On a disputed argument in Galileo's *Discorsi*

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Abstract: Laying the foundations of the free fall motion in his well-known and latest book, the *Discorsi*, Galileo sets out an argument by which he aims to disprove that the velocity acquired by a naturally falling body increases proportionally to the distance descended through. In this paper, we shall contextualise the relevance of the study of free fall motion at the time his work was published, as well as review what different historians of science have stated about Galileo's reasoning.

I. INTRODUCTION

Galileo Galilei (Pisa, 1564; Arcetri, 1642) is commonly regarded as one of the major representatives in the genesis of a much-celebrated physical theory: Newtonian mechanics. In 1638, after his trial and condemnation by the Holy Office, Galileo managed to publish his last work, *Discorsi e Dimostrazioni Matematiche intorno à due nuove scienze*, also known as the '*Discorsi*' or 'Two New Sciences'. The contents of this book are spread over four days and the expository form is a dialogue in which three interlocutors take part: SALVIATI, who represents Galileo; SAGREDO, a learned spirit of his time; and SIMPLICIO, an Aristotelian philosopher [1]. This was considered by its author to be the principal of all his scientific labours, as it contained his most mature conclusions on the new science of motion he had established. In his treatise on naturally accelerated motion, Galileo presents a dissertation that appears to stand in the way of one of his most acclaimed results: the law of free fall. In this discussion, he assesses the question as to whether the velocity of a body in free fall increases uniformly with distance or with time. To get rid of the erroneous proportionality relation, $v \propto d$, and bearing in mind that Galileo never used the algebraic notation to which all physicists are nowadays accustomed, Galileo presents a logico-mathematical argument by which he aims to demonstrate the untenability of the speed-distance law of free fall. The essence of this reasoning is to point out that the latter hypothesis entails the existence of a logical contradiction, thus allowing the reader to overcome this misconception. However, it seems that the readers of this passage did not find it as "evident" and "easy" as Galileo had claimed [2].

The present article falls within the field of History of Physics and aims to offer a standard historical review of some of the readings of Galileo's published argument made (by various scholars) over the last 350 years. From the 17th century, the interpretations of Pierre Le Cazre and Alexandre Le Tenneur will be analysed. From the 19th century, that of Ernst Mach. From the 20th century, those of Bernard Cohen, Rupert Hall, and Stillman Drake. In the 18th century, the disputed passage had also been examined by at least three academics. However, I have not been able to obtain their references and, therefore, their insights will not be provided here. All this survey will be preceded by a brief section, which is intended to contextualise the increasing necessity of studying free fall motion in the 16th century, as well as to quickly retrace Galileo's evolution on this subject. Finally, some conclusions will be extended.

As for bibliography consulted, the Spanish translation of Galileo's *Discorsi* by Teófilo Isnardi and José San Roman Villasante has been my main source [1]. As far as I have been able to ascertain, this version strives to preserve Galileo's archaic wording and is updated from previous miss translations. Nevertheless, all quotations from Galileo's book found throughout this work belong to the English translation made by Stillman Drake [2], an American historian of science considered by many as one of the greatest authorities on Galileo in the 20th century.

II. FREE FALL MOTION

The understanding of the general concept of motion has always been a subject of great interest. This strong fascination for this topic can be exemplified in the old precept "To be ignorant about motion is to be ignorant about nature" [3]. For almost two thousand years, the basic ideas about motion proposed by the Greek philosopher Aristotle were at the very heart of the study and constant development of the science of motion. These notions were part of a complete system of science that was particularly well suited to the conception of a universe at the centre of which the Earth is at rest. In 1543, Nicolas Copernicus proposed an alteration of this picture of the universe in his well-known book *De revolutionibus orbium coelestium* [On the Revolutions of the Celestial Orbs]. The main feature of this work was the introduction of a new astronomical system, which rejected Earth's central position in the universe and considered our planet to be in motion. The (gradual) acceptance of this new approach not only ended up causing a deep transformation in the purely scientific sphere but also meant a real confrontation to the serious intellectual challenges (concerning the conception of ourselves) that arose in opposing the Aristotelian singularity of the Earth, based on its central fixed position in the universe. As Cohen writes in his book: "The shift from the concept of a stationary Earth to a moving Earth necessarily involved the birth of a new physics" [3]. I would only add that the fruit and expression of

