Groundwater extraction for irrigation purposes: the case of asymmetric players

Valeriia Chukaeva* Julia de Frutos Cachorro† Jesús Marín-Solano†

Article published in *Water Economics and Policy* (2024)
DOI: 10.1142/S2382624X24500048

Abstract

We address the problem of groundwater exploitation by heterogeneous farmers for irrigation purposes. In particular, we study the possible inefficiencies that can arise in this type of common resource problem by considering the dynamic and strategic interactions between groundwater users. To this end, we build a two-player differential game in which two types of farmers (or many farmers grouped into two types, with a representative farmer for each group) display different characteristics related to their agricultural activity. More precisely, they can have different water demand functions, extraction costs, crop productivity, land types and time-preferences. Conditions are studied for the existence and uniqueness of the cooperative and non-cooperative solutions asymptotically converging to a steady state. The model is then applied to the case study of the Western La Mancha aquifer. Effects of the different heterogeneities on the degree of inefficiency of non-cooperative solutions with respect to cooperative solutions are analyzed. Numerical results show that cooperation is always beneficial for the environment and for the agents: it results in higher levels of groundwater stock and total welfare. Moreover, considering heterogeneous time preferences is crucial for reducing the inefficiency of non-cooperation with respect to cooperation, regardless of the other asymmetries between farmers.

Keywords: Groundwater resource; Cooperative vs Non-cooperative solutions; Asymmetric players; Differential game.

JEL: C70, Q15, Q25

*Department of Economics, School of Business, Economics and Social Sciences, University of Graz, Universitätsstraße 15 / F4 8010 Graz, Austria. E-mail: valeriia.chukaeva@uni-graz.at (Corresponding author).

†Departament de Matemàtica Econòmica, Financera i Actuarial and BEAT, Universitat de Barcelona, Avinguda Diagonal 690, 08034 Barcelona, Spain.

1 Introduction

Groundwater is one of the essential life resources not only for humans, but for the whole ecosystem in general, constituting 98% of world's available fresh water (not counting icebergs and glaciers) (UNESCO World Water Assessment Programme, 2009). Groundwater is mainly used for irrigation of the crops (accounting for more than 70% of total water withdrawals (Siebert et al., 2010)), for consumption in urban areas, or as an important component of certain industrial processes. In recent decades, the growing population and expanding industrial activities have increased the pressure on water quantity and quality. As a result of excessive withdrawals of water, water scarcity and the degradation of water quality have become widespread problems in most arid and semiarid regions around the world (Esteban and Albiac, 2011). For instance, among other cases, the Indus-Ganges (India, Pakistan, Bangladesh, Nepal) basins, the Ogallala aquifer in the North America, and many aquifers in Northern China plain and Europe (Spain, France and Portugal, for example) are suffering from groundwater depletion.

Access to groundwater common-pool resources is often restricted to land owners overlying the aquifer who compete for the limited resource, as explained in Roseta-Palma (2003). Hence, private exploitation by competing users, or in other words, non-cooperative behavior between groundwater users, creates externalities, such as cost and strategic externalities, which can lead to the inefficiency of non-cooperative (or competitive) solutions with respect to the socially optimal (or Pareto-optimal) cooperative solution[1] (Rubio and Casino, 2001). The cost externality arises because pumping by one user lowers the water table and, therefore, increases the cost of extraction for all other users of the aquifer. The strategic externality is a result of competition for the limited resource among users over time, in the sense that, what is not pumped by one groundwater user today, will be pumped by the other users (Sears, Lim, and Lawell, 2019).

Focusing on groundwater used for irrigation (the most common use in the case of most aquifers), this paper investigates whether cooperation between the groundwater users (i.e., the farmers) is beneficial for the environment (in terms of groundwater stock), as well as for the farmers themselves (in terms of welfare) with respect to non-cooperation, when farmers show different asymmetries related to their agricultural activity.

One of the main studies on groundwater management for irrigation use was published

[1]In the socially optimal cooperative solution, a social planner decides on the agents' extraction behavior that maximizes the sum of discounted collective profits. It is computed by solving an optimal control problem.

in 1980 by Gisser and Sánchez (1980). In this classic paper, the authors analytically compared the competitive (free market) and the socially optimal solutions of the water-table heights and extractions and concluded that, when the storage capacity of the aquifer is relatively large, the difference between solutions is negligible. Their theoretical results were illustrated numerically for the case of the Pecos River Basin in New Mexico. Therefore, regulation through the intervention of a social planner would not be justified if the capacity of the aquifer is big enough. This effect is known as Gisser-Sánchez Effect (GSE) and it has been discussed and challenged in subsequent works, such as Negri (1989), Provencher and Burt (1993), Rubio and Casino (2001), and Esteban and Albiac (2011). For instance, Esteban and Albiac (2011) extended the Gisser and Sánchez model by adding an environmental externality, which arises because the depletion of large aquifer systems causes environmental damages in linked ecosystems, and argued that socially optimal outcomes are preferable to competitive outcomes even for large aquifers. However, in Gisser and Sánchez (1980) and Esteban and Albiac (2011), the competitive solution was computed by considering that agents behave myopically, that is, making a decision over a short period of time without taking into account the impact of other agents' decision on the stock of the aquifer.

When considering the rationality of competing users by using game theory, numerous studies (e.g., Negri (1989), Provencher and Burt (1993), Rubio and Casino (2001), and de Frutos Cachorro, Erdlenbruch, and Tidball (2019)) have compared non-cooperative (or competitive) solutions with the cooperative (or socially optimal) solution and concluded that non-cooperative solutions are less favorable for the sustainability of the resource (i.e., lower stock levels are obtained under non-cooperation) in comparison to the cooperative solution. In the papers mentioned above, the competitive solutions were studied under different information structures, namely open-loop and Markov perfect (also called feedback) Nash equilibria[2], which capture the dynamic and strategic externalities that arise when groundwater users share the resource. In particular, Negri (1989) explained that while the open-loop equilibrium only captures the extraction cost externality, the Markov perfect Nash equilibrium also captures the strategic externality. As a result, the difference

[2] As described in the literature (e.g., Negri (1989) and Rubio and Casino (2001)), in the open-loop Nash equilibrium, farmers commit at the beginning of the planning horizon about their extraction behavior over time that maximizes the present value of the sum of future individual profits taking as given the extraction strategy of the others farmers. When commitment is not possible and the water-table level (or equivalently, the stock level) can be observed by the farmers at every moment, it is more realistic to assume that farmers' strategies do not only depend on time but also on the stock level, so the farmers take their decision according to their Markov perfect (or feedback) Nash equilibria

between the socially optimal (or cooperative) solution and the Markov Perfect equilibrium, in terms of the water table level (or equivalently in terms of the stock) at the steady state, is positive, and as defined in previous studies in the literature (Negri, 1989; Rubio and Casino, 2001; de Frutos Cachorro, Erdlenbruch, and Tidball, 2019), represents the level of (stock) inefficiency of non-cooperation with respect to cooperation. Similarly, the difference in total welfare obtained under cooperation and non-cooperation represents the welfare inefficiency, or in other words, the gain from cooperation. Most of previous studies (Negri, 1989; Rubio and Casino, 2001; de Frutos Cachorro, Erdlenbruch, and Tidball, 2019) confirmed that cooperative and non-cooperative solutions get closer under the important assumption of large aquifers (i.e., the GSE effect).

Focusing on the literature that applies dynamic game theory, while previous studies (Negri, 1989; Rubio and Casino, 2001; de Frutos Cachorro, Erdlenbruch, and Tidball, 2019) consider identical agents, the level of (stock and/or welfare) inefficiency of non-cooperation has been shown to vary depending on the type and level of asymmetries between the players in theoretical studies (e.g., Roseta-Palma and Brasão (2004) and Erdlenbruch, Tidball, and van Soest (2008)) and/or by computing numerical simulations in real study cases (e.g., de Frutos Cachorro, Marín-Solano, and Navas (2021) and Sears, Lim, and Lawell (2019)). Following the idea of Gisser and Sánchez (1980), cooperation could not be justified if the level of inefficiency of non-cooperation in terms of stock and/or welfare is not sufficiently large.

Roseta-Palma and Brasão (2004) and de Frutos Cachorro, Marín-Solano, and Navas (2021) demonstrated that asymmetries in demand between the players resulting from different water use (irrigation and public supply of urban areas) lead to higher (stock) inefficiency of non-cooperation with respect to cooperation. While Erdlenbruch, Tidball, and van Soest (2008) arrived to a slightly different result: they considered heterogeneity in opportunity costs of resource harvesting and concluded that the scope for cooperation is largest for intermediate levels of heterogeneities. Using calibrated data for the particular case of California, Sears, Lim, and Lawell (2019) considered different farmers' heterogeneities and concluded that the benefits from cooperation are particularly important when crop prices are high. At the same time, de Frutos Cachorro, Marín-Solano, and Navas (2021) showed that, when agents exhibit different time preferences during a fixed finite period of time, the inefficiency of the Markov Perfect non-cooperative solution in terms of stock and welfare decreases for the Western La Mancha aquifer in Spain.

This study contributes to this ongoing discussion. We investigate how the level of inefficiency of non-cooperative solutions in terms of stock and welfare is influenced by an increase in the level of different types of asymmetries between the farmers. Firstly, unlike Roseta-Palma and Brasão (2004) and de Frutos Cachorro, Marín-Solano, and Navas (2021), we assume that farmers use groundwater exclusively for irrigating their crops. Secondly, in contrast to the work of Erdlenbruch, Tidball, and van Soest (2008), the cost asymmetry we consider concerns the extraction costs, not the opportunity costs. Next, similar to Sears, Lim, and Lawell (2019), we allow for other differences in crop productivity and the type of the land the farmers own (i.e., different water percolation of the land) while considering a more simplified dynamics in the model[3], like most of the theoretical literature on groundwater resources, e.g., Rubio and Casino (2001). Finally, in contrast to the above mentioned papers, we assume that the agents can have different time preferences on the whole infinite planning horizon.

To address the research question, we analytically solve a differential game in which asymmetric farmers decide how much water to extract subject to the stock dynamics of the aquifer under conditions of cooperation and non-cooperation between farmers. To obtain the non-cooperative solution, we firstly characterize the Markov perfect Nash equilibrium and study conditions for its existence and uniqueness. We then study the cooperative solution. When farmers exhibit different time preferences, and we aggregate the discounted payoffs of all farmers, a problem of time inconsistency arises. This means that the optimal solution computed at time τ is no longer optimal at time $\tau' > \tau$. As a result, standard dynamic optimization techniques cannot be used to obtain an optimal time-consistent solution. A natural way to construct time-consistent cooperative decision rules is to apply a dynamic programming approach, by computing, at every time τ, what is optimal for the coalition, constrained to the future behavior of the coalition for $\tau' > \tau$. Karp (2007) derived the corresponding modified dynamic programming equations in a setting with one decision-maker under non constant discounting. Its natural extension to the general case of several heterogeneous agents can be found in, for instance, de-Paz, Marín-Solano, and Navas (2013), Ekeland, Long, and Zhou (2013), and Marín-Solano and Shevkoplyas (2011). This is the approach that we follow in the present paper for the derivation of the (time-consistent) cooperative solution.

To quantify the degree of inefficiency of non-cooperation, we apply the model to the case study of the Western La Mancha aquifer in Spain. We compare the numerical results between the cooperative and non-cooperative equilibria in terms of stock, individual profitability, and total welfare. We firstly study how the inefficiency of non-cooperation

[3]Indeed, Sears, Lim, and Lawell (2019) take into account a more complex groundwater recharge function by adding the possibility of spatial movements between patches owned by the different farmers.

is influenced by different types of asymmetries in isolation. We then extend the analysis by particularly focusing on the effects of the time preference asymmetry when combined with other heterogeneities.

The results suggest that the fact of considering different degrees of impatience (i.e., different time preferences) between farmers will counteract the effects of other heterogeneities on groundwater exploitation, resulting in a reduction of the inefficiency of non cooperation in terms of stock and total welfare. However, inducing cooperation by the intervention of a social planner could be not justified in some situations when slight differences between solutions are observed and/or when it is not individually profitable for one farmer to cooperate. This will depend on the heterogeneities between farmers.

