condition, we need to compute the value of  $\bar{H}(a, \phi, \phi')$  and  $\bar{H}'(a, \phi, \phi')$  at each scale parameter step to be able to evaluate Eq. (29). From a convenient rephrasing of the Friedmann equation, we get,

$$\bar{H}^2 = \frac{\bar{\kappa} \bar{\phi}^{-\lambda}(a) + 1.2 \cdot 10^5 (\omega_m a^{-3} + \omega_r a^{-4})}{12 - a^2 \bar{\phi}'(a)^2}, \quad (31)$$

and using also the equations of conservation of each component, we obtain:

$$\bar{H}' = \frac{-3}{2a\bar{H}} \left( \frac{(a\bar{H}\bar{\phi}')^2}{6} + 10^4 (\omega_m a^{-3} + \frac{4}{3} \omega_r a^{-4}) \right) \quad (32)$$

For further details see [3]. We choose the Runge-Kutta 4 finite step method to solve this system of equations.

The equations displayed above are parameterised in terms of  $w_m$ ,  $\lambda$  and  $\bar{\kappa}$ , and obtain  $H_0$  as a derived parameter. We use a complete data set to constrain our model: (i) Supernovae (SNIa) apparent magnitudes  $m$ , whose theoretical value can be computed as follows,

$$m_{th}(z) = M + 25 + 5 \log_{10}(D_L(z)), \quad (33)$$

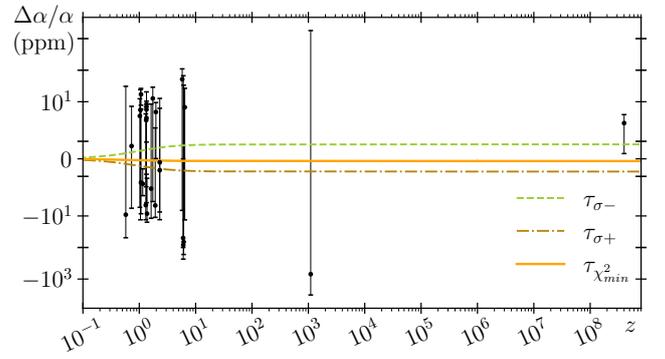


FIG. 2: Cosmological variation of  $\alpha$  as a function of the redshift  $z$ , see Eq. (16). The solid line corresponds to the theoretical curve calculated with the best-fit parameters, and the dashed lines represent those calculated with the  $\pm 1\sigma$  confidence level values of  $\tau$ . We also show the constraints employed in our fitting analysis.

where  $D_L(z) = (1+z)c \int_0^z \frac{dz'}{H(z')}$  is the luminosity distance.  $M$  is the absolute magnitude of SNIa and is treated as a *nuisance* parameter; (ii) data on baryonic acoustic oscillations (BAO) and the Hubble parameter at different redshifts, which include a set of angular distances and values of  $H(z)$ ; (iii) a CMB distance prior on the angular diameter distance to the last scattering surface; (iv) data on  $\Delta\alpha/\alpha$  from distant quasar absorption line spectra over the redshift range  $1.1 < z < 2.3$ , galactic and extragalactic measurements for  $5.8 < z < 6.4$ , and bounds from CMB and from Big Bang Nucleosynthesis (BBN). Finally, we also use the ratio of variation of  $\alpha$  at the present measured with atomic clocks,  $|\dot{\alpha}/\alpha| < 4.2 \cdot 10^{-15} \text{yr}^{-1}$ . This final dataset will allow us to constrain the values of the coupling  $\tau$ , cf. Eq. (16).

To constrain the models under study we use Bayesian statistics, which is based on the Bayes' Theorem,

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}, \quad (34)$$

for any two events  $A$  and  $B$ . We can identify  $A$  as our model and  $B$  as the data. The likelihood quantifies the probability of the data given a model,  $\mathcal{L} = P(B|A)$ .  $P(A)$  is our prior and quantifies the information we have about  $A$  before the employment of the new data. The quantity we are interested in is the so-called posterior distribution  $P(A|B)$ , the probability of having the model given the data, and  $P(B)$  is just a normalization factor. We can evaluate the likelihood given some set of values of our parameters as  $\mathcal{L} = \mathcal{N} \exp(-\chi^2/2)$ , with  $\chi^2 = (\vec{x}_B - \vec{x}_A)^T C^{-1} (\vec{x}_B - \vec{x}_A)$ .  $\vec{x}_B$  is the vector of data,  $C$  its corresponding covariance matrix, and  $\vec{x}_A$  the theoretical predictions of the model. We want to underscore that we are not interested in obtaining the normalization factor  $\mathcal{N}$ , only the shape of the distribution; therefore, the factors  $\mathcal{N}$  and  $P(B)$  are not important in this calculation.