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this major change was Newtonian physics. Thus, although Copernicus did not elaborate a new structure of physical principles that provided an adequate resolution to the kind of problems involved in considering our planet in motion, his book proved to contain, in the course of time, the seeds of the so-called great Scientific Revolution that took place during the 16th and 17th centuries.

One of the big intellectual hurdles facing the idea of a moving Earth was the nature of the free fall motion of a body. Galileo's first approach to this problem is described in *De motu antiquora,* in which he addresses this motion in a resistant medium and states that the speed of fall is proportional to the difference between the specific weight of the falling body and that of a like volume of the medium through which it fell. Galileo's final conclusion is that the acceleration of the falling body is a temporary event at the beginning of fall and that it is quickly replaced by a constant velocity. It must be mentioned that this work was never actually published by him, nor even mentioned in his extensive correspondence. As time went by, Galileo came to view acceleration as a continuing effect rather than the evanescent one he had previously assumed. Thereby, the acceleration of fall would eventually occupy a central place in his new insight of the problem and the continuous increase in velocity experienced by the body would become the essential characteristic of the free fall motion. With this remark in mind, Galileo set out to find the 'essence' of this process of variation that would enable him to deduce the properties of naturally accelerated motion; that is to say, what was the fundamental principle on which to build all subsequent structure. Finally (but not easily), Galileo became convinced of the great affinity between motion and time, and in his *Discorsi* he established that for naturally accelerated motion velocity increases uniformly with time. The establishment of the new correct law of falling bodies was one of Galileo's greatest achievements.

III. READINGS OVER GALILEO'S REFUTATION

On the third day of the *Discorsi*, Galileo begins his section on naturally accelerated motion by saying that "it seems desirable to find and explain a definition best fitting natural phenomena". Following some discussion of this stated aim, he finally suggests that "a motion is said to be uniformly accelerated, when starting from rest, it acquires, during equal time-intervals, equal increments of speed". Although this proposal is already correct, the dialogue between Salviati, Sagredo and Simplicio on this question shows that the acceptance of this definition is preceded by the examination of another principle. Sagredo opens the discussion and suggests that the nature of this motion could be made clearer by saying that "speed increases in proportion to the space traversed", a principle-definition with which Simplicio admits to be in full agreement. Then, Salviati asserts that in uniform accelerated motion this speed-distance law is "as false and impossible as [it is] that motion should be made instantaneously" and promises to provide "a very clear proof of it". Galileo's famous refutation on the proportionality of speeds in free fall to the distances traversed goes as follows:

> "If the velocities [passed through] have the same ratio as the spaces passed or to be passed, those spaces come to be passed in equal times; thus if the [instantaneous] velocities with which the falling body passed the space of four *braccia* were doubles of those with which it passed the first two *braccia* (as one distance is double the other), then the times required for these passages [over the spaces named] would be equal; but for the same moveable to pass the four *braccia* and the two in the same time cannot take place except in instantaneous motion. But we see that the falling heavy body makes its motion in time, and passes the two *braccia* in less [time] than the four; therefore, it is false that its speed increases as the space." [2]

Since its original publication in 1638, there have been many different views about what reasoning Galileo relied on in this argument.

