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Working Papers

Col·lecció d'Economia E24/477

# REGULATING A SOCIAL MEDIA PLATFORM IN THE DATA ECONOMY

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ISSN 1136-8365

UNIVERSITAT DE BARCELONA
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Working Papers

UB Economics *Working Paper No. 477*

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**JEL Codes:** C11, D62, D83, H23, L51, L88

**Keywords:** Bayesian signalling, data, social media platform, user welfare

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**Date:** December 2024

**Acknowledgements:** I would like to thank Fernando Vega-Redondo for his invaluable advice throughout my PhD. I am grateful to Christoph Carnehl, Nenad Kos, Massimo Morelli and Nicolas Serrano-Velarde for their constant guidance. I would also like to thank Francesco Decarolis, Benjamin Golub, Leonardo Madio, Marco Ottaviani, Fabrizio Panebianco, Martin Peitz, Paolo Pin, Markus Reisinger, Carlo Schwarz, David Yang, Noam Yuchtman, participants at Bocconi University, University of Leicester, University of Amsterdam, University of Barcelona, Annual MaCCI Conference 2024, 15th Paris Conference on Digital Economics 2024, Barcelona School of Economics MicroLab 2024, Oligo Workshop 2024 and 23rd LAGV Conference 2024 for their helpful comments.

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# 1 Introduction

The impact of social media platforms (SMPs) on their users has come under heavy scrutiny recently. Users have reported concerns about their privacy and have expressed feeling a lack of control over their information (see Auxier et al. (2019)). The recent involvement of SMPs in scandals has further amplified these concerns. The *Cambridge Analytica* scandal has raised allegations of data misuse by Facebook and a whistleblower has alleged lack of user privacy and data protection on Twitter[1]. To address these concerns, policymakers have implemented regulations like the General Data Protection Regulation (GDPR) and are contemplating regulations like the American Data Privacy and Protection Act in the USA[2]. While there are several facets to be considered when designing such regulations, two key factors affecting users of an SMP stand out. First, the payoff effect of data-based personalization by an SMP on users and second, the effect of data externalities on users' data control rights.

I study these two key factors by considering a network model (as in Jackson and Rogers (2005) and Jackson (2010)) that connects users on an SMP. Any pair of connected users interact with each other on the SMP. This generates data about both users. The SMP offers a horizontally differentiated product to its users and the SMP earns revenue from data-based personalized advertising. For an arbitrary fixed user, the SMP offers a personalized product to the user based on the data collected about the user (see Bergemann et al. (2022)). I provide a microfoundation that pins down the corresponding payoff of the user and of the SMP, wherein the SMP uses data to steer the user by

a) recommending the best matching product to the user (documented by Ali et al. (2019) and Kozyreva et al. (2021)) and

b) personalizing the price of the recommended product (documented by Zuiderveen Borgesius and Poort (2017) and Bourreau and De Streel (2018)).

The microfounded payoffs show that while data analysis always benefits the SMP, it can be both harmful and beneficial for the user. Consequently, the SMP wants to attain all the data of the user and the user wants to share a limited amount of data with the SMP. To collect data, the SMP offers a price for data to each user.

Specifically, I study a two-stage game form consisting of the SMP and users connected on a network. In the first stage, the SMP offers a price for data to each user. Users simultaneously decide whether to accept or reject the offer and payoffs are realised. If a user

[1]Facebook https://www.nytimes.com/2018/04/04/us/politics/cambridge-analytica-scandal-fallout.html
Twitter https://www.washingtonpost.com/technology/interactive/2022/twitter-whistleblower-sec-spam/

[2]GDPR https://gdpr.eu/
American Data Privacy and Protection Act https://www.congress.gov/bill/117th-congress/house-bill/8152/actions

accepts the offer he gets the price for data and the SMP attains all the data generated about that user. If a user rejects the offer then he does not attain the price for data. However, the presence of data externalities implies that the SMP attains some of the user's data through *data leakage*, wherein friends of the user who consent to share data inadvertently reveal information about the user. Since users do not internalize *data leakage*, the SMP attains too much data in equilibrium Acemoglu et al. (2022).

In equilibrium, the SMP attains all the data generated on the user network. Data externalities dilute data control of each user and the SMP needs to compensate users (for price personalization) on the intensive margin and not on the extensive margin. Therefore, in equilibrium, each user consents to share his data and the SMP attains all the data generated on the user network.

Based on the equilibrium outcome, I analyze the effect of policy regulations on welfare. In order to do so, we first need to understand the structure of the microfounded payoffs. Revenue of the SMP is increasing in the number of data points attained about a user as each data point improves the product recommendation, thereby increasing the probability of sale. Further, the surplus of the SMP in case of sale also increases due to higher rent extraction via personalized pricing. User payoff is non-negative and non-monotonic in number of data points attained about the user. When the SMP has few data points about the user, an additional data point improves product recommendation significantly and the marginal effect of an additional data point is positive. When the SMP has many data points about the user, an additional data point improves product recommendation only slightly. The negative effect of personalized pricing is stronger and the marginal effect of an additional data point is negative. This gives the non-monotonicity of user payoff. User payoff is non-negative since data analysis improves user-product matching, relative to a random match under no data analysis.

Given these payoff structures, I evaluate the effect of two policy regulations. First, I find that a strict consent policy like GDPR, that stops all *data leakage*, reduces user payoff weakly. Without *data leakage* the positive payoff attained from better recommendations is lost. Consequently, each user gives consent at zero price to attain some positive payoff from data analysis. Therefore, under a strict consent policy none of the users attain any compensation and user payoff is always weakly lower. User payoff is strictly lower if users are sufficiently connected on the network.

Second, I analyze the effect of implementing the data minimization principle. The principle mandates that an SMP collects and analyzes the minimum amount of data required by the platform. An SMP is also expected to be able to demonstrate that it is analyzing only the necessary amount of data. Recall that the microfounded payoff of the user tells us that a user only wants to share an intermediate amount of data with the SMP. The presence of data externalities results in too much data being shared with the SMP. However, the data minimization principle enforces an upper bound on the amount of data that can be analyzed by the SMP and counteracts the negative impact of data leakage. Consequently, each user shares less data with the SMP and the payoff of each user and overall user welfare improves. However, this result requires that sufficient data is generated about each user. Otherwise the data minimization principle limits data sharing when very little user data is generated and this decreases user payoff. Therefore, applying the data minimization principle improves user welfare, with the caveat that the user network should not be too small or too sparsely connected. In other words, the data minimization principle should be mandated for big and established SMPs and not for up and coming startup SMPs.

**Related Literature:** This paper relates to three strands of literature, namely, privacy, regulation of social media platforms and contracting externalities. The need for better control over one's data has been studied in Posner and Weyl (2018) and Zuboff (2023). The importance of data externalities has long been recognised by MacCarthy (2010) and Fairfield and Engel (2015). Acemoglu et al. (2022) emphasize that heterogeneity in privacy concerns is a critical reason for inefficiency in a data market. Since privacy is their main concern, data analysis always has a negative effect in their paper. My paper gives a microfoundation of payoffs when data is used to steer users. Hence, user payoff is non-monotonic in number of data points and data analysis can have both a positive and a negative effect. In Choi et al. (2019) excessive loss of privacy due to data externalities is highlighted. The paper further shows that GDPR may fall short in limiting the collection of personal data to the socially optimal level. My paper points to the potential backfiring of a strict consent policy, not because the amount of data attained by the SMP in equilibrium is different but because the price for data under a strict consent policy reduces. Additionally, my paper highlights a regulation that increases user welfare, namely, breaking-up an SMP whenever the user network is sufficiently large and connected. Acquisti et al. (2016) provide a comprehensive review of the economics of privacy. They conclude that privacy concerns occur under many different scenarios and these concerns cannot be addressed by one blanket policy. This paper reaffirms this conclusion by showing that a strict consent policy like GDPR may reduce user welfare and breaking-up an SMP increases user welfare if the user network is sufficiently large and connected. This paper is also related to the literature of contracting externalities. Seminal work by Segal (1999) and more recent work by Jackson and Wilkie (2005) and Ellingsen and Paltseva (2016) have analyzed inefficiencies in contracting with externalities. The results of this paper, when comparing strict consent policy and loose consent policy are consistent with the above papers.

The rest of the paper is organized as follows. The model is described in Section 2. The equilibrium outcome is detailed in Section 3. The welfare effects of different policies is analyzed in Section 4. Section 5 concludes. All proofs are in the appendix.