|                        | $\bar{H}_0$                 | $\omega_m$                        | $\bar{\kappa} \cdot 10^{-4}$  | $\lambda$         | $\tau \cdot 10^6$         | $M$                               | $\chi^2_{\min}/\text{dof}$ |
|------------------------|-----------------------------|-----------------------------------|-------------------------------|-------------------|---------------------------|-----------------------------------|----------------------------|
| $\Lambda\text{CDM}$    | $68.8^{+0.7}_{-0.6}$ (68.8) | $0.142^{+0.004}_{-0.004}$ (0.141) | -                             | -                 | -                         | $-19.39^{+0.02}_{-0.02}$ (-19.39) | 1579.35/1746               |
| $\phi\text{CDM}$       | $68.4^{+0.7}_{-0.8}$ (68.6) | $0.141^{+0.002}_{-0.002}$ (0.142) | $3.71^{+0.26}_{-0.28}$ (3.89) | $< 0.099$ (0.007) | -                         | $-19.40^{+0.02}_{-0.02}$ (-19.39) | 1579.31/1745               |
| $\alpha\phi\text{CDM}$ | $68.4^{+0.7}_{-0.7}$ (68.5) | $0.141^{+0.001}_{-0.001}$ (0.141) | $3.71^{+0.28}_{-0.29}$ (3.83) | $< 0.110$ (0.012) | $0.0^{+5.0}_{-5.7}$ (0.9) | $-19.39^{+0.02}_{-0.02}$ (-19.39) | 1578.97/1744               |

TABLE I: Fitting results for the  $\Lambda\text{CDM}$  as well as the uncoupled and coupled  $\phi\text{CDM}$ . We display the mean values and  $1\sigma$  uncertainties together with the best-fit values (between brackets) of the various parameters.

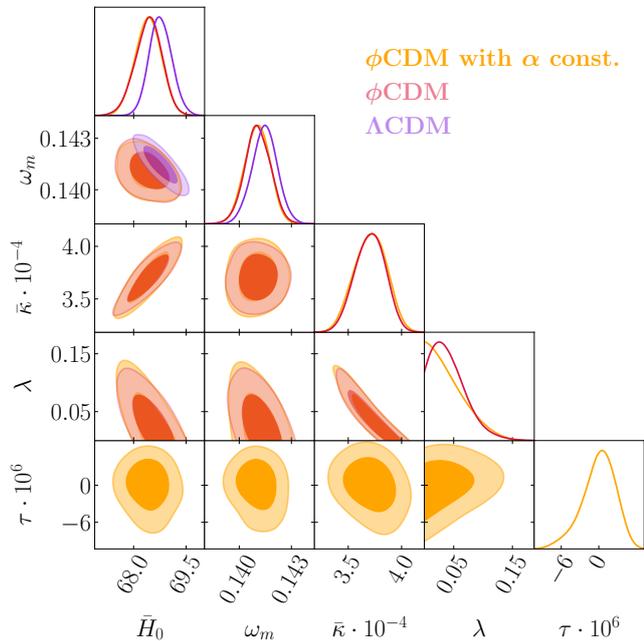


FIG. 3: Likelihood contour plots of the parameters of  $\Lambda\text{CDM}$  and  $\phi\text{CDM}$  with and without the coupling with  $\alpha$ .

With this setting, we use the Metropolis-Hastings algorithm to explore efficiently the parameter space. It essentially consists of choosing an arbitrary value for our parameter set and evaluating the posterior  $P(A|B)$ , then proposing a movement in the parameter space and calculating the new value  $P(A|B)'$ . If the ratio between them,  $P(A|B)'/P(A|B)$ , is larger than a uniformly generated random value between 0 and 1, the step is accepted; otherwise, it is not. The process is repeated until accumulating enough steps in the Monte Carlo Markov chain, leading to the convergence of the distribution of the parameters. Analysing this chain we extract the mean and best-fit values, and their associated uncertainties. For the priors, we choose for all the parameters wide enough uniform distributions that do not affect the shape of the posterior.

#### IV. DISCUSSION AND CONCLUSIONS

Our results are presented in Table I and Fig. (3). The employed data only allows us to find an upper bound on  $\lambda$  and  $\tau$ , showing no significant preference for dynamical dark energy nor the evolution of  $\alpha$ . The slight imbalance towards negative values of the coupling is induced mostly by the BBN constraint on  $\alpha$ , see Fig. (2). Weaker constraints on  $\tau$  for small values of  $\lambda$  is also depicted in Fig. (3) since in this limit the model reduces to non-dynamical DE, so the variation of  $\alpha$  goes to zero for any value of  $\tau$ , see Eq. (16). The constraints on the parameters shared by the three models are compatible in all cases, but they are looser in  $\phi\text{CDM}$  due to the higher dimensionality of its parameter space.

The discrepancy between the values obtained for  $H_0$  from measurements of the CMB and nearby redshifts is known as Hubble tension. It has been discussed in the past the fact that quintessence scalar fields do not help to alleviate this tension, but exacerbate it even further. This resonates well with Fig. (3), in which we show the existing anti-correlation between the Hubble parameter and  $\lambda$ . The distribution of  $H_0$  shifts to smaller values in the  $\phi\text{CDM}$  model.

Although the results of this study do not provide evidence supporting new physics, they also do not preclude it. An extended study involving CMB anisotropies and structure formation data could be performed in the future in order to go beyond the background level.

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