A. 17th century

The whole controversy started in 1642, a few months after Galileo's death, with Pierre Le Cazre, a French Jesuit who quickly developed a sharp criticism towards Galileo's science of motion, which depended, as he stated, upon false principles. Regarding Galileo's refutation, I have not been able to find his reference but all the historians I have consulted agree on the fact that Cazre accused Galileo of having erroneously applied a previously formulated law for uniform motion to an accelerated one. As already suggested above, this was one of the many reservations Cazre had about Galileo's discussions of free fall, and perhaps in that spirit of objection, he translated Galileo's 'velocities' as just 'velocity' without attaching any importance to it. As Drake conveniently points out in his analysis of this topic: *traduttore-traditore* (translators are traitors) [4]. This mistranslation will reappear in the 20th century section, where this kind of vision is developed in more detail and the consequences of this little change are clarified.

Over the next few years, this subject became hotly debated and Marin Mersenne, thanks to his epistolary network and judgement, turned out to be the active director of this controversy. Knowing well that Tenneur was a staunch believer in Galileo's theory, Mersenne wrote to him at the end of 1645, asking to support Galileo's theory against Cazre's attacks. Replying to Mersenne, Tenneur appears to have successfully pushed back Cazre's criticisms. Moved by the strength of his convictions, he ultimately decided to incorporate his visions in *De motu naturaliter accelerato tractatus physico-mathematicus* (Paris, 1649). This turned out to be Tenneur's most important work as it shows him to be one of the few (if not the only) mathematician-physicist of his time to appreciate Galileo's reasoning in his rejection of space-proportionality. Although an understanding of this latter work is required for a standard historical evaluation of Galileo's argument, it will not be provided until later in this paper. The reason for this procedure is that Tenneur's original reference has been impossible for me to find; instead, I found out about his explanations from a reconstruction carried out by Drake in the 1970s. Therefore, this discussion will be presented alongside others of its corresponding century.

B. 19th century

Next on our list is Ernst Mach, a very well-known physicist and philosopher from the nineteenth century, who renewed academic interest in Galileo's argument by presenting a modern resolution to the problem of an acceleration in which v is proportional to s. Just as a physicist would do today, Mach stated that Galileo's initial assumption in contemporary (post-calculus) language read as:

$$\frac{ds}{dt} = as \tag{1}$$

being s the distance and t the time. Following from this, he then derived the formula:

$$s = Ae^{at} \tag{2}$$

where a is a constant of experience and A a constant of integration. Mach's remark on this expression was that it appeared to be "an entirely different conclusion from that drawn by Galileo" [5]. On one hand, he recognized that his expression was also contradicted by ocular evidence. He showed this by setting $t = 0$ and noticing that s would then be different from zero, a result that is clearly inconsistent with experience. On the other hand, however, he claimed that "in itself" the assumption was "by no means self-contradictory" [5]. Mach therefore concluded that Galileo's reasonings were not in accordance with a modern discussion of the problem, suggesting that the initial assumption Galileo sought to disprove was not inherently absurd, but simply did not match experience. In short, he said that Galileo's passage contained a "peculiar fallacy" [5].

After many years in which the weight of Mach's name seemed to have led most writers to accept his analysis uncritically, Cohen and Drake finally argued (in the mid-twentieth century) that Mach's formula was not free of self-contradiction as he stated. Although they both reasoned quite similarly, Cohen's objections will be here the main emphasis. In his article, Cohen describes Mach's statements as "extremely interesting" but "anachronistic" [5]. Having contextualised these words, he then draws attention to the consequences of considering the initial conditions that free fall motion must fulfil. Following his steps, if we first take equation (2) and set $s = 0$ when $t = 0$, then constant A must also be zero. This implies that if s is ever zero, it must be always zero. Furthermore, if we now consider equation (1) and set the velocity $\frac{ds}{dt} = 0$ when $t = 0$, then either $A = 0$ or $a = 0$. If we first consider $a = 0$, we find that there is no velocity and we have a universe at rest, a contradiction of the basic condition of a falling body. If we now consider $A = 0$, then $s = 0$ and the same holds true. All these observations made by Cohen share a common outcome: Mach's formula for natural acceleration in fall under space-proportionality assumption was much more complicated and meaningless than he originally suggested. These ideas are also very well outlined in Drake's own words. In the correspondence to Cohen's article, we find Drake strongly supporting him by saying that "any acceptable definition of uniform acceleration motion must allow s and t to be simultaneously zero by a suitable choice of coordinates" [6], but Mach's formula $s = Ae^{at}$ is only free of physical contradiction if $s \neq 0$ and $t \neq 0$.