# 2 Model

The model consists of an SMP and its users. The SMP advertises and sells a horizontally differentiated product. Users are connected on the SMP via a social network (described in section 2.1) and data about each user is generated through the network (section 2.2). The SMP shows a personalized advertisement to each user based on the data it collects about a user. The SMP recommends the best matching product to a user at a price based on its estimation of user's willingness to pay for the product. In section 2.3, a bayesian updating model is developed and it provides a microfoundation for the SMP's and the user's payoff from personalized advertising. The corresponding payoffs are such that the SMP wants to collect all the data generated about a user but a user wants to share only an intermediate amount of data with the SMP. Consequently, in order to collect user data, the SMP offers its services to users. This is modeled in a two-stage game in section 2.4 wherein the SMP offers its service to each user and each user either consents to share his data for this service or denies consent.

The network on which users of the SMP are connected and through which data about users is generated is described below.

## 2.1 The Network

Users are connected on an *island network* (as in Jackson and Rogers (2005) and Jackson (2010)), which is a special case of the multi-type random network. Given a set of $n$ users $N = \{1, ..., n\}$, a *network* is represented via its adjacency matrix: a symmetric $n$-by-$n$ matrix $\mathbf{G}$ with entries in $\{0, 1\}$. If $G_{ij} = G_{ji} = 1$ then users i and j are connected, and the assumption of symmetry restricts attention to undirected networks.

Users have "types", which are the distinguishing features that affect their propensities to connect to each other. Types might be based on any characteristics that influence users' probabilities of connecting to each other, including age, race, gender, profession, education level, and political leaning.

Suppose there are $m$ different types in the society. Let $N_k \subset N$ denote the users of type $k$, so the society is partitioned into $m$ groups, $(N_1, ..., N_m)$. Let $n_k = |N_k|$ denote the size of group $k$.

A *multi-type random network* is defined by the cardinality vector $\mathbf{n}$ together with a symmetric $m$-by-$m$ matrix $\mathbf{P}$, whose entries in $[0,1]$ describes the probability of connection between various types. The entry $P_{kl}$ captures the probability that a user of type $k$ is connected to a user of type $l$.

The island network is a special case of the multi-type random network, where each user is of the same type. Formally, the multi-type random network $\mathbf{G}(\mathbf{n},\mathbf{P})$ is an *island network* $G(n,p)$ if the network has $n$ nodes and any two nodes are connected with probability $p$. Consequently, expected number of connections of any user is $pn$. Henceforth, any island network $G(n,p)$ is referred to as a *network*.

## 2.2 Data Generated

Consider a network $G(n,p)$. Any two users that are connected on this network interact with each other and this generates *information* or *data* about both users. The data generated is modelled as follows. Each user generates some *personal data* on the SMP and it is of two types -

- an *individual data* point, which is generated by the user only and provides the SMP information about said user only and

- an *interactive data* point, which is generated between the user and a connected user. This data point gives the SMP information about *both users*.

Information like the location of a user, time at which a user logs on and logs off from the SMP and videos and photos posted by a user about himself are considered individual data points. Information like videos and photos posted by either of two connected users that has both connected users are considered interactive data points.
While individual data about a user can be attained only from that particular user, interactive data about two connected users can be attained from any one of the two connected users. Overall, one individual data point and $pn$ interactive data points are generated about a user in expectation. Therefore, the expected number of data points generated about each user on the SMP is $1+pn$.

Note that the interactive data points capture the presence of *data externalities* on the user network. For any two connected users $i$ and $j$, if the SMP attains data generated about $i$ then it also attains some data about $j$, specifically the interactive data point generated by $i$ and $j$. These *data externalities* play a crucial role in the equilibrium outcome.

## 2.3 Payoffs from personalized advertising

For any arbitrary fixed user, the SMP analyzes the data of the user (generated on the network) and shows him a personalized advertisement of a product. Before analyzing the service-for-data game, it is vital to understand the payoff attained by the SMP and by the user from personalized advertising. This is done by considering the following signaling game.

Consider the SMP and an arbitrary fixed user. The SMP advertises and sells a horizontally differentiated product. The user has preference $t$ over the type of the product, where $t \sim N(0,1)$. The user has willingness to pay (wtp) $w$ for the product, where $w \sim Pa(1,3)$. The SMP shows a personalized advertisement to the user by analyzing user data

a) to estimate the most preferred product-type for the user and

b) to estimate the user's wtp for the product.

The timeline of the game is as follows. Prior distributions $t$ and $w$ are common knowledge. The user learns his taste $t_r$ and his wtp $w_r$ for the product. Simultaneously, the SMP attains some $x \geq 0$ data points about the user (from the network), where each data point comprises of

- a signal $s_t$ about the taste of the user, where $s_t|t \sim N(t,1)$ and
- a signal $s_w$ about the wtp of the user, where $s_w|w \sim U(0,w)$

The SMP uses these signals to *steer* the user; the SMP shows an advertisement that consists of a personalized product-type $\hat{t}$ and if the user clicks on the advertisement then the SMP offers the product at a personalized price $\hat{w}^*$ to the user[3]. It is assumed that the user is more likely to click on the advertisement if $\hat{t}$ is close to $t_r$. Once the user clicks on the advertisement, the user buys the product if $\hat{w}^* \leq w_r$. Consequently, the SMP analyzes user data to maximize its payoff by estimating the taste of the user and the wtp of the user.

In order to derive the payoff attained by the SMP and the user from personalized advertising, the payoffs attained by the SMP and the user under personalized pricing *only* are derived. Using these payoffs, the payoffs attained by the SMP and the user when the SMP personalizes both the price and the type of the product are derived. These are the payoffs from personalized advertising.

**Proposition 1.** *Suppose the SMP attains $x \geq 0$ data points about the wtp $w$ of the user. Then the payoff from personalized pricing is $\frac{x+2}{x+3}$ for the SMP and $\frac{1}{x+3}$ for the user.*

[3]The SMP is implicitly assumed to be the seller of the product. The results remain unchanged when the SMP and the seller are distinct and the SMP obtains some fixed percentage of the seller revenue.

The payoff functions clearly show that when data is used for personalized pricing, the payoff of the SMP increases with number of data points attained and payoff of the user decreases with number of data points attained. Intuitively, as the SMP attains more data about the user, it improves its estimate of user's wtp. Consequently, as the number of data points attained by the SMP increases, the SMP extracts more surplus from the user by charging a higher price.

Interestingly, if a cost of production $c > 0$ is introduced for the SMP, then the price charged by the SMP decreases as it attains more data points but the probability that $\hat{w}^* \leq w_r$ increases. The latter effect dominates the former and the SMP extracts more surplus from the user as it attains more data points. Thus, even under positive cost of production, payoff of the SMP increases with number of data points attained and payoff of the user decreases with the number of data points attained, under personalized pricing.

Next, the payoffs attained by the SMP and the user when the SMP personalizes both the price and the type of product are derived.

**Proposition 2.** *Suppose the SMP attains $x \geq 0$ data points about the taste and the wtp of the user. The payoff from personalized advertising is $\frac{x(x+2)}{(x+1)(x+3)}$ for the SMP and $\frac{x}{(x+1)(x+3)}$ for the user.*

The payoff of the SMP is denoted by function $g$ and user payoff is denoted by function $f$. Consequently,

$$g(x) = \frac{x}{x+1}\frac{x+2}{x+3} = \frac{x(x+2)}{(x+1)(x+3)}$$
$$f(x) = \frac{x}{x+1}\frac{1}{x+3} = \frac{x}{(x+1)(x+3)}$$

Note that $g$ is increasing in $x$ and $f$ is non-monotonic with a unique maximizer. The unique maximizer of $f$ is denoted by $x_f^*$. The functional properties of $g$ and $f$ are crucial for the subsequent analysis and are stated below.

**Corollary 1.** *Suppose the SMP attains $x$ data points about a user. Payoff of the SMP from personalized advertising is $g(x)$, which is increasing and concave in $x$.*

**Corollary 2.** *Suppose the SMP attains $x$ data points about a user. User payoff from personalized advertising is $f(x)$, which is non-monotonic in $x$ and is maximized at a unique value $x_f^*$.*

The two results are clearly illustrated in Figure 1, where $g$ is represented in the first part of the figure and $f$ is represented in the second part of the figure.