The work is organized as follows. In Section 2, the model is introduced. In Section 3, we solve the model in a non-cooperative framework and, in Section 4, we study the derivation of the (time-consistent) cooperative solution. Section 5 presents and discusses the results of the numerical application. Section 6 concludes the paper.

2 The model

In this work, the models of Rubio and Casino (2001) and de Frutos Cachorro, Marín-Solano, and Navas (2021) are adapted to the case of asymmetric players during the infinite planning horizon. First of all, it is quite natural to assume that farmers using groundwater from the same aquifer for irrigating their crops may own lands of different sizes and qualities, resulting in different revenues, costs and/or aquifer dynamics. Secondly, depending on the financial facilities, subjective perspectives or time preferences (or in other words, discount rates) among farmers may be different.

In our model, we consider two types of farmers (or two groups of farmers with a representative farmer for each group) that we denote by $i \in \{1, 2\}$. In line with Rubio and Casino (2001), we assume that farmers compete in a competitive market, so that the price of water p is equal to the value of the marginal product of water. In addition, the agricultural production function exhibits constant returns to scale, and production factors other than water and land are optimized conditioned to the water extraction rate. Moreover, as in the adaptation by Rubio and Casino (2001) of the model of Gisser and Sánchez (1980) to the case of several farmers, the water demand function of farmer i, $i \in \{1, 2\}$, is a negatively sloped linear function $g_i = a_i - b_i\, p$, where $a_i, b_i > 0$. In the real world, different water demand functions could be motivated by different crop yield outputs and/or different crop productivities due to different characteristics of the soil (or climate conditions in case that the farmers are situated in different locations). The corresponding revenue function of each farmer is the area under the demand curve for irrigation water, i.e., the integral of the inverse of the demand function, and is given by

$$\int_0^{g_i} p(x_i)\ dx_i = \frac{a_i}{b_i} g_i - \frac{1}{2b_i} g_i^2\ . \tag{1}$$

Moreover, in the same way as the previous literature, we assume that the marginal cost of water extraction is a linear function in the stock of the aquifer G (or the amount of water that can be stored). Total costs of extraction of farmer i depend on the quantity of water extracted:

$$C_i = (z_i - c_i\, G) g_i, \qquad z_i, c_i > 0\ (i = 1, 2), \tag{2}$$

where c_i is the slope of the marginal pumping cost function and z_i is the maximum marginal pumping cost (see, for example, Gisser and Sánchez (1980), Negri (1989), and Rubio and Casino (2001)). Pumping costs mainly correspond to the cost of energy required to pump water to the topsoil. Although, in previous theoretical works, it is common to assume that the marginal pumping costs are the same for both agents, in our theoretical model, we relax this condition to consider situations in which farmers can make use of different well installations, with different degrees of efficiency, leading to different per-unit extraction costs.

Next, as is standard in the theoretical literature of groundwater exploitation (Gisser and Sánchez, 1980; Negri, 1989; Rubio and Casino, 2001), the dynamics of the stock of the aquifer is driven by

$$\dot{G} = r - \sum_{i=1}^{2} (1 - \gamma_i)\, g_i\ , \tag{3}$$

where r is the natural recharge rate and γ_i is the return flow coefficient, hence $\gamma_i \in [0, 1)$. The natural recharge mainly refers to the rain water that moves from the land surface to the aquifer. In contrast to other studies such as Sears, Lim, and Lawell (2019), which considers the possibility of spatial movements between patches owned by the different farmers, we consider a more simple recharge function. Hence, we assume that the natural recharge rate does not depend on the resource stock, G, and that the flow of rain is constant and deterministic[4] such as in the previously mentioned theoretical literature (Gisser and Sánchez, 1980; Negri, 1989; Rubio and Casino, 2001). The return flow coefficient, in

[4]Introducing a random recharge rate would complicate the model and we think that it would not significantly modify our main qualitative results.

turn, describes the proportion of water returned to the aquifer from the cultivated area, which may depend on the quality and/or the type of the soil. In this work, we consider the possibility that both players can have different soils, so return flows can be different. However, we maintain the standard and simplifying assumption that they are constant. A more realistic model should take into account the fact that, when the soil is very humid, most additional irrigation water may flow back into the aquifer. Hence, γ_i would be an increasing function in g_i[5].

Lastly, one of the main contributions of this paper is that we allow for the possibility that, unlike the standard assumption of a unique discount rate for all farmers, farmers can exhibit different time preferences. Indeed, there is a lot of evidence that discount rates are typically non-unique in real-life applications (for two recent references on the topic, see, e.g., Bozio, Laroque, and O'Dea (2017) and Matousek, Havranek, and Irsova (2022) and references therein). For example, in a setting with big and small farmers (or firms, in general), it seems reasonable to assume that big farmers can have advantages in terms of financial facilities, which can have an impact by lowering the discount rates applied to future profits. Unlike de Frutos Cachorro, Marín-Solano, and Navas (2021), who analyzed a related model of competition between two types of uses (urban and agriculture), in our model, the discount rates are set to be different during the infinite planning horizon, not just during a finite period of time. This complicates the search for equilibria and may, in principle, lead to the existence of multiple solutions both under competition and under cooperation (due to the time inconsistency of the cooperative problem), as we will study in Sections 3 and 4.

Let

$$F_i(G, g_i) = \frac{a_i}{b_i} g_i - \frac{1}{2b_i} g_i^2 - (z_i - c_i G) g_i \tag{4}$$

denote the profit function of farmer i, for $i \in \{1, 2\}$, given by the difference between revenues (1) and costs (2). Then, in the non-cooperating setting, farmer $i \in \{1, 2\}$ aims to maximize individual welfare, defined as the present value of their future profits, subject to the dynamics of the resource (3), and given initial conditions and positivity constraints:

$$\max_{g_i} \int_0^\infty F_i(G, g_i) e^{-\rho_i t} dt \tag{5}$$

[5]We thank an anonymous referee for this suggestion.

$$\dot{G} = r - \sum_{i=1}^{2} (1 - \gamma_i)\, g_i \ ,$$

$$G(0) = G_0 \text{ given and } g_1 \geq 0,\ g_2 \geq 0,\ G \geq 0.$$

3 Non-cooperative solution: Markov Perfect Nash Equilibrium

In this section, we solve the model under non-cooperation between farmers, as defined in problem (5). In particular, we will assume that both agents have access to water monitoring systems and thus can observe the level of the water table at all times. As a result, the solution concept that we use is that of the Markov perfect Nash equilibrium (MPNE). In the corresponding strategies, the past influences the present game through its effect on the current value of the stock of groundwater. Hence, the strategies will be functions depending on time t and on the stock G. The MPNE is said to be subgame perfect, meaning that the equilibrium strategies in the whole game are also the MPNE in every proper subgame.

To calculate the MPNE, we will proceed in the standard way (for more details, refer to Dockner et al. (2000) or Haurie, Krawczyk, and Zaccour (2012), for example).

The dynamic programming equation to solve by each user $i \in \{1, 2\}$ is

$$\rho_i V_i^{NC}(G) = \max_{\{g_i\}} \left\{ F_i(G, g_i) + \left(V_i^{NC}(G)\right)' (r - (1 - \gamma_i)(g_i + \phi_j^{NC}(G)) \right\}. \tag{6}$$

In (6), $\phi_j^{NC}(G)$ denotes the strategy of player j, for $j \neq i$. In this work, we will focus on stationary linear (affine) strategies in this linear-quadratic differential game, so that $\phi_i^{NC}(G) = \alpha_i^{NC} G + \beta_i^{NC}$ and $V_i^{NC}(G) = A_i^{NC} G^2 + B_i^{NC} G + C_i^{NC}$.

In Appendix A, by using (6), we express the coefficients of the value functions V_i^{NC}, $i \in \{1, 2\}$, in terms of α_j^{NC} and β_j^{NC}, for $i, j \in \{1, 2\}$. As for the values of these parameters, as shown in Appendix A, they should solve a system of nonlinear equations. In particular, there can be up to four solutions. However, not all of them are admissible, since they must satisfy certain transversality conditions. More precisely, we are interested in interior solutions converging to a steady state G_∞^{NC}, which has several implications. First of all, it means that when solving $\dot{G} = r - \left[(1-\gamma_1)\alpha_1^{NC} + (1-\gamma_2)\alpha_2^{NC}\right] G - \left[(1-\gamma_1)\beta_1^{NC} + (1-\gamma_2)\beta_2^{NC}\right]$ we must impose the condition $(1-\gamma_1)\alpha_1^{NC} + (1-\gamma_2)\alpha_2^{NC} > 0$. Second, we must have $r \geq \sum_{i=1}^{2} (1 - \gamma_i)\beta_i^{NC}$ to ensure that the water resource will not be exhausted in finite time. The corresponding asymptotically stable steady state level of the stock of the aquifer is given by

$$G_\infty^{NC} = \frac{r - \sum_{i=1}^{2}(1-\gamma_i)\beta_i^{NC}}{\sum_{j=1}^{2}(1-\gamma_j)\alpha_j^{NC}} \ . \tag{7}$$

In Appendix A, we characterize when interior MPNE, satisfying conditions $(1-\gamma_1)\alpha_1^{NC} + (1-\gamma_2)\alpha_2^{NC} > 0$ and $r < \sum_{i=1}^{2}(1-\gamma_i)\beta_i^{NC}$, exist. The following theorem summarizes the main results.

Theorem 1. *MPNE of Problem (4-5) are characterized by the solutions to the system of nonlinear equations*

$$\frac{\rho_i}{2(1-\gamma_i)}\left(c_i - \frac{\alpha_i^{NC}}{b_i}\right) = -\frac{(\alpha_i^{NC})^2}{2b_i} + c_i\alpha_i^{NC} - \left(c_i - \frac{\alpha_i^{NC}}{b_i}\right)\sum_{k=1}^{2}\alpha_k \ , \tag{8}$$

$$\frac{1}{b_i}\left(\rho_i + \sum_{k=1}^{2}(1-\gamma_k)\alpha_k^{NC}\right)\beta_i^{NC} + (1-\gamma_i)\left(\frac{\alpha_i^{NC}}{b_i} - c_i\right)\beta_j^{NC} = \left(\frac{a_i}{b_i} - z_i\right)\left(\rho_i + \alpha_j^{NC}\right) + r\left(\frac{\alpha_i^{NC}}{b_i} - c_i\right) \tag{9}$$

for $i, j \in \{1, 2\}$, $i \neq j$, satisfying the conditions

$$\sum_{k=1}^{2}(1-\gamma_k)\alpha_k^{NC} > 0 \quad \textit{and} \tag{10}$$

$$r \geq \sum_{k=1}^{2}(1-\gamma_k)\beta_k^{NC} \ . \tag{11}$$

There exists at most one solution to (8) satisfying (10). If a (unique) solution to (8) and (10) exists, MPNE are obtained by solving the linear equation system (9), provided that its solution satisfies (11).

Proof: See Appendix A.

4 Cooperative solution

In this section, we analyze the problem in which farmers cooperate by maximizing total welfare, defined as the present value of the sum of their individual future profits, subject to the dynamics of the resource (equation (3)). When the time preferences of the two agents are set to be different, the cooperative problem becomes time inconsistent. As explained in the introduction, this means that the optimal solution at time τ, computed with the use of the standard dynamic optimization techniques, is no longer optimal at time $\tau' > \tau$. More precisely, the functional to be maximized at an arbitrary instant of time τ is

$$\int_{\tau}^{\infty} F_1(G, g_1)e^{-\rho_1(t-\tau)}\, dt + \int_{\tau}^{\infty} F_2(G, g_2)e^{-\rho_2(t-\tau)}\, dt \, . \tag{12}$$

For example, at initial time, (12) becomes $\int_0^{\infty} F_1(G, g_1)e^{-\rho_1 t}\, dt + \int_0^{\infty} F_2(G, g_2)e^{-\rho_2 t}\, dt$. Here, the discounted utilities of each player at a future moment s from the perspective of the coalition at $\tau = 0$ are $F_1(G, g_1)e^{-\rho_1 s}$ and $F_2(G, g_2)e^{-\rho_2 s}$, respectively. Therefore, the relative weight of the discounted utility of player 1 with respect to player 2 is $e^{-\rho_1 s}/e^{-\rho_2 s} = e^{-(\rho_1-\rho_2)s}$.