C. 20th century

Modern commentators of the last century have also provided new insights into Galileo's disputed passage. These recent proposals may be better understood by looking at the context in which they were developed. In the 20th century, the evolution of Galileo's law of falling bodies became the subject of many studies and much debate. Reconstructing Galileo's line of thought on accelerated motion is, by no means, an easy task, but science historians venture to try it from two different paths. One is by undertaking a deep study of his manuscripts and correspondence and other such documents; the other is by attempting a summary of the public presentation that he published in his 'Two New Sciences'. The two main discussions that emerged in the twentieth century were sparked by these two distinct approaches of attempting to trace the development of his ideas.

At the beginning of that century, most scholars dedicated their efforts to analysing Galileo's published works. As the texts of such books may be interpreted from certain philosophical or historiographical conceptions, most of these studies were mainly guided by the general continuity criteria in the history of ideas; that is to say, by reconstructing Galileo's train of thought as following that of his predecessors. Thus, many academics such as Pierre Duhem and Alexandre Koyré focused their labours on a better understanding of the pre-Galilean kinematics and dynamics, a topic that Mach was completely unaware of when he drafted his renowned book on mechanics. As a result, a widely accepted picture of Galileo was built up, in which he was perceived as the heir of a well-established medieval tradition in the mathematical analysis of the problems of motion. A useful example that nicely captures and strengthens this commonly prevailing belief has to do with the so-called 'Merton Rule' and the renowned 'Galileo's 1604 Fragment on Falling Bodies' manuscript. This unpublished document is particularly famous as it was Galileo's first attempted derivation of the law relating space and time in free fall motion; that is, $d \propto t^2$. As for the Merton Rule, this was a theorem used by medieval mathematicians in their study of uniformly accelerated motion. This proposition, developed by the Oxford Calculators of Merton College, states that "the distance traversed in any given time-interval by a uniformly accelerated body is the same as if during that time-interval the body moved with a constant speed equal to the mean speed in the accelerated case" [5]. In short, it establishes a clear relationship between uniformly accelerated motion and uniform motion. This rule stands out as a significant example of scientific thought in the Middle Ages, showing that medieval scholars made important contributions to the development of physics and mathematics. According to the Aristotelian concept of change, medieval writers regarded specific change of any magnitude or 'form' as a transition from a definite *terminus a quo* to a definite *terminus ad quem;* in other words, change was considered to be a finite concept, as it had a beginning and an end. From a philosophical perspective, this problem was solved by identifying a single measure of the total change, and this is precisely what the Merton Rule did: it allowed them to represent an over-all uniform change by a single value. Under the widespread conviction that Galileo's study of certain medieval writings served him as an inspiration for his development of the free fall law, some historians such as Duhem, Koyré and Hall paid attention to the above-mentioned manuscript and reasoned that Galileo had used the Merton Rule in it. This way, their own preconceived vision was reinforced.

The whole context just outlined above plays an essential role in analysing the early modern translations of the passage in hand. Careful evaluation of these versions is, in turn, of utmost importance, as it provides the key towards a clearer comprehension of the first discussion arising in the 20th century. A simple comparison of the German (1891), English (1914), and French (1939) translations with the original Italian one, shows that their wordings were unfaithful to Galileo's precise words. Even more significant for the history of scientific thought is the fact that they all three agreed on the same mistake, but let us return to this fact later. This mistranslation has already been mentioned before and it is that of translating Galileo's 'velocities' as just 'velocity'. One might question the truthfulness of this accusation since the Italian word *velocitá* remains the same whether it is used in the singular or the plural; but the verb forms, relative pronouns, and definite articles that Galileo employed prove us right. Thus, we notice that the passage from which Cohen and Hall based his claim is:

> "If the velocities are in proportion to the spaces traversed, or to be traversed, then these spaces are traversed in equal intervals of time; if, therefore, <u>the velocity</u> with which the falling body traverses a space of eight feet were double…" [5]

From this wording, both historians suggested that Galileo's passage could be properly understood as a clever one-line argument by assuming a (mis)use of the Merton Rule.