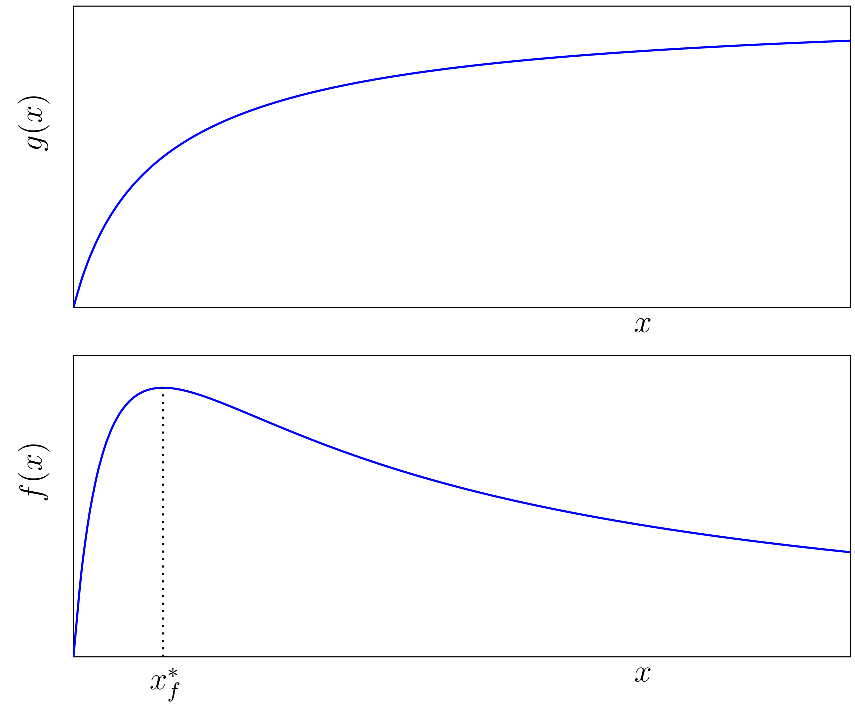



Figure 1: When the SMP analyzes $x$ data points about the user, payoff from personalized advertising is $g(x) = \frac{x(x+2)}{(x+1)(x+3)}$ for the SMP and $f(x) = \frac{x}{(x+1)(x+3)}$ for the user.

Intuitively, as the SMP attains more data points its estimate of user's taste improves and the probability that the user clicks on the advertisement increases. Furthermore, the SMP improves its estimate of the user's wtp and the surplus extracted from the user increases. Therefore, payoff of the SMP increases as it attains more data points about the user, as seen in Proposition 2.

When the SMP has few data points about the user, $x < x_f^*$, the product-type offered by the SMP matches poorly with the taste of the user and it is unlikely that the user clicks on the advertisement. In such a case, if the SMP obtains an additional data about the user then the marginal benefit from improved product recommendation outweighs the marginal loss from surplus extraction, and user payoff increases. Alternatively, when the SMP has enough data points about the user, $x \geq x_f^*$, then the product-type offered by the SMP is already a good match to the taste of the user and the user is highly likely to click on the advertisement. In such a case, if the SMP obtains an additional data about the user then the marginal benefit from improved product recommendation is outweighed by the marginal loss due to surplus extraction, and user payoff decreases. Thus, user payoff $f(x)$ is non-monotonic in $x$, initially increasing and subsequently decreasing as $x$ increases. Consequently, user payoff is maximized when the SMP attains an optimal, intermediate amount of user data, denoted by $x_f^*$.

Therefore, the SMP wants to attain all the data generated about a user and a user wants to share only $x_f^*$ data points with the SMP. In order to obtain the data of users, the SMP offers its services to users. This service-for-consent game is studied below.

## 2.4 Service-for-consent game

As the previous results have shown, an SMP aims to acquire all the data generated by its users, while each user prefers to share only an intermediate amount of data. Consequently, an SMP may offer services to users - enable users to maintain connections, to stay updated with news and events, to explore new interests - in an attempt to acquire users' data. This interaction is modeled in a two-stage game below.

The SMP and its $n$ users connected on a network $G(n, p)$ play the following two-stage game.

- First stage: The SMP offers service $s_i \geq 0$ to each user $i \in N$.
- Second stage: Each user $i$ either accepts the offer and consents to share his data, which is denoted by $a_i = 1$ or rejects the offer and refuses consent, which is denoted by $a_i = 0$.

The users choose whether to accept or reject the offer simultaneously and subsequently, payoffs are realized. The service offered by the SMP is modeled in the simplest form, as a price. This implicitly assumes that the SMP functions with a certain level of efficiency, allowing it to effectively convert the cost of building services into services for users. Indeed, the equilibrium outcome remains unchanged if the cost of building the services were scaled down by any constant factor.

The amount of data revealed to the SMP based on user choices is determined by the *consent rule* that governs the SMP. Consider a standard consent rule in which: if a user consents, all of his data is shared with the SMP; however, if the user does not consent, his individual data is not shared, although his interactive data may still be shared (leaked) by his neighbors. Specifically, for any two connected users, if one user consents while the other does not, the interactive data generated between them is shared with the SMP. This is referred to as the *unilateral consent rule*, which is formally defined below.

**Definition 2.1.** *Under unilateral consent, interactive data generated between any two connected users $i$ and $j$ is acquired by the SMP if $a_i = 1$ or $a_j = 1$.*

Under unilateral consent, for any price vector $s = (s_i)_{i \in N}$ offered by the SMP and any decision vector $a = (a_i)_{i \in N}$ of the users, the payoff of the SMP is $u_{P,uni}(s, a)$, where

$$u_{P,uni}(s,a) = -\sum_{i \in N} \underbrace{s_i}_{\substack{\text{price paid}\\ \text{for data}}} a_i + \sum_{i \in N} \left[ \underbrace{g(1+pn)}_{\substack{\text{all data obtained}\\ \text{under acceptance}}} a_i + \underbrace{g(m_i)}_{\substack{\text{only leaked data}\\ \text{obtained}\\ \text{under rejection}}} (1-a_i) \right]$$

The payoff of any user $i \in N$ is $u_i^{uni}(s, a)$, where

$$u_{i,uni}(s,a) = \underbrace{s_i}_{\substack{\text{price obtained}\\ \text{for data}}} a_i + \underbrace{f(1+pn)}_{\substack{\text{all data shared}\\ \text{under acceptance}}} a_i + \underbrace{f(m_i)}_{\substack{\text{only leaked}\\ \text{data shared}\\ \text{under rejection}}} (1-a_i)$$

where $m_i = |\{j \in N_i : a_j = 1\}|$ is the number of neighbors of $i$ who share their data.

The SMP provides service of value $s_i$ to a consenting user $i$, and in return, it attains all the $1 + pn$ data points about $i$. The resulting SMP payoff from personalized advertising is $g(1 + pn)$. The SMP does not provide any service to a non-consenting user $i$. However, $m_i$ data points about $i$ are leaked to the SMP by the consenting neighbors of $i$. The resulting SMP payoff from personalized advertising is $g(m_i)$.

A consenting user $i$ obtains service of value $s_i$ for his data and attains payoff $f(1 + pn)$ from personalized advertising, as all $1 + pn$ data points about $i$ are analyzed by the SMP. A non-consenting user $i$ does not attain any service for his data but he attains payoff $f(m_i)$ from personalized advertising, as $m_i$ data points of $i$ are analyzed by the SMP.

The presence of data externalities play an important role in the equilibrium outcome of this game. Although the SMP prefers to acquire all the data of an arbitrary user, its incentive to acquire the data of all users is affected by data externalities. The intuition for this is the following. Suppose that some user $i$ consents to share his data. Then the SMP has a lower incentive to obtain the data of his neighbors, as some of their interactive data is already leaked to the SMP by $i$. Conversely, the SMP has a higher incentive to obtain the data of his neighbors, as the consequent data leakage would reduce the price offered to $i$. These two opposing forces are analyzed in the equilibrium section below.

# 3 Equilibrium

The unique equilibrium outcome of the game is described below.

**Proposition 3.** *Suppose that users are connected on a network $G(n, p)$ on an SMP. In equilibrium, all users consent to share their data and each user obtains service of value*

$\max\{0, f(pn) - f(1+pn)\}$ *for his data.*

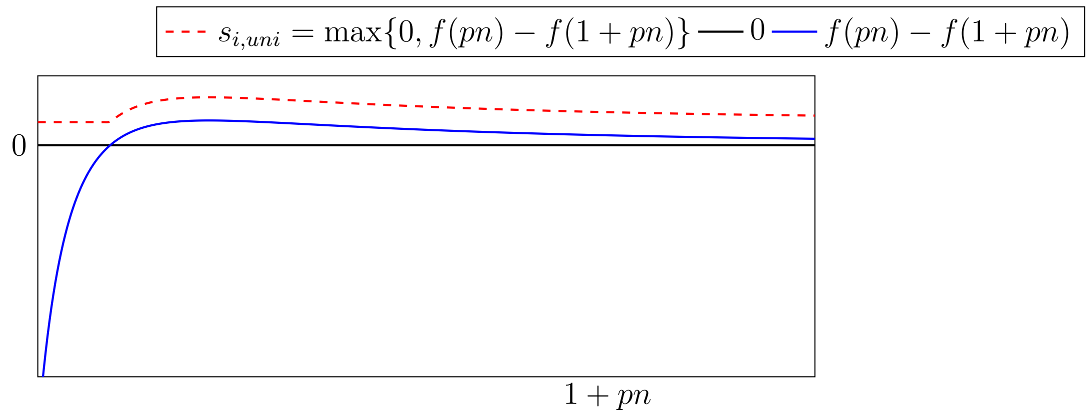



Figure 2: In equilibrium, all users consent to share their data and each user obtains price $\max\{0, f(pn) - f(1+pn)\}$ for his data.