If the coalition decides to recompute what is optimal at a future moment $\tau' = s$, the cooperative problem (12) becomes

$$\int_{s}^{\infty} F_1(G, g_1)e^{-\rho_1(t-s)}\, dt + \int_{s}^{\infty} F_2(G, g_2)e^{-\rho_2(t-s)}\, dt.$$

Note that the sum of (discounted) utilities obtained at period s is now $F_1(G, g_1)+F_2(G, g_2)$, i.e., the coalition assigns the same weights to both players. As a result, the optimization problem is different, and the optimal solution computed at time $\tau = 0$ will not coincide with the optimal solution computed at time $\tau' = s$.

Indeed, the relative weights of the utilities remain constant (and equal to one) if, and only if, $\rho_1 = \rho_2$. The key point is that, when $\rho_1 = \rho_2 = \rho$, the functional to be maximized at every moment τ, i.e., problem (12), can be written as

$$e^{\rho\tau} \int_{\tau}^{\infty} \left(F_1(G, g_1, g_2) + F_2(G, g_1, g_2)\right) e^{-\rho t}\, dt \, .$$

But this is not possible if $\rho_1 \neq \rho_2$. As a result, the cooperative problem is said to be time inconsistent. As in Karp (2007), time consistent decision rules can be derived by computing, at every time τ, what is optimal for the coalition, but taking into account its future decisions for $\tau' > \tau$. The corresponding dynamic programming equations were studied in de-Paz, Marín-Solano, and Navas (2013), Ekeland, Long, and Zhou (2013), and Marín-Solano and Shevkoplyas (2011).

More precisely, we have to solve

$$\max_{g_1, g_2} \left\{ \sum_{i=1}^{2} F_i(G, g_i) + \left(\sum_{i=1}^{2} \left(V_i^C(G)\right)' \right) \left(r - \sum_{j=1}^{2} (1-\gamma_j) g_j^C \right) \right\} \, . \tag{13}$$

If $g_i^C = \phi_i^C(G)$, for $i \in \{1,2\}$, are the corresponding strategies, then the value functions of both players must satisfy

$$\rho_i V_i^C(G) = F_i(G, \phi_i^C(G)) + \left(V_i^C(G)\right)' \left(r - \sum_{j=1}^{2} (1-\gamma_j)\phi_j^C(G) \right) . \tag{14}$$

As in the non-cooperative case, we focus our attention on stationary linear decision rules. Hence, the extraction strategies are affine functions $\phi_i^C(G) = \alpha_i^C G + \beta_i^C$, for $i \in \{1,2\}$, and the value functions are second-degree polynomials $V_i^C(G) = A_i^C G^2 + B_i^C G + C_i^C$.

From the first-order optimality conditions in (13) we have

$$\alpha_i^C = b_i \left[c_i - 2(1-\gamma_i) \sum_{j=1}^{2} \left(A_j^C\right) \right] , \qquad \beta_i^C = a_i - b_i z_i - b_i(1-\gamma_i) \sum_{j=1}^{2} B_j^C . \tag{15}$$

As shown in Appendix B, to derive the parameters of the value function and to calculate the extraction rules, we have to solve a system of 6 equations with 6 unknown variables. In particular, the (time-consistent) cooperative solutions should satisfy

$$\rho_i A_i^C = -\frac{1}{2b_i}(\alpha_i^C)^2 + c_i \alpha_i^C - 2(1-\gamma_i) A_i^C \sum_{j=1}^{2} \alpha_j^C , \quad \text{for} \quad i = 1, 2 , \tag{16}$$

with α_i^C given by (15). As in the case of the non-cooperative MPNE, we can study the number of possible solutions and their convergence to a steady state. The above equations in (16) are quadratic and depend only on A_1^C and A_2^C. Therefore, we can be sure that there will be no more than four solutions. For these solutions to converge to a steady state, the following inequality must be satisfied:

$$\sum_{i=1}^{2} (1-\gamma_i)\alpha_i^C = \sum_{i=1}^{2} (1-\gamma_i) b_i \left[c_i - 2(1-\gamma_i) \sum_{j=1}^{2} \left(A_j^C\right) \right] > 0 . \tag{17}$$

However, due to the time inconsistency of the cooperative problem, it is unclear how many (out of 4) solutions to the equation system (16) satisfy condition (17). As a complete theoretical analysis of the number of cooperative solutions goes beyond the scope of this paper, we solve the model numerically and check the above condition to find solutions converging to the steady state. For realistic values of the parameters, we obtain uniqueness.

5 Numerical analysis: the case of the Western La Mancha aquifer

In this section, the theoretical model described previously is applied to the case study of the Western La Mancha aquifer in the Upper Guadiana River Basin. The Western La Mancha aquifer is situated in central-southern Spain, it covers around 5000 square kilometers in the provinces of Ciudad Real (80%), Albacete, and Cuenca, where dry periods are frequent (Agencia del Agua de Castilla la Mancha, n.d.). Unfortunately, this aquifer has suffered from several droughts and gross mismanagement in the last decades of the 20^{th} century, which led to a decrease in the water tables dramatically impacting the wetlands in the Mancha Húmeda Biosphere Reserve (Hernández-Mora et al., 2007). Taking into account that up to 92% of water extracted is used for irrigation (Unión de Uniones de Castilla la Mancha, n.d.), this study addresses an important issue in terms of environmental policy's implication in the possible benefits of cooperation as opposed to non-cooperation when farmers exhibit different types of heterogeneities.

The parameters necessary for the simulations, previously used in de Frutos Cachorro, Erdlenbruch, and Tidball (2019) and adapted from previous works in the study area such as Esteban and Albiac (2011) and Esteban and Dinar (2016), are presented in Table 1[6].

We focus our attention on the case of two types of farmers, Type 1 farmer (denoted with subscript 1) and Type 2 farmer (denoted with subscript 2). We examine how the introduction of different asymmetries in the farmer's objective function influences the inefficiency of the non-cooperative solution with respect to the cooperative solution in terms of stock and total welfare. Here, by inefficiency, we simply mean the difference in stock (stock inefficiency) and total welfare (welfare inefficiency) between the cooperative and non-cooperative solutions, or in other words, the gain from cooperation. Moreover, we analyze the profitability of cooperation for each farmer, which is defined as the difference in individual welfare between cooperation and non-cooperation.

Farmer's demand functions defined in Section 2 have been adapted for the numerical simulations. In Table 1, the parameter θ is related to the demand asymmetry. Hence, being the aggregated water demand $g = g_1 + g_2 = a - bp$ estimated in previous literature (Esteban and Albiac (2011) and Esteban and Dinar (2016)) where p denotes the price

[6] Please note that the original data source is Esteban and Albiac (2011), although the parameter values used in de Frutos Cachorro, Erdlenbruch, and Tidball (2019), and therefore in this study, have been slightly adapted for a similar groundwater model with several farmers in which the state variable is the water stock (in Millions of m^3) instead of the water table height (in m).

Table 1: Values of parameters from the Western La Mancha aquifer reported in de Frutos Cachorro, Erdlenbruch, and Tidball (2019) and adapted from Esteban and Albiac (2011) and Esteban and Dinar (2016). Original Source: Esteban and Albiac (2011).

Parameter	Description	Units	Value
a	Water demand intercept	Million cubic meters / year	4400.73
b	Water demand slope	(Million cubic meters / year)^{2}Euro^{-1}	0.097
c	Pumping cost slope	Euros / Million cubic meters^2	3.162
z_1	Type 1 farmer's pumping cost intercept	Euros / Million cubic meters	266000
G_0	Initial stock level (in volume)	Million cubic meters	80960
r	Natural recharge rate	Million cubic meters / year	360
γ_1	Type 1 farmer's return flow coefficient	unitless	0.2
ρ_1	Type 1 farmer's discount rate	Year^{-1}	0.05
θ	Type 2 farmer's water demand proportion	unitless	$\theta \in [\frac{1}{6}, \frac{1}{2}]$
b_2 (and b_1)	Type 2 (resp. Type 1) farmer's water demand slope	(Million cubic meters / year)^{2}Euro^{-1}	$b_2 \in [\frac{b}{2}, 1.09\frac{b}{2}]$ (resp. $b_1 = b - b_2$)
z_2	Type 2 farmer's pumping cost intercept	Euros / Million cubic meters	$z_2 \in [\frac{97}{100}z_1, z_1]$
γ_2	Type 2 farmer's return flow coefficient	unitless	$\gamma_2 \in [0.05, 0.2]$
ρ_2	Type 2 farmer's discount rate	Year^{-1}	$\rho_2 \in [0.05, 0.09]$

of water, when analyzing the demand asymmetry we can rewrite the farmer's demand function as $g_1 = a_1 - b_1 p = (1-\theta)\ (a - b\,p)$, and $g_2 = a_2 - b_2 p = \theta\ (a - b\,p)$, where $\theta \in (0,1)$, respectively $(1-\theta) \in (0,1)$ represents the fraction of total aggregated water demand allocated to the Type 2 farmer, respectively the Type 1 farmer. With respect to the different values of b_1 and b_2 related to the asymmetry in the crop productivity, we consider that $b_1 = b - b_2$, $a_1 = a_2 = \frac{a}{2}$ and the aggregated water demand is again

$g = g_1 + g_2 = (\frac{a}{2} - b_1\, p) + (\frac{a}{2} - b_2\, p) = a - b\, p$. For the rest of asymmetries, we consider that $\theta = \frac{1}{2}$, $a_1 = a_2 = \frac{a}{2}$ and $b_1 = b_2 = \frac{b}{2}$, therefore $g = g_1 + g_2 = (\frac{a}{2} - \frac{b}{2}\, p) + (\frac{a}{2} - \frac{b}{2}\, p) = a - b\, p$.

In the subsequent sections, we perform numerical simulations for different cases. Firstly, we describe results obtained for the benchmark case in which both players are completely symmetric. Next, we analyze numerical results separately for water demand and discount rate asymmetries. We then progressively introduce asymmetries on the land type (i.e., different return flow coefficient due to different soil properties), crop productivity, and extraction costs, and discuss simulated results for isolated and combined asymmetries.

5.1 The benchmark case: symmetric farmers

Tables 2 and 3 show simulated results for the benchmark case, that is the symmetric scenario, and therefore $\theta = \frac{1}{2}$, $\rho_1 = \rho_2 = 0.05$, $\gamma_1 = \gamma_2 = 0.2$, $b_1 = b_2 = \frac{b}{2} = 0.048$ and $z_1 = z_2 = 266000$.

In line with previous studies (Negri, 1989; Rubio and Casino, 2001; de Frutos Cachorro, Erdlenbruch, and Tidball, 2019), non-cooperative strategies are inefficient (in terms of stock and welfare) compared to cooperative solutions. In other words, the stock at the steady state and total welfare are higher under cooperation than under non-cooperation (see last columns in the tables).

With respect to individual welfare, it is always worth cooperating for both farmers (see columns 5 and 6 of Table 3).

Table 2: Stock volume (in million cubic meters) at the steady state in the symmetric setting.

(1) G^C_∞	(2) G^{NC}_∞	Stock inefficiency (1) - (2)
78397	72502	5895

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

5.2 Different types of asymmetries (isolated effects)

First of all, we study how the inefficiency of the non-cooperative solution and the individual profitability of cooperation change when we introduce the demand and the discount

Table 3: Total welfare (in thousand euros) in the symmetric setting.