In his paper, Cohen specifically writes that "Galileo's peculiar fallacy lay in an unjust application of this rule" [5]. First, he begins by showing that if one assumes that "velocity increases uniformly with respect to distance" [5], the Merton Rule gives as a result that the time required for a body to traverse any given distance (starting from rest) is always the same. This can be quickly demonstrated by the following example. Let us first assume a body falling from rest through a distance D_1 in a time T_1. Now, let us assume the same body starting from rest and falling through a distance D_2 in a time T_2. Since the velocity "is assumed to be proportional to the distance" [5], we can therefore write:

$$V_1 = kD_1 \qquad \text{and} \qquad V_2 = kD_2.$$

Let us now suppose that "we may apply the Merton Rule", so that:

$$D_1 = \frac{V_1}{2} \times T_1 \qquad \text{and} \qquad D_2 = \frac{V_2}{2} \times T_2.$$

Finally, gathering all the information, we get:

$$T_1 = \frac{2D_1}{V_1} = \frac{2D_1}{kD_1} = \frac{2}{k} \qquad \text{and} \qquad T_2 = \frac{2D_2}{V_2} = \frac{2D_2}{kD_2} = \frac{2}{k}.$$

This proof sets out very clearly that, under the space-proportionality hypothesis, a falling body must be in two places at the same time. Thus, not only is the existence of a logical contradiction proved but it is also demonstrated that this hypothesis is shown to be experimentally inconsistent, for it forbids a proper conception of duration. According to Cohen, this is how Galileo derived his result. The actual problem with this demonstration is that the Merton Rule can only be applied for a velocity increasing uniformly with respect to time. Indeed, this last assumption alone guarantees that "the area under the graph of velocity as ordinate plotted against time as abscissa yields distance" [5]. In other words: $D = \int VdT$. Finally, in his concluding lines, Cohen states that if the reasoning he has ascribed to Galileo is correct, then we would observe a "closer kinship" between Galileo and his fourteenth-century predecessors in kinematics than "is often recognized" [5].

In Hall's article, called 'Galileo's Fallacy', he also claims that Galileo, in his demonstration of the untenability of the speed-distance law for ordinary fall, relied on some mean-speed concept. However, although Cohen's mathematical result is perfectly valid, he asserts that Galileo is not interested in setting up a function relating time, velocity and distance; the algebraic notation is completely unfamiliar to Galileo, and he specifically says that Cohen's use of mathematical notation seems to him "hardly less alien to Galileo's than Mach's" [6]. Thus, his proposal on this matter is a geometrical visualization of the Merton Rule. Indeed, one only has to glance over Galileo's book to see that this geometrical method was "almost second nature to him" in tackling problems of this kind [6]. Moreover, this 'triangular notation' allows him to link this fallacy (for such it is, since the Merton Rule is not valid for the assumption of equal increments of velocity with equal increments of distance) with the other well-known embodied in 'Galileo's 1604 Fragment on Falling Bodies'. He therefore draws the conclusion that Galileo commits the same conceptual error in the *Discorsi* as he did in 1604.

As already mentioned at the beginning of this section, the second discussion that emerged in the twentieth century was highly shaped by an in-depth study of Galileo's manuscripts. This scrupulously researched study was carried out by Stillman Drake in the 1970s, and his research has much to say about the topics raised. However, our focus here will only be on two of them. On the one hand, Drake claimed that Cohen and Hall's readings were inappropriate as they were based on a mistranslation of the passage at issue. This conclusion is carefully elaborated in his article [4], where he convincingly shows that modern translations had altered Galileo's own words inadvertently and stresses that this is "hardly a mere coincidence". In the history of scientific thought, concordance in error often has a reason, and Drake assures that strong predictions were at work in this case. He particularly remarks that the modern translators who analysed these lines were "so deeply rooted" in the theory of historical continuity and "so well informed about the truths of physics" that "Galileo's strange syntax" did not catch their interest [4].