The result is illustrated in Figure 2. Service of value $f(pn) - f(1+pn)$ is represented in blue and service of value zero is represented in black. The service obtained by a user in equilibrium is the maximum of these two values and it is represented by the dotted red line.

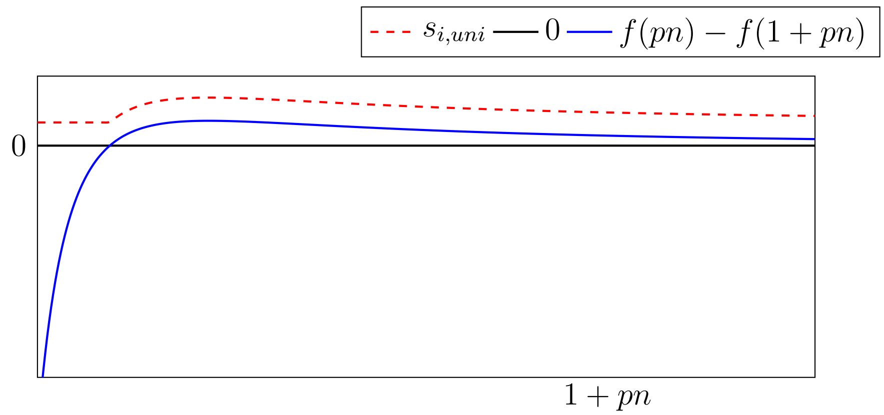
The intuition of the result is as follows. Suppose that users are connected on a network $G(n,p)$ such that $f(pn) \leq f(1+pn)$ is satisfied. Then very little data is generated about each user and the marginal payoff of a user from personalized advertising is positive; as the effect of product recommendation outweighs the effect of personalized pricing. Consequently, the incentive of a user is aligned with the SMP and a user consents to share his data for free in equilibrium. Thus, a user obtains zero service in equilibrium.

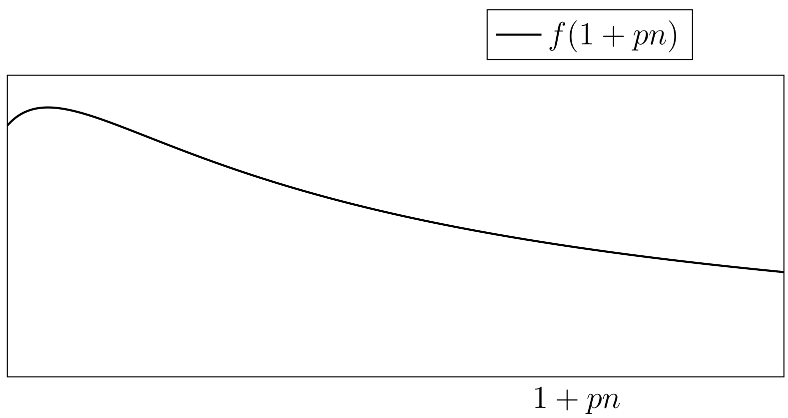
Suppose that users are connected on a network $G(n,p)$ such that $f(pn) > f(1+pn)$ is satisfied. Then too much data is generated about each user and the marginal user payoff from personalized advertising is negative; as the effect of product recommendation is outweighed by the effect of personalized pricing. Consequently, a user does not share his data for free in equilibrium. However, the SMP only provides a service of value $f(pn) - f(1+pn)$ to the user and attains all the data of the user. This happens due to the presence of data externalities on the network. A user is compensated for his individual data only and the interactive data of a user is attained by the SMP for free, even though sharing this data is detrimental to the user. Thus, a user obtains service of value $f(pn) - f(1+pn)$ in equilibrium.

The above proposition elucidates that in equilibrium, all $1 + pn$ data points of a user are shared with the SMP. Consequently, payoff of a user from personalized advertising is $f(1+pn)$ and value of the service obtained by a user is $\max\{0, f(pn) - f(1+pn)\}$. User payoff is the combined value of these two payoffs, as seen below.

**Corollary 3.** *Suppose that users are connected on a network $G(n,p)$ on an SMP. In equilibrium, user payoff is* $\max\{f(1+pn), f(pn)\}$.

For any user $i$, the value of service $s_i$ (as seen in Figure 2) and the payoff from personalized advertising $f(1+pn)$ (as seen in Figure 1) are illustrated below. User payoff in equilibrium, henceforth $u_{i,uni}(1+pn)$, is the sum of these two values and is illustrated in the final figure.



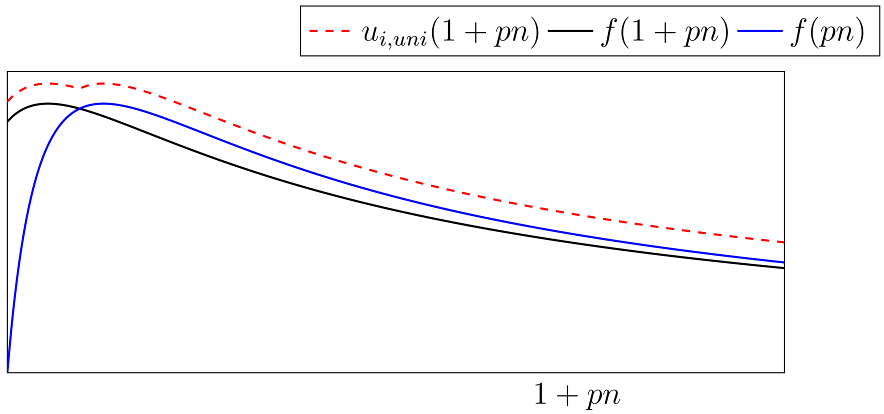



Figure 3: In equilibrium, a user obtains service of value $\max\{0, f(pn) - f(1+pn)\}$, as seen in Proposition 3. User payoff from personalized advertising is $f(1+pn)$. Consequently, user payoff in equilibrium is $\max\{f(1+pn), f(pn)\}$.

The above result is obtained as follows. User payoff under unilateral consent is $u_{i,uni} = s_i + f(1+pn)a_i + f(m_i)(1-a_i)$. From Proposition 3, the equilibrium outcome is $a_i = 1$ and $s_i = \max\{0, f(pn) - f(1+pn)\}$, therefore, user payoff in equilibrium is

$$u_{i,uni} = \begin{cases} 0 + f(1+pn) & \text{if } 0 \geq f(pn) - f(1+pn), \\ f(pn) - f(1+pn) + f(1+pn) & \text{if } 0 < f(pn) - f(1+pn). \end{cases}$$

Rewriting the expressions gives,

$$u_{i,uni} = \begin{cases} f(1+pn) & \text{if } f(1+pn) \geq f(pn), \\ f(pn) & \text{if } f(1+pn) < f(pn). \end{cases}$$

thereby proving Corollary 3.

Having analyzed the payoff attained by a user in equilibrium, the payoff attained by an SMP in equilibrium is now analyzed. The *per capita SMP payoff* is of particular interest, as it enables a direct comparison between the payoff attained by a user and the payoff attained by an SMP.

**Definition 3.1.** *The per capita SMP payoff is the utility attained by the SMP per user. Mathematically, it is* $u_{P,uni}/n$.

**Corollary 4.** *Suppose that users are connected on a network* $G(n,p)$ *on an SMP. In equilibrium, the per capita SMP payoff is* $\min\{g(1+pn), g(1+pn) - (f(pn) - f(1+pn)\}$ *and it is increasing in* $pn$.