(1) W_1^C	(2) W_2^C	(3) W_1^{NC}	(4) W_2^{NC}	Profitability cooperation		Welfare inefficiency
				Type 1 farmer (1)-(3)	Type 2 farmer (2)-(4)	(1)+(2) - ((3)+(4))
153534	153534	112269	112269	41265	41265	82530

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

rate asymmetries separately. We disentangle the effect of each asymmetry by performing numerical simulations for lower values of θ in the demand function of Type 2 farmer ($\theta \in [\frac{1}{6}, \frac{1}{2}]$), and for higher values of the discount rate of Type 2 farmer ($\rho_2 \in [0.05, 0.09]$). We next introduce other types of asymmetries in the parameter values of the Type 2 farmer: the asymmetry in the return flow coefficient (for $\gamma_2 \in [0.05, 0.2]$), in the crop productivity ($b_2 \in [\frac{b}{2}, 1.09\frac{b}{2}]$ and $b_1 = b - b_2$) and in the maximum marginal extraction costs (for $z_2 \in [\frac{97}{100}z_1, z_1]$). For asymmetries related to the parameters of the water demand functions, i.e., for the demand asymmetry ($\theta \downarrow$) and for the crop productivity asymmetry ($b_2 \uparrow$ and $b_1 \downarrow$), numerical simulations are performed in such a way that aggregated water demand is maintained. Hence, $g_1 + g_2 = a - bp$. Tables 4 and 5 summarize simulated results (i.e., tendencies and percentages) when asymmetries increase (see footnote [7] to ease the reading and interpretation of the Tables).

Demand asymmetry. The demand asymmetry is captured by θ, which represents the fraction of total aggregated water demand allocated to the Type 2 farmer. Hence, $1 - \theta$ remains the proportion of total water demand associated to the Type 1 farmer. For example, this could be explained by the fact that the Type 2 farmer owns a smaller plot of land and therefore, demands a lower quantity of water with respect to the Type 1 farmer.

In Table 4 (column 1), we observe that the higher the demand asymmetry between the players (i.e., the lower θ), the lower the inefficiency of the non-cooperative solution in terms of stock. In other words, when θ goes down, the difference in stock between cooperation and non-cooperation decreases compared to the difference in stock between cooperation and non-cooperation when agents are completely symmetric (benchmark case). Indeed, Table 7 in the Appendix shows that, when θ decreases from $\frac{1}{2}$ to $\frac{1}{6}$, steady-state stock does not vary under cooperation while increasing by 1.5% under non-cooperation (compare rows

[7] For example, tendencies and percentages in Table 4 (e.g. column (1)), rows 1-2) means that stock inefficiency decreases by around 18.7% for a high value of the demand asymmetry, i.e., for $\theta = \frac{1}{6}$, while welfare inefficiency decreases by more than 30.7% for the same parameter value of θ

1 and 4, columns 1 and 2), which brings the values of the stock under non-cooperation closer to those achieved under cooperation.

At the same time, with higher asymmetry (lower θ), the profitability of cooperation increases by 57.8% for the Type 1 farmer and decreases by 119% for the Type 2 farmer, to the extent that it is not worth cooperating for the farmer who shows a very low water demand (see Table 4 last line, column 1). As the magnitude of the effect is higher for Type 2 farmer than for Type 1 farmer, the welfare inefficiency of the non-cooperative solution decreases (see row 2, column 1).

Time preference asymmetry. Time preference asymmetry is expressed through different discount rates ρ_1 and ρ_2. As explained in Section 2, it is quite reasonable to assume that different types of farmer will exhibit different time preferences (or degrees of impatience), as a result of their personal preferences or financial opportunities. We consider that $\rho_1 \leq \rho_2$. This could be the case if, for example, the Type 2 farmer owns a smaller plot of land and might be disadvantaged in terms of financial opportunities, which will make this farmer more impatient. Higher asymmetry is now associated with higher values of ρ_2.

As the Type 2 farmer becomes more impatient (i.e., the higher ρ_2), lower levels of stock are obtained both under cooperation and non-cooperation (see Table 7 in Appendix). Indeed, as illustrated in Table 16 in the Appendix, the extraction behavior of the Type 2 farmer is more aggressive when this farmer exhibits higher impatience. Note that, when ρ_2 increases from 0.05 to 0.09, the steady state stock decreases by around 2.4% under cooperation and only by 0.2% under non-cooperation (see columns 1 and 2, first rows, in Table 7), which implies that the stock inefficiency of non-cooperation decreases (see Table 4 last column, second row). In fact, when farmers cooperate, the higher impatience of the Type 2 farmer intensifies the total extractions of both farmers throughout the planning horizon (see first columns, Table 16 in Appendix). When farmers compete, however, the higher extractions of the Type farmer 2 can be partly balanced out by the lower extractions of the more patient (Type 1) farmer and thus the stock does not decrease as fast, as observed in the last two columns of Table 16.

Indeed, cooperation "forces" the impatient (Type 2) farmer to extract less and the relatively more patient (Type 1) farmer to extract more than they would have preferred to satisfy their individual needs under competition. Moreover, when asymmetry increases, extraction costs rise due to the lower level of stock, making it individually less profitable for both farmers to cooperate. This means that the welfare inefficiency of non-cooperation also diminishes (see Table 4, last column, for a summary of results).

Return flow coefficient asymmetry. Asymmetry in the return flow coefficient is captured by lower values of γ_2. A lower return flow coefficient implies that less water percolates into the aquifer (i.e., a lower groundwater recharge), which results in lower stock levels both under cooperation and non-cooperation (see Table 9 in the Appendix for detailed results).

Unlike demand and time preference asymmetries, the asymmetry in the return flow coefficient between farmers slightly increases the stock inefficiency of non-cooperation (see Table 5, column 3, row 2). This is because the stock under cooperation decreases less significantly than under non-cooperation. Indeed, as Table 17 shows, when farmers cooperate to preserve higher stock levels, the total extractions of both farmers decrease at the same speed as γ_2 decreases from 0.2 to 0.05 (due to a corresponding increase in extraction costs). Under non-cooperation, although individual extractions also decrease, the Type 2 farmer —i.e., the farmer with a lower return flow coefficient—extracts more than the Type 1 farmer, trying to compensate for the fact that a lower value of γ_2 implies a lower percentage of their extractions percolating the aquifer.[8]

Moreover, since under cooperation, the Type 2 farmer is induced to extract less than under non-cooperation, their profitability of cooperation decreases by 7.6% with higher asymmetry. In contrast, the profitability of cooperation of the Type 1 farmer shows a slight increase of only 0.4%. Since the reduction in the profitability of cooperation for the Type 2 farmer is stronger than the gain for the Type 1 farmer, the welfare inefficiency of non-cooperation decreases (see Table 5, column (3) for a summary of results).

Crop productivity asymmetry. Crop productivity asymmetry is expressed through b_2 (and $b_1 = b - b_2$), with higher values of b_2 (respectively lower values of b_1) being associated with lower (respectively higher) crop productivity. For example, this could be due to a lower quality of the land of the Type 2 farmer with respect to the Type 1 farmer.[9]

Lower (and higher) crop productivity of the Type 2 (resp. Type 1) farmer (i.e., higher values of b_2 and lower values of b_1) gives rise to lower stock levels under non-cooperation, while maintaining stock levels under cooperation (see Table 11 in the Appendix). Consequently, this leads to the increased stock inefficiency of the non-cooperation (see Table 5, column 4, row 2).

At the same time, when the asymmetry increases, the farmer with higher crop produc-

[8]Note that the dynamics is described by $\dot{G} = r - g_1 - g_2 + \gamma_1 g_1 + \gamma_2 g_2$, as explained in Section 2.

[9]Since the farmer's demand function is $g_i = a_i - b_i p,\ i = 1..2$, if we fix g_i and a_i, higher b_i suggests that a lower price will be paid for that quantity of water g_i. As the farmer's income is computed by integrating the inverse of the demand function, this entails lower revenues. Hence higher b_i, is associated with lower crop productivity.

tivity (Type 1) is able to increase their extractions faster under cooperation than under non-cooperation to the detriment of the farmer with lower crop productivity (Type 2). In fact, in contrast to the extraction behavior of the Type 2 farmer, Table 18 shows that total extractions of the Type 1 farmer over the first 20 years now increase by around 59% under cooperation and by 18% under non-cooperation from $b_2 = \frac{b}{2}$ to $b_2 = 1.09\frac{b}{2}$. Hence, as we observe in Table 5 in column (4), for the Type 1 farmer, cooperation becomes more profitable as the crop productivity asymmetry increases, while the opposite happens for the Type 2 farmer, to the extent that it is not beneficial for the Type 2 farmer to co-operate. As a result, since the Type 1 farmer benefits more from the cooperation than the Type 2 farmer loses, the welfare inefficiency of the non-cooperative solution slightly increases with higher values of b_2 (and lower values of b_1).

Cost asymmetry. Cost asymmetry is expressed through lower maximum (per-unit) extraction costs of the Type 2 farmer, z_2, with respect to the costs of the Type 1 farmer, z_1, which could be a result of the implementation of agricultural subsidies for small businesses.

Lower z_2 allows the Type 2 farmer to considerably increase their total extractions, while the Type 1 farmer exhibits a less aggressive extraction behavior in the cooperative and non-cooperative cases (see Table 19 in the Appendix for an example). With the magnitude of the effect for the Type 2 farmer being bigger than that for the Type 1 farmer, this results in lower steady-state levels of stock both under cooperation and non-cooperation (see Table 13 in the Appendix). Interestingly, the steady-state stock decreases proportionately under cooperation and under non-cooperation compared to the benchmark case, thus keeping the level of the stock inefficiency roughly constant (see column 5 in Table 5 for a summary of the results).

Since cooperation aims to maximize joint welfare, it favors the farmer with lower costs, who can extract more than under non-cooperation and hence drive the total welfare up. As illustrated in the example in Table 19, total extractions over the first 20 years of the Type 2 farmer increase by around 77% under cooperation and by 29% under non-cooperation when z_2 decrease by only 3%. Thus, more competitive costs for the Type 2 farmer enhance their interest in cooperation (increasing by 191%) at the expense of the interests of the Type 1 farmer (decreasing by 164%) to the point that it is not worth cooperating for the farmer with higher costs (Type 1). Since the Type 2 farmer benefits more from the cooperation than the Type 1 farmer loses, the welfare inefficiency of the non-cooperation increases with higher asymmetry in costs, i.e., with lower values of z_2 (see column 5 in Table 5).

Table 4: Results of stock ($G_\infty^C - G_\infty^{NC}$) and total welfare ($\sum_i W_i^C - W_i^{NC}$) inefficiencies, and farmer's profitability of cooperation ($W_i^C - W_i^{NC},\ i = 1..2$) for an increase in water demand and/or discount rate asymmetries.

	Type of asymmetry	Water demand (1) $\theta \downarrow$		Discount rate (2) $\rho_2 \uparrow$	
Type of inefficiency	Stock	$\searrow$	[-18.7%,0]	$\searrow$	[-29%,0]
	Welfare	$\searrow$	[-30.7%,0]	$\searrow$	[-34.4%,0]
Profitability of cooperation	Type 1 farmer	$\nearrow$	[0, 57.8%]	$\nearrow$	[0,4.6%]
	Type 2 farmer	$\searrow_{\text{NP}}$	[-119%,0]	$\searrow$	[-73.4%,0]

Terms in brackets [l, h]: maximum decrease (l) and increase (h) (in percentage) of simulated results for the highest asymmetry considered with respect to the benchmark case.
$_{NP}$ means not profitable, i.e., farmer's individual welfare is lower under cooperation than under non-cooperation.

Table 5: Results of stock ($G_\infty^C - G_\infty^{NC}$) and total welfare ($\sum_i W_i^C - W_i^{NC}$) inefficiencies, and farmer's profitability of cooperation ($W_i^C - W_i^{NC},\ i = 1..2$) for different types of land, crop productivity and water cost asymmetries.