On the other hand, Drake thought that Tenneur had already "understood Galileo's reasoning exactly". In his article [4], he translates Tenneur's writings and incorporates the diagram in Figure 1. In short, Tenneur's reasoning goes as follows. First, he starts by assuming that the same heavy body Z runs through the two different spaces AC and XY in two different times, the space AC being twice the space XY. Then, he proposes to perform two consecutive bisections of the segments AC and XY. Next, he uses the hypothesis that the speeds are acquired as the spaces and, since AC is double XY, he reasons that there is no part in AC at the end of which the speed is not double the speed at the end of some homologous part in the space XY. Thus, by establishing the notion of one-to-one correspondence between two infinite aggregates he obtains that AC and XY are run through equal times by the same heavy body Z.

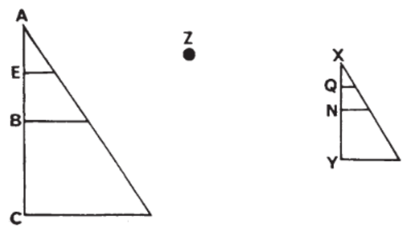

FIG. 1: Tenneur's diagram

Tenneur's argument ends here, and one might rightly argue that this result of covering two different spaces in equal time may be satisfied by either instantaneous or infinite motion. Drake addresses this matter by recalling that just before the 'famous' argument, Galileo had introduced the concept of a free-falling body passing through infinite degrees of velocity in a finite time. Thus, the equal time condition is only compatible with instantaneous motion, as Galileo had stated.

IV. CONCLUSIONS

In this paper, we have briefly outlined how Galileo's disputed passage has been interpreted by various science historians. Tenneur's reasoning, a contemporary of Galileo, allows us to read Galileo's text as a cogent argument. Mach's view mirrors a very common habit of translating Galileo's words into modern mathematical symbolism, leading to a false fallacy. Finally, Cazre's, Cohen's and Hall's readings show us how a tiny mistranslation may lead the history of science astray.

This work on the History of Physics also allows us to reflect on the evolution of scientific ideas and, therefore, on its concept of progress. Nowadays, the law of falling bodies is a simple physical law where the basic concepts of velocity and acceleration are easily related to the fundamental quantities of space and time. However, when we make a genuine effort to recover the stages of exploration and discovery of this law, a task that belongs to the historian, we realize that for such admired scholars as Galileo, this process was truly costly and fraught with errors. This should not puzzle us but rather warn us that the simplicity of such laws is only apparent. In other words, this law is not so clear except within a certain system of axioms and from a certain set of conceptions – about space, about motion, about the void, about the idea of mathematical instant – which are, by no means, simple. As stated by Koyré: "Is it not that they are too simple, like any basic notion, and therefore difficult to acquire?" [7].

On the other hand, the plurality of readings shown in Galileo's argument may be used as an example to illustrate how unstable it is to reconstruct the intentions of an author (in this case, Galileo) on a particular subject (in this case, on a reasoning) based solely on his public presentation. Thus, if we really want to re-examine the actual steps in an author's own thought processes, the study of his manuscripts, correspondences, and other such documents must also play a fundamental part. Borrowing the terminologies used by Alfred North Whitehead, it should be borne in mind that in most published presentations the contents are displayed in a preferred logical order, i.e. following a 'logic of the discovered' that often (if not mostly) fails to coincide with the 'logic of discovery'. Certainly, given more time and resources, I would have dedicated my efforts towards a greater understanding of Galileo's 'logic of discovery' [3].

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