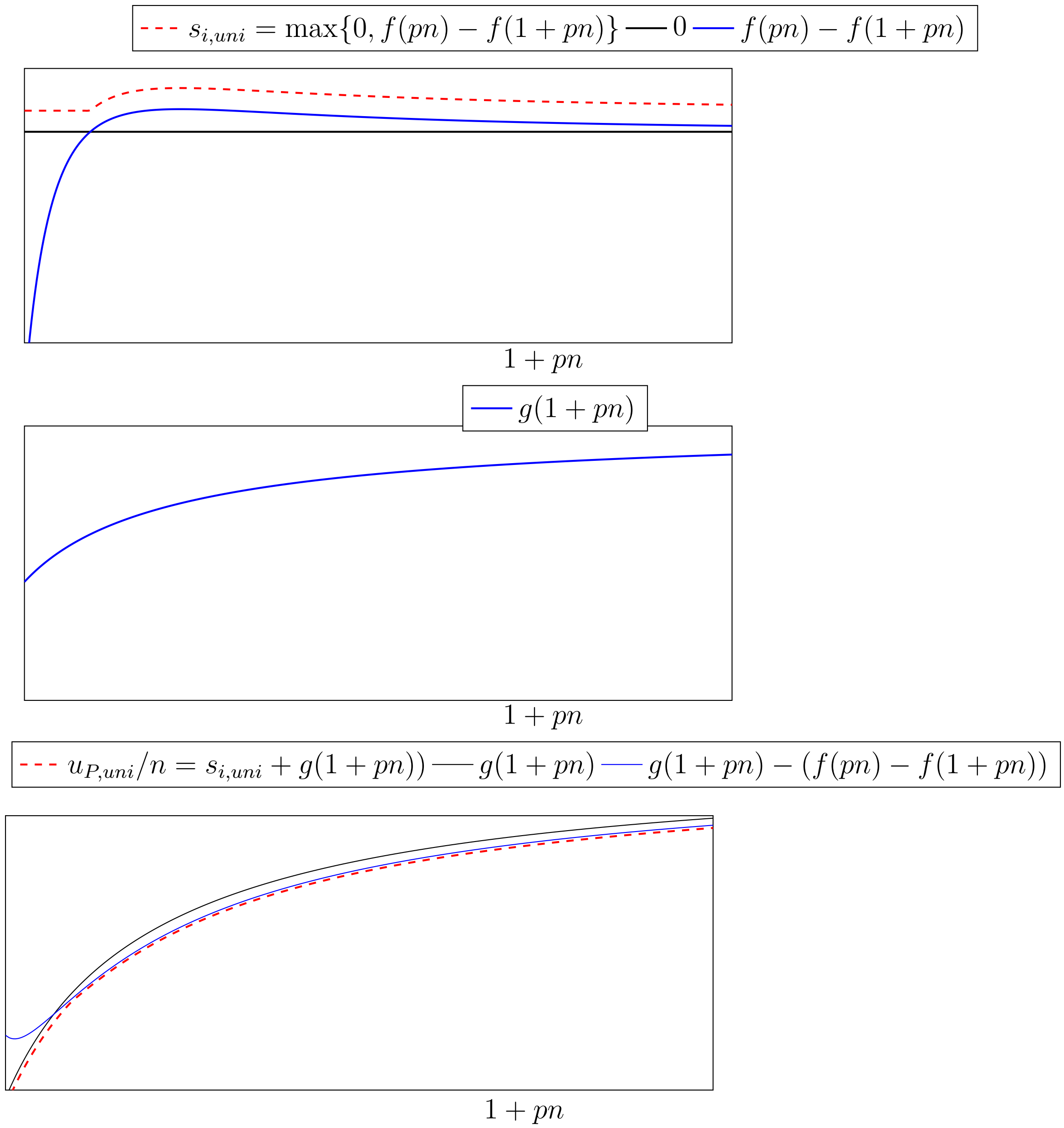



Figure 4: In equilibrium, an SMP provides service of value $s_i = \max\{0, f(pn) - f(1+pn)\}$ to each user, as seen in Proposition 3. The SMP attains payoff of $g(1+pn)$ from each user through personalized advertising. Consequently, the per capita SMP payoff is $u_{P,uni} = \min\{g(1+pn), g(1+pn) - (f(pn) - f(1+pn)\}$.

Recall that, $u_{P,uni} = -\sum_{i \in N} s_i a_i + \sum_{i \in N} [g(1+pn)a_i + g(m_i)(1-a_i)]$. From Proposition 3, $a_i = 1$ and $s_i = \max\{0, f(pn) - f(1+pn)\}$ for all $i$. Therefore,

$$\frac{u_{P,uni}}{n} = \begin{cases} 0 + g(1+pn) = g(1+pn) & \text{if } 0 \geq f(pn) - f(1+pn), \\ -(f(pn) - f(1+pn)) + g(1+pn) & \text{if } 0 < f(pn) - f(1+pn). \end{cases}$$

Rewriting the above inequalities gives,

$$\frac{u_{P,uni}}{n} = \begin{cases} g(1+pn) & \text{if } f(1+pn) \geq f(pn), \\ g(1+pn) - (f(pn) - f(1+pn)) & \text{if } f(1+pn) < f(pn). \end{cases}$$

Note that the per capita SMP payoff is increasing in $pn$ as $g$ is an increasing function and as $g(1+pn) - (f(pn) - f(1+pn))$ is increasing in $pn$ whenever $f(1+pn) < f(pn)$ is satisfied. This provides the intuition for Corollary 4.

Having analyzed the payoffs of the players, the per capita total welfare is now described.

**Definition 3.2.** *The per capita total welfare is the total utility attained by all players per user. Mathematically, it is $u_{P,uni}/n + u_{i,uni}$, for any user $i \in N$.*

**Corollary 5.** *Suppose that users are connected on a network $G(n,p)$ on an SMP. In equilibrium, the per capita total welfare is $g(1+pn) + f(1+pn)$ and it is increasing in $pn$.*

In summary, the per capita SMP payoff and the per capita total welfare increase as $pn$ increases. These results are further analyzed in the comparative statics section below, along with the effect of $pn$ on user payoff.

## 3.1 Comparative statics

In this section, the comparative statics analysis of players' payoffs is conducted. To facilitate this, an SMP is first characterized by its size.

An SMP is said to be *extremely small* if very little data is generated about a user on the SMP. More formally:

**Definition 3.3.** *An SMP with user network $G(n,p)$ is said to be extremely small if $(n,p)$ satisfies $f(pn) \leq f(1+pn)$.*

Suppose now that an SMP is not extremely small - meaning $f(pn) > f(1+pn)$ - then more data is generated on the SMP than on an extremely small SMP. Nevertheless, if $pn$ is not excessively large, the SMP compensates users sufficiently in equilibrium, ensuring that the payoff of a user in equilibrium is the same as that of a user on some extremely small SMP. Consequently, such an SMP attains limited amount of user data; in the sense that user payoff on the SMP is equal to user payoff on some extremely small SMP. Such an SMP is defined as a *small SMP*. To formalize this definition, a threshold that determines a small SMP is used.

**Definition 3.4.** *The threshold that characterizes an SMP with user network $G(n,p)$ is the largest value $t_0$ for which the following holds - for every $(n,p)$ that satisfies $1+pn \leq t_0$ there exists an extremely small SMP with user network $G(n_{eq},p_{eq})$ such that $u_{i,uni}(1+pn) = u_{i,uni}(1+p_{eq}n_{eq})$.*

An SMP is classified as small if it is an extremely small SMP or if the equilibrium user payoff is equal to equilibrium user payoff of some extremely small SMP. More formally:

**Definition 3.5.** *An SMP is characterized as small if its user network $G(n,p)$ satisfies $1+pn \leq t_0$. An SMP is characterized as large if its user network $G(n,p)$ satisfies $1+pn > t_0$. As $pn$ increases (decreases), the SMP is said to become larger (smaller).*

The threshold $t_0$ and the classification of an SMP is illustrated in the figure below. Intuitively, an SMP is classified as small if limited amount of data is generated on the SMP and an SMP is classified as large if excessive amount of data is generated on the SMP.

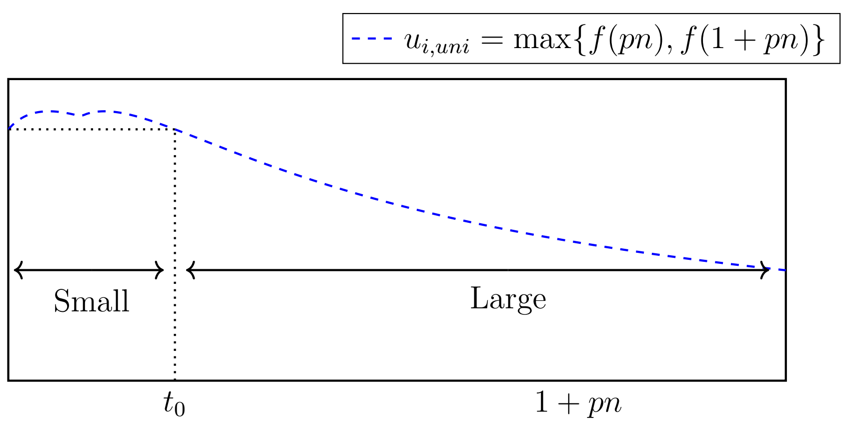



Figure 5: An SMP with user network $G(n,p)$ is called small if $1+pn \leq t_0$ and is called large if $1+pn > t_0$.