	Type of asymmetry	Return flow coefficient (3) $\gamma_2 \downarrow$		Crop productivity (4) $b_2 \uparrow$ (and $b_1 \downarrow$)		Extraction cost (5) $z_2 \downarrow$	
Type of inefficiency	Stock	$\nearrow$	[0, 0.9%]	$\nearrow$	[0, 1.1%]	$\longrightarrow$	
	Welfare	$\searrow$	[-3.6%,0]	$\nearrow$	[0, 4.5%]	$\nearrow$	[0, 13.4%]
Profitability of cooperation	Type 1 farmer	$\nearrow$	[0, 0.4%]	$\nearrow$	[0, 166%]	$\searrow_{\text{NP}}$	[-164%,0]
	Type 2 farmer	$\searrow$	[-7.6%,0]	$\searrow_{\text{NP}}$	[-157%,0]	$\nearrow$	[0, 191%]

Terms in brackets [l, h]: maximum decrease (l) and increase (h) (in percentage) of simulated results for the highest asymmetry considered with respect to the benchmark case.
$_{NP}$ means not profitable, i.e., farmer's individual welfare is lower under cooperation than under non-cooperation.

5.3 Combined asymmetries

In this section, we study the effects of the combined asymmetries on the stock and welfare inefficiency of non-cooperation, as well as on the individual profitability of cooperation (see a summary of results in Table 6). In particular, we choose to combine the discount rate asymmetry ($\rho_2 > \rho_1$) only with asymmetries in return flow coefficient ($\gamma_2 < \gamma_1$), crop productivity ($b_2 > b_1$), and marginal extraction cost ($z_2 < z_1$). In fact, one of the main contributions of our study to the literature on optimal groundwater extraction is based on assuming different discount rates throughout the infinite planning horizon, therefore we are especially interested in the analysis of how time preference asymmetry can be helpful for defining relevant policy implications. Moreover, the effect of the discount rate asymmetry and the demand asymmetry on the inefficiency of non-cooperation is similar.

When analyzing isolated effects in the previous section, we have shown that the discount rate asymmetry results in a decrease in both the stock and welfare inefficiency of the non-cooperative solution, while the asymmetry in the return flow coefficient leads to a decrease in the inefficiency only in terms of welfare but not in terms of stock. In Table 6, we can see that the introduction of the discount rate asymmetry seems to counteract the effects of the three asymmetries on the stock inefficiency, making non-cooperation less (stock) inefficient as the asymmetries increase. As for the welfare inefficiency, the combination of the discount rate asymmetry with the return flow coefficient asymmetry ($\rho_2 \uparrow + \gamma_2 \downarrow$) leads to a decrease in the welfare inefficiency, which is not surprising given that, in isolation, the two asymmetries had the same effect. In addition, the combinations $\rho_2 \uparrow + b_2 \uparrow$ and $\rho_2 \uparrow + z_2 \downarrow$ result in a decrease in stock and welfare inefficiencies, due to the predominant influence of the discount rate asymmetry on the stock levels and total welfare described in the previous section (see last column, Table 6).

Finally, the farmer's profitability of cooperation is not significantly affected by the introduction of the discount rate asymmetry when combined with the other asymmetries. In fact, if we compare the two last lines of Table 5 summarizing results for isolated asymmetries with the two last lines of Table 6 for combined asymmetries, we observe the same patterns.

Table 6: Results of stock ($G_\infty^C - G_\infty^{NC}$) and total welfare ($\sum_i W_i^C - W_i^{NC}$) inefficiencies, and farmer's profitability of cooperation ($W_i^C - W_i^{NC},\ i = 1..2$) for different types of land, crop productivity, and water cost asymmetries combined with the discount rate asymmetry.

	Type of asymmetry	(2)+(3) $\rho_2 \uparrow + \gamma_2 \downarrow$		(2)+(4) $\rho_2 \uparrow + b_2 \uparrow$		(2)+(5) $\rho_2 \uparrow + z_2 \downarrow$	
Type of inefficiency	Stock	$\searrow$	[-27.1%,0]	$\searrow$	[-21.9%,0]	$\searrow$	[-37%,0]
	Welfare	$\searrow$	[-33.5%,0]	$\searrow$	[-0.3%,0]	$\searrow$	[-55%,0]
Profitability of cooperation	Type 1 farmer	$\nearrow$	[0,5.5%]	$\nearrow$	[0,141%]	$\searrow_{\text{NP}}$	[-114%,0]
	Type 2 farmer	$\searrow$	[-72.5%,0]	$\searrow_{\text{NP}}$	[-141%,0]	$\nearrow$	[0,3.6%]

Terms in brackets [l, h]: maximum decrease (l) and increase (h) (in percentage) of simulated results for the highest asymmetry considered with respect to the benchmark case.
$_{NP}$ means not profitable, i.e., farmer's individual welfare is lower under cooperation than under non-cooperation.

6 Conclusions

This paper analyzes how different types of heterogeneities (asymmetries) between farmers who use groundwater for irrigation affect the exploitation of the natural resource and farmers' welfare by using a differential game. In particular, we consider asymmetries in water demand, time preferences (i.e., different future discount rates), extraction costs, crop productivity, and land type (i.e., different return flow coefficients due to the properties of the soil).

Focusing on the case of two types of farmers, we analytically solve the model under non-cooperation and under cooperation (i.e., the social planner problem). Firstly, we study conditions for the existence and uniqueness of the solutions. Next, we apply the model to the case study of the Western La Mancha aquifer in Spain to quantify the level of inefficiency of the non-cooperative solution with respect to the cooperative one (in terms of stock and total welfare) as heterogeneities related to farmers' agricultural activity intensify.

Numerical results show that as farmers differ more in their total demand and time preferences, cooperation loses its advantage in terms of both stock and welfare; in other words, the inefficiency of the non-cooperative solution compared to the cooperative one decreases. The opposite is observed, in terms of stock and total welfare, when farmers differ in their crop productivity, and only for the stock levels when farmers differ in the return flow coefficients. Lastly, when one farmer has lower marginal extraction costs, the difference between the stock levels under cooperation and non-cooperation stays roughly the same, while the difference in welfare increases.

In this paper, we are particularly interested in studying how the introduction of asymmetries in time preferences over the infinite planning horizon intensify or counteract the effect of other asymmetries on the inefficiency of non-cooperation. We observed that considering different degrees of impatience between farmers will counteract the effects of other heterogeneities (in the type of land in use, crop productivity, or costs) on groundwater exploitation and reduce the inefficiency of non-cooperation in terms of stock and total welfare.

Summarizing, this study shows that, generally, cooperation is more desirable both for the environment and the agents: it results in higher levels of stock and total welfare. However, when agents differ in the type of land used or crop productivity, only slight differences (of around 0-4.5%) between solutions in terms of total welfare are achieved for higher levels of heterogeneities between farmers. As explained in Gisser and Sánchez (1980), cooperation may not be justified under these circumstances.

On the other hand, when agents exhibit different time preferences together with the aforementioned asymmetries, differences in stock and total welfare between solutions decrease significantly, reaching up to 55% (in the case of heterogeneous costs). Despite this, when a farmer shows low levels of crop productivity or high marginal extraction costs, it is not profitable for this farmer to cooperate, irrespective of the degree of their impatience with water consumption.

This could mean that, if policy-makers want to induce cooperation of groundwater users to protect the environment, ensuring a stable economic situation through individual discount rates may not be sufficient to convince a specific farmer to cooperate. While instruments such as crop insurances to secure their future crop productivity need to be provided, lowering the costs of extraction for some farmers by introducing subsidies might actually hinder cooperation due to its negative impact on profitability of farmers not eligible for these subsidies.

Several extensions of this work are possible. Firstly, we could solve the model for other non-cooperative cases: open-loop, myopic behavior or farsighted vs. myopic farmers to estimate how solutions differ with respect to the cooperative case. Secondly, we could include other externalities such as environmental externality in the cost function as in Esteban and Albiac (2011), to account for the possible ecosystem damage caused by excessive extractions. Next, as performed in de Frutos Cachorro, Erdlenbruch, and Tidball (2019), it would be interesting to compute what would be the optimal quota to impose to the non-cooperative farmers in order to reach the stock levels under cooperation. Finally, it would be useful to implement the theoretical results on the aquifer data from a developing country where the problem of general water management is quite critical.

Acknowledgments

This work is supported by the Spanish State Research Agency under projects PID2020-112509GB-100 (AEI) and TED2021-130390B-I00.

Appendix

A Markov Perfect Nash Equilibrium. Proof of Theorem 1

First of all, by defining $\bar{g}_i = (1-\gamma_i)g_i$, $\bar{a}_i = (1-\gamma_i)a_i$, $\bar{b}_i = (1-\gamma_i)^2 b_i$, $\bar{z}_i = \dfrac{z_i}{1-\gamma_i}$ and $\bar{c}_i = \dfrac{c_i}{1-\gamma_i}$, functions F_i in Equation (4) and the stock dynamics (3) become

$$F_i(G, \bar{g}_i) = \frac{\bar{a}_i}{\bar{b}_i}\bar{g}_i - \frac{1}{2\bar{b}_i}\bar{g}_i^2 - (\bar{z}_i - \bar{c}_i G)\bar{g}_i \ ,$$

$$\dot{G} = r - \sum_{j=1}^{2} \bar{g}_j \ .$$

As a result, the dynamic programming equations (6) become

$$\rho_i V_i^{NC}(G) = \max_{\{\bar{g}_i\}} \left\{ F_i(G, \bar{g}_i) + \left(V_i^{NC}(G)\right)' \left(r - (\bar{g}_i + \bar{\phi}_j^{NC}(G)\right) \right\} \tag{18}$$

for $i, j \in \{1, 2\}$, $i \neq j$, where $\bar{\phi}_i^{NC}(G) = \bar{\alpha}_i^{NC} G + \bar{\beta}_i^{NC}$, with $\bar{\alpha}_i^{NC} = (1-\gamma_i)\alpha_i^{NC}$ and $\bar{\beta}_i^{NC} = (1-\gamma_i)\beta_i^{NC}$, and $V_i^{NC}(G) = A_i^{NC} G^2 + B_i^{NC} G + C_i^{NC}$.

From the First Order Conditions of a maximum in the right hand side of (18) we obtain

$$\frac{1}{\bar{b}_i}\bar{\phi}_i^{NC} = \left(\bar{c}_i - 2A_i^{NC}\right) G + \frac{\bar{a}_i}{\bar{b}_i} - \bar{z}_i - B_i^{NC} \ .$$

Therefore,

$$\bar{\alpha}_i^{NC} = \bar{b}_i \left(\bar{c}_i - 2A_i^{NC}\right) \ , \qquad \bar{\beta}_i^{NC} = \bar{a}_i - \bar{z}_i \bar{b}_i - \bar{b}_i B_i^{NC} \ ,$$

so

$$A_i^{NC} = \frac{1}{2}\left(\bar{c}_i - \frac{\bar{\alpha}_i^{NC}}{\bar{b}_i}\right) \ , \qquad B_i^{NC} = \frac{1}{\bar{b}_i}\left(\bar{a}_i - \bar{b}_i \bar{z}_i - \bar{\beta}_i^{NC}\right) \ . \tag{19}$$

Since, from (18),

$$A_i^{NC} = \frac{-\frac{(\bar{\alpha}_i^{NC})^2}{2\bar{b}_i} + \bar{c}_i\bar{\alpha}_i^{NC}}{\rho_i + 2\sum_{j=1}^2 \bar{\alpha}_j^{NC}} \ , \tag{20}$$

$$B_i^{NC} = \frac{\left(\frac{\bar{a}_i}{\bar{b}_i} - \bar{z}_i\right)\bar{\alpha}_i^{NC} - \frac{\bar{\alpha}_i^{NC}\bar{\beta}_i^{NC}}{\bar{b}_i} + \bar{c}_i\bar{\beta}_i^{NC} + 2A_i^{NC}\left(r - \sum_{j=1}^2 \bar{\beta}_j^{NC}\right)}{\rho_i + \sum_{j=1}^2 \bar{\alpha}_j^{NC}} \ , \tag{21}$$

$$C_i^{NC} = \frac{-\frac{1}{2\bar{b}_i}(\bar{\beta}_i^{NC})^2 + \left(\frac{\bar{a}_i}{\bar{b}_i} - \bar{z}_i\right)\bar{\beta}_i^{NC} + B_i^{NC}\left(r - \sum_{j=1}^2 \bar{\beta}_j^{NC}\right)}{\rho_i} \ , \tag{22}$$

by substituting (20)-(22) into equations (19), parameters $\bar{\alpha}_i^{NC}$ and $\bar{\beta}_i^{NC}$ solve the following system of nonlinear equations:

$$\frac{\rho_i}{2}\left(\bar{c}_i - \frac{\bar{\alpha}_i^{NC}}{\bar{b}_i}\right) = -\frac{(\bar{\alpha}_i^{NC})^2}{2\bar{b}_i} + \bar{c}_i\bar{\alpha}_i^{NC} - \left(\bar{c}_i - \frac{\bar{\alpha}_i^{NC}}{\bar{b}_i}\right)\sum_{k=1}^2 \bar{\alpha}_k \ , \tag{23}$$

$$\frac{1}{\bar{b}_i}\left(\rho_i + \sum_{k=1}^2 \bar{\alpha}_k^{NC}\right)\bar{\beta}_i^{NC} + \left(\frac{\bar{\alpha}_i^{NC}}{\bar{b}_i} - \bar{c}_i\right)\bar{\beta}_j^{NC} = \left(\frac{\bar{a}_i}{\bar{b}_i} - \bar{z}_i\right)\left(\rho_i + \bar{\alpha}_j^{NC}\right) + r\left(\frac{\bar{\alpha}_i^{NC}}{\bar{b}_i} - \bar{c}_i\right) \ , \tag{24}$$

for $i, j \in \{1, 2\}$, $i \neq j$. In addition, from Theorem 1, the conditions

$$\sum_{j=1}^2 \bar{\alpha}_j^{NC} > 0 \quad \text{and} \quad r \geq \sum_{j=1}^2 \bar{\beta}_j^{NC} \tag{25}$$

must be satisfied.

Next, we analyze first the nonlinear equations system (23) subject to (25).

First, we write equations (23) as

$$\rho_i(\bar{b}_i\bar{c}_i - \bar{\alpha}_i^{NC}) = -(\bar{\alpha}_i^{NC})^2 + 2\bar{b}_i\bar{c}_i\bar{\alpha}_i^{NC} - 2(\bar{b}_i\bar{c}_i - \bar{\alpha}_i^{NC})\sum_{j=1}^2 \bar{\alpha}_j^{NC} \ ,$$

for $i = 1, 2$. Next, by defining $x_i = \bar{\alpha}_i^{NC} - \bar{b}_i\bar{c}_i$, the previous equations can be rewritten as

$$-\rho_i x_i = -(x_i^2 + 2\bar{b}_i\bar{c}_i x_i + \bar{b}_i^2\bar{c}_i^2) + 2\bar{b}_i\bar{c}_i(x_i + \bar{b}_i\bar{c}_i) + 2x_i\sum_{j=1}^2 (x_j + \bar{b}_j\bar{c}_j)$$

and, rearranging the terms, we obtain

$$x_i^2 + \left(\rho_i + 2\sum_{j=1}^2 \bar{b}_j\bar{c}_j\right)x_i + 2x_1x_2 + \bar{b}_i^2\bar{c}_i^2 = 0 \tag{26}$$

Let

$$y_i = x_1 + x_2 + \frac{\rho_i}{2} + \sum_{j=1}^{2} \bar{b}_j \bar{c}_j \ . \tag{27}$$

Then, for $i, j \in \{1, 2\}$, $i \neq j$,

$$x_j = y_i - x_i - \frac{\rho_i}{2} - \sum_{k=1}^{2} \bar{b}_k \bar{c}_k \ . \tag{28}$$

By substituting (28) into (26), we obtain $x_i^2 - 2y_i x_i - \bar{b}_i^2 \bar{c}_i^2 = 0$

Summarizing, from (25), we have to solve, for $i = 1, 2$,

$$\left. \begin{array}{rcl} x_i^2 - 2y_i x_i - \bar{b}_i^2 \bar{c}_i^2 & = & 0 \\ \sum_{j=1}^{2} (x_j + \bar{b}_j \bar{c}_j) & > & 0 \end{array} \right\} . \tag{29}$$

In order to determine the number of Markov Perfect Nash Equilibria, first of all we have to check the number of roots of equation (29). Note that we can write

$$x_i = y_i \pm \sqrt{y_i^2 + \bar{b}_i^2 \bar{c}_i^2} \ ,$$

for $i = 1, 2$. From these 4 solutions, we are interested in those satisfying the condition $\sum_{j=1}^{2} (x_j + \bar{b}_j \bar{c}_j) > 0$.

Without loss of generality, we can assume that $\rho_1 \geq \rho_2$.

<u>Case 1</u>. For $i = 1, 2$, assume that $x_i = y_i + \sqrt{y_i^2 + \bar{b}_i^2 \bar{c}_i^2}$.

Then, taking into account that $y_i = x_1 + x_2 + \frac{\rho_i}{2} + \sum_{k=1}^{2} \bar{b}_k \bar{c}_k$ and, also, $y_i = x_i - \sqrt{y_i^2 + \bar{b}_i^2 \bar{c}_i^2})$, for $i, j = 1, 2$, $i \neq j$, we obtain

$$x_j + \bar{b}_j \bar{c}_j = -\frac{\rho_i}{2} - \bar{b}_i \bar{c}_i - \sqrt{y_i^2 + \bar{b}_i^2 \bar{c}_i^2} < 0 \ .$$

But this is in contradiction with the convergence condition $\sum_{j=1}^{2} (x_j + \bar{b}_j \bar{c}_j) > 0$.

<u>Case 2</u>. If $x_1 = y_1 + \sqrt{y_1^2 + \bar{b}_1^2 \bar{c}_1^2}$ and $x_2 = y_2 - \sqrt{y_2^2 + \bar{b}_2^2 \bar{c}_2^2}$, by proceeding as in the previous case, we obtain

$$x_1 + \bar{b}_1 \bar{c}_1 = -\frac{\rho_2}{2} - \bar{b}_2 \bar{c}_2 + \sqrt{y_2^2 + \bar{b}_2^2 \bar{c}_2^2} \quad \text{and}$$

$$x_2 + \bar{b}_2 \bar{c}_2 = -\frac{\rho_1}{2} - \bar{b}_1 \bar{c}_1 - \sqrt{y_1^2 + \bar{b}_1^2 \bar{c}_1^2} \ .$$

Therefore,

$$\sum_{k=1}^{2}(x_k + \bar{b}_k\bar{c}_k) = -\sum_{k=1}^{2}(\rho_k + \bar{b}_k\bar{c}_k) + \sqrt{y_2^2 + \bar{b}_2^2\bar{c}_2^2} - \sqrt{y_1^2 + \bar{b}_1^2\bar{c}_1^2}\ .$$

Note that, since $\rho_i \geq \rho_2$, then $y_1 \geq y_2$.

If $\bar{b}_1\bar{c}_1 \geq \bar{b}_2\bar{c}_2$, then it is clear that $\sqrt{y_1^2 + \bar{b}_1^2\bar{c}_1^2} \geq \sqrt{y_2^2 + \bar{b}_2^2\bar{c}_2^2}$. Hence, $\sum_{k=1}^{2}(x_k + \bar{b}_k\bar{c}_k) < 0$, that contradicts the convergence condition.

It remains to consider the case $\bar{b}_1\bar{c}_1 < \bar{b}_2\bar{c}_2$. Let us write

$$y_2^2 + \bar{b}_2^2\bar{c}_2^2 = \left(x_1 + x_2 + \frac{\rho_2}{2} + \sum_{k=1}^{2}\bar{b}_k\bar{c}_k\right)^2 + \bar{b}_2^2\bar{c}_2^2\ .$$

If the convergence condition is satisfied, necessarily $x_1 + x_2 + \frac{\rho_2}{2} + \sum_{k=1}^{2}\bar{b}_k\bar{c}_k > 0$, hence

$$(x_1 + x_2 + \frac{\rho_2}{2} + \sum_{k=1}^{2}\bar{b}_k\bar{c}_k)^2 + \bar{b}_2^2\bar{c}_2^2 < (x_1 + x_2 + \frac{\rho_2}{2} + \sum_{k=1}^{2}\bar{b}_k\bar{c}_k + \bar{b}_2\bar{c}_2)^2\ ,$$

so

$$\sqrt{y_2^2 + \bar{b}_2^2\bar{c}_2^2} < x_1 + x_2 + \frac{\rho_2}{2} + \sum_{k=1}^{2}\bar{b}_k\bar{c}_k + \bar{b}_2\bar{c}_2\ ,$$

i.e.,

$$x_1 + x_2 + \sum_{k=1}^{2}\bar{b}_k\bar{c}_k < -\sum_{k=1}^{2}(\frac{\rho_k}{2} + \bar{b}_k\bar{c}_k) - \sqrt{y_1^2 + \bar{b}_1^2\bar{c}_1^2} + x_1 + x_2 + \frac{\rho_2}{2} + \sum_{k=1}^{2}\bar{b}_k\bar{c}_k + \bar{b}_2\bar{c}_2\ .$$

This implies that $0 < -\frac{\rho_1}{2} - 2\bar{b}_1\bar{c}_1 - \sqrt{y_1^2 + \bar{b}_1^2\bar{c}_1^2}$, that is not possible.

<u>Case 3</u>. If $x_1 = y_1 - \sqrt{y_1^2 + \bar{b}_1^2\bar{c}_1^2}$ and $x_2 = y_2 + \sqrt{y_2^2 + \bar{b}_2^2\bar{c}_2^2}$, then

$$\sum_{k=1}^{2}(x_k + \bar{b}_k\bar{c}_k) = -\sum_{k=1}^{2}(\rho_k + \bar{b}_k\bar{c}_k) - \sqrt{y_2^2 + \bar{b}_2^2\bar{c}_2^2} + \sqrt{y_1^2 + \bar{b}_1^2\bar{c}_1^2}\ .$$

Again, by reproducing the analysis of the previous case, we arrive to a contradiction.

<u>Case 4</u>. There is one case left for consideration:

$$x_1 = y_1 - \sqrt{y_1^2 + \bar{b}_1^2\bar{c}_1^2} \quad \text{and} \tag{30}$$

$$x_2 = y_2 - \sqrt{y_2^2 + \bar{b}_2^2\bar{c}_2^2}\ . \tag{31}$$

By taking the sum of expressions (30) and (31), and from (27), noting that $y_2 = y_1 + \frac{1}{2}(\rho_2 - \rho_1)$, we obtain

$$y_1 - \sqrt{y_1^2 + \bar{b}_1^2 \bar{c}_1^2} - \sqrt{\left(y_1 + \frac{1}{2}(\rho_2 - \rho_1)\right)^2 + \bar{b}_2^2 \bar{c}_2^2} = -\frac{\rho_2}{2} - \bar{b}_1 \bar{c}_1 - \bar{b}_2 \bar{c}_2 \ ,$$

that can be rewritten as

$$y_1 = f(y_1) \ , \quad \text{for} \quad f(y_1) = \sqrt{y_1^2 + \bar{b}_1^2 \bar{c}_1^2} + \sqrt{\left(y_1 + \frac{1}{2}(\rho_2 - \rho_1)\right)^2 + \bar{b}_2^2 \bar{c}_2^2} - \frac{\rho_2}{2} - \bar{b}_1 \bar{c}_1 - \bar{b}_2 \bar{c}_2 \ . \tag{32}$$

If the condition $\sum_{k=1}^{2}(x_k + \bar{b}_k \bar{c}_k) > 0$ is met, then $y_1 > 0$ and, from $f(y_1) < 0$, the (strict) convexity of function $f(y_1)$ and that

$$\lim_{y_1 \to \infty} \left[f(y_1) - \left(2y_1 - \frac{\rho_2}{2} - \bar{b}_1 \bar{c}_1 - \bar{b}_2 \bar{c}_2 \right) \right] = 0 \ ,$$

then equation (32) has a unique solution. This implies that x_1 and x_2 in (30) and (31) are unique. Therefore, if the condition $\sum_{k=1}^{2}(x_k + \bar{b}_k \bar{c}_k) > 0$ is met, there will be a unique solution (x_1, x_2) to the equation system (29). By undoing the changes of variables, coefficients α_i^{NC} in the equilibrium strategies will be $\alpha_i^{NC} = x_i - (1 - \gamma_i) b_i c_i$. On the contrary, if the condition $\sum_{k=1}^{2}(x_k + \bar{b}_k \bar{c}_k) > 0$ is not verified for the unique solution of the nonlinear equation system, there will not be stationary linear MPNE.