Having classified SMPs by their size, the comparative statics analysis of players' equilibrium payoffs by size is conducted below.

**Corollary 6.** *Suppose that users are connected on a network $G(n,p)$ on a large SMP. As the SMP becomes larger, the per capita SMP payoff and the per capita total welfare increases but user payoff decreases.*

The above result is illustrated in Figure 6 below.

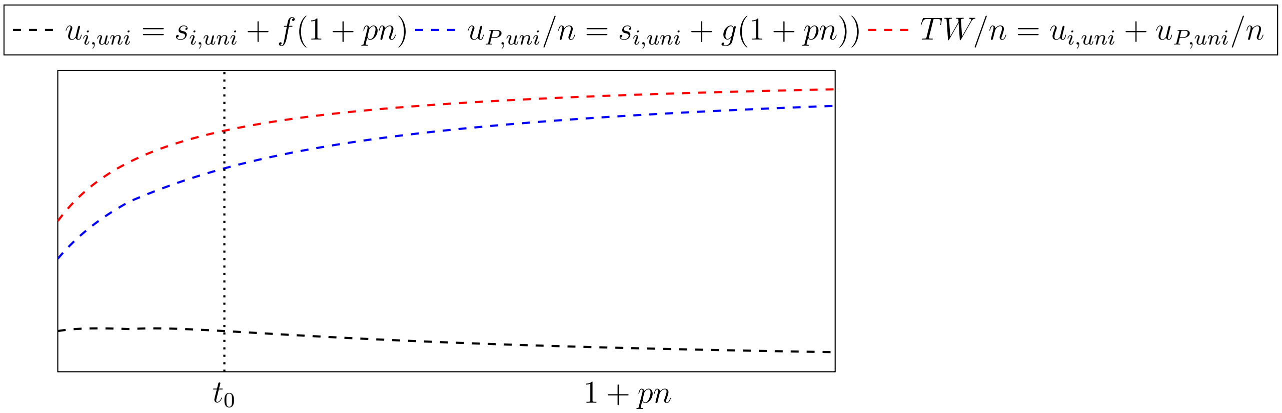



Figure 6: As a large SMP becomes larger, the per capita SMP payoff and the per capita total welfare increase but user payoff decreases.

The above result highlights a key conflict regarding regulation. If policymakers are only concerned about the total welfare generated on an SMP, then any regulation that hampers the growth of a large SMP is detrimental, since per capita total welfare increases as $pn$ increases. Instead, if policymakers are also concerned about the distribution of welfare between the SMP and its users, then the unregulated growth of a large SMP is detrimental since user payoff decreases when a large SMP becomes larger. Notably, Art. 5 of GDPR states that user data should be "...processed lawfully, *fairly* and in a transparent manner in relation to the data subject"[4]; implying that user interest is a key focus area for policymakers. Therefore, it is vital to analyze the impact of policy interventions on welfare, specifically on user welfare. This analysis is conducted in the following section.

# 4 Effect of policy interventions

The results of this paper have shown that, in equilibrium, users on an SMP consent to share their data but are compensated for their individual data only. This outcome is a result of data externalities, which enable the SMP to attain users' interactive data at no cost. Consequently, a natural approach to enhance user welfare is to give users more control over their data. The GDPR aims to do this through several measures, with two key policy measures being:

1. data minimization principle (Article 5(1)(c) of the GDPR and Article 4(1)(c) of Regulation (EU) 2018/1725),

2. stronger consent rights (Article 4,6,7,8 of the GDPR).

[4]https://gdpr-info.eu/art-5-gdpr/

The welfare effects of these two measures is examined below.

## 4.1 Data minimization principle

As stated in Art. 5 of GDPR, the data minimization principle requires that collection of user data should be adequate, relevant and limited to what is necessary. In the context of this model, it is defined as follows.

**Definition 4.1.** *The data minimization principle is said to be weakly implemented on an SMP if the SMP is constrained to analyzing at most $\bar{x}$ data points about any user, for some $\bar{x} \geq 0$.*

**Definition 4.2.** *The data minimization principle is said to be implemented on an SMP with user network $G(n,p)$ if the SMP is constrained to analyzing at most $0 \leq \bar{x} < 1 + pn$ data points about any user.*

Since users of the SMP are homogeneous, constraining the SMP to analyzing $\bar{x}$ data points per user is equivalent to constraining the SMP to analyzing $n\bar{x}$ data points on the SMP. Therefore, the data minimization principle can also be implemented by setting an aggregate threshold instead of a per user threshold on the SMP.

The welfare effects of the data minimization principle are now analyzed. Suppose the SMP is constrained to analyzing at most $0 \leq \bar{x} < 1 + pn$ data points about each user. Post implementation, the SMP and the users play the two stage game described in section 2.4.

The results below show that the data minimization principle may decrease or increase user welfare and this crucially depends on whether the SMP is small or large, respectively.

**Lemma 1.** *Suppose users are connected on a network $G(n,p)$ on a small SMP. For almost each value of $1 + pn$, there exists a $\bar{x}_{(n,p)} < 1 + pn$ such that implementation of the data minimization principle with threshold $\bar{x}_{(n,p)}$ decreases user welfare.*

The above result is illustrated in Figure 7 below, where restricting the SMP to analyze some $\bar{x}$ data points decreases user payoff. As a result, user welfare also decreases.

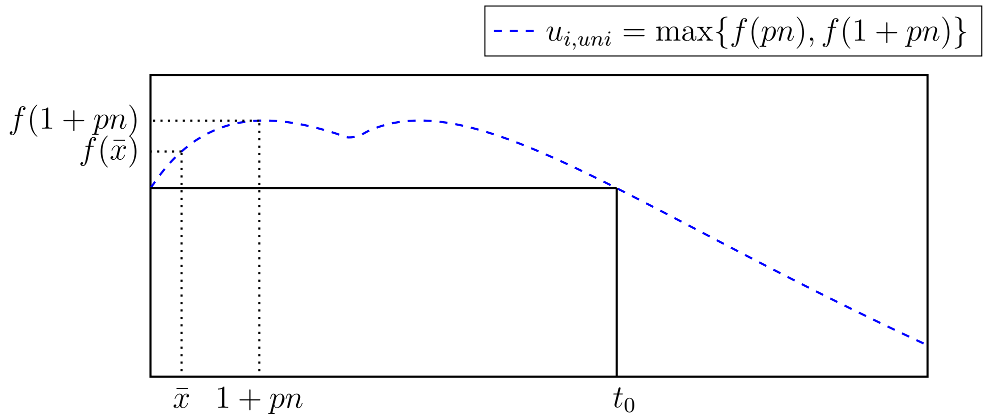



Figure 7: The figure illustrates that for a small SMP ($1 + pn \leq t_0$), there exists a threshold $\bar{x}$ under which user payoff decreases. As a result, user welfare also decreases.

When the data minimization principle is not implemented, an extremely small SMP obtains $1 + pn$ data points about each user and each user's payoff is $f(1 + pn)$, as seen in Proposition 3. By definition of an extremely small SMP, $1 + pn$ is small enough for the marginal effect of personalized advertising to be positive. Consequently, if the SMP is restricted to analyzing any $\bar{x} < 1 + pn$ data points, user payoff decreases, as the positive effect from improved recommendation is curtailed. As a result, implementing the data minimization principle on an extremely small SMP decreases user welfare.

Consider a small SMP with user network $G(n, p)$, where $1 + pn \in (1, t_0)$. By definition, the small SMP is user-payoff-equivalent to some extremely small SMP with user network $G(n_{eq}, p_{eq})$. Now consider any threshold $\bar{x}$ that satisfies $\bar{x} < 1 + p_{eq}n_{eq}$. By the argument in the paragraph above, user payoff attained from analysis of $\bar{x}$ data points is lower than user payoff attained from analysis of $1+p_{eq}n_{eq}$ data points. Moreover, by user-payoff-equivalence, user payoff attained from analysis of $\bar{x}$ data points is lower than user payoff attained from analysis of $1 + pn$ data points. Therefore, for almost any small SMP, there exists some threshold $\bar{x}$, such that implementing the data minimization principle decreases user payoff and consequently, decreases user welfare.

Having described the effect of the data minimization principle on a small SMP, its effect on a large SMP is now analyzed.

**Lemma 2.** *Suppose users are connected on a network $G(n, p)$ on a large SMP. Implementing the data minimization principle with any threshold $\bar{x}$ increases user welfare.*

The above result is illustrated in Figure 8 below, where restricting the SMP to analyze any $\bar{x} < 1 + pn$ data points increases user payoff.