B Cooperative Solution

First of all, as in Appendix A, we define $\bar{g}_i = (1 - \gamma_i) g_i$, $\bar{a}_i = (1 - \gamma_i) a_i$, $\bar{b}_i = (1 - \gamma_i)^2 b_i$, $\bar{z}_i = \dfrac{z_i}{1 - \gamma_i}$ and $\bar{c}_i = \dfrac{c_i}{1 - \gamma_i}$.

For the strategies $\bar{\phi}_i^C(G) = \bar{\alpha}_i^C G + \beta_i^C$, where $\bar{\alpha}_i^C = (1 - \gamma_i) \alpha_i^C$ and $\bar{\beta}_i^C = (1 - \gamma_i) \beta_i^C$, for $i = 1, 2$, from Equation (14) we have to solve

$$\rho_i \left(A_i^C G^2 + B_i^C G + C_i^C \right) = \frac{\bar{a}_i}{\bar{b}_i} \bar{\alpha}_i^C G + \frac{\bar{a}_i}{\bar{b}_i} \bar{\beta}_i^C - \frac{1}{2\bar{b}_i} \left(\bar{\alpha}_i^C G + \bar{\beta}_i^C \right)^2 - (\bar{z}_i - \bar{c}_i G) \left(\bar{\alpha}_i^C G + \bar{\beta}_i^C \right)$$
$$+ \left(2A_i^C G + B_i^C \right) \left[r - \sum_{j=1}^{2} \left(\bar{\alpha}_j^C G + \bar{\beta}_j^C \right) \right] \ ,$$

for $i = 1, 2$. Using (15), we can derive the following system of 6 equations in the unknown variables A_1^C, A_2^C, B_1^C, B_2^C, C_1^C and C_2^C:

$$\rho_i A_i^C = -\frac{1}{2\bar{b}_i}\left(\bar{\alpha}_i^C\right)^2 + \bar{c}_i\bar{\alpha}_i^C - 2A_i^C\sum_{j=1}^{2}\bar{\alpha}_j^C\ , \tag{33}$$

$$\rho_i B_i^C = \frac{\bar{a}_i}{\bar{b}_i}\bar{\alpha}_i^C - \frac{1}{\bar{b}_i}\bar{\alpha}_i^C\bar{\beta}_i^C - \bar{z}_i\bar{\alpha}_i^C + \bar{c}_i\bar{\beta}_i^C + 2rA_i^C - 2A_i^C\sum_{j=1}^{2}\bar{\beta}_i^C - B_i^C\sum_{j=1}^{2}\bar{\alpha}_j^C\ , \tag{34}$$

and

$$\rho_i C_i^C = \frac{\bar{a}_i}{\bar{b}_i}\bar{\beta}_i^C - \frac{1}{\bar{b}_i}\left(\bar{\beta}_i^C\right)^2 - \bar{z}_i\bar{\beta}_i^C + rB_i^C - B_i^C\sum_{j=1}^{2}\bar{\beta}_j^C\ , \tag{35}$$

for $i = 1, 2$, where

$$\bar{\alpha}_i^C = \bar{b}_i\left[\bar{c}_i - 2\sum_{j=1}^{2}\left(A_j^C\right)\right]\ , \qquad \bar{\beta}_i^C = \bar{a}_i - \bar{b}_i\bar{z}_i - \bar{b}_i\sum_{j=1}^{2}B_j^C\ . \tag{36}$$

The convergence condition (17) becomes $\bar{\alpha}_1^C + \bar{\alpha}_2^C > 0$.

In the following, we assume that $\bar{b}_1 = \bar{b}_2 = \bar{b}$ and $\bar{c}_1 = \bar{c}_2 = \bar{c}$. In that case, $\bar{\alpha}_1^C = \bar{\alpha}_2^C = \bar{\alpha}^C$. Equation (33) can be rewritten as

$$\frac{1}{2\bar{b}}\left(\bar{\alpha}^C\right)^2 + \left(4A_i^C - \bar{c}\right)\bar{\alpha}^C + \rho_i A_i^C = 0\ , \tag{37}$$

with $\bar{\alpha}^C = \bar{b}\left[\bar{c} - 2\left(A_1^C + A_2^C\right)\right] > 0$. In that case we have the following bounds for the coefficients A_i^C.

Lemma 1. *For $i = 1, 2$, if solutions exist to Equation (37) satisfying $\bar{\alpha}^C > 0$, then $0 < A_i^C < \dfrac{\bar{c}}{4}$.*

Proof: First of all, from (37), we can solve

$$\alpha = \bar{b}\left(\bar{c} - 4A_i^C\right) \pm \sqrt{\bar{b}^2\left(\bar{c} - 4A_i^C\right)^2 - 2\bar{b}\rho_i A_i^C}\ . \tag{38}$$

If $A_i^C > 0$, since $\bar{\alpha} > 0$, then $\bar{c} - 4A_i^C > 0$, so $A_i^C < \frac{c}{4}$.

Next, note that $A_1^C(\rho_1 + 4\bar{\alpha}) = A_2^C(\rho_2 + 4\bar{\alpha}) = -\frac{\bar{\alpha}^2}{2\bar{b}} + \bar{c}\bar{\alpha}$. From the positivity of ρ_1, ρ_2 and $\bar{\alpha}$, necessarily $\mathrm{Sign}A_1^C = \mathrm{Sign}A_2^C$.

If $A_i^C = 0$, then $A_j^C = 0$ and $-\frac{\bar{\alpha}}{2\bar{b}} + \bar{c} = 0$. But if $A_1^C = A_2^C = 0$, then $\bar{\alpha}^C = \bar{b}\bar{c}$ and the latter condition implies that $\bar{b}\bar{c} = 0$, that is not possible.

It remains to check that, it is not possible that $A_1^C < 0$ and $A_2^C < 0$. For $i = 1, 2$, by writing $\Delta_i = \sqrt{\bar{b}^2\left(\bar{c} - 4A_i^C\right)^2 - 2\bar{b}\rho_i A_i^C}$ in Equation (38), if $A_i^C < 0$, then $\Delta_i >$

$\bar{b}\left(\bar{c}-4A_i^C\right)>0$. Since $\bar{\alpha}>0$, necessarily $\bar{\alpha}=\bar{b}\left(\bar{c}-4A_i^C\right)+\Delta_i$, i.e., $\bar{\alpha}-\bar{b}\bar{c}=-4A_i^C+\Delta_i$. Next, recall that $\bar{\alpha}-\bar{b}\bar{c}=-2\bar{b}\left(A_1^C+A_2^C\right)$. Therefore, $-2\bar{b}\left(A_1^C+A_2^C\right)=-4A_i^C+\Delta_i$, for $i=1,2$. This implies that $\Delta_1+\Delta_2=0$, in contradiction with the condition $\Delta_i>0$. $\square$

C Tables corresponding to numerical simulations

C.1 Demand or/and discount asymmetries

Table 7: Stock volume (in millions cubic meters) at the steady state for different values of ρ_2 and for different θ with $\gamma_1 = \gamma_2, b_1 = b_2$ and $z_1 = z_2$.

	Parameters	(1) G_∞^C	(2) G_∞^{NC}	(1)-(2)
$\theta = \frac{1}{2}$	$\rho_1 = \rho_2 = 0.05$	78397	72502	5895
	$\rho_1 = 0.05, \rho_2 = 0.07$	77263	72408	4855
	$\rho_1 = 0.05, \rho_2 = 0.09$	76519	72332	4187
$\theta = \frac{1}{6}$	$\rho_1 = \rho_2 = 0.05$	78397	73603	4794
	$\rho_1 = 0.05, \rho_2 = 0.07$	77992	73566	4426
	$\rho_1 = 0.05, \rho_2 = 0.09$	77695	73535	4160

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

Table 8: Welfare analysis (in thousand euros) for different values of ρ_2 and for different θ with $\gamma_1 = \gamma_2, b_1 = b_2$ and $z_1 = z_2$.

	Parameters	(2) W_1^C	(3) W_2^C	(4) W_1^{NC}	(5) W_2^{NC}	(6) (2)-(4)	(7) (3)-(5)	(8) (2)+(3)	(9) (4)+(5)	(10) (8)-(9)
$\theta = \frac{1}{2}$	$\rho_1 = \rho_2 = 0.05$	153534	153534	112269	112269	41265	41265	307068	224538	82530
	$\rho_1 = 0.05, \rho_2 = 0.07$	152585	118532	109721	98199	42864	20333	271117	207920	63197
	$\rho_1 = 0.05, \rho_2 = 0.09$	150819	99778	107633	88808	43186	10970	250597	196441	54156
$\theta = \frac{1}{6}$	$\rho_1 = \rho_2 = 0.05$	255890	51178	190791	59052	65099	-7874	307068	249843	57225
	$\rho_1 = 0.05, \rho_2 = 0.07$	255695	39094	189715	49403	65980	-10309	294789	239118	55671
	$\rho_1 = 0.05, \rho_2 = 0.09$	255293	32298	188827	43243	66466	-10945	287591	232070	55521

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

C.2 Return flow coefficient or/and discount asymmetries

Table 9: Stock volume (in millions cubic meters) at the steady state for different values of ρ_2 and for different γ_2 with $\theta = \frac{1}{2}$, $b_1 = b_2$ and $z_1 = z_2$.

Parameters		(1) G_∞^C	(2) G_∞^{NC}	(1)-(2)
$\gamma_1 = \gamma_2 = 0.2$	$\rho_1 = \rho_2 = 0.05$	78397	72502	5895
	$\rho_1 = 0.05, \rho_2 = 0.07$	77263	72408	4855
	$\rho_1 = 0.05, \rho_2 = 0.09$	76519	72332	4187
$\gamma_2 = 0.05$	$\rho_1 = \rho_2 = 0.05$	78252	72305	5947
	$\rho_1 = 0.05, \rho_2 = 0.07$	77173	72224	4949
	$\rho_1 = 0.05, \rho_2 = 0.09$	76453	72156	4297

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

Table 10: Welfare analysis (in thousand euros) for different values of ρ_2 and for different γ_2 with $\theta = \frac{1}{2}$, $b_1 = b_2$ and $z_1 = z_2$.

Parameters		(2) W_1^C	(3) W_2^C	(4) W_1^{NC}	(5) W_2^{NC}	(6) (2)-(4)	(7) (3)-(5)	(8) (2)+(3)	(9) (4)+(5)	(10) (8)-(9)
$\gamma_1 = \gamma_2 = 0.2$	$\rho_1 = \rho_2 = 0.05$	153534	153534	112269	112269	41265	41265	307068	224538	82530
	$\rho_1 = 0.05, \rho_2 = 0.07$	152585	118532	109721	98199	42864	20333	271117	207920	63197
	$\rho_1 = 0.05, \rho_2 = 0.09$	150819	99778	107633	88808	43186	10970	250597	196441	54156
$\gamma_2 = 0.05$	$\rho_1 = \rho_2 = 0.05$	153496	131465	112074	93329	41422	38136	284961	205403	79558
	$\rho_1 = 0.05, \rho_2 = 0.07$	152991	102549	109894	82786	43097	19763	255540	192680	62860
	$\rho_1 = 0.05, \rho_2 = 0.09$	151598	87020	108062	75687	43536	11333	238618	183749	54869

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

C.3 Crop productivity or/and discount asymmetries

Table 11: Stock volume (in millions cubic meters) at the steady state for different values of ρ_2 and for different b_2 (and b_1) with $b_1 = b - b_2$, $\theta = \frac{1}{2}$, $\gamma_1 = \gamma_2$ and $z_1 = z_2$.

Parameters		(1) G_∞^C	(2) G_∞^{NC}	(1)-(2)
$b_2 = b_1 = \frac{b}{2}$	$\rho_1 = \rho_2 = 0.05$	78397	72502	5895
	$\rho_1 = 0.05, \rho_2 = 0.07$	77263	72408	4855
	$\rho_1 = 0.05, \rho_2 = 0.09$	76519	72332	4187
$b_2 = 1.09\frac{b}{2}$	$\rho_1 = \rho_2 = 0.05$	78397	72437	5960
	$\rho_1 = 0.05, \rho_2 = 0.07$	77513	72372	5141
	$\rho_1 = 0.05, \rho_2 = 0.09$	76921	72319	4602

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

Table 12: Welfare analysis (in thousand euros) for different values of ρ_2 and for different b_2 (and b_1) with $b_1 = b - b_2$, $\theta = \frac{1}{2}$, $\gamma_1 = \gamma_2$ and $z_1 = z_2$.

Parameters		(2) W_1^C	(3) W_2^C	(4) W_1^{NC}	(5) W_2^{NC}	(6) (2)-(4)	(7) (3)-(5)	(8) (2)+(3)	(9) (4)+(5)	(10) (8)-(9)
$b_2 = b_1 = \frac{b}{2}$	$\rho_1 = \rho_2 = 0.05$	153534	153534	112269	112269	41265	41265	307068	224538	82530
	$\rho_1 = 0.05, \rho_2 = 0.07$	152585	118532	109721	98199	42864	20333	271117	207920	63197
	$\rho_1 = 0.05, \rho_2 = 0.09$	150819	99778	107633	88808	43186	10970	250597	196441	54156
$b_2 = 1.09\frac{b}{2}$	$\rho_1 = \rho_2 = 0.05$	268841	54589	158962	78246	109879	-23657	323430	237208	86222
	$\rho_1 = 0.05, \rho_2 = 0.07$	260495	51438	156512	71214	103983	-19776	311933	227726	84207
	$\rho_1 = 0.05, \rho_2 = 0.09$	253951	48818	154486	66035	99465	-17217	302769	220521	82248

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

C.4 Extraction costs or/and discount asymmetries

Table 13: Stock volume (in millions cubic meters) at the steady state for different values of ρ_2 and for different z_2 with $\theta = \frac{1}{2}$, $\gamma_1 = \gamma_2$ and $b_1 = b_2$.

Parameters		(1) G_∞^C	(2) G_∞^{NC}	(1)-(2)
$z_2 = z_1$	$\rho_1 = \rho_2 = 0.05$	78397	72502	5895
	$\rho_1 = 0.05, \rho_2 = 0.07$	77263	72408	4855
	$\rho_1 = 0.05, \rho_2 = 0.09$	76519	72332	4187
$z_2 = \frac{97}{100} z_1$	$\rho_1 = \rho_2 = 0.05$	77135	71240	5895
	$\rho_1 = 0.05, \rho_2 = 0.07$	75664	71111	4553
	$\rho_1 = 0.05, \rho_2 = 0.09$	74720	71006	3714

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

Table 14: Welfare analysis (in thousand euros) for different values of ρ_2 and for different z_2 with $\theta = \frac{1}{2}$, $\gamma_1 = \gamma_2$ and $b_1 = b_2$.

Parameters		(2) W_1^C	(3) W_2^C	(4) W_1^{NC}	(5) W_2^{NC}	(6) (2)-(4)	(7) (3)-(5)	(8) (2)+(3)	(9) (4)+(5)	(10) (8)-(9)
$z_2 = z_1$	$\rho_1 = \rho_2 = 0.05$	153534	153534	112269	112269	41265	41265	307068	224538	82530
	$\rho_1 = 0.05, \rho_2 = 0.07$	152585	118532	109721	98199	42864	20333	271117	207920	63197
	$\rho_1 = 0.05, \rho_2 = 0.09$	150819	99778	107633	88808	43186	10970	250597	196441	54156
$z_2 = \frac{97}{100} z_1$	$\rho_1 = \rho_2 = 0.05$	61659	306878	88312	186639	-26653	120239	368537	274951	93586
	$\rho_1 = 0.05, \rho_2 = 0.07$	72491	227216	85729	160185	-13238	67031	299707	245914	53793
	$\rho_1 = 0.05, \rho_2 = 0.09$	77969	185884	83645	143112	-5676	42772	263853	226757	37096

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

C.5 Extraction results. Detailed calculations

Table 15: Total extractions (in cubic meters) over the first 20 years for different θ with $\rho_1 = \rho_2 = 0.05, \gamma_1 = \gamma_2 = 0.2, b_1 = b_2 = \frac{b}{2} = 0.0485$ and $z_1 = z_2 = 266000$.

Parameters	Total extractions (Tg)			
	Tg_1^C	Tg_2^C	Tg_1^{NC}	Tg_2^{NC}
$\theta = \frac{1}{2}$	5828	5828	9630	9630
$\theta = \frac{1}{6}$	9713	1943	12372	5306

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

Table 16: Total extractions (in cubic meters) over the first 20 years for different ρ_2 with $\theta = \frac{1}{2}, \gamma_1 = \gamma_2 = 0.2, b_1 = b_2 = \frac{b}{2} = 0.0485$ and $z_1 = z_2 = 266000$.

Parameters	Total extractions (Tg)			
	Tg_1^C	Tg_2^C	Tg_1^{NC}	Tg_2^{NC}
$\rho_1 = \rho_2 = 0.05$	5828	5828	9630	9630
$\rho_1 = 0.05, \rho_2 = 0.09$	6897	6897	9438	10050

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

Table 17: Total extractions (in cubic meters) over the first 20 years for different γ_2 with $\theta = \frac{1}{2}, \rho_1 = \rho_2 = 0.05, b_1 = b_2 = \frac{b}{2} = 0.0485$ and $z_1 = z_2 = 266000$.

Parameters	Total extractions (Tg)			
	Tg_1^C	Tg_2^C	Tg_1^{NC}	Tg_2^{NC}
$\gamma_1 = \gamma_2 = 0.2$	5828	5828	9630	9630
$\gamma_2 = 0.05$	5423	5423	8924	8976

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

Table 18: Total extractions (in cubic meters) over the first 20 years for different b_2 (and b_1) with $b_1 = b - b_2$, $\theta = \frac{1}{2}, \rho_1 = \rho_2 = 0.05, \gamma_1 = \gamma_2 = 0.2$ and $z_1 = z_2 = 266000$.

Parameters	Total extractions (Tg)			
	Tg_1^C	Tg_2^C	Tg_1^{NC}	Tg_2^{NC}
$b_2 = b_1 = \frac{b}{2}$	5828	5828	9630	9630
$b_2 = 1.09\frac{b}{2}$	9264	2392	11327	8008

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

Table 19: Total extractions (in cubic meters) over the first 20 years for different z_2 with $\theta = \frac{1}{2}, \rho_1 = \rho_2 = 0.05, \gamma_1 = \gamma_2 = 0.2$ and $b_2 = b_1 = \frac{b}{2} = 0.0485$.

Parameters	Total extractions (Tg)			
	Tg_1^C	Tg_2^C	Tg_1^{NC}	Tg_2^{NC}
$z_2 = z_1$	5828	5828	9630	9630
$z_2 = \frac{97}{100}z_1$	2625	10339	8350	12440

^{C} The cooperative solution, ^{NC} the non-cooperative solution.

References

Agencia del Agua de Castilla la Mancha (n.d.). *Acuíferos*. URL: https://agenciadelagua.castillalamancha.es/el-agua-en-castilla-la-mancha/situacion-del-agua-en-clm/acuiferos. (accessed: 21.05.2020).

Bozio, A, G Laroque, and C O'Dea (2017). "Discount rate heterogeneity among older households: a puzzle?" In: *Journal of Population Economics* 30, pp. 647–680.

de Frutos Cachorro, J, K Erdlenbruch, and M Tidball (2019). "Sharing a groundwater resource in a context of regime shifts". In: *Environmental and Resource Economics* 72, pp. 913–940.

de Frutos Cachorro, J, J Marín-Solano, and J Navas (2021). "Competition between different groundwater uses under water scarcity". In: *Water Resources and Economics* 33, p. 100173.

Dockner, EJ, S Jorgensen, N Van Long, and G Sorger (2000). *Differential Games in Economics and Management Science*. Cambridge: Cambridge University Press.

Ekeland, I, Y Long, and Q Zhou (2013). "A new class of problems in the calculus of variations". In: *Regular and Chaotic Dynamics* 18(6), pp. 558–589.

Erdlenbruch, K, M Tidball, and D van Soest (2008). "Renewable resource management, user heterogeneity, and the scope for cooperation". In: *Ecological Economics* 64(3), pp. 597–602.

Esteban, E and J Albiac (2011). "Groundwater and ecosystems damages: Questioning the Gisser-Sánchez effect". In: *Ecological Economics* 70(11), pp. 2062–2069.

Esteban, E and A Dinar (2016). "The role of groundwater-dependent ecosystems in groundwater management". In: *Natural Resource Modeling* 29(1), pp. 98–129.

Gisser, M and DA Sánchez (1980). "Competition versus optimal control in groundwater pumping". In: *Water Resources Research* 16(4), pp. 638–642.

Haurie, A, JB Krawczyk, and G Zaccour (2012). *Games and Dynamic Games*. Now Publishers Series in Business: Vol. 1. Singapore: World Scientific.

Hernández-Mora, N, L Martínez-Cortina, E Gimena, and M LLamas (2007). *Groundwater Issues in Southern EU Member States. Spain Country Report*. Technical Report.

Karp, L (2007). "Non-constant discounting in continuous time". In: *Journal of Economic Theory* 132.1, pp. 557–568. ISSN: 0022-0531.

Marín-Solano, J and EV Shevkoplyas (2011). "Non-constant discounting and differential games with random time horizon". In: *Automatica* 47(12), pp. 2626–2638.

Matousek, J, T Havranek, and Z Irsova (2022). “Individual discount rates: a meta-analysis of experimental evidence”. In: *Experimental Economics* 25, pp. 318–358.

Negri, DH (1989). “The common property aquifer as a differential game”. In: *Water Resources Research* 25(1), pp. 9–15.

de-Paz, A, J Marín-Solano, and J Navas (2013). “Time-consistent equilibria in common access resource games with asymmetric players under partial cooperation”. In: *Environmental Modeling and Assessment* 18, pp. 171–184.

Provencher, B and O Burt (1993). “The Externalities Associated with the Common Property Exploitation of Groundwater”. In: *Journal of Environmental Economics and Management* 24, pp. 139–158.

Roseta-Palma, C (2003). “Joint quantity/quality management of groundwater”. In: *Environmental and Resource Economics* 26, pp. 89–106.

Roseta-Palma, C and A Brasão (2004). “Strategic games in groundwater management”. In: *Working paper* 2004/39.

Rubio, SJ and B Casino (2001). “Competitive versus efficient extraction of a common property resource: The groundwater case”. In: *Journal of Economic Dynamics and Control* 25(8), pp. 1117–1137.

Sears, L, D Lim, and CY Cynthia Lin Lawell (2019). “Spatial groundwater management: A dynamic game framework and application to California”. In: *Water Economics and Policy* 5(1), p. 1850019.

Siebert, S, J Burke, JM Faures, K Frenken, J Hoogeveen, P Döll, and FT Portmann (2010). “Groundwater use for irrigation a global inventory”. In: *Hydrology and Earth System Sciences* 14.10, pp. 1863–1880. DOI: 10.5194/hess-14-1863-2010. URL: https://hess.copernicus.org/articles/14/1863/2010/.

UNESCO World Water Assessment Programme (2009). *Water In a Changing World: the United Nations World Water Development Report 3*. Report.

Unión de Uniones de Castilla la Mancha (n.d.). *Anuncio de 01/12/2017, de la Confederación Hidrográfica del Guadiana, sobre el Programa de Actuación de la Masa de Agua Subterránea Mancha Occidental 1*. URL: https://unionclm.org/2018/01/15/11475/#more-11475. (accessed: 21.05.2020).