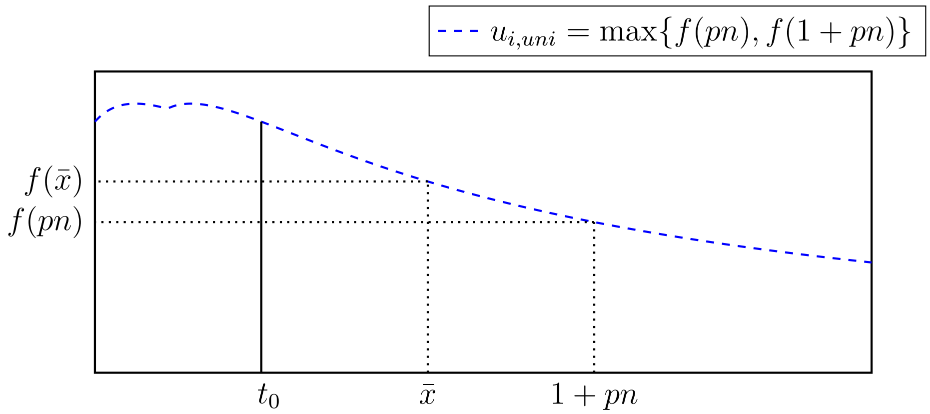



Figure 8: Implementing the data minimization principle on a large SMP ($1 + pn > t_0$) increases user payoff from $f(pn)$ to $f(\bar{x})$. As a result, user welfare also increases.

When the data minimization principle is not implemented, a large SMP obtains $1 + pn$ data points about each user and each user's payoff is $f(pn)$, as seen in Proposition 3. By definition of a large SMP, $pn$ is large enough for the marginal effect of personalized advertising to be negative. Consequently, if the data minimization principle is implemented and the SMP is restricted to analyzing any $\bar{x} < 1 + pn$ data points, user payoff increases as the negative effect from personalized pricing is curtailed. Consequently, the data minimization principle increases payoff of each user and as a result, increases user welfare.

Therefore, the size of the SMP is the key factor which determines the effect of the data minimization principle on user welfare. Implementing the data minimization principle increases user welfare when the SMP is large but user welfare may increase or decrease when the SMP is small. Naturally, it is important to examine the effect of the data minimization principle on total welfare. This analysis is provided below.

**Lemma 3.** *Suppose that users are connected on a network $G(n, p)$ on an SMP. Implementing the data minimization principle decreases per capita SMP payoff and per capita total welfare.*

Since the per capita SMP payoff and the per capita total welfare both increase in $pn$ (see Corollary 6 and Figure 6), restricting the SMP to analyze some $\bar{x} < 1 + pn$ data points decreases the per capita SMP payoff and the per capita total welfare.

Based on these lemmas, the overall impact of implementing the data minimization principle is summarized in the two propositions below.

**Proposition 4.** *Suppose that users are connected on a network $G(n, p)$ on a small SMP. Implementing the data minimization principle always decreases the per capita total welfare and user welfare almost always decreases for some threshold.*

Thus, implementing the data minimization principle on a small SMP is detrimental as the per capital total welfare always decreases and user welfare almost always decreases for some threshold.

Instead, for a large SMP, the effect of the data minimization principle on user welfare is always beneficial.

**Proposition 5.** *Suppose that users are connected on a network $G(n,p)$ on a large SMP. Implementing the data minimization principle decreases the per capita total welfare but increases user welfare.*

Therefore, implementing the data minimization principle on a large SMP enhances user welfare, thereby addressing policymakers' distribution concerns. The focus on user welfare, at the expense of total welfare, is especially significant in the context of personalized advertising, where user payoff decreases as the SMP becomes larger. In such a scenario, implementing a policy to limit the decline in user payoff is warranted. For a small SMP however, data minimization is clearly harmful, as it reduces both total welfare and user welfare. Overall, the data minimization principle is an effective policy intervention when applied to a large SMP, but it is ineffective when applied to a small SMP.

## 4.2 Stronger consent rights

This section analyzes the effect of providing users greater control over their data.

According to Art. 6 of the GDPR, user data may be processed if *consent* of a user is obtained[5]. Therefore, the GDPR enhances users' control over their data. This is represented through the *bilateral consent rule* outlined below, and the effect of the above policy is analyzed by comparing welfare under unilateral consent with welfare under bilateral consent.

**Definition 4.3.** *Under bilateral consent, interactive data generated between any two connected users $i$ and $j$ is shared with the SMP if and only if $a_i = 1$ and $a_j = 1$.*

Under bilateral consent, an SMP attains the data point generated between two users if and only if both users consent to share their data. If user $i$ does not consent to share data, $a_i = 0$, then utility of $i$ is unaffected by the decision of his neighbors. Therefore, data externalities are completely shut down under $a_i = 0$. Instead, if user $i$ consents to share data, $a_i = 1$, then the interactive data points generated between $i$ and other consenting neighbors of $i$ are shared with the SMP. Thus, under bilateral consent the SMP only attains the interactive data generated between $i$ and other consenting neighbors of $i$.

[5] see https://gdpr-info.eu/art-6-gdpr/

The SMP and its $n$ users connected on a network $G(n,p)$ play the two stage service-for-consent game, as described in section 2.4 under the bilateral consent rule. For any price vector $s = (s_i)_{i \in N}$ offered by the SMP and any decision vector $a = (a_i)_{i \in N}$ of the users, the payoff of the SMP is $u_{P,bil}(s,a)$, where

$$u_{P,bil}(s,a) = -\sum_{i \in N} \underbrace{s_i}_{\substack{\text{price paid}\\\text{for data}}} a_i + \sum_{i \in N} \left[ \underbrace{g(1+m_i)}_{\substack{\text{personal data and data between}\\\text{consenting users obtained}\\\text{under acceptance}}} a_i + \underbrace{g(0)}_{\substack{\text{no data}\\\text{obtained}\\\text{under rejection}}} (1-a_i) \right]$$

The payoff of any user $i \in N$ is $u_{i,bil}(s,a)$, where

$$u_{i,bil}(s,a) = \underbrace{s_i}_{\substack{\text{price obtained}\\\text{for data}}} a_i + \underbrace{f(1+m_i)}_{\substack{\text{personal data and data between}\\\text{consenting users shared}\\\text{under acceptance}}} a_i + \underbrace{f(0)}_{\substack{\text{no data}\\\text{shared}\\\text{under rejection}}} (1-a_i)$$

Recall that $m_i = |\{j \in N_i : a_j = 1\}|$ is the number of neighbors of $i$ who consent to share their data.

The SMP provides service of value $s_i$ to a consenting user $i$, and in return, it attains the individual data point of $i$ and the interactive data points generated between $i$ and consenting neighbors of $i$. As a result, SMP payoff from personalized advertising is $g(1+m_i)$. The SMP does not provide any service to a non-consenting user $i$ and does not attain any data about $i$, since data externalities are shut down under the bilateral consent rule. As a result, payoff of the SMP is $g(0) = 0$ from personalized advertising.

A consenting user $i$ obtains service of value $s_i$ for his data and attains payoff $f(1+m_i)$ from personalized advertising, as $1+m_i$ data points about $i$ are analyzed by the SMP. A non-consenting user $i$ does not obtain any service from the SMP and attains payoff $f(0) = 0$ from personalized advertising, as no data points about $i$ are analyzed by the SMP.

Having described the payoffs attained by the players under bilateral consent, equilibrium outcome of the two-stage game is now analyzed.

**Proposition 6.** *Suppose that users are connected on a network $G(n,p)$ on an SMP. In equilibrium, all users consent to share their data and each user obtains service of zero value.*

The intuition of the result is as follows. Under the bilateral consent rule, a user $i$ consents to share his data if the following inequality is satisfied.

$$s_i + f(1+m_i) \geq 0 + f(0) = 0$$

If user $i$ denies consent, the SMP obtains no data about the user as there is no data leakage. Since $f(x)$ is *positive* for all $x > 0$, a user $i$ prefers to consent and share his data in order to obtain (possibly a very small) positive payoff from personalized advertising. Consequently, all users consent to share their data in equilibrium, in return for no service.

### 4.2.1 Welfare impact of stronger consent rights

Having described the equilibrium outcome under bilateral consent, the welfare consequences of stronger consent rights are analyzed. This is done by comparing welfare under unilateral consent to welfare under bilateral consent.

In order to do so, the per capita total welfare under bilateral consent is defined below.

**Definition 4.4.** *The per capita total welfare under bilateral consent is the total utility attained by all players per user. Mathematically, it is $u_{P,bil}/n + u_{i,bil}$, for any user $i \in N$.*

The results below elucidate that the bilateral consent rule backfires, in the sense that both user welfare and the per capita total welfare are lower under bilateral consent than under unilateral consent.

**Corollary 7.** *Suppose that users are connected on a network $G(n, p)$ on an SMP. In equilibrium, user welfare is lower under bilateral consent than under unilateral consent.*

Since the SMP attains data of all users under both unilateral consent and bilateral consent, each user attains the same payoff from personalized advertising under both consent rules. However, the SMP provides service of positive value to users under unilateral consent rule (Proposition 3) but it provides service of zero value to users under bilateral consent rule (Proposition 6). Therefore, payoff of each user is lower under bilateral consent rule than under unilateral consent rule. Consequently, user welfare is lower under bilateral consent rule than under unilateral consent rule.

As the next result elucidates, the per capita total welfare is equal under both consent rules.

**Corollary 8.** *Suppose that users are connected on a network $G(n, p)$ on an SMP. In equilibrium, total welfare under bilateral consent is equal to total welfare under unilateral consent.*

Unlike user welfare, the per capita total welfare is unaffected by the service provided by the SMP in equilibrium. Therefore, the per capita total welfare only depends on the payoff that the players attain from personalized advertising. Since the SMP attains data of all users under both consent rules, the payoff of each player (from personalized advertising) is equal under both rules. Therefore, in equilibrium, the per capital total welfare under unilateral consent and the per capita total welfare under bilateral consent are equal.

In summary, providing stronger consent rights to users does not affect total welfare but decreases user welfare. Thus, although GDPR aims to enhance user benefit by giving users more control over their data, this policy may backfire as users are given *too much control* over their data.

# 5 Conclusion

This paper analyzes the effect of policy regulations on a data economy consisting of a social media platform and users embedded on a network. The data economy consists of an SMP that generates revenue from data through personalized advertising. Using a microfounded model I show that the payoff of the SMP is increasing in number of data points and user payoff is non-monotonic in number of data points. I model the consequent misaligned incentives in a service-for-data model and study the effect of two policy implications. I find that, first, stronger consent rules backfire and decrease user welfare. Second, wen the SMP is large enough then implementing the data minimization principle improves user welfare. Consequently, the data minimization principle is a better policy regulation than stronger consent rules like GDPR, when it comes to improving user welfare.

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# A Proof of Proposition 1:

*Proof.* Since $w \sim Pa(1,3)$, $s_w|w \sim U(0,w)$ and Pareto distribution is the conjugate prior of uniform distribution, the Bayesian updated estimate of $w$ also has Pareto distribution, in particular,

$$w|s_1, s_2, ..., s_k \sim Pa(c_k, k+3)$$

where $c_k = max\{s_1, ...s_k, 1\}$.
The distribution of $w|s_1, ...s_k$ is

$$F(w|s_1, ...s_k) = \begin{cases} 1 - \left(\frac{c_k}{w}\right)^{k+3} & w \geq c_k \\ 0 & w < c_k \end{cases}$$

The SMP wants to maximise its payoff from selling a product.

Since the user learns his wtp when the SMP attains $k$ signals, the SMP maximizes its payoff by choosing $p$ that maximizes $p[1 - F(p|s_1, ...s_k)]$.

$$p[1 - F(p)] = p\left(\frac{c_k}{p}\right)^{k+3} = \frac{c_k^{k+3}}{p^{k+2}}$$

for $p \geq c_k$ and is maximised at minimum $p$, which implies $p = c_k$.
$p[1 - F(p)] = p$ for $p < c_k$, which is maximised at maximum $p$, which implies $p = c_k$.
Therefore, after getting $k$ signals, the SMP sets price $c_k = max\{s_1, ..., s_k, 1\}$ and

$$PltPayoff_{wtp} = c_k$$

where $PltPayoff_{wtp}$ is the *wtp Part of the SMP Payoff.*

A user $i$ buys the product if and only if $w_r > c_k$. The ex-ante payoff of the user is then

$$UserPayoff_{wtp} = E(w_r - c_k)$$

where $UserPayoff_{wtp}$ is the *wtp Part of User Payoff.* To calculate $E(c_k)$ first define the random variable $x_k := max\{s_1, s_2, ..., s_k\}$.
Since the signals are iid $U(0,w)$,

$$F_{x_k|w}(x) = \begin{cases} 0 & x < 0 \\ \left(\frac{x}{w}\right)^k & 0 \leq x < w \\ 1 & x \geq w \end{cases}$$

Since $c_k = max\{x_k, 1\}$,

$$\begin{aligned}E(c_k|w) = E(x_k|X_k > 1)P(X_k > 1) + E(1|x_k \leq 1)P(x_k \leq 1)\\ = \int_1^\infty x \frac{kx^{k-1}}{w^k} I_{(0,w)}\, dx + \frac{1}{w^k}\\ = \int_1^w \frac{k}{w^k} x^k\, dx + \frac{1}{w^k}\\ = \frac{k}{w^k}\frac{w^{k+1}-1}{k+1} + \frac{1}{w^k}\end{aligned}$$

Using $w \sim Pa(1,3)$ take expectation again to get

$$\begin{aligned}E(c_k) = E(E(c_k|w))\\ = \frac{k}{k+1}\int_1^\infty \frac{w^{k+1}-1}{w^k}\frac{3}{w^4}\, dw + \int_1^\infty \frac{1}{w^k}\frac{3}{w^4}\, dw\\ = \frac{3k}{k+1}\left[\int_1^\infty \frac{1}{w^3}\, dw - \int_1^\infty \frac{1}{w^{4+k}}\, dw\right] + \int_1^\infty \frac{3}{w^{k+4}}\, dw\\ = \frac{3k}{k+1}\left[\frac{1}{2} - \frac{1}{k+3}\right] + \frac{3}{k+3}\\ = \frac{3(k+2)}{2(k+3)} = \frac{3}{2}\left[1 - \frac{1}{k+3}\right]\end{aligned}$$

$$UserPayoff_{wtp} = E(w_r - c_k) = \frac{3}{2} - \frac{3}{2}\left[1 - \frac{1}{k+3}\right] = \frac{3}{2(k+3)}$$

which clearly decreases as the number of data points $k$ increases. Thus, the decreasing part of user payoff has been attained.

Recall that $wtpPlatformPayoff = c_k$. The ex-ante payoff of the SMP is then

$$wtpPlatformPayoff = E(c_k) = \frac{3(k+2)}{2(k+3)} = \frac{3}{2}\left[1 - \frac{1}{k+3}\right]$$

which clearly increases as the number of signals / data points $k$ increases. □

# B Proof of Proposition 2:

*Proof.* Since $t \sim N(0,1)$, $s_t|t \sim N(t,1)$ and normal distribution is the conjugate prior of itself, the Bayesian updated estimate of $t$ also has normal distribution, in particular,

$$t|s_1, s_2, ..., s_k \sim N\left(\frac{\sum_{i=1}^{k} s_i}{1+k}, \frac{1}{1+k}\right)$$

When the SMP attains $k$ data points about $t$ and the user learns his type $t_r$, the user is more likely to click on an advertisement if the recommended product is closer to his type. I formulate this probability by

$$1 - \frac{Var(t|s_1, ...s_k)}{Var(t)}$$
$$= 1 - \frac{1}{1+k} = \frac{k}{k+1}$$

Note that this formulation is independent of the realized values of signals. Therefore, the ex-ante SMP payoff from personalized advertising is

$$PlatformPayoff = \frac{3(k+2)}{2(k+3)}\frac{k}{k+1} = \frac{3k(k+2)}{2(k+1)(k+3)}$$

Similarly, the ex-ante user payoff from personalized advertising is

$$UserPayoff = \frac{k}{(k+1)}\frac{3}{2(k+3)} = \frac{3k}{2(k+1)(k+3)}$$

Extending the function obtained to any $x \geq 0$ and normalizing the scalar to one gives,

$$PlatformPayoff = g(x) = \frac{(x+2)}{(x+3)}\frac{x}{x+1} = \frac{x(x+2)}{(x+1)(x+3)}$$

$$UserPayoff = f(x) = \frac{x}{(x+1)}\frac{1}{(x+3)} = \frac{x}{(x+1)(x+3)}$$

□

# C Proof of Proposition 3

*Proof.* to be added □

# D Proof of Proposition 4

*Proof.* The individual rationality condition for a user $i$ is

$$IR_i : p_i + f(1 + |m_i|) \geq 0$$

If $i$ refuses to share data, then no data about $i$ is revealed to the SMP and user payoff is zero. Since $f(x) > 0$, $i$ is better off giving his data to the SMP for free, irrespective of what other users do. This is true for all users. Thus, all users share their data for free in equilibrium. Equilibrium price and action are $p^*_{i,Bil} = 0$ and $a^*_{i,Bil} = 1 \; \forall \; i \in N$. □